THE NEWSVENDOR PROBLEM WITH MULTIPLE INPUTS AND ENVIRONMENT SENSITIVE CUSTOMERS

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By
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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Motivated by the global aim and trends to reduce carbon emissions, in this thesis we investigate the effects of carbon sensitivity on the operations management in the context of inventory management. We assume the newsvendor setting under multiple substitutable inputs with varying carbon emission levels, and carbon sensitive random demand. The Cobb-Douglas production function is used which provides a link between the production quantity and the inputs. Our goal is to determine the optimal production quantity under two different supply chain models. In the decentralized model, we consider an independent manufacturer and a retailer, where the retailer orders Q units to the manufacturer and the manufacturer produces these items in such a way that he minimizes his production cost. In the integrated production or the centralized model, the manufacturer and the retailer act as a centralized system and the aim is to find the production quantity that maximizes the expected profit of the integrated system. Exact expressions for the expected profits of both models are derived and analytical results regarding the optimal solutions are presented. Numerical results are also provided to illustrate the effects of the system parameters and carbon sensitivity levels.

Keywords: Newsvendor, Environment Sensitive Customers, Carbon Emissions, Inventory Management, Carbon Sensitive Demand, Multiple Inputs, Cobb-Douglas, Operations Management.

ÖZET

BİRDEN FAZLA GİRDİNİN VE ÇEVREYE DUYARLI MÜŞTERİLERİN OLDUĞU GAZETECİ ÇOCUK PROBLEMİ

Nazlı Sönmez
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Bu çalışmada, dünya çapındaki karbon emisyonunu düşürme eğilimi göz önünde bulundurularak, karbona duyarlılığın operasyon yönetimine etkisi incelenmiştir. Karbon emisyon miktarları değişken olan ve ikame edilebilen birden çok girdinin kullanıldığı, karbona duyarlı ve rassal bir talebin olduğu gazeteci çocuk modeli ele alınmıştır. Üretim miktarı, Cobb-Douglas üretim fonksiyonu aracılığıyla ilişkilendirilmiştir. Çalışmanın amacı, iki farklı model yapısının altındaki amaç fonksiyonlarını en iyileyen üretim miktarlarını bulmaktır. Merkezi olmayan modelde, satıcıdan bağımsız bir üretici olduğu kabul edilmiştir. Bu modelde, satıcı üreticiye Q miktarında ürün sipariş etmekte ve bağımsız üretici kendi üretim maliyetini en aza indirgeyecek girdi dağılımına karar vermektedir. Toplam üretim modeli ya da merkezi modelde ise üretici ve satıcı merkezi bir sistem olarak hareket etmekte ve bütün sistemin beklenen karını en yüksek düzeye çıkartan üretim miktarını belirlemektedir. Her iki sistem için beklenen kar miktarını belirten açık ifadeler türetilmiş, en iyi çözümler hakkında analitik sonuçlar sunulmuştur. Sistem parametrelerinin ve karbona duyarlılık seviyesinin etkilerini göstermek için sayısal örnekler verilmiştir.

Anahtar sözcükler: Gazeteci Çocuk Problemi, Çevreye Duyarlı Müşteriler, Envanter Yönetimi, Karbon Emisyonu, Karbona Duyarlı Talep, Birden Fazla Girdi, Cobb-Douglas, Operasyon Yönetimi.

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Chapter 1

Introduction

Global warming is one of the critical problems that the world is currently facing. Greenhouse gas emission levels due to human activities have continuously increased since pre-industrial era and among these gases the carbon dioxide has the most significant effect on global warming [1]. The report of the United States Environmental Protection Agency states that the main sources of greenhouse gas emissions in the United States are electricity production, transportation, industry, commercial and residential, agriculture, land use and forestry [2]. The concentration of carbon dioxide in the earth has recently became 403.26 ppm (parts per million) and for the past decade $2005 - 2014$, the average annual increase is known as 2.1 ppm per year while it was 1.9 ppm per year in prior decade $1995 - 2004$. Therefore, it can be said that the concentrations of carbon dioxide in the atmosphere follows an increasing trend at an accelerating rate from decade to decade. This trend alert us to the possible effects of the global warming like more health related illnesses or diseases, increased risk of drought, fire and floods, higher temperatures, rising seas, wildlife at risk, economic losses [3].

Due to these possible disastrous effects, environment protection has recently become an important factor in human decision making. Environmental consciousness therefore become a commonly encountered concept, which is defined as the inclination to behave with pro-environmental intent in a general sense [4]. Chua et al. stated that the consumers who have environmental and ecological concerns have been described in several ways such as environmentally-sensitive, environmentally conscious or environmentalists [5]. There are different theories which explain the motivation behind the pro-environmental behavior under the disciplines of psychology and sociology. These can be briefly stated as the planned behavior theories, value theories, theories of altruistic behavior and the theories which assume environmental consciousness as a worldview or paradigm.

Environmentally conscious consumers, also known as carbon sensitive consumers, aim to reduce the carbon footprints of the products or activities. Even if there are different definitions for carbon footprint, it can be defined as a measure of the total amount of carbon dioxide emission that is directly or indirectly caused by everyday activities [6]. It includes for instance the carbon emission cause by driving a car as well as the carbon emitted while producing a good that we purchase. In order to reduce the carbon emissions, environmentally conscious consumers adopt environmentally friendly alternatives for their everyday activities such as buying green cars and preferring products which has low carbon footprint.

Complementary to sociology and psychology disciplines, consumer behavior research also focuses on pro-environmental consumer behavior. Different empirical studies are undertaken to understand the characteristics of the consumers who prefer green goods. These studies generally state that the environmentally conscious consumers are likely to be young (pre-middle age adult), well-educated and with high socioeconomic status [7, 8]. Another stream of empirical studies done to understand the consumer behavior aim to estimate the importance of environmental sensitivity in purchasing behavior. They indicate that price and quality trade-offs can be obstacles for people who have propensity to reduce his/her carbon footprint. The studies under various disciplines disclose that environmental consciousness or carbon sensitivity has became an important factor in determining consumer behavior.

The high carbon emission level in the atmosphere and its possible disastrous effects do not only affect the customer behavior, they are also high on the agenda of many countries. The Kyoto Protocol signed in 1997 by 196 membership countries was the most important attempt which aims to reduce the carbon emission levels all over the world.It was an international consensus related to the United Nations Framework Convention on Climate Change which commits all parties to reduce greenhouse gas emissions based on binding emission reduction targets.The Kyoto Protocol has three main mechanisms which are emission trading, the clean development mechanism (CDM) and joint implementation to reach targeted emission levels [9].

The emission trading mechanism enables countries which have carbon emission levels to spare to sell their excess capacity to the countries which need extra credits. Therefore, by this mechanism the carbon is tracked and traded like other commodities and a new concept of "carbon market" was created [10].The other mechanism: Clean Development Mechanism enables countries which ratify the Kyoto Protocol to do an emission reduction project on developing countries. As a consequence of these projects, certified emission reduction (CER) credits started to be sold, where one CRE credit is equivalent to one tonne of carbon dioxide, can be earned. The Joint Implementation mechanism provides the parties, which ratified Kyoto Protocol, a flexible and cost-efficient way of satisfying a part of their Kyoto commitments. It is a project-based mechanism which provides opportunity of doing projects between two countries which are the members of the Kyoto Protocol. It enables the country who support an emission-reduction or emission removal project in another country to earn emission reduction units (ERUs) where each emission reduction unit is equivalent to one tonne of carbon dioxide. By this mechanism, the host country can benefit from foreign investment and technology transfer while the investor finds an alternative way of fulfilling its commitment related to carbon emission levels [10].

These macro level approaches also emphasize that there is a growing attention to the carbon emission levels and global trend of environmental consciousness. It can be understood that the governments, companies, customers struggle to reduce their carbon footprint.

Motivated by the global aim and trends to reduce carbon emissions, in this thesis we investigate the effects of carbon sensitivity on operations management. In particular we consider the well known newsvendor inventory setting under the random product demand. To investigate the impact of carbon emitting inputs on the final product,we assume that there are multiple substitutable inputs that make up a product. We considered the well known Cobb-Douglas production function to model the link between the product and the inputs. Hence our basic model is a newsvendor model where the random demand is effected by the carbon sensitivity of the customers and the product is composed of multiple inputs which have different carbon emission coefficients. Our goal is to determine the optimal production quantities. To this end we considered two models regarding the underlying supply chain. In the first model, we consider an independent manufacturer and a retailer, where the retailer orders Q units to the manufacturer and the manufacturer produces these items in such a way that he minimizes his production cost. The retailer then sells these items to the end customer. We will refer to his model as the *independent manufacturer* or the *decentralized* model. In the second model that we consider, the manufacturer and the retailer act as a centralized system so that the order quantity is obtained so as to maximize the objective function of the integrated system. We refer to this model as *integrated* model or the *centralized* model.

In the independent manufacturer model, the retailer's problem is formulated as a newsvendor model with multiple inputs and a random, carbon sensitive demand. In the setting of the problem, the retailer orders Q units of product to the manufacturer who applies a cost-plus approach and finds the optimal input mixture which will minimize his own production cost with a Cobb-Douglas production function. Since the final product has inputs which differ in their emission levels, the production process will result in a carbon emission level which depends on used input mixture. The resulting carbon emission level of the product can be considered as the carbon footprint of the product. As we will present the details in the following chapters, this emission level will have an impact on the demand function of the customer who are assumed to be sensitive to the carbon footprint of the products that they purchase. For this model, we derive the optimal production policy and show that under some conditions the retailer's problem has a unique solution.

In the integrated model, we consider a supply chain where the decisions of the manufacturer and the retailer are controlled centrally. In this setting the optimal production quantity is determined so as to maximize the expected profit of the supply chain. The objective function is derived analytically and the first order conditions for the optimal solution are also derived.

In the numerical studies, we present a sensitivity analysis for the two that are considered in our work under four different parameter sets. Focusing on customers with different carbon emission sensitivity levels provides us the opportunity of investigating the effect of the carbon emission sensitivity on the inventory management. The real data from the work of Hatirli et.al. [11] related to tomato production is also used to analyze the optimal order quantity level under the retailer's problem with an independent manufacturer. In this example, five inputs are used in the tomato production which are fertilizer, chemical, labor, machinery and water for irrigation. The sensitivity analyses indicate that the integrated problem results in a higher optimal order quantity with a higher corresponding expected profit under the same conditions for each carbon sensitivity parameter under same input cost, carbon emission parameters. In addition, in the integrated model, we clearly see a decrease in the usage of the high cost-high carbon emitted input when the customers are carbon sensitive by investigating the change in the input allocation ratios when the price or the carbon emission parameter of the inputs change. On the contrary, in the independent manufacturer problem, the allocation of inputs are determined by only considering the price of the inputs. The real-life application of the agricultural data showed that the effect of the change in the ratio of shortage cost and selling price on the optimal order quantity is higher when the demand uncertainty increases under each customer sensitivity level. The effect of demand variation on the input allocation is also evaluated and it is concluded that the increase in optimal order quantity and input allocation are higher when demand uncertainty is high.

The rest of the thesis is organized as follows: In Chapter 2, we review the literature of the classical newsvendor problem with its extensions, basic demand models which include sensitivity to a product attribute, empirical studies related to carbon sensitive consumer behavior. In Chapter 3, we present a detailed review on the structure of the classical newsvendor problem, newsvendor problems with multiple inputs with the Cobb-Douglas production function and our carbon sensitive demand function. In Chapter 4, we cover independent manufacturer model which is the retailer's problem with an independent manufacturer and provide analysis of the problem. In Chapter 5, the integrated model of the retailer and the manufacturer is introduced in detail. In Chapter 6, numerical analysis is done by focusing on sensitiveness of the problems to the parameters of the problem. The final chapter: Chapter 7 includes the concluding remarks.

Chapter 2

Literature Review

In this chapter, we review the literature related to our work. We summarize the existing works under four subsections including the results for (i) the classical newsvendor model and its extensions, and (ii) the demand models which represent different customer behaviors, (iii) empirical works that investigate the environment sensitive customer behavior and (iv) carbon sensitive inventory models.

2.1 The Classical Newsvendor Problem and its Extensions

The classical newsvendor problem has been widely studied in the inventory management literature since it is one of the earliest models and provides a building block for other extensions. This model also applies to many real life situations and is used in decision making process of the fashion, sporting industries, manufacturing and service industry [12].

In the classical newsvendor problem, there is a single product subject to a probabilistic demand with a known distribution. The retailer aims to satisfy the demand of the customers for a single period and it regulates its inventory by replenishment at the beginning of the period. The suppliers apply a unit purchasing cost to the retailer in the replenishment process. In the model, there is a constant revenue per unit sold. If the demand of the period exceeds the inventory level, the retailer incur a cost called shortage cost per each unit shortage. Any leftover units at the end of the period are hold as excess inventory and associated holding cost per unit is incurred per each excess inventory at the end of the period. The ultimate goal of the classical newsvendor problem under this setting is to determine the optimal order quantity which maximizes the expected profit function of the retailer.

Khouja [12] classifies the extensions of the classical newsvendor problem into different categories in his review. In particular, extensions to various objective functions, pricing policies, discounting structures, to multiple products with constraints or substitution, to multiple locations qand to different demand functions are discussed. Since our models involve multiple inputs which with a Cobb-Douglas production function, we would like to focus on the literature with similar properties. However, to our knowledge, there have been no study that consider multiple inputs in the newsvendor setting with a production function. Therefore, we consider other newsvendor extensions which can be associated with our models in terms of model setting in this part. These extensions involve multiple products or resources with a budget, capacity or resource constraint and multiple products with substitution.

Hadley and Whitin [13] who are known for their classical model of what is known today as the newsvendor problem, provide an extension of their original model involving the multiple products with constraints. They use Lagrange multipliers, Leibniz Rule and dynamic programming approaches to solve their,however, they have difficulties when the number of the products is large.

The work of Lau and Lau [14] deals with multiple products, and several resource constraints under a single-period. The study is motivated from a real life problem. The authors notice that by providing an extension of the newsvendor problem involving multiple products with multiple resource constraints they successfully represent many large fresh food businesses which directly sell to the consumers like the bakery firms that sell different kinds of products through various company-owned outlets in shopping malls, subway stations. The restaurant chains which are supplied from a single center and provide fresh food also encounter with the same problem. At the beginning of each period or workday in these cases, they need to determine the optimal production quantity level for each product at their center facility and the optimal quantity to send each shop. Their solution algorithm is developed based on determination of the Lagrange multiplier for each constraint that satisfy the necessary condition. However, the limitation of the study is that the Lagrange multipliers for each constraint shows whether a resource should be expanded but how much it can be expanded can not be explained. Therefore as an extension, the authors suggests that the resources can be modeled as nonlinear cost components in the objective function instead of constraints.

The work of Moon and Silver [15] suggests a multiple product newsvendor problem subject to a budget constraint on the total value of the replenishment quantities. In addition to the cost parameters of the classical newsvendor problem, they consider the fixed costs for non-zero replenishment. The solution is obtained by dynamic programming where two different cases are considered with known demand distribution and the distribution free approach where only the first two moments of the distributions are known. Besides dynamic programming, simple and efficient heuristic algorithms are provided to represent more realistic sized problems.

Erlebacher [16] develops another extension of the newsvendor problem involving multiple products newsvendor with one capacity constraint. Both optimal and heuristic solutions for the problem is found. He begins by considering two special cases. In the first case, it is assumed that the cost structure is the same for all considered products and for the second case, uniform probability density function for the demand distribution of each product is taken. After proving the optimality of the order quantities under these two special cases, he develops heuristics for some general probability distributions.

Abdel-Malek et al. [17] develop exact, approximate and iterative models to solve the multiple products newsvendor problem with budget constraint. The developed models provide exact solutions to the problem when the demand has uniform distribution and near optimal solution when the demand has a distribution other than the uniform. For the cases where the demand is not uniformly distributed, an iterative method is provided, with an estimate for the error at each iteration.

The paper of Vairaktarakis [18] presents an alternative approach for the multiple item newsboy model with a budget constraint and demand uncertainty. As in the above papers, the traditional approach to describing uncertainty is by means of probability density functions. In this paper they present an alternative approach and use deterministic optimization models. Two types of scenarios, interval and discrete, are used to describe the demand characteristics. For the interval case, lower and upper bounds for the uncertain demand of each item, while for the discrete scenarios, a set of likely demand outcomes for each item are assumed available. Using these two scenarios, they develop several minimax regret formulations for the newsvendor problem with multiple products under a budget constraint and present a robust newsvendor model for uncertain demand. Their approach is found very suitable for the industries which plan to launch new products.

Regrading the models with substitutable demands, Khouja et al [19] formulate a two product newsvendor problem with substitutability. They obtain the upper and lower bounds for the optimal order quantity level of the each product. In addition, Monte Carlo simulation method is used to determine the exact optimal solution of the problem. Parlar and Goyar [20] also deal with a two product newsvendor problem in which in case of a shortage, the products can substitute each other. They assume the parameters salvage value and lost sales cost as zero. They derive optimality conditions for the problem under an assumption that the substitution occurs by fixed probabilities.

The extensions of the newsvendor model involving multiple products with substitution also focus on the competition between retailers. Therefore, there are studies that consider the cases in which the customers substitute from the competitor retailer. The work of Lippman and McCardle [21] is one of the examples under this category. According to their model, the aggregate demand, which is independent from the number of the firms, is split into retailers according to a rule known to all, each firm's strategy is the order quantity they choose and there is no price competition. After the initial allocation, the excess demand for each firm is reallocated to other firms. For the duopoly case in which two firms compete, it is found that competition never leads to decrease in the total industry inventory. Moreover, for the multiple case, it is shown that under herd behavior in which all excess demand goes to one firm, the expected industry profit converges to zero as the number of the firms increases.

Another related type of models is the component commonality models where multiple products and multiple resources exist such as assemble-to-order systems.

Baker et al. [22] deal with two product, two level model to understand the effect of commonality on the safety stock of the components. The link between the service level and safety stock provides a constraint for the problem. As a result, the model shows that commonality make achieving the target service level with less amount of safety stock easier. In addition, under the component commonality, the models results that optimal safety stock for unique parts increases in contrast to other parts. The same result is also proved by Sauer [23] who suggests a newsvendor model with commonality in multiple products.

In the work of Harrison and Mieghem [24], there is a retailer who sells multiple inputs that are produced by using common resources. Therefore, instead of commonality in the components, now they have commonality in the resources and the authors aim to determine the optimal investment by their multi-dimensional newsvendor model under uncertain demand for products. Their analysis also focus on the difference in optimal investment strategies under deterministic and stochastic models.

The work of Mohebbi and Choobineh [25] investigates the effect of component commonality on an assemble-to-order system with demand and supply variability by being motivated from the fact that most manufacturing systems face encounter with different uncertainties in product demand, lead times, performance of the production, resulted quality. The environment is simulated and it is found that commonality in components becomes beneficial for the companies when uncertainties in both supply and demand occurs. ANOVA results also support the outcome.

Johnnson and Silver [26] also focus on common component inventory problem in which there are multiple end items which requires assembly of the different components and some of them are common whereas others are unique to specific products. In the model, components are ordered at the beginning of the period when the demand, which are assumed to be normally distributed, for products are not known. However, the assembly process can be done after realization of the demand. Since the budget is limited, the problem has a budget constraint and it aim to maximize the expected total sold end products and finds the optimal allocation of the budget in order to achieve its aim. For a simple commonality structure, the optimal allocation of the budget found. After that, a heuristic approach is designed and it is shown that the heuristic gives successful results under different parameters. After this work, the authors extended their work [27] to find the optimal allocation under same conditions for the cases when number of the products and components are large. They provide two-stage stochastic programming models to solve the problem. Since the problem is very difficult to solve, heuristics and bounding methods which give successful outcomes.

2.2 Demand Models which Represent Different Customer Behaviors

In this part we consider the literature related to demand models where the customers may be sensitive to a specific feature of the product like quality, durability and price. To our knowledge, there is no demand model which explicitly indicates how the customer demand is affected by the carbon sensitivity level of the customers. Therefore we benefit from the models which involve price or quality sensitivity to model the customer sensitivity to carbon emissions.

According to Kotler and Armstrong [28], the purchasing decision of the customers consists of five sequential stages described as problem recognition, search of information where they investigate the brands or products which are appropriate for their need, evaluation of alternatives, purchase decision and post purchase behavior. After an extensive information search, at the stage of evaluation of the alternatives, the products are evaluated by the customers according to various product attributes like price, quality, durability, brand equity and carbon emission levels as in our case. Therefore, this stage is the one where we can determine the demand function of the customers by focusing on the attributes they give importance. After the evaluation of the alternatives, purchase decision occurs which is generally based on choosing the product which provides maximum utility. After the purchasing decision, the post purchase behavior stage refers to activities which may result from the feelings after purchasing like satisfaction or displeasure. It can be exemplified as advising the product to others, returning the product and applying technical service.

According to the review paper of Tang [29], commonly used demand models can be listed as exogenous demand models, constant-utility attraction models, constant-utility choice models and random utility multinomial models. These different demand models consider price, location, quality and other various attributes related to the product as factors which determine the demand function. Exogenous demand models generally consider the attributes of the products like price as an exogenous variable and by determining a parameter to denote the sensitivity level of the customers to the attribute and if there is competition, by considering how the customers are sensitive to the difference in the level of attributes, the model forms a demand function called exogenous demand function. The second category, constant utility attraction models develop a deterministic utility function that indicates the utility that the customer derives from buying the product. In the models under this category, the utility function is affected from different product attributes like price, quality, functionality, service attribute. In the constant utility choice models, the multiple attributes of each product is represented by a vector where each customer has his/her own ideal vector of multiple attributes. By benefiting from a function called distance, the gap between the attributes of the product and the ideal vector of the customers is determined and it is used as a factor that affect the utility of the customers derived by purchasing a product. After determining the ideal points for the customers by comparing the different utility functions, the demand function for each product is determined. The last category, random utility multinomial models, assumes that the utility for each customer which is derived from buying a specific product is a function which consists of a deterministic and a stochastic component. By using the utility functions, the choice probabilities are obtained and based on these choice probabilities, the demand function for each product is determined.

There are different extensions of the demand models categorized above. However, the demand structures given in the article of Petruzzi and Dada [30] as additive and multiplicative demand cases, which are formed by considering effects of price on demand, inspired us to determine the structure of our demand function. In the additive demand case, the demand function is determined as the summation of a function called $y(p)$ which is a decreasing function that captures the dependency between demand and price and a random error term. In the multiplicative case, the demand is formulated as multiplication of the same function and a random error term. The structure of the function $y(p)$ which represents the relationship between price and demand is formulated in different ways. In the first one, it represents a linear demand curve which is common in the literature related to economics and for the second case, the function represents an iso-elastic demand curve. The additive (2.1) and multiplicative demand models (2.2) of Petruzzi and Dada are given as follows

$$D(p, \epsilon) = y(p) + \epsilon \tag{2.1}$$

where y(p)=$a - bp$(a>0, b>0) in the additive case

$$D(p, \epsilon) = y(p)\epsilon \tag{2.2}$$

where y(p)=ap^{-b}(a>0, b>1) in the multiplicative case

In addition to general demand models which use product attributes like price, quality, functionality in determining demand functions under different settings, the following two models focus on environmentally conscious and price sensitive customers.

One of the recent publications of Giri and Bardhan [31] develop a demand model under a two-echelon supply chain with environmentally aware consumers. Like Petruzzi and Dada [30] they use two types of price dependent demand pattern which are linear and iso-elastic. Demand is associated with environmentally friendliness of the product. The difference from our models is that they assume an additional cost for the manufacturer which is associated with making environmentally friendly products. Since this additional cost raise the price of the products, the cost to achieve a specific level of environmentally friendliness is quadratic with the level itself. Therefore, by assuming that spending money on the environmental friendliness of the product has an additive effect on linear demand while a multiplicative effect on the iso-elastic demand, they reconstruct their demand functions with inclusion of the cost to achieve a specific level of the environmental friendliness. In both linear and iso-elastic demands, a constant also represent the inclination of the customers to purchase green products. The linear and iso-elastic expected demands are given in (2.3) and (2.4) respectively.

$$D = a - bp + \gamma e \tag{2.3}$$

$$D = Ap^{-\alpha}e^{\gamma} \tag{2.4}$$

where a>0, α>1, b>0, A>0, γ is a non-negative constant which represents the customer awareness or the inclination of customers towards eco-friendly products, p is the unit retail price, e is the eco-friendliness of the product.

A recent work of Chen et al. [32] focuses on coordination in a two-level supply chain with environmentally conscious and price sensitive customers. Price sensitivity is shown by a simple linear demand function as in Petruzzi and Dada's and Giri and Bardhan's papers. Then, it is assumed that if the demand is not price sensitive and merely sensitive to environment, the expected demand can be represented as a multiplication of a positive constant and an environmental protection satisfaction constant which represent the quasi-environmental protection elasticity. The important term of the model which is environmental protection satisfaction is calculated by the expenses of the manufacturer and the retailer to protect the environment. By combining the demand models which represent price sensitive customers and environmentally conscious customers, they obtain a demand formulation which is the product of the environment sensitive demand formulation and the linear price sensitive demand function. Their expected demand function $Q(s,p)$ is given as follows

$$Q(s,p) = ws^a(\alpha - \beta p) \tag{2.5}$$

where w is a positive constant, s is the environmental protection satisfaction, a represents the quasi-environmental-protection elasticity, p is the price charged to customers, α is a scaling parameter, β is the price elasticity.

Despite the fact that these two models reflect environmentally conscious customers, they generally focus on the alteration in the price of the product caused by the expenditures for environmental protection. In the first one [31], the spending of the companies on the environmental protection is directly added to the model as a positive effect and in the second study [32],the term customer protection satisfaction is used in the demand model which again resulted from the expenditure of the manufacturer and retailer on the environmental protection. Therefore, in contrast to our demand model settings, they generally perceive environmental consciousness as a cause of an additional cost for the companies and make relation with the retailer's and manufacturer's spending on environment and customer demand.

The work of Glock et al. [33] also focuses on the customers who are price and environmentally sensitive. In contrast to the two models above which provide an association between environmental sensitivity and the expenditures for environmental protection, they assume environmental impact of the production process as a quality characteristic and the customers attribute a higher quality to products which have less environmental impact. To make a connection between the quality characteristic and the environmental impact, they introduce a sustainability indicator SI to measure the quality of the product by considering two types of pollutants which are emissions (Em) and scrap (Sc). Their end customer demand $D(p,q)$ is formulated as a linear function of both price and quality as follows

$$D(p,q) = a - bp + cq \tag{2.6}$$

where $D(p,q) > 0$ $\forall$p and q$\in [0,1]$, p is the price charged to customers, and the quality characteristic q = SI = Em. Sc

2.3 Empirical Studies on Environmentally Conscious Customer Behavior

Several empirical studies are done to understand the effect of environmental consciousness on purchasing behavior. These studies generally focus on the car purchasing behavior via which one easily see the effect of environmental concerns in purchasing behavior. Since conventional cars (fuel based) have common pro-environmental alternatives like hybrid and electric cars, investigating car purchasing behavior may imply results related to the effect of environmental consciousness in purchasing decisions. Again considering Kotler's stages of purchasing behavior, the studies inquire whether consumers take environmental attributes of the goods into account when purchasing.

The work of Laurence and Macharis [34], deals with the same question and investigates the relationship between environmental consciousness and car purchasing behavior. Under three research methodologies classified as attitudinal surveys, experimental and quasi-experimental studies and preference valuation techniques, the author presents results from different studies to clarify the role of environmental consciousness in car purchasing behavior.

Among the studies that Laurence and Macharis [34] report, the most preferred method was preference valuation technique. The preference valuation technique is generally used by economists to analyze the potential demand for a service or product by measuring the consumer preferences for those products/services. The two methodologies under preference valuation technique were applied in the studies. These are the Stated Preference Technique (SP) which is survey based and help researchers understand the value given by the people to different attributes of the products/services like quality, price, design, environmental attributes and Revealed Preference technique in which real market data from observations on actual choices are used to measure the preferences of the people. The most common used SP techniques are the Choice Modeling (CM) and Contingent Valuation Method (CVM). They work in following mechanism. The CM uses a choice experiment and consumers are asked for their preferences for hypothetical vehicles which are described by specific attributes. By evaluating their responds and statistical techniques, the analysis determines a value for each attribute of the vehicle. On the other hand, CVM asks respondents their maximum willingness to pay (WTP) for an increase or their minimum Willingness to Accept (WTA) for a decrease in a specific attribute.

The study by Bunch et al. [35] uses Stated Preference Techniques, to predict the market penetration of pro-environmental cars in California with seven hundred Californian respondents. The attributes of the empirical study tested in the design were fuel cost, range, price, performance, fuel availability and vehicle emissions. As a result, it is found that consumers are willing to pay 9000 more for a vehicle which cause less carbon emission levels up to ninety percent. However, other studies showed that the situation is different in another states like New York and it is found that Californians are 1.5 times more likely to pay for a car with reduced carbon emission level [36].

Potoglou and Kanaroglou [37] assess the car purchasing behavior in Montreal by considering three different attributes: monetary (price of the car, fuel cost, operating cost), non-monetary (quality, safe, power), environmental attributes where the pollution level caused by carbon emission assumed as the determinant for the pro-environmental feature of the car. As a result, it is found that despite the sensitivity of the consumers to environmental attributes, the elevated prices of the pro-environmental cars are the main obstacles. The survey done by OIVO [38] also found that three most important attributes consumers consider when evaluating car alternatives are price, operating cost and quality of the car. Therefore, even if environmental consciousness as an intent to buy environmental friendly goods exists among consumers, the price and quality tradeoffs of the consumers change the dynamics. Therefore, it can be concluded that incentives and public support is very important to prevent the price and quality trade-offs of the consumers who are environmentally conscious.

After briefly reviewing the studies related to the effect of environmental consciousness on purchasing behavior, we next move on the literature related to environment sensitive consumer behavior. The studies under this context generally focus on the motivations and factors that determine the consumer behavior. According to the study of Bamberg [39], the effect of environmental consciousness on pro-environmental consumer behavior is disappointing since reviews of many studies suggests that there is low to moderate relationship between the environmental consciousness and pro-environmental consumer behavior. He stated that environmental concerns seem to explain not more than ten percent variance of specific pro-environmental behaviors. Therefore, the studies realize the possibility of the existence of other motivations behind the environmentally conscious purchasing and conduct empirical studies to understand the factors that determine environmentally conscious consumer behavior.

Chua et al. [5] summarize the intrinsic and extrinsic motivations behind pro-environmental consumption.Intrinsic motives are the real environmental concerns related to consequences of the purchasing decisions and they are expected to be the main motivation behind pro-environmental consumer behavior. However, he reported that the extrinsic rewards like popularity, image, status may be the most significant reasons for some consumers to choose pro-environmental products. Griskevicius, Tybur and Van den Bergh [40] conduct experiments to understand the motivations behind pro-environmental consumer behavior. The study indicated that the pro-environmental consumption behavior have a relation with conspicuous consumption characteristics since the results showed that people have the propensity to choose pro-environmental products which are more expensive than other green cheaper ones and the desire for green products increases when shopping in public (not private). By purchasing pro-environmental products they try to buy an identity, be called as the environmentalist, which is considered as altruistic, sensitive, unselfish, pro-social from the rest of the society.

The study of Chua et. al. [5] investigates the motivations behind pro-environmental consumer behavior by focusing on the hybrid car buyers and find that the buyers of hybrid cars value social-image factors more than the quality and appeal of the cars. They stated that to be seen in a pro-environmental car is important for them, in other words, the “green image” is very important. The results of the study suggests that intrinsic motivations do not enter the evaluation sets of the hybrid car buyers and hybrid car buyer may show themselves as being more environmentalist than they really are and choose the hybrid cars to show their environmentalist or green identity.

The study of Barr [41] also focuses on gaps between environmental consciousness and pro-environmental consumer behavior and he states that most of people have learned the semantics of environmentalism and know that the environmentalism is the socially accepted manner. He advocates that there are different other extrinsic motivations behind green purchasing in addition to intrinsic ones and concludes with a sentence: “it might be stated that some of us are environmentalist, but rest of us know how to sound like environmentalist.”

2.4 Carbon Sensitive Inventory Models

The growing attention to the carbon emission levels and global trend of environmental consciousness yields to the consideration of carbon sensitivity in operations management. The researchers start to construct inventory models which include carbon emission concerns by modifying the well-known settings such as EOQ and newsvendor models.

Chen et. al. [42] show that without significantly increasing cost, the carbon emission levels can be reduced by modifying order quantities in EOQ model. The model is investigated under different environmental regulations such as strict carbon cap, carbon tax, cap-and-offset and cap-and-price.

Benjaafar et. al. [43] show that carbon emission concerns can be considered in widely used inventory, procurement, production models. The traditional models are modified in such a way that accounts for both cost and carbon footprint. Instead of costly applications to reduce carbon emission levels, they propose some modifications to well-known models to satisfy carbon reduction requirements. The case of multiple firms within the same supply chain is also investigated. The impact of the collaboration between these firms on their costs and carbon emission levels is taken into account under different environmental regulations. As a result of the study, the significant effects of the operational decisions and the environmental regulation policy on the carbon emission levels are shown.

Hua et. al. [44] investigate the inventory management under the carbon emission trading mechanism. They analyze a EOQ setting where carbon trading mechanism exists. Optimal order quantity, impacts of carbon trade, carbon cap and carbon price on decisions of the company, carbon emissions and total costs are derived as findings of the study.

Sözüer [45] considers two problems under the newsvendor setting with multiple inputs, a carbon emission constraint and non-linear production functions such as Leontief and Cobb-Douglas production functions. In the first problem, the optimal order quantity and input allocation that maximize the expected profit of the retailer under a strict carbon cap are found. In the second problem, an emission trading scheme is assumed where purchase of carbon emission permits is available before the demand is realized. For this problem, the optimal allocation of the inputs and the carbon trading policy is found which maximizes the expected profit. A random demand is assumed for both problems and the customers are not environment sensitive. Our study is motivated by her work and we extend her work to address the behavior of a customers who are environment sensitive. Instead of a strict carbon cap, we formulate a carbon sensitive demand structure to represent environment sensitive customers and construct newsvendor models under two different scenarios which will be explained in the following sections.

In the Master of Science thesis of Özüm Korkmaz [46], considering the chances of facing unexpected losses due to demand uncertainty, two different problems are investigated with a single product newsvendor under CVAR maximization objective. In the first problem, newsvendor problem is investigated under two different carbon emission reduction policies and in the second problem, newsvendor problem with multiple resource constraints is considered where there exists a quota for each resource and trade options are available.

Chapter 3

Preliminaries

Before introducing our model, we shall briefly review the classical newsvendor problem, the newsvendor problem with multiple inputs, the Cobb-Douglas production function and the carbon sensitive demand structure models.

3.1 The Classical Newsvendor Problem

The classical newsvendor problem refers to the replenishment or production decision for a single item with random demand in a single period. The cost parameters of the problem are the unit ordering cost c, unit selling price p, unit excess/holding cost c_e and the unit shortage cost c_s. The demand D is assumed to be continuous with p.d.f f(.) and c.d.f F(.). The decision variable is the Q and the aim is to find the optimal value of Q that maximizes the expected profit. The profit function $\pi(Q)$ and expected profit function E[$\pi(Q)$]=$\overline{\pi}(Q)$ are written as:

$$\Pi(Q) = s\min(Q, D) - c_s\max(0, (D - Q)) - c_e\max(0, (Q - D)) - cQ \quad (3.1)$$

$$
\begin{aligned}
\overline{\Pi}(Q) =& sE[\min(Q, D)] - c_s E[\max(0, (D - Q))] - c_e E[\max(0, (Q - D))] - cQ \\
=& s\int_0^Q uf(u)du + s\int_Q^\infty Qf(u)du - c_s\int_Q^\infty (u - Q)f(u)du \\
& - c_e\int_0^Q (Q - u)f(u)du - cQ
\end{aligned} \tag{3.2}
$$

The classical newsvendor problem solves the following optimization problem

$$
\begin{aligned}
\max_{Q} \quad & \overline{\Pi}(Q) \\
s.t. \quad & Q \geq 0
\end{aligned}
$$

The concavity of the objective function of the classical newsvendor problem is proven and the optimal order quantity Q^* is given by:

$$
F(Q^*) = \frac{s + c_e - c}{s + c_e - c_s} \tag{3.3}
$$

3.2 Newsvendor Problem with Multiple Inputs

The newsvendor problem with multiple inputs differs from the classical newsvendor problem in the usage of multiple inputs instead of a single input and a production function for transforming inputs into outputs. In the thesis, to construct a newsvendor problem with multiple inputs, the Cobb-Douglas production function is used. Below we briefly introduce the Cobb-Douglas production function.

3.2.1 The Cobb Douglas Production Function

In economics, production functions represent the relationship between the output and the combination of inputs, factors which are used to obtain it. As a general representation, a production function is given as

$$Q = \phi(\overrightarrow{x}) = \phi(x_1, x_2, ...x_n)$$

where x_i denotes the input quantity for resource i for $i = 1, 2...n$. Above, $\phi()$ represents the link between the total amount produced and the input quantities.

The Cobb-Douglas production function is most widely used production function. It was proposed by Knut Wicksell and tested by Charles Cobb and Paul Douglas in 1928. Cobb and Douglas published a study in which they use Cobb-Douglas production function to model the growth of American economy between 1899 - 1922. They considered a simple version of Cobb-Douglas production function in which the output is determined by the amount of labor and the amount of capital. The form they suggested was as follows:

$$Q = AK^{\alpha}L^{\beta}$$

where Q is the production quantity, K is the capital invested as an input, L is the labor input, A is a positive coefficient which represents the technology level for the process, α and β are the input elasticities of labor and capital, respectively.

For the multiple inputs, the Cobb-Douglas production form is generalized as follows:

$$Q = \phi(\overrightarrow{x}) = A\prod_{j=1}^{n} {x_i}^{\alpha_i} \tag{3.4}$$

where Q and A are defined as defined before, x_i denotes the input quantity for resource $i = 1, 2...n$ and α_i represents input elasticity of resources $i = 1, 2...n$.

The Cobb-Douglas function in two kinds above allows to model the contribution of the inputs to the output via a concept referred to as "returns to scale".This technical term determines the amount of change in the output, caused by a proportional change in all inputs. In particular, if all the inputs increase by a constant factor, and in result the output increases by the same proportion, this implies a constant returns to scale (CRS). If the output increases by less than the proportional increase in the inputs, then there is a decreasing returns to scale (DRS). Finally, if the output increases by more than the proportional increase in the inputs, it is called increasing returns to scales (IRS). For the Cobb-Douglas production function, returns to scale is determined by the: $r = \sum_{i=1}^{n} \alpha_i$, where $r < 1$ represents DRS, $r > 1$ represents IRS and $r = 1$ represents CRS.

In the thesis, we assume that $r < 1$, implying a DRS setting. Suppose the inputs used to produce an item have different emissions. In particular, let β_i be the carbon emitted when one unit of input i is used for production. Consequently, if $Q = \phi(\overrightarrow{x}) = A \prod_{j=1}^{n} {x_i}^{\alpha_i}$ holds, then x_i units of input i is used and the total emission for this particular choice of inputs is

$$\xi(Q(\overrightarrow{x})) = \sum_{i=1}^{n} \beta_i x_i \tag{3.5}$$

Exploiting the product form of the production function, we note that if Q is fixed, any one of the other inputs can be expressed in terms of Q and the remaining inputs, without loss of generality, let us represent x_n in the way as

$$x_n = \left(\frac{Q}{A \prod_{i=1}^{n-1} x_i^{\alpha_i}} \right)^{\frac{1}{\alpha_n}} \tag{3.6}$$

Then the total emission for Q units of products that uses x_1,....,x_n amounts from inputs $1, ..., n$ respectively is expressed as

$$\xi(Q(\overrightarrow{x})) = \sum_{i=1}^{n-1} \beta_i x_i + \beta_n \left(\frac{Q}{A \prod_{i=1}^{n-1} x_i^{\alpha_i}} \right)^{\frac{1}{\alpha_n}} \tag{3.7}$$

3.3 Demand Structure of Carbon Sensitive Customers

As discussed earlier we assume in our work that customers are environmentally conscious. In order to represent such environmentally conscious customers, we form an additive demand function which reflects the effect at the level of the product on the demand. Our demand function, D, is assumed to be a decreasing function of the per unit carbon emission level caused by the production.

