# CENTRALIZED AND DECENTRALIZED DETECTION WITH COST-CONSTRAINED MEASUREMENTS

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By

Eray Laz

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Centralized and Decentralized Detection with Cost-Constrained Measurements
By Eray Laz
May 2016

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Sinan Gezici (Advisor)

Orhan Arıkan

Umut Orguner

Approved for the Graduate School of Engineering and Science:

Levent Onural
Director of the Graduate School

# ABSTRACT

## CENTRALIZED AND DECENTRALIZED DETECTION WITH COST-CONSTRAINED MEASUREMENTS

Eray Laz
M.S. in Electrical and Electronics Engineering
Advisor: Sinan Gezici
May 2016

In this thesis, optimal detection performance of centralized and decentralized detection systems is investigated in the presence of cost constrained measurements. For the evaluation of detection performance, Bayesian, Neyman-Pearson and $J$-divergence criteria are considered. The main goal for the Bayesian criterion is to minimize the probability of error (more generally, the Bayes risk) under a constraint on the total cost of the measurement devices. In the Neyman-Pearson framework, the probability of detection is to be maximized under a given cost constraint. In the distance based criterion, the $J$-divergence between the distributions of the decision statistics under different hypotheses is maximized subject to a total cost constraint. The probability of error expressions are obtained for both centralized and decentralized detection systems, and the optimization problems are proposed for the Bayesian criterion. The probability of detection and probability of false alarm expressions are obtained for the Neyman-Pearson strategy and the optimization problems are presented. In addition, $J$-divergences for both centralized and decentralized detection systems are calculated and the corresponding optimization problems are formulated. The solutions of these problems indicate how to allocate the cost budget among the measurement devices in order to achieve the optimum performance. Numerical examples are presented to discuss the results.

*Keywords:* Hypothesis testing, measurement cost, decentralized detection, centralized detection, sensor networks.

# ÖZET

# MALİYET KISITLI ÖLÇÜMLERLE MERKEZİ VE DAĞITILMIŞ SEZİM

Eray Laz
Elektrik ve Elektronik Mühendisliği, Yüksek Lisans
Tez Danışmanı: Sinan Gezici
Mayıs 2016

Bu tezde merkezi ve dağıtılmış sezim sistemlerinin eniyi sezim performansları maliyet kısıtlı ölçümlerin varlığında araştırılmaktadır. Sezim performansının değerlendirilmesi için Bayes, Neyman-Pearson ve $J$-ıraksaklık kriterleri göz önünde bulundurulmaktadır. Bayes kriterindeki temel amaç, ölçüm aletlerinin toplam maliyeti üzerindeki kısıt altında hata olasılığını (daha genel olarak, Bayes riski) azaltmaktır. Neyman-Pearson kriterinde tespit olasılığı verilen maliyet kısıtı altında en yüksek seviyeye çıkarılmaktadır. Uzaklık tabanlı kriterde ise farklı hipotezler altındaki karar istatistiği dağılımları arasındaki $J$-ıraksaklık, toplam maliyet kısıtı altında maksimuma çıkarılmaktadır. Bayes kriterinde hem merkezi hem de dağıtılmış sezim sistemleri için hata olasılıkları elde edilmekte ve eniyileme problemleri önerilmektedir. Neyman-Pearson stratejisi için tespit ve yanlış alarm olasılıkları elde edilmekte ve eniyileme problemleri sunulmaktadır. Ek olarak, hem merkezi hem de dağıtılmış sezim sistemleri için $J$-ıraksaklık değerleri hesaplanmakta ve bunlara uygun eniyileme problemleri formüle edilmektedir. Bu problemlerin çözümleri en iyi performansı elde etmek için maliyet bütçesinin ölçüm aletlerine nasıl dağıtılacağını göstermektedir. Sonuçları tartışmak amacıyla sayısal örnekler sunulmaktadır.

*Anahtar sözcükler*: Hipotez sınama, ölçüm maliyeti, dağıtılmış sezim, merkezi sezim, algılayıcı ağlar.

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# Chapter 1

# Introduction

In this thesis, centralized and decentralized hypothesis-testing (detection) problems are investigated in the presence of cost constrained measurements. In such systems, decisions are performed based on measurements gathered by multiple sensors, the qualities of which are determined according to assigned cost values. The aim is to develop optimal cost allocation strategies for the Bayesian, Neyman-Pearson, and $J$-divergence criteria under a total cost constraint. In the case of centralized detection, a set of geographically separated sensors send all of their measurements to a fusion center, and the fusion center decides on one of the hypotheses [1]. On the other hand, in decentralized detection, sensors transmit a summary of their measurements to the fusion center [2]. For quantifying the costs of measurement devices (sensors), the model in [3] is employed in this study. According to [3], the cost of a measurement device is basically determined by the number of amplitude levels that it can reliably distinguish.

Detection and estimation problems considering system resource constraints have extensively been studied in the literature [4–20]. In [4], measurement cost minimization is performed under various estimation accuracy constraints. In [5], optimal distributed detection strategies are studied for wireless sensor networks by considering network resource constraints, where it is assumed that observations at the sensors are spatially and temporally independent and identically distributed (i.i.d.). Two types of constraints are taken into consideration related to the transmission power and the communication channel. For the communication channel, there exist two options, which are multiple access and parallel access channels. It is shown that using a multiple access channel with analog communication of local likelihood ratios (soft decisions) is asymptotically optimal when each sensor communicates with a constant power [5]. In [6], binary decentralized detection problem is investigated under the constraint of wireless channel capacity. It is proved that having a set of identical sensor is asymptotically optimal when the observations conditioned on the hypothesis are i.i.d. and the number of observations per sensor goes to infinity. In [7], a decentralized detection problem is studied, where the sensors have side information that affects the statistics of their measurements and the network has a cost constraint. The author examines wireless sensor networks with a cost constraint and a capacity constraint separately. In both scenarios, the error exponent is minimized under the specified constraints. The study in [7] produces a similar result to that in [6] for the scenario with the capacity constraint. In addition, [7] and [8] have the same results for scenario with the power constraint. It is obtained that having identical sensors which use the same transmission scheme is asymptotically optimal when the observations are conditionally independent given the state of the nature.

In [9], the decentralized detection problem is studied in the presence of system level costs. These costs stem from processing the received signal and transmitting the local outputs to the fusion center. It is shown that the optimum detection performance can be obtained by performing randomization over the measurements and over the choice of the transmission time. In [10], the aim is to minimize the probability of error under communication rate constraints, where the sensors can censor their observations. The optimum result is obtained by censoring uninformative observations and sending informative observations to the fusion center. In [11], the aim is to obtain a network configuration that satisfies the optimum detection performance under a given cost constraint. The cost constraint depends on the number of sensors employed in the network. In [12], the optimal power allocation for distributed detection is studied, where both individual and joint constraints on the power that sensors use while transmitting their decisions to the fusion center are taken into consideration. The optimal detection performance is obtained for the proposed power allocation scheme. In [13], a binary hypothesis testing problem is investigated under communication constraints. The proposed algorithm determines a data reduction rate for transmitting a reduced version of data and finds the performance of the best test based on the reduced data. In [14], the decentralized detection problem is investigated under both power and bandwidth constraints. It is shown that combining many 'not so good' local decisions is better than combining a few very good local decisions in the case of large sensor systems. In [15–17], the decentralized detection problem is studied with fusion of Gaussian signals. It is stated that there is an optimal number of local sensors that achieves the highest performance under a given global power constraint, and increasing the number of sensors beyond the optimal number degrades the performance. In [18], the authors investigate decentralized detection and fusion performance of a sensor network under a total power constraint. It is shown that using non-orthogonal communication between local sensors and the fusion center improves fusion performance monotonically. In [19], the optimization of detection performance of a sensor network is studied under communication constraints, and it is found that the optimal fusion rule is similar to the majority-voting rule for binary decentralized detection.

Based on the cost function proposed in [3] for obtaining measurements, various studies have been performed on estimation with cost constraints [4, 20]. In particular, [4] considers the costs of measurements and aims to minimize the total cost under various estimation accuracy constraints. In [20], average Fisher information maximization is studied under cost constrained measurements. On the other hand, [21] investigates the tradeoff between reducing the measurement cost and keeping the estimation accuracy within acceptable levels in continuous time linear filtering problems. In [22], the channel switching problem is studied, where the aim is to minimize the probability of error between a transmitter and a receiver that are connected via multiple channels and only one channel can be used at a given time. In that study, a logarithmic cost function similar to that in [3] is employed for specifying the cost of using a certain channel.

Although costs of measurements have been considered in various estimation and channel switching problems such as [4, 20–22], there exist no studies in the literature that consider the optimization of both centralized and decentralized detection systems in the presence of cost constrained measurements based on a specific cost function as in [3]. In this study, we first consider the centralized detection problem and propose a general formulation for allocating the cost budget to measurement devices in order to achieve the optimum performance according to the Bayesian criterion. Also, a closed-form expression is obtained for binary hypothesis testing with Gaussian observations. In addition, it is shown that the probability of error expression for the Gaussian case is convex with respect to the total cost constraint in the case of equally likely binary hypotheses. Then, we investigate the decentralized detection problem in the Bayesian framework with some common fusion rules, and present a generic formulation that aims to minimize the probability of error by optimally allocating the cost budget to measurement devices. A numerical solution is proposed for binary hypothesis testing with Gaussian observations. As convexity is an important property for the optimization problems, the convexity property is explored for the case of two measurement devices. Furthermore, the Neyman-Pearson and $J$-divergence criteria are investigated for the cost allocation problem in order to achieve the optimum detection performance. The general optimization problems are proposed for both criteria and the Gaussian scenario is investigated as a special case. As for the Bayesian criterion, both centralized and decentralized detection systems are taken into consideration.

The remainder of the thesis is organized as follows: In Chapter 2, the optimal cost allocation among measurement devices is studied for the Bayesian criterion. In Chapter 3, the problem is investigated in the Neyman-Pearson framework. In Chapter 4, the optimization problems obtained according to $J$-divergence are examined. In Chapter 5, numerical examples that illustrate the obtained results are presented. Finally, conclusions are presented in Chapter 6.

# Chapter 2

# Cost Allocation for Bayesian Criterion

In this chapter, the cost allocation problem is investigated for hypothesis-testing problems based on the Bayesian criterion. When it is possible to assign costs to the decisions and when the prior probabilities of the states of nature are known, the Bayesian approach is a well-suited candidate for detection criterion [23]. The aim in this section is to minimize the Bayes risk for both centralized and decentralized detection systems under a total cost constraint on measurements.

## 2.1 Centralized Detection

In centralized detection problems, all sensor nodes transmit their observations to the fusion center, and the decision is performed in the fusion center based on the data from all the sensors. The system model for centralized detection is shown in Figure 2.1.

