JOINT LOT SIZING AND TOOL MANAGEMENT IN A SINGLE CNC ENVIRONMENT

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCES

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Siraceddin ÖNEN

October, 1996

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. M. Selim Aktürk (Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Sinan Kayalıgil

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. İhsan Sabuncuoğlu

Approved for the Institute of Engineering and Sciences:

Prof. Mehmet Baray
Director of Institute of Engineering and Science

ABSTRACT

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Siraceddin ÖNEN
M.S. in Industrial Engineering
Supervisor: Assist. Prof. M. Selim Aktürk
October, 1996

In most of the studies on tool management, lot sizes are taken as predetermined input while deciding on tool allocations and machining parameters. In this study, we considered the integration of lot sizing and tool management problems for single and multi period cases. For the single period case, we proposed a new algorithm. By this algorithm we not only improved the overall solution by exploiting interactions, but also prevented any infeasibility that might occur for the tool management problem due to the decisions made at the lot sizing level. The computational experiments showed that in a set of randomly generated problems 22.5% of solutions found by a traditional approach were infeasible and the proposed joint approach improved the overall solution by 6.8%. For the multi period case, we proposed five new algorithms. Among these algorithms, the most promising one was the Look Ahead-LUC algorithm, which improved the overall solution on the average by 6.5% compared to the best known algorithm, Wagner-Whitin, used in traditional approach, over a set of randomly generated problems.

Key words: Flexible Manufacturing Systems, Lot Sizing, Tool Management

ÖZET

BİLGİSAYAR KONTROLLÜ İMALAT SİSTEMLERİNDE KAFİLE BÜYÜKLÜĞÜ VE KESİCİ UÇ İŞLETİMİ PROBLEMLERİNİN ENİYİLEMESİ

Siraceddin ÖNEN
Endüstri Mühendisliği Bölümü Yüksek Lisans
Tez Yöneticisi: Yard. Doç. Dr. M. Selim Aktürk
Ekim, 1996

Kesici uç işletimi ile ilgili yapılmış pek çok çalışmada, kesici uç dağılımı ve üretim şartları belirlenirken kafile büyüklükleri önceden belirlenmiş sabit değerler olarak alınmışlardır. Bu çalışmada tek veya çok dönem üretim durumlarında kafile büyüklüğü ve kesici uç işletimi problemlerinin birada eniyilenmesi amaçlanmıştır. Tek dönem modelinin çözümü için önerdiğimiz yeni metodu farklı şartlar gözönünde bulundurularak üretilen bir küme problem üzerinde test ettik. Bu metot ile klasik yaklaşımda %22.5 olan olursuzluğu önlemekle kalmayıp, ortalama %6.8 lik bir maliyet indirimi gerçekleştirdik. Çok dönem modelinin çözümü için ise beş farklı çözüm metodu önerdik. Önerdiğimiz çözüm metotlarını farklı şartlar gözönünde bulundurularak üretilmiş bir küme problem üzerinde test ettik. Önerdiğimiz çözüm metotları arasında özellikle ikincisi çözüm süresi ve ortalama maliyet indirimi gibi performans ölçütleri dikkate alındığında en iyi metot olarak göze çarpmaktadır. Bu metot ile klasik yaklaşımda en iyi sonucu veren Wagner-Whitin metoduna kıyasla %6.5'lik bir maliyet indirimi gerçekleştirdik.

Anahtar sözcükler: Esnek Üretim Sistemleri, Kafile Büyüklüğü Belirlenmesi, Kesici Uç İşletimi

To my small nieces and nephews

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Chapter 1

Introduction

Many companies have realized that in order to compete in today's world market, they must rely on innovative developments in manufacturing technology. To increase productivity, companies are applying computer controlled machine tools, automated materials handling and storage systems. Due to the progress in manufacturing technology and organization, the concept of flexible manufacturing systems (FMSs) has emerged. FMSs can be defined as computer controlled production systems capable of processing a variety of part types. The main components of such a system are,

- Computer numerically controlled (CNC) machines including the tools to operate these machines,
- An automated material handling system (MHS) to move the workpieces through the system, and
- On line computer control to manage the entire FMS, including CNC machines and the MHS.

These systems may differ enormously in the extent of automation and the diversity of the parts. An FMS is designed to achieve the efficiency of both automated high volume mass production and flexibility of low volume job shop production. Due to the complex nature of FMSs, the related production management problems are also more complex than other manufacturing systems. Therefore, the efficient operation of an FMS is a very difficult task, and in many implementations the available capacity is underutilized. In view of the high initial cost of the FMSs, it is important to operate these systems efficiently as much as possible in order to get expected benefits of flexibility and economy.

In FMSs tool related issues are the key factors for the overall system performance due to their impact on both cost figures and the operational considerations. The cutting tool utilization is important for the entire system performance especially in metal cutting industries due to the high metal removal rate in metal cutting processes, and the consequent increased tool consumption rates and tool replacement frequencies.

In manufacturing industry, due to the complexity of the planning problem, materials requirement planning (MRP) based systems are extensively used for managing production. In such environments, the lot sizing and tool management problems are solved independently. The lot sizing problem is considered as a planning problem and is assumed to be solved at a higher level in an organization than is the tool management problem, whereas the tool management problem is considered a low level detailed decision problem that should be solved after the lot sizing problem. Consequently, these two problems are solved independently in a two-level approach ignoring the interactions such as production rates, tooling costs, tool and machine hour capacity constraints, between these two levels. However, in a two-level approach determining lot sizes prior to the tool management decisions may result in either suboptimal or infeasible solutions.

