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O.R. Applications

Hyperbolic efficiency and return to the dollar

Rolf Färe [a,b,*], Shawna Grosskopf [a], Osman Zaim [c]

[a] *Department of Economics, Oregon State University, Corvallis, OR 97331, USA*
[b] *Department of Agricultural Economics, Oregon State University, Corvallis, OR 97331, USA*
[c] *Department of Economics, Bilkent University, 06533 Bilkent, Ankara, Turkey*

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Abstract

This paper, after establishing the relations between hyperbolic graph measure of technical efficiency and the radial measures of technical efficiency, shows that the dual, cost and revenue interpretation of the hyperbolic efficiency measure is related to Georgescu-Roegen's notion of "return to the dollar" [N. Georgescu-Roegen, in: Koopmans, T. (Ed.), Activity Analysis of Production and Allocation, Wiley, New York, 1951, pp. 98–115]. Once this relation is established, it leads to a derivation of an allocative efficiency index, which measures the price distortions using data on observed costs and revenues without requiring information on prices.

Keywords: Technical efficiency; Return to the dollar

1. Introduction

Farrell (1957), while setting up a framework for the measurement of efficiency, distinguishes between output and input oriented technical efficiency measures. It has been shown (see Färe and Lovell, 1978) that under constant returns to scale (CRS) these measures are reciprocal to each other. [1] While the input oriented measure of technical efficiency seeks a scalar by which one can equiproportionately scale down (contract) the inputs while maintaining the production of a given level of outputs, the output oriented measure of technical efficiency searches for a scalar by which one can scale up (expand) the outputs while maintaining the same level of input use. In order to allow for simultaneous scaling of inputs and outputs, or desirable and undesirable outputs, Färe et al. (1985, 1994) introduced the hyperbolic approach to efficiency measurement. This approach allows for a simultaneous and equiproportionate expansion of outputs and contraction of inputs (or undesirable outputs), and thus, performs simultaneously what the two Farrell measures do. The hyperbolic measure has been used in environmental economics by e.g., Färe et al. (1989), Zaim and Taskin (2000) and Taskin and Zaim (2000). It has also found its way into the literature on the measurement of productivity, see e.g., (Arocena

* Corresponding author. Tel.: +1-541-737-8184; fax: +1-541-737-5917.

E-mail address: rolf.fare@orst.edu (R. Färe).

[1] Finn Førsund has in private conversations pointed out to us that Farrell defines his output oriented technical efficiency measure as the reciprocal of Färe et al. (1985).

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and Waddams Price, 1999; Lovell and Zofio, 1957).

An important property of the two Farrell measures of technical efficiency is that they have a dual representation. The input oriented measure has a dual in the cost-efficiency measure and the output oriented measure has its dual in the revenue measure of efficiency. [2] One purpose of this paper is to provide a dual, cost and revenue interpretation of the hyperbolic efficiency measure. This dual representation is shown to be Georgescu-Roegen's (1951) notion of "return to the dollar". Althin et al. (1996) show that the return to the dollar provides a profitability interpretation of the Malmquist productivity index, thus here we provide a second use of Georgescu-Roegen's profitability concept in the modern performance measurement literature. A second purpose of our paper is to show how the primal and dual hyperbolic efficiency measures can be used to evaluate profitability, technical and allocative efficiency using data envelopment analysis (DEA) [3] or activity analysis. [4]

The paper unfolds as follows. In Section 2 we present the theoretical foundations. Section 3 is reserved for an application and in Section 4 we conclude.

2. Theoretical foundations

We start this section by developing the hyperbolic measure for technology or equivalently the graph. Then we show that under constant returns to scale, the Farrell input oriented measure of technical efficiency equals the square of the hyperbolic measure and that return to the dollar can be seen as the dual to the hyperbolic technical efficiency measure.

