Joint Source-Channel Coding and Guessing[1]

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Abstract — **We consider the joint source-channel guessing problem, define measures of optimum performance, and give single-letter characterizations. As an application, sequential decoding is considered.**

I. Introduction and Main Result

Let P be a discrete memoryless source over a finite alphabet $\mathcal{U}$, $\hat{\mathcal{U}}$ a reconstruction alphabet, and d a single-letter distortion measure defined on $\mathcal{U} \times \hat{\mathcal{U}}$. A D-admissible guessing strategy for $\mathcal{U}^N$ is a possibly infinite ordered list $\mathcal{G}_N = \{\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \ldots\} \subset \hat{\mathcal{U}}^N$ such that for each $\mathbf{u} \in \mathcal{U}^N$ there exists $\hat{\mathbf{u}}_i \in \mathcal{G}_N$ with $d(\mathbf{u}, \hat{\mathbf{u}}_i) \leq ND$. The guessing function $G_N(\cdot)$ induced by a guessing strategy $\mathcal{G}_N$ is the function that maps each $\mathbf{u} \in \mathcal{U}^N$ into the index i of the first $\hat{\mathbf{u}}_i \in \mathcal{G}_N$ such that $d(\mathbf{u}, \hat{\mathbf{u}}_i) \leq ND$. Thus, $G_N(\mathbf{u})$ is the number of guesses required to find a reconstruction of $\mathbf{u}$ within distortion level ND by sequentially probing from the list $\mathcal{G}_N$. The moments $\mathbf{E}[G_N(\mathbf{U})^\rho]$, $\rho \geq 0$, serve as measures of complexity for the guessing effort. Arikan and Merhav [1] defined the *guessing exponent* as

$$E(D,\rho) = \lim_{N\to\infty} \frac{1}{N} \inf_{\mathcal{G}_N} \ln \mathbf{E}[G_N(\mathbf{U})^\rho] \qquad (1)$$

for $\rho \geq 0$, and showed that it has a single-letter form given by

$$E(D,\rho) = \max_Q [\rho R(D,Q) - D(Q\|P)] \qquad (2)$$

where $R(D,Q)$ is the rate-distortion function, $D(Q\|P)$ is the relative entropy, and the maximum is over all probability distributions on $\mathcal{U}$.

The aim of this talk is to consider the guessing problem in a joint source-channel setting, in which one is allowed to send information about $\mathbf{U}$ to the guesser over some discrete memoryless channel W, using the channel λ times for each source symbol. We assume W has a finite input alphabet $\mathcal{X}$ and a finite output alphabet $\mathcal{Y}$. The source output $\mathbf{U} \in \mathcal{U}^N$ is encoded into a channel input block $\mathbf{X} \in \mathcal{X}^K$, $K = \lceil \lambda N \rceil$, using an encoder $e_N : \mathcal{U}^N \to \mathcal{X}^K$, $\mathbf{X}$ is transmitted over W, and the guesser observes the channel output $\mathbf{Y} \in \mathcal{Y}^K$. A D-admissible guessing scheme, in this situation, is a collection $\{\mathcal{G}_N(\mathbf{y}),\ \mathbf{y} \in \mathcal{Y}^K\}$, such that for each $\mathbf{y} \in \mathcal{Y}^K$, $\mathcal{G}_N(\mathbf{y}) \subset \hat{\mathcal{U}}^N$ is a D-admissible guessing scheme for $\mathcal{U}^N$ in the previously defined sense. We define the *joint source-channel guessing exponent* as

$$E_{sc}(D,\rho) = \lim_{N\to\infty} \frac{1}{N} \inf_{e_N,\ \mathcal{G}_N} \ln \mathbf{E}[G_N(\mathbf{U}|\mathbf{Y})^\rho]. \qquad (3)$$

Here, $G_N(\mathbf{U}|\mathbf{Y})$ denotes the guessing function induced by $\mathcal{G}_N(\mathbf{Y})$. Our main result is the following.

[1]The work of N. Merhav was partially supported by the Wolfson Research Awards administered by the Israel Academy of Sciences and Humanities.

Theorem 1 *The joint source-channel guessing exponent is given by*

$$E_{sc}(D,\rho) = \max\{0,\ E(D,\rho) - \lambda E_0(\rho)\} \qquad (4)$$

where $E_0(\rho)$ is Gallager's function [2, p.138] for W.

II. List-Error Exponent

Consider list-decoding in the above situation so that given the channel output $\mathbf{Y}$ one is allowed to generate ℓ estimates of the source output $\mathbf{U}$ and suppose an error occurs only if none of the estimates is within distortion level ND of $\mathbf{U}$. Let $P_{e,N}$ denote the minimum possible value of the list decoding error probability over all encoders e_N and all list-ℓ decoders. The asymptotic behavior of $P_{e,N}$ for $\ell = 1$ has been considered by Csiszár, but it remains only partially known. Here, we consider exponential list sizes, $\ell = e^{NL}$, and define the *joint source-channel list-error exponent* as

$$F_{sc}(L,D) = \lim_{N\to\infty} -\frac{1}{N} \log P_{e,N}. \qquad (5)$$

Our second result is the following.

Theorem 2 *The joint source-channel list-error exponent is given by*

$$F_{sc}(L,D) = \inf_{R\geq L} [F(R,D) + \lambda E_{sp}[(R-L)/\lambda]] \qquad (6)$$

where $F(R,D)$ is Marton's source coding exponent [4] and $E_{sp}(\cdot)$ is the sphere-packing exponent [2, p. 157].

III. Application to Sequential Decoding

Koshelev [5] considered using sequential decoding in joint source-channel coding systems for the lossless case $D = 0$. Here, we prove the following converse which complements his result, and applies to the lossy case $D > 0$ as well.

Theorem 3 *For any $\rho \geq 0$, the ρth moment of computation in sequential decoding in a joint source-channel coding system must grow exponentially with the number of symbols correctly decoded if $E(D,\rho) > \lambda E_0(\rho)$.*

References

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