As illustrated in Figure 2.1, $x_1, x_2, \ldots, x_K$ represent the scalar observations, and $\mathrm{s}_1, \mathrm{s}_2, \ldots, \mathrm{s}_K$ denote the sensors by which the measurements are taken. The

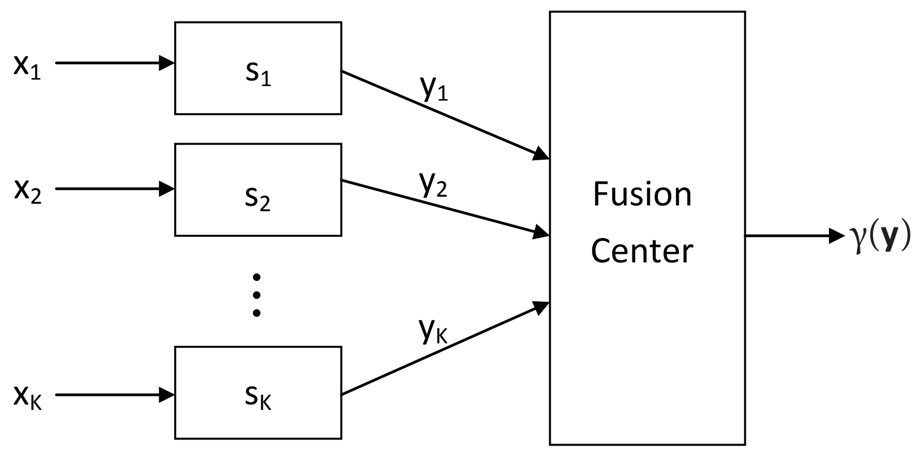


Figure 2.1: Centralized detection system model.

measurement at sensor $i$ is represented as $y_i = x_i + m_i$, where $m_i$ is the measurement noise. The measurement $\mathbf{y} \in \mathbb{R}^K$ is processed by the fusion center to produce the final decision $\gamma(\mathbf{y})$, where $\mathbf{y} = [y_1, y_2, \ldots, y_K]^T$ and $\gamma(\mathbf{y})$ takes values from $\{0, 1, \ldots, M-1\}$ for $M$-ary hypothesis testing.

In the Bayesian hypothesis-testing framework, the optimum decision rule is the one that minimizes the Bayes risk, which is defined as the average of the conditional risks [23]. The conditional risk for a decision rule $\delta(\cdot)$ when the state of nature is $H_j$ is given by

$$R_j(\delta) = \sum_{i=0}^{M-1} \tilde{c}_{ij} P_j(\Gamma_i) \tag{2.1}$$

where $\tilde{c}_{ij}$ is the cost of choosing hypothesis $H_i$ when the state of nature is $H_j$, and $P_j(\Gamma_i)$ is the probability of deciding hypothesis $H_i$ when $H_j$ is correct, with $\Gamma_i$ denoting the decision region for hypothesis $H_i$. Then, the Bayes risk can be expressed as

$$r(\delta) = \sum_{j=0}^{M-1} \pi_j R_j(\delta) \tag{2.2}$$

where $\pi_j$ is the prior probability of hypothesis $H_j$. For the values of $\tilde{c}_{ij}$, uniform cost assignment (UCA) is commonly employed, which is stated as [23]

$$\tilde{c}_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}. \tag{2.3}$$

For UCA, the Bayes rule, which minimizes the Bayes risk specified by (2.1) and (2.2), reduces to choosing the hypothesis with the maximum a-posteriori probability (MAP), and the corresponding Bayes risk can be stated, after some manipulation, as

$$r(\delta_B) = 1 - \int_{\mathbb{R}^K} \max_{l=\{0,1,\ldots,M-1\}} \pi_l \, p_l(\mathbf{y}) \, d\mathbf{y} \tag{2.4}$$

where $\delta_B$ denotes the Bayes rule, and $p_l(\mathbf{y})$ is the probability distribution of $\mathbf{y}$ under hypothesis $H_l$ [23].

In this section, the aim is to perform the optimal cost allocation among the sensors in Figure 2.1 in order to minimize the Bayes risk expression in (2.4) under a total cost constraint. The cost of measuring the $i$th component of the observation vector, $x_i$, is given by $C_i = 0.5 \log_2(1 + \sigma_{x_i}^2/\sigma_{m_i}^2)$, where $\sigma_{x_i}^2$ is the variance of $x_i$ and $\sigma_{m_i}^2$ is the variance of the noise introduced by the $i$th sensor [3]. Then, the total cost is expressed as

$$C = \sum_{i=1}^{K} C_i = \frac{1}{2} \sum_{i=1}^{K} \log_2 \left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right). \tag{2.5}$$

The cost function for each sensor is monotonically decreasing, nonnegative, and convex with respect to $\sigma_{m_i}^2$ for $\forall \sigma_{m_i}^2 > 0$ and $\forall \sigma_{x_i}^2 > 0$. (The convexity property of the cost function can easily be shown by examining its Hessian matrix [24].) In addition, when the measurement noise variance is low, the cost is high since the number of amplitude levels that the device can distinguish gets high [3]. When $\sigma_{m_i}^2$ goes to infinity, the cost converges to zero and when $\sigma_{m_i}^2$ goes to zero, the cost approaches infinity.

Based on (2.4) and (2.5), the following optimization problem is proposed for centralized detection problems:

$$\begin{aligned} \max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & \int_{\mathbb{R}^K} \max_{l=\{0,1,\ldots,M-1\}} \pi_l \, p_l(\mathbf{y}) \, d\mathbf{y} \\ \text{subject to} \quad & \frac{1}{2} \sum_{i=1}^{K} \log_2 \left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \leq C_T \end{aligned} \tag{2.6}$$

where $C_T$ is the (total) cost constraint. Hence, the optimal allocation of the measurement noise variances, $\sigma_{m_i}^2$, (equivalently, the costs, $C_i$) is to be performed under the total cost constraint. It is also noted that the maximization of the objective function in (2.6) corresponds to the minimization of the Bayes risk in (2.4), which represents the probability of error for the Bayes rule. When the optimization problem proposed in (2.6) is solved, the optimum cost values for the measurement devices (sensors) are obtained and these values achieve the optimum performance for centralized detection.

In practical systems, the observations, $\mathbf{x} = [x_1, \ldots, x_K]^T$, are independent of the measurement noise, $\mathbf{m} = [m_1, \ldots, m_K]^T$. Hence, the conditional probability density function (PDF) of the measurement vector when hypothesis $H_l$ is true can be obtained as the convolution of the PDFs of $\mathbf{m}$ and $\mathbf{x}$ as follows:

$$p_l(\mathbf{y}) = \int_{\mathbb{R}^K} p_{\mathbf{M}}(\mathbf{m}) p_{\mathbf{X}}(\mathbf{y} - \mathbf{m} | H_l) d\mathbf{m} \, . \tag{2.7}$$

In addition, if the sensors have independent noise, $p_{\mathbf{M}}(\mathbf{m})$ can be expressed as $p_{\mathbf{M}}(\mathbf{m}) = p_{M_1}(m_1) \cdots p_{M_K}(m_K)$.

As a special case, a centralized binary hypothesis-testing problem is investigated in the presence of Gaussian observations and measurement noise, which is a common scenario in practice. In this case, the distribution of observation $\mathbf{x}$ under hypothesis $H_0$ is Gaussian with mean vector $\boldsymbol{\mu_0}$ and covariance matrix $\boldsymbol{\Sigma}$, which is denoted by $\mathcal{N}(\boldsymbol{\mu_0}, \boldsymbol{\Sigma})$. Similarly, $\mathbf{x}$ is distributed as $\mathcal{N}(\boldsymbol{\mu_1}, \boldsymbol{\Sigma})$ under hypothesis $H_1$. In addition, the measurement noise vector, $\mathbf{m}$, is distributed as $\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma_m})$, where $\boldsymbol{\Sigma_m} = \mathrm{diag}\{\sigma^2_{m_1}, \sigma^2_{m_2}, \ldots, \sigma^2_{m_K}\}$; that is, the measurement noise is independent for different sensors [3]. Considering that $\mathbf{x}$ and $\mathbf{m}$ are independent, the distribution of the measurement, $\mathbf{y} = \mathbf{x} + \mathbf{m}$, is denoted by $\mathcal{N}(\boldsymbol{\mu_0}, \boldsymbol{\Sigma} + \boldsymbol{\Sigma_m})$ under hypothesis $H_0$ and by $\mathcal{N}(\boldsymbol{\mu_1}, \boldsymbol{\Sigma} + \boldsymbol{\Sigma_m})$ under $H_1$.

For the hypothesis-testing problem specified in the previous paragraph, the Bayes risk corresponding to the Bayes rule can be obtained as follows in the case of UCA [23, Chapter 3]:

$$r(\delta_B) = \pi_0 Q\left(\frac{\ln(\pi_0/\pi_1)}{d} + \frac{d}{2}\right) + \pi_1 Q\left(\frac{d}{2} - \frac{\ln(\pi_0/\pi_1)}{d}\right) \tag{2.8}$$

where

$$d \triangleq \sqrt{(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})^T (\boldsymbol{\Sigma} + \boldsymbol{\Sigma_m})^{-1} (\boldsymbol{\mu_1} - \boldsymbol{\mu_0})} \tag{2.9}$$

and $Q(x) = (1/\sqrt{2\pi}\,) \int_x^\infty \mathrm{e}^{-0.5t^2} dt$ denotes the $Q$-function. It can be shown that the derivative of $r(\delta_B)$ in (2.8) with respect to $d$ is negative for all values of $d$; hence, $r(\delta_B)$ is a monotone decreasing function of $d$. Therefore, the minimization of $r(\delta_B)$ can be achieved by maximizing $d$. If the observations are assumed to be independent; that is, if $\mathbf{\Sigma} = \mathrm{diag}\{\sigma_{x_1}^2, \sigma_{x_2}^2, \ldots, \sigma_{x_K}^2\}$, then $d$ can be expressed as

$$d = \sqrt{\sum_{i=1}^{K} \frac{\mu_i^2}{\sigma_{x_i}^2 + \sigma_{m_i}^2}} \tag{2.10}$$

where $\mu_i$ represents the $i$th component of the vector $\boldsymbol{\mu_1} - \boldsymbol{\mu_0}$. Hence, the optimization problem in (2.6) for this case is stated as follows:

$$\begin{aligned} \max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & \sum_{i=1}^{K} \frac{\mu_i^2}{\sigma_{x_i}^2 + \sigma_{m_i}^2} \\ \text{subject to} \quad & \frac{1}{2} \sum_{i=1}^{K} \log_2 \left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \le C_T \end{aligned} \tag{2.11}$$