As discussed in the literature review part, several models have been introduced where customer demands are sensitive to specific features of the product such as the price or quality. However we have not encountered any model that directly reflects the carbon emission sensitivity of the customers to the demand function.

In this study we assume that the carbon emissions of the product in general negatively affects the customer demand. The specification of the particular form of this impact in fact is not very straightforward. To come up with a reasonable functional relationship we assumed that the emission quantity due to the production of a unit is available to the customer and the demand is negatively affected by this emission. Following commonly used models in the literature we adopted an additive demand function as follows

$$D = y(\frac{\xi(Q(\overrightarrow{x}))}{Q}) + \epsilon \tag{3.8}$$

where y is a known function as will be discussed below, $\xi(Q(\overrightarrow{x}))$ is the carbon emission quantity for producing Q units of products as given in 3.7. Hence $\xi(Q(\overrightarrow{x}))/Q$ is the emission per unit, and ϵ is a random term with mean 0 and variance σ^2, known distribution function $f_\epsilon(.)$, and cumulative distribution function $F_\epsilon(.)$. As we observe from 3.7, $\xi(Q(\overrightarrow{x}))$ directly depends on the input mixture to produce Q units of product. Hence the mixture choice will have a significant impact on the total amount of emitted carbon.

As mentioned previously, we consider two settings for the supply chain. We first consider the case where the manufacturer acts independently to minimize his cost and in the second setting the supply chain is managed centrally. Therefore, in these two models, the input choice and consequently the total emission will differ which in turn will effect the customer demand differently. For both settings, the function y in the demand model 3.8 is explicitly given as follows

$$y(Q(\overrightarrow{x})) = B(1 - a(\frac{\xi(Q(\overrightarrow{x}))}{Q})^b) \tag{3.9}$$

where B represents the mean demand, b denotes the carbon sensitivity level of the customer assumed to be less than one, a is a positive coefficient, ϵ is the random error term with mean 0 and variance σ^2. The different structures of the $(\xi(Q(\overrightarrow{x}))/Q)^b$ depending on the setting of the supply chain will be covered in following chapters.

Chapter 4

Retailer's Problem with an Independent Manufacturer

In this chapter, we consider a setting where the retailer and the manufacturer behave independently, which we refer as the "decentralized" setting. In this setting, the retailer orders Q units to the manufacturer, the manufacturer produces the Q units using the n inputs in an optimal way that minimizes his expected cost. As discussed above, the ordered quantity and the inputs have the relationship as given in (3.4) as follows

$$Q = \phi(\overrightarrow{x}) = A \prod_{j=1}^{n} {x_i}^{\alpha_i} \tag{4.1}$$

where x_i is the input quantity for the ith input, α_i is the elasticity of input i, such that $0 < \alpha_i \leq 1$ for $i = 1, ...n$ and the returns to scale is $r = \sum_{i=1}^{n} \alpha_i$.

The production process by the manufacturer results in a carbon emission level, which can be considered as the carbon footprint of the product. The level of the carbon emission is determined depending on the selected input mix. We assume that input i emits β_i units of carbon per unit and the procurement cost of input i is assumed to be p_i per unit. The emission level, which is caused by the production, is calculated as $\xi(\overrightarrow{x}) = \sum_{i=1}^{n} \beta_i x_i$ where $\overrightarrow{x} = (x_1, x_2, ..., x_n)$ denotes the amounts used from each input. Furthermore, as we will see in the following pages, the optimal input quantities that will minimize the manufacturer's expected cost will also be a function of the order quantity Q. Hence we explicitly state the dependence of the emissions per a lot of size Q as $\xi(Q(\overrightarrow{x}))$.

Our aim is to find the optimal production quantity that maximizes the expected total profit function of the retailer. Our study is motivated by an earlier work by Sozuer [45] who also considered a similar model, however with customers who are not environmentally conscious. In this study we extend her work to address the behavior of a customers who are environment sensitive. Since the customers are sensitive to the carbon emission levels, as it is discussed in the previous part, the demand function is assumed to be a decreasing function of the carbon emission level per unit, which is denoted as $\xi(Q(\overrightarrow{x}))/Q$. An additive demand model is formulated as:

$$D = y(\frac{\xi(Q(\overrightarrow{x}))}{Q}) + \epsilon$$

where

$$y(Q(\overrightarrow{x})) = B(1 - a(\frac{\xi(Q(\overrightarrow{x}))}{Q})^b),$$

B represents the mean demand, b denotes the carbon sensitivity level of the customer assumed to be less than one, a is a positive coefficient, ϵ is the random error term with mean 0 and variance σ^2.

We incur a shortage cost of c_s per unit, for unsatisfied demand and excess cost of c_e per unit for each unsold item. The fixed cost for unit item is denoted as c. The selling price is s per unit. We assume that the manufacturer applies a cost-plus approach, so that he sells his products with a price which is a $(1+\delta)$ multiple of its total production cost. The profit function of the retailer is given by the following expression.

$$\Pi(Q) = s\min(Q,D) - c_s\max(0,(D-Q)) - c_e\max(0,(Q-D))$$
$$- cQ - (1+\delta)(\sum_{i=1}^{n} p_i x_i(Q))$$

The expected profit $\overline{\Pi}(Q) \equiv \mathrm{E}[\Pi(Q)]$ of the retailer, is given as follows

$$E[\Pi(Q)] \equiv \overline{\Pi}(Q) = sE[\min(Q,D)] - c_sE[\max(0,(D-Q))]$$
$$- c_eE[\max(0,(Q-D))] - cQ$$
$$- (1+\delta)(\sum_{i=1}^{n} p_i x_i(Q))$$

$$= s\int_{-\infty}^{Q} uf(u)du + s\int_{Q}^{\infty} Qf(u)du$$
$$- c_s\int_{Q}^{\infty}(u-Q)f(u)du - c_e\int_{-\infty}^{Q}(Q-u)f(u)du$$
$$- cQ - (1+\delta)(\sum_{i=1}^{n} p_i x_i(Q))$$

The retailer's problem is as follows

$$\underset{Q}{Max} \quad \overline{\Pi}(Q)$$
$$s.t. \quad \overrightarrow{x}, Q \geq 0$$

This problem is considered in two stages. First, for any given Q, the optimal input mix is determined as a function of Q. This stage is assumed to be undertaken by the manufacturer who minimizes his costs which yield an optimal input mixture for any given Q. This stage of the problem is solved in Sozuer [45] where the optimal values of input quantities that minimize the total costs of the manufacturer are obtained, denoted by x_i^*, $i = 1, ..., n$. The x_i^* values turn out to be a polynomial function of the production quantity Q. This problem solved in Sozuer has the following form which has the following form.

$$\begin{aligned} \underset{\overrightarrow{x}}{Min} \quad & \sum_{i=1}^{n} p_i x_i \\ s.t. \quad & A \prod_{i=1}^{n} {x_i}^{\alpha_i} = Q \\ & \overrightarrow{x} \geq 0 \end{aligned}$$

For completeness we report the following results from Sozuer [45].

Theorem 1: For a given Q,

(i) the unique optimal solution to problem is

$$x_i{}^*(Q) = \psi_i Q^{\frac{1}{r}} \quad \text{for all} \quad i = 1, ...n \text{ where } \psi_i = \tfrac{\alpha_i}{p_i} A^{-\frac{1}{r}} \textstyle\prod_{i=1}^{n} \left(\tfrac{p_i}{\alpha_i}\right)^{\frac{\alpha_i}{r}} \tag{4.2}$$

(ii)The emission level at the optimal input allocation for a given Q is

$$\xi^*(Q) = \sum_{i=1}^{n} \frac{\beta_i \alpha_i}{p_i} \prod_{j=1}^{n} (\frac{p_i}{\alpha_i})^{\frac{\alpha_i}{r}} (\frac{Q}{A})^{\frac{1}{r}} \tag{4.3}$$

Once the optimal x_i^* values are obtained, we are able to find how much carbon is emitted using the carbon emission coefficients of the inputs. The per unit carbon emission level at the optimal allocation for a given Q is found as follows.

Corollary 1: Per unit emission level at the optimal input allocation for a given Q is

$$\frac{\xi^*(Q)}{Q} = A^{-\frac{1}{r}} C Q^{\frac{1-r}{r}} \tag{4.4}$$

where $C = \sum_{i=1}^{n} \frac{\beta_i \alpha_i}{p_i} \prod_{j=1}^{n} (\frac{p_j}{\alpha_j})^{\frac{\alpha_j}{r}}$

We note that the amount of carbon emission per unit production, $\xi^*(Q)/Q$, depends on the production quantity, Q and it decreases or increases according to the value of r. In this work, the return to scale, r, is assumed to be less than one and the customer demand is assumed to be a decreasing function of the carbon emission amount per unit product. This results in a model where the firm faces a demand that also depends on the production quantity Q in a specific way. Using the above result, the demand function previously expressed as $D = y(\frac{\xi(Q(\overrightarrow{x}))}{Q}) + \epsilon$ is explicitly written as follows:

$$D = y(Q(\overrightarrow{x})) + \epsilon$$

where $y(Q(\overrightarrow{x})) = B(1 - a(Q^{\frac{1-r}{r}} C A^{-\frac{1}{r}})^b)$

This specific structure is a distinguishing feature of the present model where the customer demand depends on the production quantity as well as the composition of the inputs used in production. Such a behavior reflects a customer set who are considerate about the environmental issues.This set is gradually increasing in recent years as societies become more sensitive to the quality where they are living in as well as the official efforts to reduce the carbon emissions through several mechanisms such as Kyoto Protocol.

Under the carbon sensitive demand structure, the objective function of the problem is now explicitly written as follows:

$$\begin{aligned}\overline{\Pi}(Q) = &\int_{-\infty}^{Q-y(Q)} (s(y(Q)+u) - c_e(Q - y(Q) - u) - cQ) dF\epsilon(u) \\ &+ \int_{Q-y(Q)}^{\infty} (sQ - cQ - c_s(y(Q) + u - Q)) dF\epsilon(u) - (1+\delta)(\sum_{i=1}^{n} p_i x_i^*(Q))\end{aligned}$$

$$ \tag{4.5} $$

Then the optimization problem of the retailer is written as follows

$$
\begin{aligned}
&\underset{Q}{Max} && \overline{\Pi}(Q) \\
&s.t. && \overrightarrow{x}, Q \geq 0
\end{aligned}
\tag{4.6}
$$

4.1 Analytical Results

To provide the optimality results for the problem 4.6 we begin with deriving the first order conditions. Using Leibniz rule we have

$$
\begin{aligned}
\frac{d\overline{\Pi}(Q)}{dQ} =& \frac{d(Q - y(Q))}{dQ} (sQ - cQ)\, f_x(Q - y(Q)) \\
&+ \int_{-\infty}^{Q-y(Q)} (sy'(Q) - c_e(1 - y'(Q)) - c) dF\epsilon(u) \\
&- (1 - y'(Q))[sQ - cQ] f_x(Q - y(Q)) \\
&+ \int_{Q-y(Q)}^{\infty} [s - c - cs(y'(Q) - 1)] dF\epsilon(u) - \frac{d}{dQ}\left((1+\delta)\sum_{i=1}^{n} p_i x_i^*(Q)\right)
\end{aligned}
$$

$$
\begin{aligned}
=& \int_{-\infty}^{Q-y(Q)} (sy'(Q) - c_e(1 - y'(Q)) - c) dF\epsilon(u) \\
&+ \int_{Q-y(Q)}^{\infty} [s - c - cs(y'(Q) - 1)] dF\epsilon(u) \\
&- \frac{(1+\delta)\sum_{i=1}^{n} p_i x_i^*(Q)}{rQ}
\end{aligned}
$$

$$
\begin{aligned}
=&F(Q-y(Q))\left(-(c+c_e)+sy'(Q)+c_e y'(Q)\right)\\
&+\overline{F}(Q-y(Q))(s-c+c_s-c_s y'(Q))\\
&-\frac{(1+\delta)\sum_{i=1}^{n}p_i\psi_i Q^{\frac{1}{r}}}{rQ}
\end{aligned}
$$

$$
\begin{aligned}
=&F(Q-y(Q))[(c_e+c_s+s)(y'(Q)-1)]+(s-c+c_s(1-y'(Q)))\\
&-\frac{(1+\delta)\sum_{i=1}^{n}p_i\psi_i Q^{\frac{1}{r}}}{rQ}
\end{aligned}
$$

The following expressions are frequently used in the study.

$$
y(Q)=B(1-a(Q^{\frac{1-r}{r}}CA^{-\frac{1}{r}})^b) \tag{4.7}
$$

$$
\begin{aligned}
y'(Q)&=-abBA^{-\frac{b}{r}}C^b\left(\frac{1-r}{r}\right)Q^{b(\frac{1-r}{r})-1} \\
&\Rightarrow y'(Q)<0
\end{aligned} \tag{4.8}
$$

$$
w(Q)=Q-y(Q)=Q-B+Ba\left(Q^{\frac{1-r}{r}}CA^{-\frac{1}{r}}\right)^b \tag{4.9}
$$

$$
\begin{aligned}
w'(Q)=1-y'(Q)&=1+abBA^{-\frac{b}{r}}C^b\left(\frac{1-r}{r}\right)Q^{b(\frac{1-r}{r})-1} \\
&\Rightarrow w'(Q)>0
\end{aligned} \tag{4.10}
$$

$$
\begin{aligned}
w'(Q)-1&=-y'(Q)>0 \\
&\Rightarrow w'(Q)-1>0
\end{aligned} \tag{4.11}
$$

$$
w''(Q)=abBC^bA^{-\frac{b}{r}}(\frac{1-r}{r})\left(\frac{b}{r}-b-1\right)Q^{b(\frac{1-r}{r})-2}. \tag{4.12}
$$

$$w''(Q) = \left(\frac{w'(Q)-1}{Q}\right)\left(\frac{b}{r} - b - 1\right). \tag{4.13}$$

Also let

$$H(Q) = \frac{s-c}{(s+c_e+c_s)w'(Q)} + \frac{c_s}{s+c_e+c_s} - \frac{(1+\delta)\psi Q^{\frac{1}{r}-1}}{r(s+c_e+c_s)w'(Q)}. \tag{4.14}$$

where $\sum_{i=1}^{n} p_i\psi_i Q^{\frac{1}{r}} = \psi Q^{\frac{1}{r}}$ and $\psi = A^{-\frac{1}{r}} \prod_{i=1}^{n} \left(\frac{p_i}{\alpha_i}\right)^{\frac{\alpha_i}{r}} \sum_{i=1}^{n} \alpha_i$

$$H(0) = \frac{s-c+c_s}{(s+c_e+c_s)} \tag{4.15}$$

$$G(Q) = \frac{(1+\delta)\psi Q^{\frac{1}{r}-1}}{r(b(1-r)-r)}[\frac{(r-1)Q^{\frac{-b}{r}+b+1}}{beC^b A^{-\frac{b}{r}}(\frac{1-r}{r})} + (b(1-r)-1)] \tag{4.16}$$

$$G(0) = 0 \tag{4.17}$$

Lemma 1: The first order condition for the problem is given by

$$F(w(Q)) = H(Q) \tag{4.18}$$

Proof: Directly follows from setting $d\overline{\Pi}(Q)/dQ = 0$.

The Theorem 1 is also valid for the case where $r = 1$. Therefore, we also investigate the first order condition for the problem under the case $r = 1$.

Corollary 2: Suppose $r = 1$. Then the first order condition for the problem is given by

$$F(w(Q)) = \frac{1}{(s + c_e + c_s)}[(s - c + c_s) - (1 + \delta)A^{-1}\prod_{i=1}^{n}(\frac{p_i}{\alpha_i})^{\alpha_i}]$$

where $w(Q) = Q - B(1 - a(A^{-1}\sum_{i=1}^{n}\frac{\beta_i\alpha_i}{p_i}\prod_{j=1}^{n}(\frac{p_j}{\alpha_j})^{\alpha_j})^b)$

Proof: Directly follows from equation 4.18.

We also investigate the special case where there is only one input with elasticity α, carbon emission parameter β, per unit price p and the customers are all carbon sensitive such that $b = 1$. The first order condition for the problem under this case is given as follows.

Corollary 3: Suppose $n = 1$ and $b = 1$. Then the first order condition for the problem is given by

$$F(w(Q)) = \frac{1}{(s + c_e + c_s)w'(Q)}[(s - c) - \frac{(1 + \delta)pA^{\frac{-1}{\alpha}}Q^{\frac{1}{\alpha}-1}}{\alpha}] + \frac{c_s}{s + c_e + c_s}$$

where $w(Q) = Q - B(1 - a\beta Q^{\frac{1}{\alpha}-1}A^{\frac{-1}{\alpha}})$, $w'(Q) = 1 + Ba\beta A^{\frac{-1}{\alpha}}(\frac{1}{\alpha} - 1)Q^{\frac{1}{\alpha}-2}$

Proof: Directly follows from equation 4.18.

We next elaborate on the relationship (4.18). $F(w(Q))$ is an increasing function which starts at 0 and converges to 1 as it is a distribution function. We next investigate the behavior of $H(Q)$. The derivative of the $H(Q)$ with respect to Q is:

$$
\begin{aligned}
\frac{d(H(Q))}{dQ} =& \frac{(s-c)(s+c_e+c_s)(w''(Q))}{(s+c_e+c_s)^2(w'(Q))^2} \\
&- \frac{(1+\delta)\psi Q^{\frac{1}{r}} w'(Q)(s+c_e+c_s)}{r^2Q^2(s+c_e+c_s)^2(w'(Q))^2} \\
&+ \frac{(1+\delta)\psi Q^{\frac{1}{r}}(rw'(Q)(s+c_e+c_s)+rQw''(Q)(s+c_e+c_s))}{r^2Q^2(s+c_e+c_s)^2(w'(Q))^2}
\end{aligned}
$$

Referring to 4.13, the first derivative of the $H(Q)$ is rewritten as

$$
\begin{aligned}
\frac{d(H(Q))}{dQ} =& \frac{-(s-c)(s+c_e+c_s)(w'(Q)-1)(\frac{b}{r}-b-1)}{Q(s+c_e+c_s)^2(w'(Q))^2} \\
&- \frac{(1+\delta)\psi Q^{\frac{1}{r}}(s+c_e+c_s)(w'(Q)(1-r)-r(w'(Q)-1)(\frac{b}{r}-b-1)}{r^2Q^2(s+c_e+c_s)^2(w'(Q))^2}
\end{aligned}
$$

$$
\begin{aligned}
=& \frac{-(s-c)(s+c_e+c_s)(w'(Q)-1)(\frac{b}{r}-b-1)r^2Q}{r^2Q^2(s+c_e+c_s)^2(w'(Q))^2} \\
&- \frac{(1+\delta)\psi Q^{\frac{1}{r}}(s+c_e+c_s)[w'(Q)(1-r)-r(w'(Q)-1)(\frac{b}{r}-b-1)]}{r^2Q^2(s+c_e+c_s)^2(w'(Q))^2}
\end{aligned}
$$

$$
\equiv (A+D)/B
$$

where

$$
\begin{aligned}
A =& (s+c_e+c_s)((1+\delta)\psi Q^{\frac{1}{r}})[r(\frac{b}{r}-b-1)(w'(Q)-1)-w'(Q)(1-r)] \\
=& (s+c_e+c_s)((1+\delta)\psi Q^{\frac{1}{r}})[b(1-r)(w'(Q)-1)-(w'(Q)-r)]
\end{aligned}
\tag{4.19}
$$

$$
D = -(s-c)r^2Q(s+c_e+c_s)(w'(Q)-1)[b(\frac{1-r}{r})-1] \tag{4.20}
$$

$$
B = r^2Q^2(w'(Q))^2(s+c_e+c_s)^2 \tag{4.21}
$$

Clearly, $B \geq 0$, since it is the square of a quantity. For evaluating both A and D, we focus on two possible cases as follows

Case 1: $b \geq \frac{r}{1-r}$

Since $b((1-r)/r) \geq 1$, $w'(Q) - 1 \geq 0$ referring to 4.11, and $(s-c) \geq 0$, it follows that $D \leq 0$.

Noting that $(s + c_e + c_s)$ and $(1+\delta)\psi_i Q^{\frac{1}{r}}$ are non-negative, we focus on the sign of $[b(1-r)(w'(Q)-1) - (w'(Q)-r)]$ to understand A.

As $r < 1$ and $b < 1$ it follows that $(w'(Q)-1) < (w'(Q)-r)$ and $b(1-r) \leq 1$. Thus, we conclude that $[b(1-r)(w'(Q)-1) - (w'(Q)-r)] \leq 0$ and as a result $A \leq 0$ for any $b, r \leq 1$.

Therefore we conclude that $H(Q)$ is a decreasing function of Q. We now state our first main result:

Theorem 2: If $b \geq \frac{r}{1-r}$, there exists a unique Q^* that satisfies 4.18.

Proof: We check the initial points of both sides to understand whether these two functions intersect or not. As discussed above $H(Q)$ decreases in Q with the intercept $(s - c + c_s)/(s + c_s + c_e)$ which is less than one while $F(w(Q))$ is an increasing function of Q and takes values between 0 and 1. Therefore they intersect at exactly one point.

We next discuss the case of $b < r/(1-r)$.

Case 2: $b < \frac{r}{1-r}$

Lemma 2: Assume $b < \frac{r}{1-r}$. Then, H(Q) increases(decreases) in Q if

$$G(Q) \underset{(\geq)}{\leq} (s-c) \tag{4.22}$$

Proof: Consider A, D, B given in 4.19, 4.20, and 4.21.

Since $b < r/(1-r)$, $b((1-r)/r)-1 < 0$. Then, $D \geq 0$. We also know from the previous discussion that $A \geq 0$, $B \geq 0$. Hence the derivative of $H(Q)$ of (4.18) is positive (negative) if the following relation holds.

$$((1+\delta)\psi Q^{\frac{1}{r}})[b(1-r)(w'(Q)-1)-(w'(Q)-r)] \underset{(\leq)}{\geq} (s-c)r^2 Q(w'(Q)-1)[b(\frac{1-r}{r})-1]$$

By referring to the 4.10, the above inequality is simplified as follows:

$$\frac{((1+\delta)\psi Q^{\frac{1}{r}})[b(1-r)(w'(Q)-1)-(w'(Q)-r)]}{r^2 Q(w'(Q)-1)[b(\frac{1-r}{r})-1]} \underset{(\geq)}{\leq} (s-c)$$

$$\frac{((1+\delta)\psi Q^{\frac{1}{r}-1})[(1+abC^bA^{-\frac{b}{r}}(\frac{1-r}{r})Q^{b(\frac{1-r}{r})-1})(b-br-1)-(b-br-r)]}{r^2(abC^bA^{-\frac{b}{r}}(\frac{1-r}{r})Q^{b(\frac{1-r}{r})-1})[b(\frac{1-r}{r})-1]} \underset{(\geq)}{\leq} (s-c)$$

$$\frac{(1+\delta)\psi Q^{\frac{1}{r}-1}}{r(b(1-r)-r)}[\frac{(r-1)Q^{\frac{-b}{r}+b+1}}{beC^bA^{-\frac{b}{r}}(\frac{1-r}{r})}+(b(1-r)-1)] \underset{(\geq)}{\leq} (s-c)$$

Then by referring to 4.16, 4.22 is obtained.

We next present our second main result, regarding to the optimal solution of the problem.

Theorem 3: Suppose $b < (\frac{r}{1-r})$ and $H(Q)$ is a concave function. Then at most three points may satisfy the first order condition of the problem (4.18).

In accordance with Lemma 2, let $Q_0 = G^{-1}(s-c)$.
(i) First order condition (4.18) is satisfied at a unique point if

(a) $F(w(Q_0)) > H(Q_0)$ (See Figure 4.1).

(b) If $F(w(Q_0)) \leq H(Q_0)$ and $F(w(Q)) \neq H(Q)$ for $Q < Q_0$ (See Figure 4.2).

(ii) First order condition (4.18) is satisfied at two distinct points $Q_1 < Q_0$ and $Q_2 > Q_0$ s.t $F(w(Q_0)) < H(Q_0)$; $F(w(Q_i)) = H(Q_i)$ $i = 1, 2$ (See Figure 4.3).

(iii) First order condition (4.18) is satisfied at three points Q_1, Q_2, Q_3 where $Q_1 < Q_2 < Q_0$ and $Q_3 > Q_0$ s.t $F(w(Q_0)) < H(Q_0)$; $F(w(Q_i)) = H(Q_i)$ $i = 1, 2, 3$ (See Figure 4.4).

Proof:
First, since F is concave $f' = F'' < 0$. Then $[F(w(Q))]' = f(w(Q))w'(Q)$ and $[F(w(Q))]'' = f'(w(Q))(w'(Q))^2 + w''(Q)f(w(Q))$. Since $f'(w(Q)) < 0$, $(w'(Q))^2 > 0$, $w''(Q) < 0$ and $f(w(Q)) > 0$, $[F(w(Q))]'' < 0$.

Then, $F(w(Q))$ is also concave and increasing with $F(w(Q)) = 0$. By assumption $H(Q)$ is also concave. By Lemma 2, $H(Q)$ is increasing if $Q \leq Q_0$ and decreasing if $Q > Q_0$. Hence we inspect how many times two function, one is concave increasing-decreasing can intersect. Then

(i) Figure 4.1 clearly shows that under given conditions in part a has a unique intersection point. Figure 4.2 also shows the uniqueness of the solution under conditions given in part b.

(ii) Figure 4.3 shows that two distinct points $Q_1 < Q_0$ and $Q_2 > Q_0$ are the intersection points of F(w(Q)) and H(Q) under given conditions in part ii.

(iii) Figure 4.4 shows that there are three points Q_1, Q_2, Q_3 where $Q_1 < Q_2 < Q_0$ and $Q_3 > Q_0$ that satisfy the conditions of the problem given in part iii.

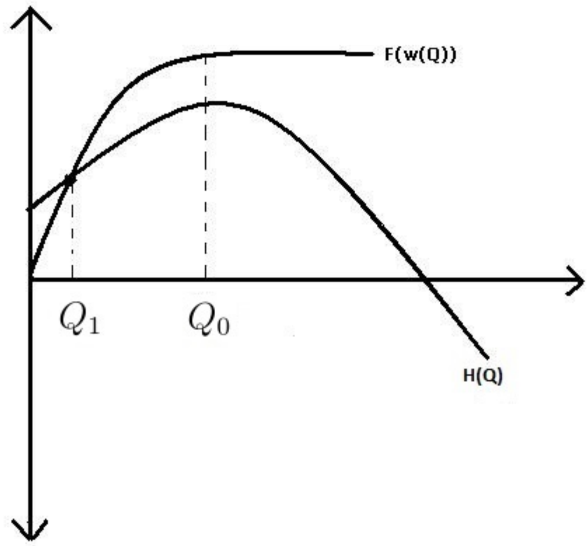


Figure 4.1: $F(w(Q))$ and $H(Q)$ under the conditions given in part a of i

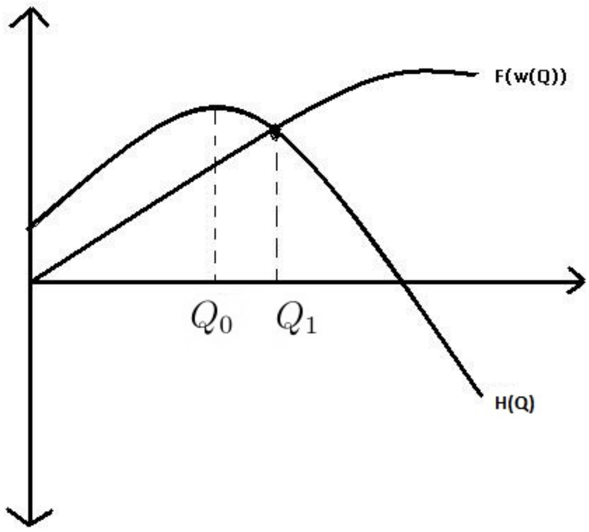


Figure 4.2: $F(w(Q))$ and $H(Q)$ under the conditions given in part b of i

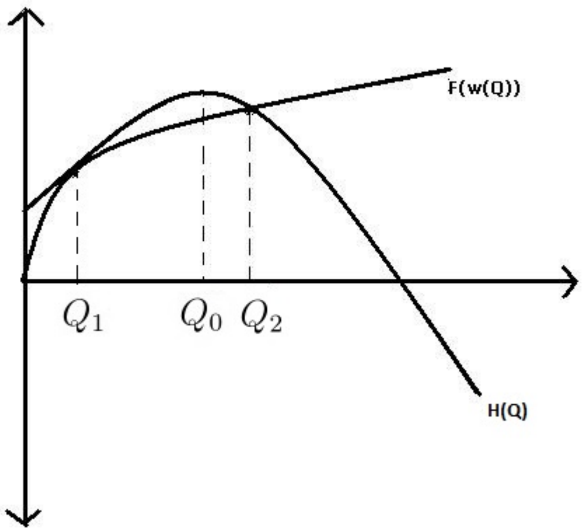


Figure 4.3: $F(w(Q))$ and $H(Q)$ under the conditions given in part *ii*

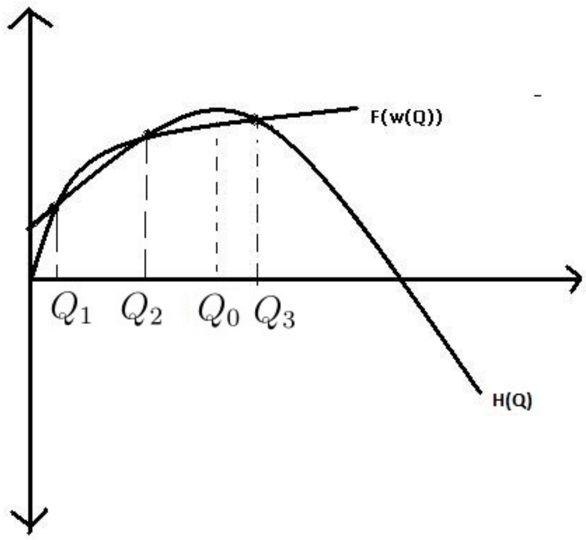


Figure 4.4: $F(w(Q))$ and $H(Q)$ under the conditions given in part *iii*

Chapter 5

Integrated Problem of the Retailer and the Manufacturer

In the previous chapter, we considered the newsvendor problem with multiple inputs and environment sensitive customers. The retailer aims to maximize her expected profit under random customer demand which is sensitive to the carbon emission levels. Under this setting, the retailer orders Q units to the manufacturer who applies a cost plus approach and the manufacturer produces the required Q units of product with an optimal selection of the input quantities that minimizes his production cost. The retailer then optimizes her production quantity that maximizes her expected profit.

In the integrated model, the retailer produces its own products instead of ordering from a manufacturer who applies a cost-plus approach. In this setting, the retailer does not know the carbon emission level caused by the production before determining the optimal production quantity. The retailer acts as a single manager and she jointly optimizes the order quantity and the input mix. The demand structure has the similar form given as follows

$$D = y(Q(\overrightarrow{x})) + \epsilon \text{ and } y(Q(\overrightarrow{x})) = B(1 - a(\tfrac{\xi(Q(\overrightarrow{x}))}{Q})^b)$$

where as before $(\xi(Q(\overrightarrow{x}))/Q)$ denotes the per unit carbon emission level caused by the production, ϵ is a random error term with mean 0 and variance σ^2, b denotes the carbon emission level sensitivity of the customers, B is the mean demand and a is a positive coefficient.

The profit function of the retailer under this new setting is as follows.

$$\begin{aligned}\Pi(Q) =& s\min(Q,D) - c_s\max(0,(D-Q)) - c_e\max(0,(Q-D)) \\ & - cQ - (\sum_{i=1}^{n} p_i x_i(Q))\end{aligned}$$

The expected profit $\overline{\Pi}(Q)\equiv\mathrm{E}[\Pi(Q)]$ of the retailer, is given as follows

$$\begin{aligned}\mathrm{E}[\Pi(Q)]\equiv \overline{\Pi}(Q) &= sE[\min(Q,D)] - c_sE[\max(0,(D-Q))] \\ & - c_eE[\max(0,(Q-D))] - cQ - (\sum_{i=1}^{n} p_i x_i(Q)) \\ &= s\int_{-\infty}^{Q} uf(u)du + s\int_{Q}^{\infty} Qf(u)du \\ & - c_s\int_{Q}^{\infty}(u-Q)f(u)du - c_e\int_{-\infty}^{Q}(Q-u)f(u)du \\ & - cQ - (\sum_{i=1}^{n} p_i x_i(Q))\end{aligned}$$

Under the carbon sensitive demand structure, the objective function of the problem is now explicitly written as follows:

$$\begin{aligned}\overline{\Pi}(Q) = & \int_{-\infty}^{Q-y(Q)} (s(y(Q)+u) - c_e(Q-y(Q)-u) - cQ)dF\epsilon(u) \\ & \int_{Q-y(Q)}^{\infty} (sQ - cQ - c_s(y(Q)+u-Q))dF\epsilon(u) - (\sum_{i=1}^{n} p_i x_i(Q))\end{aligned}$$

Then the optimization problem of the retailer is written as follows

$$
\begin{aligned}
\underset{Q,\overrightarrow{x}}{Max} \quad & \overline{\Pi}(Q) \\
s.t. \quad & A\prod_{i=1}^{n} {x_i}^{\alpha_i} = Q \\
& Q, \overrightarrow{x} \geq 0
\end{aligned}
$$

where the constraint again corresponds to the Cobb-Douglas production function.

We first consider the case $n = 2$ with two inputs. Then the Cobb-Douglas production function is written as

$$Q = Ax_1^{\alpha_1} x_2^{\alpha_2}$$

Hence, $x_2 = (Q/(Ax_1^{\alpha_1}))^{\frac{1}{\alpha_2}}$ which is a function of Q and x_1. The objective function then becomes a function of two decision variables, (Q,x_1). We now state our first main result.

Theorem 4: Suppose $n = 2$. Then the optimal order quantity, Q^* and x_1 satisfy the following equations

$$a(c_s-(s+c_e+c_s)F(Q-y(Q))) = \frac{p_1+p_2\left(\frac{Q}{Ax_1^{\alpha 1}}\right)^{\frac{1}{\alpha 2}} \frac{-\alpha_1}{\alpha_2}\frac{1}{x_1}}{b\left(\frac{\beta_1 x_1+\beta_2\left(\frac{Q}{Ax_1^{\alpha 1}}\right)^{\frac{1}{\alpha 2}}}{Q}\right)^{b-1}\left(\frac{\beta_1+\beta_2\left(\frac{Q}{Ax_1^{\alpha 1}}\right)^{\frac{1}{\alpha 2}} \frac{-\alpha_1}{\alpha_2}\frac{1}{x_1}}{Q}\right)} \tag{5.1}$$

$$F(Q-y(Q)) = \frac{s-c+c_s(1-y'(Q))+\frac{p_2}{\alpha_2 Q}\left(\frac{Q}{Ax_1^{\alpha}1}\right)^{\frac{1}{\alpha_2}}}{(s+c_e+c_s)(1-y'(Q))} \tag{5.2}$$

Proof: For n = 2 and given Q, the partial derivative of the objective function with respect to x_1 results in following equation

$$\frac{\partial\overline{\Pi}(Q)}{\partial x_1} = a(cs-(s+c_e+c_s)F(Q-y(Q)))\frac{d\delta}{dx_1} - p_1 + \frac{p_2}{x_1}\left(\frac{Q}{Ax_{1}^{\alpha}{}_1}\right)^{\frac{1}{\alpha_2}}\frac{\alpha_1}{\alpha_2} \quad (5.3)$$

where $\frac{\partial\delta}{\partial x_1} = b\left(\frac{\beta_1 x_1+\beta_2\left(\frac{Q}{Ax_1^{\alpha_1}}\right)^{\frac{1}{\alpha_2}}}{Q}\right)^{b-1}\left(\frac{\beta_1+\beta_2(\frac{Q}{Ax_1^{\alpha_1}})^{\frac{1}{\alpha_2}}\frac{-\alpha_1}{\alpha_2}\frac{1}{x_1}}{Q}\right)$.

Then, the partial derivative set to zero results in

$$a(c_s-(s+c_e+c_s)F(Q-y(Q))) = \frac{p_1+p_2\left(\frac{Q}{Ax_1^{\alpha_1}}\right)^{\frac{1}{\alpha_2}}\frac{-\alpha_1}{\alpha_2}\frac{1}{x_1}}{b\left(\frac{\beta_1 x_1+\beta_2\left(\frac{Q}{Ax_1^{\alpha_1}}\right)^{\frac{1}{\alpha_2}}}{Q}\right)^{b-1}\left(\frac{\beta_1+\beta_2\left(\frac{Q}{Ax_1^{\alpha_1}}\right)^{\frac{1}{\alpha_2}}\frac{-\alpha_1}{\alpha_2}\frac{1}{x_1}}{Q}\right)} \quad (5.4)$$

In a similar fashion, the partial derivative of the objective function with respect to Q is found as follows:

$$\frac{\partial\overline{\Pi}(Q)}{\partial Q} = F(Q-y(Q))[(y'(Q)-1)(s+c_e+c_s)]+(s-c+c_s(1-y'(Q)))+\frac{p_2 Q^{\frac{1}{\alpha_2}-1}}{\alpha_2\left(Ax_{1}^{\alpha}{}_1\right)^{\frac{1}{\alpha_2}}} \quad (5.5)$$

Setting the above equation to zero results as follows

$$F(Q-y(Q)) = \frac{s-c+c_s(1-y'(Q))+\frac{p_2}{\alpha_2 Q}(\frac{Q}{Ax_{1}^{\alpha}{}_1})^{\frac{1}{\alpha_2}}}{(s+c_e+c_s)(1-y'(Q))}$$

We next consider the case $n = 3$ with three inputs. Then the Cobb-Douglas production function is written as

$$Q = Ax_1^{\alpha_1} x_2^{\alpha_2} x_3^{\alpha_3}$$

Hence, $x_3 = (Q/(Ax_1^{\alpha_1} x_2^{\alpha_2}))^{\frac{1}{\alpha_3}}$ which is a function of Q,x_1 and x_2. The objective function then becomes a function of three decision variables, (Q,x_1,x_2). We now state our second main result.

