Dynamical instability of a two component Bose-Einstein Condensate in an optical lattice

To cite this article: M Emre Tagin and M Ö Oktel 2006 *J. Phys.: Conf. Ser.* **36** 200

Journal of Physics: Conference Series **36** (2006) 200–204
The 12th Central European Workshop on Quantum Optics

Dynamical instability of a two component Bose-Einstein Condensate in an optical lattice

M. Emre Taşgın

Department of Physics, Bilkent University, Bilkent, 06800 Ankara, Turkey

E-mail: metasgin@fen.bilkent.edu.tr

M. Ö. Oktel

Department of Physics, Bilkent University, Bilkent, 06800 Ankara, Turkey

E-mail: oktel@fen.bilkent.edu.tr

Abstract. We investigate the dynamical instability for a two component Bose-Einstein Condensate (BEC) in an optical lattice. We assume that the two components are in sinusoidal potentials with the same period but different phases. We find a swallow-tail type instability near the band edge, however an extra tail is formed due to the inter-component interaction. Depending on the phase difference of the potentials, the instability region may expand or shrink with increasing inter-component interaction.

1. Introduction

One of the most promising directions in ultra cold atom research is the investigation of the optical lattices. Behavior of bosons [1], as well fermions[2] in periodic potentials have been investigated both theoretically and experimentally. As a result, many phenomena familiar from solid-state systems, such as Josephson effect [3], non-linear tunneling [4, 5], Mott transition [6] and Bloch oscillations [7] have been obtained. One of the novel phenomena displayed by Bose-Einstein condensates (BEC) in an optical lattice is the presence of dynamical instabilities near band edges [8, 9, 10].

It has been found by analytical and numerical calculations that the chemical potential for a BEC in an optical lattice may become multi-valued near the Brillouin zone boundary. The resulting swallow-tail type band structure signifies a dynamical instability of the condensate. It has been suggested that this instability is a direct consequence of the superfluidity of the BEC [11]. This instability mechanism is also shown to be connected to the Mott transition in a continuous manner [12]. Further investigation of the dynamical instability is needed to clarify the physics of BEC in optical lattices.

The versatility of cold-atom experiments made it possible to create novel superfluids, such as mixtures or spinor BECs. These systems facilitate investigation of superfluidity in a broader domain. Thus, we study wether a two component BEC system in an optical lattice has dynamical instabilities. The connection of these instabilities to superfluidity of multicomponent condensates will be discussed elsewhere.

In this paper, variational method is used to determine approximate band structure of a two-component BEC near the Brillouin zone boundary. Components are assumed to be trapped separately in periodic potentials which have the same period, but with a phase difference with respect to each other. We show that the dynamical instability also appears in a two-component BEC, with an additional tail. We also investigate the dependence of the width of the instability region on the inter-component interaction.

2. Calculation and Results

We consider the shifted periodic potentials forming the optical lattice as

$$V_1(x) = V_{01} sin^2\left(\frac{\pi x}{d}\right) \quad \text{and} \quad V_2(x) = V_{02} sin^2\left(\frac{\pi x}{d} + \phi\right), \tag{1}$$

where $\theta = 2\phi$ is the phase difference induced spatially. V_{01} and V_{02} are the maximum magnitudes of the potentials and d is the lattice constant.

Since the two component gas can be described by coupled Gross-Piteaevskii equations, the mean energy functional is

$$E = \frac{1}{d}\int_{-d/2}^{-d/2} dx \quad \left[\frac{\hbar^2}{2m_1}\left|\frac{d\Psi_1}{dx}\right|^2 + V_1(x)|\Psi_1(x)|^2 + \frac{1}{2}g_{11}|\Psi_1(x)|^4 + \frac{\hbar^2}{2m_2}\left|\frac{d\Psi_2}{dx}\right|^2\right.$$
$$\left. + \quad V_2(x)|\Psi_2(x)|^2 + \frac{1}{2}g_{22}|\Psi_2(x)|^4 + g_{12}|\Psi_1(x)|^2|\Psi_2(x)|^2\right], \tag{2}$$

where $g_{ij} = \frac{2\pi\hbar^2 a_{ij}}{m_{ij}}$ is the coupling strength between components i and j, m_{ij} is the reduced mass and a_{ij} is the scattering length. Mean number densities are

$$n_i = \frac{1}{d}\int_{-d/2}^{d/2} dx|\Psi_i|^2, \quad i = 1, 2. \tag{3}$$

The periodic potential produces a band structure. Since we are interested in the solutions near the zone boundary, we use trial wave functions of the form

$$\Psi_1(x) = \sqrt{n_1}\left(\cos\alpha e^{ikx} + \sin\alpha e^{\left(ik - \frac{2\pi}{d}\right)x}\right) \tag{4}$$