The objective function in (2.11) is convex with respect to $\sigma_{m_i}^2$ for $\forall \sigma_{m_i}^2 > 0$ and $\forall \sigma_{x_i}^2 > 0$ since the Hessian matrix of the objective function, $\mathbf{H} = \mathrm{diag}\{2\mu_1^2/(\sigma_{x_1}^2 + \sigma_{m_1}^2)^3, 2\mu_2^2/(\sigma_{x_2}^2 + \sigma_{m_2}^2)^3, \ldots, 2\mu_K^2/(\sigma_{x_K}^2 + \sigma_{m_K}^2)^3\}$, is positive definite. Since a convex objective function is maximized over a convex set, the solution lies at the boundary [20, 25]. Therefore, the constraint function becomes an equality constraint and the optimization problem can be solved by using the Lagrange multipliers method [24], [25]. Based on this approach, the optimal cost allocation algorithm is obtained as follows:

$$\sigma_{m_i}^2 = \begin{cases} \frac{\sigma_{x_i}^4}{\mu_i^2 \alpha - \sigma_{x_i}^2}\,, & \text{if } \sigma_{x_i}^2 < \mu_i^2 \alpha \\ \infty\,, & \text{if } \sigma_{x_i}^2 \ge \mu_i^2 \alpha \end{cases} \tag{2.12}$$

with

$$\alpha = \left(2^{2C_T} \prod_{i \in S_K} \frac{\sigma_{x_i}^2}{\mu_i^2}\right)^{1/|S_K|} \tag{2.13}$$

where set $S_K$ is given by $S_K = \{i \in \{1, 2, \ldots, K\} : \sigma_{m_i}^2 \ne \infty\}$, and $|S_K|$ represents the number of elements in the set $S_K$. The algorithm in (2.12) implies that if the observation variance $\sigma_{x_i}^2$ is greater than $\mu_i^2\alpha$, the variance of the measurement device (sensor) is set to infinity; that is, the observation is not measured at all, and the cost of the measurement device is zero. If the observation variance is smaller than the specified threshold, the variance of the measurement noise is calculated according to the expression in (2.12), which states that if the observation variance is low, the variance of the measurement device is assigned to be low. In other words, if the observation variance is low, a device with a high cost is considered to take measurements. Moreover, if the difference between the means of the observations for the two hypotheses, $\mu_i$, is high and $\sigma_{x_i}^2 < \mu_i^2\alpha$ is satisfied, a low measurement noise variance is assigned to the measurement device. If $\mu_i$ is close to zero such that $\sigma_{x_i}^2 \geq \mu_i^2\alpha$, a measurement device with zero cost is considered. Apart from this, if the observations are i.i.d. given the hypothesis, the variances of the measurement devices are chosen as equal, meaning that all the devices are required to have equal costs in order to achieve the optimum performance. The variances of the measurement devices become $\sigma_m^2 = \sigma_x^2/(2^{2C_T/K} - 1)$ for i.i.d. observations.

In the following lemma, the probability of error corresponding to the optimal cost allocation in (2.12) is shown to be convex with respective to the total cost constraint, $C_T$, for the case of equal priors.

**Lemma 1.** *Consider a binary hypothesis-testing problem in the presence of independent Gaussian observations and measurement noise. Then, for the optimal cost allocation strategy in* (2.12)*, the probability of error in* (2.8) *is a convex monotone decreasing function of the total cost constraint $C_T$ in the case of equal priors; i.e., $\pi_0 = \pi_1 = 0.5$.*

*Proof.* In the case of equal priors, the probability of error in (2.8) reduces to $Q(d/2)$. Assume, without loss of generality, that the first $N$ of $K$ sensors have finite measurement noise variances; that is, $\sigma_{m_i}^2 < \infty$ for $i \in \{1, \ldots, N\}$. Then, from (2.10), the probability of error can be written as $P_e = Q\left(\frac{1}{2}\sqrt{\sum_{i=1}^{N} \frac{\mu_i^2}{\sigma_{x_i}^2 + \sigma_{m_i}^2}}\right)$. When the optimal $\sigma_{m_i}^2$ values obtained from (2.12) and (2.13) are inserted into the probability of error expression, the optimal probability of error is stated as

$$P_e^* = Q\left(\frac{1}{2}\sqrt{\left(\sum_{i=1}^{N}\frac{\mu_i^2}{\sigma_{x_i}^2}\right) - \tau\, 2^{-2C_T/N}}\right) \tag{2.14}$$

where $\tau \triangleq N\left(\frac{\mu_1^2\cdots\mu_N^2}{\sigma_{x_1}^2\cdots\sigma_{x_N}^2}\right)^{1/N}$. The first order derivative of $P_e^*$ with respect to the total cost $C_T$ is obtained as

$$\frac{\partial P_e^*}{\partial C_T} = -\frac{(\ln 2)\tau 2^{-2C_T/N}\exp\big(-(\beta-\tau 2^{-2C_T/N})/8\big)}{2\sqrt{2\pi}N\sqrt{\beta-\tau 2^{-2C_T/N}}} \tag{2.15}$$

where $\beta \triangleq \frac{\mu_1^2}{\sigma_{x_1}^2} + \cdots + \frac{\mu_N^2}{\sigma_{x_N}^2}$. Then, the second order derivative of $P_e^*$ with respect to the total cost $C_T$ is calculated, after some manipulation, as follows:

$$\begin{aligned}\frac{\partial^2 P_e^*}{\partial C_T^2} &= \frac{\tau}{\sqrt{2\pi}}\left(\frac{\ln 2}{N}\right)^2 2^{-4C_T/N} \\ &\quad\times(\beta-\tau 2^{-2C_T/N})^{-1/2}\exp\left(-\frac{(\beta-\tau 2^{-2C_T/N})}{8}\right) \\ &\quad\times\left(\frac{\tau}{8}+2^{2C_T/N}+\frac{\tau}{2}(\beta-\tau 2^{-2C_T/N})^{-1}\right)\end{aligned} \tag{2.16}$$

As the arithmetic mean is larger than or equal to the geometric mean, $\beta \geq \tau$ is obtained. Then, $\beta > \tau 2^{-2C_T/N}$ since $2^{-2C_T/N} < 1$. Therefore, it is observed from (2.15) and (2.16) that the first and the second order derivatives of $P_e^*$ with respect to $C_T$ are negative and positive, respectively. Hence, $P_e^*$ is a convex and monotone decreasing function of the total cost constraint $C_T$ for all $C_T > 0$. □

Lemma 1 states the convexity property of the probability of error corresponding to the optimal cost allocation strategy in (2.12) for equally likely binary hypotheses and in the presence of independent Gaussian observations and measurement noise. It should be noted that the convexity property in Lemma 1 is specific for the case of equal priors and non-convex behavior can be observed for some $C_T$ for hypotheses with unequal priors.

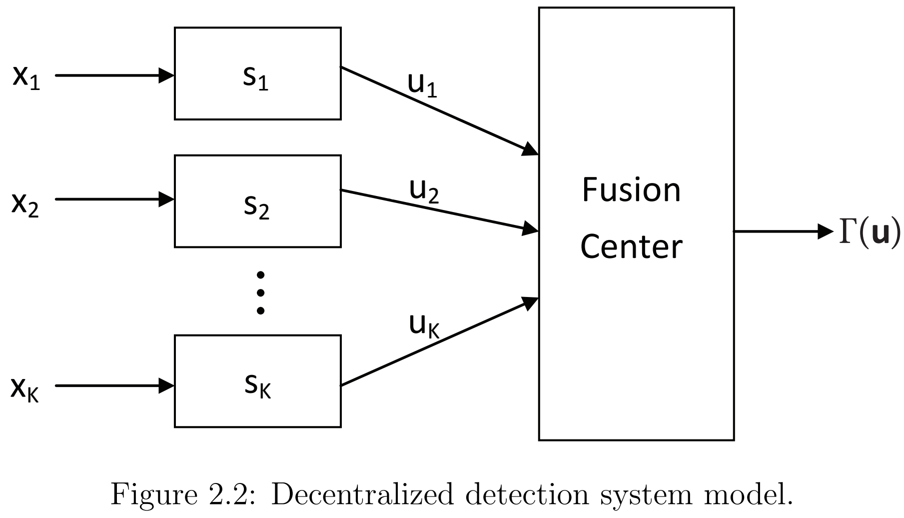


Figure 2.2: Decentralized detection system model.

## 2.2 Decentralized Detection

In contrast to centralized detection, local sensors send a summary of their observations to the fusion center in decentralized detection. For binary hypothesis-testing, local sensors can send their binary decisions about the true hypothesis (0 or 1) to the fusion center. The fusion center collects the binary decisions of the sensors and decides on the hypothesis. The fusion center can employ, e.g., OR, AND, or majority rules [26], as discussed in the following. The system model in this scenario is presented in Figure 2.2. As in centralized detection, sensor $i$, $\mathrm{s}_i$, measures the observation as $y_i = x_i + m_i$. Then, the sensors make local decisions about one of the two hypotheses as $\gamma_i(y_i) = u_i$, where $u_i$ is equal to 0 for hypothesis $H_0$ and 1 for hypothesis $H_1$. The outputs of the sensors, $u_1, u_2, \ldots, u_K$, are provided as inputs to the fusion center, which makes the final decision denoted by $\Gamma(\mathbf{u})$. The fusion rule that is employed in this section is the majority rule [26]. The majority rule is optimal when the noise components of the sensors are i.i.d., the hypotheses are equally likely, and the observations are i.i.d. and independent of the noise of the sensors [27]. The expression for the majority rule is given by

$$\Gamma(u_1, u_2, \ldots, u_K) = \begin{cases} 1, & \text{if } \sum_{i=1}^{K} u_i \geq t \\ 0, & \text{if } \sum_{i=1}^{K} u_i < t \end{cases} \tag{2.17}$$

with $t = \lfloor K/2 \rfloor + 1$, where $\lfloor \cdot \rfloor$ represents the floor operator that maps a real number to the largest integer lower than or equal to itself. Although the majority rule is considered in the following analysis, the results can easily be extended for generic integer values of $t$ in (2.17). (For $t = 1$ and $t = K$, the rule in (2.17) reduces to the OR fusion rule and the AND fusion rule, respectively.)

Considering independent but not necessarily identically distributed measurements ($y_i$'s), the probability of error (i.e., the Bayes risk for UCA) for the fusion rule in (2.17) can be calculated as

$$r(\Gamma) = \pi_0 \sum_{z=t}^{K} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} p^i_{l_{(z,c,i)}0} + \pi_1 \sum_{z=0}^{t-1} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} p^i_{l_{(z,c,i)}1} \tag{2.18}$$

where $p^i_{l_{(z,c,i)}j}$ denotes, for the $i$th sensor, the probability of choosing hypothesis $H_{l_{(z,c,i)}}$ when hypothesis $H_j$ is true, and $l_{(z,c,i)}$ corresponds to the element at the $c$th row and the $i$th column of matrix $\mathbf{L}(z)$, which has a dimension of $\binom{K}{z} \times K$ and is formed as follows: The numbers of 1's and 0's in a row are $z$ and $K - z$, respectively, and the rows of the matrix contain all possible combinations of $z$ 1's and $K - z$ 0's. For example, matrix $\mathbf{L}(z)$ for $K = 5$ and $z = 3$ can be given as follows:

$$\mathbf{L}(z) = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

where, e.g., $l_{(3,1,3)} = 1$, $l_{(3,4,2)} = 0$, and $l_{(3,3,3)} = 1$. Although matrix $\mathbf{L}(z)$ is not unique (e.g., the orders of the rows can be changed), all the $\mathbf{L}(z)$ matrices result in the same probability of error in (2.18).