In this study, we will consider the integration of lot sizing and tool management problems on a single CNC turning machine and discuss the joint problem for both single and multi period cases. After giving the mathematical formulation of the problem, we will present some new algorithms to solve this problem for both of the cases. All of the proposed algorithms will be tested on a set of randomly generated problems, and step-by-step execution of the algorithms will be given on numerical examples.

The remainder of the thesis can be outlined as follows. In the next chapter, we will give a short review of the literature on the lot sizing and tool management problems including existing few studies on the integration of these two problems. In Chapter 3, we will give the definition of the problem and the underlying assumptions in addition to the notation used throughout the thesis. In this chapter, we will concentrate on independent lot sizing and tool management problems. In Chapters 4 and 5, the joint problem will be studied for single and multi period cases, respectively. In these chapters, we will give the mathematical models as well as the algorithms and the computational results of the experimental designs. Finally, in Chapter 6, some concluding remarks and suggestions for future research are provided.

Chapter 2

Literature Review

Flexible manufacturing systems (FMSs) typically contain a set of numerically controlled workstations and a material handling system coordinated by a central controller for the purpose of simultaneous production of a variety of part types. The FMS represents a significant investment in training, hardware and software. This investment is justified by the ability of the system to produce a variety of high quality parts with short lead times while requiring less floor space than traditional systems [33]. An FMS has the ability to produce a family of parts in a flexible way. To realize these potential benefits, careful attention must be paid not only to design but also to planning of the system once it has been installed. Since FMSs are usually a part of a multistage manufacturing system, inputs and outputs are dictated by the master production plan. This plan specifies availability dates for raw materials and components, and due dates for finished products. The FMS production planning problem consists of organizing production such as to satisfy the master production plan as well as to obtain an efficient use of system resources (machines, pallets, fixtures, tools etc.). In an FMS of reasonable size, this planning process is quite complex. Again, it is helpful to decompose the problem into a set of smaller and manageable subproblems [2].

There are critical tool management issues that affect the productivity of many automated and flexible manufacturing systems. Manufacturers and machine tool suppliers recognize that a lack of attention to such tool management issues is a primary reason for the poor performance of many facilities. Gray et al. [17] classify tool management issues into tool-level, machine-level and system-level issues. The "tools" they are concerned with are the cutting or shaping tools residing in an automated computer numerical control (CNC) "machine" used to remove metal from castings. A "system" is an integrated production facility with several automated machines and, perhaps, automated handling of parts and tools. The classification of tool, machine and system level issues allows one to portray how individual tool-related models may fit into machine level models and how technological constraints directly affect key operational decisions at all levels.

In the literature it is stated that approximately 50% of U.S. annual expenditures on manufacturing is in the metal-working industry, and two-thirds of metal-working is metal cutting. In addition 75% of the dollar volume of metal worked products is manufactured in batches of less than 50 parts [34]. Besides being a critical issue in factory integration, tool management has direct cost implications. Veeramani et al. [44] emphasized that the lack of tooling considerations has resulted in the poor performance of automated manufacturing systems. Kouvelis [25] identified cutting tool utilization as an important parameter for the overall system performance. In this study, the cost of tooling has been reported to be 25-30% of the fixed and variable cost of production. Manufacturing management publications have recently paid considerable attention to the benefits of improving the integration of tool management within total system design, planning, scheduling and control [17].

Most of the existing studies in the literature on tool management ignore the lot sizing decision at system level and take it as a predetermined input while deciding on tool allocations and machining parameters. Gray et al. [17] pointed out that, efforts in tool management focus on single level decisions, although a decision made at a higher level without considering its impact on the lower levels can lead to either infeasible or inferior results. Lot sizing is such a decision which is taken at system level ignoring its impact on lower level tool allocation and machining conditions decisions. In most of the existing studies on tool management, lot sizes are taken as predetermined input while deciding on tool allocations and machining parameters. In an automated manufacturing environment operational problems such as machining conditions, tool availability and tool life should be taken into account for the reliable modeling of CNCs, or the absence of such crucial constraints may lead to infeasible results. It has been shown in several studies that significant cost savings can be realized by controlling production rates (Cheng et al. [7], Jones and Inman [21], Silver [31], Strusevich [35]). Consequently, total production cost can be decreased and any infeasibility due to machine hour capacity limitation can be avoided.

For solving the tool allocation problem at the system level, most of the published studies use 0-1 binary variables, i.e. a particular tool j is assigned to operation i, to represent tool requirements. Stecke [34] formulates the FMS loading problem as a nonlinear mixed integer programming (MIP) problem and solves it through linearization techniques. Sodhi et al. [33] propose a four level hierarchy for production control of FMSs, including part type selection and loading, and present various models at each level. Sarin and Chen [30] give an MIP formulation under the assumption that the total machining costs depend upon the tool-machine combination. The tool life is considered as a constraint in the formulation. Unfortunately, all of these studies assume constant lot sizes, production rates as well as processing times. Furthermore, these studies determine the tool requirements for each operation independently, and fail to relate the contention among the operations for a limited number of tools.

At the machine level, there exist several studies paying attention to tooling issues like tool selection, tool magazine loading and minimization of tool switches due to a change in a part mix, at both the long term planning and operational level (Bard and Feo [4], Crama et al. [8], Kouvelis [25], Tang and Denardo [37]). Unfortunately, these studies also assume constant lot sizes, processing times and tool lives, even though the tool replacement frequency is directly related with the machining conditions selections. Further, in the multiple operation case, non-machining time components, such as the tool replacements due to tool wear, can have a significant impact on the total cost of production and the throughput of parts as shown by Tetzlaff [38]. Gray et al. [17] reported that tools are changed ten times more often due to tool wear than to part mix because of the relatively short tool lives of many turning tools.

