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Optimal pickup point location on material handling networks

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The optimal location of a pickup point on a material handling network is considered. The pickup point is defined as the material exchange point between the material handling system (MHS) and a station. The problem is defined as that of choosing the location of the pickup point to minimize the total cost of material movement in the MHS. A facility location model on directed networks has been developed, and strongly polynomial solution methods are presented.

1. Introduction

We consider the optimal location of a pickup point on a material handling network. The pickup point may connect the material handling network to any one of the following: a machining or assembly station, load, unload or inspection station, central or local storage. Sometimes the pickup point may serve as a transfer point to or from another material handling network. The problem arises in manufacturing systems where an automated material handling system (MHS) transports the parts between the stations. Material flow is often restricted by unidirectional movements of conveyors, carts or vehicles.

We use the term 'pickup point' in a generic sense. The term may represent a variety of scenarios. Some examples are the following. (1) A new workstation with a multipurpose machine tool is being added to the system. The pickup point is the point of material exchange between the new station and other stations. (2) A new coordinate measuring station is added to the system for the purpose of checking certain critical dimensions before continuing with the remaining operations. The pickup point is the point where the coordinate measuring machine interacts with the MHS. (3) The work in progress (WIP) that cannot be stored at local buffer areas of work stations have to be taken out of the MHS to be stored temporarily at a central storage area. Eventually, when there is sufficient space available at the local buffer storage of a workstation, these parts will be fetched from the central storage to be returned to the MHS. The pickup point in this scenario represents the point where WIP exits or enters the MHS. Our analysis and results are not restricted to any one of the above scenarios. Whenever it is necessary to interpret the model, we will use the last scenario.

The problem is important in both conventional and flexible manufacturing systems. In a conventional manufacturing setting addition of a new workstation, inspection station or storage area may be required due to changes in demand, part design or manufacturing technology. A new multipurpose station may replace existing stations, etc. The location decision at the time of such a change has considerable cost consequences. In a flexible manufacturing system where workstations are connected and operated under a central computer control, the consequences of the station locations may be even more important. In such a case, the operational performance of

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the system is more sensitive to layout design due to the effects of transportation times on blocking and starvation probabilities.

In some cases, the question of where to locate the pickup point may actually be a part of the 'what if' analysis that must be performed during the design of the system. A small number of key stations may be evaluated one at a time as an addition to the conceptually existing system. Alternative designs can be developed by examining different workstations. This requires efficient and robust techniques.

The problem can be defined as that of choosing the location of the pickup point so that the total cost of the material movement is minimized. Our model does not consider the movements outside the MHS as part of the total cost. We have two reasons for this. Either such movements are negligibly small in comparison to the movements within the MHS, and can be ignored, or (when they are not small) a minor modification of our analysis handles the extra material movement cost outside the MHS.

The problem is modelled as a facility location problem that seeks to minimize the sum of weighted network distances. The MHS is represented by a network consisting of directed arcs that correspond to conveyor belts, tow lines, monorail or wire paths. Because the network is directed, the distances (as defined by shortest path lengths) are asymmetric. Associated with each work station is a pair of weights that represent two types of flow: from workstations to the pickup point and from pickup point to workstations. The flows are also asymmetric. These asymmetric distances and flows will be considered in this paper.

Although they are not directly related to the problem we address here, some layout-related problems in manufacturing systems have been addressed in the literature (see, for example, Tansel *et al.* 1983, Hansen *et al.* 1985). Material handling system and machine layout problems are described in Kusiak (1985) and Kusiak and Heragu (1987). Afentakis (1986) developed a graph-theoretic model and proposed algorithms based on 'switch-and-check' techniques. Heragu and Kusiak (1986) proposed clustering-based heuristic methods for machine layout problems. Gaskins and Tanchoco (1987) defined a material handling network design problem. Tansel and Kiran (1988) provided solutions for storage location in flexible manufacturing cells.

2. Notation and definitions

Let MHN $=(N,A)$ be the material handling network. MHN is defined by node set $N=\{1,\dots,n\}$ that represents workstations and junction points and (directed) arc set A that corresponds to material handling paths of the MHS. An example of a MHS layout and corresponding MHN are shown in Fig. 1. We assume that there is a directed path from any node to any other node, and we allow loops and parallel arcs.