Theorem 5: Suppose n = 3. Then the optimal order quantity, Q^*, x_1, and x_2 satisfy the following equations

$$a(c_s - (s + c_e + c_s)F(Q - y(Q))) = \frac{p_1 + p_3 \left(\frac{Q}{Ax_1^{\alpha_1} x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}} \frac{-\alpha_1}{\alpha_3} \frac{1}{x_1}}{b \left(\frac{\beta_1 x_1 + \beta_2 x_2 + \beta_3 \left(\frac{Q}{Ax_1^{\alpha_1} x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_2}}}{Q}\right)^{b-1} \left(\frac{\beta_1 + \beta_3 \left(\frac{Q}{Ax_1^{\alpha_1} x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}} \frac{-\alpha_1}{\alpha_2} \frac{1}{x_1}}{Q}\right)} \tag{5.6}$$

$$\frac{\alpha_2}{x_2}\left(\frac{Q}{Ax_1^{\alpha_1} x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}} \frac{1}{\alpha_3}(\beta_1 p_3 - p_1 \beta_3) + \frac{\alpha_1}{x_1}\left(\frac{Q}{Ax_1^{\alpha_1} x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}} \frac{1}{\alpha_3}(\beta_3 p_2 - p_3 \beta_2) = \beta_1 p_2 - p_1 \beta_2 \tag{5.7}$$

$$F(Q - y(Q)) = \frac{s - c + c_s(1 - y'(Q)) + \frac{p_3}{\alpha_3 Q}\left(\frac{Q}{Ax_1^{\alpha} 1 x_2^{\alpha} 2}\right)^{\frac{1}{\alpha_3}}}{(s + c_e + c_s)(1 - y'(Q))} \tag{5.8}$$

Proof: For $n = 3$, the partial derivative of the objective function with respect to x_1 results in following equation

$$\frac{\partial\overline{\Pi}(Q)}{\partial x_1} = a(cs-(s+c_e+c_s)F(Q-y(Q)))\frac{d\delta}{dx_1} - p_1 + \frac{p_3}{x_1}\left(\frac{Q}{Ax^{\alpha}_{1\,1}x^{\alpha}_{2\,2}}\right)^{\frac{1}{\alpha_3}}\frac{\alpha_1}{\alpha_3} \quad (5.9)$$

where $\frac{\partial\delta}{\partial x_1} = b\left(\frac{\beta_1x_1+\beta_2x_2+\beta_3\left(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}}}{Q}\right)^{b-1}\left(\frac{\beta_1+\beta_3(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_3}}\frac{-\alpha_1}{\alpha_3}\frac{1}{x_1}}{Q}\right)$

Then the partial derivative set to zero results in

$$a(c_s-(s+c_e+c_s)F(Q-y(Q))) = \frac{p_1+p_3\left(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}}\frac{-\alpha_1}{\alpha_3}\frac{1}{x_1}}{b\left(\frac{\beta_1x_1+\beta_2x_2+\beta_3(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_2}}}{Q}\right)^{b-1}\left(\frac{\beta_1+\beta_3(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_3}}\frac{-\alpha_1}{\alpha_2}\frac{1}{x_1}}{Q}\right)} \quad (5.10)$$

In a similar fashion, the partial derivative of the objective function with respect to x_2 is given by:

$$\frac{\partial\overline{\Pi}(Q)}{\partial x_2} = a(cs-(s+c_e+c_s)F(Q-y(Q)))\frac{d\delta}{dx_2} - p_2 + \frac{p_3}{x_2}\left(\frac{Q}{A\prod_{k=1}^{2}x^{\alpha}_{k\,k}}\right)^{\frac{1}{\alpha_3}}\frac{\alpha_2}{\alpha_3} \quad (5.11)$$

where $\frac{\partial\delta}{\partial x_2} = b\left(\frac{\beta_1x_1+\beta_2x_2+\beta_3\left(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}}}{Q}\right)^{b-1}\left(\frac{\beta_2+\beta_3(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_3}}\frac{-\alpha_2}{\alpha_3}\frac{1}{x_2}}{Q}\right)$

Then the partial derivative set to zero results in

$$
a(c_s-(s+c_e+c_s)F(Q-y(Q)))=\frac{p_2+p_3\left(\frac{Q}{Ax_1^{\alpha_1}x_1^{\alpha_1}}\right)^{\frac{1}{\alpha_3}}\frac{-\alpha_2}{\alpha_3}\frac{1}{x_2}}{b\left(\frac{\beta_1x_1+\beta_2x_2+\beta_3\left(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}}\right)^{\frac{1}{\alpha_3}}}{Q}\right)^{b-1}\left(\frac{\beta_2+\beta_3(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_3}}\frac{-\alpha_2}{\alpha_3}\frac{1}{x_2}}{Q}\right)} \tag{5.12}
$$

To find a ratio between x_1 and x_2 values, the equations (5.10) and (5.12) are used and after the division the following relation is found.

$$
1=\frac{p_1+p_3\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}\frac{-\alpha_1}{\alpha_3}\frac{1}{x_1}}{\left(\beta_1+\beta_3(\frac{Q}{A\prod_{k=1}^{2}x_k^{\alpha_k}})^{\frac{1}{\alpha_3}}\frac{-\alpha_1}{\alpha_3}\frac{1}{x_1}\right)}.\frac{\left(\beta_2+\beta_3(\frac{Q}{A\prod_{k=1}^{2}x_k^{\alpha_k}})^{\frac{1}{\alpha_3}}\frac{-\alpha_2}{\alpha_3}\frac{1}{x_2}\right)}{p_2+p_3\left(\frac{Q}{A\prod_{k=1}^{2}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_3}}\frac{-\alpha_2}{\alpha_3}\frac{1}{x_2}} \tag{5.13}
$$

Then using the above ratio, the resulting equation is found as follows:

$$
\frac{\alpha_2}{x_2}(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_3}}\frac{1}{\alpha_3}(\beta_1p_3-p_1\beta_3)+\frac{\alpha_1}{x_1}(\frac{Q}{Ax_1^{\alpha_1}x_2^{\alpha_2}})^{\frac{1}{\alpha_3}}\frac{1}{\alpha_3}(\beta_3p_2-p_3\beta_2)=\beta_1p_2-p_1\beta_2
$$

In a similar fashion, the partial derivative of the objective function with respect to Q results in following equation

$$
\begin{aligned}
\frac{\partial\overline{\Pi}(Q)}{\partial Q}=&F(Q-y(Q))[(y'(Q)-1)(s+c_e+c_s)]+(s-c+c_s(1-y'(Q))) \\
&+\frac{p_3Q^{\frac{1}{\alpha_3}-1}}{\alpha_3\left(\prod_{k=1}^{2}Ax_k^{\alpha}{}_k\right)^{\frac{1}{\alpha_3}}}
\end{aligned} \tag{5.14}
$$

Then, the partial derivative set to zero results in

$$F(Q-y(Q)) = \frac{s-c+c_s(1-y'(Q))+\frac{p_3}{\alpha_3 Q}(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2})^{\frac{1}{\alpha_3}}}{(s+c_e+c_s)(1-y'(Q))}$$

We next consider the case $n > 3$ with more than three inputs. The following result is obtained for this case.

Theorem 6: Suppose n>3. Then the optimal order quantity, Q^*, x_1, and x_2 satisfy the following equation.

$$a(c_s-(s+c_e+c_s)F(Q-y(Q))) = \frac{p_1-\sum_{k=3}^{n-1} p_k F(x_1,x_2)+p_n I(x_1,x_2)H(x_1,x_2)}{\frac{b}{Q}(\beta(x_1,x_2,Q))^{b-1}[\beta_1-\sum_{k=3}^{n-1}\beta_k F(x_1,x_2)+\beta_n I(x_1,x_2)H(x_1,x_2)]} \tag{5.15}$$

where $\gamma = (\beta_1 p_n - p_1\beta_n)(\beta_1 p_2 - p_1\beta_2), \delta = (\beta_1 p_n - p_1\beta_n)(\beta_1 p_k - p_1\beta_k)$,
$z = (\beta_1 p_k - p_1\beta_k)(\beta_2 p_n - p_n\beta_2) - (\beta_1 p_2 - p_1\beta_2)(\beta_k p_n - p_k\beta_n)$,
$\beta(x_1,x_2,Q) = (\frac{\beta_1 x_1+\beta_2 x_2+\beta_k\sum_{k=3}^{n-1}\frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}}+\beta_n(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2\prod_{k=3}^{n-1}(\frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}})^{\alpha_k}})^{\frac{1}{\alpha_n}}}{Q})$,
$F(x_1,x_2) = \frac{\gamma\alpha_k}{\left(\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}\right)^2}, H(x_1,x_2) = \frac{1}{\alpha_n x_1}[\frac{\alpha_k(n-3)\alpha_1 z}{\delta\frac{\alpha_2 x_1}{x_2}-z\alpha_1} - \alpha_1]$
$I(x_1,x_2) = (\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2(\frac{\gamma\alpha_k}{\delta\alpha_k\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}})^{\alpha_k}})^{\frac{1}{\alpha_n}}$.

Proof: The partial derivatives of the objective function with respect to x_i, $i = 1,..n-1$ are given by

$$\frac{d\overline{\Pi}(Q)}{dx_i} = a(cs-(s+c_e+c_s)F(Q-y(Q)))\frac{d\delta}{dx_i}-p_i+\frac{p_n}{x_i}\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha}{}_k}\right)^{\frac{1}{\alpha_n}}\frac{\alpha_i}{\alpha_n} \tag{5.16}$$

where $\frac{d\delta}{dx_i} = b\left(\frac{\sum_{i=1}^{n-1}\beta_i x_i(Q)+\beta_n\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}}{Q}\right)^{b-1}\left(\frac{\beta_i+\beta_n(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}})^{\frac{1}{\alpha_n}}\frac{-\alpha_i}{\alpha_n}\frac{1}{x_i}}{Q}\right)$

Setting (5.16) to zero we get

$$a(c_s-(s+c_e+c_s)F(Q-y(Q)))=\frac{p_i+p_n\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}\frac{-\alpha_i}{\alpha_n}\frac{1}{x_i}}{b\left(\frac{\sum_{i=1}^{n-1}\beta_i x_i(Q)+\beta_n\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}}{Q}\right)^{b-1}\left(\frac{\beta_i+\beta_n(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}})^{\frac{1}{\alpha_n}}\frac{-\alpha_i}{\alpha_n}\frac{1}{x_i}}{Q}\right)} \tag{5.17}$$

Note that in the above equation the LHS is independent of i and the first term in the denominator of the RHS is also independent of i. Hence the RHS of equation (5.17) holds for $x_i \neq x_j$.

We observe that the following relation should hold for any $i \neq j$ where $i, j \neq n$.

$$1=\frac{p_i+p_n\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}\frac{-\alpha_i}{\alpha_n}\frac{1}{x_i}}{\left(\beta_i+\beta_n(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}})^{\frac{1}{\alpha_n}}\frac{-\alpha_i}{\alpha_n}\frac{1}{x_i}\right)}.\frac{\left(\beta_j+\beta_n(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}})^{\frac{1}{\alpha_n}}\frac{-\alpha_j}{\alpha_n}\frac{1}{x_j}\right)}{p_j+p_n\left(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}\frac{-\alpha_j}{\alpha_n}\frac{1}{x_j}} \tag{5.18}$$

Then referring to the 5.18, the resulting equation is found for any $i \neq j$ where $i, j \neq n$ as follows:

$$\begin{aligned}\beta_i p_j - p_i\beta_j =& \frac{\alpha_j}{x_j}(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}})^{\frac{1}{\alpha_n}}\frac{1}{\alpha_n}(\beta_i p_n - p_i\beta_n)\\ &+\frac{\alpha_i}{x_i}(\frac{Q}{A\prod_{k=1}^{n-1}x_k^{\alpha_k}})^{\frac{1}{\alpha_n}}\frac{1}{\alpha_n}(\beta_n p_j - p_n\beta_j)\end{aligned} \tag{5.19}$$

Then referring to the 5.19, we have the following ratio

$$\frac{\left(\frac{Q}{A\prod_{k=1}^{n-1} x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}} \frac{1}{\alpha_n}\left[\frac{\alpha_j}{x_j}(\beta_1 p_n - p_1\beta_n) - \frac{\alpha_1}{x_1}(\beta_j p_n - p_j\beta_n)\right]}{\left(\frac{Q}{A\prod_{k=1}^{n-1} x_k^{\alpha_k}}\right)^{\frac{1}{\alpha_n}} \frac{1}{\alpha_n}\left[\frac{\alpha_k}{x_k}(\beta_1 p_n - p_1\beta_n) - \frac{\alpha_1}{x_1}(\beta_k p_n - p_k\beta_n)\right]} = \frac{\beta_1 p_j - p_1\beta_j}{\beta_1 p_k - p_1\beta_k} \quad (5.20)$$

Then, x_k for $k = i+1, \ldots n-1$ is written as

$$x_k = \frac{a_{12}a_{1n}\alpha_k}{a_{1n}a_{1k}\frac{\alpha_2}{x_2} - (a_{1k}a_{2n} - a_{12}a_{kn})\frac{\alpha_1}{x_1}} \quad (5.21)$$

where $a_{ij} = (\beta_i p_j - p_i\beta_j)$ for all 1 ≤ i ≤ j ≤ n

Substituting x_k, which is a function of x_1 and x_2, into our existing objective function transforms our objective function into a function of three variables, x_1, x_2 and Q, independent of n. The new form of the $((\sum_{i=1}^{n}\beta_i x_i(Q))/Q)^b$ is written as follows:

$$\left(\frac{\beta_1 x_1 + \beta_2 x_2 + \sum_{k=3}^{n-1}\frac{\beta_k a_{12}a_{1n}\alpha_k}{a_{1n}a_{1k}\frac{\alpha_2}{x_2} - (a_{1k}a_{2n} - a_{12}a_{kn})\frac{\alpha_1}{x_1}} + \beta_n\left(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2\prod_{k=3}^{n-1}\left(\frac{a_{12}a_{1n}\alpha_k}{a_{1n}a_{1k}\frac{\alpha_2}{x_2} - (a_{1k}a_{2n} - a_{12}a_{kn})\frac{\alpha_1}{x_1}}\right)^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}}{Q}\right)^b \quad (5.22)$$

To simplify the notation of the $((\sum_{i=1}^{n}\beta_i x_i(Q))/Q)^b$,let $a_{1n}a_{12}=\gamma$, $a_{1n}a_{1k}=\delta$, $a_{1k}a_{2n}$-$a_{12}a_{kn}= z$. Then, $(\sum_{i=1}^{n}\beta_i x_i(Q)/Q)^b$ is written as follows

$$(\frac{\sum_{i=1}^{n}\beta_i x_i(Q)}{Q})^b = \left(\frac{\beta_1 x_1+\beta_2 x_2+\sum_{k=3}^{n-1}\beta_k \frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}}+\beta_n\left(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2\prod_{k=3}^{n-1}\left(\frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}}\right)^{\alpha_k}}\right)^{\frac{1}{\alpha_n}}}{Q}\right)^b \tag{5.23}$$

In a similar fashion, under this setting, $(\sum_{i=1}^{n} p_i x_i(Q))$ is written as follows

$$\begin{aligned}(\sum_{i=1}^{n} p_i x_i(Q)) =& p_1 x_1 + p_2 x_2 + \sum_{k=3}^{n-1} p_k \frac{a_{12}a_{1n}\alpha_k}{a_{1n}a_{1k}\frac{\alpha_2}{x_2}-(a_{1k}a_{2n}-a_{12}a_{kn})\frac{\alpha_1}{x_1}} \\ &+ p_n\left(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2\prod_{k=3}^{n-1}\left(\frac{a_{12}a_{1n}\alpha_k}{a_{1n}a_{1k}\frac{\alpha_2}{x_2}-(a_{1k}a_{2n}-a_{12}a_{kn})\frac{\alpha_1}{x_1}}\right)^{\alpha_k}}\right)^{(\frac{1}{\alpha_n})}\end{aligned}$$

Again, to simplify the notation of the $(\sum_{i=1}^{n} p_i x_i(Q))$, let $a_{1n}a_{12}=\gamma$, $a_{1k}a_{2n}$-$a_{12}a_{kn}= z$, $a_{1n}a_{1k}= \delta$. Then, $(\sum_{i=1}^{n} p_i x_i(Q))$ is written as follows

$$(\sum_{i=1}^{n} p_i x_i(Q)) = p_1 x_1 + p_2 x_2 + \sum_{k=3}^{n-1} p_k \frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}} + p_n\left(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2\prod_{k=3}^{n-1}\left(\frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2}-z\frac{\alpha_1}{x_1}}\right)^{\alpha_k}}\right)^{\frac{1}{\alpha_n}} \tag{5.24}$$

Under this setting, the expected profit function and its partial derivative with respect to x_1 are given by following equations. Since $(\sum_{i=1}^{n} p_i x_i(Q))$ and

$((\sum_{i=1}^{n} \beta_i x_i(Q))/Q)^b$ became functions of x_1,x_2 and Q independent of n, we reduced the number of variables in the objective function from n: x_1,x_2,...x_{n-1},Q into three: x_1,x_2,Q.

$$\begin{aligned}\overline{\Pi}(Q, x_1, x_2) =& sE[\min(Q, D)] - c_s E[\max(0, (D - Q))] - c_e E[\max(0, (Q - D))] \\ & - cQ - (\sum_{i=1}^{n} p_i x_i(Q))\end{aligned}$$

where $D = y(Q) + \epsilon$ and $y(Q) = B(1 - a(\frac{\sum_{i=1}^{n} \beta_i x_i(Q)}{Q})^b)$

$$\begin{aligned}\frac{\partial \overline{\Pi}(Q, x_1, x_2)}{\partial x_1} =& a(c_s - (s + c_e + c_s)F(Q - y(Q)))(\frac{\partial (\frac{\sum_{i=1}^{n} \beta_i x_i(Q)}{Q})^b}{\partial x_1}) \\ & - p_1 + \sum_{k=3}^{n-1} p_k \frac{\gamma \alpha_k}{(\delta \frac{\alpha_2}{x_2} - z \frac{\alpha_1}{x_1})^2} \frac{\alpha_1 z}{x_1^2} \\ & - p_n \left(\frac{Q}{A x_1^{\alpha} {}_1 x_2^{\alpha} {}_2 \left(\frac{\gamma \alpha_k}{\delta \frac{\alpha_2}{x_2} - z \frac{\alpha_1}{x_1}} \right)^{\alpha_k}} \right)^{\frac{1}{\alpha_n}} \frac{1}{\alpha_n x_1} [\frac{\alpha_k (n-3) \alpha_1 z}{\delta \frac{\alpha_2 x_1}{x_2} - z \alpha_1} - \alpha_1]\end{aligned}$$

The expression $(\partial (\frac{\sum_{i=1}^{n} \beta_i x_i(Q)}{Q})^b / \partial x_1)$ is written as follows by using appropriate simplifications.

$$\frac{d(\frac{\sum_{i=1}^{n} \beta_i x_i(Q)}{Q})^b}{dx_1} = \frac{b}{Q} \left(\beta(x_1, x_2, Q)\right)^{b-1} \left(\beta_1 - \sum_{k=3}^{n-1} \beta_k F(x_1, x_2) + \beta_n I(x_1, x_2) H(x_1, x_2)\right) \tag{5.25}$$

where $\beta(x_1, x_2, Q)$,$F(x_1, x_2)$, $I(x_1, x_2)$ are functions such that

$$\beta(x_1, x_2, Q) = (\frac{\beta_1 x_1 + \beta_2 x_2 + \beta_k \sum_{k=3}^{n-1} \frac{\gamma \alpha_k}{\delta \frac{\alpha_2}{x_2} - z \frac{\alpha_1}{x_1}} + \beta_n (\frac{Q}{A x_1^{\alpha} {}_1 x_2^{\alpha} {}_2 \prod_{k=3}^{n-1} (\frac{\gamma \alpha_k}{\delta \frac{\alpha_2}{x_2} - z \frac{\alpha_1}{x_1}})^{\alpha_k}})^{\frac{1}{\alpha_n}}}{Q}),$$

$F(x_1,x_2) = \frac{\gamma\alpha_k}{\left(\delta\frac{\alpha_2}{x_2} - z\frac{\alpha_1}{x_1}\right)^2}, H(x_1,x_2) = \frac{1}{\alpha_n x_1}[\frac{\alpha_k(n-3)\alpha_1 z}{\delta\frac{\alpha_2 x_1}{x_2} - z\alpha_1} - \alpha_1]$
$I(x_1,x_2) = (\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2 (\frac{\gamma\alpha_k}{\delta\alpha_k\frac{\alpha_2}{x_2} - z\frac{\alpha_1}{x_1}})^{\alpha_k}})^{\frac{1}{\alpha_n}}$

Then, by using the functions $\beta(x_1,x_2,Q)$,$F(x_1,x_2)$, $I(x_1,x_2)$, the partial derivative of the objective function with respect to x_1 is given in the following equation.

$\frac{\partial\overline{\Pi}(Q,x_1,x_2)}{\partial x_1} =$
$(\mathrm{c}_s - (s+c_e+c_s)F(Q-y(Q)))\frac{ab}{Q}\left(\beta_1 - \sum_{k=3}^{n-1}\beta_k F(x_1,x_2) + \beta_n I(x_1,x_2)H(x_1,x_2)\right)\beta(x_1,x_2,Q)^{b-1}$
$-p_1 + \sum_{k=3}^{n-1} p_k F(x_1,x_2,Q)$
$-p_n I(x_1,x_2,Q)H(x_1,x_2,Q)$

Setting the above equation to zero results in

$$a(c_s - (s+c_e+c_s)F(Q-y(Q))) = \frac{p_1 - \sum_{k=3}^{n-1} p_k F(x_1,x_2) + p_n I(x_1,x_2)H(x_1,x_2)}{\frac{b}{Q}(\beta(x_1,x_2,Q))^{b-1}[\beta_1 - \sum_{k=3}^{n-1}\beta_k F(x_1,x_2) + \beta_n I(x_1,x_2)H(x_1,x_2)]} \quad (5.26)$$

where $\gamma = (\beta_1 p_n - p_1\beta_n)(\beta_1 p_2 - p_1\beta_2), \delta = (\beta_1 p_n - p_1\beta_n)(\beta_1 p_k - p_1\beta_k)$,
$z = (\beta_1 p_k - p_1\beta_k)(\beta_2 p_n - p_n\beta_2) - (\beta_1 p_2 - p_1\beta_2)(\beta_k p_n - p_k\beta_n)$,
$\beta(x_1,x_2,Q) = (\frac{\beta_1 x_1 + \beta_2 x_2 + \beta_k \sum_{k=3}^{n-1}\frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2} - z\frac{\alpha_1}{x_1}} + \beta_n(\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2 \prod_{k=3}^{n-1}(\frac{\gamma\alpha_k}{\delta\frac{\alpha_2}{x_2} - z\frac{\alpha_1}{x_1}})^{\alpha_k}})^{\frac{1}{\alpha_n}}}{Q})$,
$F(x_1,x_2) = \frac{\gamma\alpha_k}{\left(\delta\frac{\alpha_2}{x_2} - z\frac{\alpha_1}{x_1}\right)^2}, H(x_1,x_2) = \frac{1}{\alpha_n x_1}[\frac{\alpha_k(n-3)\alpha_1 z}{\delta\frac{\alpha_2 x_1}{x_2} - z\alpha_1} - \alpha_1]$
$I(x_1,x_2) = (\frac{Q}{Ax_1^{\alpha}{}_1 x_2^{\alpha}{}_2 (\frac{\gamma\alpha_k}{\delta\alpha_k\frac{\alpha_2}{x_2} - z\frac{\alpha_1}{x_1}})^{\alpha_k}})^{\frac{1}{\alpha_n}}$.

Chapter 6

Numerical Studies

In this chapter we provide the results of the numerical experiments conducted to analyze the impacts of problem parameters on the optimal policies of the problems we discussed in previous chapters. The numerical study findings and discussions are provided under three sections.

In section 6.1, the effects of input cost parameters and carbon coefficients on the optimal order quantity, expected profit and allocation of the inputs are investigated for a supply chain with two inputs under the decentralized problem, Retailer's Problem with an Independent Manufacturer. A real agricultural production example of Hatirli et al. [11] is also examined under the decentralized model. In section 6.2, we investigate similar effects for a supply chain with two inputs under the centralized problem, Integrated Problem of the Retailer and the Manufacturer.

In section 6.3, a comparison between the numerical results of the centralized model and decentralized is made.

6.1 Decentralized Model

In this section, we first present the experimental settings used for the sensitivity analysis of the decentralized model in the subsection 6.1.1. The results of the sensitivity analysis of the decentralized model are given in the subsection 6.1.2. As a last part, a real-life agricultural production application is done under the decentralized model and results are presented in the subsection 6.1.3.

6.1.1 Experimental Settings for the Sensitivity Analysis

We consider a supply chain with two inputs, $n = 2$ and decreasing rate of return, $r < 1$ where both elasticities $\alpha_1 = 0.45$ and $\alpha_2 = 0.45$. We assume the technology level $A = 1$. The $(1+\delta)$ parameter which was the manufacturer's expected profit coefficient is set 1.2, and he set the positive coefficient $a = 0.1$ for the cases where the customers are sensitive to the carbon emission levels, $b > 0$. For the insensitive customers, the positive coefficient $a = 0$. Therefore we represent the carbon sensitivity parameters as (b, a) where $(0, 0)$ represents the customers insensitive to the carbon emission, $(0.3, 0.1)$, $(0.5, 0.1)$, $(0.7, 0.1)$, $(1, 0.1)$ denote the cases where the customers are sensitive to the carbon emission level of the product.

We set the selling price $s = 50$, the fixed acquisition cost $c = 1$, the shortage cost $c_s = 10$ and we calculate the excess cost $c_e = 3.157$ based on the assumption of 95% service level in the classical newsvendor problem. The mean demand B is assumed to be 100 and the error term ϵ normally distributed with mean 0 and $\sigma = 30$.

To understand the sensitivity of the optimal policy parameters with respect to change in input cost parameters and carbon coefficients, four parameter sets are determined.

1) $p_1 = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$

$p_2 = 0.4,\ \beta_1 = 0.1,\ \beta_2 = 0.25.$

2) $p_2 = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$

$p_1 = 0.4,\ \beta_1 = 0.1,\ \beta_2 = 0.25.$

3) $\beta_1 = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$

$p_1 = 0.3,\ p_2 = 0.4,\ \beta_2 = 0.25.$

4) $\beta_2 = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$

$p_1 = 0.3,\ p_2 = 0.4,\ \beta_1 = 0.25.$

Each of these four parameter sets are investigated under five different carbon sensitivity parameters, i.e. (b, a) which are $(0, 0)$, $(0.3, 0.1)$, $(0.5, 0.1)$, $(0.7, 0.1)$, $(1, 0.1)$ in the sensitivity analysis part.

6.1.2 Sensitivity Analysis

In the sensitivity analysis of the decentralized problem, the experimental settings presented in the previous subsection are used and under the assumption that the error term has a normal distribution with mean zero and variance σ^2, we write the expected profit of the decentralized problem (4.5) as follows

$$\begin{aligned}\overline{\Pi}(Q) = &\int_{-\infty}^{Q-y(Q)} (s(y(Q)+u) - c_e(Q - y(Q) - u) - cQ)\frac{e^{-\frac{u^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}du \\ &\int_{Q-y(Q)}^{\infty} (sQ - cQ - c_s(y(Q) + u - Q))\frac{e^{-\frac{u^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}du \\ &- (1+\delta)(\sum_{i=1}^{n} p_i x_i(Q))\end{aligned} \tag{6.1}$$

Then the optimization problem of the retailer (4.6) is written as follows

$$\begin{aligned} &\underset{Q}{Max} && \overline{\Pi}(Q) \\ &s.t. && \overrightarrow{x}, Q \geq 0 \end{aligned} \tag{6.2}$$

and the first order condition of the problem (4.18) becomes:

$$\frac{c_s}{s + c_e + c_s} = F(w(Q)) - \frac{1}{(s + c_e + c_s)w'(Q)} \left((s - c) - \frac{(1+\delta)\psi Q^{\frac{1}{r}-1}}{r} \right) \tag{6.3}$$

For computations, we use MATLAB to examine each of four parameter set given in 6.1.1 under five different carbon sensitivity parameters, $(0,0)$,$(0.3,0.1)$,$(0.5,0.1)$,$(0.7,0.1)$, $(1,0.1)$. Below, we provide a pseudocode that we use in MATLAB for the analysis.

Algorithm 1 Sensitivity Analysis of the Decentralized Problem

```
BEGIN
INITIALIZE parameters s,c_s,c,c_e, δ,B,σ,α_1,α_2,p_1,p_2,β_1,β_2,a,b.
SET Qunc := Q* of classical EOQ Model under Given Parameters
SET X := Q* of classical EOQ Model under Given Parameters
for Each sensitivity level (b,a):=(0,0),(0.3,0.1),(0.5,0.1),(0.7,0.1),(1,0.1) do
    Set Qunc := FoundRoot of Equation 6.3.
end for
for Each sensitivity level (b,a):=(0,0),(0.3,0.1),(0.5,0.1),(0.7,0.1),(1,0.1) do
    Set X = Found Order Quantity which Maximizes the Problem 6.2.
end for
```

In this algorithm, we first initialize the parameters a and b according to the carbon sensitivity level we work on from our set $(0,0)$,$(0.3,0.1)$,$(0.5,0.1)$,$(0.7,0.1)$, $(1,0.1)$. Then, by considering the four input cost and carbon emission parameter sets of the study given in 6.1.1, for each set, the p_1,p_2,β_1,β_2 are initialized. To provide an initial point for conducting search, we start with the optimal order quantity which is obtained under the classical EOQ model and our given parameters. Then, Equation 6.3 is solved by $fsolve$ function of MATLAB and as an alternative, starting from the same initial point, the optimization problem in 6.2 is solved by $fmincon$ function of the MATLAB. It also provides us to check the correctness of the solution we found.

The detailed results of the study are presented in Figure 6.1, 6.2 and 6.3 for $(b, a) = (0, 0)$, Figure 6.4, 6.5 and 6.6 for $(b, a) = (0.3, 0.1)$, Figure 6.7, 6.8 and 6.9 for $(b, a) = (0.5, 0.1)$, Figure 6.10, 6.11 and 6.12 for $(b, a) = (0.7, 0.1)$, Figure 6.13, 6.14 and 6.15 for $(b, a) = (1, 0.1)$. The exact results are also presented in Table 6.1 for $(b, a) = (0, 0)$, 6.2 for $(b, a) = (0.3, 0.1)$, 6.3 for $(b, a) = (0.5, 0.1)$, 6.4 for $(b, a) = (0.7, 0.1)$, 6.5 for $(b, a) = (1, 0.1)$. The behavior of the expected profit function is also investigated by focusing on the curve of the expected profit function under four different cases. The Figure 6.31 highlights that along the search interval we used, the expected profit of the problem generally behaves like a concave function. This situation prevents ignoring some possible local maximum points in the search interval.

First, we observe the effect of changes in p_1, p_2, β_1, β_2 on the optimal order quantity, optimal input allocation and expected profit where the carbon sensitivity level of the customers, $(b, a) = (0, 0)$. In other words, under this setting, the customers are insensitive to carbon emission level of the products they buy. As it is expected, for $(b, a) = (0, 0)$, the effective demand is found as equal to mean demand B and since the customers are insensitive, the changes in the carbon emission parameters β_1, β_2 do not cause an alteration in the optimal order quantity, allocation of inputs and expected profit as we can observe from I.c, II.c, I.d, II.d of Figure 6.1, 6.3 and Table 6.1. The changes in p_1 and p_2 are also investigated under $(b, a) = (0, 0)$ and it is found that the optimal order quantity and corresponding expected profit follow a decreasing trend when the p_1 or p_2 increases. Note that the change in p_1 and p_2 result in the same ratio between optimal input allocation, x_1 and x_2 without an alteration based on the carbon emission level parameters. It is also expected under this setting since both the manufacturer does the optimization by only considering the cost of inputs he use and the customers are assumed to be insensitive to carbon emission level of the product. It can be observed in I.a, I.b, II.a, II.b of Figure 6.1 and 6.2and Table 6.1.

For the customers with carbon sensitivity level $(b, a) = (0.3, 0.1)$, Table 6.2 clearly shows that the ratio of x_1 and x_2 gives the same result if the carbon emission level parameter β_1 or β_2 changes while all other parameters are constant. It again implies that the optimal input allocation is only affected from the change in the cost of inputs, p_1 and p_2 even if the each input has different carbon emission level and the customers are sensitive to carbon emission levels. The effective demand under this setting becomes a value which is less than mean demand since the the customers are now sensitive to the carbon footprint of the product. The optimal order quantity and corresponding expected profit level follows a decreasing pattern when the costs of inputs or carbon emission parameters change. The same results hold for the other carbon sensitivity levels $(b, a) = (0.5, 0.1)$, $(b, a) = (0.7, 0.1)$ as it is shown in Figure 6.7 and 6.10.

From the Figure 6.13 and Table 6.5 we observe the customers which have carbon sensitivity level, $(b, a) = (1, 0.1)$.This case is expected to be most affected from the changes in the input parameters. From II.a, II.b, II.c, II.d of Figure 6.13, we can clearly observe that the expected profit values associated with the optimal order quantities become more nonlinear. The effect of changes in the cost of input one, p_1 shows that the effective demand decreases when the cost of input with low carbon emitted increases while other parameters are constant. On the other hand, the change in the cost of input two, p_2 indicates that the effective demand rate of the customers increases when the cost of input with high carbon emission level increases. The logic behind both results is explained as follows. From previous discussion, we know that the production process relies on the low cost input since the manufacturer does his own optimization to minimize his own production cost. If the input type selected to be mainly used is the one with low carbon emission level, the effective demand increases if the input with high carbon emission level also have a higher cost.

6.1.3 Real-Life Agricultural Production Application

In this section, we investigate the work of Hatirli et. al. [11] and the numerical study of Sözüer [45] on this work. The study of Hatirli et. al focus on the relationship between energy inputs and crop yield for tomato production in Antalya, Turkey. We use the example based on the work of Hatirli et. al. [11] and constructed by Sözüer [45] and investigate it under our decentralized model.

In the example, the order quantity, Q represents the greenhouse tomato production where the inputs, fertilizer (x_1), chemicals (x_2), labor (x_3), machinery (x_4) and water for irrigation (x_5). The measured unit of each input is taken as their energy equivalents in mega joules during the study.

The vector of input elasticities (α), input prices (p) and carbon coefficients (β) are taken from the data provided in [11]. The α vector is scaled to provide a DRS setting. The corresponding vectors are as follows α=[0.1714 0.642 0.107 0.4 0.164],β=[0.0197 0.0504 0.0398 0.3542 0.2951],p=[0.1278 0.5929 1.3043 1.7217 0.2689].

The technology level is taken $A = 1.34$, the selling price for tomato $s = 2.875 TL/MJ$ from [45]. The mean demand B is assumed to be $130000 MJ$,$(1 + \delta) = 1$.

In the analysis, we investigate this example for each of five carbon sensitivity parameter in a similar procedure expressed in the previous chapter. We set the carbon sensitivity parameter (b, a) as $(0, 0)$ for insensitive customers, (0.3,0.1),(0.5,0.1),(0.7,0.1),(1,0.1) for sensitive customers. For each carbon sensitivity level, it is assumed that the shortage cost is given by a ratio of the selling price, s and the excess cost is calculated by considering the 95% service level. We conduct our analysis with three different c_s/s ratios: 0.01, 0.02 and 0.04.

We also study the effect of the variance of the random error term of our demand function on the optimal policy parameters. Two different σ values are considered, which are 5000 and 15000. Therefore, we investigate the following cases.

1) (b, a)=$(0, 0)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 5000$

2) (b, a)=$(0, 0)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 15000$

3) (b, a)=$(0.3, 0.1)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 5000$

4) (b, a)=$(0.3, 0.1)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 15000$

5) (b, a)=$(0.5, 0.1)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 5000$

6) (b, a)=$(0.5, 0.1)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 15000$

7) (b, a)=$(0.7, 0.1)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 5000$

8) (b, a)=$(0.7, 0.1)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 15000$

9) (b, a)=$(1, 0)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 5000$

10) (b, a)=$(1, 0)$ and $c_s = \{0.01, 0.02, 0.04\}$

where $\sigma = 15000$

For the computation, the same algorithm explained in the previous section is used and the results are shown in Table 6.11 and 6.12.

Table 6.12 shows that if the customers are insensitive to carbon emission levels, for each c_s/s ratio, we obtain same expected profit values under two different demand variations. For other carbon sensitivity levels, we observe that as c_s/s increases, the expected profit decreases and the percentage of the decrease in expected profit is much higher when the demand variation is high.

The effect of the change in c_s/s ratio on the optimal order quantity is also observed from the Table 6.12. It is understood that an increase in c_s/s from 0.01 to 0.04 results in approximately 4% increase in the optimal order quantity Q^* for each carbon sensitivity level under the given $\sigma = 5000$. When the uncertainty in demand increases and $\sigma = 15000$, an increase in c_s/s from 0.01 to 0.04 leads to approximately 10% increase in the optimal order quantity Q^* under each carbon sensitivity level. It shows the effect of uncertainty in demand on the optimal order quantity.

We observe from the Table 6.11 that the percentage of the changes in the input quantities for different cs/s ratios under a given carbon sensitivity level is approximately same for a given demand variation. The increase in the input quantities is less than 6% with $\sigma = 5000$. When the uncertainty in demand increases and $\sigma = 15000$, the increase in the input quantities is less than 11%. In the other chapters of the numerical study, we could make comments related to the usage of the low cost-low carbon emitted inputs however since the elasticity parameters of the inputs are also different in this setting, we can not easily observe a production system relies on the low carbon emitted inputs when the sensitivity level increases. However we can observe the optimal policy parameters under each carbon sensitivity level and conclude that the optimal strategies for the retailer can be determined under this model.

6.2 Centralized Model

For the Integrated Problem of the Retailer and the Manufacturer, we again observe the sensitivity of the optimal policy parameters with respect to change in input costs, carbon coefficients.

6.2.1 Experimental Settings for the Sensitivity Analysis

We consider a supply chain with two inputs. We assume technology level $A = 1$, $r < 1$ where both elasticities $\alpha_1 = 0.45$ and $\alpha_2 = 0.45$. We set the same cost parameters, the selling price $s = 50$, the fixed acquisition cost $c = 1$, the shortage cost $c_s = 10$ and we calculate $c_e = 3.157$ based on the assumption of 95% service level in the classical newsvendor problem. The mean demand B is assumed to be 100 and the error term ϵ is again assumed to be normally distributed with mean 0 and $\sigma = 30$. The retailer set positive coefficient $a = 0.1$ for the cases where the customers are sensitive to the carbon emission levels, $b > 0$. For the insensitive customers, the positive coefficient $a = 0$. Carbon sensitivity parameters are again as follows $(0, 0)$, $(0.3, 0.1)$, $(0.5, 0.1)$, $(0.7, 0.1)$, $(1, 0.1)$.

We do the sensitivity analysis by following the same procedure explained in the previous problem. Each of four input cost and carbon coefficient parameter set in 6.1.1 are investigated under each carbon sensitivity parameter.

6.2.2 Sensitivity Analysis

For the computations, we use MATLAB. Since the first order condition of the centralized problem results in functions of two variables, Q and x_1, another algorithm with same logic is used. The algorithm is as follows:

Algorithm 2 Sensitivity Analysis of the Centralized Problem

BEGIN
INITIALIZE parameters s,c_s,c,c_e,B,σ,α_1,α_2,p_1,p_2,β_1,β_2,a,b.
SET Qunc := Q^* of classical EOQ Model under Given Parameters
SET X := Q^* of classical EOQ Model under Given Parameters
SET $X_1 := x_1$ estimated for Q^* of classical EOQ Model under Given Parameters (from Eq.4.2)
SET $x_1 := x_1$ estimated for Q^* of classical EOQ Model under Given Parameters (from Eq.4.2)

for Each sensitivity level (b, a):=$(0, 0)$,$(0.3, 0.1)$,$(0.5, 0.1)$,$(0.7, 0.1)$,$(1, 0.1)$ **do**
  Set $Qunc, X_1$:= FoundRoot of Equation 5.1 and 5.2.
end for
for Each sensitivity level (b, a):=$(0, 0)$,$(0.3, 0.1)$,$(0.5, 0.1)$,$(0.7, 0.1)$,$(1, 0.1)$ **do**
  Set X, x_1 = Found Order Quantity and Input One Allocation which Maximizes the Centralized Problem.
end for

For each customer sensitivity parameter, the relationship between the optimal order quantity and the changing input cost/carbon emission parameter with corresponding expected profit are presented in in Figure 6.16, 6.17 and 6.18 for $(b, a) = (0, 0)$, Figure 6.19, 6.20 and 6.21 for $(b, a) = (0.3, 0.1)$, Figure 6.22, 6.23 and 6.24 for $(b, a) = (0.5, 0.1)$, Figure 6.25, 6.26 and 6.27 for $(b, a) = (0.7, 0.1)$, Figure 6.28, 6.29 and 6.30 for $(b, a) = (1, 0.1)$. Table 6.6, 6.7, 6.8, 6.9 and 6.10 also provide exact results of the sensitivity analysis.

We start with observing the effect of changes in p_1, p_2, β_1, β_2 on the optimal order quantity, optimal input allocation and expected profit where the carbon sensitivity level of the customers, $(b, a) = (0, 0)$. In other words, under this setting, the customers are insensitive to carbon emission level of the products they buy. As it is expected, for $(b, a) = (0, 0)$, the effective demand is found as equal to mean demand B and since the customers are insensitive, the changes in the carbon emission parameters β_1, β_2 do not cause an alteration in the optimal order quantity, allocation of inputs and expected profit as we can observe from I.c, I.d of Figure 6.16, 6.18 and Table 6.6. The changes in p_1 and p_2 are also investigated under $(b, a) = (0, 0)$ and it is found that the optimal production quantity follows an increasing pattern whereas the corresponding expected profit follows a decreasing trend when the p_1 or p_2 increases while all other parameters are constant. Note that the change in p_1 and p_2 again result in the same ratio between x_1 and x_2 without an alteration based on the carbon emission level parameters and it can be observed in Figure 6.17. It is also expected under this setting since the customers are insensitive to the carbon emission levels and the retailer chooses the inputs according to its price.