The machining conditions optimization for a single operation is a well known problem, where the decision variables are the cutting speed and feed rate. Several models and solution methodologies have been developed in the literature (Ermer [11], Gopalakrishnan and Al-Khayyal [14], Tan and Creese [36]). However, these models consider only the contribution of machining time and tooling cost to the total cost of operation, usually ignoring the contribution of non-machining time components to the operating cost, which could be very significant for the multiple operation case. Further, these studies exclude the tooling issues such as tool availability and tool life capacity limitations.

In a recent study, Akturk and Avci [1] proposed a new solution procedure to make tool allocation and machining conditions selection decisions simultaneously by considering the related tooling considerations of tool wear, tool availability, and tool replacing and loading times, since they affect both the machining and non-machining time components, hence the total cost of production. In this study they extended single machining operation problem (SMOP) formulation by adding a new tool life constraint, which enabled them to include tooling issues like tool wear and tool availability. Furthermore, they proposed a new cost measure to exploit the interaction between the number of tools required with the machining, tool replacing and loading times, and tool waste cost in conjunction with the optimum machining conditions for alternative operation-tool pairs. Consequently, they prevented any infeasibility that might occur for the tool allocation problem at the system level due to tool contention among tool life restrictions through a feedback mechanism.

The importance of effective lot sizing is well recognized by both practitioners and researchers, and the lot sizing problem, in a variety of forms, has received much attention in the literature. Lot sizing problems play an important role in modern production planning systems, such as materials requirement planning (MRP), hierarchical production planning (HPP) and just in time (JIT) manufacturing. Therefore research to develop and improve solution procedures for lot sizing problems is of eminent importance. Although many interesting real life problems in lot sizing remain unsolved, a still growing number of them can be solved successfully as stated by Salomon [29] in his Ph.D. thesis in which he studied many deterministic lot sizing models.

For relatively simple manufacturing environments, under the assumption that there is a single product and an infinite production capacity, efficient solution procedures have been developed in the literature. The famous economic order quantity (EOQ) model was developed by Harris [19] in 1913. This model assumes a simple single product, single machine situation with instantaneous replenishments in which demand is stable and inventory holding costs and machine setup costs are deterministic and constant over time. After introduction of the EOQ model, Wagner and Whitin [45] developed an extension of this model in 1958, in which time phased dynamic demand over a finite planning horizon was considered. Nonetheless, when the manufacturing process is more complicated, as will often be the case in practice, problem complexity may increase formidably. Some of complicating factors in the manufacturing process are,

- Dynamic demand
- Nonlinear cost structure
- Capacity restrictions
- Setup times

In the late fifties Manne [28], and in the sixties and seventies Lasdon and Terjung [26] and Zangwill [47] among others started working on lot sizing models for more complex manufacturing situations, in which multiple products, capacity restrictions and machine setup times play an important role. Besides the more theoretical work on lot sizing models, a large number of researchers have worked on solving lot sizing problems in practice.

The considerable number of publications in international specialist journals reporting on successful lot sizing applications in industry, e.g. Gunther [18], Van Nunen and Wessels [42], Van Wassenhove and De Bodt [43], demonstrates that research on lot sizing models is not only of theoretical interest, but also of large practical value.

In the uncapacitated dynamic lot sizing problem, the objective is to minimize the total of fixed costs and holding costs. In each period t of a T-period horizon, lot size (Q_t) values are to be determined for a given demand (d_t). This choice also determines the inventory quantity (I_t) at the end of period t. In the process of developing a dynamic programming procedure for finding the optimal solution to this problem, assuming nonnegative inventory position and consequently disallowing backlogging, Wagner and Whitin [45] showed that there exists an optimal solution such that $I_{t-1}.Q_t = 0$ for every t. This means that each order quantity covers demand for an integral number of periods, a characteristic sometimes called the Integrality Property. For such dynamic lot sizing problems some heuristic procedures, that retain the integrality property, such as least unit cost (LUC), Silver-Meal or least period cost (LPC), part period balancing and marginal cost difference heuristics are developed as summarized by Baker [3]. Gorham [16] states that the most commonly used heuristic was the LUC procedure. Under LUC, the various alternative lot sizing decisions for the first period, Q_1, are evaluated according to their cost per unit of demand, $C(t)/D_t$, where D_t denotes the cumulative demand of t periods and $C(t)$ denotes the corresponding total cost. The stopping rule calls for setting $Q_1 = D_t$ when this ratio first reaches a local minimum, that is, when $C(t)/D_t < C(t+1)/D_{t+1}$. Silver and Meal [32] proposed a stopping rule sometimes called the least period cost (LPC) procedure. Their approach is similar to LUC, but the basis for stopping rule is cost per period rather than cost per unit. Under LPC, the various alternatives for Q_1 are evaluated according to their cost per period, $C(t)/t$. The stopping rule calls for setting $Q_1 = D_t$ when this ratio first reaches a local minimum, that is, when $C(t)/t < C(t+1)/(t+1)$.