To introduce some notation let inputs be denoted by $x = (x_1, \ldots, x_N) \in R_+^N$ and outputs by $y = (y_1, \ldots, y_M) \in R_+^M$. Together they define the technology or graph as

$$T = \{(x, y) : x \text{ can produce } y\}. \tag{1}$$

Following Färe et al. (1985, 1994) the hyperbolic "Farrell" measure of technical efficiency is defined as

$$F_g(x, y) = \min\left\{\lambda : \left(\lambda x, \frac{y}{\lambda}\right) \in T\right\}. \tag{2}$$

This measure while expanding the outputs proportionally, simultaneously contracts inputs proportionally. The proportionality or scaling factor λ is the same for both inputs $x$and outputs y .

We say that the technology T satisfies CRS if it is a cone, i.e., if T has the property

$$\lambda T = T, \quad \lambda > 0. \tag{3}$$

Under this condition, the hyperbolic measure takes a simple form, namely,

$$\begin{aligned} F_g(x, y) &= \min\left\{\lambda : \left(\lambda x, \frac{y}{\lambda}\right) \in T\right\} \\ &= \min\left\{\lambda : (\lambda^2 x, y) \in T\right\} \quad \text{(by CRS)} \\ &= \min\left\{\sqrt{\lambda^2} : (\lambda^2 x, y) \in T\right\} \\ &= \min\left\{\sqrt{\gamma} : (\gamma x, y) \in T\right\}, \end{aligned} \tag{4}$$

where at the optimum γ^*

$$F_g(x, y) = \sqrt{\gamma^*}. \tag{5}$$

Following Färe et al. (1985, 1994) the input oriented Farrell measure is defined as

$$F_i(x, y) = \min\{\lambda : (\lambda x, y) \in T\}, \tag{6}$$

and at the optimum λ^*

$$F_i(x, y) = \lambda^*. \tag{7}$$

It follows from (4) and (6) that at the optimum

$$\gamma^* = \lambda^*, \tag{8}$$

thus by (5) and (7)

$$\left(F_g(x, y)\right)^2 = F_i(x, y)$$

showing that under CRS, the input measure $F_i(y, x)$ is equal to the square of the hyperbolic measure $F_g(x, y)$. This generalizes Färe et al. (1985,

[2] See Färe and Grosskopf (1995) for the derivation of these measure from Mahler inequality (Mahler, 1939).

[3] This terminology was introduced by Charnes et al. (1978).

[4] This theory has its foundation in von Neumann (1938).

1994) who obtained the same result for an activity analysis model.

Following the same procedure as above one can show that $(F_g(x,y))^2 = 1/F_0(x,y)$, where

$$F_0(x,y) = \max\{\lambda : (x, \lambda y) \in T\} \tag{9}$$

is the Farrell output oriented measure of technical efficiency. Thus CRS implies

$$(F_g(x,y))^2 = F_i(x,y), \quad (F_g(x,y))^2 = \frac{1}{F_0(x,y)}. \tag{10}$$

Now if (10) holds then it follows from Färe and Lovell (1978) that the technology exhibits CRS (they showed that $F_i(y,x) = 1/F_0(x,y)$ if and only if technology exhibits CRS). Hence (10) holds if and only if technology is CRS.

In the next step we relate the hyperbolic measure to "return to the dollar". For this we introduce prices and profit. Let $p \in R^M_+$ denote output prices and $w \in R^N_+$ input prices. Profit is then defined as

$$\pi(p,w) = \max\{py - wx : (x,y) \in T\}. \tag{11}$$

From this maximization problem it follows that

$$\pi(p,w) \geqslant py - wx \text{ for all } (x,y) \in T,$$

$$\text{and since } \left(xF_g(x,y), \frac{y}{F_g(x,y)}\right) \in T,$$

we have

$$\pi(p,w) \geqslant \frac{py}{F_g(x,y)} - F_g(x,y)wx. \tag{12}$$

Under CRS, maximum feasible profit is zero, i.e., $\pi(p,w) = 0$. Note, however, that observed profit and therefore py/wx may not be maximal due to inefficiency (and deviations from CRS). Taking account of the maximum feasible profit equal to zero, by (12) we obtain

$$\frac{py}{wx} \leqslant (F_g(x,y))^2, \tag{13}$$

where py/wx is Georgescu-Roegen's "return to the dollar" measure. Note that when py is interpreted as observed revenue and wx as observed cost, there is no need to know output or input prices in estimating the return to the dollar.