$$\Psi_2(x) = \sqrt{n_2}\left(\cos\beta e^{ik\left(x + \frac{\phi}{\pi}d\right)} + \sin\beta e^{\left(ik - \frac{2\pi}{d}\right)\left(x + \frac{\phi}{\pi}d\right)}\right), \tag{5}$$

where α and β are to be determined by energy minimization at a specific k value. Here the choice of two different variational parameters (α and β) is needed, as shown by the results below. In a similar calculation, [13], β was assumed to be equal to $\alpha + \pi/2$. Energy minimization shows that β assumes values other than $\alpha + \pi/2$, and the variational wavefunction used in [13] is too restrictive.

Inserting trial wave functions (4) and (5) into the energy functional (2), we obtain the parameterized energy

$$E_k(\alpha,\beta) = \frac{\hbar^2}{2m_1}n_1\left(k^2\cos^2\alpha + \left(k - \frac{2\pi}{d}\right)^2\sin^2\alpha\right) + \frac{V_{01}n_1}{2}\left(1 - \frac{1}{2}\sin 2\alpha\right)$$
$$+\frac{g_{11}{n_1}^2}{2}\left(1 + \frac{1}{2}\sin^2\alpha\right) + \frac{\hbar^2}{2m_2}n_2\left(k^2\sin^2\beta + \left(k - \frac{2\pi}{d}\right)^2\cos^2\beta\right) + \frac{V_{02}n_2}{2}\left(1 - \frac{1}{2}\sin 2\beta\right)$$
$$+\frac{g_{22}{n_2}^2}{2}\left(1 + \frac{1}{2}\sin^2\beta\right) + g_{12}n_1n_2\left(1 + \frac{1}{2}\cos\theta\sin 2\alpha\sin 2\beta\right), \tag{6}$$

to be minimized. Minimization with respect to α and β leads to two coupled equations. These are only analytically solvable when $k = \pi/d$, on the zone edge. There exist two type of solutions

$$i) \quad \cos 2\alpha = 0 \qquad and \qquad \cos 2\beta = 0, \tag{7}$$

$$ii) \quad \sin 2\alpha = \frac{1}{2n_1}\frac{g_{22}V_{01} - \cos\theta g_{12}V_{02}}{g_{11}g_{22} - \cos^2\theta g_{12}^2} \qquad and \qquad \sin 2\beta = \frac{1}{2n_2}\frac{g_{11}V_{02} - \cos\theta g_{12}V_{01}}{g_{11}g_{22} - \cos^2\theta g_{12}^2}, \tag{8}$$

where solution i corresponds to the standing waves of the upper and lower band, and solution ii corresponds the tails in between. For the existence of tails (type ii), both particle densities must exceed the critical densities, such as, $n_1 > n_c^1$ and $n_2 > n_c^2$. n_c^1 and n_c^2 are obtained by putting the maximum value of 1 for sin function in (8). Following equations (8), critical densities increase with increasing phase difference. So it is harder to observe the tails in out phase two-component BEC.

Solutions for the arbitrary values of k (about $\frac{\pi}{d}$), can be studied numerically. For an arbitrary value of k, the derivatives

$$\begin{aligned}
\frac{1}{n_1}\frac{\partial E_k}{\partial \alpha} &= \frac{\hbar^2}{2m_1}\left[\left(k - \frac{2\pi}{d}\right)^2 - k^2\right]\sin 2\alpha - \frac{1}{2}V_{01}\cos 2\alpha \\
&+ \frac{1}{2}g_{11}n_1 \sin 4\alpha + \cos\theta g_{12}n_2\cos 2\alpha \sin 2\beta \qquad (9)\\
\frac{1}{n_2}\frac{\partial E_k}{\partial \beta} &= \frac{\hbar^2}{2m_1}\left[\left(k - \frac{2\pi}{d}\right)^2 - k^2\right]\sin 2\beta - \frac{1}{2}V_{02}\cos 2\beta \\
&+ \frac{1}{2}g_{22}n_2 \sin 4\beta + \cos\theta g_{12}n_1\cos 2\beta \sin 2\alpha. \qquad (10)
\end{aligned}$$

are equated to zero to determine α and β.