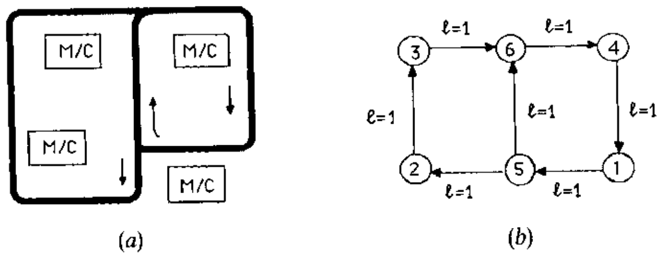

Figure 1. (*a*) Material handling system layout; (*b*) corresponding material handling network.

For each node j, let u_j, v_j be a pair of non-negative weights where u_j represents the flow (unit loads/unit time) from node j to pickup point, and v_j represents the flow from pickup point to node j. Parameters u_j and v_j can be estimated from the routing information as described in Tansel and Kiran (1988).

Note that $u_j \neq v_j$ in general. It will be convenient to define

$$U = \sum_{j=1}^{n} u_j \tag{1}$$

and

$$V = \sum_{j=1}^{n} v_j, \tag{2}$$

the total flows to and from the pickup point, respectively. The case with $U = V$ will be called the 'balanced case'. In most automated manufacturing systems the 'balancedness' assumption holds. Any part coming to the pickup point will eventually leave the pickup point. This explains why the total inflow U is equal to the total outflow V. The balancedness assumption may fail when losses or gains occur at the pickup point. For example, defective parts can be removed from the system at an inspection station. A station may be allocated only to load parts. In a conventional manufacturing setting the balancedness is harder to justify due to loss and removal of the parts at the stations and occasional manual transport of the parts. We will develop our results for the case with $U \neq V$. Some interesting special results will be given for the balanced case.

All points along all arcs are eligible for locating the pickup point. Let $d(x, y)$ be the length of a shortest path from x to y. We note that $d(x, y) \geqslant 0$ with $d(x, y) = 0$ if $x = y$. Further, $\forall x, y, z \in$ MHN, shortest path lengths satisfy $d(x, y) + d(y, z) \geqslant d(x, z)$, and $d(x, y) \neq d(y, x)$ in general.

For a pickup point located at x the total cost of material flow is given by

$$f(x) = f_1(x) + f_2(x), \tag{3}$$

where $f_1(x)$ is the total cost of material flow from stations to the pickup point, and $f_2(x)$ is the total cost of material flow from the pickup point to stations, i.e.

$$f_1(x) = \sum_{j=1}^{n} u_j d(j, x), \tag{4}$$

$$f_2(x) = \sum_{j=1}^{n} v_j d(x, j). \tag{5}$$

The problem is to find x such that $f(x) \leqslant f(y)$ for all points y on MHN.

3. Node optimality

We now prove that there exists at least one node that minimizes f on MHN.

Theorem 1 (also see Handler and Mirchandani (1979) and Mirchandani (1975)): Let (a, b) be an arc and let $x \in (a, b)$, then

$$f(x) \geqslant \min\{f(a), f(b)\}. \tag{6}$$

Proof: Let l be the length of arc (a, b). Let λ be the distance between a and x (see Fig. 2). If $x = a$ or $x = b$, the claim is true. For $x \neq a, b$, we have

$$f(x) = \sum_{j=1}^{n} u_j [d(j, a) + \lambda] + \sum_{j=1}^{n} v_j [l - \lambda + d(b, j)]. \tag{7}$$

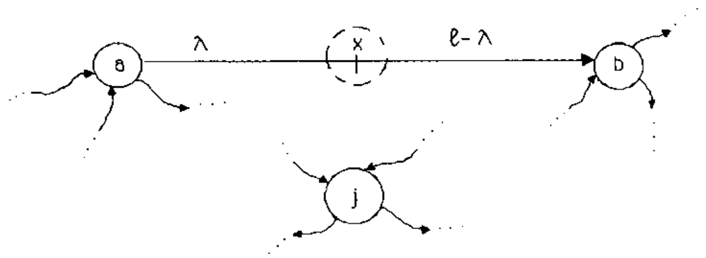

Figure 2. Theorem 1.