For the customers with sensitivity level $(b, a) = (0.3, 0.1)$, $(b, a) = (0.5, 0.1)$, $(b, a) = (0.7, 0.1)$ $(b, a) = (1, 0.1)$ by observing Tables 6.7, 6.8, 6.9 and 6.10, we understand that the production process relies more on the input with both low price and low carbon emission level. Despite that the price of the input plays a more important role in determining the input mixture, we also observe the effect of carbon emission parameters of the inputs on the optimal input allocation by x_1^*/x_2^* ratios of the given tables. As the sensitivity level of the customers, b increases, it becomes more clear.

As another result, for the sensitivity parameters $(b, a) = (0.5, 0.1)$, $(b, a) = (0.7, 0.1)$ and $(b, a) = (1, 0.1)$, we observe that the optimal order quantity with respect to change in price of input one, p_1 follows a decreasing pattern. It is clearly shown in Figure I.a of 6.22 for $(b, a) = (0.5, 0.1)$, I.a of Figure 6.25 for $(b, a) = (0.7, 0.1)$, I.a of Figure 6.28 for $(b, a) = (1, 0.1)$. The reason lies behind the carbon emission parameter associated with each input during the sensitivity analysis. When solving the model for different costs of input one, the input one has constantly the low carbon emission level if it is compared with the input two (See Table 6.8, 6.9, 6.10). Since the production process have an inclination to rely on the low cost-low carbon emission parameter input, the optimal production quantity decreases when an increase in the cost of the low cost-low carbon emitted input occurs, p_1.The situation is same if we observe the optimal order quantity with respect to change in the carbon emission parameter of the input β_2. Since the input two is associated with a lower price if it is compared with the price of the input one during the analysis, the retailer's optimal production quantity decreases when the carbon emission level of the low cost-low carbon emitting input increases.

Our final observation is that the expected profit follows a decreasing pattern when we change an input cost starting from 0.1 to 1 or carbon emission parameter from 0 to 1 under each carbon sensitivity parameter if we investigate II.a,II.b,II.c,II.d parts of the Figure 6.16, 6.19, 6.22, 6.25, 6.28.

6.3 Comparison of the Centralized and the Decentralized Models

A comparison between Table 6.1 and 6.6 highlights that the centralized version of the problem generally results in a higher optimal order quantity with a higher corresponding expected profit under each carbon sensitivity parameter and same input cost, carbon emission parameters. The increase in the expected profit under the same conditions is approximately 1% and the increase in optimal order quantity is approximately 10%

In the centralized version of the problem, we understand that the production process relies more on the input with both low price and low carbon emission level for the cases where the customers are carbon sensitive. Despite that the price of the input plays a more important role in determining the input mixture, we also observe the effect of carbon emission parameters of the inputs on the optimal input allocation by x_1^*/x_2^* ratios of the given tables, Table 6.1, 6.2, 6.3, 6.4, 6.5 for decentralized, 6.6, 6.7, 6.8, 6.9, 6.10 for centralized problem. As the sensitivity level of the customers, b increases, it becomes more clear. However, the sensitivity of the input allocations to the carbon emission parameters in the decentralized problem shows that since the first stage optimization is done by the manufacturer who selects the input mixture which minimizes his production cost, the amount of each input in the allocation do not change when the carbon emission parameters change.

Figures of The Numerical Study

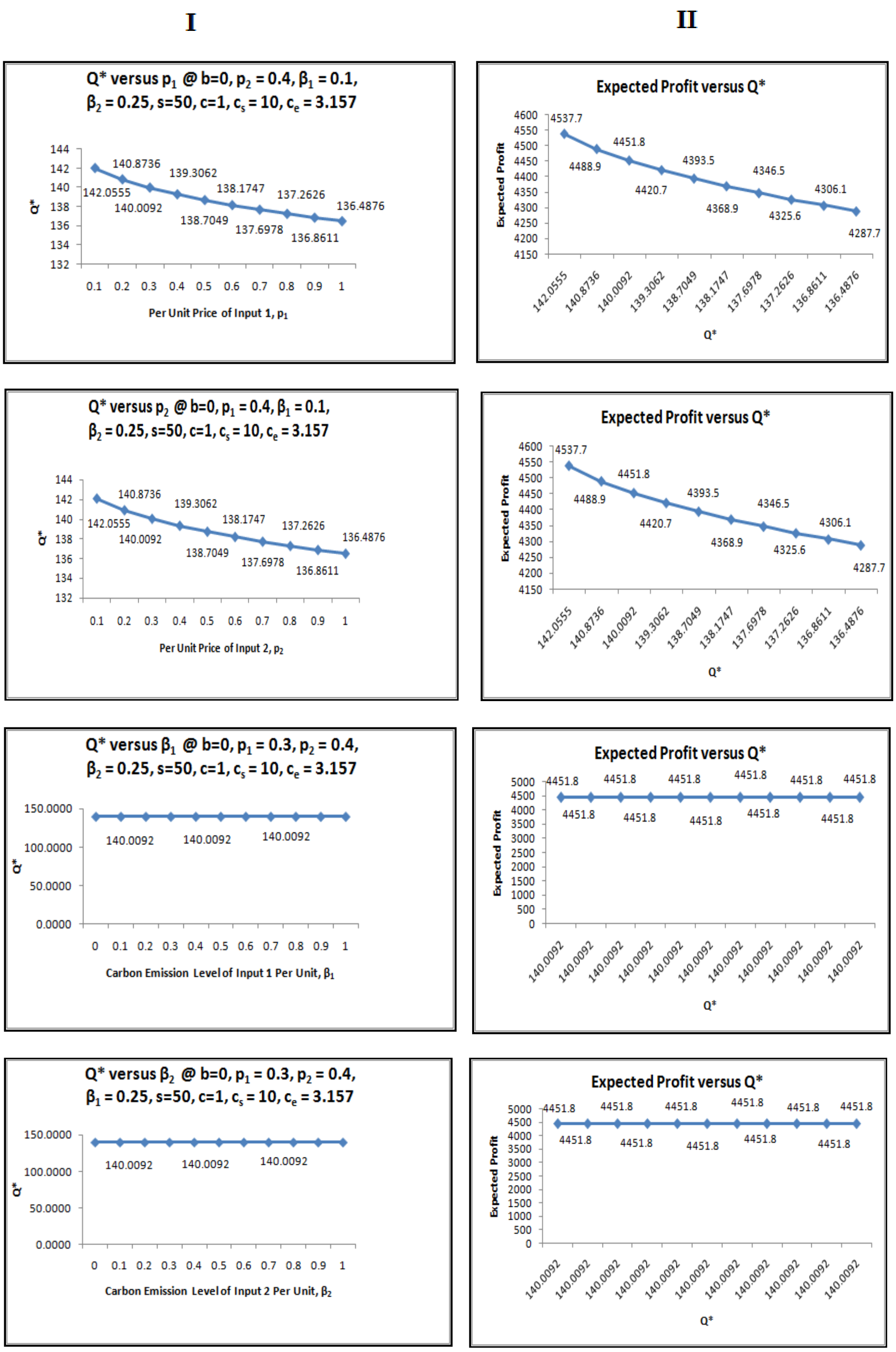


Figure 6.1: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0, 0)$, c=1, s=50, c_s=10, c_e=3.157 with $(1+\delta)$ = 1.2 under the Retailer's Problem with an Independent Manufacturer.

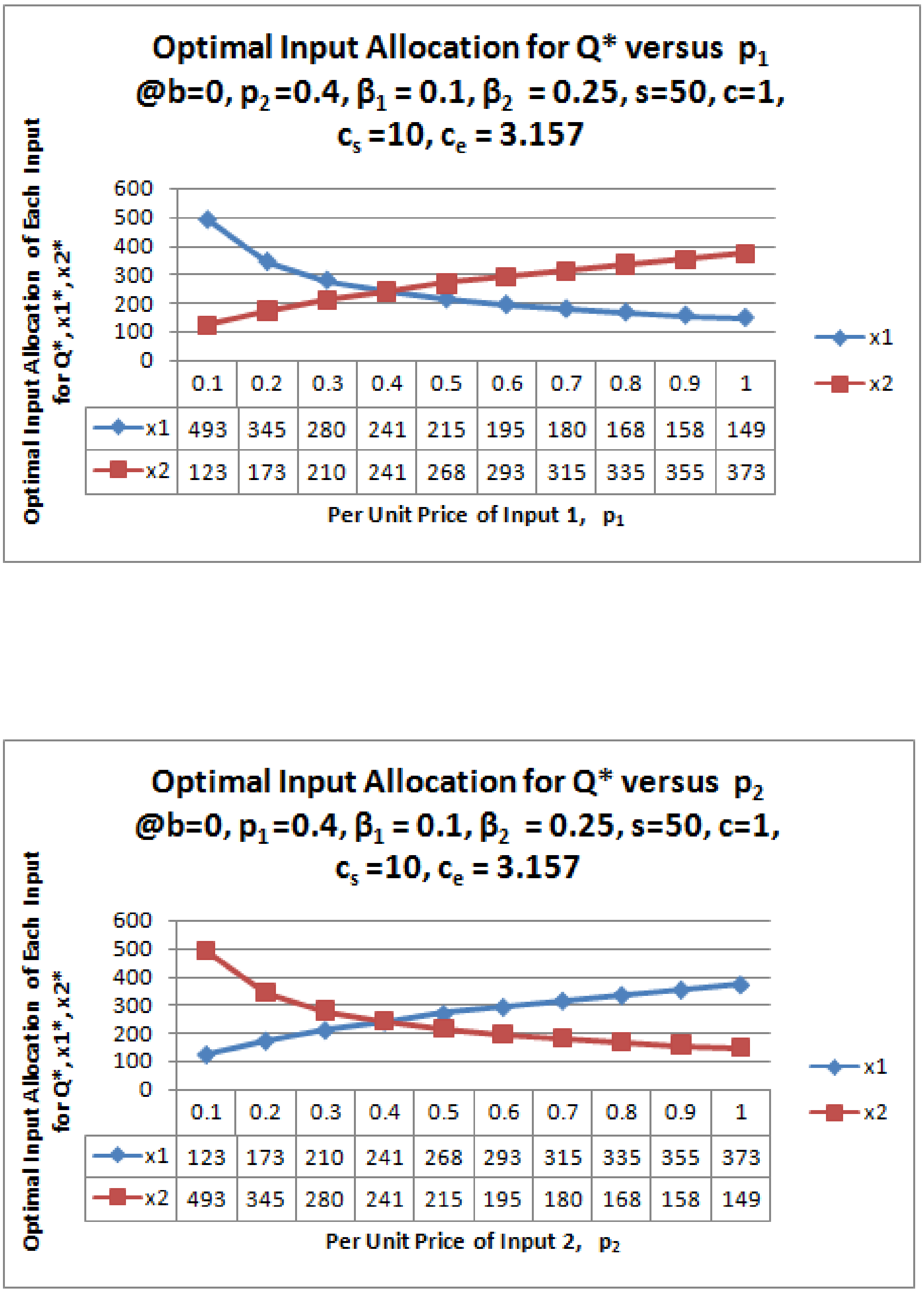
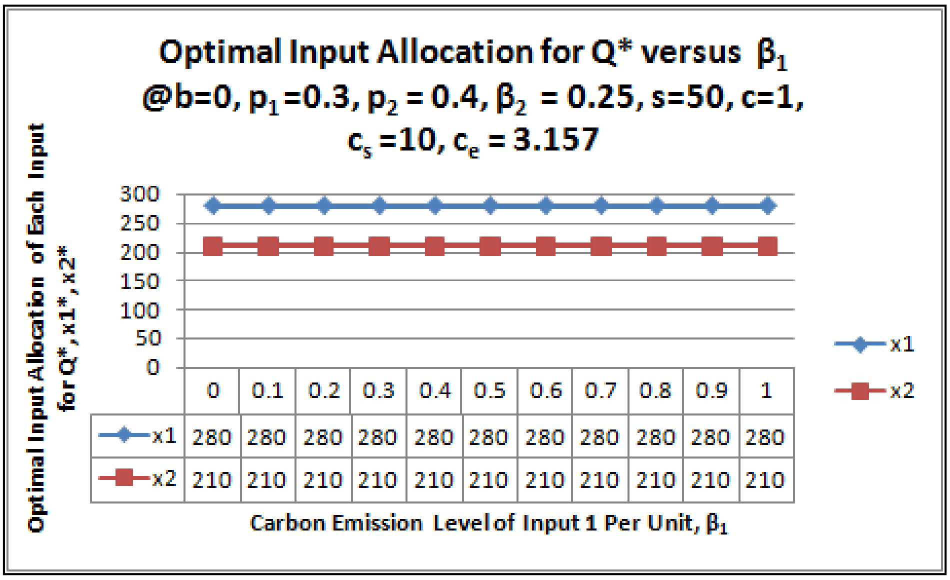


Figure 6.2: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0, 0)$, $\beta_1 = 0.1$, $\beta_2 = 0.25$, c=1, s=50, c_e=3.157, c_s=10 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



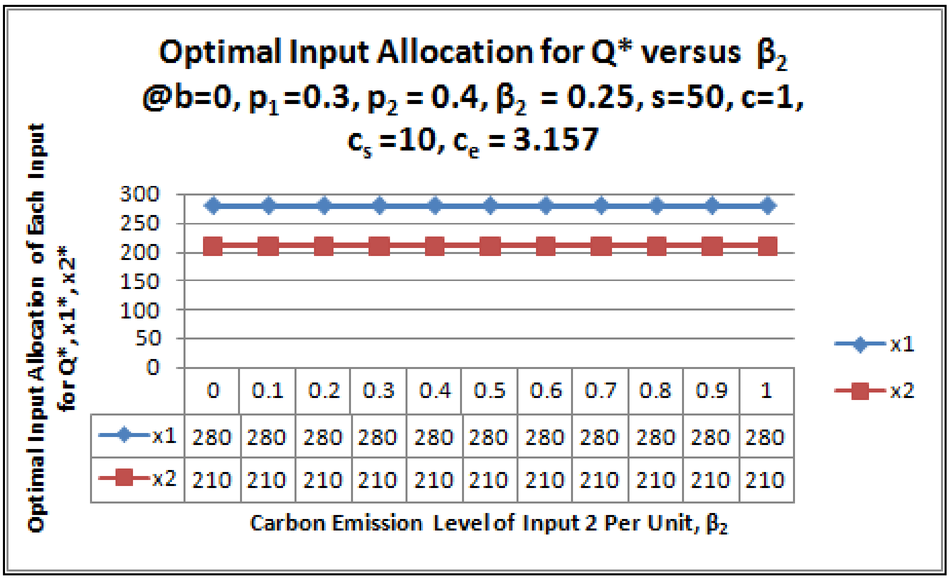


Figure 6.3: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0, 0)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_e=3.157, c_s=10 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.

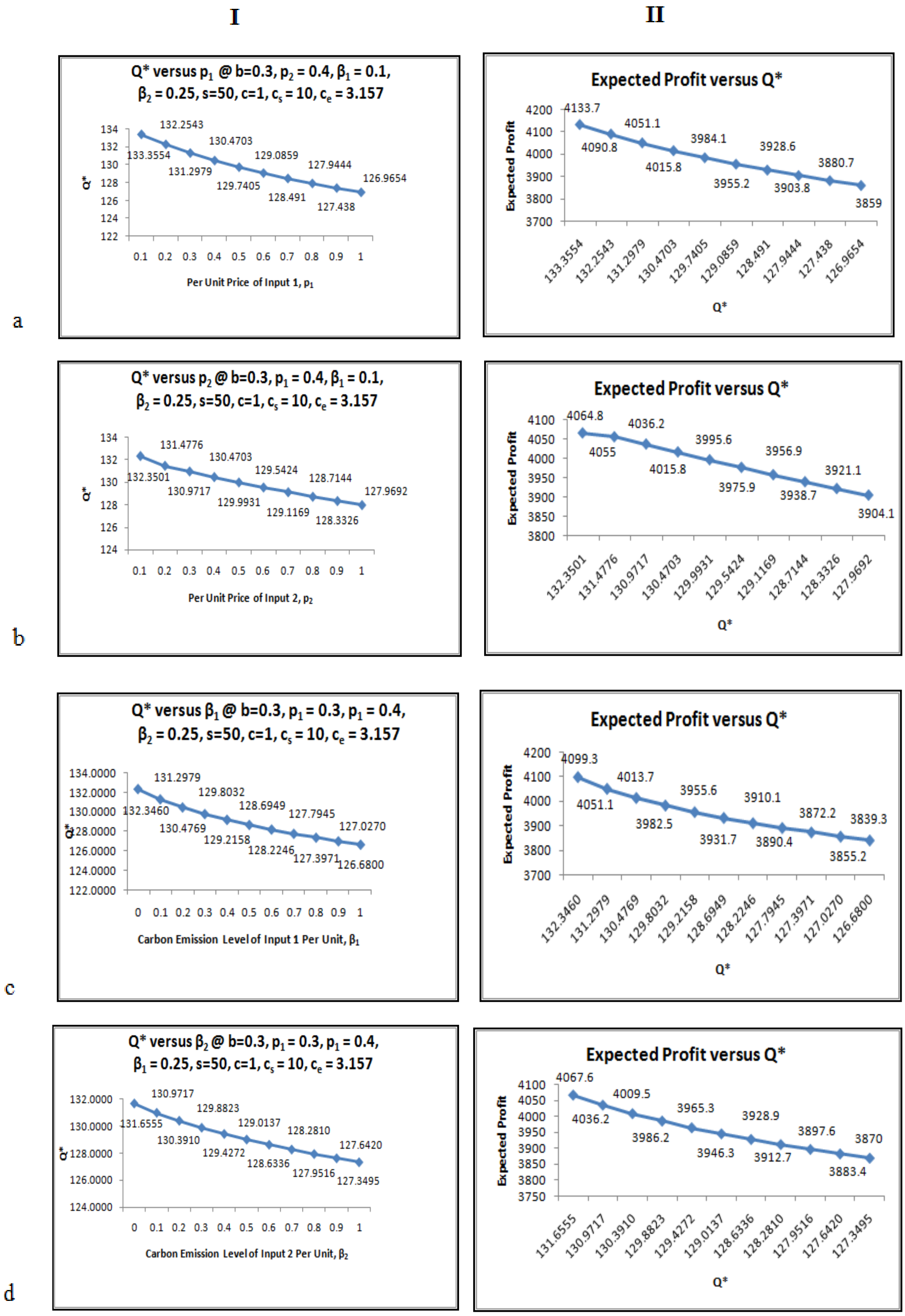


Figure 6.4: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0.3, 0.1)$, c=1, s=50, c_s=10,c_e=3.157 with $(1+\delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.

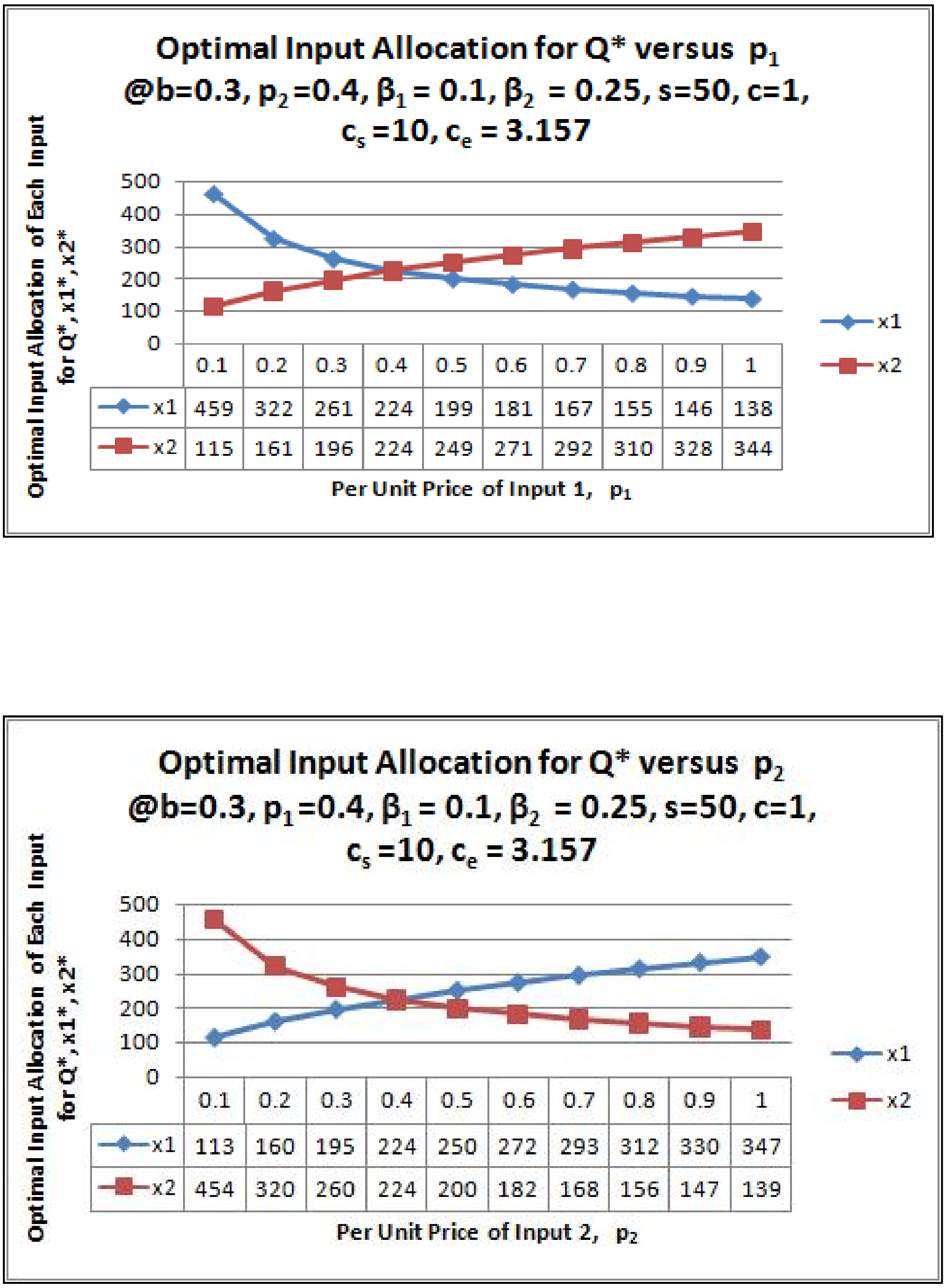
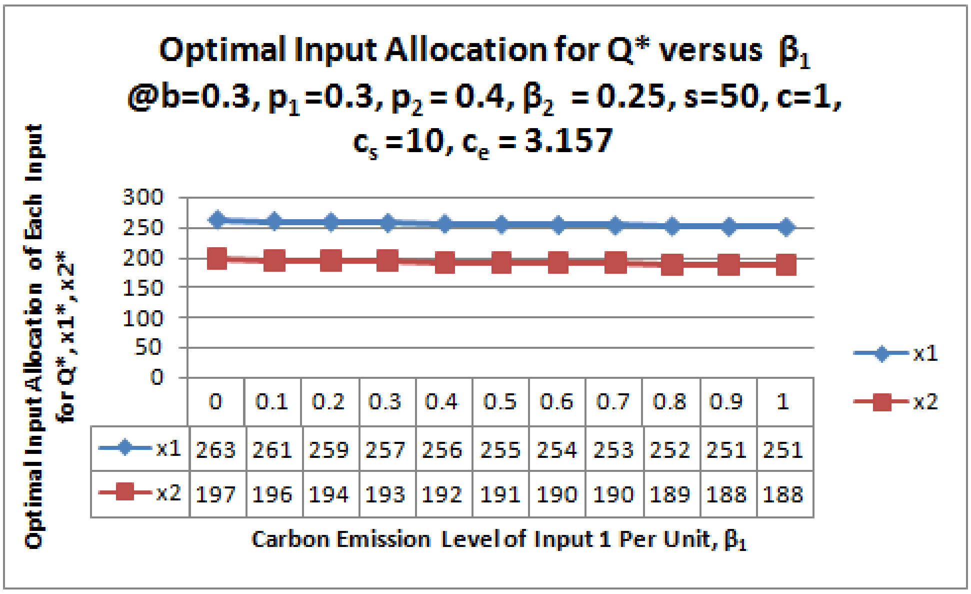


Figure 6.5: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0.3, 0.1)$, $\beta_1 = 0.1$, $\beta_2 = 0.25$, c=1, s=50,, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



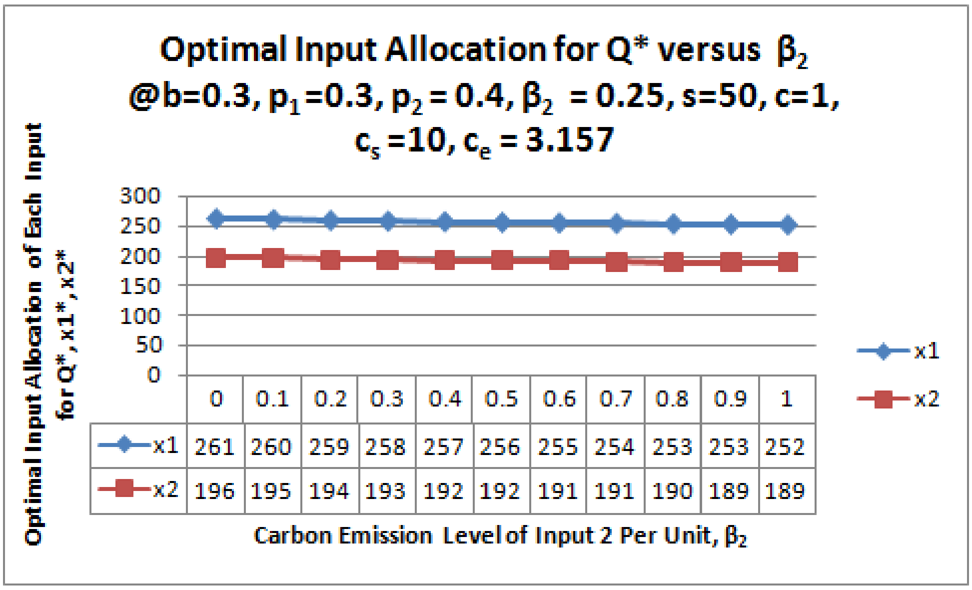


Figure 6.6: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0.3, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157, with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.

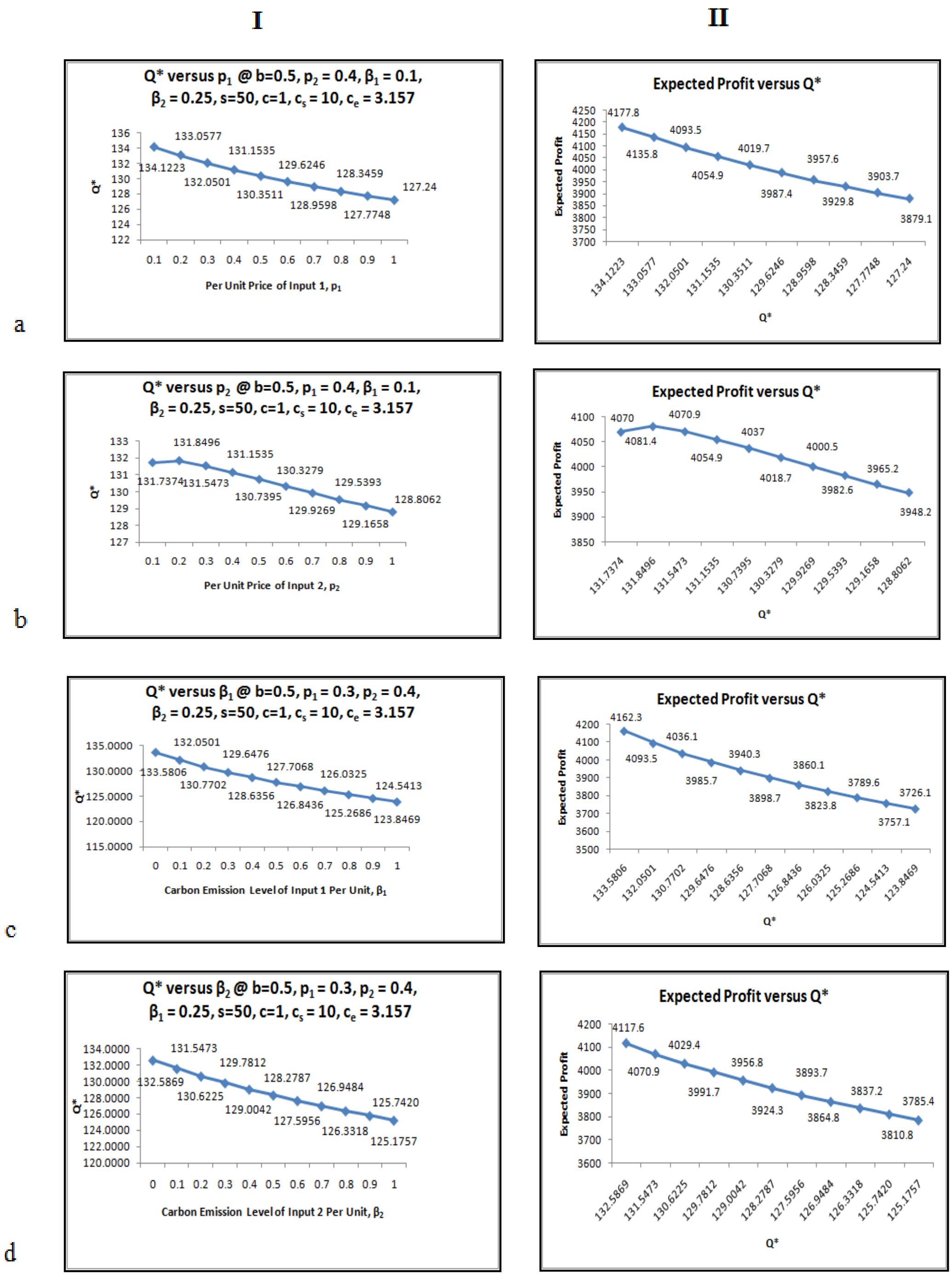
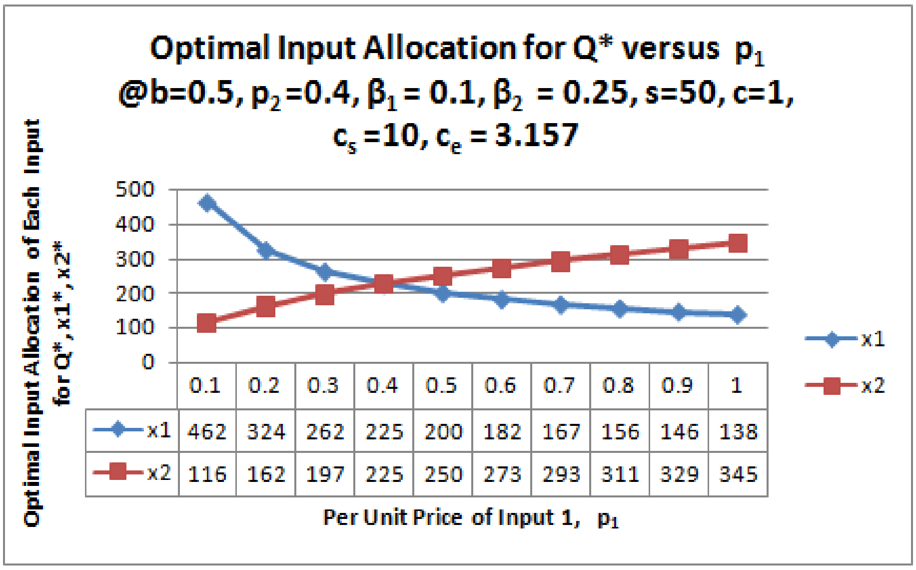


Figure 6.7: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0.5, 0.1)$, c=1, s=50, c_s=10,c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



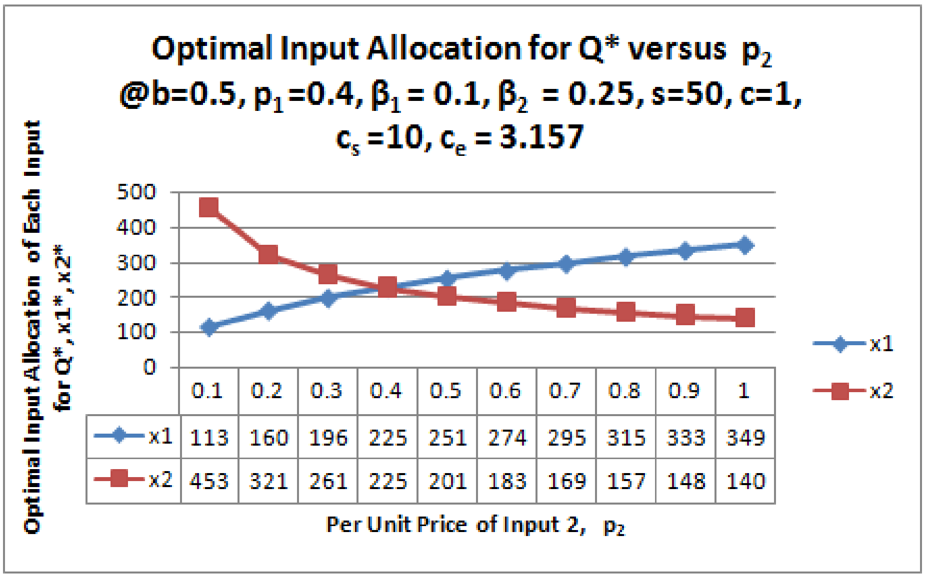
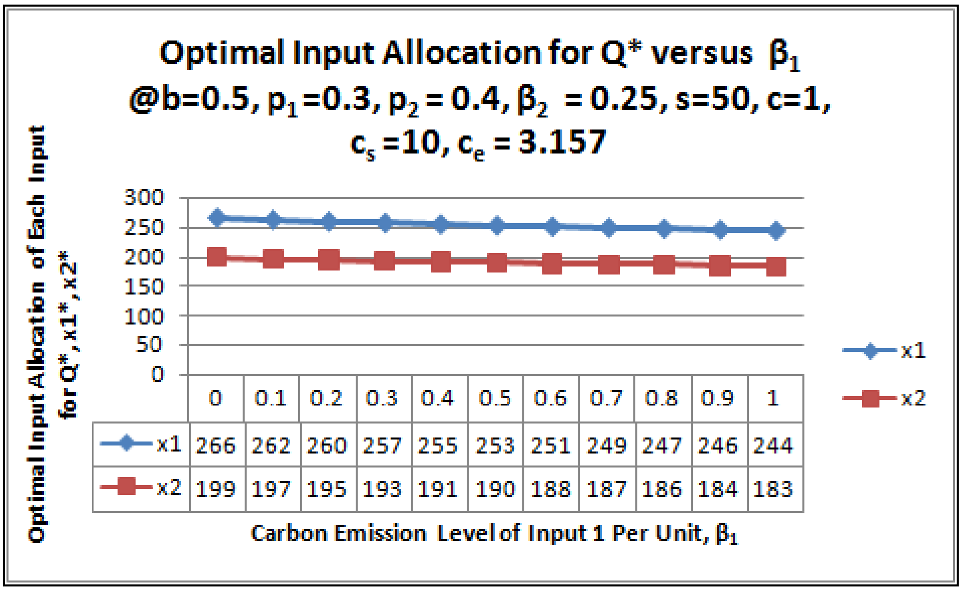


Figure 6.8: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0.5, 0.1)$, $\beta_1 = 0.1$, $\beta_2 = 0.25$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



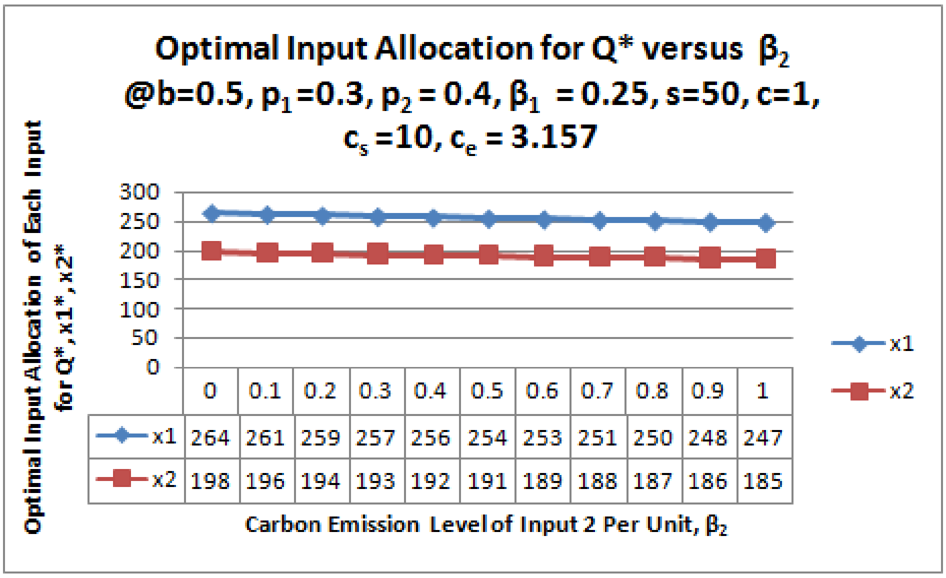


Figure 6.9: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0.5, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.

