

Strategic information revelation in fundraising

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Abstract

We consider a model of voluntary contributions for a public project with random number of potential contributors. The fundraiser, who observes this number, has to decide whether to reveal or suppress the information before contributions are given. The fundraiser's objective is to collect maximal contributions. We show that whether the public project is convex or non-convex can be the key to the fundraiser's announcement decision. In the convex case, this number is always revealed. In the non-convex case the number may not be revealed at all or sometimes revealed only when it is in an intermediate range. In the presence of multiple equilibria, total contributions increase with the extent of concealment. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Professional charities and organizations providing various public goods and services to the community primarily rely on voluntary contributions. In this context, the *number* of potential beneficiaries who may contribute is an important piece of information for both the charities and contributors. The total funds raised,

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hence the amount of the public good supplied, will depend on the number of contributors. Individual contribution strategies also depend on the expected number of contributors and how strongly each contributor perceives himself to be pivotal in the contribution process. However, fundraisers are often much better informed about the number of potential contributors, through their pre-fundraising research or professional expertise.¹ This raises the question, would a fundraiser, whose objective is to collect maximal contributions for a public project, reveal or suppress the information about the number of potential contributors before contributions are given? We address this question for two types of public good production technologies — a convex technology that involves zero minimal production, and a non-convex technology that requires a positive minimal production for the project to be viable. The distinction, as we show, turns out to be crucial.

Some opposing evidences and intuitions about information revelation issues motivate our work. A fundraiser may prefer concealing the presence of a sizeable number of potential contributors so that individual contributors do not engage in free-riding. On the other hand, there is also the view that sometimes the organizers want to assure the prospective contributors of a critical mass of committed contributors, so that the public project is almost guaranteed to take off (Andreoni, 1998). Intuitively, two countervailing scenarios may play on an individual donor's mind: if he knows there are too few potential contributors, the project may not be successfully launched even with the maximum amount of contribution he is willing to make, thus discouraging him against contribution; on the other hand, if he knows there are many potential contributors, the importance of his individual contribution for the project's viability as well as his own marginal benefit become negligible, again discouraging contribution. Thus, the fundraiser's strategy to inform or not to inform potential contributors should depend on the expected reaction of the contributors, which in turn depends, as we will argue, on whether the public good technology is convex or non-convex.

Only a few papers examine the issue of information revelation in fundraising. Vesterlund (1999) considers imperfect information about the quality of a public good (or charity) to analyze how sequential fundraising with announcement of contributions (as opposed to a simultaneous contribution arrangement with no announcement) helps signal the information about quality. She shows that for low cost of information gathering, a high quality charity strictly prefers to announce contribution by an initial donor in order to induce her to gather and signal quality related information and thereby influence a second donor's contribution. In our

¹Fundraisers may obtain potential donor information, besides their own means, through several institutions that specialize in tracking important data about donors. To give a few examples, a software integrating donor databases and email, called DonorLink IT, is available to nonprofit organizations (source: *NonProfit Times*, August 2000). National Catholic Reporter sells donor lists at <http://www.natcath.com/listrent/listrent.html>.

paper the quality of the public good is not an issue. Nor do we consider the question of sequential versus simultaneous contribution arrangement. Our work rather builds on Andreoni (1998).² He considers a two-phase fundraising arrangement where in the first phase the fundraiser's principal objective is to secure the promises of contributions from the government or 'leaders' and then announce it to the public, to remove any uncertainty for the public that the project might not take off, which then jump-start the main contributions phase from the public. We complement Andreoni's work by shifting attention to another important phenomenon, though somewhat of a contrasting flavor than the one pointed out by Andreoni, that of appeals to the public by fundraising organizations that without their help some proposed project(s) might not take off. Implicit in such appeals is the message that the number of potential contributors is not large and therefore each contributor must offer generously to rescue the project. Thus, the worry that the project might not take off may guide a fundraiser to choose different strategies: Andreoni highlights the need for early assurances about the project's success to the yet untapped contributors, whereas we consider the possibility of a failed project to motivate potential contributors for generous contributions; there are no 'leaders' in our model. Specifically, we focus on the fundraiser's strategic use of the private information about the number of potential contributors. For large number of potential contributors a rough intuition gained from Andreoni (1998) suggests that the fundraiser should announce this information, whereas the intuition for free-riding with many contributors exemplified in another paper by Andreoni (1988) suggests that the fundraiser should hide the information. For small number of potential contributors, again symmetrically opposite intuitions appear puzzling.

We show the following results. If the public good production technology is *convex* with zero minimal production requirement, the fundraiser will *always reveal* the number of potential contributors. The intuition is as follows. Given convexity, the amount of public good production can be varied continuously. Now if there is any (non-singleton) set of numbers that the fundraiser may want to suppress, then following non-revelation the contributors would put strictly positive beliefs to all the numbers within this set and choose an 'average' level of contribution by balancing between large values and small values of the suppressed numbers. This means, if the true number happens to be one of the small values, then the fundraiser will be able to induce contributions above the 'average' level by announcing this number. Hence the concealment set must be empty. The results and their intuitions in the non-convex case are strikingly different. When the technology is *non-convex* with a strictly positive minimal production requirement, there can be multiple equilibria with different symmetric contributions. First, there always exists a revealing equilibrium in which the fundraiser announces if the number of potential contributors (weakly) exceeds a threshold number and

²Andreoni also surveys several other interesting works on fundraising and their links to the public goods literature.

suppresses otherwise; total contributions following announcement are just enough to make the project viable, but collapse to zero following suppression. In addition, there may exist partially revealing and/or non-revealing equilibria. In a partially revealing equilibrium, the fundraiser suppresses small turnouts to avoid the zero contributions outcome and suppresses large turnouts to induce contributions exceeding the minimal viable level, but announces intermediate turnouts so that each gives just enough to make the project viable. In a non-revealing equilibrium, the number of potential contributors is always suppressed so that contributors with the expectation of ‘high enough’ turnouts would make positive contributions and the fundraiser gets to collect high overall contributions in the case of large turnouts. Total contribution is *largest* in a non-revealing equilibrium, followed by a partially revealing equilibrium, and *lowest* in a revealing equilibrium. Thus, charities should always like to conceal the number of potential contributors and induce the non-revealing equilibrium, if such an equilibrium exists.³

Our results shed some light on whether and how a credible fundraiser may strategically use his private information about the number of potential contributors to maximize total contributions. This information is important especially when the target population of contributors is relatively homogeneous, as for instance in the case of the alumni of a well-established, reputable high school, listeners of a specialized radio station in a small community, blood and organ donations, or beneficiaries of a local public good (e.g. a sports facility or a concert hall) living in the same neighborhood. Announcement strategies may also indirectly give hints as to the number of potential contributors. Charities have the option of announcing the number of mailings or members in their solicitation letter.⁴ As a strategy for recruiting more blood donors, American Red Cross may announce the number of donors and total donations received in previous years. If we observe a credible and established fundraiser giving no hints to potential contributors about their likely number, this is either because the fundraiser does not have an informational advantage relative to the public (which is unlikely) or the target population of contributors is so heterogeneous that the number of people who may contribute is almost void of relevance for individual contributors, hence the fundraiser. The

³The fundraiser’s preference for the non-revealing equilibrium is similar in spirit to a result in the auction literature, by McAfee and McMillan (1987) and Matthews (1987): with a random number of bidders who have constant or decreasing absolute risk aversion, if the auctioneer can commit to a particular announcement policy then in the first-price auction the auctioneer would gain by concealing the number of bidders. With constant absolute risk aversion while the bidders are indifferent between a policy of concealment and a policy of announcement, the contributors in our case (who also have constant absolute risk aversion) can become worse off or better off ex-post due to concealment by the fundraiser. Despite some similarities, however the auction and fundraising contexts differ in obvious ways.