Hence,

$$f(x)-f(a)=\sum_{j=1}^{n} v_j[d(b,j)-d(a,j)]+lV+\lambda(U-V). \tag{8}$$

In Equation (8), $d(a,j)\leqslant l+d(b,j)$, since arc (a,b) may or may not be on the shortest path from a to j. Hence,

$$f(x)-f(a)\geqslant\sum_{j=1}^{n} v_j(-l)+lV+\lambda(U-V)=\lambda(U-V). \tag{9}$$

A similar derivation gives

$$f(x)-f(b)\geqslant(V-U)(l-\lambda). \tag{10}$$

Equations (9) and (10) imply that either $f(x)\geqslant f(a)$ or $f(x)\geqslant f(b)$. ■

Theorem 1 reduces the candidate solution set to node locations. This means that the minimum total cost can be obtained by locating the pickup point next to an existing workstation or at an intersection point of the MHN. The minimum cost node can be found by evaluating $f(j)$ at each node j, and selecting the minimum $f(j)$. Given the node-to-node distance matrix, finding the optimal node in worst case takes $O(n^2)$ time.

In the event the transport cost outside the MHN is not negligible, a minor modification is required. Using the scenario with the central storage we now explain this modification. For node j, let s_j be the cost of moving one unit from the pickup point to central storage and let t_j be the cost of moving one unit from central storage to the pickup point. The parameters s_j, t_j can be estimated *a priori* for each node j and their values depend on the particular mode of transportation between the pickup point and the central storage. Given these numbers, the total cost associated with node j is $f(j)+s_jU+t_jV$. Defining this cost to be $g(j)$, the minimum cost node is obtained by choosing the smallest $g(j)$.

The $O(n^2)$ enumeration will now be improved to $O(kn)$ where k is the number of nodes whose in or out degree is greater than one. This algorithm is based on the following observation.

Property 1: Let (a,b) be the only arc connecting nodes a and b (Fig. 3). We then have

$$f_1(b)=f_1(a)+lU-u_b(l+d(b,a)), \tag{11}$$

$$f_2(a)=f_2(b)+lV-v_a(l+d(b,a)). \tag{12}$$

Proof:

$$f_1(b)=\sum_{j\in N} u_j d(j,b),$$

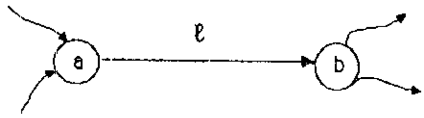

Figure 3. Property 1.

where $d(j,b)=d(j,a)+l$ for $j \neq b$. Hence

$$f_1(b)=\sum_{\substack{j\in N\\ j\neq b}} u_j\,[d(j,a)+l]=f_1(a)-u_b\,d(b,a)+l\sum_{\substack{j\in N\\ j\neq b}} u_j,$$

or

$$f_1(b)=f_1(a)-u_b\,d(b,a)+lU-lu_b, \tag{13}$$

which is equivalent to Equation (11). Equation (12) can be shown in a similar way. ■

Let $N'\subset N$ be the set of nodes with indegree greater than one. After calculating $f_1(j)$ for $j\in N'$, f_1 values for the other nodes can be calculated using Equation (11). Similarly, if $N''\subset N$ is the set of nodes with a outdegree greater than one, f_2 values of the nodes, $j\in\{N-N''\}$, can be calculated using Equation (12). Hence f values of all nodes can be calculated in $O(kn)$ time where $k=|N'\cup N''|$. If k is small, as is often the case in many systems, then the computation time is substantially smaller than $O(n^2)$. The calculations are illustrated in Example 1.

3.1. *Example 1*

Problem data are given below. Graph MHN$=(N,A)$ is given in Fig. 4. Node-to-node distances are given in matrix D.

j	1	2	3	4	5
u_j	5	10	15	10	0
v_j	15	5	10	15	0

$$D=\begin{bmatrix}0&2&2&3&1\\1&0&3&4&2\\4&3&0&1&2\\3&2&2&0&1\\2&1&1&2&0\end{bmatrix}$$

$N'=\{5\}$. We calculate $f_1(5)$ using equation (4):

$$f_1(5)=5(1)+10(2)+15(2)+10(1)=65.$$

Now $f_1(j), j\in\{1,2,3,4\}$, can be calculated using Equation (11).

$$f_1(2)=65+1\ (40-10)-10(2)=75,$$

$$f_1(1)=75+1\ (40-5)-5(2)=100,$$

$$f_1(3)=65+1\ (40-15)-15(2)=60,$$

$$f_1(4)=60+1\ (40-10)-10(2)=70.$$

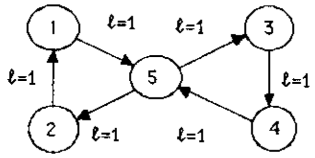

Figure 4. MHN for Example 1.