For capacitated problems the situation is quite different. Florian et al. [13]

and Bitran and Yanasse [5] have shown that the single item capacitated lot sizing problem (CLSP) is NP-hard even in many special cases. Multi item CLSP is also NP-hard except for a few special cases (e.g. when all setup costs are zero) as stated in [29]. However, some polynomial algorithms exist for some special capacitated problems. Florian and Klein [12] developed an $O(T^4)$ dynamic programming based shortest path algorithm for a model with concave costs and constant capacities. Love [27] provided an $O(T^3)$ algorithm that searched the extreme points of the solution space for a model with piecewise linear concave cost functions and bounds on inventories. In a recent study, Van Hoesel and Wagelmans [41] developed an algorithm that solves the constant capacity economic lot sizing problem with concave production costs and linear holding costs in $O(T^3)$ time. This algorithm is based on the standard dynamic programming approach which requires the computation of the minimal costs for all possible subplans of the production plan. Instead of computing these costs in a straightforward manner, they used structural properties of the optimal subplans to arrive at a more efficient implementation. Due to the complexity of more general CLSPs, most of the literature on this problem focus on heuristic solution procedures.

The heuristic approaches reported in the literature are classified by Kirca and Kokten [22] into two groups as mathematical programming and common sense approaches. The heuristics suggested by Thizy and Van Wassenhove [39], Trigeiro [40] and Cattrysse et al. [6] belong to the first class. The first two heuristics are Lagrangian relaxation based procedures. The procedures suggested in Cattrysse et al. [6] use the set partitioning formulation of Manne [28] and generate candidate plans for the items by several well known heuristics. The feasible schedules are obtained by rounding off the LP-relaxation of the set partitioning problem. Most common sense heuristics use of a period-by-period approach, in which CLSP is solved on a period by period basis. In each period, lot sizes for all items are determined on the basis of a cost savings criterion. In a given period, future demand of items are scheduled to be produced in that period until no further cost savings are possible or until all the capacity at that period is exhausted. Some of the heuristics reported in the literature which use this approach are the ones due to Dixon and Silver [9], Dogramaci et al. [10], and Gunther [18]. An alternative item-by-item approach for generating solutions to CLSP is proposed by Kirca and Kokten [22]. In this approach, solutions are generated iteratively. In each iteration, a set of items among the items not scheduled is selected and production schedules over the planning horizon for this set of items are determined.

In a machining environment especially due to the tool and machine hour capacity limitations unit production costs as well as unit resource consumption rates cannot be determined unless tool allocation and machining conditions optimization problems are solved for each possible lot size value, which could be a very difficult task due to the computational burden. Also existence of these constraints cause production cost function to be convex. However, in most of the models developed in the literature unit production costs and unit resource consumption rates are assumed to be fixed and apriori known, whereas these are decision variables in a machining environment. Furthermore, in most of these studies the production cost function is assumed to be either concave or linear due to the economies of scale. Consequently, existence of such problems makes the applicability of available lot sizing procedures almost impossible for machining environments.

In the literature there exist few studies on the integration of lot sizing and tool management problems. Wysk et al. [46] introduce lot size considerations in determining the optimal cutting speed in a single item, single machine, single period problem. Koulamas [23] presents a queueing model for determining analytically the optimal lot size in a machining economics problem under stochastic tool life considerations. Koulamas [24] proposes an iterative procedure for the simultaneous determination of the cutting speed and lot size values in machining systems for single and multiple part cases using the Lagrangian technique, while the feed rate is taken as a constant. In this study, parts are assumed to be composed of single operation. Consequently, parts are machined by a single cutting tool and tool allocation decisions are not considered. The author also has not considered machine horsepower, surface finish and tool availability constraints, although in many real life problems the machining parameters are constrained by these limitations. Furthermore, in all of these studies only single period case is considered and consequently demand is assumed to be fixed.

The objective of the research reported in this thesis is to show that there is a close relationship between the lot sizing and tool management decisions by proposing some algorithms for single and multi period cases of the joint problem. In the next chapter, we give the definition and the underlying assumptions of the problem as well as the details of the algorithm proposed by Akturk and Avci [1] to solve tool allocation and machining conditions optimization problems simultaneously. Our joint algorithms for the integration of lot sizing problem to the tool management problem for single and multi period cases are given in Chapters 4 and 5, respectively. Finally in Chapter 6, some concluding remarks and future research suggestions are provided.

Chapter 3

Problem Statement

Due to the complex nature of flexible manufacturing systems (FMSs), the related production management problems are also more complex than other manufacturing systems. Therefore the efficient operation of an FMS is a very difficult task, and in many implementations the available capacity has been underutilized. In view of the high initial cost of the FMSs, it is very important to operate these systems efficiently as much as possible in order to get expected benefits of flexibility and economy.

In FMSs, lot sizing is one of the important issues which needs further consideration. In the traditional approaches, lot sizing decisions are given at system level, independent of tool management decisions. However all of these decisions are closely related and lot sizing decisions affect tool allocations as well as machining conditions such as cutting speed and feed rate. Furthermore, the interaction between lot sizing decisions and production rates in addition to the tool and machine hour capacity constraints cannot be ignored. Integration of these decisions can result in reductions in the total production cost and prevent any infeasibility due to the tool and machine hour capacity constraints.

The organization of this chapter is as follows. In §3.1 the definition of the problem and underlying assumptions are given. In §3.2 the lot sizing decision is summarized along with a description of some well-known uncapacitated lot sizing heuristics. In §3.3 the tool management decisions including a mathematical model and an exact algorithm to solve tool allocation and machining conditions optimization problems simultaneously are discussed. Finally, some concluding remarks are provided in §3.4.