⁴Plan International UK, a charity helping children and poor families in developing countries, sends the following message in its leaflet: ‘When you join PLAN’s ‘world family’, you become a member of a worldwide community in the fullest sense. There are over 830,000 PLAN International sponsors around the world.’

third possibility, as shown in this paper, is that such concealment is part of an equilibrium announcement strategy by the fundraiser that, through its effects on contributors' beliefs, ultimately results in higher total contributions.

The paper is organized as follows. Section 2 presents the fundraising game. The equilibrium of this game for the case of a convex technology is analyzed in Section 3. Section 4 introduces non-convexity and reviews its implications for information revelation and contribution decisions. Section 5 concludes. Appendix A contains the proofs of Propositions 1 and 2.

2. The fundraising game

A public good project will have to be financed through private contributions. The number of potential contributors, hereafter called players, is random: $p(k)$ is the probability that there are $k \in \{1, \dots, N\}$ players chosen by Nature, $\sum_{k=1}^N p(k) = 1$, and $p(\cdot)$ is common knowledge. Given any number of players k drawn by Nature, each of the potential N players will be assumed to have an equal chance, k/N , to be included among the k -players. This implies players' names will have no importance in fundraising.

As in most of the fundraising literature, we assume that the fundraiser, hereafter referred as the 'planner', maximizes total contributions. The players maximize (net) expected utilities and have identical quasi-linear preferences:

$$u(m_i - g_i, G) = v(G) + m_i - g_i, \text{ with } v'(\cdot) > 0, v''(\cdot) < 0 \text{ and } v'(0) > 1$$

where m_i is player i 's endowment of a single private good (or wealth), g_i is i 's contribution, and G is the level of public good.⁵ The level, G , depends on an underlying technology, thus total contribution and the level of public good provided may differ. We assume that potential contributors are not wealth constrained so that m_i s are ignored in the rest of the analysis.

Below we consider a fundraising game in which the planner may choose to announce or conceal the number of players. If the decision is to announce, it must be truthful; nontruthful announcements are illegal and/or seriously undermine the planner's authority. We assume zero cost of information announcements; similar results obtain in the presence of small, fixed operational costs.⁶

⁵These assumptions imply that players have constant absolute risk aversion (CARA) — see the condition in Matthews (1987, p. 638).

⁶We do not model fundraising costs, only a small component of which is information announcement costs. While the latter costs can be lump sum and negligible — say, a charity can make the announcement at a preliminary event or in a widely noticeable forum — overall fundraising costs that include the costs of gathering information about potential donors, contacting each donor individually by phone or mail etc. can be non-trivial and vary with the number of donors solicited. As pointed out by one of the referees, analyzing fundraising costs would require a more elaborate modeling.

2.1. The game Γ

The fundraising game, denoted Γ , consists of *two* stages. Stage 1 can be called the preparations stage, while Stage 2 is the contributions stage when the fund drive is launched.

Stage 1. The planner observes k and then decides whether to announce or suppress k .

Stage 2. Having observed the planner’s announcement or no announcement, the players update their beliefs $p(k)$ and simultaneously decide on their contributions.

The strategies in Γ are defined as follows. The announcement strategy of the planner is a map $a:\{1, \dots, N\} \rightarrow \{1, \dots, N\} \cup \{\emptyset\}$ such that $a(k) \in \{k, \emptyset\}$; $a(k) = k$ if k is announced and $a(k) = \emptyset$ if k is suppressed. Given an announcement strategy $a(\cdot)$, the set of announced k s is denoted by \mathcal{A} and the complementary set is denoted by \mathcal{C} . Players’ (common) beliefs following planner’s announcement decision is a probability distribution $\mu(\cdot)$ over the set $\{1, \dots, N\}$ and must be *consistent*; that is, $\mu(\cdot)$ is derived from the priors using Bayes’ rule, whenever possible. Thus, given an announcement strategy $a(\cdot)$, if the planner announces k , then $\mu(k) = 1$ and $\mu(n) = 0, n \neq k$. If the planner makes no announcement, each player revises the probability that k players are in the game conditional on his own presence, as:⁷

$$\mu(k) = \begin{cases} \text{pr}\{n = k\} \cap \{n \in \mathcal{C}\} \cap \{\Gamma \text{ m in the game}\}, & \text{for } k \in \mathcal{C} \neq \emptyset \\ 0, & \text{for } k \in \mathcal{A} \end{cases}$$

$$= \begin{cases} \frac{\text{pr}\{n = k\} \cap \{n \in \mathcal{C}\} \cap \{\Gamma \text{ m in the game}\}}{\text{pr}\{n \in \mathcal{C}\} \cap \{\Gamma \text{ m in the game}\}}, & \text{for } k \in \mathcal{C} \neq \emptyset \\ 0, & \text{for } k \in \mathcal{A}. \end{cases}$$

Finally, the contribution strategies of the players depend on the planner’s announcement $a(\cdot)$ and beliefs $\mu(\cdot)$, that is, $g_i:\{1, \dots, N\} \cup \{\emptyset\} \times [0, 1]^N \rightarrow \mathcal{R}_+$. We do not consider mixed contribution strategies.

We require the strategies and beliefs, $(a^*(\cdot), \{g_i^*(\cdot, \cdot)\}, \mu(\cdot))$, to form a *perfect Bayesian* equilibrium. The analysis will make use of the following assumptions about the equilibrium strategies.

Assumption 1 (Symmetry). Whenever there are both a symmetric contributions equilibrium and an asymmetric contributions equilibrium yielding the same total

⁷To give an example of how to calculate the posterior, for $N = 4, \mathcal{C} = \{1,4\}, \mathcal{A} = \{2,3\}$,

$$\mu(1) = \frac{\text{pr}\{n = 1\} \cap \{\Gamma \text{ m in the game}\}}{\text{pr}\{n \in \{1,4\}\} \cap \{\Gamma \text{ m in the game}\}} = \frac{(1/4) \cdot p(1)}{(1/4) \cdot p(1) + (4/4) \cdot p(4)}.$$

contribution, players always play the symmetric equilibrium, so that $g_i^* = g_j^*$ for all i, j .