$N'' = \{5\}$. Hence,

$$f_2(5) = 15(2) + 5(1) + 10(1) + 15(2) = 75.$$

Now using Equation (12) we obtain

$$f_2(1) = 75, \quad f_2(2) = 105, \quad f_2(4) = 75, \quad f_2(3) = 90.$$

Adding f_1 and f_2 for each node we have

$$f(1) = 175,$$
$$f(2) = 180,$$
$$f(3) = 150,$$
$$f(4) = 145,$$
$$f(5) = 140.$$

Node 5 is the optimal node location.

4. Unicyclic case

In the case of a unicyclic MHN a more efficient solution method can be developed. A unicyclic MHN is an appropriate model for a closed loop conveyor system which is common material handling equipment in many manufacturing systems. In a unicyclic MHN all arcs are oriented in the same direction to form a cycle. Let us number the nodes from 1 to n, starting at an arbitrary node, and following the arcs (see Fig. 5).

Let a and b be two adjacent nodes. Let l be the length of arc (a, b) and c be the cycle length (i.e. total length of all arcs in the network). We observe.

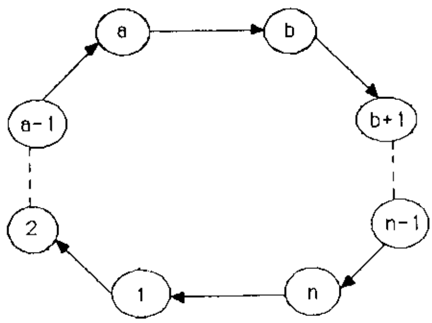

Figure 5. Numbering a unicyclic graph.

Property 2:

$$f(a)-f(b)=c(u_b-v_a)-l(V-U). \tag{14}$$

Proof:

$$f(a)-f(b)=f_1(a)+f_2(a)-f_1(b)-f_2(b).$$

Using equations (11) and (12) we have

$$f(a)-f(b)=f_1(a)+f_2(b)+lV-v_ac-f_1(a)-lU+u_bc-f_2(b),$$

which is the desired result. ∎

We now use Equation (14) to give an $O(n)$ algorithm that finds the optimal pickup point in the case of unicyclic MNHs. In the algorithm, $f^*(i)$ is defined to be $f(i)=f(1)$ for $i=1,\dots,n$.

Algorithm ONLUN (Optimal Node Location on Unicyclic Network)

INITIAL Number nodes from 1 to n. Set $i=1, f^*(1)=0$, MIN $=0$, OPNODE $=1$.

MAIN: $i \leftarrow i+1$, compute

$$f^*(i=f^*(i-1)+c(-u_i+v_{i-1})-l_{i-1,i}(V-U).$$

If $f^*(i)<$ MIN then MIN $=f^*(i)$, and OPNODE $=i$.

If $i=n$ stop, otherwise repeat MAIN.

5. Optimal pickup point location on arcs

The previous results indicate that the minimum cost location coincides with the location of another station. This may not be possible in practice due to computer control difficulties or size constraints. In most real world situations, the pickup point may have to be located away from existing stations, at an interior point of some arc. The following result simplifies the search on the arcs.

Property 3. Let (a,b) be an arc with length l (see Fig. 6). Let $x \in (a,b)$ be a point with $d(a,x)=\lambda$. We have

$$f(x)=f_1(a)+f_2(b)+\lambda U+(l-\lambda)V, \quad \text{for } 0<\lambda<l. \tag{15}$$

Proof: Rewriting Equation (3) for $x \neq a,b$, we have

$$f(x)=f_1(x)+f_2(x)=\sum_{j=1}^{n} u_j d(j,x)+\sum_{j=1}^{n} v_j d(x,j). \tag{16}$$