Expression (13) shows that there is a dual relationship between "return to the dollar" and the hyperbolic graph measure. It is a type of Mahler inequality which is used as the basis for a Farrell-type efficiency decomposition, as we shall demonstrate.

The right-hand side of (13) is interpreted as a measure of technical efficiency and by expression (10) it is the reciprocal of the output oriented Farrell measure of technical efficiency. [5]

Following the tradition of Farrell (1957) we may define allocative efficiency AE as a residual, i.e.,

$$\mathrm{AE} = \frac{py}{wx}\frac{1}{(F_g(x,y))^2}, \tag{14}$$

thus

$$\frac{py}{wx} = \mathrm{AE} \cdot \mathrm{TE}. \tag{15}$$

To interpret expression (15) we note that the technical efficiency component TE equals $(F_g(x,y))^2$ which is the square of the hyperbolic distance function from (x,y) to the boundary of the technology. This takes values between zero and one. Return to the dollar may take values bigger than one when a firm incurs losses. Hence our allocative measure AE may also take values smaller and bigger than one, with one signaling allocative efficiency. To interpret AE, rearrange the terms in (14) as

$$\mathrm{AE} = \frac{p(y/F_g(x,y))}{w(xF_g(x,y))} = \frac{p\hat{y}}{w\hat{x}},$$

where $\hat{y} = y/F_g(x,y)$ and $\hat{x} = xF_g(x,y)$ are technically efficient quantities of outputs and inputs, respectively (i.e., projections of the observed outputs and inputs on to the efficient frontier, respectively), and noting that $\mathrm{AE}(py, wx, y, x) = 1$ when evaluated using prices $\hat{p}$ and $\hat{w}$ that support $\hat{y}$ and $\hat{x}$ (here convexity of T is assumed), i.e.,

$$\mathrm{AE}(py, wx, y, x) = \frac{\hat{p}\hat{y}}{\hat{w}\hat{x}} = 1,$$

simple division implies that

[5] Expression (14) can also be derived from the duality in Färe and Primont (1995).

$$\mathrm{AE}(py, wx, y, x) = \left(\frac{p\hat{y}}{\hat{p}\hat{y}} \Big/ \frac{w\hat{x}}{\hat{w}\hat{x}}\right) = \left(\frac{\sum p_m \hat{y}_m}{\sum \hat{p}_m \hat{y}_m} \Big/ \frac{\sum w_n \hat{x}_n}{\sum \hat{w}_n \hat{x}_n}\right).$$

This expression shows that allocative efficiency, $\mathrm{AE}(py, wx, y, x)$, is a ratio of an output price index to an input price index where output and input prices are weighted with technically efficient quantities of outputs $\hat{y}$ and inputs $\hat{x}$, respectively. In this expression, while the numerator shows the extent to which short-run output prices p deviate from their optimal long-run values $\hat{p}$ (i.e., prices that support $\hat{y}$) the denominator shows how short-run input prices w deviate from their optimal long-run values, $\hat{w}$ (i.e., prices that support $\hat{x}$).

We now have the following decomposition of return to the dollar, where

$$\frac{py}{wx} = \mathrm{AE} \cdot \mathrm{TE}, \tag{16}$$

and

$$\mathrm{TE} = (F_g(x, y))^2 = F_i(x, y) = 1/F_0(x, y). \tag{17}$$

Expression (17) may be used to derive shadow prices. Since these prices are known for $F_i(x, y)$ and $F_0(x, y)$, the shadow prices for $F_g(x, y)$ follow directly.

The last expressions also give us a measure of profitability that can be used for a CRS technology. This complements the profit efficiency measures by Banker and Maindiratta (1988) and Berger and Mester (1997) who define the profit efficiency as the ratio of maximal to observed profit. The latter measures are not of course necessarily well defined under CRS technologies. [6]

To continue, under CRS, maximal revenue

$$R(x^*, p) = \max\{py : (x, y) \in T\} = py^* \tag{18}$$

equals minimal cost [7]

$$C(y^*, w) = \min\{wx : (x, y) \in T\} = wx^*. \tag{19}$$

Thus (16) also equals the ratio of the Farrell revenue efficiency measure

$$\mathrm{OE}_o = \frac{R(p, x)}{py}, \tag{20}$$

and the cost efficiency measure

$$\mathrm{OE}_i = \frac{C(y, w)}{wx}, \tag{21}$$

namely,

$$\frac{\mathrm{OE}_i}{\mathrm{OE}_o} = \mathrm{AE} \cdot \mathrm{TE}, \tag{22}$$

since under CRS $R(x^*, p) = C(y^*, w)$. [8]

In the next section of the paper we show how (16) or equivalently (22) may be used to measure the profitability under CRS and how to decompose it into allocative and technical efficiencies.