Assumption 2 (Tie-breaking). If announcing and suppressing k yield the same payoff for the planner, then he suppresses k .

Assumption 2 is a working assumption; even a slight cost of making an announcement would break the tie in favor of not making an announcement. Assumption 1, which can be justified by the focal point argument, simplifies the analysis. Hereafter we shall drop the subscript i from equilibrium contribution strategies and refer to a perfect Bayesian equilibrium satisfying Assumptions 1 and 2 simply as ‘equilibrium’.

3. The convex technology case

In this section, we consider a convex technology for the public project. For simplicity we assume that the level of public good is the sum total of individual contributions.⁸

To begin with the analysis of the game Γ , suppose that the planner announces the presence of k players. In the continuation game (Stage 2), the symmetric individual Nash equilibrium contribution $\bar{g}^*(k)$ is determined by the first-order condition $v'(kg^*(k)) = 1$ which, defining \bar{G} through $v'(\bar{G}) = 1$, can be written as:

$$v'(kg^*(k)) = v'(\bar{G}). \tag{1}$$

Therefore, $g^*(k) = \bar{G}/k$ and the aggregate Nash contribution equals \bar{G} . By $v'(0) > 1$ and strict concavity of $v(\cdot)$, the equilibrium is immune to deviations. Note that given quasi-linear preferences, in any interior Nash equilibrium (i.e. where each contributor makes a strictly positive donation) the aggregate level of public good, \bar{G} , is independent of the number of contributors, k .⁹

Consider now a *non-revelation* policy. If the planner adopts the strategy of *never announcing* the number of players whatever be this number, then in Stage 2 the symmetric Bayes’ Nash equilibrium contribution of a player, $g^*(\emptyset)$, will satisfy:

⁸This is much stronger than needed for our results. Letting $f(\Sigma g_i) = G$ denote the public good production technology, a continuous production function $f(\cdot)$ that preserves strict concavity of $v(f(x))$ in x is sufficient.

⁹This is a special case of the so-called ‘Neutrality Theorem’ (see Bergstrom et al. (1986)) — dollar-for-dollar substitution in the contribution of each player — which makes players’ equilibrium contributions indeterminate given any k . But we invoke Assumption 1 to focus only on the symmetric equilibrium.

$$\mu(1) v'(g^*(\emptyset)) + \mu(2) v'(2g^*(\emptyset)) + \dots + \mu(N) v'(Ng^*(\emptyset)) = v'(\bar{G}). \quad (2)$$

Clearly, $\bar{G}/N < g^*(\emptyset) < \bar{G}$. Therefore, there exists some $k_0 \leq N$ such that for all $k \geq k_0$, $g^*(\emptyset) > \bar{G}/k$, and for all $k < k_0$, $g^*(\emptyset) \leq \bar{G}/k$. The following result is implied.

Lemma 1. *Compared to the revelation policy, the policy of non-revelation leads to lower public good level ex-post when less than k_0 number of players turn up, but higher public good level ex-post when at least k_0 number of players turn up.*

The intuition behind Lemma 1 is simple. Non-revelation of the number of players creates uncertainties for an individual player whether he is among a ‘few’ (that is, $k < k_0$) or one among ‘many’ (that is, $k \geq k_0$). The prospect of many other contributions enhances free-riding incentives and results in a lower public good level when the number of players turns out to be small. On the other hand, the players also consider the possibility that there may be only a few of them, which induces each player not to rely too much on free-riding opportunities and hence insure himself by contributing more than what he would contribute under perfect information of a large number of players. This induces a higher public good level ex-post, when the number of players turns out to be large.¹⁰ Thus, if the planner can commit to following a policy of never revealing k then he would commit to such a policy if the *expected* number of players is sufficiently large.

Interestingly, lacking commitment power, the non-revelation policy of never announcing k or a partial revelation policy in which the planner suppresses a (non-singleton) proper subset of N potential numbers of players, cannot be an equilibrium strategy. To see this, suppose that there is a set \mathcal{C} of numbers (containing at least two elements¹¹) such that the planner does not announce k if $k \in \mathcal{C}$. Each player, on observing no announcement by the planner, would determine his own contribution according to condition (2), modified to take into consideration only those k s that belong to \mathcal{C} . By Lemma 1, the planner will prefer revealing small turnouts from the set \mathcal{C} , thus will deviate from the strategy of not revealing *any* k from the set \mathcal{C} . We have the following result.

Proposition 1. *Suppose that the public good production technology is convex. Then in the unique equilibrium of Γ , the planner announces all $k \in \{1, \dots, N-1\}$ and suppresses $k = N$. Following no announcement, the players correctly infer their number: $\mu(N) = 1$, and $\mu(n) = 0$ for $n \neq N$. Each player contributes \bar{G}/k if k*

¹⁰Whether by not revealing the number of players the planner is likely to induce total contribution that exceeds the socially optimal level, is not relevant in our context. Our planner is interested in maximizing total contributions.

¹¹If \mathcal{C} is singleton, the announcement strategy is fully revealing.

is announced, and contributes \bar{G}/N if there is no announcement. Thus, the equilibrium is fully revealing.

The equilibrium unraveling result of Proposition 1 is closely related to a well-known result due to Okuno-Fujiwara et al. (1990), who provide various sufficient conditions for complete revelation of all private information for a broadly defined information revelation game.¹² Their main sufficient condition (see their Theorem 1) requires that following a preliminary information revelation stage, the equilibrium expected payoffs of the agents in the subsequent actions stage are *weakly positive-monotone in beliefs*. This roughly means that *any* (type of any) agent will be strictly better off if the other agents believe him to be the ‘highest’ type with probability one. In our context, for the convex technology, the planner who observes the smallest of a likely set of numbers corresponds to the ‘highest’ type in the terminology of Okuno-Fujiwara et al. The total contributions by the players will be maximal if they believe that the planner is of the highest type. Thus, the sufficient condition of Okuno-Fujiwara et al. is satisfied.¹³ However, our proof uses a more direct argument than the one in Okuno-Fujiwara et al. and is included in Appendix A. The driving force behind the information revelation result in this paper, Okuno-Fujiwara et al. and the papers mentioned in Footnote 12, is what is known as ‘due skepticism’: the uninformed agent(s) assume that if the informed agent does not announce the realized value of a variable affecting different agents’ payoffs, this value is the one which the informed agent must be most reluctant to announce. In our context, failure by the planner to announce the number of players is interpreted by the players as an evidence of maximal turnouts, inducing full revelation.