But

$$d(j,x)=d(j,a)+\lambda, \tag{17}$$

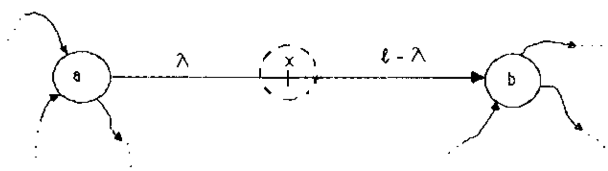

Figure 6. Property 3.

and

$$d(x,j) = d(b,j) + (l - \lambda). \tag{18}$$

By substituting Equations (17) and (18) into (16) we obtain Equation (15). ■

In a balanced case, Equation (15) becomes

$$f(x) = f_1(a) + f_2(b) + lV \tag{19}$$

Equation (19) indicates that in the balanced case the total cost of the locating pickup point at interior point x is independent of λ. This means that once the minimum cost arc of MHN is determined, the pickup point can be located anywhere on this arc. The minimum cost location can be found by evaluating Equation (19) for all arcs in the network.

In the general case of $U \neq V$, the minimum cost location of the pickup point x depends on the distances λ and $l - \lambda$. But the minimum cost location can be easily determined. For any interior point x of arc(a, b), let us write Equation (15) as

$$f(x) = f_1(a) + f_2(b) + lV + \lambda(U - V), \quad \text{for } 0 < \lambda < l, \tag{20}$$

and observe that $f(x)$ is a linear function of λ with slope $U - V$. The slope of $f(x)$ is constant on any arc of the MHN. This indicates that, if node locations are not eligible, then the pickup point must be located on arc (a, b) as close as possible to a if $U > V$, and as close as possible to b if $U < V$. Hence, if a lower limit on λ (or $l - \lambda$) is given because of size limitations, there is only one candidate point on each directed arc(a, b). The optimum location can be determined by evaluating Equation (20) at candidate points.

5.1. *Example 2*

For the problem data in Example 1, the evaluation of arcs is given in Example 2. Suppose, due to size limitations, the pickup point must be located at least 0·2 unit distance away from nodes 1, 2, 3, 4 and 0·1 unit distance away from node 5. Let us find the optimal pickup point location. Note that $U = 40 < V = 45$. Hence, we will consider the points close to b on directed arcs (a, b). These are evaluated in Table 1. The optimum location is on arc(3, 4), 0·8 unit distance away from node 3.

6. Modifying the MHN

In this section we will analyse the situations when a new arc is added to the original MHN. There are two possible applications of this analysis. First, as we will show, we may omit loops and parallel arcs during the initial analysis for optimal locations. These

Arc (a, b)	Node b	$f_1(a)$	$f_2(b)$	lV	λ	$f(x)$
(1, 5)	5	100	75	45	0·9	215·5
(2, 1)	1	75	75	45	0·8	191
(3, 4)	4	60	75	45	0·8	176
(4, 5)	5	70	75	45	0·9	185·5
(5, 2)	2	65	105	45	0·8	211
(5, 3)	3	65	90	45	0·8	196

Table 1. Computations for Example 2.

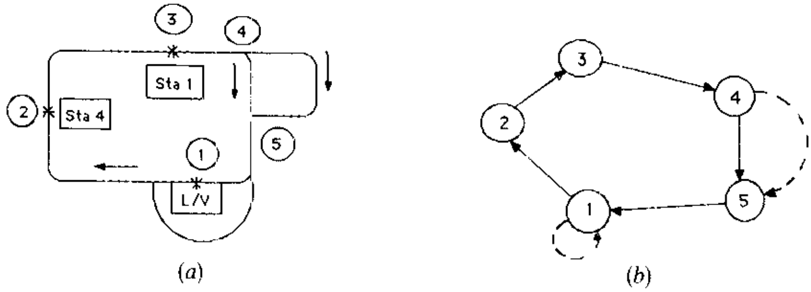

Figure 7. (a) MHS layout with a loop and parallel arcs; (b) corresponding MHS.

omitted arcs may be added to MHN if they are candidates for the new facility. The second possible application is the case where it is necessary to add a new arc to the network when it is not possible to locate the new station on the original MHN.