3. An application

A recent paper by Voyvoda and Yeldan (1999), on the phases of macroeconomic adjustments in the Turkish economy, and their implications for the functional distribution of income, identifies 1989 as being the year where export oriented growth strategy of the 1980s (which basically relied on wage suppression and price subsidies to exporters of manufactured goods) reached its economic and political limits. The authors, after noting an annual rate of 10.2% increase in real wages between 1989 and 1993, for which populist policies followed and union gains shared equal responsibility, describe the private and public sector response to this new era, which ultimately resulted in a profit explosion in the private manufacturing industry. [9] As they describe it, while the private sector tried to match wage increases with large layoffs which

[6] For an alternative additive profit measure, see Chambers et al. (1998).

[7] The linear programming or DEA formulation of (18) and (19) is found in Shephard (1970, p. 288).

[8] This theory may be extended to non-constant returns to scale by using the idea of Tone (1993). See also Cooper et al. (2000).

[9] They observe that the profit margins in fact followed a rising trend, and reached 47% in 1994, from its average of 33.5% in 1989.

increased average labor productivity, the public sector's stance was to delay the restructuring of public prices against this inflationary background in an attempt to provide low-cost intermediary inputs to the private sector.

This background provides a valuable laboratory for both measuring the extent of distortions created in markets, and also for creating an opportunity to demonstrate the usefulness of the proposed indices in this paper. As we will demonstrate, the proposed index is particularly helpful in measuring distortions especially when one has to work with "Annual Manufacturing Industry Statistics" where there is no price-related information.

To this end, we choose to concentrate on manufactures of machinery and equipment (ISIC 38) not only because of its importance in total manufacturing output but also considering the linkage effects this industry may have over the manufacturing industry as a whole. Since this industry is a major supplier of the capital inputs to other industries, any distortion created within this industry will spread over the entire manufacturing industry. The data related to this industry are compiled from Annual Manufacturing Industry Statistics published by the State Institute of Statistics and cover all four-digit industries under ISIC 38 for the year 1991. A desirable feature of the data is that, except in a few cases, both government and private activities coexist allowing for a comprehensive analysis of relative distortions created by each ownership type.

Our measure of the aggregate output of a subsector is the real value of the output of the industry. [10] The four input proxies chosen are: labor as measured by number of hours worked, electricity used in kW/h, real value of fuel purchased and real value of raw materials. Our measure of profit rate is the ratio of revenues from sales to the cost of production as measured by the total cost of all the variable inputs including packaging material, wage payments and maintenance costs. Since cost of fixed inputs may not be entirely represented in our total costs, our profit measure may carry a slight upward bias.

Maintaining the CRS assumption, our computation strategy is simple: first, we construct a production frontier for a two-digit industrial classification ISIC 38 (the manufactures of machinery and equipment) using data on 22 subsectors (defined at the four-digit level according to the International Standard Industrial Classification) where public and private sectors are recorded separately, and then computing the technical efficiency of each four-digit sector with respect to this common frontier. Once the technical efficiency is computed, allocative efficiency can easily be obtained as a residual from (16).