4. The non-convex technology case

In this section we consider the case of a non-convex technology: any production of the public good must meet a minimum threshold quantity requirement. This is due to an initial lumpiness in the production process, for example, a minimal amount of capital investment is essential for the construction of a local public

¹²See also Milgrom (1981), Matthews and Postlewaite (1985), and Milgrom and Roberts (1986) for similar results in specific, product quality disclosure games between an informed seller and uninformed buyer(s).

¹³Assumption 1 of Theorem 1 of Okuno-Fujiwara et al., a specific condition on the nature of certifiable reports, is also satisfied in our context, because our planner can make only truthful announcements. However, for the non-convex technology case considered next, the weakly positive-monotone in beliefs condition will be violated and full revelation of information need no longer be the unique equilibrium.

school or hospital. This case has been the main focus of Andreoni (1998).¹⁴ We assume the same non-convex technology:

$$G = \begin{cases} \sum_{i=1}^n g_i, & \text{if } \sum_{i=1}^n g_i \geq G_{\min} \\ 0, & \text{if } \sum_{i=1}^n g_i < G_{\min} \end{cases}$$

and adopt the fixed costs interpretation of non-convexity.¹⁵

The occurrence of zero contributions, a likely outcome in our model under complete information, can be avoided through strategic concealment of information by the planner. In this context, we focus on the impact of non-convexity on two key aspects: the planner's strategy of whether to announce or conceal the number of potential contributors, and its impact on the equilibrium level of contributions.

We need to specify what happens to players' contributions if total contributions fall short of G_{\min} . We assume, as is the case in most fund drives, that insufficient contributions will *not* be refunded and the planner will use the funds for some other project that yields no benefits to the players.¹⁶

4.1. Equilibria of the fundraising game Γ

Let us denote the equilibria of Γ by $(\tilde{a}(\cdot), \tilde{g}(\cdot, \cdot), \mu(\cdot))$. If $G_{\min} < \bar{G}$ then whenever the planner reveals k the threshold level will have no bite because the players would voluntarily contribute in excess of the threshold level. So, hereafter we consider the interesting case and assume:

Assumption 3. $G_{\min} > \bar{G}$.

Assumption 3 implies that the equilibrium derived in Proposition 1 for the convex technology, under which all k are revealed and the planner collects the aggregate

¹⁴Andreoni shows that the occurrence of a zero contributions outcome due to non-convexity can be avoided by a two-phase fundraising arrangement where the 'leader' contributors give early assurances by providing 'seed grants'. Marx and Matthews (2000) also consider a similar non-convexity where benefits jump discontinuously after a minimal production. However, they are not concerned with the strategic issues of fundraising; they examine, in a direct contribution game setting, whether the opportunity to make repeated contributions can eliminate inefficiencies.

¹⁵As Andreoni (1998) notes, non-convexity may be due to increasing returns to scale over some range of total contributions. It can also stem from the players' preferences. Our results would be similar in these cases.

¹⁶Similar results can be derived if we assume $G = \lambda \sum_i g_i$, where $0 < \lambda < 1$, if $\sum_i g_i < G_{\min}$. The parameter λ would capture the diminished benefits of the players due to an alternative use of total funds that fall short of G_{\min} .

contribution \bar{G} , is no longer an equilibrium of Γ under the non-convex technology. Now, when the planner reveals a $k \in \{1, \dots, N\}$, the contributions stage of Γ will have a symmetric equilibrium $\{\tilde{g}(k)\}$ with $k\tilde{g}(k) = G_{\min}$ if and only if individual participation constraints:

$$v(G_{\min}) - \tilde{g}(k) \geq v(0) \tag{3}$$

are satisfied. This is easy to check: No player has an incentive to deviate to a lower contribution, $0 \leq g_i < \tilde{g}(k)$, as each player is pivotal for the supply of the threshold level; nor would any player contribute more than $\tilde{g}(k)$ because the marginal cost exceeds the marginal benefit: $1 > v'(G_{\min})$. Thus, announcing k induces an equilibrium where each player contributes an equal share for a guaranteed supply of G_{\min} . The following assumption ensures that such an ‘interior’ (symmetric) contributions equilibrium exists if and only if k is sufficiently large.

Assumption 4. $G_{\min} > v(G_{\min}) - v(0) > G_{\min}/N$.

Thus, we can define:

Definition 1. \underline{k} is the minimum k such that $v(G_{\min}) - G_{\min}/k \geq v(0)$.

While the interior contributions equilibrium with each player contributing G_{\min}/k is now guaranteed for announcements of $k \geq \underline{k}$, for the same k announcements there is also a second equilibrium in which players make zero contributions. This happens because by the first inequality in Assumption 4 no single player would be willing to put up the entire G_{\min} alone. Below we assume that the players avoid coordination failures and never play the zero contributions equilibrium for $k \geq \underline{k}$, i.e. whenever a positive contributions equilibrium exists. Note also that the positive contributions equilibrium Pareto-dominates the zero contributions equilibrium for $k \geq \underline{k}$.

Under the non-convex technology, the game Γ has equilibria in which the planner conceals a (non-singleton) set of numbers of players. We offer two examples to provide intuition about the structure of such equilibria.

Example 1. Let $v(G) = 10(G)^{0.5}$, $N = 5$, and $p(1) = 0.1$, $p(2) = 0.2$, $p(3) = 0.1$, $p(4) = 0.3$, $p(5) = 0.3$. In the case of convex technology, equilibrium total contribution G equals 25. In the case of non-convex technology, which is our concern, suppose $G_{\min} = 200$. Check that $\underline{k} = 2$. Below we verify the following equilibrium:

The planner announces $k \in \{2, 3\}$ and suppresses $k \in \{1, 4, 5\}$, and the players contribute, respectively, 100 and 200/3 for announcement of $k = 2$ and $k = 3$ and contribute $g^*(\emptyset) = 50$ when there is no announcement.

This is a ‘partially revealing’ equilibrium. When $k = 2$ is announced, contributing 100 given that the other player contributes 100 is clearly optimal, as it just ensures the threshold level and yields a net utility of 41.42136, while deviating to zero contribution yields zero net utility. Similarly, contributing $200/3$ when $k = 3$ is announced is optimal for the players. We next check that in the no-announcement continuation game no player deviates to zero contribution. In the absence of any announcement under the proposed equilibrium, we calculate the posteriors as follows:

$$\mu(1) = \frac{(1/5) p(1)}{(1/5) p(1) + (4/5) p(4) + (1) p(5)} = 1/28.$$

Similarly,

$$\mu(2) = \mu(3) = 0, \quad \mu(4) = 3/7, \quad \mu(5) = 15/28.$$

Given these posteriors, the net expected utility of a player from contributing 50 is $(3/7) \cdot 10(200)^{0.5} + (15/28) \cdot 10(250)^{0.5} - 50 = 145.313$, whereas net expected utility by deviating to zero contribution equals $(15/28) \cdot 10(200)^{0.5} = 75.761$. Thus, deviation to zero contribution will not occur. All other deviations can similarly be ruled out.