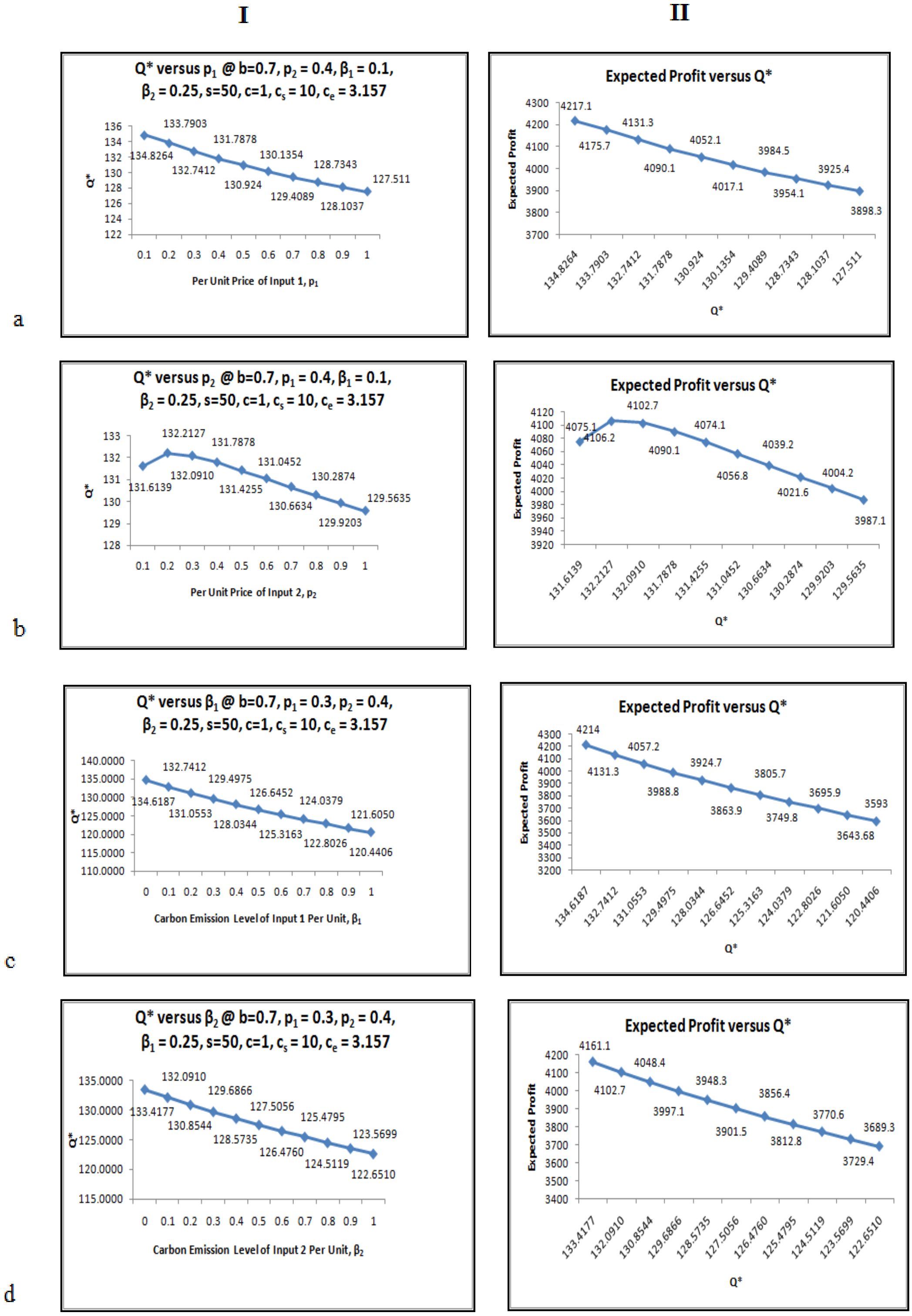
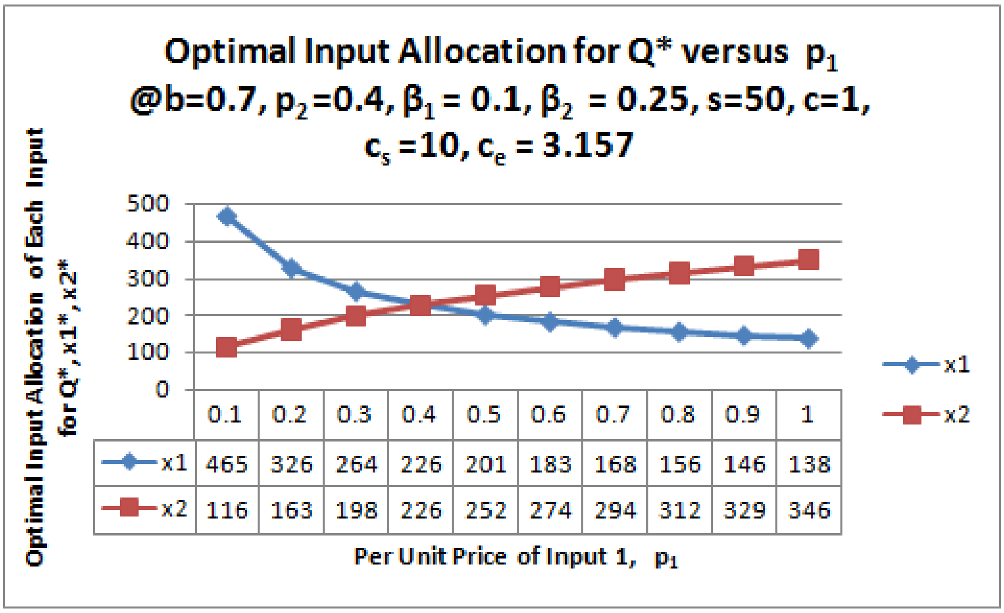


Figure 6.10: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0.7, 0.1)$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



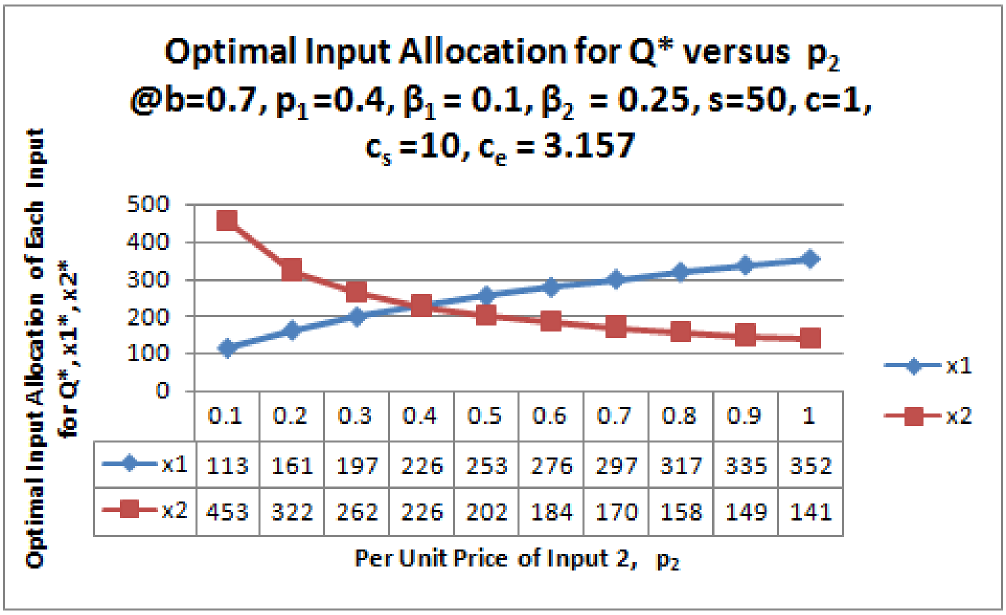
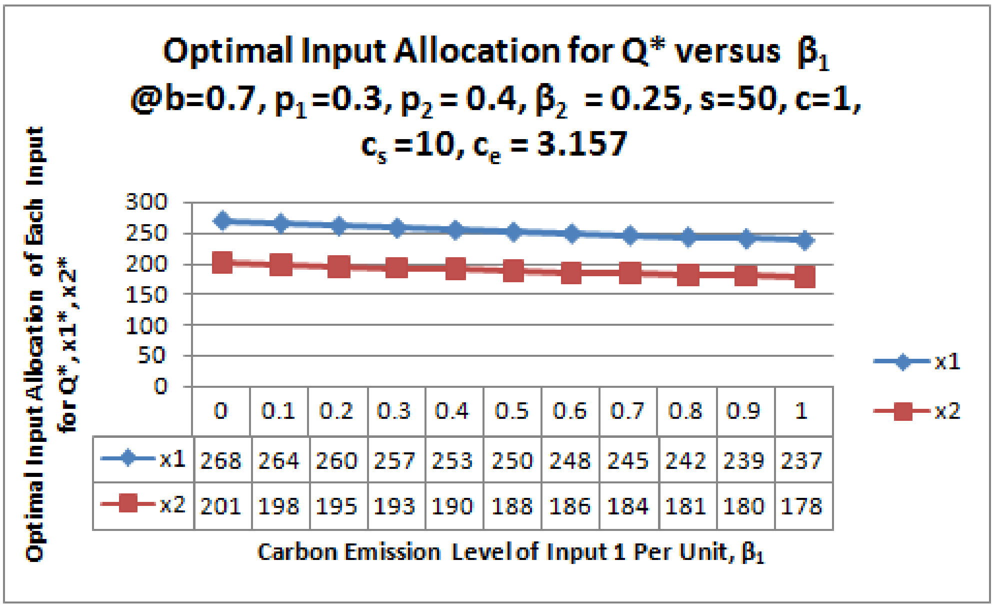


Figure 6.11: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0.7, 0.1)$, $\beta_1 = 0.1$, $\beta_2 = 0.25$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



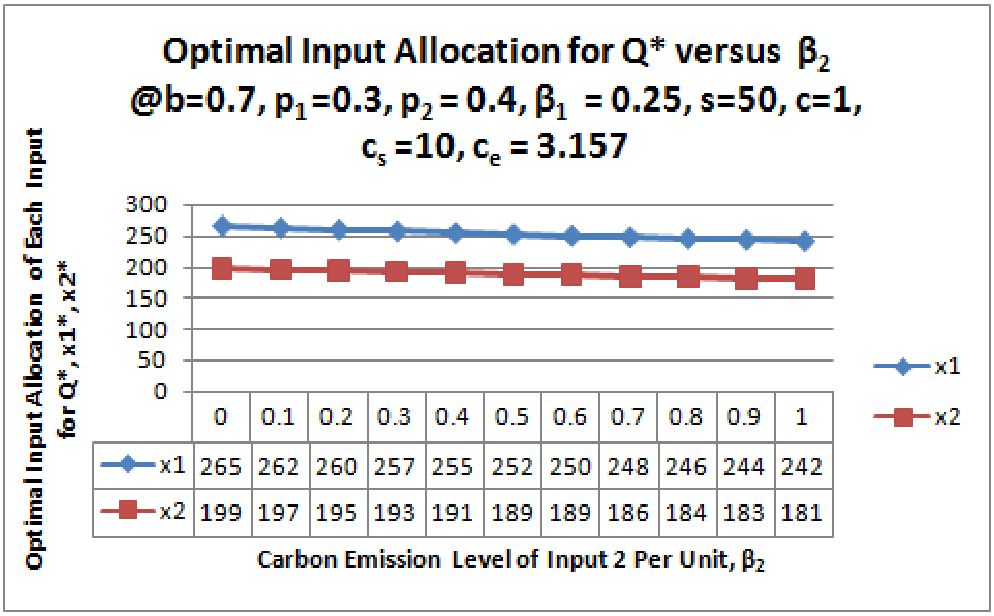


Figure 6.12: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0.7, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.

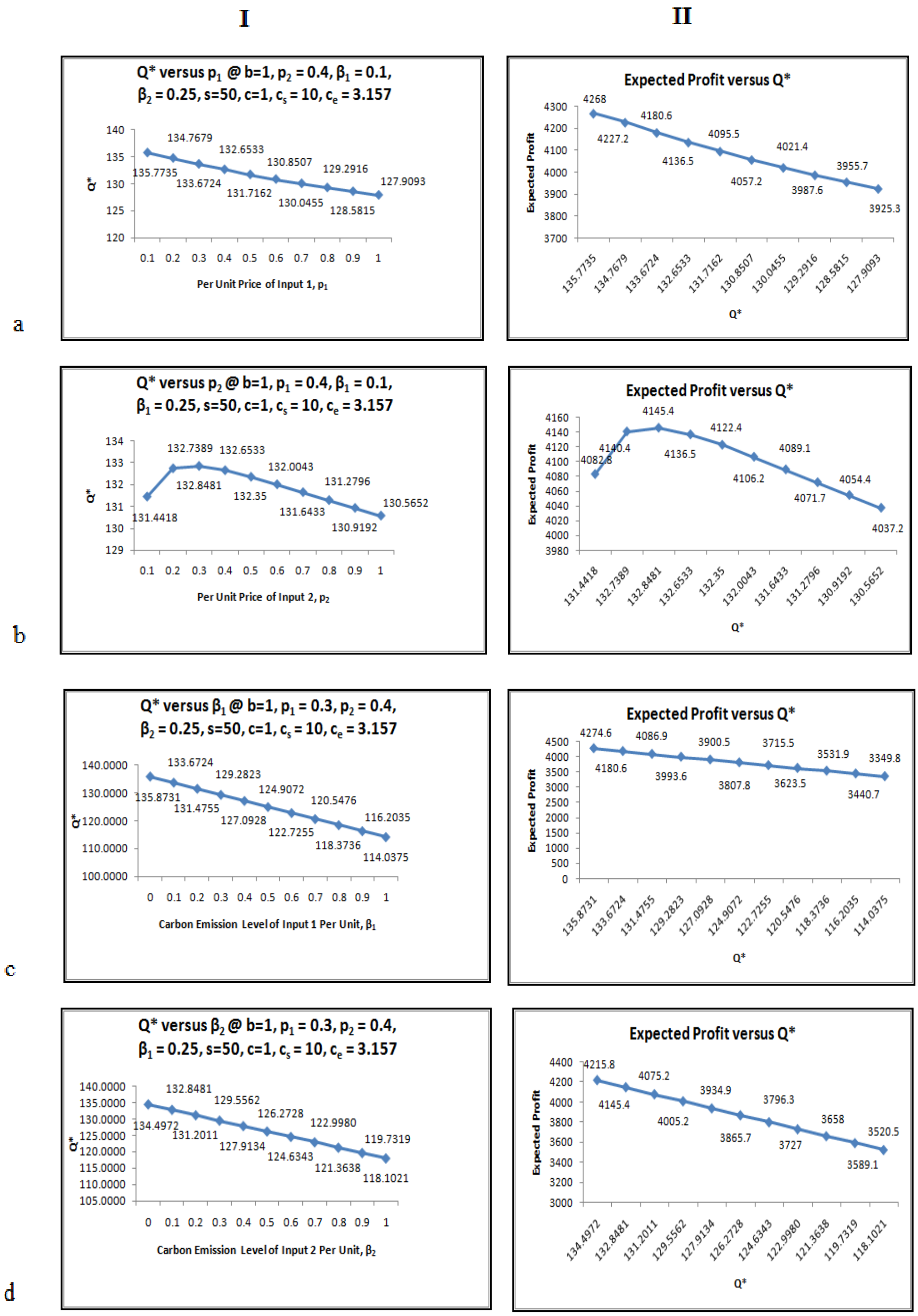
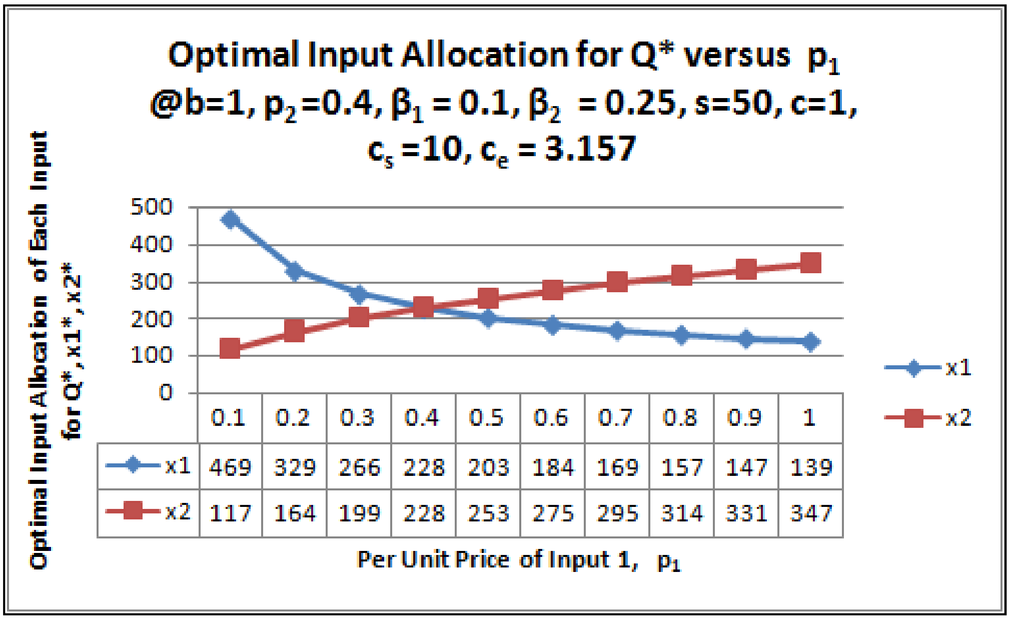


Figure 6.13: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (1, 0.1)$, c=1, s=50, c_s=10, c_e=3.157 with $(1+\delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



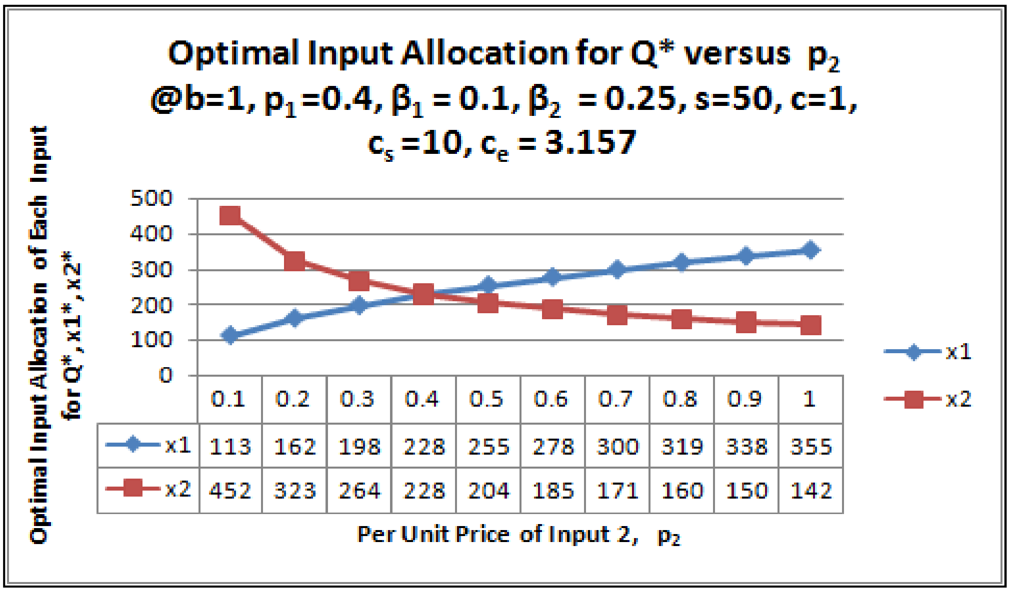
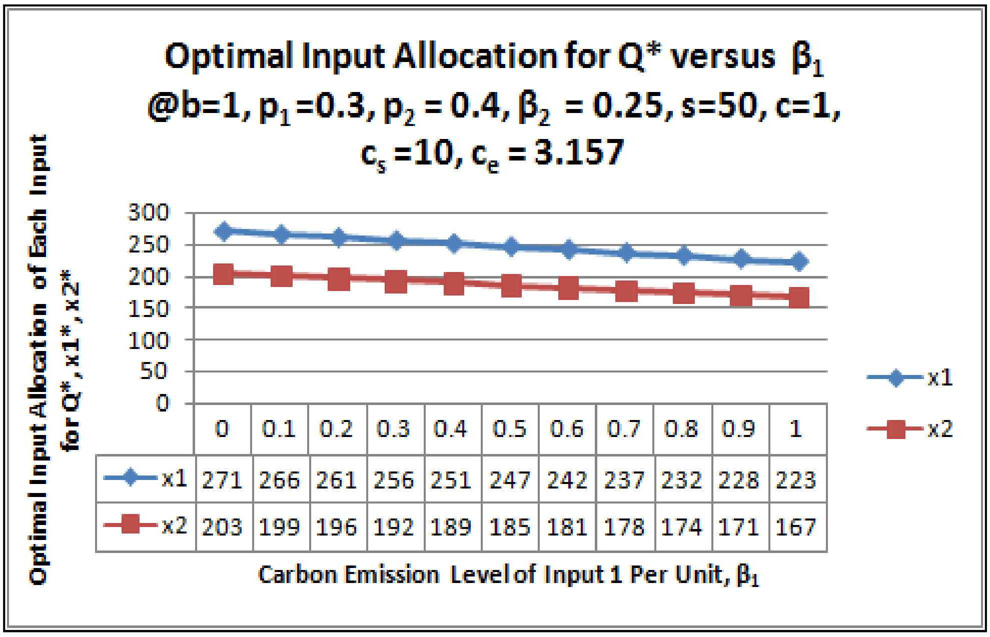
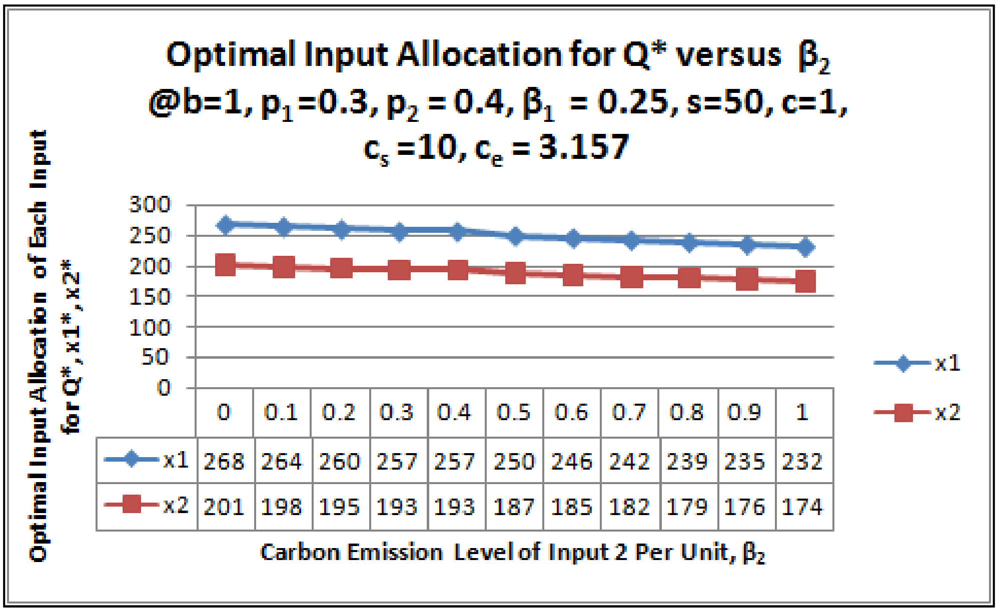


Figure 6.14: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (1, 0.1)$, $\beta_1 = 0.1$, $\beta_2 = 0.25$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	271	266	261	256	251	247	242	237	232	228	223
x2	203	199	196	192	189	185	181	178	174	171	167



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	268	264	260	257	257	250	246	242	239	235	232
x2	201	198	195	193	193	187	185	182	179	176	174

Figure 6.15: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (1, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 with $(1 + \delta) = 1.2$ under the Retailer's Problem with an Independent Manufacturer.

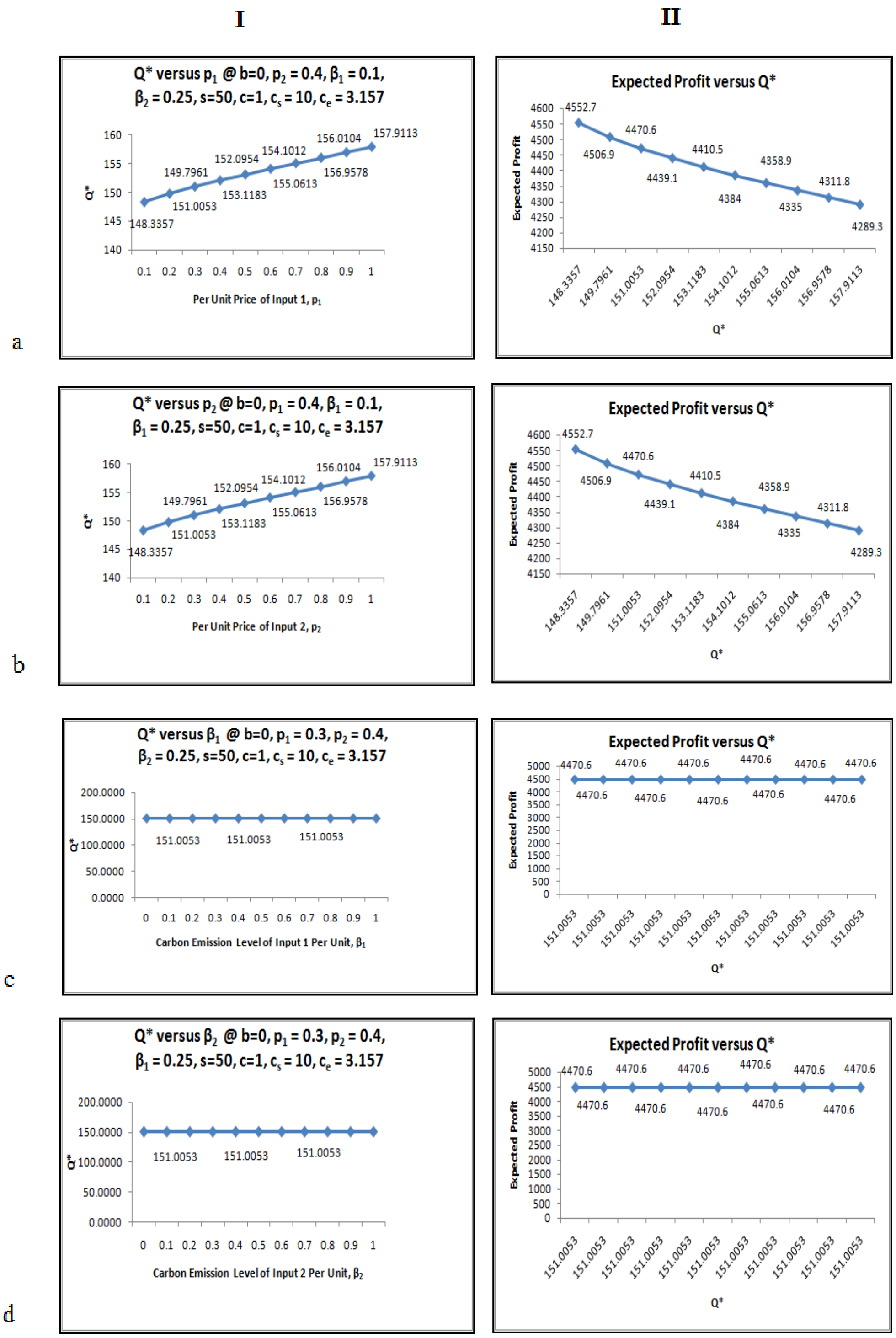
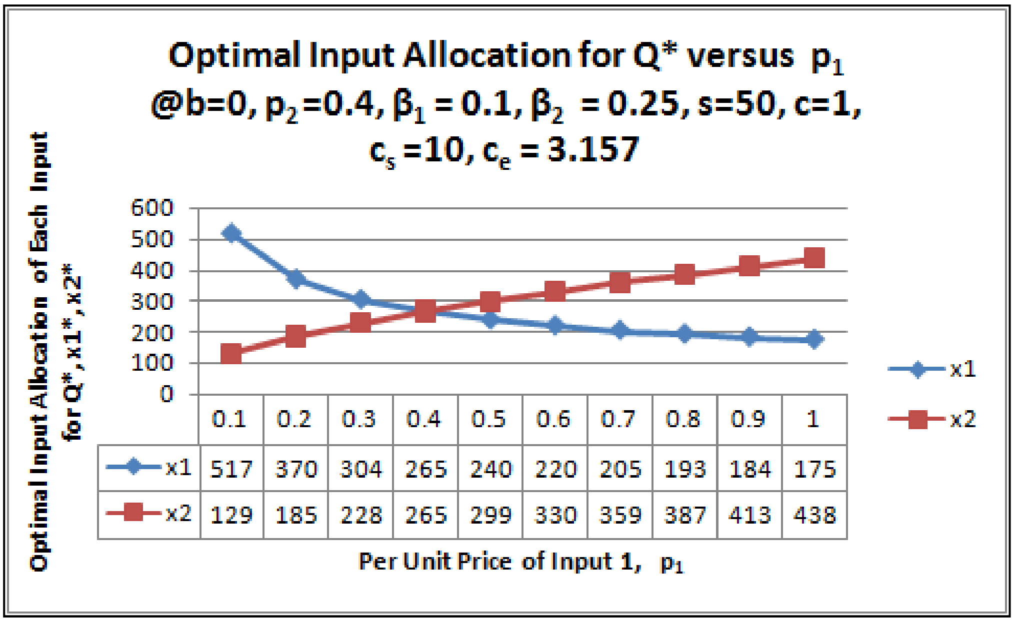
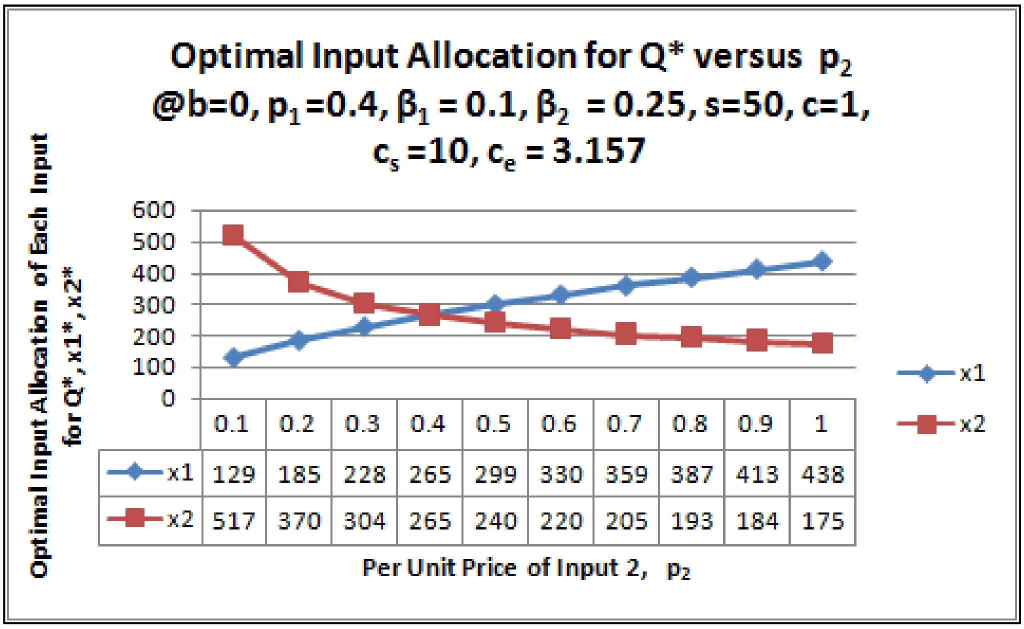


Figure 6.16: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0, 0)$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

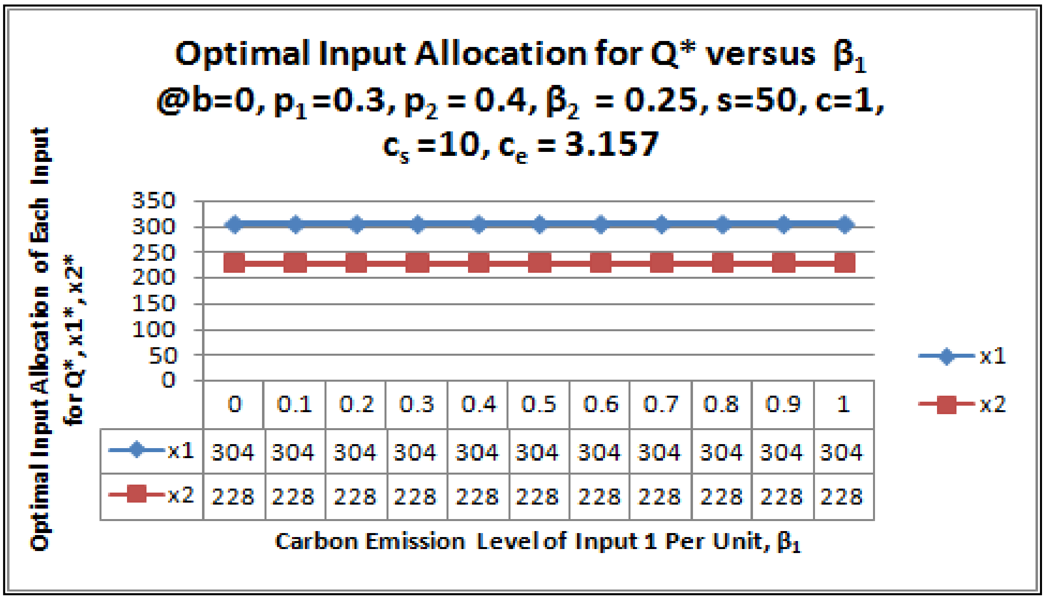


	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	517	370	304	265	240	220	205	193	184	175
x2	129	185	228	265	299	330	359	387	413	438

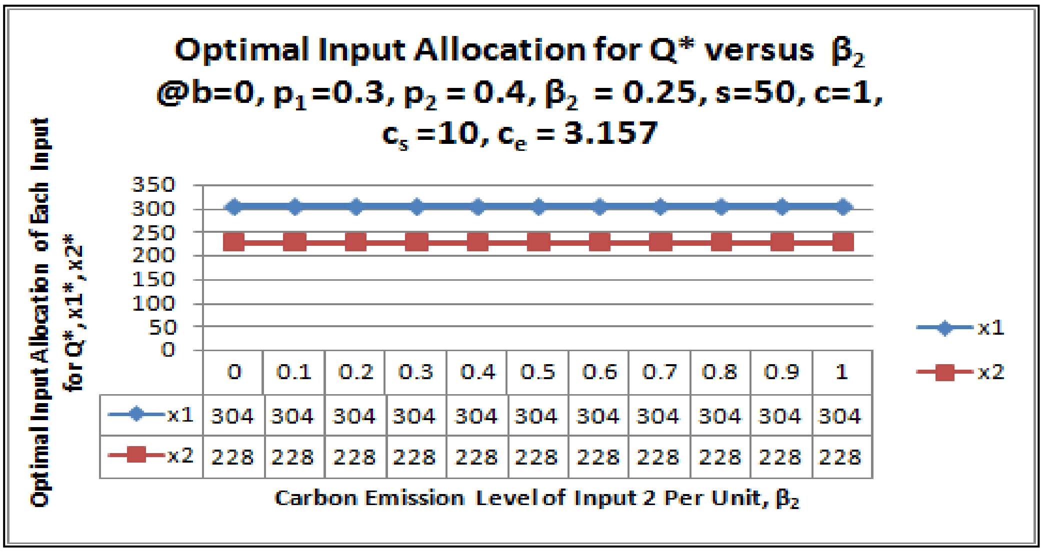


	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	129	185	228	265	299	330	359	387	413	438
x2	517	370	304	265	240	220	205	193	184	175

Figure 6.17: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0, 0)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 with under the Integrated Problem of the Retailer and the Manufacturer.



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	304	304	304	304	304	304	304	304	304	304	304
x2	228	228	228	228	228	228	228	228	228	228	228



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	304	304	304	304	304	304	304	304	304	304	304
x2	228	228	228	228	228	228	228	228	228	228	228

Figure 6.18: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0, 0)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

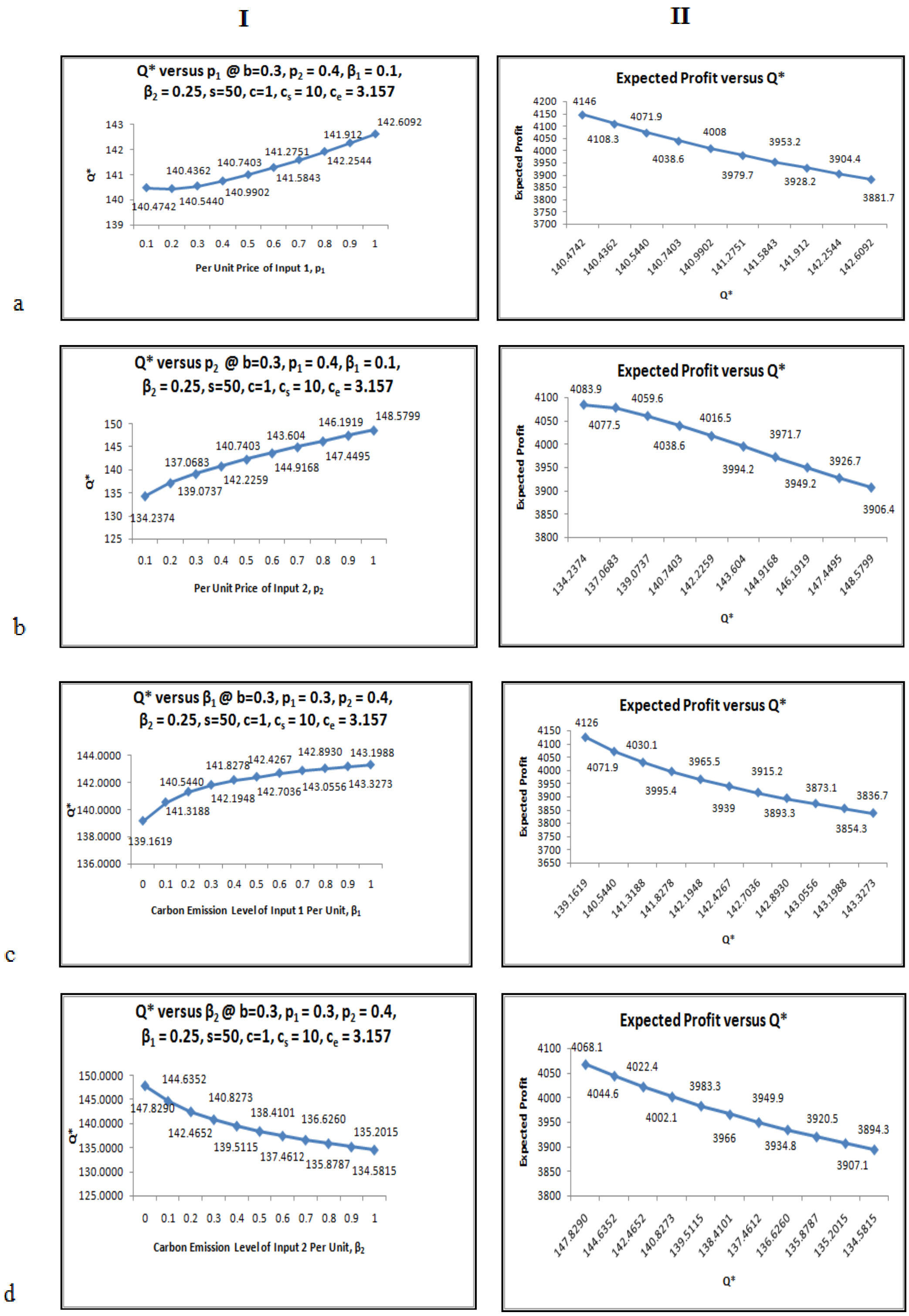
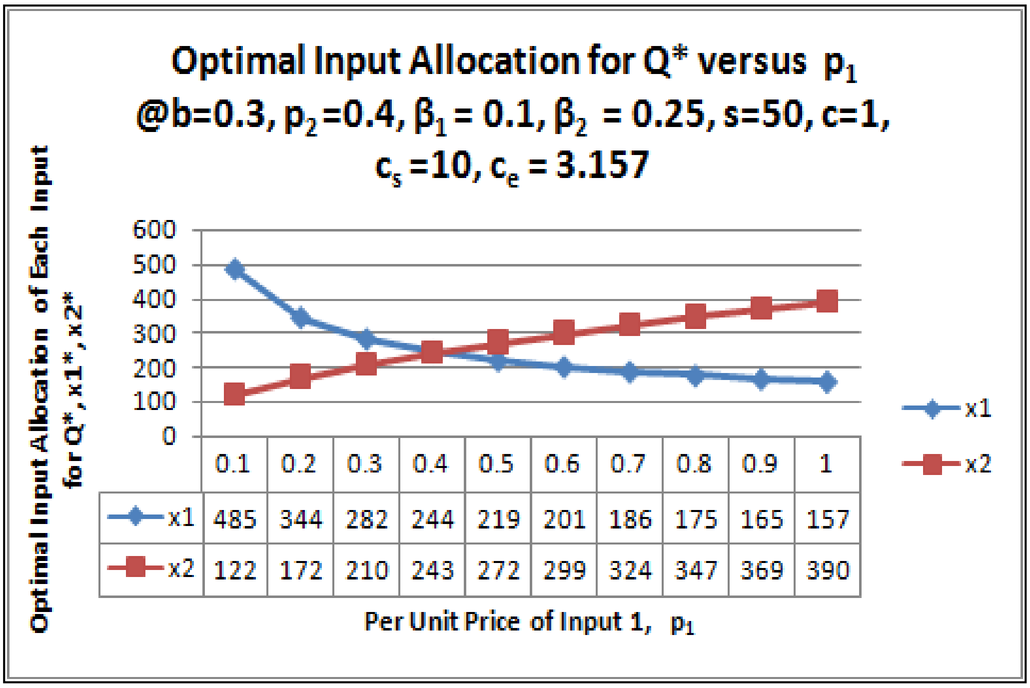
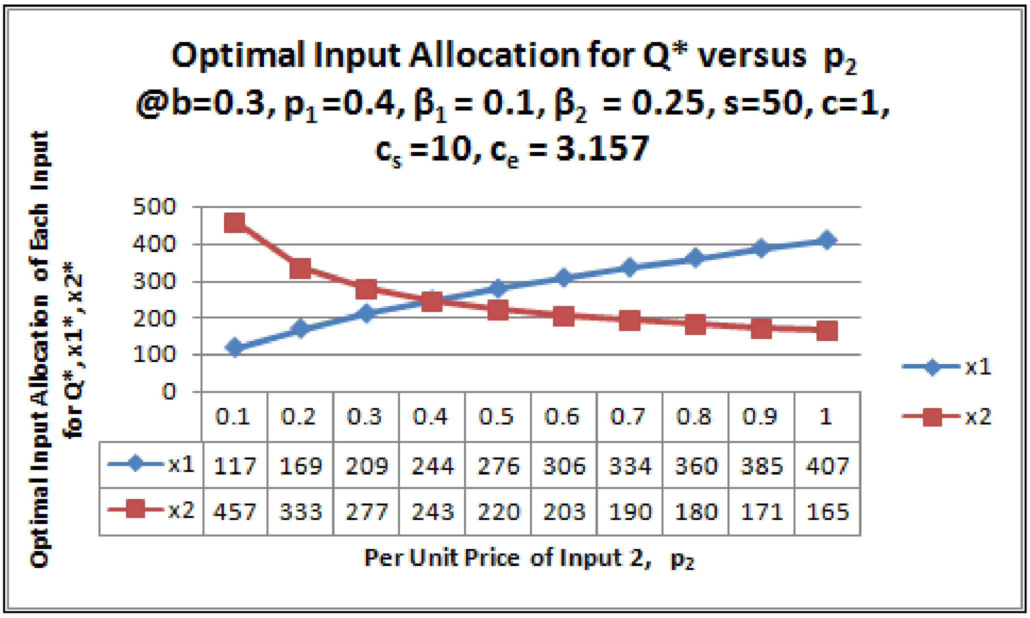