For the case of i.i.d. measurements ($y_i$'s) and identical decision rules at the sensors, the probability of error for the fusion rule in (2.17) can be expressed, as a special case of (2.18), as follows:

$$r(\Gamma) = \pi_0 \sum_{z=t}^{K} \binom{K}{z} (p_{10})^z (p_{00})^{K-z} + \pi_1 \sum_{z=0}^{t-1} \binom{K}{z} (p_{11})^z (p_{01})^{K-z} \tag{2.19}$$

where $p_{lj}$ represents, for each sensor, the probability of deciding for hypothesis $H_l$ when hypothesis $H_j$ is true.

In the decentralized detection framework, the aim is to minimize the probability of error in (2.18) under the total cost constraint; that is,

$$\begin{aligned} \min_{\{\sigma^2_{m_i}\}_{i=1}^K} \quad & \pi_0 \sum_{z=t}^{K} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} p^i_{l_{(z,c,i)}0} + \pi_1 \sum_{z=0}^{t-1} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} p^i_{l_{(z,c,i)}1} \\ \text{subject to} \quad & \frac{1}{2} \sum_{i=1}^{K} \log_2 \left(1 + \frac{\sigma^2_{x_i}}{\sigma^2_{m_i}}\right) \leq C_T \end{aligned} \tag{2.20}$$

The solution of (2.20) provides the optimum cost allocation strategy for the considered decentralized detection system.

As a special case, the Gaussian scenario is investigated. Suppose that the probability distributions of the observations are independent when the hypothesis is given, and the distribution of the $i$th observation is denoted by $\mathcal{N}(\mu_{i0}, \sigma^2_{x_i})$ and $\mathcal{N}(\mu_{i1}, \sigma^2_{x_i})$ under hypothesis $H_0$ and hypothesis $H_1$, respectively. In addition, the distribution of the $i$th measurement noise is given by $\mathcal{N}(0, \sigma^2_{m_i})$, and the observations are independent of the measurement noise. For the sensors, the Bayes rule is employed assuming UCA and equally likely priors [23]. In this setting, the probability distribution of $u_i$ (i.e., the decision of the $i$th sensor) given the hypotheses can be specified as follows:

$$p_j(u_i) = \begin{cases} Q\left(\frac{(-1)^j(\mu_{i0}-\mu_{i1})}{2\sqrt{\sigma^2_{x_i}+\sigma^2_{m_i}}}\right), & \text{if } u_i = 0 \\ Q\left(\frac{(-1)^j(\mu_{i1}-\mu_{i0})}{2\sqrt{\sigma^2_{x_i}+\sigma^2_{m_i}}}\right), & \text{if } u_i = 1 \end{cases} \tag{2.21}$$

for $j \in \{0, 1\}$, where $p_j(u_i)$ represents the probability of $u_i$ under hypothesis $H_j$. Hence, the optimization problem can be expressed for the Gaussian case as follows:

$$
\begin{aligned}
\min_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & \frac{1}{2}\sum_{z=t}^{K}\sum_{c=1}^{\binom{K}{z}}\prod_{i=1}^{K} Q\left(\beta_{(z,c,i)}\frac{\mu_{i1}-\mu_{i0}}{2\sqrt{\sigma_{x_i}^2+\sigma_{m_i}^2}}\right) \\
& + \frac{1}{2}\sum_{z=0}^{t-1}\sum_{c=1}^{\binom{K}{z}}\prod_{i=1}^{K} Q\left(-\beta_{(z,c,i)}\frac{\mu_{i1}-\mu_{i0}}{2\sqrt{\sigma_{x_i}^2+\sigma_{m_i}^2}}\right) \\
\text{subject to} \quad & \frac{1}{2}\sum_{i=1}^{K}\log_2\left(1+\frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \le C_T
\end{aligned}
\tag{2.22}
$$

where $\beta_{(z,c,i)} = 2l_{(z,c,i)} - 1$. The solution of this optimization problem leads to the optimal performance for the considered decentralized detection system by optimally allocating the cost values to the measurement devices (sensors).

In the following lemma, the convexity of the optimization problem in (2.22) is investigated for the special case of two sensors.

**Lemma 2.** *Consider the Gaussian scenario that leads to the optimization problem in* (2.22)*. In addition, suppose that* $K = 2$*,* $\mu_{i0} = 0$*, and* $\mu_{i1} = \mu > 0$ *for* $i = 1, 2$*. Then, the problem in* (2.22) *is a convex optimization problem if* $\sigma_{x_i}^2 + \sigma_{m_i}^2 \le \mu^2/12$ *for* $i = 1, 2$ *and for all values of* $\sigma_{m_i}^2$ *under the total cost constraint.*

*Proof.* Under the assumptions specified in the lemma, the objective function in (2.22) can be expressed as

$$
\begin{aligned}
r(\Gamma) = & \frac{1}{2}Q\left(\frac{\mu}{2\sqrt{\sigma_{x_1}^2+\sigma_{m_1}^2}}\right)Q\left(\frac{\mu}{2\sqrt{\sigma_{x_2}^2+\sigma_{m_2}^2}}\right) \\
& + \frac{1}{2}\left(1 - Q\left(-\frac{\mu}{2\sqrt{\sigma_{x_1}^2+\sigma_{m_1}^2}}\right)Q\left(-\frac{\mu}{2\sqrt{\sigma_{x_2}^2+\sigma_{m_2}^2}}\right)\right).
\end{aligned}
\tag{2.23}
$$

The Hessian matrix $\mathbf{H}$ of $r(\Gamma)$ is stated as follows: $\mathbf{H} = \begin{pmatrix} r_{\sigma_{m_1}^2,\sigma_{m_1}^2} & r_{\sigma_{m_1}^2,\sigma_{m_2}^2} \\ r_{\sigma_{m_2}^2,\sigma_{m_1}^2} & r_{\sigma_{m_2}^2,\sigma_{m_2}^2} \end{pmatrix}$, where $r_{\sigma_{m_i}^2,\sigma_{m_j}^2}$ represents second-order derivative of $r(\Gamma)$ with respect to $\sigma_{m_i}^2$ and $\sigma_{m_j}^2$. It can be shown that $r_{\sigma_{m_1}^2,\sigma_{m_2}^2}$ and $r_{\sigma_{m_2}^2,\sigma_{m_1}^2}$ are zero. Hence, the diagonal terms must be positive for the convexity of $r(\Gamma)$ with respect to $\sigma_{m_1}^2$ and $\sigma_{m_2}^2$.

After some manipulation, $r_{\sigma^2_{m_i},\sigma^2_{m_i}}$ can be expressed for $i \in \{1, 2\}$ as

$$\begin{aligned} r_{\sigma^2_{m_i},\sigma^2_{m_i}} = & \frac{\mu}{8\sqrt{2\pi}} \exp\left( - \frac{\mu^2}{8(\sigma^2_{x_i} + \sigma^2_{m_i})} \right) \\ & \times \frac{1}{(\sigma^2_{x_i} + \sigma^2_{m_i})^{5/2}} \left( \frac{\mu^2}{8(\sigma^2_{x_i} + \sigma^2_{m_i})} - \frac{3}{2} \right). \end{aligned} \tag{2.24}$$

From (2.24), the convexity condition for $r(\Gamma)$ can be obtained as $\frac{\mu^2}{\sigma^2_{x_i}+\sigma^2_{m_i}} \geq 12$ for $i = 1, 2$. That is, if this condition is satisfied for all values of $\sigma^2_{m_i}$ under the total cost constraint, the optimization problem becomes a convex optimization problem as the constraint is already convex as discussed previously. □

Lemma 2 presents conditions under which the optimal cost allocation problem in (2.22) becomes a convex optimization problem. In that case, the problem can be solved based on convex optimization algorithms such as the interior-point algorithm [24].

# Chapter 3

# Cost Allocation for Neyman-Pearson Criterion

The Bayesian criterion considered in the previous chapter is well-suited in the presence of prior probabilities of the hypotheses and cost assignments for possible decisions (see (2.1)–(2.3)). However, in some cases, the information about the prior probabilities of the hypotheses may not be available or assigning costs to possible decisions may not be suitable. In such scenarios, the Neyman-Pearson approach can be adopted for binary hypothesis-testing problems, where the aim is to maximize the probability of detection while satisfying a constraint on the probability of false alarm [23]. In this chapter, the Neyman-Pearson approach is employed for designing optimum centralized and decentralized detection systems in the presence of a cost constraint on measurement devices.

## 3.1 Centralized Detection

As described in Section 2.1, the sensors in a centralized detection system transmit all of their observations to the fusion center and the fusion center decides on the hypothesis. Therefore, it suffices to apply the Neyman-Pearson criterion to the fusion center only. In this context, the aim is to maximize the probability of detection subject to the constraints on the probability of false alarm and the total cost, which is stated by the following optimization problem:

$$\begin{aligned} \max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & \int_{\Gamma_1} p_1(\mathbf{y}) d\mathbf{y} \\ \text{subject to} \quad & \int_{\Gamma_1} p_0(\mathbf{y}) d\mathbf{y} \leq \alpha_{fc} \\ & \frac{1}{2}\sum_{i=1}^{K} \log_2\left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \leq C_T \end{aligned} \tag{3.1}$$

where $\Gamma_1$ is the decision region for hypotheses $H_1$, $p_i(\mathbf{y})$ is the probability distribution of the observation under $H_i$, where $i \in \{0, 1\}$, and $\alpha_{fc}$ is the false alarm constraint. The solution of (3.1) yields the maximum value of the probability of detection via optimal cost assignments for the local sensors under the false alarm and total cost constraints.

Next, the Gaussian scenario is investigated as a special case based on the same distributions and assumptions employed in Section 2.1. Due to the presence of separate constraints in (3.1), the optimal NP decision rule can be obtained first, which leads to a likelihood ratio test with the probability of false alarm set to $\alpha_{fc}$ [23]. For the considered Gaussian scenario, the corresponding probability of detection can be obtained as $P_D = Q(Q^{-1}(\alpha_{fc}) - d)$, where $d$ is given by (2.9) [23]. Therefore, the optimization problem in (3.1) can be expressed as follows:

$$\begin{aligned} \max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & Q\big(Q^{-1}(\alpha_{fc}) - d\big) \\ \text{subject to} \quad & \frac{1}{2}\sum_{i=1}^{K} \log_2\left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \leq C_T \end{aligned} \tag{3.2}$$

In order to maximize the objective function, the term inside $Q$ function should be minimized which can be achieved by increasing $d$ in (2.9). This results in the same optimization problem proposed in Section 2.1; hence, the cost values of the sensors are determined according to the algorithm given in (2.12).