3.1 Problem Definition and Assumptions

In this study an automated machining environment consisting of a single CNC machine is considered. The limits of the problem are defined by stating operating policy and characteristics of the system. The following assumptions are made to define the scope of the study:

- Production may take place in a single period under static demand or in multiple periods under dynamic demand.
- There are multiple parts in demanded quantities and each part is composed of multiple operations.
- Each operation can be performed by a set of alternative tool types with limited quantities on hand.
- Backlogging is not allowed.
- Initial and final inventory levels are assumed to be zero without loss of generality.
- CNC machine can work for a limited number of hours.
- The tool switching is only allowed during the part changing and only a single tool can be changed at a time. This assumption implies that tool changing time occurred in a particular part loading/unloading event is additive. So tool changing times of different tools can be summed to find the total tool changing time occurred.

- For the machining operations, the cutting speed and the feed rate will be taken as the decision variables, and the depth of cut is assumed to be given as an input. This assumption particularly limits our attention to single pass machining. Therefore, if a material removal requires more than one pass, those should be prespecified as different volumes with their depths.

- Each machining operation of a part should be completed by a single tool type throughout the manufacturing of whole lot. Therefore, during the manufacturing of a lot, the same operation of a part is always machined by the same tool type.

- After completion of a lot, remaining tool lives can be used for manufacturing of another lot. Therefore the actual usage of tools are included in the tooling cost and tool availability related calculations.

Under these assumptions we want to solve lot sizing, tool allocation and machining conditions optimization problems simultaneously to determine the following decision variables:

- **Lot sizing decisions:** In what quantities each part will be produced.
- **Tool management decisions:**
  - **Tool allocation:** How tools will be allocated to parts in terms of quantities and allocation scheme.
  - **Machining conditions selection:** What the cutting speed and feed rate will be for each operation of each part.

Traditional approach for the determination of these decision variables consists of a two-level optimization procedure. In the first level, lot sizing decision is given using some lot sizing algorithms and in the second level, taking the lot sizes found in the first level as input, tool management decisions are given. In the next sections these decisions will be explained in more detail.

3.2 Lot Sizing Decision

The question of "How much to produce ?" is an important problem that almost every manufacturing business must deal for smooth and efficient running of its operation. The lot sizing decision answers this question by determining the optimum amount that should be produced to satisfy the demand. In a manufacturing environment using a traditional two level approach, lot sizes are determined by minimizing the total inventory related cost which is usually expressed as the sum of setup and inventory holding costs assuming that unit manufacturing cost is fixed and no shortage cost occurs. The setup cost represents the fixed charge incurred when an order to manufacture is placed. Thus, to satisfy the demand for a given time period, the more frequent manufacturing of smaller quantities will result in higher setup cost during the period than if the demand is satisfied by less frequent manufacturing of larger quantities. The holding cost, which represents the cost of carrying inventory in stock (e.g. interest on invested capital, storage handling, depreciation and maintenance) normally increases with the level of inventory.

Different lot sizing models have been proposed according to some demand characteristics. A deterministic demand may be static, in the sense that consumption rate remains constant with time, or dynamic, where demand is known with certainty but varies from one time period to the next. Without capacity considerations, a multiple part lot sizing problem can be solved for each item independently for both of the static and dynamic cases. For the static demand case a simple economic order quantity (EOQ) model and for dynamic multi period case Wagner-Whitin (WW), least unit cost (LUC) and least period cost (LPC) algorithms are the most widely used ones. A more detailed discussion on these approaches can be found in [3] and [20]. Now, we will briefly explain these models below.

3.2.1 Economic Order Quantity (EOQ) Model

This model is based on the assumption of continuous and steady demand rate. It performs well only where actual demand approximates this assumption. In a single period two level approach EOQ model can be used at system level to find the lot sizes. For the EOQ model, the optimal lot size (Q) is given by the following expression:

$$Q = \sqrt{\frac{2 \cdot S \cdot D}{h \cdot (1 - D/P)}}$$

where,

S : Setup cost per lot,

h : Inventory holding cost per unit per period,

D : Demand rate per period, and

P : Production rate per period ($P > D$).

For nonintegral lot size values, the optimal Q value can be rounded off. However it is difficult to achieve zero final inventory condition in a single period by producing equal lots of size Q. Because the optimum lot size (Q) found may not exactly divide the demand. In this case it is reasonable to apply the following procedure to get rid of this problem. Let $r = \lfloor D/Q \rfloor$, then $Q^* = D - Q \cdot r$ gives the remaining quantity to be produced to satisfy the demand. If $Q^* > Q/2$, then we produce r lots of size Q and a single lot of size Q^*. Otherwise we produce $r - 1$ lots of size Q and a single lot of size $Q + Q^*$.

3.2.2 Wagner-Whitin (WW) Model

This model can be used in a two level approach at system level to find the optimum lot sizes for multi period dynamic demand cases. Under the assumption of concave or linear production costs, the optimum production quantity in any period is one of these values: 0 or $\sum_{t=m}^{n} D_t$ for $n = m, m+1, \cdots, T$ where D_t and T denote the demand in period t and the planning horizon, respectively. In another words, if production takes place in any period, then the entering inventory for that period should be zero. Based on this fact forward and backward dynamic programming algorithms have been developed. The solution procedure and a more detailed discussion of this model can be found in [20].