To simplify our calculations, we considered the same parameters for both components: $g_{11} = g_{22} = g_0$, $m_1 = m_2 = m$, $n_1 = n_2 = n$ and $V_{01} = V_{02} = V_0$. We varied the interspecies interaction g_{12} to observe the resulting band structures. We observed that band diagram has a more complex structure compared to the single-component case [10]. A typical band diagram is given in figure 1. In part a of the figure, the band diagram is depicted including the energetically unstable ($\frac{\partial^2 E}{\partial\alpha^2}$ *or* $\frac{\partial^2 E}{\partial\beta^2} < 0$) tails. The part b of the figure only contains the stable ($\frac{\partial^2 E}{\partial\alpha^2}, \frac{\partial^2 E}{\partial\beta^2} < 0$) tails for visual clarity. In comparison to a single-component BEC, two-component BEC gives one more stable band tail, OA.

There is a simple physical picture for the doubling of the number of modes near the band edge. If the components of the BEC were assumed to be independent condensates, both of them would have separate swallow-tail structures. Without inter-component interaction, these modes would be degenerate. However, the presence of the interaction between the components couples these two structures and determines the complex shape of the bands. Of the two energetically stable tails, one is controlled by the in phase oscillations of the two components, while the other tail corresponds to the out of phase oscillations.

We also investigated the width of the instability region as a function of interaction strength. The unstable region has opposite behaviors for the two separate regions of the phase difference, θ. While in the region $\theta \in [0, \pi/2]$ the increase in the inter-species coupling g_{12} results in spreading tails(over k), in the region $\theta \in [\pi/2, \pi]$ it results in a narrower instability region. This is depicted in the figure 2. This behavior supports our picture that the dynamical instability in different branches are controlled by different oscillations of the two component condensate.

3. Conclusion

We investigated the dynamical instability of a two component BEC in an optical lattice. We found that the dynamical instability takes place for the two component BEC, however the band

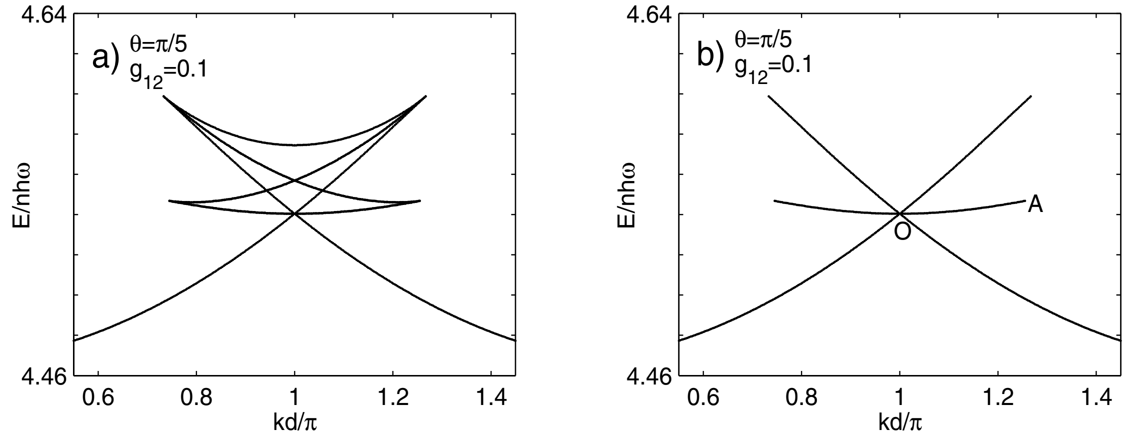

Figure 1. The tails in the band diagram of two-component BEC near the zone edge. In part (b), energetically unstable curves are eliminated for visual clarity. BA is the extra loop compared to single-componet [10]. Energy per particle is scaled with $\hbar\omega = 2\sqrt{\varepsilon_r V}$, the energy of small harmonic oscillations in the trap. ε_r is the recoil energy. Trap depth is chosen to be $\sqrt{\frac{V}{\epsilon_r}} = 6$.

structure near the band edge has one more tail compared to a single component BEC. We investigated the width of the instability region as a function of inter-component interaction and found two different behaviors for different optical lattices. The behavior near the band edge suggests that the instability is governed by in-phase and out-of phase modes of the two components.

References

[1] B. P. Anderson and M. A. Kasevich, Science **282**,1686 (1998) ; S. Burger, F. S. Cataliotti, C. Fort, F. Minardi, M. Inguscio, M. L.Chiofalo, and M. P. Tosi, Phys. Rev. Lett. **86**, 4447 (2001).

[2] W. Hofstetter, J. I. Cirac, P. Zoller, E. Demler, and M. D. Lukin, Phys. Rev. Lett. **89**, 220407 (2002); X.-J. Liu, P. D. Drummond, and H. Hu, Phys. Rev. Lett. **94**, 136406 (2005).

[3] F. S. Cataliotti, S. Burger, C. Fort, P. Maddaloni, F. Minardi, A. Trombettoni, A. Smerzi, and M. Inguscio, Science **293**, 843 (2001).