First, let us consider a MHS layout with loops and parallel arcs (Fig. 7 (a)). We can use the previous results to simplify the analysis of such a network. Since we have defined $d(a, b)$ as the shortest path distance between a and b, loops and all but the shortest of the parallel arcs will never enter into the calculation of f_1 and f_2. Hence, for analysis purposes all loops and all but the shortest of parallel arcs can be removed. For example, in Fig. 7 (a) the loop around load/unload station and one of the parallel arcs between junction points 4 and 5 may be removed. The resulting unicyclic network can easily be analysed. If loops or parallel arcs are candidate locations for the new facility, $f(x)$ on these arcs can be calculated using Equation (15). For example, for a loop (a, a) and $x \in (a, a)$ with $l = l_{aa}$ and $\lambda = d(a, x)$,

$$f(x) = f_1(a) + f_2(a) + \lambda U + (l - \lambda)V, \quad \text{for } 0 < \lambda < l.$$

$f(x)$ for any parallel arc can be calculated by directly applying Equation (15).

The second case is encountered where the physical configuration of the MHS, space and size limitations, accessibility, safety considerations, etc., may not permit locating a new station on the original MHN. There are usually a few alternative arcs that can be added to MHN. Figure 8 shows such a situation.

Let (x, y) be the new arc added to the original network where x and y are any points on the original MHN and let z be a point of this arc. Denoting the length of arc (x, y) by l and the length of the subarc (x, z) by λ, we observe that every unit transported from j to point z must visit point x during its journey. This implies

$$f_1(z) = \sum_{i=1}^{n} u_j [d(j, x) + \lambda] = f_1(x) + \lambda U. \tag{21}$$

Arc (a, b)	$f_1(a)$	$f_2(b)$	λU	$(l-\lambda)V$	$f(z)$
(1, 3)	100	90	56	9	255
(3, 1)	60	75	56	9	200
(2, 4)	75	75	56	9	215
(4, 2)	70	105	56	9	240

Table 2. Computations for Example 3.

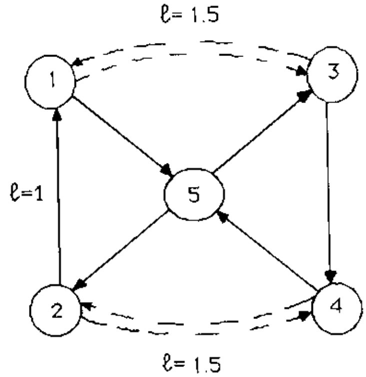

Figure 8. Alternative new arcs.

Similarly, every unit transported from z to node j must visit y. This gives

$$f_2(z)=f_2(y)+(l-\lambda)V. \tag{22}$$

It follows that $f(z)=f_1(x)+f_2(y)+l(U-V)+lV$. Hence, if $U-V=0$, all points on the new arc have equal merit for being the pickup point. If $U-V>0$ then the pickup point must be as close to x as permitted, and if $U-V<0$, it must be as close to y as permitted.

6.1. *Example 3*

Let us consider the alternative arcs (1, 3) (3, 1), (2, 4) and (4, 2) for the data given in Example 1 (Fig. 8). Recall that $U=40<V=45$. Hence, we will consider the points close to the head of the directed arcs (a, b). On a given arc there is a unique candidate point which is 0·2 distance unit away from the points head of the arc. Evaluation of the candidate points is given in Table 2. The optimal point is on new arc (3, 1), 0·2 unit distance away from node 3.

7. Summary and conclusions

We have considered the problem of locating a pickup point on the existing MHN of a manufacturing system. We have shown that the pickup point may be located at a node of the MHN to minimize the total cost function defined as the sum of the products of material flow and travel distances. In a real manufacturing environment, locating the pickup point at existing nodes may not be possible due to constraints on computer controls and size limitations. We analysed cases in which the pickup point must be located on an arc of the MHN. The actual location of the pickup point on the arc does not affect the objective function under a reasonable 'balancedness' assumption. If MHN consists of a unicycle, the optimum node location can be found in $O(n)$ time. Otherwise, it is found in $O(kn)$ time where k is the number of nodes whose indegree or outdegree or both are greater than one.

We have considered modifications in the existing MHN to accommodate a node or to relieve the congestion in the system. Results have been obtained to calculate the effect of such a modification on the cost function. The results in this section can also be used to evaluate limited alternative layouts that differ from one another in only one arc.