3.2.3 Least Unit Cost (LUC) Model

This model proposes a heuristic procedure based on the assumptions of the WW model. This procedure can be used in a two level approach to find the lot sizes at system level for multi period dynamic demand cases. For the first period a lot size that covers requirements of t periods is $Q_{1t} = D_1 + D_2 + \cdots + D_t$. If the fixed setup cost for this lot is S and its holding cost is $H_t = h.(D_2 + 2.D_3 + \cdots + (t-1).D_t)$, then its cost per unit produced is $LUC_1 = \frac{S+H_t}{Q_{1t}}$. This algorithm begins with LUC_1 (corresponding to Q_{11}) and calculates LUC_2, LUC_3 etc. until $LUC_{k+1} > LUC_k$. Finding this condition, the procedure sets the size of the first lot to Q_{1k}. In other words, the procedure locates the first local minimum of cost per unit. It then fixes the first lot size at this quantity and proceeds to determine the second lot size. This is accomplished by starting over at time $k+1$ and searching for the least unit cost over periods $k+1$, $k+2$, etc.

3.2.4 Least Period Cost (LPC) Model

This model proposes a procedure similar to the one given in the previous section. However, in this method the stopping criterion for the local optimum search is cost per period rather than cost per unit. Using the notations of the previous section, the minimum cost per period is $LPC = \frac{S+H_t}{t}$. The LPC method uses this quantity to find a local minimum. That is, the stopping rule dictates that the lot should not be extended further if $LPC_{k+1} > LPC_k$.

Similar algorithms also exist in the literature that can be used to determine the lot sizes at system level. However these are the most widely used ones and experimentally it has been shown that, these procedures give the best results on the average among available similar procedures as stated by Baker [3].

3.3 Tool Management Decisions

Tool management problems are mainly composed of decisions related to tool allocations and machining conditions selection. In this section assuming a fixed lot size, we will give a mathematical model for the simultaneous solution of these two problems and an exact solution algorithm proposed by Akturk and Avci [1] for the tool management problem with necessary explanations will follow.

Advances in cutting tool materials and designs will increase the cutting speeds at which machining is carried out, consequently reduce the machining time, but the initial tooling cost might be higher. Therefore we consider a set of alternative cutting tool types for each machining operation, since no one cutting tool type is best for all purposes. Moreover, the same tool may be used in several machining operations, each one with different machining conditions.

The machining conditions optimization for a single operation is a well known problem and several methods have been developed as discussed in Chapter 2. However, these methods only consider the contribution of machining time and tooling cost to the total cost of the operation, where the decision variables are the cutting speed, depth of cut and feed rate. However, in our study of multi operation case, non-cutting time components resulting from different sources, like tool tuning, workpiece loading/unloading etc. have also significant contribution to the total cost of production via operating cost. All these time consuming events except the actual cutting operation are called non-machining time components. In our models we only consider tool replacing and loading times, since these are the only ones that can be expressed as a function of both the machining conditions and alternative operation-tool pairs.

3.3.1 Notation

The following notation is used throughout the thesis.

α_j, β_j, γ_j : Speed, feed, depth of cut exponents for tool j

C_m, b, c, e : Specific coefficient and exponents of the machine power constraint

C_o : Operating cost of the CNC machine, (\$/min)

C_s, g, h, l : Specific coefficient and exponents of the surface roughness constraint

C_{t_j} : Cost of tool j, (\$/tool)

d_i : Depth of cut for operation i, (in.)

f_{ij} : Feed rate for operation i using tool j, (ipr)

G_i : Diameter of the generated surface for operation i, (in.)

HP_{max} : Maximum available machine power, (hp)

I : Set of all operations

J : Set of the available tool types

J_i : Set of the candidate tool types for the operation i

L_i : Length of the generated surface for operation i, (in.)

n_{ij} : Number of tool type j required for completion of operation i

N_j : Number of available tools of type j

Q : Lot size, (parts)

q_{ij} : Number of times that an operation i can be performed by a tool type j

SFM_i : Maximum allowable surface roughness for the operation i, (μin.)

TC_j : Taylor's tool life constant for tool j

t_{l_j} : Tool magazine loading time for a single tool j, (min.)

t_{r_j} : Tool replacing time for tool j, (min.)

v_{ij} : Cutting speed for operation i using tool j, (fpm)

x_{ij} : 0-1 binary decision variable which is equal to 1, if tool j is assigned to operation i

y_{ij} : 0-1 binary indicator which is equal to 1, if tool j is a candidate tool for operation i

3.3.2 Mathematical Model

As an introduction, we are going to define some possible time components that should be included in the objective function of total cost for the manufacturing of a fixed lot of a single part. There exist two distinct categories for the time components, namely machining time (cutting time) and non-machining time (non-cutting or idle time).

- **Machining time** ($t_{m_{ij}}$): Time required to complete a metal cutting operation. For example, the cutting time expression for a turning operation is given in [15] as follows:

$$t_{m_{ij}} = \frac{\pi.G_i.L_i}{12.v_{ij}.f_{ij}}$$

  Similar expressions for a wide variety of machining operations are available in the literature. However, for the machining economics studies the above expression has been preferred to study on since it is a common expression to all researchers and easy to extend to some other operations.

- **Taylor's tool life expression** (T_{ij}): The relationship between machining time and tool life can be expressed as a function of the machining conditions by using an extended form of Taylor's tool life equation as follows:

$$T_{ij} = \frac{TC_j}{v_{ij}^{\alpha_j}.f_{ij}^{\beta_j}.d_i^{\gamma_j}}$$

  This expression is frequently used in the machining economics literature, especially in the cases where there exist more than one machining condition as the controllable variable.