[4] B. Wu and Q. Niu, Phys. Rev. A **61**, 023402 (2000).

[5] O. Zobay and B. M. Garraway, Phys. Rev. A **61**, 036303 (2000).

[6] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).

[7] K. Berg-Sϕrensen and K. Mϕlmer, Phys. Rev. A **58**, 1480 (1998).

[8] B. Wu and Q. Niu, Phys. Rev. A **64**, 061603 (2001).

[9] B. Wu, R. B. Diener, and Q. Niu, Phys. Rev. A, **65**, 025601 (2002).

[10] D. Diakonov, L.M. Jensen, C. J. Pethick, and H. Smith, Phys. Rev. A, **66**, 013604 (2002).

[11] Erich J. Mueller, Phys. Rev. A **66**, 063603 (2002).

[12] A. Polkovnikov, E. Altman, E. Demler, B. Halperin, and M. D. Lukin, Phys. Rev. A **71**, 063613 (2005).

[13] Vikash Ranjan, Aranya B. Bhattacherjee and Man Mohan, cond-mat/0502399, *unpublished.*

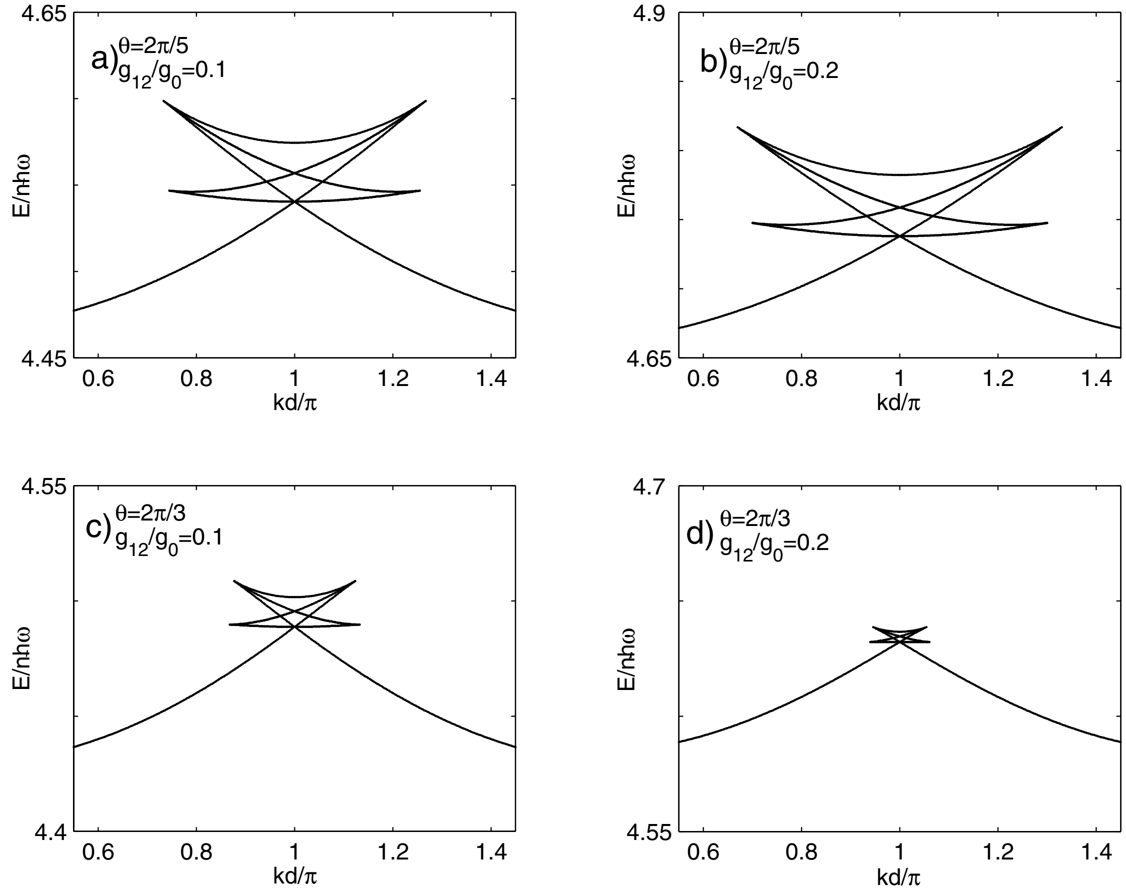

Figure 2. Change of the spread of tails by increasing inter-components coupling g_{12}. While growing g_{12} results in the broadening of the tails for $\theta = 2\pi/5 < \pi/2$((a) and (b)), it results the opposite for $\theta = 3\pi/5 > \pi/2$ ((c) and (d)).