To check that the strategy of announcing if and only if $k \in \{2,3\}$ is optimal for the planner, consider the possible deviations. If the planner suppresses $k \in \{2,3\}$, each player contributes $g^*(\emptyset) = 50$ and total contribution falls short of 200 achieved by announcing k . If the planner announces $k = 1$, the only player contributes zero (contributing G_{\min} yields $v(G_{\min}) - G_{\min} = -58.578644$) which is less than the contribution 50 under no announcement. If $k = 4$ or $k = 5$ is announced, total contribution equals 200, whereas total contribution under no announcement equals 200 when $k = 4$, and 250 when $k = 5$. ||

Example 2. Again let $v(G) = 10(G)^{0.5}$, $N = 5$, $G_{\min} = 200$, but $p(1) = 0.1$, $p(2) = 0.8$, $p(3) = 0.05$, $p(4) = 0.025$, $p(5) = 0.025$. Recall, $\underline{k} = 2$. Below we verify a ‘non-revealing’ equilibrium in which the planner suppresses all k and the players contribute a positive amount.

Suppose the players believe that in equilibrium the planner would always conceal the number of players. With no announcement, the updated beliefs need to be calculated. Given the priors,

$$\begin{aligned} \mu(1) &= \frac{(1/5) p(1)}{(1/5) p(1) + (2/5) p(2) + (3/5) p(3) + (4/5) p(4) + (1) p(5)} \\ &= 4/83. \end{aligned}$$

Similarly,

$$\mu(2) = 64/83, \quad \mu(3) = 6/83, \quad \mu(4) = 4/83, \quad \mu(5) = 5/83.$$

We now check that the equilibrium symmetric contribution under no announce-

ment is $G_{\min}/\underline{k} = 100$. The net expected utility from contributing $g(\emptyset) = 100$ is $(64/83) \cdot 10(200)^{0.5} + (6/83) \cdot 10(300)^{0.5} + (4/83) \cdot 10(400)^{0.5} + (5/83) \cdot 10(500)^{0.5} - 100 = 44.677$, whereas net expected utility from deviation to zero contribution equals $(6/83) \cdot 10(200)^{0.5} + (4/83) \cdot 10(300)^{0.5} + (5/83) \cdot 10(400)^{0.5} = 30.619$. Thus, a deviation to zero contribution is not beneficial. Similarly, it can be shown that other deviations are not beneficial either. For the planner to deviate to announce k is not beneficial for any k . ||

The equilibria of the game Γ are stated formally in the proposition below. Equilibrium characterizations can be found in the proof.

Proposition 2. *Suppose that the public good production technology is non-convex. The game Γ always has a revealing equilibrium in which the planner announces all k higher than or equal to the threshold number of players, \underline{k} given in Definition 1, and suppresses all $k < \underline{k}$. Each player makes the positive contribution $\tilde{g}(k) = G_{\min}/k$ if $k \geq \underline{k}$ is announced, a zero contribution $\tilde{g}(k) = 0$ if either $k < \underline{k}$ is announced or if no announcement is made.*

Any other equilibria of Γ must be one of the following two types:

Partially Revealing Equilibrium: The planner announces all k in an intermediate range $\mathcal{A} = \{\underline{k}, \dots, \bar{k}\}$ and suppresses all other k s. Each player contributes $\tilde{g}(k) = G_{\min}/k$ if $k \geq \underline{k}$ is announced, $\tilde{g}(k) = 0$ if $k < \underline{k}$ is announced. If the planner makes no announcement, each player makes the positive contribution $\tilde{g}(\emptyset) = G_{\min}/(\bar{k} + 1)$.

Non-revealing Equilibrium: The planner suppresses all $k \in \{1, \dots, N\}$. Each player contributes $\tilde{g}(k) = G_{\min}/k$ if $k \geq \underline{k}$ is announced, $\tilde{g}(k) = 0$ if $k < \underline{k}$ is announced. If no announcement is made, each player contributes $\tilde{g}(\emptyset) = G_{\min}/\hat{k}$ where $\hat{k} \leq \underline{k}$.

4.2. Intuitions

In all three types of equilibria, the supply of G_{\min} through symmetric contributions is guaranteed if at least a threshold number of players turn up.

In a revealing equilibrium the planner is not able to collect any funds by suppressing the number of players, because players interpret no announcement as evidence of sufficiently small turnouts so that positive contributions by the players to guarantee the threshold level G_{\min} are no longer individually rational. Therefore only sufficiently large turnouts will be announced, leaving small turnouts to be correctly inferred.

In a partially revealing equilibrium the planner suppresses large turnouts ($k > \bar{k}$) to counter pessimistic beliefs arising from the suppression of small turnouts ($k < \underline{k}$), and each player makes a symmetric positive contribution with total contribution matching, and often exceeding, the threshold level G_{\min} for large

turnouts, but failing to meet the threshold level for small turnouts. Intermediate turnouts ($\underline{k} \leq k \leq \bar{k}$) are announced, inducing the threshold level G_{\min} .

In a non-revealing equilibrium, the planner never announces the number of players. As stated in Proposition 2, corresponding individual contributions must generate at least the required sum G_{\min} whenever a minimum number \hat{k} of players turn up, where $\hat{k} \leq \underline{k}$ (recall, \underline{k} is the (minimum) number of players which, under complete information, will induce each player to make non-zero contributions equal to G_{\min}/\underline{k}). The number \hat{k} cannot exceed \underline{k} because, if it were to exceed, the planner would announce observations of numbers between \underline{k} and \hat{k} and increase total contribution to G_{\min} . This would upset the non-revealing equilibrium. Given $\hat{k} \leq \underline{k}$, note that it is optimal for the planner not to reveal any $k < \hat{k}$ for this would generate zero contributions, nor is it optimal to reveal any $k \geq \hat{k}$ for this can only reduce total contribution (to zero if $\hat{k} < \underline{k}$ and $k \in \{\hat{k}, \dots, \underline{k} - 1\}$ is announced, to G_{\min} if $k \geq \underline{k}$ is announced). To see why \hat{k} can be strictly lower than \underline{k} , suppose that the players put very high probability on the event $k = \underline{k}$. Anticipating with high probability a turnout of \underline{k} , the players would rather be cautious and be protective of a discrete drop in utility resulting from k being close to, yet smaller than, \underline{k} , as opposed to a small increase in utility from k slightly exceeding \underline{k} . Risk-aversion of the players (by strict concavity of $v(\cdot)$) would prompt each player to contribute the amount G_{\min}/\hat{k} which exceeds G_{\min}/\underline{k} , realizing the project for numbers of players even lower than \underline{k} , the minimum number required under complete information.