- **Usage rate expression** (U_{ij}): For the turning operation by combining above two time expressions, the following expression can be derived for the machining time to tool life ratio.

$$U_{ij} = \frac{t_{m_{ij}}}{T_{ij}} = \frac{\pi.G_i.L_i.d_i^{\gamma_j}}{12.TC_j.v_{ij}^{(1-\alpha_j)}.f_{ij}^{(1-\beta_j)}}$$

  It is possible to derive similar expressions for other operations.

Consequently, $q_{ij} = \lfloor 1/U_{ij} \rfloor$ and $n_{ij} = \lceil Q/q_{ij} \rceil$. For practical purposes, q_{ij} must be selected in order to instruct either the CNC program or the operator to change tools after predetermined number of pieces have been machined. Before introducing the mathematical model some remarks on our assumptions about tool usages will be helpful. As we stated in §3.1, only the used amounts of tools are considered in cost calculations and tool availability related constraints. Therefore we used an actual tool usage expression which is equal to $Q \cdot U_{ij}$ for a lot size Q and corresponding U_{ij} values.

A mathematical formulation of the tool allocation and machining conditions optimization problem can be as follows:

$$\text{Minimize} \quad C_{tm} = Q.C_o.\left(\sum_{i\in I}\sum_{j\in J} x_{ij}.t_{m_{ij}}\right) + C_o.\left(\sum_{i\in I}\sum_{j\in J} x_{ij}.\left((n_{ij}-1).t_{r_j} + t_{t_j}\right)\right)$$
$$+\sum_{i\in I}\sum_{j\in J} x_{ij}\,.\,Q\,.\,U_{ij}\,.\,C_{t_j}$$

Subject to:

- Tool Assignment Constraints:

  $$\sum_{j\in J} x_{ij} = 1, \text{ for every } i \in I$$

  $$\sum_{i\in I}\sum_{j\in J}(1-y_{ij}).x_{ij} = 0$$

- Tool Availability Constraints:

  $$\sum_{i\in I} x_{ij}\,.\,Q\,.\,U_{ij} \leq N_j, \text{ for every } j \in J$$

- Tool Life Constraints:

  $$x_{ij}\,.\,U_{ij}\,.\,q_{ij} \leq 1, \text{ for every } i \in I,\ j \in J$$

- Machine Power Constraint:

  $$x_{ij}.C_m.v_{ij}^b.f_{ij}^c.d_i^e \leq HP_{max}, \text{ for every } i \in I,\ j \in J$$

- Surface Roughness Constraints:

  $x_{ij}.C_s.v_{ij}^g.f_{ij}^h.d_i^l \leq SFM_i$, for every $i \in I$, $j \in J$

- Nonnegativity and Integrality Constraints:

  $v_{ij} > 0$, $f_{ij} > 0$, for every $i \in I$, $j \in J$

  $x_{ij} \in \{0,1\}$ and n_{ij} nonnegative integer for every $i \in I$, $j \in J$

In the above mathematical formulation of the problem, the total cost (C_{tm}) of manufacturing of a particular lot (Q) is expressed as the sum of operating cost due to machining time and non-machining time components, and tooling costs, respectively. There are four sets of decision variables. The first set of decision variables, x_{ij}, represents the tool allocation decisions. The second set of decision variables, n_{ij}, depicts the number of tools of a given type allocated to an operation. Finally, the third and fourth sets of decision variables, v_{ij} and f_{ij}, represent the machining conditions selection decisions.

In the presented nonlinear MIP formulation, there exist three types of constraints, namely, operational, tool related and machining operation constraints. The first set of constraints represents the operational constraints which ensure that each operation is assigned to a single tool type of its candidate tools set. The tool availability and tool life constraints are the tool related constraints which guarantee that the solution will not exceed the available quantity on hand and the available tool life capacity for any tool type, respectively. Finally, last two set of constraints represent the usual machining operation constraints. The surface roughness presents the quality requirement on the operation and the machine power constraint provides to operate machine tool without being subject to any damage. The complexity of the above problem was discussed by Akturk and Avci [1], and they showed that the tool allocation and machining conditions optimization problem is $\mathcal{NP}$-complete and presented a solution algorithm to this problem using the classical single machining operation problem, which will be discussed in the next section, as a starting point.

3.3.3 Single Machining Operation Problem (SMOP)

In SMOP, the objective function includes the tooling cost and operating cost due to the machining time, and it is possible to impose the machining operation constraints on the problem together with a tool life constraint. The following standard mathematical formulation of geometric programming (GP) can be written for the SMOP for every possible operation-tool pair:

$$\text{Minimize} \quad M_{ij} = C_1.v_{ij}^{-1}.f_{ij}^{-1} + C_2.v_{ij}^{(\alpha_j-1)}.f_{ij}^{(\beta_j-1)}$$

$$\text{Subject to:} \quad C_t'.v_{ij}^{(\alpha_j-1)}.f_{ij}^{(\beta_j-1)} \leq 1 \qquad \text{(Tool Life Constraint)}$$

$$C_m'.v_{ij}^{b}.f_{ij}^{c} \leq 1 \qquad \text{(Machine Power Constraint)}$$

$$C_s'.v_{ij}^{g}.f_{ij}^{h} \leq 1 \qquad \text{(Surface Roughness Constraint)}$$

$$v_{ij}, f_{ij} > 0$$

where,

$$C_1 = \frac{\pi.G_i.L_i.C_o}{12}, \quad C_2 = \frac{\pi.G_i.L_i.d_i^{\gamma_j}.C_{t_j}}{12.TC_j}$$

$$C_t' = \frac{\pi.G_i.L_i.d_i^{\gamma_j}.q_{ij}}{12.TC_j}, \quad C_m' = \frac{C_m.d_i^{e}}{HP_{max}}, \text{ and } C_s' = \frac{C_s.d_i^{l}}{SFM_i}$$