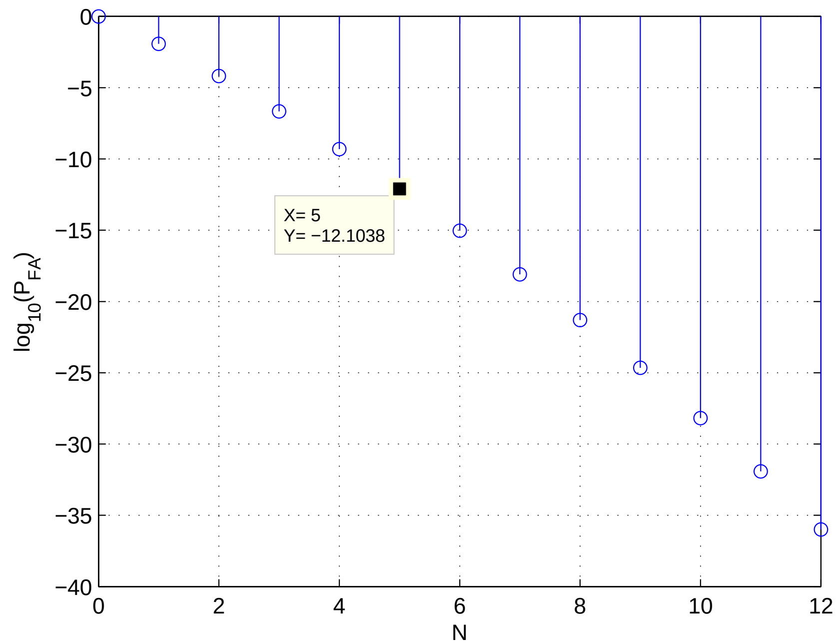


Figure 3.1: Probability of false alarm versus $N$ for the $N$ out $K$ fusion rule.

## 3.2 Decentralized Detection

In decentralized detection, all local sensors make their own decisions, which are processed in the fusion center to decide on the hypothesis. In Section 2.2, local sensors make a decision according to the Bayes rule and the majority fusion rule is employed at the fusion center. In this part, decisions are made according to the Neyman-Pearson criterion in the local sensors and the fusion center uses a counting rule [28]. The counting rule is specified in such a way that the probability of false alarm is lower than a specified threshold. As an example, the probability of false alarm in the fusion center versus the value of $N$ (for the $N$ out of $K$ rule) is illustrated in Figure 3.1 for a sensor network with 12 local sensors. In the figure, the probability of false alarm for the local sensors is $10^{-3}$ and the measurements of the sensors are independent. For such a system to achieve an overall probability of false alarm lower than $10^{-12}$, the best fusion rule becomes 5 out of 12. Moreover, it is observed that the probability of false alarm is a decreasing function of $N$ similar to the probability of detection. In order to achieve the maximum probability of detection, $N$ is chosen to be the minimum of possible value that satisfies constraint on the probability of false alarm, $\alpha_{fc}$.

The same assumptions and the probability distributions used in Section 2.2 are employed in this section. Then, the probability of false alarm $P_{FA_{fc}}$ at the fusion center for the $N$ out of $K$ strategy is calculated as follows:

$$P_{FA_{fc}} = \sum_{z=N}^{K} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} |l_{(z,c,i)} - 1| + (2l_{(z,c,i)} - 1)\alpha_i \tag{3.3}$$

where $\alpha_i$ is the probability of false alarm at the $i$th sensor, and $l_{(z,c,i)}$ corresponds to the element at the $c$th row and the $i$th column of matrix $\mathbf{L}(z)$, as defined in Section 2.2.

The proposed optimization problem aims to maximize the probability of detection while keeping the total cost of the sensors under a certain limit and guaranteeing that the probability of false alarm is below the specified false alarm constraint. Based on (3.3), the optimization problem is stated as

$$\begin{aligned} \max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & \sum_{z=N}^{K} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} |l_{(z,c,i)} - 1| + (2l_{(z,c,i)} - 1)P_{D_i} \\ \text{subject to} \quad & \frac{1}{2}\sum_{i=1}^{K} \log_2\left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \leq C_T \end{aligned} \tag{3.4}$$

where $P_{D_i}$ is the probability of detection of the $i$th sensor, and the value of $N$ is equal to the minimum integer number that satisfies $P_{FA_{fc}} \leq \alpha_{fc}$ for the $N$ out of $K$ decision rule.

As a special case, the Gaussian scenario in Section 2.2 is investigated. In this case, the detection threshold is calculated based on the given $\alpha_i$ value by equating the probability of false alarm to $\alpha_i$. Then, the probability of detection is determined for the obtained detection threshold. In particular, the probability of detection for the $i$th sensor is calculated as follows:

$$P_{D_i} = Q\left(Q^{-1}(\alpha_i) - \frac{\mu_{i1} - \mu_{i0}}{\sqrt{\sigma_{x_i}^2 + \sigma_{m_i}^2}}\right) \tag{3.5}$$

From (3.5), the optimization problem in (3.4) can be specified as follows:

$$\max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad \sum_{z=N}^{K} \sum_{c=1}^{\binom{K}{z}} \prod_{i=1}^{K} |l_{(z,c,i)} - 1| + (2l_{(z,c,i)} - 1) Q\left(Q^{-1}(\alpha_i) - \frac{\mu_{i1} - \mu_{i0}}{\sqrt{\sigma_{x_i}^2 + \sigma_{m_i}^2}}\right)$$

$$\text{subject to} \quad \frac{1}{2}\sum_{i=1}^{K} \log_2\left(1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2}\right) \leq C_T \tag{3.6}$$

where $N$ is chosen as stated above. The solution of (3.6) results in the maximum probability of detection for the given cost and false alarm constraints.

# Chapter 4

# Cost Allocation for $J$-divergence Criterion

In some centralized and decentralized detection problems, it can be difficult and complex to calculate the probability of detection, the probability of false alarm, or the probability of error. In such scenarios, distance related bounds are commonly used for quantifying detection performance. The distance related bounds provide upper and lower bounds on the probabilities of detection and false alarm (or, the probability of error). Some examples of these bounds are the Bhattacharrya bound, $J$-divergence and Chernoff bound [23]. These bounds belong to the Ali-Silvey class of distance measures [29]. In this chapter, we employ $J$-divergence, firstly introduced by Jeffreys [30], for the cost allocation problem. The $J$-divergence is a commonly used metric for detection performance [31–34]. It introduces a lower bound on the probability of error $P_e$ [33] as follows:

$$P_e > \pi_0 \pi_1 e^{-J/2} \tag{4.1}$$

where $\pi_0$ and $\pi_1$ are the prior probabilities of hypothesis $H_0$ and hypothesis $H_1$, respectively, and $J$ denotes the $J$-divergence, which is the symmetric version of the Kullback-Leibler (KL) distance [35]. The $J$-divergence is defined between two probability densities, $p$ and $q$, as follows:

$$J(p, q) = D(p\|q) + D(q\|p) \tag{4.2}$$

where $D(p\|q)$ is the KL distance between $p$ and $q$, which is calculated as

$$D(p\|q) = \int p(x) \ln \frac{p(x)}{q(x)} dx \,. \tag{4.3}$$

According to the formula in (4.3), the $J$-divergence is obtained as follows:

$$J(p, q) = \int (p(x) - q(x)) \ln \frac{p(x)}{q(x)} dx \tag{4.4}$$

In this section, the cost allocation problem is investigated based on the $J$-divergence criterion for both centralized and decentralized detection systems.

## 4.1 Centralized Detection

The aim is to maximize the detection performance at the fusion center under a total cost constraint. To this aim, the $J$-divergence between $p_1(\mathbf{y})$ and $p_0(\mathbf{y})$ is to be maximized. The optimization problem for centralized detection can be written as follows:

$$\begin{aligned} \max_{\{\sigma_{m_i}^2\}_{i=1}^K} \quad & J(p_1(\mathbf{y}), p_0(\mathbf{y})) \\ \text{subject to} \quad & \frac{1}{2} \sum_{i=1}^{K} \log_2 \left( 1 + \frac{\sigma_{x_i}^2}{\sigma_{m_i}^2} \right) \leq C_T. \end{aligned} \tag{4.5}$$

As in the previous section, the Gaussian scenario is investigated in detail. The $J$ divergence between densities $p$ and $q$ with distributions $\mathcal{N}(\boldsymbol{\mu_0}, \boldsymbol{\Sigma_0})$ and $\mathcal{N}(\boldsymbol{\mu_1}, \boldsymbol{\Sigma_1})$, respectively, is given as follows [36]:

$$\begin{aligned} J(p, q) = & \frac{1}{2}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})^T ({\boldsymbol{\Sigma_0}}^{-1} + {\boldsymbol{\Sigma_1}}^{-1})(\boldsymbol{\mu_1} - \boldsymbol{\mu_0}) \\ & + \frac{1}{2} \mathrm{tr}\{{\boldsymbol{\Sigma_0}}^{-1}\boldsymbol{\Sigma_1} + {\boldsymbol{\Sigma_1}}^{-1}\boldsymbol{\Sigma_0} - 2\mathbf{I}\} \end{aligned} \tag{4.6}$$

where $\mathbf{I}$ is the identity matrix with the same size as the covariance matrices. For the Gaussian scenario described in Section 2.1, the $J$-divergence is calculated as

$$J(p_1(\mathbf{y}), p_0(\mathbf{y})) = (\boldsymbol{\mu_1} - \boldsymbol{\mu_0})^T {\boldsymbol{\Sigma_T}}^{-1} (\boldsymbol{\mu_1} - \boldsymbol{\mu_0}) \tag{4.7}$$

which is the same as the objective function in (2.11). Therefore, the same optimization problem as in Section 2.1 and 3.1 is obtained. As a result, the cost allocation strategy is determined according to the algorithm in (2.12).

## 4.2 Decentralized Detection

In this part, a decentralized detection system is examined based on the $J$-divergence criterion. The aim is to maximize the $J$-divergence between $p_1(\mathbf{u})$ and $p_0(\mathbf{u})$ under a total cost constraint. The mathematical description of the problem is given by

$$\begin{aligned} \max_{\{\sigma^2_{m_i}\}_{i=1}^K} \quad & J(p_1(\mathbf{u}), p_0(\mathbf{u})) \\ \text{subject to} \quad & \frac{1}{2}\sum_{i=1}^{K} \log_2\left(1+\frac{\sigma^2_{x_i}}{\sigma^2_{m_i}}\right) \leq C_T \end{aligned} \tag{4.8}$$

In order to solve this problem, the conditional density functions of the local decisions should be determined. These densities are given as follows:

$$p_1(\mathbf{u}) = \prod_{i=1}^{K} P_{D_i}^{u_i}(1-P_{D_i})^{1-u_i} \tag{4.9}$$

$$p_0(\mathbf{u}) = \prod_{i=1}^{K} P_{FA_i}^{u_i}(1-P_{FA_i})^{1-u_i} \tag{4.10}$$

where $P_{FA_i}$ and $P_{D_i}$ represent the probability of false alarm and the probability of detection at the $i$th sensor, respectively. The information about $P_{FA_i}$ and $P_{D_i}$ can be obtained by using the Neyman-Pearson rule. The objective function in the optimization problem can be expressed as follows:

$$\begin{aligned} J(p_1(\boldsymbol{u}), p_0(\boldsymbol{u})) = & \sum_{u_1=0}^{1}\sum_{u_2=0}^{1}\cdots\sum_{u_k=0}^{1} \\ & \left(\prod_{i=1}^{K} P_{D_i}^{u_i}(1-P_{D_i})^{1-u_i} - \prod_{i=1}^{K} P_{FA_i}^{u_i}(1-P_{FA_i})^{1-u_i}\right) \\ & \times \ln\frac{\prod_{i=1}^{K} P_{D_i}^{u_i}(1-P_{D_i})^{1-u_i}}{\prod_{i=1}^{K} P_{FA_i}^{u_i}(1-P_{FA_i})^{1-u_i}} \end{aligned} \tag{4.11}$$

In order to examine the Gaussian scenario, $P_{D_i}$ is determined in terms of the specified probability of false alarm as in (3.5). Then, the given $P_{FA_i}$ and the calculated $P_{D_i}$ values can be inserted into (4.11) in order to determine the $J$-divergence between $p_1(\mathbf{u})$ and $p_0(\mathbf{u})$. At this point, the obtained $J$-divergence between $p_1(\mathbf{u})$ and $p_0(\mathbf{u})$ is inserted into (4.8) and the optimization problem is solved numerically in order to obtain the optimum detection performance in the sense of $J$-divergence.