The types of equilibria in Proposition 2 are exhaustive. The only possible type of revealing equilibrium is the one stated in Proposition 2, given the tie-breaking Assumption 2 and ruling out the possibility of zero contributions equilibrium due to coordination failures following announcement of $k \geq \underline{k}$. We note that the planner would announce all $k \in \{1, \dots, N\}$ instead if we replace Assumption 2 by the opposite tie-breaking rule, but the equilibrium contributions will be no different. In Appendix A we show that in any partially revealing equilibrium the set of announced numbers must be an intermediate range of $\{1, 2, \dots, N\}$.

The following proposition compares the planner's payoffs in different types of equilibria.

Proposition 3. *The planner's payoffs or total contribution P^j in a type- j equilibrium of Γ are as follows:*

Revealing: $P^R = 0$ if $k < \underline{k}$, $P^R = G_{\min}$ if $k \geq \underline{k}$

Partially Revealing: $P^P = (k/(\bar{k} + 1))G_{\min}$ if $k < \underline{k}$ or $k > \bar{k}$, and $P^P = G_{\min}$ if $k \in \{\underline{k}, \dots, \bar{k}\}$

Non-revealing: $P^N = (k/\hat{k})G_{\min}$ where $\hat{k} \leq \underline{k}$.

Thus, total contribution is (weakly) largest in a non-revealing equilibrium, (weakly) lowest in the revealing equilibrium.

It is worth noting that while the planner would always prefer an equilibrium with greater concealment because total contribution increases, players' utilities do not necessarily improve.¹⁷

4.3. The role of non-convexity

A comparison of the non-convex case with the convex case may help understand how non-convexity makes non-revealing equilibrium an attractive outcome for the planner. In both cases, the complete information equilibrium resulting from revelation of k has each player contributing some fixed amount divided by the number of players (or else contributing zero). This amount is \bar{G} in the convex case, G_{\min} in the non-convex case. The difference in the results comes from the discontinuity induced by the threshold G_{\min} . In both cases, if a player cuts his contribution by ϵ , he causes a discrete drop in the probability that contributions sum to the respective amounts, \bar{G} and G_{\min} . In the convex case where the first-order conditions do apply and players' contributions change continuously with the beliefs, this does *not* cause a discrete payoff reduction. In particular, if the planner were to suppress in equilibrium at least two different k values, then a player's contribution under incomplete information must lie between the smallest and largest complete information contributions corresponding to these k values.

In contrast, in the non-convex case, *nothing* is provided if contributions drop below G_{\min} , causing a discrete drop in the players' payoffs. As a result, optimal contributions need not be continuous in beliefs. This can create a situation where a player's contribution under incomplete information lies (weakly) above all possible complete information contributions. Thus, *in the non-convex case, the planner can only benefit from the players' uncertainty concerning their number.* This feature has a nice parallel with a result by McAfee and McMillan (1987) and Matthews (1987) in the auction literature, that the seller of an indivisible (private) good holding a first-price sealed-bid auction will extract a higher expected price if the bidders are kept uninformed about the number of their competitors. Their intuition primarily relies on bidder *risk aversion*: the bidders face greater risks from revelation of information (of intense vs. weak competition) than from concealment, as a result risk-averse bidders bid less aggressively on average when the seller commits to a revelation policy. Likewise, in our setting under the non-revealing equilibrium risk-averse players try to protect themselves from the downside risks of no benefits due to insufficient total contribution by raising

¹⁷In fact, for $k < \underline{k}$ the players' ranking is the exact opposite of that of the planner: the revealing equilibrium, followed by the partially revealing equilibrium and, finally, the non-revealing equilibrium. The reason is straightforward — there are no benefits to the players from the public good when total contributions fall short of G_{\min} .

individual contributions well and above their maximal complete information (i.e. revealing equilibrium) levels.

5. Concluding remarks

The theoretical literature on fundraising by charities has recently focused on the strategic aspects of particular fundraising procedures with the fundraiser as a key player. In this paper we address the issue as to whether the fundraiser should announce or conceal the number of players in order to influence the players' contribution decisions. Our analysis considered a non-altruistic model of donations. The main intuitions behind our results should remain valid for additional motivations of donations as well, such as warm-glow (Andreoni, 1989) and prestige (Harbaugh, 1998).

We made several assumptions in our analysis. Quasi-linearity of preferences is a standard assumption in the related literature on public goods. The assumptions of symmetry of contributions equilibria and tie-breaking for the planner's announcement strategy, both serve to simplify the analysis with no qualitative impact on our results. Because we assumed symmetric potential contributors, the announcement of the information about their number makes the contribution game one of complete information. In this context, the assumption that the fundraiser's announcement is truthful though he can keep silent and conceal his private information is important. Credibility of various announcements by fundraisers is understandably a controversial issue. However, the insights developed in this paper should be complementary to parallel insights for the fundraising procedures analyzed by Andreoni (1998) and Vesterlund (1999). For instance, the same non-convex technology in our paper and in Andreoni (1998) yield somewhat different implications for the fundraiser's announcement strategies, albeit with respect to two different types of information; Vesterlund (1999) analyzes, compared to ours, less direct but more credible means of communication in that some players, better informed than others, try to signal a different type of information, viz. the quality of public good, through the choice of their gifts. Overall, our paper should be viewed as part of these other initiatives in rationalizing various strategic decisions by charities and contributors and their interdependence.

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Appendix A

Proof of Proposition 1. We will consider announcement strategies that are less than fully revealing and ask whether these can be supported in equilibrium.

Suppose that Γ has an equilibrium such that for some $\mathcal{A} \subset \{1, 2, \dots, N\}$, \mathcal{A} possibly empty and containing at most $N - 2$ elements, the planner reveals k if $k \in \mathcal{A}$ and suppresses k if $k \in \mathcal{C}$. By construction, \mathcal{C} is nonempty and has at least two elements. Now order the elements of \mathcal{C} from the lowest to the highest, k_1, k_2, \dots, \hat{k} . Given the beliefs $\mu(\cdot)$ consistent with the proposed strategy of the planner, the symmetric contribution of each player following no announcement, $g(\emptyset)$, is determined by the first-order condition:¹⁸

$$\mu(k_1) v'(k_1 g(\emptyset)) + \mu(k_2) v'(k_2 g(\emptyset)) + \dots + \mu(\hat{k}) v'(\hat{k} g(\emptyset)) = v'(\bar{G}).^{19}$$

Since $k_1 < k_2 < \dots < \hat{k}$, it must be that:

$$k_1 g(\emptyset) < \bar{G} \quad \text{and} \quad \hat{k} g(\emptyset) > \bar{G}$$

But if $k = k_1$, the planner prefers revealing k , so that each player will increase his contribution from $g(\emptyset)$ to $g^*(k_1)$ according to:

$$v'(k_1 g^*(k_1)) = v'(\bar{G})$$

and aggregate contribution equals $\bar{G} > k_1 g(\emptyset)$. This contradicts the hypothesis that the planner will suppress any $k \in \mathcal{C}$.