To compute technical efficiency scores with respect to a CRS production frontier, as (16) shows, one can either choose to utilize the hyperbolic graph measure of technical efficiency or one of the radial measures, commonly referred to as input oriented technical efficiency measure and output oriented technical efficiency measure. Among competing methodological alternatives to the measurement of efficiency, we use DEA [11] or activity analysis since the computational procedure does not require specifying a parametric functional form. Hence, if one chooses to use the input oriented technical efficiency measure, for each observation $k' = 1, \ldots, K$, it can be computed as the solution to the following programming problem:

$$F_i(x_{k'}, y_{k'}) = \min \lambda$$

subject to

$$\begin{aligned} \sum_k z_k y_{km} &\geqslant y_{k'm}, \quad m = 1, \ldots, M, \\ \sum_k z_k x_{kn} &\leqslant \lambda x_{k'n}, \quad n = 1, \ldots, N, \\ z_k &\geqslant 0, \quad k = 1, \ldots, K, \end{aligned}$$

where $[F_g(y_{k'}, x_{k'})]^2 = F_i(x_{k'}, y_{k'})$.

We note that we have restricted the z variables to be non-negative. This effectively allows the reference technology frontier to satisfy CRS, which

[10] Nominal values are deflated by four-digit sectoral price indices.

[11] This terminology was introduced by Charnes et al. (1978).

means that maximal feasible profit (but not necessarily observed profit) is equal to zero, which we need for our decomposition.

In this particular application, we used the linear programming formulation above to measure the technical efficiency of production for the subsectors of the manufactures of machinery and equipment and then used (16) to decompose Georgescu-Roegen's "return to the dollar" measure into its allocative and technical efficiency components. Table 1 shows this decomposition categorized by the ownership status.

The analysis of the table reveals some interesting results. Commensurate with the observations of Voyvoda and Yeldan (1999), there is a large discrepancy between profit margins in the public and private sectors. While in the private sector the average profit margin is almost 41%, public sector

Table 1
Decomposition of "return to dollar"

ISIC	Manufacture of:	TR/TC		Technical efficiency		Allocative efficiency	
		Private	Public	Private	Public	Private	Public
3811	Hand tools and general hardware	1.408		0.848		1.661	
3812	Furniture and fixtures primarily of metal	1.282		0.807		1.589	
3813	Structural metal products	1.334	1.220	0.709	0.613	1.881	1.991
3819	Fabricated metal products	1.539	1.210	0.672	0.699	2.290	1.731
3821	Engines and tribunes	1.398	1.064	0.602	0.576	2.322	1.846
3822	Agricultural machinery and equipment	1.173	0.675	0.551	0.639	2.129	1.056
3823	Metal and wood working machinery	1.415	0.495	0.807	0.639	1.754	0.775
3824	Special industrial machinery and equipment	1.327	1.283	0.756	1.000	1.755	1.283
3825	Office computing and accounting machinery	1.383		1.000		1.383	
3829	Machinery and equipment except electrical	1.369	0.694	1.000	0.527	1.369	1.318
3831	Electrical industrial machinery and apparatus	1.543	1.265	0.854	0.537	1.807	2.356
3832	Radio, TV and communication equipment	1.514	1.024	1.000	0.533	1.514	1.921
3833	Electrical appliances and housewares	1.308		0.744		1.758	
3839	Electrical apparatus not elsewhere classified	1.289	1.250	0.647	0.716	1.993	1.746
3841	Ship building and repairing	2.122	0.721	1.000	0.933	2.122	0.773
3842	Railroad equipment and repairing		1.179		0.739		1.596
3843	Motor vehicles and repairing	1.340	0.461	0.691	0.538	1.940	0.857
3844	Motorcycles and repairing	1.616		1.000		1.616	
3845	Aircrafts and repairing	1.014		0.642		1.579	
3851	Professional scientific measuring equipment	1.501	0.944	0.742	0.458	2.024	2.061
3852	Photographic and optical goods	1.570		1.000		1.570	
3854	Other	1.430		0.386		3.705	
Geometric mean		1.409	0.913	0.763	0.638	1.846	1.432
Geometric mean		1.185		0.710		1.667	