Figure 6.19: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0.3, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

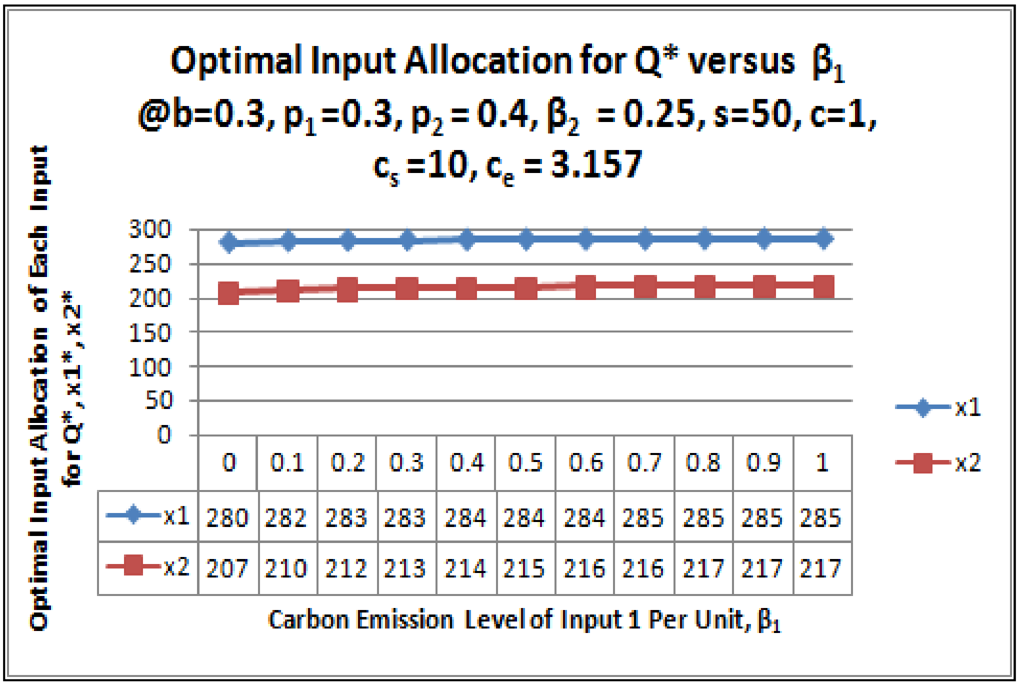


	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	485	344	282	244	219	201	186	175	165	157
x2	122	172	210	243	272	299	324	347	369	390

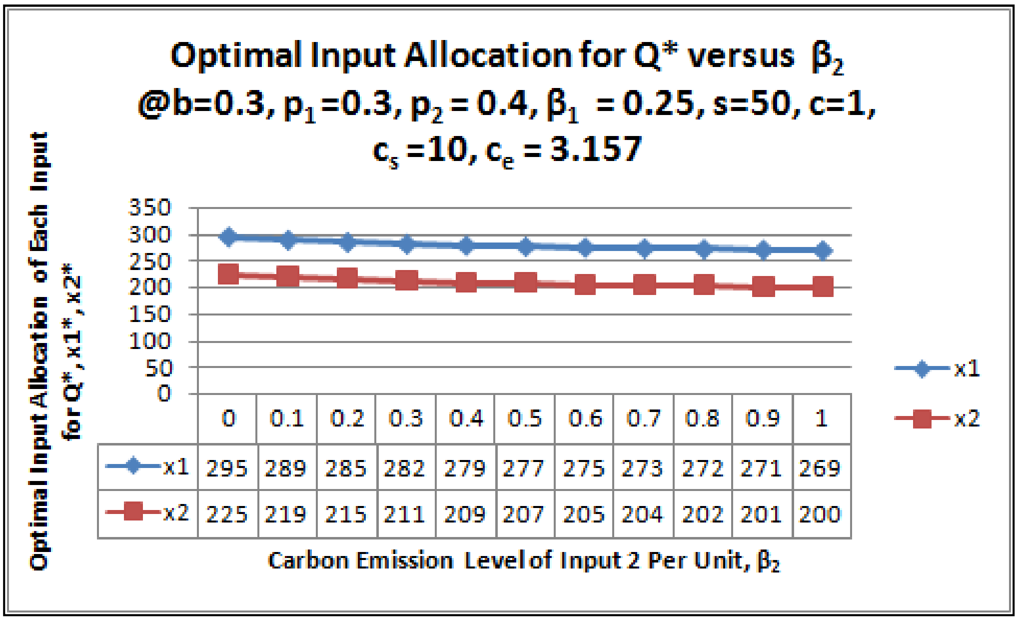


	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	117	169	209	244	276	306	334	360	385	407
x2	457	333	277	243	220	203	190	180	171	165

Figure 6.20: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0.3, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10,c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	280	282	283	283	284	284	284	285	285	285	285
x2	207	210	212	213	214	215	216	216	217	217	217



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	295	289	285	282	279	277	275	273	272	271	269
x2	225	219	215	211	209	207	205	204	202	201	200

Figure 6.21: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0.3, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

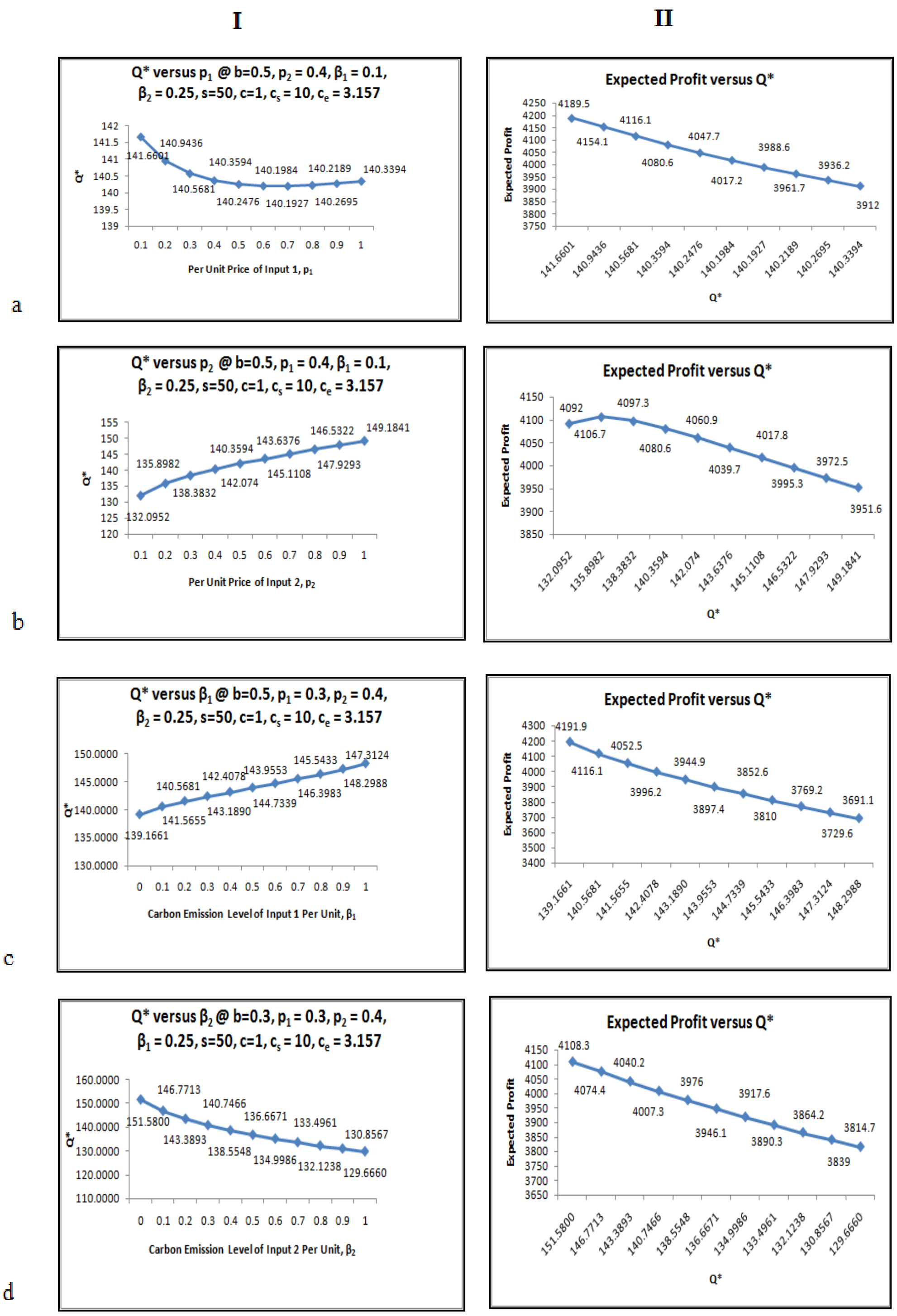
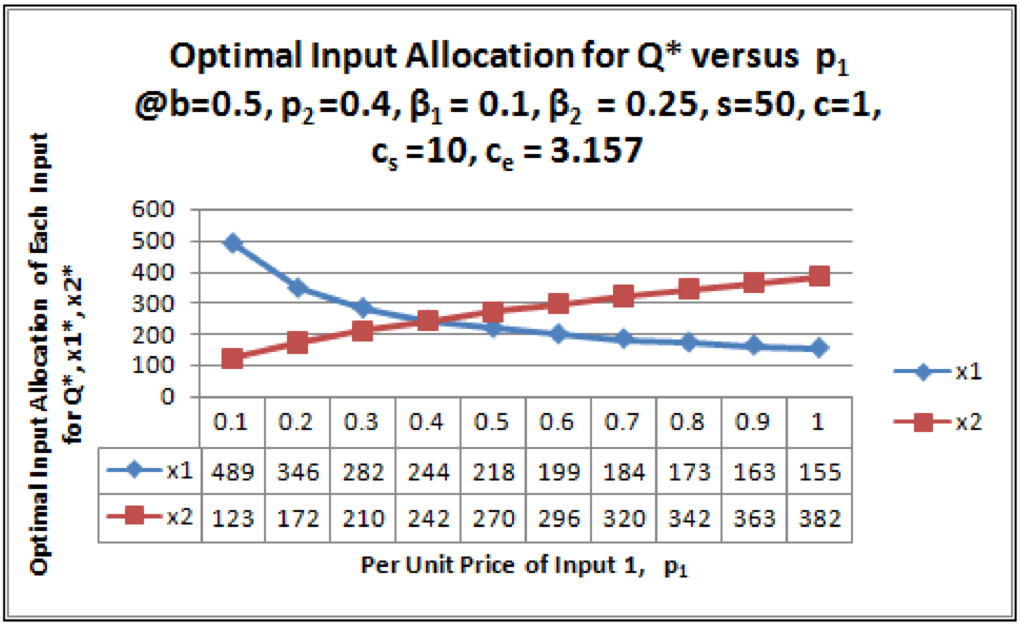


Figure 6.22: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0.5, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



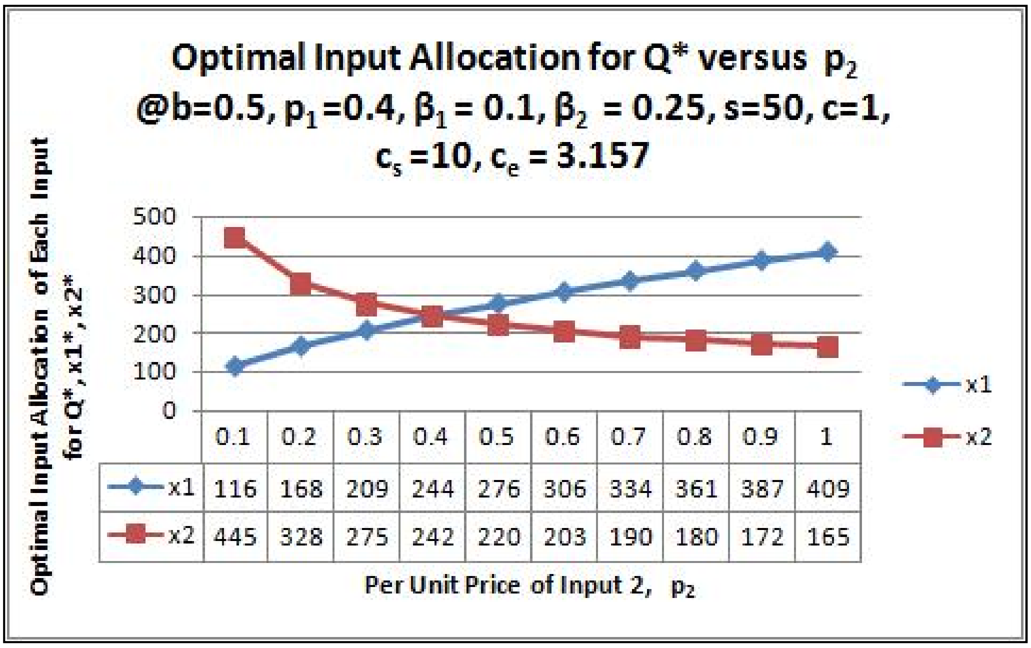
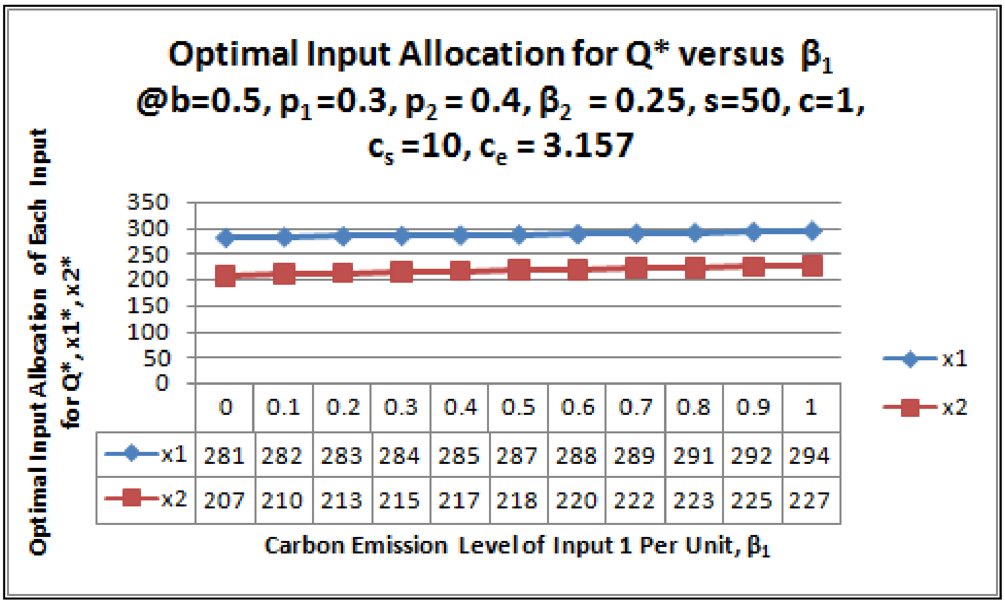


Figure 6.23: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0.5, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



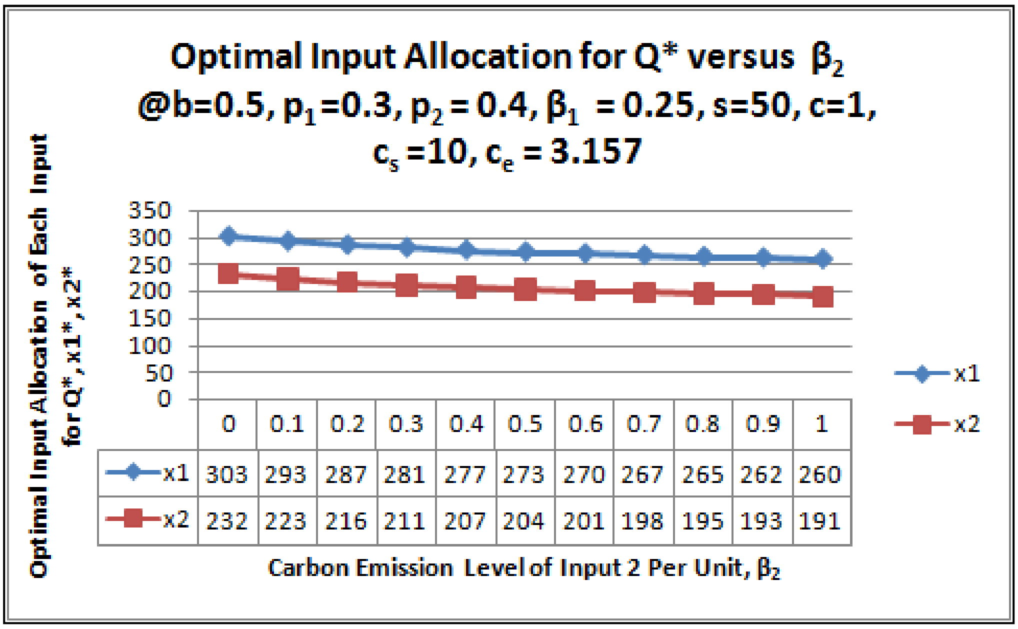


Figure 6.24: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0.5, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

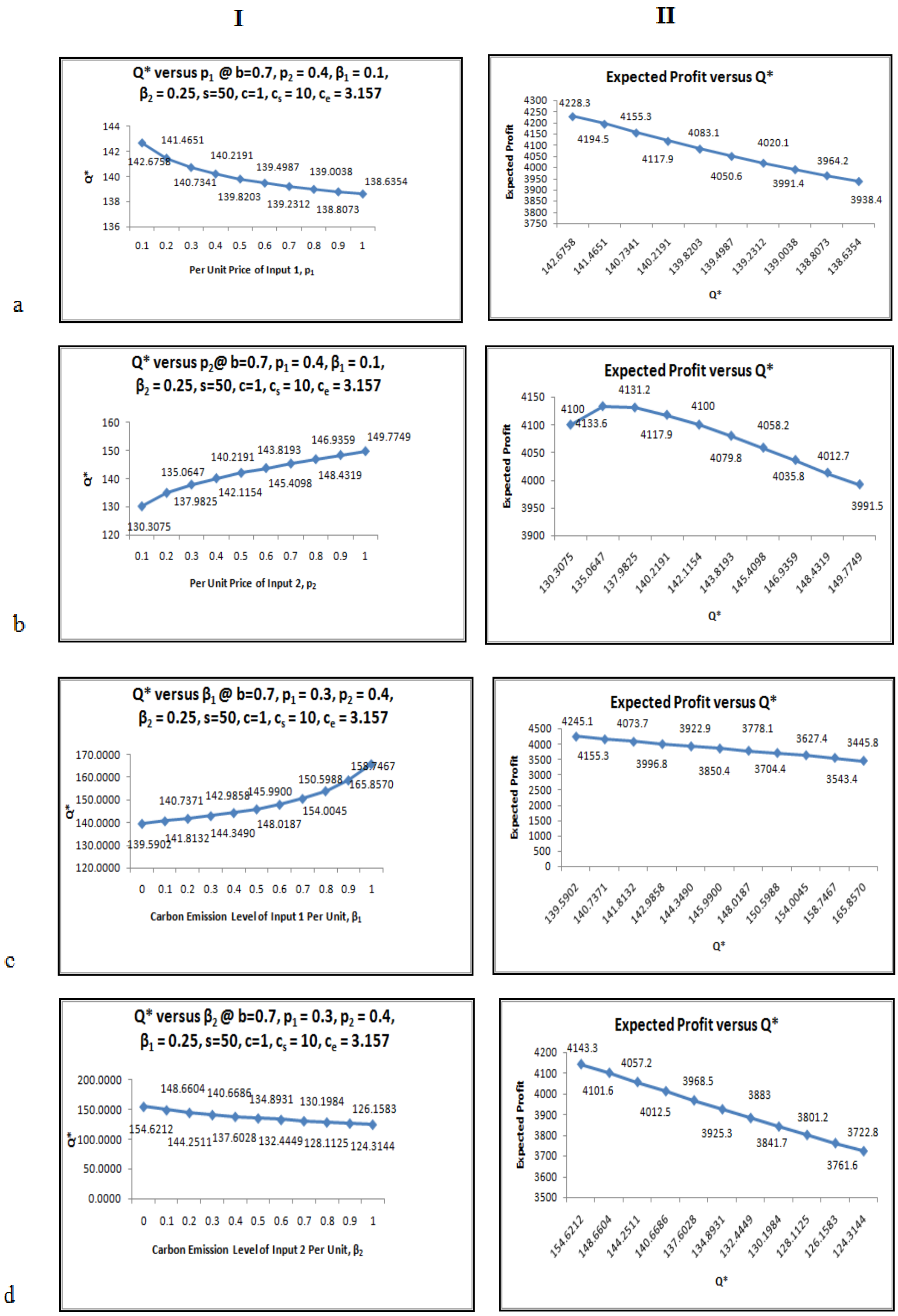
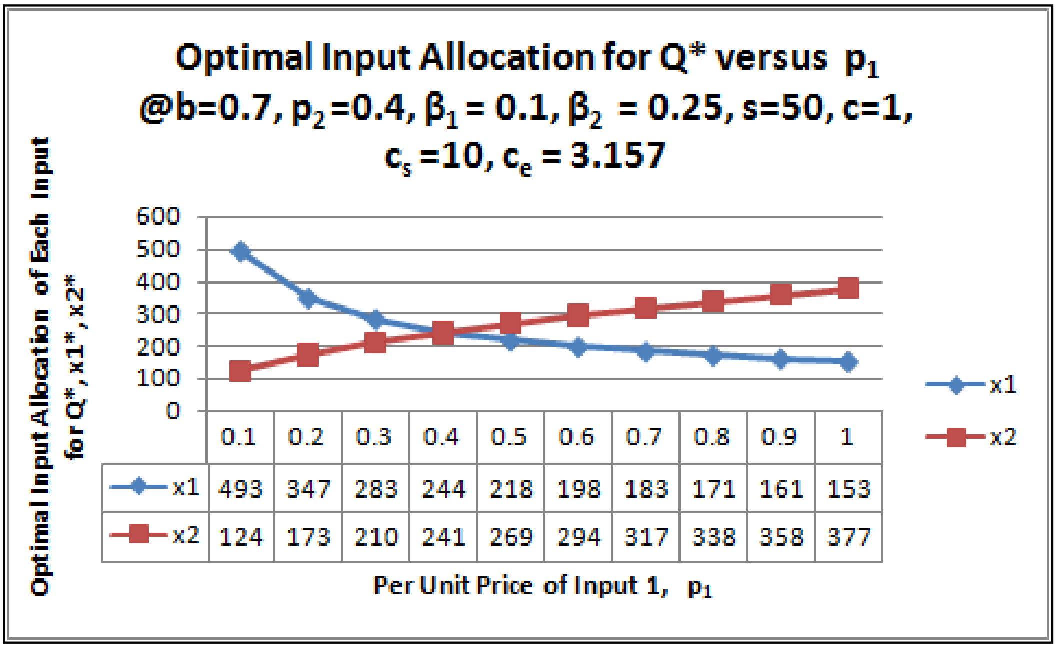


Figure 6.25: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (0.7, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



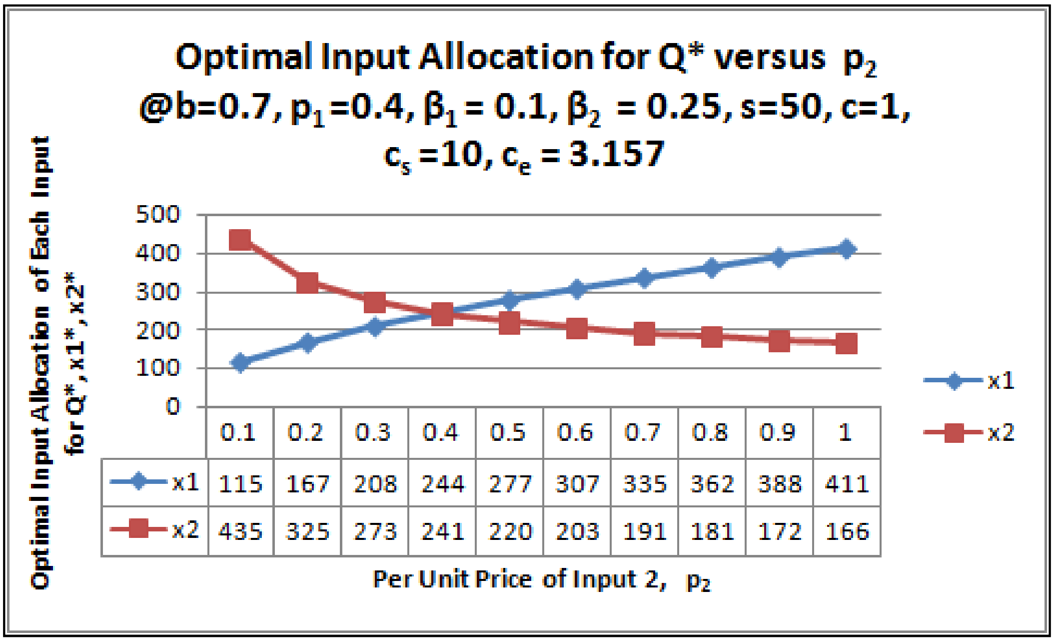
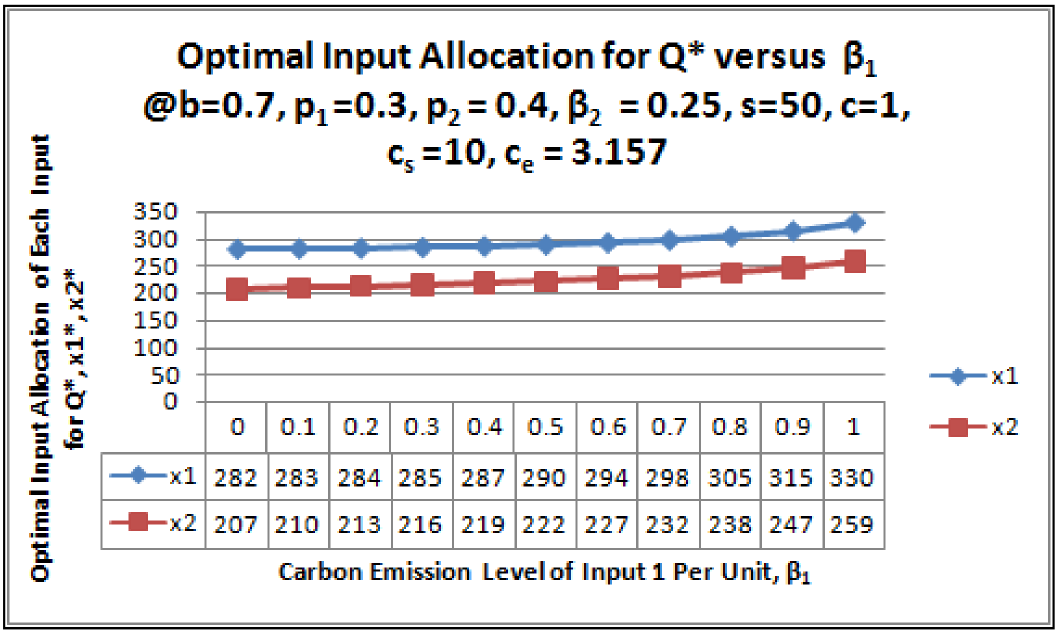
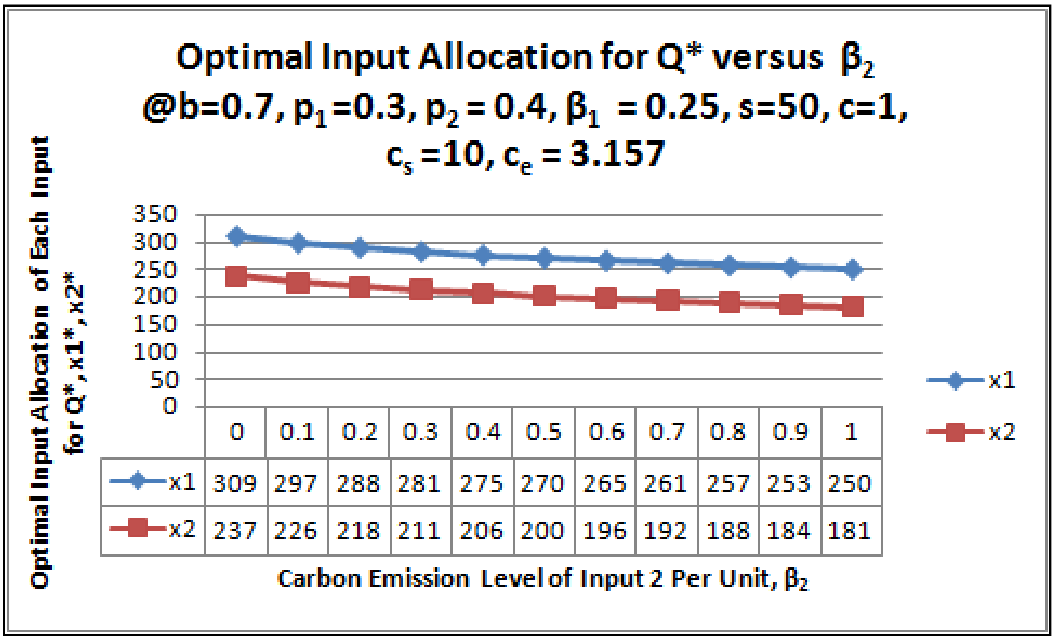


Figure 6.26: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (0.7, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	282	283	284	285	287	290	294	298	305	315	330
x2	207	210	213	216	219	222	227	232	238	247	259



	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x1	309	297	288	281	275	270	265	261	257	253	250
x2	237	226	218	211	206	200	196	192	188	184	181

Figure 6.27: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (0.7, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

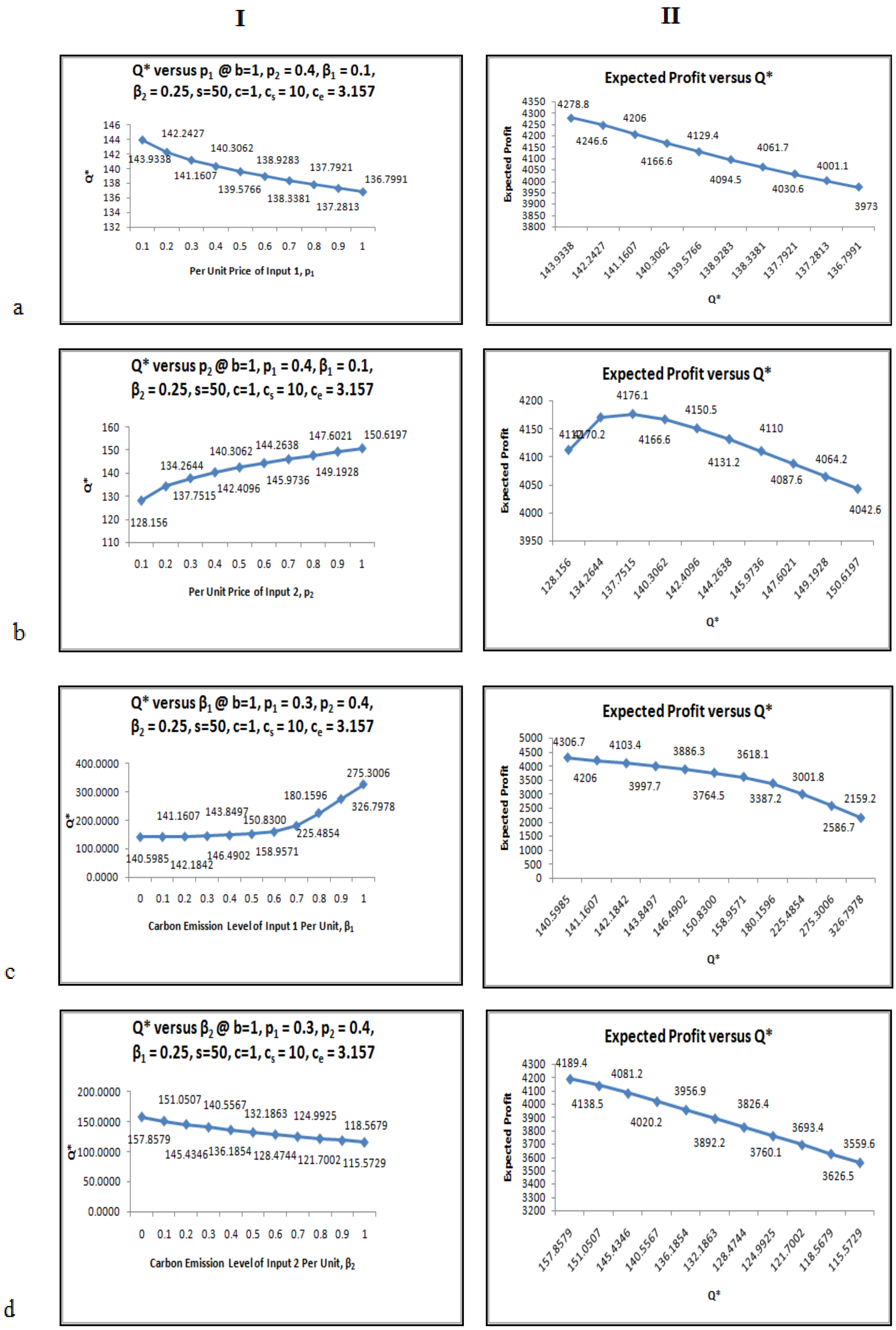
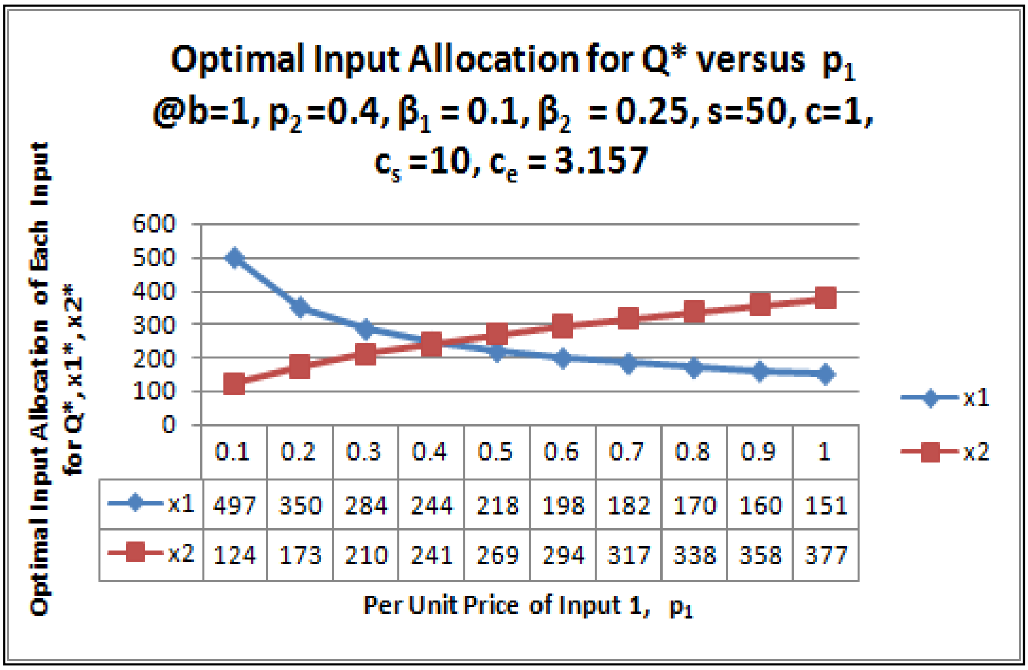


Figure 6.28: Q^* vs. p_1, p_2, β_1, β_2 and Expected Profit vs the Q* values at $(b, a) = (1, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



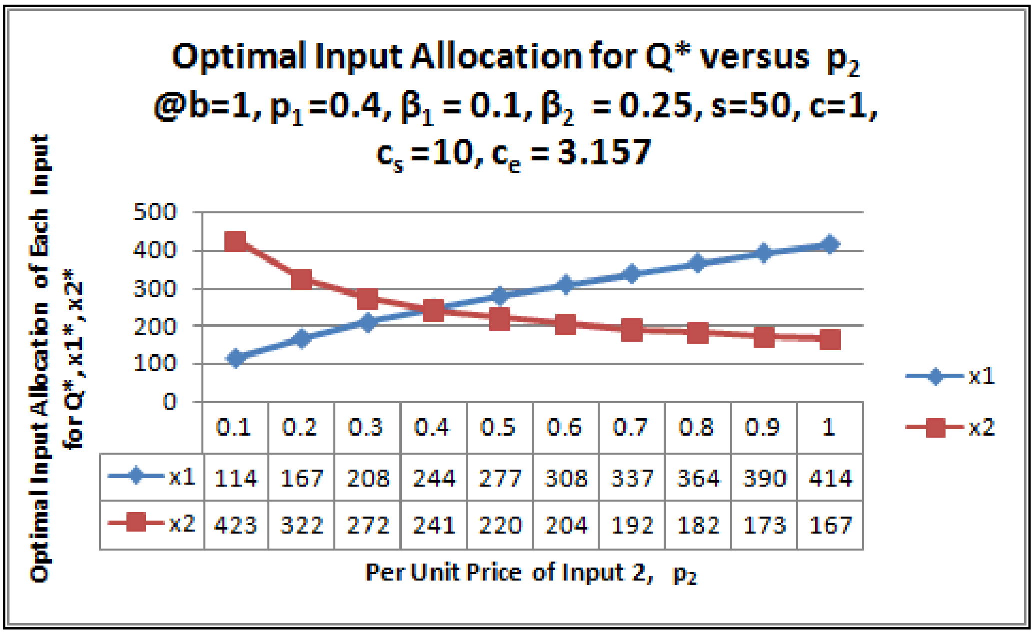
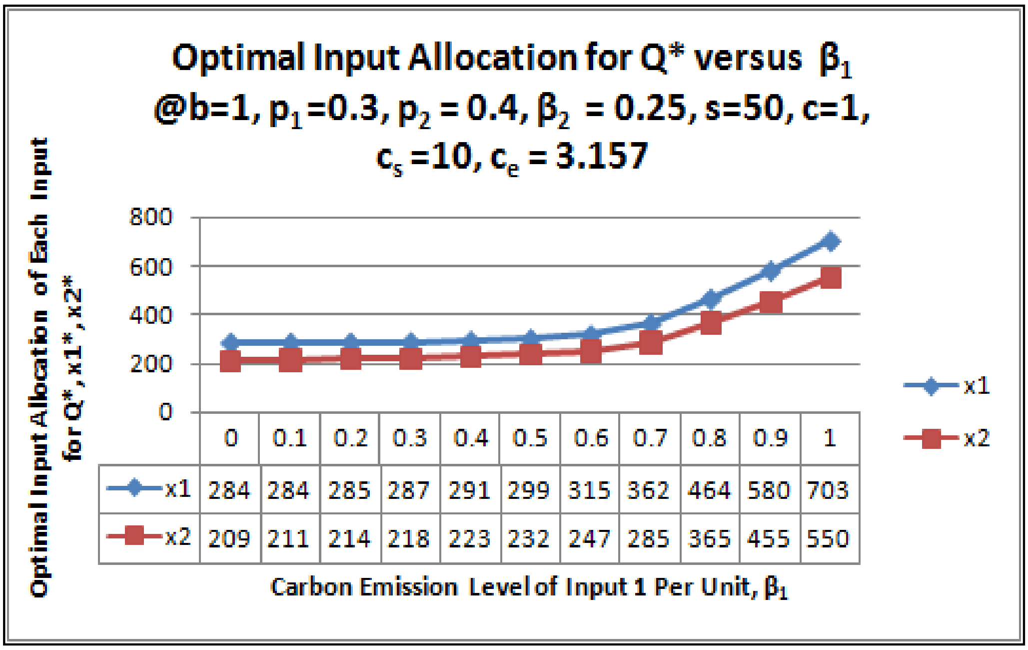


Figure 6.29: x_1^* and x_2^* vs. p_1, p_2 at $(b, a) = (1, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.