# Chapter 5

# Numerical Results

In this part, the performance of the proposed optimal cost allocation strategies is evaluated via numerical examples. Firstly, the results for centralized detection in the Bayesian framework are presented. The distribution of the observation $\mathbf{x}$ under hypothesis $H_0$ is given by $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$, where $\mathbf{0} = [0, 0, 0]^T$. Similarly, the distribution of $\mathbf{x}$ under hypothesis $H_1$ is modeled as $\mathcal{N}(\mathbf{1}, \mathbf{\Sigma})$, where $\mathbf{1} = [1, 1, 1]^T$. In these distributions, $\mathbf{\Sigma}$ represents the covariance matrix, which is expressed as $\text{diag}\{\sigma_{x_1}^2, \sigma_{x_2}^2, \sigma_{x_3}^2\}$. The values of the variances $\sigma_{x_1}^2$, $\sigma_{x_2}^2$ and $\sigma_{x_3}^2$ are set to 0.2, 0.7, and 1.2, respectively. Measurement noise $\mathbf{m}$ also has Gaussian distribution denoted by $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma_m})$, where $\mathbf{\Sigma_m} = \text{diag}\{\sigma_{m_1}^2, \sigma_{m_2}^2, \sigma_{m_3}^2\}$. Lastly, the hypotheses are equally likely; i.e., $\pi_0 = \pi_1 = 0.5$.

The strategies that are compared with the proposed optimal cost allocation strategy are

- assignment of equal measurement variances to the measurement devices (sensors), and
- assignment of all the cost to the sensor with the best observation.

When the measurement devices have equal measurement noise variances; i.e., $\sigma_m^2 = \sigma_{m_1}^2 = \sigma_{m_2}^2 = \sigma_{m_3}^2$, the variance $\sigma_m^2$ can be calculated by using the formula

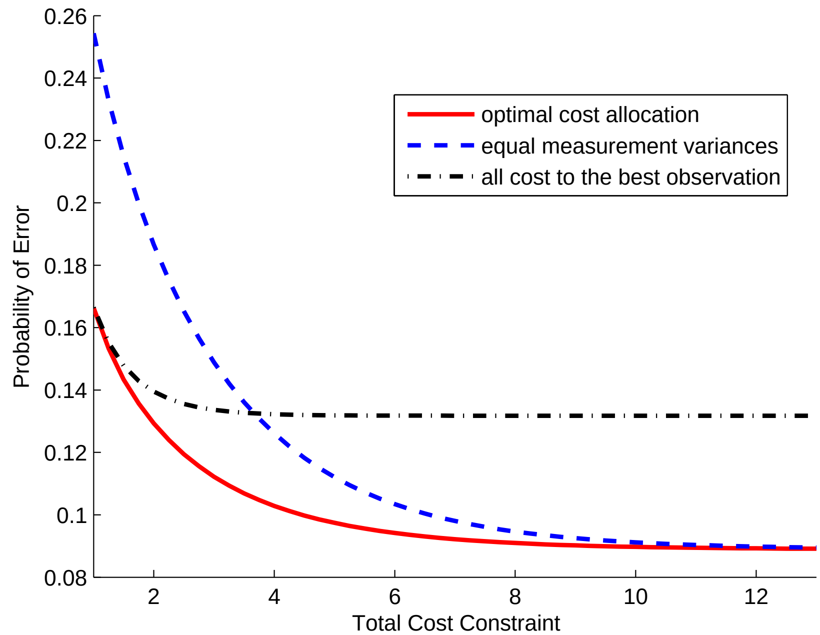


Figure 5.1: Probability of error vs. total cost constraint for Bayesian centralized detection.

$\prod_{i=1}^{3}(1 + \sigma_{x_i}^2/\sigma_m^2) = 2^{2C_T}$, where the variance $\sigma_m^2$ corresponds to the smallest positive root of this equation. After finding $\sigma_m^2$, the probability of error is calculated as $r(\delta_B) = Q(0.5\sqrt{\sum_{i=1}^{3} 1/(\sigma_{x_i}^2 + \sigma_m^2)})$. In the second strategy, all the available cost is assigned to the measurement device having the observation with the smallest variance. In this example, $\sigma_{x_1}^2$ has the smallest variance; hence, all the cost is assigned to sensor 1 and $\sigma_{m_1}^2 = \sigma_{x_1}^2/(2^{2C_T} - 1)$. The other variances $\sigma_{m_2}^2$ and $\sigma_{m_3}^2$ are set to infinity, and no measurements are taken from the corresponding measurement devices. The probability of error is obtained for this case as $r(\delta_B) = Q(0.5\sqrt{2^{2C_T} - 1}/\sqrt{2^{2C_T}\sigma_{x_1}^2})$. The results obtained for the centralized detection in the Bayesian framework are presented in Figure 5.1. In addition, Table 5.1 shows the measurement variances and corresponding probability of error values for various total cost constraints. In the table, EMV represents the equal measurement variances strategy and ACBO corresponds to the all cost to the best observation strategy.

Figure 5.1 illustrates the probability of error versus the total cost constraint,

Table 5.1: Measurement variances and corresponding probability of error values for all strategies and various total cost constraints for Bayesian centralized detection.

| $C_T$ | Strategy | $\sigma^2_{m_1}$ | $\sigma^2_{m_2}$ | $\sigma^2_{m_3}$ | $P_e$ |
|---|---|---|---|---|---|
| 2.5 | Optimal | 0.0258 | 0.4659 | 2.6096 | 0.1194 |
| | EMV | 0.2787 | 0.2787 | 0.2787 | 0.1653 |
| | ACBO | 0.0065 | $\infty$ | $\infty$ | 0.1356 |
| 5 | Optimal | 0.0075 | 0.1008 | 0.3302 | 0.0974 |
| | EMV | 0.0628 | 0.0628 | 0.0628 | 0.1121 |
| | ACBO | $1.96 \times 10^{-4}$ | $\infty$ | $\infty$ | 0.1319 |
| 10 | Optimal | $7.16 \times 10^{-4}$ | 0.0089 | 0.0262 | 0.0897 |
| | EMV | 0.0055 | 0.0055 | 0.0055 | 0.0912 |
| | ACBO | $1.91 \times 10^{-7}$ | $\infty$ | $\infty$ | 0.1318 |

$C_T$, for the optimal cost allocation strategy and the two strategies described above. For small values of $C_T$, assigning all the cost to the sensor with the best observation converges to the optimal solution since, when $C_T$ is small, the optimal strategy allocates the total cost to the sensors with the best observations. Moreover, the probability of error for assigning all the cost to the sensor with the best observation converges to $Q(0.5/\sqrt{\sigma^2_{x_1}})$, which is equal to $Q(0.5/\sqrt{0.2}) = 0.1318$ since $\sigma^2_{m_1}$ goes to zero as $C_T$ increases. For high total cost constraints, the equal measurement variances strategy converges to the optimal strategy. Similar to the strategy that assigns all the cost to the sensor with the best observation, when $C_T$ is high, the measurement noise variances become low and the probability of error converges to $r(\delta_B) = Q(0.5\sqrt{1/\sigma^2_{x_1} + 1/\sigma^2_{x_2} + 1/\sigma^2_{x_3}})$ which is equal to 0.0889 for the values specified above. Overall, the proposed optimal cost allocation strategy yields the lowest probabilities of error. In other words, the optimum performance according to the Bayesian criterion is attained with the optimal cost allocation strategy.

For the same setting as in Figure 5.1, the results for decentralized detection in the Bayesian framework are presented in Figure 5.2. Moreover, Table 5.2 shows the measurement variances and corresponding probability of error values

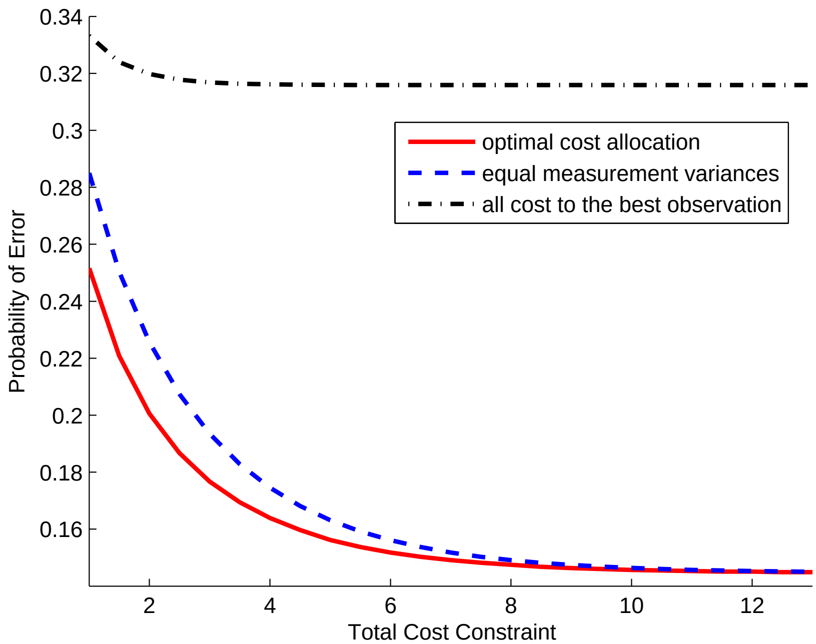


Figure 5.2: Probability of error vs. total cost constraint for Bayesian decentralized detection.

for various total cost constraints.