Now suppose Γ has an equilibrium in which $\mathcal{C} = \emptyset$ and let $\{\hat{\mu}(n)\}_{n=1}^N \gg 0$ be the players' strictly positive beliefs when they receive no announcement (which would be off the proposed equilibrium path). By sequential rationality, the symmetric equilibrium contributions \hat{g} will satisfy the first-order condition:

$$\hat{\mu}(1) v'(\hat{g}) + \dots + \hat{\mu}(N) v'(N\hat{g}) = v'(\bar{G}) = 1.$$

Then it must be that $N\hat{g} > \bar{G}$, which implies that the planner will deviate to suppress $k = N$. Thus, $\mathcal{C} \neq \emptyset$ in equilibrium.

We already proved that \mathcal{C} cannot be empty, nor can it have two or more elements, which imply that \mathcal{C} is a singleton. We now claim that $\mathcal{C} = \{N\}$. To see this, suppose on the contrary that there is an equilibrium in which $\mathcal{C} = \{n\}$ and $n < N$. Then, if no announcement is received, by Bayes' rule $\mu(n) = 1$ and

¹⁸The first-order principle applies because of the convexity of the production technology.

¹⁹Our Assumption 1 that players always play a *symmetric* equilibrium, whenever they need to make a choice between a symmetric equilibrium and any asymmetric equilibrium with the same total contribution, is rather harmless. Here, what is important is that the *expected* marginal benefit (expectation taken over different levels of public goods corresponding to different number of players) of a player equals his marginal benefit at the public good level \bar{G} , which, it can be checked, will also be true for any asymmetric contribution equilibrium.

$\mu(k) = 0$ for $k \neq n$, and each player will contribute $g^*(n)$ which exceeds $g^*(k)$ for $k > n$. Given this, the planner will deviate to $a(k) = \emptyset$ if $k > n$ which upsets the equilibrium.

Thus, the only possibility left is $a(k) = k$ if $k \in \{1, \dots, N - 1\}$ and $a(N) = \emptyset$, which we argue is the unique equilibrium strategy for the planner. The equilibrium beliefs following no announcement will be $\mu(N) = 1$ and $\mu(n) = 0$ for $n \neq N$. The players' contribution strategies will be as stated in the proposition. Although total contribution will be exactly the same, \bar{G} , whether the planner announces $k = N$ or not, by the (tie-breaking) Assumption 2 the planner will not announce $k = N$. Q.E.D.

Proof of Proposition 2. To show that a revealing equilibrium *always* exists is straightforward: Given the players' contribution strategies as specified, the planner can do no better than revealing all $k \geq \bar{k}$ and suppressing all $k < \bar{k}$. Also, given the planner's announcement strategy, the players' contribution strategies form a symmetric Bayesian Nash equilibrium. The players' beliefs about k following the announcements (or no announcement) by the planner are correct in equilibrium.

To derive plausible conditions under which a partially revealing equilibrium exists, we first establish the following claim.

Claim 1. In any (partially revealing) equilibrium with $\mathcal{C} \neq \emptyset$, $\mathcal{A} \neq \emptyset$ and $\tilde{g}(\emptyset) > 0$, the set of announced k s is of the form $\mathcal{A} = \{\bar{k}, \dots, \bar{k}\}$ where $1 < \bar{k} \leq \bar{k} < N$ and $(\bar{k} + 1)\tilde{g}(\emptyset) = G_{\min}$.

Proof. Any partially revealing equilibrium will have two continuation games according to whether k is announced, or k is suppressed. In any continuation equilibria following the announcement of k , the players' optimal contribution strategies are straightforward: $\tilde{g}(k) = 0$ if $k < \bar{k}$; $\tilde{g}(k) = G_{\min}/k$ if $k \geq \bar{k}$.

We now show that in any equilibrium with $\mathcal{C} \neq \emptyset$, $\mathcal{A} \neq \emptyset$ and $\tilde{g}(\emptyset) > 0$, there exists $\bar{k} < N$ such that $(\bar{k} + 1)\tilde{g}(\emptyset) = G_{\min}$. Note that $k\tilde{g}(\emptyset) < G_{\min}$ for all $k = 1, \dots, N$ would imply $\tilde{g}(\emptyset) = 0$, contradicting the assumption $\tilde{g}(\emptyset) > 0$. Thus, there exists k such that $k\tilde{g}(\emptyset) \geq G_{\min}$. Let $\bar{k} + 1$ be the smallest such k . We claim that in any equilibrium with $\tilde{g}(\emptyset) > 0$, $(\bar{k} + 1)\tilde{g}(\emptyset) = G_{\min}$. Suppose on the contrary that $(\bar{k} + 1)\tilde{g}(\emptyset) > G_{\min}$, thus, $r\tilde{g}(\emptyset) < G_{\min}$ for $r < \bar{k} + 1$. Given the strategy $\tilde{g}(\emptyset) > 0$, under no announcement the expected payoff of a player is written as:

$$V(\emptyset) = \sum_{k=1}^{\bar{k}-1} \mu(k) v(0) + \sum_{k=\bar{k}+1}^N \mu(k) v(k\tilde{g}(\emptyset)) + m_i - \tilde{g}(\emptyset)$$

where $\mu(k)$ s are derived using Bayes' rule and $k\tilde{g}(\emptyset) < G_{\min}$ for $k < \bar{k} + 1$, thus, $v(k\tilde{g}(\emptyset)) = v(0)$. Since $\sum_{k \geq \bar{k}+1} \mu(k) v'(k\tilde{g}(\emptyset)) < 1 (= v'(G))$ and $(\bar{k} + 1)\tilde{g}(\emptyset) > G_{\min}$, any player can unilaterally deviate to $\tilde{g}(\emptyset) - \epsilon$, ϵ arbitrarily small and positive, to increase his individual payoff above $V(\emptyset)$. This contradicts the

assumption that $\tilde{g}(\emptyset)$ is an equilibrium strategy. Thus, $(\bar{k} + 1) \tilde{g}(\emptyset) = G_{\min}$. We next show that in any partially revealing equilibrium where $\mathcal{A} \neq \emptyset$, the planner's strategy must generate a set of announced k s of the form $\mathcal{A} = \{\underline{k}, \dots, \bar{k}\}$ where $1 < \underline{k} \leq \bar{k} < N$. It is not optimal to announce $k \geq \bar{k} + 1$ for this can only decrease total contribution from $k\tilde{g}(\emptyset)$ to G_{\min} . Suppressing any $k \in \mathcal{A} = \{\underline{k}, \dots, \bar{k}\}$ yields contributions $k\tilde{g}(\emptyset)$ which is strictly less than G_{\min} , while announcing $k < \underline{k}$ yields zero contribution instead of the positive amount $k\tilde{g}(\emptyset)$ ($< G_{\min}$). Claim 2 shows that $\bar{k} = N$ is not compatible with $\tilde{g}(\emptyset) > 0$. Finally, $\underline{k} \leq \bar{k}$ ensures that \mathcal{A} is nonempty. \parallel

The following claim is easy to check.