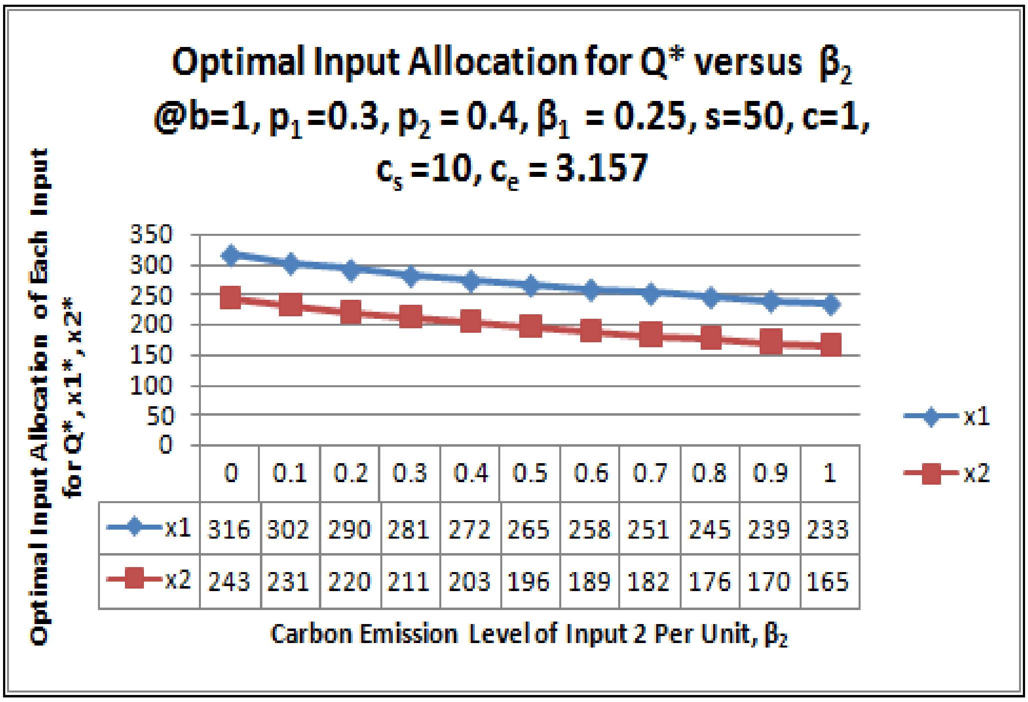


Figure 6.30: x_1^* and x_2^* vs. β_1, β_2 at $(b, a) = (1, 0.1)$, $p_1 = 0.3$, $p_2 = 0.4$, c=1, s=50, c_s=10, c_e=3.157 under the Integrated Problem of the Retailer and the Manufacturer.

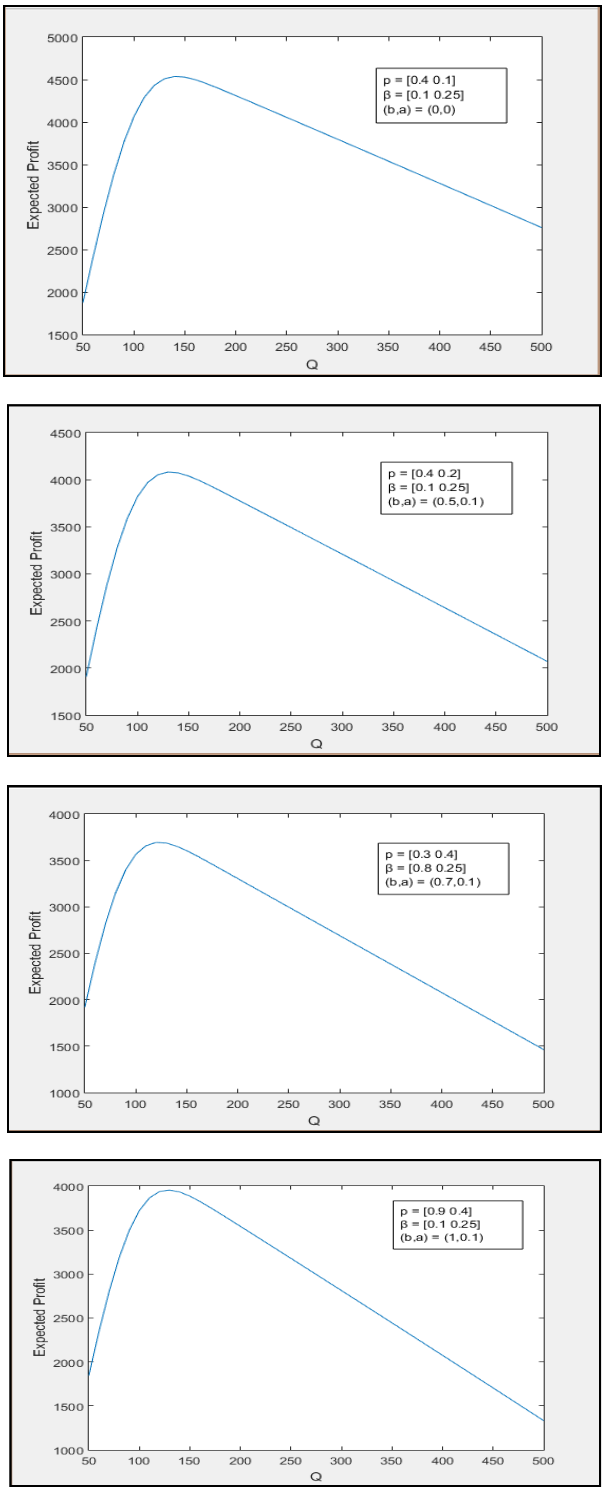


Figure 6.31: Expected Profit versus Q for Different Cases under Decentralized Problem.

b=0, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	100	142.0555	492.7767	123.1942	4	4537.7	4945.2	10.9557	136.2667	118.2664	142.0555
0.2	0.4	0.1	0.25	100	140.8736	345.226	172.613	2	4488.9	4940.3	11.9423	132.8458	165.7085	140.8736
0.3	0.4	0.1	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.7928
0.4	0.4	0.1	0.25	100	139.3062	241.0957	241.0957	1	4420.7	4933.2	13.3646	128.3454	231.4519	139.3062
0.5	0.4	0.1	0.25	100	138.7049	214.6085	268.2606	0.8	4393.5	4930.3	13.9466	126.6301	257.5302	138.7049
0.6	0.4	0.1	0.25	100	138.1747	195.078	292.617	0.66667	4368.9	4927.6	14.477	125.1234	280.9123	138.1747
0.7	0.4	0.1	0.25	100	137.6978	179.9147	314.8507	0.57143	4346.5	4925.2	14.9683	123.7726	302.2567	137.6978
0.8	0.4	0.1	0.25	100	137.2626	167.7039	335.4078	0.5	4325.6	4922.9	15.4286	122.5436	321.9915	137.2626
0.9	0.4	0.1	0.25	100	136.8611	157.599	354.5977	0.44444	4306.1	4920.7	15.8635	121.4131	340.4138	136.8611
1	0.4	0.1	0.25	100	136.4876	149.0582	372.6454	0.4	4287.7	4918.6	16.2773	120.3642	357.7396	136.4876

b=0, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	100	142.0555	123.1942	492.7767	0.25	4537.7	4945.2	10.9557	136.2667	118.2664	142.0555
0.2	0.4	0.1	0.25	100	140.8736	172.613	345.226	0.5	4488.9	4940.3	11.9423	132.8458	165.7085	140.8736
0.3	0.4	0.1	0.25	100	140.0092	209.966	279.9547	0.75	4451.8	4936.4	12.7101	130.3586	201.5674	140.7928
0.4	0.4	0.1	0.25	100	139.3062	241.0957	241.0957	1	4420.7	4933.2	13.3646	128.3454	231.4519	139.3062
0.5	0.4	0.1	0.25	100	138.7049	268.2606	214.6085	1.25	4393.5	4930.3	13.9466	126.6301	257.5302	138.7049
0.6	0.4	0.1	0.25	100	138.1747	292.617	195.078	1.5	4368.9	4927.6	14.477	125.1234	280.9123	138.1747
0.7	0.4	0.1	0.25	100	137.6978	314.8507	179.9147	1.75	4346.5	4925.2	14.9683	123.7726	302.2567	137.6978
0.8	0.4	0.1	0.25	100	137.2626	335.4078	167.7039	2	4325.6	4922.9	15.4286	122.5436	321.9915	137.2626
0.9	0.4	0.1	0.25	100	136.8611	354.5977	157.599	2.25	4306.1	4920.7	15.8635	121.4131	340.4138	136.8611
1	0.4	0.1	0.25	100	136.4876	372.6454	149.0582	2.5	4287.7	4918.6	16.2773	120.3642	357.7396	136.4876

b=0, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.1	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.2	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.3	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.4	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.5	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.6	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.7	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.8	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.9	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
1	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092

b=0, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.1	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.2	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.3	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.4	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.5	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.6	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.7	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.8	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
0.9	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092
1	0.3	0.4	0.25	100	140.0092	279.9547	209.966	1.33333	4451.8	4936.4	12.7101	130.3586	201.5674	140.0092

Table 6.1: Sensitivity Analysis of the Retailer's Problem with an Independent Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0, 0)$, $(1 + \delta) = 1.2$, c=1, s=50, c_s=10, c_e=3.157

b=0.3, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	91.5977	133.3554	459.3596	114.8399	4	4133.7	4523.9	11.1976	135.4025	110.2463	133.3554
0.2	0.4	0.1	0.25	91.6531	132.2543	321.8378	160.9189	2	4090.8	4521.8	12.18	132.0606	154.4822	132.2543
0.3	0.4	0.1	0.25	91.5484	131.2979	260.6689	195.5017	1.33333	4051.1	4512.7	12.9487	129.614	187.6816	131.2979
0.4	0.4	0.1	0.25	91.4151	130.4703	224.1654	224.1654	1	4015.8	4502.7	13.605	127.6287	215.1988	130.4703
0.5	0.4	0.1	0.25	91.2798	129.7405	199.2538	249.0673	0.8	3984.1	4493	14.1888	125.9355	239.1046	129.7405
0.6	0.4	0.1	0.25	91.1497	129.0859	180.8737	271.3105	0.66667	3955.2	4483.9	14.721	124.4472	260.4581	129.0859
0.7	0.4	0.1	0.25	91.0267	128.491	166.5991	291.5485	0.57143	3928.6	4475.3	15.2138	123.1126	279.8865	128.491
0.8	0.4	0.1	0.25	90.9108	127.9444	155.1028	310.2057	0.5	3903.8	4467.2	15.6755	121.8983	297.7974	127.9444
0.9	0.4	0.1	0.25	90.8018	127.438	145.5894	327.5761	0.44444	3880.7	4459.5	16.1116	120.7812	314.4731	127.438
1	0.4	0.1	0.25	90.6989	126.9654	137.5493	343.8733	0.4	3859	4452.3	16.5263	119.7447	330.1183	126.9654

b=0.3, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.1649	132.3501	113.4179	453.6715	0.25	4064.8	4452	11.2422	135.2451	108.8812	131.8683
0.2	0.4	0.1	0.25	90.9026	131.4776	159.8692	319.7385	0.5	4055	4484.1	12.203	131.9854	153.4745	131.4776
0.3	0.4	0.1	0.25	91.2324	130.9717	194.962	259.9493	0.75	4036.2	4496.8	12.9583	129.5843	187.1635	130.9717
0.4	0.4	0.1	0.25	91.4151	130.4703	224.1654	224.1654	1	4015.8	4502.7	13.605	127.6287	215.1988	130.4703
0.5	0.4	0.1	0.25	91.5251	129.9931	249.6061	199.6849	1.25	3995.6	4505.3	14.1815	125.9563	239.6218	129.9931
0.6	0.4	0.1	0.25	91.5934	129.5424	272.3767	181.5845	1.5	3975.9	4506.1	14.7079	124.4833	261.4816	129.5424
0.7	0.4	0.1	0.25	91.6357	129.1169	293.1271	167.5012	1.75	3956.9	4505.8	15.1959	123.1604	281.402	129.1169
0.8	0.4	0.1	0.25	91.6607	128.7144	312.2808	156.1404	2	3938.7	4504.8	15.6536	121.9551	299.7896	128.7144
0.9	0.4	0.1	0.25	91.6736	128.3326	330.1322	146.7254	2.25	3921.1	4503.2	16.0864	120.8452	316.9269	128.3326
1	0.4	0.1	0.25	91.6778	127.9692	346.8953	138.7581	2.5	3904.1	4501.4	16.4981	119.8144	333.0195	127.9692

b=0.3, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	92.5636	132.3460	262.9819	197.2364	1.33333	4099.3	4563.6	12.9183	129.7082	189.347	132.346
0.1	0.3	0.4	0.25	91.5484	131.2979	260.6689	195.5017	1.33333	4051.1	4512.7	12.9487	129.614	187.6816	131.2979
0.2	0.3	0.4	0.25	90.7585	130.4769	258.87	194.1525	1.33333	4013.7	4473.1	12.9727	129.5397	186.3864	130.4822
0.3	0.3	0.4	0.25	90.1013	129.8032	257.3737	193.0302	1.33333	3982.5	4440.1	12.993	129.4772	185.309	129.8032
0.4	0.3	0.4	0.25	89.533	129.2158	256.08	192.06	1.33333	3955.6	4411.6	13.0106	129.4226	184.3776	129.2158
0.5	0.3	0.4	0.25	89.0291	128.6949	254.9332	191.1999	1.33333	3931.7	4386.3	13.0265	129.3739	183.5519	128.6949
0.6	0.3	0.4	0.25	88.5743	128.2246	253.8983	190.4237	1.33333	3910.1	4363.5	13.0409	129.3296	182.8068	128.2246
0.7	0.3	0.4	0.25	88.1585	127.7945	252.9522	189.7142	1.33333	3890.4	4342.7	13.0541	129.2888	182.1256	127.7945
0.8	0.3	0.4	0.25	87.7743	127.3971	252.0784	189.0588	1.33333	3872.2	4323.4	13.0665	129.251	181.4964	127.3971
0.9	0.3	0.4	0.25	87.4166	127.0270	251.2648	188.4486	1.33333	3855.2	4305.4	13.078	129.2155	180.9106	127.027
1	0.3	0.4	0.25	87.0812	126.6800	250.5022	187.8767	1.33333	3839.3	4288.6	13.0889	129.1821	180.3616	126.68

b=0.3, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	91.8947	131.6555	261.4578	196.0933	1.33333	4067.6	4530	12.9383	129.6463	188.2496	131.6555
0.1	0.3	0.4	0.25	91.2324	130.9717	259.9493	194.962	1.33333	4036.2	4496.8	12.9583	129.5843	187.1635	130.9717
0.2	0.3	0.4	0.25	90.6703	130.3910	258.6691	194.0018	1.33333	4009.5	4468.6	12.9754	129.5313	186.2418	130.391
0.3	0.3	0.4	0.25	90.178	129.8823	257.5481	193.1611	1.33333	3986.2	4443.9	12.9906	129.4845	185.4347	129.8823
0.4	0.3	0.4	0.25	89.7376	129.4272	256.5455	192.4091	1.33333	3965.3	4421.9	13.0043	129.4423	184.7128	129.4272
0.5	0.3	0.4	0.25	89.3375	129.0137	255.6349	191.7262	1.33333	3946.3	4401.8	13.0168	129.4038	184.0572	129.0137
0.6	0.3	0.4	0.25	88.9698	128.6336	254.7982	191.0987	1.33333	3928.9	4383.4	13.0283	129.3682	183.4547	128.6336
0.7	0.3	0.4	0.25	88.6289	128.2810	254.0224	190.5168	1.33333	3912.7	4366.2	13.0391	129.3349	182.8962	128.281
0.8	0.3	0.4	0.25	88.3104	127.9516	253.2978	189.9733	1.33333	3897.6	4350.3	13.0493	129.3038	182.3744	127.9516
0.9	0.3	0.4	0.25	88.011	127.6420	252.6168	189.4626	1.33333	3883.4	4335.3	13.0589	129.2743	181.8841	127.642
1	0.3	0.4	0.25	87.7282	127.3495	251.9736	188.9802	1.33333	3870	4321.1	13.068	129.2464	181.421	127.3495

Table 6.2: Sensitivity Analysis of the Retailer's Problem with an Independent Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0.3, 0.1)$, $(1 + \delta) = 1.2$, c=1, s=50, c_s=10, c_e=3.157

b=0.5, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	92.516	134.1223	462.296	115.574	4	4177.8	4569.2	11.3222	134.964	110.951	134.1223
0.2	0.4	0.1	0.25	92.598	133.0577	324.0107	162.0054	2	4135.8	4568.4	12.305	131.6533	155.5252	133.0577
0.3	0.4	0.1	0.25	92.4426	132.0501	262.3286	196.7464	1.33333	4093.5	4556.7	13.0808	129.207	188.8766	132.0501
0.4	0.4	0.1	0.25	92.2431	131.1535	225.4700	225.4700	1	4054.9	4543.4	13.7454	127.2156	216.4512	131.1535
0.5	0.4	0.1	0.25	92.0386	130.3511	200.296	250.3701	0.8	4019.7	4530.2	14.3375	125.5145	240.3553	130.3511
0.6	0.4	0.1	0.25	91.8399	129.6246	181.7125	272.5688	0.66667	3987.4	4517.6	14.8777	124.0184	261.666	129.6246
0.7	0.4	0.1	0.25	91.6502	128.9598	167.2747	292.7308	0.57143	3957.6	4505.6	15.3783	122.6762	281.0215	128.9598
0.8	0.4	0.1	0.25	91.47	128.3459	155.6438	311.2876	0.5	3929.8	4494.3	15.8473	121.4548	298.8361	128.3459
0.9	0.4	0.1	0.25	91.2989	127.7748	146.017	328.5382	0.44444	3903.7	4483.5	16.2905	120.3312	315.3967	127.7748
1	0.4	0.1	0.25	91.1365	127.24	137.8798	344.6996	0.4	3879.1	4473.3	16.7119	119.2887	330.9116	127.24

b=0.5, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.2738	131.7374	113.2928	453.1712	0.25	4070	4456.5	11.4409	134.5506	108.7611	131.7374
0.2	0.4	0.1	0.25	91.4572	131.8496	160.3718	320.7437	0.5	4081.4	4511	12.3648	131.4596	153.957	131.8496
0.3	0.4	0.1	0.25	91.9665	131.5473	195.9144	261.2192	0.75	4070.9	4532.8	13.1057	129.1307	188.0778	131.5473
0.4	0.4	0.1	0.25	92.2431	131.1535	225.4700	225.4700	1	4054.9	4543.4	13.7454	127.2156	216.4512	131.1535
0.5	0.4	0.1	0.25	92.4078	130.7395	251.1991	200.9593	1.25	4037	4548.8	14.3182	125.5689	241.1511	130.7395
0.6	0.4	0.1	0.25	92.5095	130.3279	274.2125	182.8083	1.5	4018.7	4551.3	14.8428	124.1137	263.244	130.3279
0.7	0.4	0.1	0.25	92.5722	129.9269	295.1709	168.6691	1.75	4000.5	4552	15.3302	122.8035	283.3641	129.9269
0.8	0.4	0.1	0.25	92.609	129.5393	314.5052	157.2526	2	3982.6	4551.5	15.7879	121.6078	301.925	129.5393
0.9	0.4	0.1	0.25	92.628	129.1658	332.5147	147.7843	2.25	3965.2	4550.3	16.2211	120.5051	319.2141	129.1658
1	0.4	0.1	0.25	92.6342	128.8062	349.4171	139.7669	2.5	3948.2	4548.5	16.6338	119.4801	335.4404	128.8062

b=0.5, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	93.8929	133.5806	265.7091	199.2818	1.33333	4162.3	4629.6	13.0061	129.4366	191.3106	133.5806
0.1	0.3	0.4	0.25	92.4426	132.0501	262.3286	196.7464	1.33333	4093.5	4556.7	13.0808	129.207	188.8766	132.0501
0.2	0.3	0.4	0.25	91.2309	130.7702	259.505	194.6288	1.33333	4036.1	4495.8	13.1446	129.0118	186.8436	130.7702
0.3	0.3	0.4	0.25	90.169	129.6476	257.031	192.7733	1.33333	3985.7	4442.4	13.2016	128.8382	185.0623	129.6476
0.4	0.3	0.4	0.25	89.2124	128.6356	254.8027	191.102	1.33333	3940.3	4394.4	13.2538	128.6798	183.4579	128.6356
0.5	0.3	0.4	0.25	88.335	127.7068	252.759	189.5695	1.33333	3898.7	4350.2	13.3024	128.5327	181.9867	127.7068
0.6	0.3	0.4	0.25	87.5202	126.8436	250.8617	188.1463	1.33333	3860.1	4309.3	13.3483	128.3945	180.6205	126.8436
0.7	0.3	0.4	0.25	86.7562	126.0325	249.0828	186.8121	1.33333	3823.8	4270.9	13.3919	128.2635	179.3397	126.0338
0.8	0.3	0.4	0.25	86.0347	125.2686	247.4031	185.5523	1.33333	3789.6	4234.6	13.4336	128.1386	178.1302	125.2686
0.9	0.3	0.4	0.25	85.3494	124.5413	245.8077	184.3558	1.33333	3757.1	4200.1	13.4736	128.0188	176.9815	124.5413
1	0.3	0.4	0.25	84.6954	123.8469	244.2853	183.2139	1.33333	3726.1	4167.2	13.5124	127.9035	175.8854	123.8469

b=0.5, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	92.9511	132.5869	263.5137	197.6353	1.33333	4117.6	4582.3	13.0544	129.288	189.7299	132.5869
0.1	0.3	0.4	0.25	91.9665	131.5473	261.2192	195.9144	1.33333	4070.9	4532.8	13.1057	129.1307	188.0778	131.5473
0.2	0.3	0.4	0.25	91.0912	130.6225	259.1794	194.3846	1.33333	4029.4	4488.8	13.152	128.9891	186.6092	130.6225
0.3	0.3	0.4	0.25	90.2953	129.7812	257.3252	192.9939	1.33333	3991.7	4448.8	13.1947	128.859	185.2742	129.7812
0.4	0.3	0.4	0.25	89.5607	129.0042	255.614	191.7105	1.33333	3956.8	4411.9	13.2347	128.7377	184.0421	129.0042
0.5	0.3	0.4	0.25	88.8751	128.2787	254.0172	190.5129	1.33333	3924.3	4377.4	13.2724	128.6234	182.8924	128.2787
0.6	0.3	0.4	0.25	88.2301	127.5956	252.5148	189.3861	1.33333	3893.7	4345	13.3083	128.5149	181.8107	127.5956
0.7	0.3	0.4	0.25	87.6191	126.9484	251.092	188.319	1.33333	3864.8	4314.2	13.3427	128.4113	180.7862	126.9484
0.8	0.3	0.4	0.25	87.0373	126.3318	249.7374	187.303	1.33333	3837.2	4285	13.3758	128.3119	179.8109	126.3318
0.9	0.3	0.4	0.25	86.481	125.7420	248.4422	186.3316	1.33333	3810.8	4257	13.4077	128.216	178.8784	125.742
1	0.3	0.4	0.25	85.9472	125.1757	247.1993	185.3995	1.33333	3785.4	4230.2	13.4387	128.1234	177.9835	125.1757

Table 6.3: Sensitivity Analysis of the Retailer's Problem with an Independent Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0.5, 0.1)$, $(1 + \delta) = 1.2$, c=1, s=50, c_s=10, c_e=3.157

b=0.7, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	$β_1$	$β_2$	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	93.3324	134.8264	464.9933	116.2483	4	4217.1	4609.5	11.4155	134.6386	111.5984	134.8264
0.2	0.4	0.1	0.25	93.4344	133.7903	325.9935	162.9967	2	4175.7	4609.7	12.3974	131.3547	156.4769	133.7903
0.3	0.4	0.1	0.25	93.2408	132.7412	263.8547	197.891	1.33333	4131.3	4596.1	13.181	128.9008	189.9753	132.7412
0.4	0.4	0.1	0.25	92.9899	131.7878	226.6819	226.6819	1	4090.1	4580.2	13.8552	126.895	217.6146	131.7878
0.5	0.4	0.1	0.25	92.7300	130.924	201.2745	251.5931	0.8	4052.1	4564.2	14.4573	125.1783	241.5294	130.924
0.6	0.4	0.1	0.25	92.475	130.1354	182.5083	273.7624	0.66667	4017.1	4548.7	15.0074	123.6667	262.8119	130.1354
0.7	0.4	0.1	0.25	92.2292	129.4089	167.922	293.8635	0.57143	3984.5	4533.9	15.5177	122.3097	282.109	129.4089
0.8	0.4	0.1	0.25	91.9937	128.7343	156.1672	312.3343	0.5	3954.1	4519.7	15.9961	121.0744	299.841	128.7343
0.9	0.4	0.1	0.25	91.7683	128.1037	146.4347	329.4782	0.44444	3925.4	4506.2	16.4484	119.9376	316.299	128.1037
1	0.4	0.1	0.25	91.5526	127.511	138.2061	345.5153	0.4	3898.3	4493.2	16.8787	118.8828	331.6947	127.511

b=0.7, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	$β_1$	$β_2$	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.3819	131.6139	113.1748	452.6992	0.25	4075.1	4460.9	11.6356	133.8807	108.6478	131.6139
0.2	0.4	0.1	0.25	91.9771	132.2127	160.8626	321.7252	0.5	4106.2	4536.3	12.5052	131.0088	154.4281	132.2127
0.3	0.4	0.1	0.25	92.6379	132.0910	196.8141	262.4188	0.75	4102.7	4565.8	13.2256	128.7652	188.9416	132.0909
0.4	0.4	0.1	0.25	92.9899	131.7878	226.6819	226.6819	1	4090.1	4580.2	13.8552	126.895	217.6146	131.7878
0.5	0.4	0.1	0.25	93.1972	131.4255	252.664	202.1312	1.25	4074.1	4587.7	14.4226	125.2754	242.5574	131.4255
0.6	0.4	0.1	0.25	93.3243	131.0452	275.8898	183.9265	1.5	4056.8	4591.5	14.9442	123.8378	264.8542	131.0452
0.7	0.4	0.1	0.25	93.4022	130.6634	297.0306	169.7318	1.75	4039.2	4593	15.4301	122.5396	285.1494	130.6634
0.8	0.4	0.1	0.25	93.4479	130.2874	316.5239	158.262	2	4021.6	4593	15.8873	121.3523	303.863	130.2874
0.9	0.4	0.1	0.25	93.4714	129.9203	334.6735	148.7438	2.25	4004.2	4592	16.3207	120.2557	321.2866	129.9203
1	0.4	0.1	0.25	93.4792	129.5635	351.7005	140.6802	2.5	3987.1	4590.3	16.7339	119.2349	337.6324	129.5635

b=0.7, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
$β_1$	p_1	p_2	$β_2$	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	94.9832	134.6187	268.0045	201.0034	1.33333	4214	4683.9	13.0546	129.2873	192.9632	134.6187
0.1	0.3	0.4	0.25	93.2408	132.7412	263.8547	197.891	1.33333	4131.3	4596.1	13.181	128.9008	189.9753	132.7412
0.2	0.3	0.4	0.25	91.6783	131.0553	260.1337	195.1003	1.33333	4057.2	4517.4	13.2975	128.5475	187.2963	131.0553
0.3	0.3	0.4	0.25	90.2367	129.4975	256.7004	192.5253	1.33333	3988.8	4444.8	13.4078	128.2157	184.8243	129.4975
0.4	0.3	0.4	0.25	88.8843	128.0344	253.4798	190.1098	1.33333	3924.7	4376.6	13.5138	127.8993	182.5054	128.0344
0.5	0.3	0.4	0.25	87.6019	126.6452	250.4258	187.8193	1.33333	3863.9	4312	13.6166	127.5945	180.3066	126.6452
0.6	0.3	0.4	0.25	86.3767	125.3163	247.5078	185.6308	1.33333	3805.7	4250.3	13.717	127.2988	178.2056	125.3163
0.7	0.3	0.4	0.25	85.1994	124.0379	244.7039	183.5279	1.33333	3749.8	4190.9	13.8155	127.0105	176.1868	124.0379
0.8	0.3	0.4	0.25	84.0633	122.8026	241.9977	181.4983	1.33333	3695.9	4133.6	13.9126	126.7282	174.2384	122.8026
0.9	0.3	0.4	0.25	82.963	121.6050	239.3769	179.5326	1.33333	3643.68	4078.1	14.0086	126.4511	172.3513	121.605
1	0.3	0.4	0.25	81.8945	120.4406	236.8313	177.6235	1.33333	3593	4024.2	14.1037	126.1782	170.5186	120.4406

b=0.7, δ=1.2				Effective					Expected	Expected	Expected	Expected	Expected	Expected
$β_2$	p_1	p_2	$β_1$	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	93.8683	133.4177	265.3492	199.0119	1.33333	4161.1	4627.7	13.1351	129.0409	191.0514	133.4177
0.1	0.3	0.4	0.25	92.6379	132.0910	262.4188	196.8141	1.33333	4102.7	4565.8	13.2256	128.7652	188.9416	132.0909
0.2	0.3	0.4	0.25	91.4923	130.8544	259.6908	194.7681	1.33333	4048.4	4508.1	13.3116	128.505	186.9774	130.8544
0.3	0.3	0.4	0.25	90.4115	129.6866	257.1168	192.8376	1.33333	3997.1	4453.6	13.3943	128.2562	185.1241	129.6866
0.4	0.3	0.4	0.25	89.3824	128.5735	254.6659	190.9994	1.33333	3948.3	4401.7	13.4745	128.0164	183.3595	128.5735
0.5	0.3	0.4	0.25	88.396	127.5056	252.3169	189.2377	1.33333	3901.5	4352	13.5526	127.7838	181.6682	127.5056
0.6	0.3	0.4	0.25	87.4459	126.4760	250.0541	189.2377	1.32138	3856.4	4304.1	13.6292	127.5571	180.039	126.476
0.7	0.3	0.4	0.25	86.5271	125.4795	247.8659	185.8994	1.33333	3812.8	4257.8	13.7045	127.3353	178.4635	125.4795
0.8	0.3	0.4	0.25	85.6358	124.5119	245.7431	184.3073	1.33333	3770.6	4212.9	13.7788	127.1178	176.935	124.5119
0.9	0.3	0.4	0.25	84.7689	123.5699	243.6783	182.7588	1.33333	3729.4	4169.2	13.8521	126.904	175.4484	123.5699
1	0.3	0.4	0.25	83.9239	122.6510	241.6657	181.2493	1.33333	3689.3	4126.6	13.9247	126.6933	173.9993	122.651

Table 6.4: Sensitivity Analysis of the Retailer's Problem with an Independent Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0.7, 0.1)$, $(1 + \delta) = 1.2$, c=1, s=50, c_s=10, c_e=3.157

b=1, δ=1.2

p_1	p_2	β_1	β_2	Effective Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Expected Profit	Expected Revenue	Expected Shortage Cost	Expected Excess Cost	Expected Manufacturing Cost	Expected Acquisition Cost
0.1	0.4	0.1	0.25	94.3913	135.7735	468.624	117.156	4	4268	4662	11.5089	134.3151	112.4698	135.7735
0.2	0.4	0.1	0.25	94.5132	134.7679	328.6413	164.3206	2	4227.2	4663.2	12.488	131.0636	157.7478	134.7679
0.3	0.4	0.1	0.25	94.2808	133.6724	265.9121	199.4341	1.33333	4180.6	4647.6	13.2837	128.5894	191.4567	133.6724
0.4	0.4	0.1	0.25	93.9754	132.6533	228.3366	228.3366	1	4136.5	4628.9	13.9732	126.5531	219.2031	132.6533
0.5	0.4	0.1	0.25	93.6542	131.7162	202.6282	253.2852	0.8	4095.5	4609.8	14.5918	124.8038	243.1538	131.7162
0.6	0.4	0.1	0.25	93.3343	130.8507	183.6232	275.4349	0.66667	4057.2	4590.9	15.1588	123.2597	264.4175	130.8507
0.7	0.4	0.1	0.25	93.0216	130.0455	168.8402	295.4704	0.57143	4021.4	4572.6	15.6859	121.8713	283.6516	130.0455
0.8	0.4	0.1	0.25	92.7179	129.2916	156.9185	313.8371	0.5	3987.6	4555	16.1812	120.6057	301.2836	129.2916
0.9	0.4	0.1	0.25	92.4239	128.5815	147.0417	330.8438	0.44444	3955.7	4537.9	16.6501	119.44	317.61	128.5815
1	0.4	0.1	0.25	92.1392	127.9093	138.686	346.7149	0.4	3925.3	4521.5	17.0969	118.3574	332.8463	127.9093

b=1, δ=1.2

p_2	p_1	β_1	β_2	Effective Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Expected Profit	Expected Revenue	Expected Shortage Cost	Expected Excess Cost	Expected Manufacturing Cost	Expected Acquisition Cost
0.1	0.4	0.1	0.25	90.5425	131.4418	113.0104	452.0416	0.25	4082.8	4467.5	11.92	132.9201	108.49	131.4418
0.2	0.4	0.1	0.25	92.6966	132.7389	161.5741	323.1483	0.5	4140.4	4571.4	12.68	130.4535	155.1112	132.7389
0.3	0.4	0.1	0.25	93.5393	132.8481	198.0681	264.0908	0.75	4145.4	4610.2	13.3621	128.3528	190.1453	132.8481
0.4	0.4	0.1	0.25	93.9754	132.6533	228.3366	228.3366	1	4136.5	4628.9	13.9732	126.5531	219.2031	132.6533
0.5	0.4	0.1	0.25	94.2280	132.35	254.6397	203.7117	1.25	4122.4	4638.7	14.5306	124.9738	244.4541	132.35
0.6	0.4	0.1	0.25	94.3813	132.0043	278.1344	185.4229	1.5	4106.2	4643.8	15.0466	123.561	267.009	132.0043
0.7	0.4	0.1	0.25	94.4747	131.6433	299.5068	171.1468	1.75	4089.1	4646.1	15.5295	122.2788	287.5265	131.6433
0.8	0.4	0.1	0.25	94.5292	131.2796	319.2034	159.6017	2	4071.7	4646.5	15.9852	121.102	306.4353	131.2796
0.9	0.4	0.1	0.25	94.5572	130.9192	337.5338	150.015	2.25	4054.4	4645.8	16.4184	120.0123	324.0324	130.9192
1	0.4	0.1	0.25	94.5663	130.5652	354.7229	141.8892	2.5	4037.2	4644.2	16.8321	118.9959	340.534	130.5652

b=1, δ=1.2

β_1	p_1	p_2	β_2	Effective Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Expected Profit	Expected Revenue	Expected Shortage Cost	Expected Excess Cost	Expected Manufacturing Cost	Expected Acquisition Cost
0	0.3	0.4	0.25	96.2633	135.8731	270.7806	203.0855	1.33333	4274.6	4747.8	13.0787	129.2135	194.9621	135.8731
0.1	0.3	0.4	0.25	94.2808	133.6724	265.9121	199.4341	1.33333	4180.6	4647.6	13.2837	128.5894	191.4567	133.6724
0.2	0.3	0.4	0.25	92.3057	131.4755	261.0606	195.7954	1.33333	4086.9	4547.8	13.4949	127.9555	187.9636	131.4755
0.3	0.3	0.4	0.25	90.3382	129.2823	256.2264	192.1698	1.33333	3993.6	4448.3	13.7126	127.3117	184.483	129.2823
0.4	0.3	0.4	0.25	88.3783	127.0928	251.4095	188.5571	1.33333	3900.5	4349.2	13.9371	126.6576	181.0149	127.0928
0.5	0.3	0.4	0.25	86.4264	124.9072	246.6103	184.9577	1.33333	3807.8	4250.5	14.1687	125.9929	177.5594	124.9072
0.6	0.3	0.4	0.25	84.4824	122.7255	241.8287	181.3715	1.33333	3715.5	4152.1	14.4077	125.3173	174.1167	122.7255
0.7	0.3	0.4	0.25	82.5467	120.5476	237.0651	177.7988	1.33333	3623.5	4054.1	14.6545	124.6305	170.6869	120.5476
0.8	0.3	0.4	0.25	80.6194	118.3736	232.3196	174.2397	1.33333	3531.9	3956.4	14.9094	123.9321	167.2701	118.3736
0.9	0.3	0.4	0.25	78.7006	116.2035	227.5923	170.6942	1.33333	3440.7	3859.2	15.173	123.2218	163.8664	116.2035
1	0.3	0.4	0.25	76.7906	114.0375	222.8834	167.1626	1.33333	3349.8	3762.3	15.4455	122.4992	160.4761	114.0375

b=1, δ=1.2

β_2	p_1	p_2	β_1	Effective Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Expected Profit	Expected Revenue	Expected Shortage Cost	Expected Excess Cost	Expected Manufacturing Cost	Expected Acquisition Cost
0	0.3	0.4	0.25	95.0234	134.4972	267.7358	200.8018	1.33333	4215.8	4685.1	13.2061	128.8246	192.7698	134.4972
0.1	0.3	0.4	0.25	93.5393	132.8481	264.0908	198.0681	1.33333	4145.4	4610.2	13.3621	128.3528	190.1453	132.8481
0.2	0.3	0.4	0.25	92.0594	131.2011	260.4554	195.3415	1.33333	4075.2	4535.4	13.5217	127.8756	187.5279	131.2011
0.3	0.3	0.4	0.25	90.5837	129.5562	256.8297	192.6223	1.33333	4005.2	4460.8	13.685	127.3927	184.9174	129.5562
0.4	0.3	0.4	0.25	89.0969	127.9134	256.8297	192.6223	1.33333	3934.9	4386.4	13.8521	126.9041	182.3139	127.9134
0.5	0.3	0.4	0.25	87.6454	126.2728	249.6077	187.2058	1.33333	3865.7	4312.2	14.0231	126.4095	179.7176	126.2728
0.6	0.3	0.4	0.25	86.1829	124.6343	246.0116	184.5087	1.33333	3796.3	4238.2	14.1981	125.909	177.1283	124.6343
0.7	0.3	0.4	0.25	84.725	122.9980	242.4254	181.8191	1.33333	3727	4164.4	14.3774	125.4023	174.5463	122.998
0.8	0.3	0.4	0.25	83.2716	121.3638	238.8494	179.137	1.33333	3658	4090.8	14.561	124.8894	171.9715	121.3638
0.9	0.3	0.4	0.25	81.823	119.7319	235.2834	176.4625	1.33333	3589.1	4017.4	14.7491	124.3699	169.404	119.7319
1	0.3	0.4	0.25	80.379	118.1021	231.7276	173.7957	1.33333	3520.5	3944.2	14.9419	123.844	166.8439	118.1021