As observed from Figure 5.2, assigning all the cost to the sensor with the best observation yields the worst performance in this case since all the sensors make their own decisions. When zero cost is assigned to a sensor, the measurement noise variance becomes infinity and the probability of error for that measurement device becomes 0.5. Then, the probability of error converges to $r(\Gamma) = 0.75\, Q(0.5/\sqrt{\sigma^2_{x_1}}) + 0.5\, Q(-0.5/\sqrt{\sigma^2_{x_1}})$ for high cost constraints. For $\sigma^2_{x_1} = 0.2$, the probability of error converges to 0.3159. When the cost constraint is high, the equal measurement variances strategy converges to the optimal strategy. For high cost constraints, the probability of error for the equal measurement variances strategy converges to $r(\Gamma) = ab+ac+bc-2abc$ where $a = Q(0.5/\sqrt{\sigma^2_{x_1}})$, $b = Q(0.5/\sqrt{\sigma^2_{x_2}})$, and $c = Q(0.5/\sqrt{\sigma^2_{x_3}})$. For the values specified above, $r(\Gamma)$ converges to 0.1446. Overall, the optimal cost allocation strategy yields the lowest probabilities of error for decentralized detection, as well.

Table 5.2: Measurement variances and corresponding probability of error values for all strategies and various total cost constraints for Bayesian decentralized detection.

| $C_T$ | Strategy | $\sigma^2_{m_1}$ | $\sigma^2_{m_2}$ | $\sigma^2_{m_3}$ | $P_e$ |
|---|---|---|---|---|---|
| 2.5 | Optimal | 0.0600 | 0.3400 | 0.8500 | 0.1867 |
| | EMV | 0.2787 | 0.2787 | 0.2787 | 0.2074 |
| | ACBO | 0.0065 | $\infty$ | $\infty$ | 0.3178 |
| 5 | Optimal | 0.0160 | 0.0800 | 0.1770 | 0.1562 |
| | EMV | 0.0628 | 0.0628 | 0.0628 | 0.1631 |
| | ACBO | $1.96 \times 10^{-4}$ | $\infty$ | $\infty$ | 0.3159 |
| 10 | Optimal | 0.0015 | 0.0070 | 0.0158 | 0.1457 |
| | EMV | 0.0055 | 0.0055 | 0.0055 | 0.1464 |
| | ACBO | $1.91 \times 10^{-7}$ | $\infty$ | $\infty$ | 0.3159 |

In the Neyman-Pearson framework, the probability of detection achieved by the proposed algorithm is compared with the two strategies explained above (that is, assignment of equal measurement variances to the measurement devices and assignment of all the cost to the sensor with the best observation). In centralized detection, the distribution of observation $\mathbf{x}$ is specified by $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ and $\mathcal{N}(\mathbf{2}, \mathbf{\Sigma})$ for hypotheses $H_0$ and $H_1$, respectively. The covariance matrix is the same as in the previous scenario; i.e., $\mathbf{\Sigma} = \text{diag}\{0.2, 0.7, 1.2\}$. The probability of false alarm at the fusion center is required to be less than or equal to $\alpha_{fc} = 10^{-6}$. The results obtained for centralized detection in the Neyman-Pearson framework are presented in Figure 5.3. In addition, Table 5.3 shows the measurement variances and corresponding probability of detection values for various total cost constraints.

Similar to the results for the Bayesian criterion, assigning all the cost to the best observation yields similar performance to the optimal algorithm for low cost values. When the cost budget increases, $P_D$ converges to $Q(Q^{-1}(\alpha_{fc}) - \mu_1/\sigma_{x_1})$; hence, for the considered parameters, the probability of detection converges to $Q(Q^{-1}(10^{-6}) - 2/\sqrt{0.2}) = 0.3892$. On the other hand, the equal measurement variances strategy converges to the same value

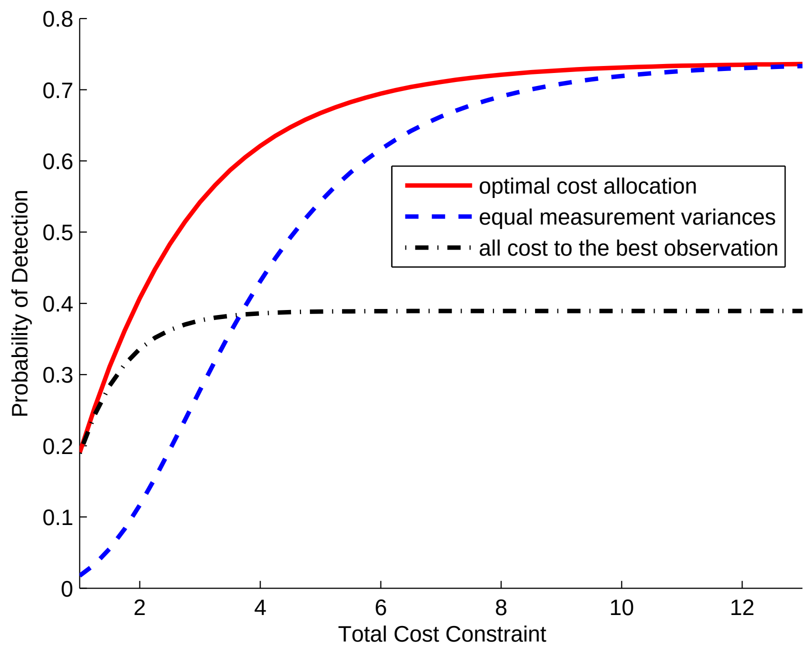


Figure 5.3: Probability of detection vs. total cost constraint for NP centralized detection.

Table 5.3: Measurement variances and corresponding probability of detection values for all strategies and various total cost constraints for Neyman-Pearson centralized detection.

| $C_T$ | Strategy | $\sigma^2_{m_1}$ | $\sigma^2_{m_2}$ | $\sigma^2_{m_3}$ | $P_D$ |
|---|---|---|---|---|---|
| 2.5 | Optimal | 0.0258 | 0.4659 | 2.6096 | 0.4833 |
| | EMV | 0.2787 | 0.2787 | 0.2787 | 0.1945 |
| | ACBO | 0.0065 | $\infty$ | $\infty$ | 0.3625 |
| 5 | Optimal | 0.0075 | 0.1008 | 0.3302 | 0.6672 |
| | EMV | 0.0628 | 0.0628 | 0.0628 | 0.5431 |
| | ACBO | $1.96 \times 10^{-4}$ | $\infty$ | $\infty$ | 0.3884 |
| 10 | Optimal | $7.16 \times 10^{-4}$ | 0.0089 | 0.0262 | 0.7311 |
| | EMV | 0.0055 | 0.0055 | 0.0055 | 0.7192 |
| | ACBO | $1.91 \times 10^{-7}$ | $\infty$ | $\infty$ | 0.3892 |

of $Q(Q^{-1}(\alpha_{fc}) - \sqrt{\mu_1^2/\sigma_{x_1}^2 + \mu_2^2/\sigma_{x_2}^2 + \mu_3^2/\sigma_{x_3}^2})$ as the optimal algorithm for high cost values. In particular, the optimal algorithm converges to $Q(Q^{-1}(10^{-6}) - \sqrt{4/0.2 + 4/0.7 + 4/1.2}) = 0.7377$ as the total cost constraint increases. As a result, the optimal cost allocation strategy produces the maximum probability of detection in all cases and outperforms the other approaches.

In the next example, the optimality of the proposed algorithm is illustrated for decentralized detection in the Neyman-Pearson framework. The distribution of observation $\mathbf{x}$ is denoted as $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ and $\mathcal{N}(\mathbf{4}, \mathbf{\Sigma})$ for hypotheses $H_0$ and $H_1$, respectively, where $\mathbf{\Sigma}$ is the same as that in the centralized detection case. All the local sensors have the same probability of false alarm given by $\alpha_1 = \alpha_2 = \alpha_3 = 10^{-4}$. It is required to achieve a false alarm probability not exceeding $10^{-7}$ at the fusion center. In order to satisfy this false alarm probability at the fusion center, the 2 out 3 fusion rule must be used. This fusion rule produces a false alarm probability of $10^{-7.5}$, which satisfies the requirement. The results related to this scenario are shown in Figure 5.4. Moreover, Table 5.4 shows the measurement variances and corresponding probability of detection values for various total cost constraints.

Table 5.4: Measurement variances and corresponding probability of detection values for all strategies and various total cost constraints for Neyman-Pearson decentralized detection.

| $C_T$ | Strategy | $\sigma_{m_1}^2$ | $\sigma_{m_2}^2$ | $\sigma_{m_3}^2$ | $P_D$ |
|---|---|---|---|---|---|
| 2.5 | Optimal | 0.2110 | 0.0610 | 3.7910 | 0.8074 |
| | EMV | 0.2787 | 0.2787 | 0.2787 | 0.7409 |
| | ACBO | 0.0065 | $\infty$ | $\infty$ | $2.00 \times 10^{-4}$ |
| 5 | Optimal | 0.1760 | 0.0193 | 0.1010 | 0.9055 |
| | EMV | 0.0628 | 0.0628 | 0.0628 | 0.8904 |
| | ACBO | $1.96 \times 10^{-4}$ | $\infty$ | $\infty$ | $2.00 \times 10^{-4}$ |
| 10 | Optimal | 0.0828 | 0.0010 | 0.0028 | 0.9234 |
| | EMV | 0.0055 | 0.0055 | 0.0055 | 0.9213 |
| | ACBO | $1.91 \times 10^{-7}$ | $\infty$ | $\infty$ | $2.00 \times 10^{-4}$ |

It is observed that assigning all the cost to the best observation has detection

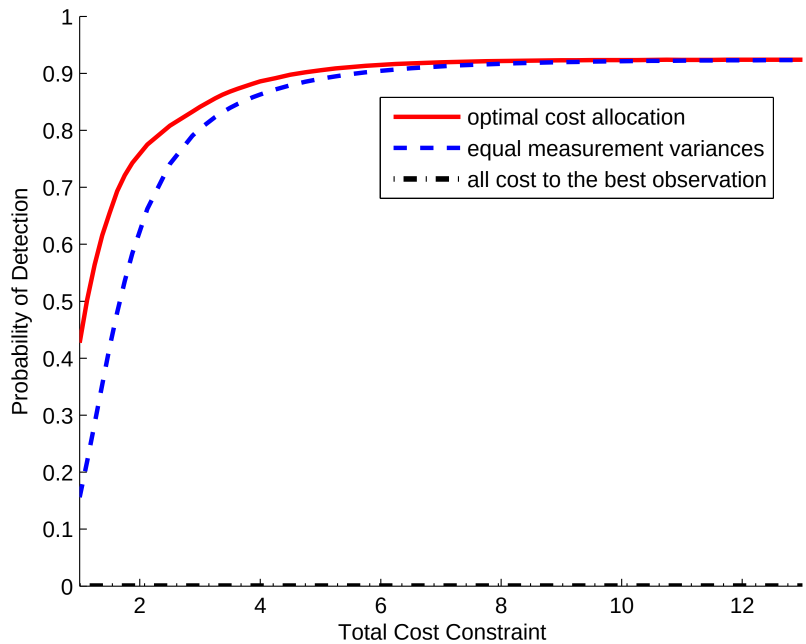


Figure 5.4: Probability of detection vs. total cost constraint for NP decentralized detection.

probability close to zero since the sensors having zero cost have infinite noise powers and the probability of detection for these sensors is $10^{-4}$. When the total cost constraint is high, the equal measurement variances strategy and the proposed algorithm converge to the same probability of detection, specified by $P_D = P_{d_1}P_{d_2} + P_{d_1}P_{d_3} + P_{d_2}P_{d_3} - 2P_{d_1}P_{d_2}P_{d_3}$, where $P_{d_1} = Q(Q^{-1}(\alpha_1) - \mu_1/\sigma_{x_1})$, $P_{d_2} = Q(Q^{-1}(\alpha_2) - \mu_2/\sigma_{x_2})$ and $P_{d_3} = Q(Q^{-1}(\alpha_3) - \mu_3/\sigma_{x_3})$. For the values given above, $P_D$ converges to 0.9240. Overall, the optimal cost allocation algorithm yields the highest probabilities of detection in this scenario.