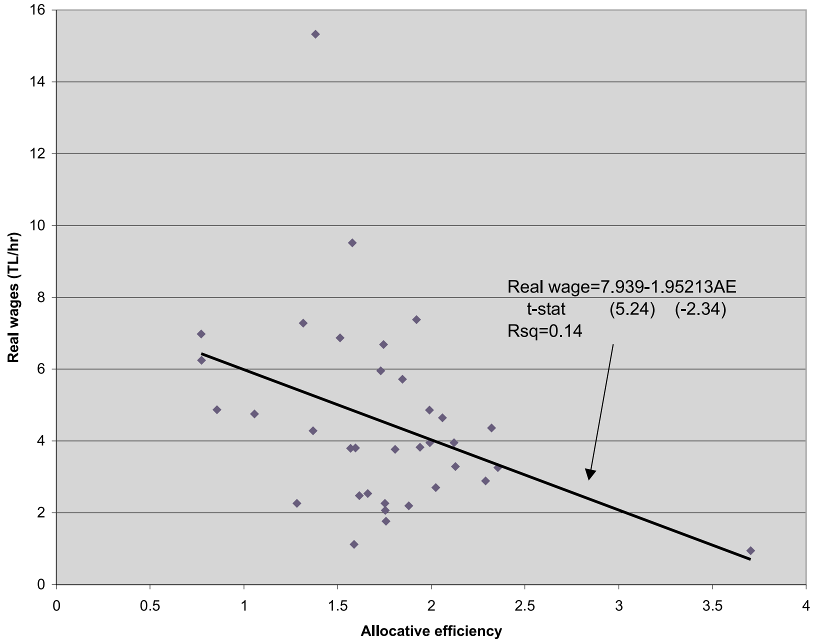

Fig. 1. Real wages and allocative efficiency.

enterprises incur losses with total costs exceeding total revenues by 8.7% on the average. As for the decomposition of the "return to dollar", the results indicate that low levels of technical efficiency in the manufactures and machinery and equipment sector are more than recuperated by distorting the relative output prices with respect to input prices. Note that, while the sector as a whole suffers from low levels of efficiency (71%), the public sector with technical efficiency score averaging 63.8% lags behind the private sector whose technical efficiency averages 76.3%. However, as the figures in the allocative efficiency columns show, in an attempt to cover the inefficiency-induced losses, both the public and private sectors resort to distorting the relative output prices with respect to input prices; the distortion being more pronounced in the private sector than in the public sector.

We should note however that our interpretation differs slightly from that of Voyvoda and Yeldan (1999) who held the private sector primarily responsible for price distortions and attribute the losses of the public sector to delays in restructuring of public prices against an inflationary background in an attempt to provide low-cost intermediary inputs to the private sector. As our indexes show, the public sector shares in the responsibility for distorting the relative output prices with respect to input prices. In fact, as Voyvoda and Yeldan (1999) claim, if the public sector could not match the wage increases with layoffs and as a result paid higher than optimal wages, [12] this implies considerable increases in the short-run output prices with respect to their long-run values in the public sector, since in this sector, allocative efficiency measure is also greater than one. However, this may have an aggravating effect on price distortions in the private sector especially in cases where the public sector's output is an input to the private sector.

In a final analysis, in order to show how our measure of allocative efficiency (which is computed

[12] Assuming that wage payments are a large component of the total costs and hence

$$\frac{\sum w_n \hat{x}_n}{\sum \hat{w}_n \hat{x}_n} > 1.$$

without price information) captures price distortions, we relate this measure to real wages, the only price information we could obtain from the data.[13] Since for the allocative efficiency measure, values greater than one imply the ability to distort output prices at higher rates than input prices, we would expect a negative relationship between real wages and allocative efficiency. This relation as plotted in Fig. 1 confirms our expectations by showing that real wages are in fact negatively and significantly related to allocative efficiency.

4. Conclusion

This paper, after establishing the relations between the hyperbolic graph measure of technical efficiency and the radial measures of technical efficiency, shows that the dual cost and revenue interpretation of the hyperbolic efficiency measure is related to Georgescu-Roegen's (1951) notion of "return to the dollar". Once this relation is established, it leads to a derivation of an allocative efficiency index, which measures price distortions using data on observed costs and revenues without requiring explicit information on prices.

An application of this technique for the Turkish manufactures of machinery and equipment industry reveals that, perhaps in an attempt to cover technical inefficiency induced losses, both the public and private sectors apparently resort to distorting relative output prices with respect to input prices; the distortion being more pronounced in the private sector than in the public sector.

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