Table 6.5: Sensitivity Analysis of the Retailer's Problem with an Independent Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (1, 0.1)$, $(1 + \delta) = 1.2$, c=1, s=50, c_s=10, c_e=3.157

	b=0,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	100	148.3357	517.0413	129.2603	4	4552.7	4966	6.7907	154.7835	103.4083	148.3357
0.2	0.4	0.1	0.25	100	149.7961	369.605	184.8025	2	4506.9	4969.8	6.0461	159.1602	147.842	149.7961
0.3	0.4	0.1	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.4	0.4	0.1	0.25	100	152.0954	265.8112	265.8112	1	4439.1	4974.9	5.0166	166.0959	212.649	152.0954
0.5	0.4	0.1	0.25	100	153.1183	239.5261	299.4076	0.8	4410.5	4977	4.6099	169.1977	239.5261	153.1183
0.6	0.4	0.1	0.25	100	154.1012	220.2166	330.3249	0.66667	4384	4978.8	4.2464	172.1869	264.2599	154.1012
0.7	0.4	0.1	0.25	100	155.0613	205.2927	359.2623	0.57143	4358.9	4980.4	3.9159	175.1143	287.4098	155.0613
0.8	0.4	0.1	0.25	100	156.0104	193.3402	386.6804	0.5	4335	4981.9	3.6114	178.0153	309.3443	156.0104
0.9	0.4	0.1	0.25	100	156.9578	183.5133	412.9048	0.44444	4311.8	4983.4	3.3284	180.9178	330.3239	156.9578
1	0.4	0.1	0.25	100	157.9113	175.2716	438.1789	0.4	4289.3	4984.7	3.0635	183.8453	350.5431	157.9113

	b=0,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	100	148.3357	129.2603	517.0413	0.25	4552.7	4853.7	29.262	96.0091	87.3831	127.4767
0.2	0.4	0.1	0.25	100	149.7961	184.8025	369.605	0.5	4506.9	4969.8	6.0461	159.1602	147.842	149.7961
0.3	0.4	0.1	0.25	100	151.0053	228.3668	304.4891	0.75	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.4	0.4	0.1	0.25	100	152.0954	265.8112	265.8112	1	4439.1	4974.9	5.0166	166.0959	212.649	152.0954
0.5	0.4	0.1	0.25	100	153.1183	299.4076	239.5261	1.25	4410.5	4977	4.6099	169.1977	239.5261	153.1183
0.6	0.4	0.1	0.25	100	154.1012	330.3249	220.2166	1.5	4384	4978.8	4.2464	172.1869	264.2599	154.1012
0.7	0.4	0.1	0.25	100	155.0613	359.2623	205.2927	1.75	4358.9	4980.4	3.9159	175.1143	287.4098	155.0613
0.8	0.4	0.1	0.25	100	156.0104	386.6804	193.3402	2	4335	4981.9	3.6114	178.0153	309.3443	156.0104
0.9	0.4	0.1	0.25	100	156.9578	412.9048	183.5133	2.25	4311.8	4983.4	3.3284	180.9178	330.3239	156.9578
1	0.4	0.1	0.25	100	157.9113	438.1789	175.2716	2.5	4289.3	4984.7	3.0635	183.8453	350.5431	157.9113

	b=0,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.1	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.2	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.3	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.4	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.5	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.6	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.7	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.8	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.9	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
1	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053

	b=0,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.1	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.2	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.3	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.4	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.5	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.6	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.7	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.8	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
0.9	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053
1	0.3	0.4	0.25	100	151.0053	304.4891	228.3668	1.33333	4470.6	4972.6	5.484	162.801	182.6934	151.0053

Table 6.6: Sensitivity Analysis of the Integrated Problem of the Retailer and the Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0, 0)$, c=1, s=50, c_s=10, c_e=3.157

b=0.3,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	91.5848	140.4742	485.2518	122.0309	3.97647	4146	4546.7	6.4996	156.4401	97.3375	140.4742
0.2	0.4	0.1	0.25	91.6367	140.4362	344.3799	171.8454	2.00401	4108.3	4549.1	6.5461	156.1709	137.6142	140.4362
0.3	0.4	0.1	0.25	91.5309	140.5440	281.7829	210.3786	1.33941	4071.9	4544.4	6.4361	156.8105	168.6863	140.544
0.4	0.4	0.1	0.25	91.3965	140.7403	244.5437	243.1684	1.00566	4038.6	4538.5	6.2688	157.8024	195.0848	140.7403
0.5	0.4	0.1	0.25	91.2597	140.9902	219.2155	272.3356	0.80495	4008	4532.6	6.078	158.963	218.542	140.9902
0.6	0.4	0.1	0.25	91.128	141.2751	200.5904	298.96	0.67096	3979.7	4527	5.878	160.2153	239.9383	141.2751
0.7	0.4	0.1	0.25	91.0031	141.5843	186.1733	323.6805	0.57518	3953.2	4521.8	5.6757	161.5225	259.7935	141.5843
0.8	0.4	0.1	0.25	90.8854	141.912	174.6003	346.9124	0.5033	3928.2	4516.9	5.4745	162.8656	278.4452	141.912
0.9	0.4	0.1	0.25	90.7743	142.2544	165.0554	368.9442	0.44737	3904.4	4512.3	5.276	164.2348	296.1275	142.2544
1	0.4	0.1	0.25	90.6694	142.6092	157.0164	389.9862	0.40262	3881.7	4508.1	5.0811	165.6248	313.0109	142.6092

b=0.3,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.1893	134.2374	117.1509	456.9433	0.25638	4083.9	4462.2	9.4478	142.0828	92.5547	134.2374
0.2	0.4	0.1	0.25	90.902	137.0683	168.5577	332.6598	0.5067	4077.5	4504.9	8.0417	148.3277	133.955	137.0683
0.3	0.4	0.1	0.25	91.2208	139.0737	209.2851	276.7121	0.75633	4059.6	4525.8	7.05367	153.3417	166.7277	139.073
0.4	0.4	0.1	0.25	91.3965	140.7403	244.5407	243.1684	1.00564	4038.6	4538.5	6.2688	157.8024	195.0848	140.7403
0.5	0.4	0.1	0.25	91.5012	142.2259	276.3696	220.2453	1.25483	4016.5	4547	5.6103	161.9547	220.6705	142.2259
0.6	0.4	0.1	0.25	91.5653	143.604	305.8179	203.3483	1.50391	3994.2	4553.1	5.04	165.9242	244.3361	143.604
0.7	0.4	0.1	0.25	91.6039	144.9168	333.5247	190.2646	1.75295	3971.7	4557.5	4.5358	169.789	266.5951	144.9168
0.8	0.4	0.1	0.25	91.6254	146.1919	359.9143	179.7801	2.00197	3949.2	4560.9	4.0833	173.6048	287.7898	146.1919
0.9	0.4	0.1	0.25	91.635	147.4495	385.2918	171.166	2.25098	3926.7	4563.4	3.6725	177.4161	308.1661	147.4495
1	0.4	0.1	0.25	91.6363	148.5799	407.4601	164.6238	2.4751	3906.4	4565.2	3.3325	180.8743	325.9616	148.5799

b=0.3,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	92.5658	139.1619	279.9136	207.1833	1.35104	4126	4589.4	7.7793	149.6022	166.8474	139.1619
0.1	0.3	0.4	0.25	91.5309	140.5440	281.7829	210.3786	1.33941	4071.9	4544.4	6.4361	156.8105	168.6863	140.544
0.2	0.3	0.4	0.25	90.734	141.3188	282.7912	212.2052	1.33263	4030.1	4508.3	5.6741	161.5334	169.7194	141.3188
0.3	0.3	0.4	0.25	90.0735	141.8278	283.425	213.429	1.32796	3995.4	4477.9	5.159	165.0638	170.3991	141.8278
0.4	0.3	0.4	0.25	89.5034	142.1948	283.8623	214.3277	1.32443	3965.5	4451.3	4.776	167.9021	170.8898	142.1948
0.5	0.3	0.4	0.25	88.9985	142.4267	284.1846	215.029	1.32161	3939	4427.6	4.4738	170.2915	171.267	142.4767
0.6	0.3	0.4	0.25	88.543	142.7036	284.4348	215.6009	1.31927	3915.2	4406	4.2253	172.3677	171.5708	142.7036
0.7	0.3	0.4	0.25	88.1267	142.8930	284.6376	216.083	1.31726	3893.3	4386.3	4.015	174.2139	171.8245	142.893
0.8	0.3	0.4	0.25	87.7423	143.0556	284.8082	216.5004	1.31551	3873.1	4367.9	3.8329	175.8842	172.0426	143.0556
0.9	0.3	0.4	0.25	87.3843	143.1988	284.9564	216.8693	1.31395	3854.3	4350.9	3.5293	172.2346	172.4071	143.327325
1	0.3	0.4	0.25	87.0488	143.3273	285.0887	217.2012	1.31256	3836.7	4334.8	3.5293	178.8362	172.4071	143.3273

b=0.3,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	91.8803	147.8290	295.31	224.6001	1.31483	4068.1	4575.9	3.6306	177.8265	178.4331	147.829
0.1	0.3	0.4	0.25	91.2092	144.6352	289.0579	218.5868	1.32239	4044.6	4538	4.4933	170.1325	174.1521	144.6352
0.2	0.3	0.4	0.25	90.6442	142.4652	284.831	214.5024	1.32787	4022.4	4506.6	5.1309	165.2656	171.2503	142.4652
0.3	0.3	0.4	0.25	90.1515	140.8273	281.6438	211.4266	1.33211	4002.1	4479.4	5.6325	161.8075	169.0638	140.8273
0.4	0.3	0.4	0.25	89.7121	139.5115	279.0815	208.9629	1.33556	3983.3	4455.4	6.0445	159.1702	167.3096	139.5115
0.5	0.3	0.4	0.25	89.3137	138.4101	276.9337	206.907	1.33845	3966	4433.7	6.3935	157.0604	165.8429	138.4101
0.6	0.3	0.4	0.25	88.9481	137.4612	275.0799	205.141	1.34093	3949.9	4413.9	6.6962	155.3139	164.5804	137.4612
0.7	0.3	0.4	0.25	88.6094	136.6260	273.4454	203.591	1.34311	3934.8	4395.7	6.9635	153.8303	163.4701	136.626
0.8	0.3	0.4	0.25	88.2932	135.8787	271.9806	202.2081	1.34505	3920.5	4378.6	7.2029	152.5446	162.4774	135.8787
0.9	0.3	0.4	0.25	87.9963	135.2015	270.6508	200.958	1.3468	3907.1	4362.7	7.4199	151.4124	161.5785	135.2015
1	0.3	0.4	0.25	87.7159	134.5815	269.4313	199.816	1.3484	3894.3	4347.7	7.6185	150.4023	160.7558	134.5815

Table 6.7: Sensitivity Analysis of the Integrated Problem of the Retailer and the Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0.3, 0.1)$, c=1, s=50, c_s=10, c_e=3.157

b=0.5,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	92.4969	141.6601	489.133	123.3454	3.96556	4189.5	4593	6.3597	157.2604	98.2514	141.6601
0.2	0.4	0.1	0.25	92.5749	140.9436	345.9235	172.4551	2.00588	4154.1	4594.9	6.773	154.8821	138.1667	140.9436
0.3	0.4	0.1	0.25	92.4202	140.5681	282.1447	210.1889	1.34234	4116.1	4586.5	6.8919	154.2225	168.719	140.5681
0.4	0.4	0.1	0.25	92.2208	140.3594	244.1519	242.0962	1.00849	4080.6	4576.6	6.897	154.1946	194.4992	140.3594
0.5	0.4	0.1	0.25	92.0157	140.2476	218.2811	270.3106	0.80752	4047.7	4566.6	6.8465	154.4733	217.2648	140.2476
0.6	0.4	0.1	0.25	91.8159	140.1984	199.2349	295.9205	0.67327	4017.2	4557	6.7657	154.9234	237.9091	140.1984
0.7	0.4	0.1	0.25	91.6247	140.1927	184.4739	319.57	0.57726	3988.6	4547.9	6.6672	155.4782	256.9597	140.1927
0.8	0.4	0.1	0.25	91.4426	140.2189	172.6094	341.6776	0.50518	3961.7	4539.3	6.5582	156.1013	274.7586	140.2189
0.9	0.4	0.1	0.25	91.2696	140.2695	162.81	362.5335	0.44909	3936.2	4531.3	6.4428	156.7711	291.5424	140.2695
1	0.4	0.1	0.25	91.1049	140.3394	154.5435	382.3487	0.4042	3912	4523.6	6.3236	157.4745	307.483	140.3394

b=0.5,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.3538	132.0952	116.0054	445.2504	0.26054	4092	4461.6	11.2109	135.3553	90.9272	132.0952
0.2	0.4	0.1	0.25	91.4722	135.8982	167.5798	328.2869	0.51047	4106.7	4527.7	9.1827	143.1926	132.6893	135.8982
0.3	0.4	0.1	0.25	91.9577	138.3832	208.5861	274.5856	0.75964	4097.3	4558.5	7.8825	149.0962	165.8101	138.3832
0.4	0.4	0.1	0.25	92.2208	140.3594	244.1519	242.0962	1.00849	4080.6	4576.6	6.897	154.1946	194.4992	140.3594
0.5	0.4	0.1	0.25	92.3763	142.074	276.3008	219.7779	1.25718	4060.9	4588.3	6.0939	158.8647	220.4093	142.074
0.6	0.4	0.1	0.25	92.4708	143.6376	306.0882	203.2743	1.50579	4039.7	4596.5	5.4124	163.2887	244.3999	143.6376
0.7	0.4	0.1	0.25	92.5272	145.1108	334.1543	190.4716	1.75435	4017.8	4602.3	4.8188	167.5751	266.9918	145.1108
0.8	0.4	0.1	0.25	92.5585	146.5322	360.9291	180.2033	2.0029	3995.3	4606.5	4.2921	171.7987	288.5343	146.5322
0.9	0.4	0.1	0.25	92.5723	147.9293	386.7248	171.7674	2.25144	3972.5	4609.5	3.8187	176.0176	309.2806	147.9293
1	0.4	0.1	0.25	92.5738	149.1841	409.3054	165.3663	2.47514	3951.6	4611.5	3.4299	179.8524	327.4348	149.1841

b=0.5,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	93.9064	139.1661	280.582	206.7035	1.35741	4191.9	4652.2	8.62	145.6476	166.856	139.1661
0.1	0.3	0.4	0.25	92.4202	140.5681	282.1447	210.1889	1.34234	4116.1	4586.5	6.8919	154.2225	168.719	140.5681
0.2	0.3	0.4	0.25	91.1922	141.5655	283.2972	212.6489	1.33223	4052.5	4530.8	5.7719	160.8962	170.0487	141.5655
0.3	0.3	0.4	0.25	90.1214	142.4078	284.3423	214.6786	1.3245	3996.2	4481.4	4.9384	166.6744	171.1742	142.4078
0.4	0.3	0.4	0.25	89.1591	143.1890	285.3927	216.5048	1.31818	3944.9	4436.6	4.2719	171.9698	172.2197	143.189
0.5	0.3	0.4	0.25	88.2777	143.9553	286.5037	218.2385	1.3128	3897.4	4395.3	3.7157	176.9974	173.2465	143.9553
0.6	0.3	0.4	0.25	87.4596	144.7339	287.7091	219.9447	1.3081	3852.6	4356.8	3.2384	181.8888	174.2906	144.7339
0.7	0.3	0.4	0.25	86.6925	145.5433	289.0341	221.6665	1.30391	3810	4320.5	2.8209	186.7352	175.3768	145.5433
0.8	0.3	0.4	0.25	85.9679	146.3983	290.501	223.4369	1.30015	3769.2	4286.1	2.4509	191.6071	176.5251	146.3983
0.9	0.3	0.4	0.25	85.2789	147.3124	292.1321	225.284	1.29673	3729.6	4253.3	2.1202	196.5647	177.7533	147.3124
1	0.3	0.4	0.25	84.6207	148.2988	293.9521	227.2344	1.29361	3691.1	4221.9	1.8227	201.6644	179.0794	148.2988

b=0.5,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	92.9336	151.5800	302.7558	231.6211	1.30712	4108.3	4632.3	2.8722	186.1059	183.4752	151.58
0.1	0.3	0.4	0.25	91.931	146.7713	293.1913	222.6419	1.31687	4074.4	4576.6	3.9899	174.4399	177.0142	146.7713
0.2	0.3	0.4	0.25	91.0485	143.3893	286.5438	216.3061	1.32471	4040.2	4527.8	4.9163	166.8394	172.4856	143.3893
0.3	0.3	0.4	0.25	90.2517	140.7466	281.3818	211.3539	1.33133	4007.3	4484	5.7155	161.2625	168.9561	140.7466
0.4	0.3	0.4	0.25	89.5199	138.5548	277.116	207.2515	1.3371	3976	4443.9	6.4249	156.8762	166.0354	138.5548
0.5	0.3	0.4	0.25	88.8396	136.6671	273.4498	203.7241	1.34226	3946.1	4406.6	7.0677	153.266	163.5246	136.6671
0.6	0.3	0.4	0.25	88.2016	134.9986	270.2137	200.6124	1.34694	3917.6	4371.8	7.6591	150.1987	161.3091	134.9986
0.7	0.3	0.4	0.25	87.599	133.4961	267.3018	197.816	1.35126	3890.3	4338.9	8.2098	147.5307	159.317	133.4961
0.8	0.3	0.4	0.25	87.0266	132.1238	264.6438	195.2673	1.35529	3864.2	4307.7	8.7273	145.1681	157.5001	132.1238
0.9	0.3	0.4	0.25	86.4805	130.8567	262.1904	192.9188	1.35907	3839	4277.9	9.2173	143.0461	155.8246	130.8567
1	0.3	0.4	0.25	85.9575	129.6660	259.9057	190.7357	1.36265	3814.7	4249.5	9.684	141.1184	154.266	129.6766

Table 6.8: Sensitivity Analysis of the Integrated Problem of the Retailer and the Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0.5, 0.1)$, c=1, s=50, c_s=10, c_e=3.157

b=0.7,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	93.3087	142.6758	492.5319	124.4546	3.95752	4228.3	4634.2	6.2572	157.872	99.035	142.6758
0.2	0.4	0.1	0.25	93.4067	141.4651	347.4646	173.1052	2.00725	4194.5	4635.6	6.9407	153.955	138.735	141.4651
0.3	0.4	0.1	0.25	93.2160	140.7341	282.7542	210.2965	1.34455	4155.3	4624.6	7.2393	152.3528	168.9449	140.7371
0.4	0.4	0.1	0.25	92.9671	140.2191	244.1479	241.5625	1.0107	4117.9	4611.4	7.393	151.5512	194.2841	140.2191
0.5	0.4	0.1	0.25	92.7083	139.8203	217.8209	269.0509	0.80959	4083.1	4598	7.474	151.1349	216.5308	139.8203
0.6	0.4	0.1	0.25	92.4536	139.4987	198.4115	293.8628	0.67518	4050.6	4585.1	7.513	150.9361	236.592	139.4987
0.7	0.4	0.1	0.25	92.2074	139.2312	183.3485	316.6516	0.57902	4020.1	4572.7	7.5255	150.8726	255.0046	139.2312
0.8	0.4	0.1	0.25	91.9711	139.0038	171.2249	337.8426	0.50682	3991.4	4561	7.5203	150.8991	272.1169	139.0038
0.9	0.4	0.1	0.25	91.7446	138.8073	161.1979	357.731	0.45061	3964.2	4549.7	7.5027	150.9883	288.1705	138.807
1	0.4	0.1	0.25	91.5274	138.6354	152.728	376.5316	0.40562	3938.4	4539	7.4764	151.123	303.3407	138.6354

b=0.7,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.5429	130.3075	115.138	435.2241	0.26455	4100	4462.5	12.9348	129.6569	89.5776	130.3075
0.2	0.4	0.1	0.25	92.014	135.0647	166.9661	325.0194	0.51371	4133.6	4549.8	10.179	139.164	131.7903	135.0647
0.3	0.4	0.1	0.25	92.6354	137.9825	208.2816	273.2207	0.76232	4131.2	4589	8.5627	145.9055	165.2789	137.9825
0.4	0.4	0.1	0.25	92.9671	140.2191	244.1479	241.5625	1.0107	4117.9	4611.4	7.393	151.5512	194.2841	140.2191
0.5	0.4	0.1	0.25	93.1614	142.1154	276.5873	219.6922	1.25898	4100	4625.7	6.4663	156.6337	220.481	142.1154
0.6	0.4	0.1	0.25	93.2786	143.8193	306.6617	203.4649	1.5072	4079.8	4635.5	5.6944	161.4003	244.7436	143.8193
0.7	0.4	0.1	0.25	93.3484	145.4098	335.0187	190.851	1.75539	4058.2	4642.3	5.0306	165.993	267.6032	145.4098
0.8	0.4	0.1	0.25	93.3868	146.9359	362.0961	180.724	2.00359	4035.8	4647.1	4.4474	170.5068	289.4176	146.9359
0.9	0.4	0.1	0.25	93.4035	148.4319	388.2143	172.4029	2.25179	4012.7	4650.5	3.9268	175.0138	310.4483	148.4319
1	0.4	0.1	0.25		149.7749	411.1096	166.093	2.47518	3991.5	4652.7	3.5017	179.1155	328.8759	149.7749

b=0.7,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	95.0048	139.5902	281.8839	207.1449	1.36081	4245.1	4704.9	9.0727	143.6612	167.4231	139.5902
0.1	0.3	0.4	0.25	93.216	140.7371	282.7542	210.2965	1.34455	4155.3	4624.6	7.2393	152.3528	168.9449	140.7371
0.2	0.3	0.4	0.25	91.6272	141.8132	283.8102	213.0908	1.33187	4073.7	4552.1	5.8597	160.3325	170.3794	141.8132
0.3	0.3	0.4	0.25	90.1684	142.9858	285.2623	215.9212	1.32114	3996.8	4484.8	4.7264	168.2844	171.9472	142.9858
0.4	0.3	0.4	0.25	88.8031	144.3490	287.2541	218.9932	1.3117	3922.9	4421.4	3.7577	176.5946	173.7735	144.349
0.5	0.3	0.4	0.25	87.5091	145.9900	289.947	222.4787	1.30326	3850.4	4360.9	2.9143	185.597	175.9756	145.99
0.6	0.3	0.4	0.25	86.2707	148.0187	293.5673	226.5779	1.29566	3778.1	4302.7	2.176	195.6808	178.7015	148.0187
0.7	0.3	0.4	0.25	85.076	150.5988	298.4732	231.5783	1.28887	3704.4	4246.1	1.5338	207.3983	182.1733	150.5988
0.8	0.3	0.4	0.25	83.914	154.0045	305.2761	237.9538	1.28292	3627.4	4190.8	0.987	221.65	186.7643	154.0045
0.9	0.3	0.4	0.25	82.7729	158.7467	315.1319	246.5827	1.278	3543.4	4135.9	0.5435	240.0891	193.1727	158.7467
1	0.3	0.4	0.25	81.6362	165.8570	330.3958	259.243	1.27446	3445.8	4080.7	0.2227	266.0308	202.8159	165.857

b=0.7,				Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	93.8490	154.6212	308.9055	237.2558	1.30199	4143.3	4680.6	2.3768	192.6629	187.574	154.6212
0.1	0.3	0.4	0.25	92.5918	148.6604	296.891	226.2059	1.31248	4101.6	4611.6	3.5934	178.1936	179.5497	148.6604
0.2	0.3	0.4	0.25	91.434	144.2511	288.1516	217.9825	1.3219	4057.2	4548.1	4.7265	168.2835	173.6385	144.2511
0.3	0.3	0.4	0.25	90.3511	140.6686	281.1285	211.2838	1.33057	4012.5	4488.6	5.7979	160.7283	168.8521	140.6686
0.4	0.3	0.4	0.25	89.3275	137.6028	275.165	205.5474	1.33869	3968.5	4432.3	6.8231	154.603	164.7685	137.6028
0.5	0.3	0.4	0.25	88.3529	134.8931	269.9242	200.4788	1.3464	3925.3	4378.6	7.813	149.4361	161.1688	134.8931
0.6	0.3	0.4	0.25	87.4197	132.4449	265.2102	195.9043	1.35377	3883	4327.1	8.7752	144.9562	157.9248	132.4449
0.7	0.3	0.4	0.25	86.5222	130.1984	260.8992	191.7126	1.36089	3841.7	4277.5	9.7151	140.9927	154.9548	130.1984
0.8	0.3	0.4	0.25	85.656	128.1125	256.9077	187.8276	1.36778	3801.2	4229.6	10.637	137.4321	152.2034	128.1125
0.9	0.3	0.4	0.25	84.8177	126.1583	253.1766	184.1952	1.3745	3761.6	4183.2	11.5439	134.1947	149.6311	126.1583
1	0.3	0.4	0.25	84.0045	124.3144	249.6626	180.7752	1.38107	3722.8	4138	12.4384	131.2225	147.2089	124.3144

Table 6.9: Sensitivity Analysis of the Integrated Problem of the Retailer and the Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (0.7, 0.1)$, c=1, s=50, c_s=10, c_e=3.157

	b=1,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_1	p_2	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	94.3629	143.9338	496.8612	125.8005	3.9496	4278.8	4687.4	6.1561	158.4837	100.0063	143.9338
0.2	0.4	0.1	0.25	94.4815	142.2427	349.7032	174.1051	2.00858	4246.6	4688.6	7.1045	153.0684	139.5827	142.2427
0.3	0.4	0.1	0.25	94.2548	141.1607	283.9367	210.824	1.34679	4206	4674.8	7.5947	150.5221	169.5106	141.1607
0.4	0.4	0.1	0.25	93.9545	140.3062	244.6012	241.4478	1.01306	4166.6	4658.1	7.9276	148.8772	194.4196	140.3062
0.5	0.4	0.1	0.25	93.6373	139.5766	217.7088	268.1479	0.8119	4129.4	4641	8.1833	147.6556	216.1136	139.5766
0.6	0.4	0.1	0.25	93.3206	138.9283	197.8345	292.0486	0.6774	4094.5	4624.1	8.3939	146.675	235.5202	138.9283
0.7	0.4	0.1	0.25	93.0104	138.3381	182.3753	313.8214	0.58114	4061.7	4607.6	8.5754	145.8482	253.1913	138.3381
0.8	0.4	0.1	0.25	92.7088	137.7921	169.9058	333.9056	0.50884	4030.6	4591.8	8.7365	145.1273	269.4869	137.7921
0.9	0.4	0.1	0.25	92.4164	137.2813	159.5717	352.6072	0.45255	4001.1	4576.4	8.8827	144.4836	284.6575	137.2813
1	0.4	0.1	0.25	92.133	136.7991	150.8255	370.1495	0.40747	3973	4561.6	9.0175	143.8985	298.8852	136.7991

	b=1,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
p_2	p_1	β_1	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0.1	0.4	0.1	0.25	90.8616	128.156	114.2255	422.7668	0.27019	4112	4466.1	15.3946	122.6332	87.9669	128.156
0.2	0.4	0.1	0.25	92.7709	134.2644	166.5064	321.6405	0.51768	4170.2	4581.5	11.416	134.637	130.9307	134.2644
0.3	0.4	0.1	0.25	93.5482	137.7515	208.3051	272.1744	0.76534	4176.1	4630.7	9.3382	142.5382	164.9744	137.7515
0.4	0.4	0.1	0.25	93.9545	140.3062	244.6012	241.4478	1.01306	4166.6	4658.1	7.9276	148.8772	194.4196	140.3062
0.5	0.4	0.1	0.25	94.1892	142.4096	277.4268	220.0364	1.26082	4150.5	4675.2	6.8527	154.4391	220.9889	142.4096
0.6	0.4	0.1	0.25	94.3296	144.2638	307.8594	204.068	1.50861	4131.2	4686.6	5.9795	159.5751	245.5846	144.2638
0.7	0.4	0.1	0.25	94.4127	145.9736	336.5611	191.6171	1.75643	4110	4694.4	5.2413	164.479	268.7564	145.9736
0.8	0.4	0.1	0.25	94.4581	147.6021	363.9821	181.6042	2.00426	4087.6	4699.9	4.6	169.2759	290.8762	147.6021
0.9	0.4	0.1	0.25	94.4777	149.1928	390.4552	173.3724	2.25212	4064.2	4703.7	4.0324	174.058	312.2172	149.1928
1	0.4	0.1	0.25	94.4793	150.6197	413.6899	167.1332	2.47521	4042.6	4706.1	3.5714	178.4133	330.9378	150.6197

	b=1,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_1	p_1	p_2	β_2	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	96.2891	140.5985	284.2953	208.6991	1.36223	4306.7	4768.1	9.2638	142.8498	168.7682	140.5985
0.1	0.3	0.4	0.25	94.2548	141.1607	283.9367	210.824	1.34679	4206	4674.8	7.5947	150.5221	169.5106	141.1607
0.2	0.3	0.4	0.25	92.2387	142.1842	284.5866	213.7468	1.33142	4103.4	4582.1	5.974	159.6093	170.8747	142.1842
0.3	0.3	0.4	0.25	90.2372	143.8497	286.6483	217.7731	1.31627	3997.7	4489.7	4.4239	170.6998	173.1037	143.8497
0.4	0.3	0.4	0.25	88.244	146.4902	290.8672	223.467	1.30161	3886.3	4397.3	2.975	184.8749	176.647	146.4902
0.5	0.3	0.4	0.25	86.2461	150.8300	298.8721	232.0591	1.28791	3764.5	4303.9	1.6753	204.4784	182.4853	150.83
0.6	0.3	0.4	0.25	84.2091	158.9571	315.3833	247.1125	1.27627	3618.1	4207.4	0.6171	236.2414	193.46	158.9571
0.7	0.3	0.4	0.25	81.999	180.1596	361.6361	284.6427	1.27049	3387.2	4099.7	0.0424	309.9942	222.3479	180.1596
0.8	0.3	0.4	0.25	79.4744	225.4854	464.4963	364.8966	1.27295	3001.8	3973.7	0	461.0873	285.3076	225.4854
0.9	0.3	0.4	0.25	76.8929	275.3006	580.4212	455.041	1.27554	2586.7	3844.6	0	626.5507	356.1428	275.3006
1	0.3	0.4	0.25	74.2855	326.7978	702.8191	550.1024	1.27762	2159.2	3714.3	0	797.4074	430.8867	326.7978

	b=1,			Effective					Expected	Expected	Expected	Expected	Expected	Expected
β_2	p_1	p_2	β_1	Demand: y(Q*)	Q*	x1*	x2*	x1*/x2*	Profit	Revenue	Shortage Cost	Excess Cost	Manufacturing Cost	Acquisition Cost
0	0.3	0.4	0.25	95.0022	157.8579	315.5768	243.1817	1.2977	4189.4	4740.3	1.9664	199.1125	191.9457	157.8579
0.1	0.3	0.4	0.25	93.4805	151.0507	301.6357	230.6813	1.30759	4138.5	4658.2	3.1561	182.7973	182.7632	151.0507
0.2	0.3	0.4	0.25	91.9793	145.4346	290.3774	220.2751	1.31825	4081.2	4576.6	4.4823	170.2218	175.2233	145.4346
0.3	0.3	0.4	0.25	90.4988	140.5567	280.7652	211.1837	1.32948	4020.2	4493.5	5.9203	159.9474	168.703	140.5567
0.4	0.3	0.4	0.25	89.0392	136.1854	272.2692	203.0084	1.34117	3956.9	4414.7	7.4541	151.2369	162.8841	136.1854
0.5	0.3	0.4	0.25	87.6005	132.1863	264.5827	195.5178	1.35324	3892.2	4334.7	9.0725	143.6623	157.5819	132.1863
0.6	0.3	0.4	0.25	86.1827	128.4744	257.5119	188.5653	1.36564	3826.4	4255.3	10.767	136.9529	152.6797	128.4744
0.7	0.3	0.4	0.25	84.7856	124.9925	250.9268	182.0519	1.37833	3760.1	4176.6	12.531	130.9263	148.0988	124.9925
0.8	0.3	0.4	0.25	83.4093	121.7002	244.7356	175.9073	1.39128	3693.4	4098.7	14.3592	125.4534	143.7836	121.7002
0.9	0.3	0.4	0.25	82.0534	118.5679	238.8711	170.0795	1.40447	3626.5	4021.4	16.2472	120.4396	139.6931	118.5679
1	0.3	0.4	0.25	80.7178	115.5729	233.2825	164.5289	1.41788	3559.6	3944.9	18.1913	115.8132	135.7963	115.5729

Table 6.10: Sensitivity Analysis of the Integrated Problem of the Retailer and the Manufacturer under Two Inputs when the Input Prices or Carbon Emission Levels of Inputs Change, $(b, a) = (1, 0.1)$, c=1, s=50, c_s=10, c_e=3.157

		b=0					b=0.3					b=0.5				
σ	cs/s	x1	x2	x3	x4	x5	x1	x2	x3	x4	x5	x1	x2	x3	x4	x5
5000	0.01	441,650	75360	27050	76490	201160	400750	68390	24540	69410	182530	401900	68580	24610	69610	183050
	0.02	458,900	78310	28100	79480	209010	410990	70130	25170	71180	187190	412230	70340	25240	71400	187760
	0.04	467440	79770	28630	80960	212900	418560	71420	25630	72490	190640	419820	71640	25710	72710	191210
15000	0.01	353,250	60280	21630	61180	160900	312350	53300	19130	54100	142270	313480	53490	19200	54290	142780
	0.02	373,870	63800	22900	64750	170280	328770	56100	20130	56940	149740	329980	56310	20210	57150	150290
	0.04	394090	67250	24130	68260	179500	346880	59190	21240	60080	157990	348130	59410	21320	60300	158560

		b=0.7					b=1				
σ	cs/s	x1	x2	x3	x4	x5	x1	x2	x3	x4	x5
5000	0.01	403020	68770	24680	69800	183560	404630	69050	24780	70080	184290
	0.02	413,430	70550	25320	71610	188310	415,180	70850	25430	71910	189100
	0.04	421040	71850	25780	72920	191770	422820	72150	25890	73230	1925800
15000	0.01	314570	53680	19260	54480	143280	316150	53950	19360	54760	143990
	0.02	331,150	56510	20280	57360	150830	332,850	56800	20380	57650	151600
	0.04	349350	59610	21390	60510	159120	351110	59920	21500	60810	159920

Table 6.11: Optimal Allocation of the Inputs under the Agricultural Production System Analysis .

σ	b	cs/s	Q*	Effective Demand: y(Q*)	Expected Profit	Expected Revenue	Expected Manufacturing Cost	Expected Acquisition Cost	Expected Shortage Cost	Expected Excess Cost
	0	0.01	113850	130000	3268	263,000	257,710	915.0339	1107.5	3.7325
		0.02	118200	130,000	2244	277,890	272,750	967.254	1917.3	10.7384
		0.04	120,360	130,000	487.1925	292,290	287,500	1018.4	3258.4	27.6822
	0.3	0.01	103,530	117500	3831.1	297,630	292,360	1035.3	401.9135	0.5911
		0.02	106,110	117500	3473.7	305,020	299,830	1061.1	655.9155	3.0172
		0.04	108,030	117500	2879.1	310,410	305,350	1080.3	1095.9	8.8194
	0.5	0.01	103,820	117830	3824.3	298,460	293,200	1038.2	402.8948	0.5777
		0.02	106,430	117820	3466.4	305,920	300,730	1064.3	656.4583	3.0005
		0.04	108,340	117820	2871.5	311,330	306,270	1083.4	1096.3	8.8003
	0.7	0.01	104100	118140	3817.70	299270	294010	1041.00	403.8646	0.5647
		0.02	106730	118140	3459.3	306,800	301,610	1067.3	656.9879	2.9842
		0.04	108,650	118130	2864	312,220	307,160	1086.5	1096.8	8.7817
	1	0.01	104510	118600	3807.90	300440	295190	1045.10	405.30	0.5460
		0.02	107170	118590	3448.9	308,070	302,890	1071.7	657.7583	2.9608
5000		0.04	109100	118590	2853.10	313,510	308,460	1091	1097.5	8.7549
	0	0.01	91503	130000	3268	263,000	257,710	915.0339	1107.5	3.7325
		0.02	96725	130000	2244	277,890	272,750	967.254	1917.3	10.7384
		0.04	101,840	130000	487.1925	292,290	287,500	1018.4	3258.4	27.6822
	0.3	0.01	81,123	117520	3384.7	263,000	227,870	811.2267	1047.6	3.7325
		0.02	85,292	117520	2402.1	277,890	239,840	852.9158	1857.9	10.7384
		0.04	89,888	117510	686.2287	292,290	253,060	898.8774	3199.2	27.6822
	0.5	0.01	81408	117860	3382.8	233,120	228690	814.0816	1049	5.6938
		0.02	85,598	117850	2399.1	244,970	240,730	855.9839	1859.5	13.0195
		0.04	90,205	117850	682.2428	257,870	253,970	902.0492	3200.8	30.2403
	0.7	0.01	81685	118190	3380.90	233940	229490	816.85	1050.50	5.64
		0.02	85897	118180	2396.2	245,850	241,580	858.9666	1861	12.9556
		0.04	90,513	118170	678.2965	258,790	254,860	905.1339	3202.3	30.1704
	1	0.01	82087	118660	3378.10	234740	230640	820.87	1052.50	5.5547
		0.02	86329	118650	2391.9	246,710	242,820	863.2865	1863.1	12.8636
15000		0.04	90,960	118630	672.457	259,670	256,140	909.603	3204.5	30.0696

Table 6.12: Optimal Production Quantities and Expected Profit, Costs under the Agricultural Production System Analysis

Chapter 7

Conclusion

Motivated by the global aim and trends to reduce carbon emissions, in this thesis we investigate the effects of carbon sensitivity on operations management by extending the well-known newsvendor problem. We considered two different supply chain settings. In the first setting we investigate the optimal ordering policy of a retailer who acts independently of the manufacturer. In the second setting, the retailer and the manufacturer are considered in a centralized perspective and the order quantity is obtained so as to maximize the total expected profit of the system.

In both models, the Cobb-Douglas production function is used to provide a link between the product and the inputs and since the customers are assumed to be carbon sensitive, the random demand is formulated so that it is affected by both the carbon emission level of the product and the carbon sensitivity level of the customers.

In the first setting with a retailer an independent manufacturer, we find the optimal order quantity of the retailer under the condition that an independent manufacturer produces the ordered items in such a way that he minimizes his total production cost. We derive the optimal production policy under different carbon sensitivity levels and provide results related to the number of solutions to the problem.

In the second setting, since the retailer produces its own products under a centralized system, she does not know the carbon emission level caused by the production before determining the optimal production quantity. She jointly optimizes the production quantity and the input mix. We derive the first order conditions for the problem for the cases where the number of inputs is two and three. For the cases where number of inputs is higher, a simplification is done and the first order conditions are expressed as a function of three variables.

In the numerical studies we considered several system parameters and investigated the impact of these parameters on the optimal order quantity decisions and the expected profit. We particularly focused on the impact of the products and their carbon emission levels. We conducted the numerical experiments for $n = 2$ and compared the optimal behaviors of the decentralized and the centralized systems. We also provided an example with the real data from Hatirli et.al. [11].

In our analytic results, we showed that under the Retailer's Problem with an Independent Manufacturer, if $b \geq (r/(1-r))$, a unique optimal order quantity exists, otherwise if $b < (r/(1-r))$ we showed that the problem has at most three solutions. For the Integrated Problem of the Retailer and the Manufacturer, we derived the first order conditions for the problem when there are two, three or more than three inputs.

In our numerical results, the sensitivity analysis on the decentralized problem indicates that the change in the carbon emission level parameters does not have an impact on the allocation of the inputs since an optimization is done by manufacturer by only considering the price of the inputs. However, we clearly see the effect of the carbon emission parameters on the effective demand. If the input type selected to be mainly used is the one with both low cost and low carbon emission level, the effective demand increases if the input with high carbon emission level also have an increasing cost.

The sensitivity analysis we conduct for the centralized problem shows that the change in the carbon emission parameters of the inputs have an important effect on the allocation of the inputs especially when the carbon emission sensitivity level of the customers is high. It is observed that the production process have an inclination to rely on the low cost-low carbon emission parameter input and the optimal production quantity decreases when the cost of the low cost-low carbon emitted input increases. The effect of the price is more noticeable but the same results are obtained when the change occurs in the carbon emission parameter of the input which has low cost. The retailer's optimal production quantity also decreases when the carbon emission level of the low cost-low carbon emitting input increases.

A comparison between the sensitivity analyses indicate that the centralized version of the problem results in a higher optimal order quantity with a higher corresponding expected profit under the same conditions for each carbon sensitivity parameter under same input cost, carbon emission parameters. In addition, in the second setting, we clearly see the effect of the carbon sensitivity level of the customers by investigating the change in the input allocation ratios when the price or the carbon emission parameter of the inputs change in contrast to independent manufacturer model where the allocation of inputs are determined by only considering the price of the inputs.

The real-life agricultural application is investigated under the Retailer's Problem with an Independent Manufacturer and the effect of variation in demand and the change in the ratio of shortage cost and selling price are observed under each carbon sensitivity level. As a result, it is found that the effect of the change in the ratio of shortage cost and selling price on the optimal order quantity is higher when the demand uncertainty increases under each carbon sensitivity level. The effect of demand variation on the input allocation is also observed.

For further study, we propose extensions to our problems. For both problems, it would be nice to consider a multiplicative demand structure and make a comparison between results. Instead of a newsvendor model under single period and single product, newsvendor problem under multiple periods or multiple products can be considered. Last extension can be the addition of the competition to the decentralized problem where more than one manufacturer can exist and compete.

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