Claim 2. In any equilibrium, $\tilde{g}(\emptyset) = 0$ if and only if $\tilde{a}(k) = k$ for all $k \geq \underline{k}$.

Armed with the result in Claim 1 concerning the structure of the set \mathcal{A} whenever $\mathcal{A} \neq \emptyset$, $\mathcal{C} \neq \emptyset$ and $\tilde{g}(\emptyset) > 0$, we focus below on the conditions for a partially revealing equilibrium where symmetric contributions are positive under no announcement. Consider first deviations to $g_i < \tilde{g}(\emptyset)$ by player i , given the planner's announcement strategy, the corresponding beliefs $\mu(\cdot)$ and the other players' contributions $\tilde{g}(\emptyset)$. If player i deviates as above, it will take at least $\bar{k} + 1$ other players, each contributing $\tilde{g}(\emptyset)$, to meet the threshold contribution G_{\min} . Clearly the best deviation strategy among all $g_i < \tilde{g}(\emptyset)$ is $g_i = 0$, which yields the expected payoff:

$$V^0 = \left(\sum_{k=1}^{\bar{k}-1} \mu(k) + \mu(\bar{k} + 1) \right) v(0) + \sum_{k=\bar{k}+1}^{N-1} \mu(k + 1) v(k\tilde{g}(\emptyset)) + m_i.$$

Thus

$$V(\emptyset) \geq V^0 \tag{A.1}$$

must hold for the prescribed strategies to constitute an equilibrium. Consider now a deviation to $g_i > \tilde{g}(\emptyset)$. Such a deviation, if not large enough, increases the potential size of the public good without affecting the probability of its positive supply. But this would not be a beneficial deviation, as shown by the marginal evaluation in the proof of Claim 1. If the deviation to $g_i > \tilde{g}(\emptyset)$ is large enough, it can increase both the probability and the potential size of a positive supply. For instance, deviating to $g^s = G_{\min} - (s - 1) \tilde{g}(\emptyset)$ for $s < \bar{k}$ will ensure the supply of G_{\min} with s players, including the deviator. This deviation yields the expected payoff:

$$V^s = \sum_{k=1}^{s-1} \mu(k) v(0) + \sum_{k \in \mathcal{C}, k \geq s} \mu(k) v((k - 1) \tilde{g}(\emptyset) + g^s) + m_i - g^s.$$

In equilibrium, we require:

$$V(\emptyset) \geq V^s, \text{ for all } s < \underline{k}. \tag{A.2}$$

The planner’s strategy of announcing all $k \in \{\underline{k}, \dots, \bar{k}\}$ and suppressing k otherwise is clearly optimal given the above strategy of the players. As for the determination of \underline{k} and the associated contribution level $\tilde{g}(\emptyset)$, there are only a finite number of choices for \bar{k} . An exhaustive verification of conditions (A.1) and (A.2) will establish whether a particular pair $(\bar{k}, \tilde{g}(\emptyset))$, where $\tilde{g}(\emptyset) = G_{\min}/(\bar{k} + 1)$ and $\bar{k} < N$, is compatible with (A.1) and (A.2).

We consider below a non-revealing equilibrium in which the planner suppresses all k s.

Claim 3. In any equilibrium in which $\mathcal{A} = \emptyset$ (thus, $\mathcal{C} = \{1, \dots, N\}$), $\tilde{g}(\emptyset) > 0$ and there exists $\hat{k} \leq \underline{k}$ such that $\hat{k}\tilde{g}(\emptyset) = G_{\min}$.

Proof. $\tilde{g}(\emptyset) = 0$ is clearly not compatible with $\mathcal{A} = \emptyset$; the planner would announce $k \geq \underline{k}$ and collect G_{\min} . Thus, $\tilde{g}(\emptyset) > 0$ in any equilibrium in which $\mathcal{A} = \emptyset$. The arguments in the proof of Claim 1 (that show the existence of $\bar{k} + 1$) can be applied to show that there must exist \hat{k} such that $\hat{k}\tilde{g}(\emptyset) = G_{\min}$. If $\hat{k} > \underline{k}$, which means $\underline{k}\tilde{g}(\emptyset) < G_{\min}$, the planner would announce \underline{k} and collect G_{\min} , contradicting the assumption that $\mathcal{A} = \emptyset$. ||

Given the strategy $a(k) = \emptyset$ for all $k \in \{1, \dots, N\} = \mathcal{C}$, we have $\mu(k) = p(k)$ for all k . Thus, $\tilde{g}(\emptyset) > 0$ will be part of a non-revealing equilibrium if:

$$\begin{aligned} \hat{V}(\emptyset) &\equiv \sum_{k=1}^{\hat{k}-1} \mu(k) v(0) + \sum_{k=\hat{k}}^N \mu(k) v(k\tilde{g}(\emptyset)) + m_i - \tilde{g}(\emptyset) \\ &\geq \sum_{k=1}^{\hat{k}} \mu(k) v(0) + \sum_{k=\hat{k}+1}^N \mu(k) v((k-1)\tilde{g}(\emptyset)) + m_i \end{aligned} \tag{A.3}$$

(which is the analogue of (A.1)), and, for all $s < \hat{k}$:

$$\hat{V}(\emptyset) \geq \sum_{k=1}^{s-1} \mu(k) v(0) + \sum_{k \geq s, k \in \mathcal{C}} \mu(k) v((k-1)\tilde{g}(\emptyset) + \hat{g}^s) + m_i - \hat{g}^s \tag{A.4}$$

where $\hat{g}^s = G_{\min} - (s - 1)\tilde{g}(\emptyset)$. Condition (A.4) is the analogue of (A.2) in a non-revealing equilibrium: it states that an individual deviation to \hat{g}^s to meet the threshold G_{\min} for $s < \hat{k}$ players is not beneficial. The verification of the conditions (A.3) and (A.4) to find if an equilibrium pair of $(\hat{k}, \tilde{g}(\emptyset))$ exists, are straightforward.

If one excludes the ‘bad’ equilibria involving zero contributions for announcements of $k \geq \underline{k}$, the remaining equilibria of the game Γ are clearly limited to the three types mentioned in the proposition. In a revealing equilibrium k is announced if $k \geq \underline{k}$, but otherwise suppressed. In a non-revealing equilibrium all k values are suppressed. By Claim 1, in a partially revealing equilibrium where the planner

suppresses some k values and announces the rest must involve announcements from an intermediate range $\mathcal{A} = \{\underline{k}, \dots, \bar{k}\}$. In particular, the set of announced k s, \mathcal{A} , cannot be the union of sets of numbers with gaps in between containing positive integer(s), nor can it be located at one of the two tails of the set of numbers, $\{1, \dots, N\}$. The list of equilibrium types is therefore exhaustive. Q.E.D.

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