Next, the $J$-divergence criterion is considered and the proposed algorithm is compared with the other two strategies. In centralized detection, the distribution of observation vector $\mathbf{x}$ is represented by $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ and $\mathcal{N}(\mathbf{2}, \mathbf{\Sigma})$ for hypotheses $H_0$ and $H_1$, respectively, where the covariance matrix is given by $\mathbf{\Sigma} = \text{diag}\{0.2, 0.7, 1.2\}$. The results for this case are shown in Figure 5.5. In addition, Table 5.5 shows the measurement variances and corresponding $J$-divergence values for various total cost constraints.

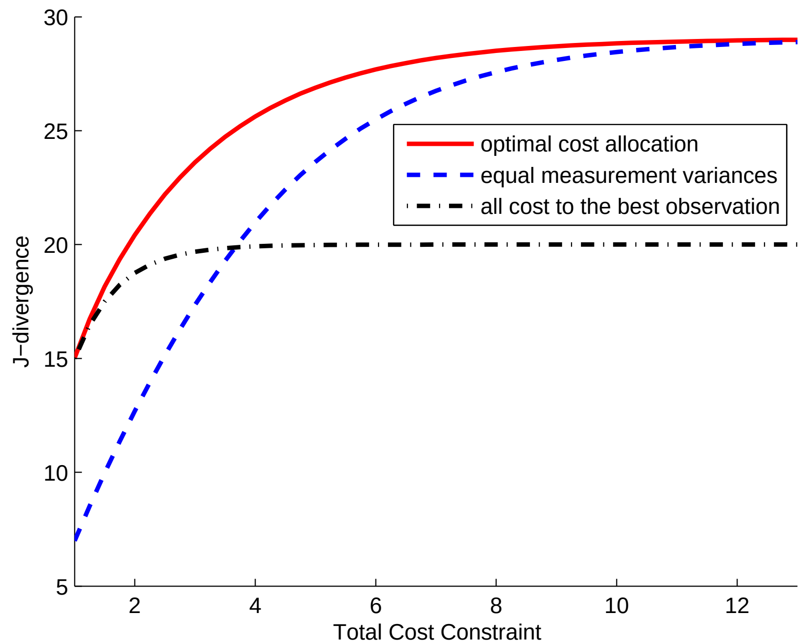


Figure 5.5: $J$-divergence versus the total cost constraint for centralized detection.

Table 5.5: Measurement variances and corresponding $J$-divergence values for all strategies and various total cost constraints for $J$-divergence based centralized detection.

| $C_T$ | Strategy | $\sigma^2_{m_1}$ | $\sigma^2_{m_2}$ | $\sigma^2_{m_3}$ | $J$-divergence |
|---|---|---|---|---|---|
| 2.5 | Optimal | 0.0258 | 0.4659 | 2.6096 | 22.1976 |
| | EMV | 0.2787 | 0.2787 | 0.2787 | 15.1476 |
| | ACBO | 0.0065 | $\infty$ | $\infty$ | 19.3750 |
| 5 | Optimal | 0.0075 | 0.1008 | 0.3302 | 26.8900 |
| | EMV | 0.0628 | 0.0628 | 0.0628 | 23.6348 |
| | ACBO | $1.96 \times 10^{-4}$ | $\infty$ | $\infty$ | 19.9805 |
| 10 | Optimal | $7.16 \times 10^{-4}$ | 0.0089 | 0.0262 | 28.8336 |
| | EMV | 0.0055 | 0.0055 | 0.0055 | 28.4515 |
| | ACBO | $1.91 \times 10^{-7}$ | $\infty$ | $\infty$ | 20.0000 |

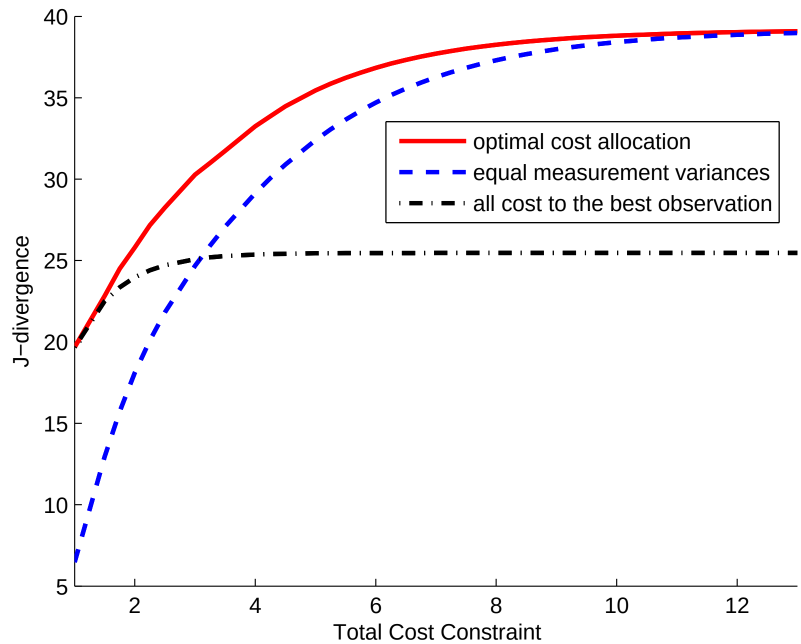


Figure 5.6: $J$-divergence vs. total cost constraint for decentralized detection.

It is observed that assigning all the cost to the best observation and the proposed optimal strategy achieve similar performance for low cost values. When the total cost increases, the $J$-divergence converges to $\mu_1^2/\sigma_{x_1}^2 = 20$ for the strategy that assigns all the cost to the best observation, which is significantly lower than that achieved by the optimal strategy. On the other hand, the performance of the equal measurement variances strategy converges to that of the optimal algorithm for high cost values; in particular, the $J$-divergence converges to $\sum_{i=1}^{3} \mu_i^2/\sigma_{x_i}^2 = 29.0476$. Overall, the proposed algorithm yields the maximum $J$-divergence for all cost values resulting in the optimum performance.

In the final example, a decentralized detection problem is considered according to the $J$-divergence criterion. The distribution of observation $\mathbf{x}$ is denoted by $\mathcal{N}(\mathbf{0}, \mathbf{\Sigma})$ and $\mathcal{N}(\mathbf{4}, \mathbf{\Sigma})$ for hypotheses $H_0$ and $H_1$, respectively, where $\mathbf{\Sigma}$ is the same as in the centralized detection case. The probability of false alarm for the local sensors is given by $\alpha_1 = \alpha_2 = \alpha_3 = 10^{-4}$. The results related to this scenario are presented in Figure 5.6. Moreover, Table 5.6 shows the measurement variances and corresponding $J$-divergence values for various total cost constraints.

Table 5.6: Measurement variances and corresponding $J$-divergence values for all strategies and various total cost constraints for $J$-divergence based decentralized detection.

| $C_T$ | Strategy | $\sigma^2_{m_1}$ | $\sigma^2_{m_2}$ | $\sigma^2_{m_3}$ | $J$-divergence |
|---|---|---|---|---|---|
| 2.5 | Optimal | 0.0260 | 0.3510 | 5.2510 | 28.2564 |
| | EMV | 0.2787 | 0.2787 | 0.2787 | 21.8077 |
| | ACBO | 0.0065 | $\infty$ | $\infty$ | 24.7139 |
| 5 | Optimal | 0.0111 | 0.0881 | 0.2391 | 35.4487 |
| | EMV | 0.0628 | 0.0628 | 0.0628 | 32.3978 |
| | ACBO | $1.96 \times 10^{-4}$ | $\infty$ | $\infty$ | 25.4419 |
| 10 | Optimal | 0.0010 | 0.0090 | 0.0184 | 38.8120 |
| | EMV | 0.0055 | 0.0055 | 0.0055 | 38.4187 |
| | ACBO | $1.91 \times 10^{-7}$ | $\infty$ | $\infty$ | 25.4655 |

It is noted that assigning all the cost to the best observation achieves improved performance in this case compared to the decentralized detection examples in the Bayesian and Neyman-Pearson frameworks Figure 5.2 and Figure 5.4, respectively. The main reason for this observation is that no counting rule is applied at the fusion center in this case. Similar to the centralized detection case, the proposed algorithm and the algorithm that assigns all the cost to the best observation yield similar results for low cost values. As the cost increases, the equal measurement variance strategy and the proposed algorithm converges to the same value of 39.177 while assigning all the cost to the best observation leads to a convergence to 25.466 for high cost values. From Figure 5.6, it is observed that the proposed algorithm yields the maximum $J$-divergence in all the cases, and achieves the optimum detection performance.

# Chapter 6

# Conclusions

In this thesis, centralized and decentralized detection systems have been investigated in the presence of cost constrained measurements. Novel cost allocation strategies that achieve the optimum detection performance according to the Bayesian, Neyman-Pearson and $J$-divergence criteria have been proposed for both centralized and decentralized detection systems. A closed form expression has been presented for the measurement noise variances by considering centralized detection in a Gaussian scenario. This expression indicates that if the observation variance is low, using a measurement device with a high cost is more beneficial. Also, the convexity property of the objective and constraint functions has been studied under certain conditions. For decentralized detection, a general probability of error expression for the Bayesian criterion and the probabilities of detection and false alarm expressions for the Neyman-Pearson framework have been presented according to the counting rules at the fusion center. In addition, the $J$-divergence has been employed for the distance based criterion. The Gaussian scenario has been investigated as a special case and the optimization problems have been proposed for all the criteria. The optimality of the proposed cost allocation strategies has been shown via numerical examples. Overall, the proposed cost allocation strategies minimize the Bayes risk for the Bayesian criterion, maximize the probability of detection (under a constraint on the probability of false alarm) for the Neyman-Pearson criterion, and maximize the $J$-divergence for the distance based criterion under given cost constraints, and they achieve the optimum performance.

Future work includes investigation of the proposed cost allocation strategies for various wireless sensor network applications. Moreover, instead of using a constant total cost budget in the optimization problems, randomized total cost values can be employed and it can be provided that the average of the randomized total cost values do not increase the specified average cost budget. Finally, in addition to the $J$-divergence criterion, other distance related bounds such as Bhattacharrya bound and Chernoff bound can also be investigated for the cost allocation problem as future work.

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