The model presented here is applicable to a variety of real world scenarios. These scenarios include addition of a new workstation, an inspection station or a central

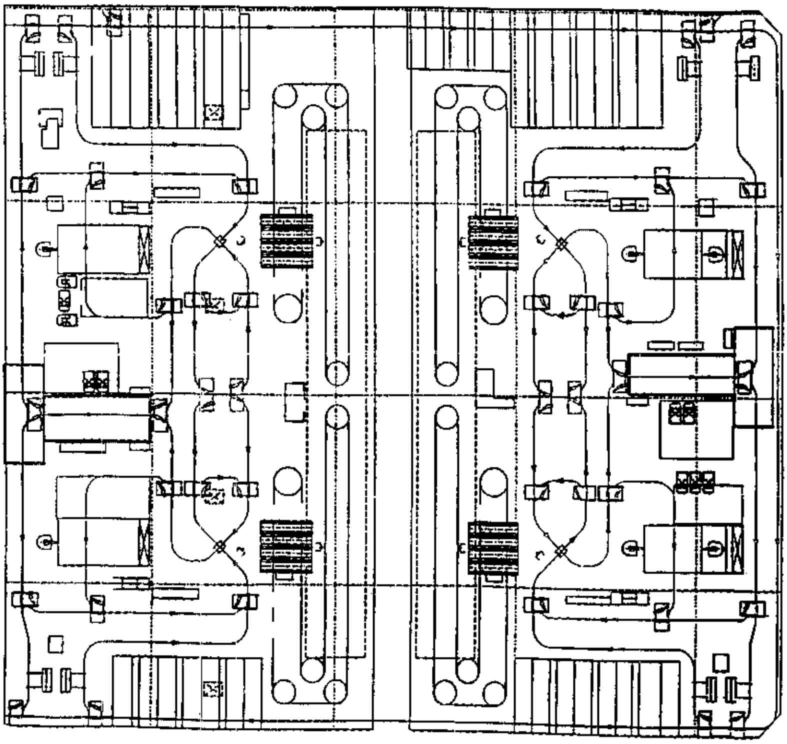

Figure 9. A sample MHS layout (courtesy of Litton Industries, Inc.).

storage. An example of an existing MHS layout where model is applicable is shown in Fig. 9. We believe that the applicability of the model and the results will encourage the research on more comprehensive models of different types of MHNs. The research for some general cases is already undertaken by the authors.

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References

AFENTAKIS, P., 1986, A model for layout design in FMS. In *Flexible Manufactoring Systems: Methods and Studies*, edited by A. Kusiak (Amsterdam: North Holland).

FRANCIS, R. L., and WHITE, J. A., 1974, *Flexible Layout and Location* (Englewood Cliffs, NJ: Prentice-Hall).

FRANCIS, R. L., MCGINNIS, L. F., and WHITE, J. A., 1983, Locational analysis. *European Journal of Operational Research*, **12**, 220–252.

GASKINS, R. J., and TANCHOCO, J. M. A., 1987, Flowpath design for automated guided vehicle systems. *International Journal of Production Research*, **25**, 667–676.

HANDLER, G. Y., and MIRCHANDANI, P. B., 1979, *Location on Networks: Theory and Algorithms* (Cambridge, MA: MIT Press).

HANSEN, P., LABBE, M., DOMINIC, P., and THISSE, J. F., 1985, Single facility location on networks. Working paper, RUTGOR, Rutgers University.

HERAGU, S. S., and KUSIAK, A., 1986, Machine layout problem in flexible manufacturing systems. Working paper 08/86, Department of Mechanical and Industrial Engineering, University of Manitoba.

Kusiak, A., 1985, Material handling in flexible manufacturing systems. *Material Flow*, **2**, 79–95.

Kusiak, A., and Heragu, S. S., 1987, The facility layout problem. *European Journal of Operational Research*, **29**, 229–251.

Mirchandani, P. B., 1985, Analysis of stochastic networks in emergency service systems. IRP-TR-15-75, Operations Research Center, MIT.

Tansel, B. C., Francis, R. L., and Lowe, T. J., 1983, Location on networks: a survey, Parts I and II. *Management Science*, **29**, 482–512.

Tansel, B. C., and Kiran, A. S., 1988, Optimum central storage location in flexible manufacturing cells. *Journal of Manufacturing Systems*, **7**, 121–129.