Denoting the dual variables by $Z_1, Z_2, .., Z_5$ the GP-Dual formulation for the above problem can be as follows:

$$\text{Maximize} \quad (\frac{C_1}{Z_1})^{Z_1}.(\frac{C_2}{Z_2})^{Z_2}.(C_t')^{Z_3}.(C_m')^{Z_4}.(C_s')^{Z_5}$$

$$\text{Subject to:} \quad Z_1 + Z_2 = 1$$

$$-Z_1 + (\alpha_j - 1).Z_2 + (\alpha_j - 1).Z_3 + b.Z_4 + g.Z_5 = 0$$

$$-Z_1 + (\beta_j - 1).Z_2 + (\beta_j - 1).Z_3 + c.Z_4 + h.Z_5 = 0$$

$$Z_1, Z_2, Z_3, Z_4, Z_5 \geq 0$$

The objective for the dual problem is still non-linear, but the constraints of the dual formulation are well-defined linear equations. The dual problem can be solved by using the complementary slackness conditions between dual variables and primal constraints, which are given below, in addition to constraints of both the primal and dual problems.

$$Z_3.(C_t'.v_{ij}^{(\alpha_j-1)}.f_{ij}^{(\beta_j-1)} - 1) = 0$$

$$Z_4.(C_m'.v_{ij}^b.f_{ij}^c - 1) = 0$$

$$Z_5.(C_s'.v_{ij}^g.f_{ij}^h - 1) = 0$$

Each of the constraints of the primal problem can be either loose or tight at optimality. If a dual feasible solution is found for a given problem then the corresponding primal solution can be evaluated in terms of its decision variables, and consequently the primal feasibility of the solution can be checked. At optimality, the corresponding solution should be feasible in both the dual and primal problems, and the objective function value for both problems should be the same. Since we have three constraints in the problem, there are eight different cases for the dual, but only six of them are feasible as shown in [1]. Therefore, we can find the solution of SMOP very quickly since the explicit analytic expressions of the solution exist for all cases.

3.3.4 Exact Solution Procedure

In this section we will give somehow modified version of the the exact algorithm proposed in [1], to solve tool allocation and machining conditions optimization problems simultaneously. The constraints and the decision variables for machining conditions and tool allocation interact with each other. In order to solve these two interrelated problems simultaneously, the set of tool availability constraints, which can be called coupling constraints, are relaxed. In this resource directed decomposition procedure, the optimum machining conditions for every possible operation-tool pair is found and then the tool giving the minimum cost measure is selected using SMOP as a key. This provides a lower bound for the tool allocation and machining conditions optimization problem.

If the required number of tools for any tool type exceeds the number of tools available on hand then different tool requirement levels for every operation-tool pair are generated. Consequently, the nonlinear MIP formulation with several set of constraints given in the previous section is polynomially transformed to a much simpler IP formulation.

The following parameters should be specified, for all tools and operations, as an input to the algorithm.

- **System related inputs:** C_o, HP_{max}, Q.
- **Tool related inputs:** J and N_j, α_j, β_j, γ_j, t_{l_j}, t_{r_j}, C_{t_j} $\forall j \in J$.
- **Part and operation related inputs:** I and y_{ij}, d_i, G_i, L_i, SFM_i $\forall i \in I$ and $\forall j \in J$.
- **Technological exponents:** C_m, b, c, e, C_s, g, h, l

The execution of the algorithm provides us with the following output decision variables:

- **Optimum tool allocations:** x_{ij}, n_{ij} $\forall i \in I$ and $\forall j \in J$.
- **Optimum machining conditions:** v_{ij}, f_{ij} $\forall i \in I$ and $\forall j \in J$.
- **Other consequence variables:**
  - * H: Sum of total machining and non-machining times.
  - * W: Sum of total machining, non-machining and tooling costs.
  - * R_j: Actual requirement of tool type j, $\forall j \in J$

Algorithm

The step by step illustration of the algorithm is as follows:

- **Step 1:** Initially, for every possible $(i,\ j)$ pair set $q_{ij} = \lceil Q/N_j \rceil$ and solve SMOP to determine optimum v_{ij}, f_{ij}, n_{ij} and U_{ij}. Then update q_{ij} and n_{ij} as follows: $q_{ij} = \lfloor 1/U_{ij} \rfloor$ and $n_{ij} = \lceil Q/q_{ij} \rceil$

- **Step 2:** Resolve SMOP for the requirement level, $k \in \{1, 2, \cdots, n_{ij}\}$, of every operation (i, j) to find v_{ij}^k, f_{ij}^k, q_{ij}^k, U_{ij}^k, and the corresponding M_{ij}^k to determine C_{ij}^k as follows:

  For every $i \in I$

  For every $j \in J_i$

  For every $k \in \{1, 2, \cdots, n_{ij}\}$

  Set $q_{ij} = \lceil Q/k \rceil$

  Solve SMOP

  Determine $C_{ij}^k = Q.M_{ij}^k + C_o.\left[(k-1).t_{r_j} + t_{l_j}\right]$

- **Step 3:** For every $i \in I$ find the $(j,\ k)$ pair giving the minimum C_{ij}^k value and using the corresponding U_{ij} value compute tool type j requirement for every $j \in J$ as $R_j = \sum_{i \in I} Q\ .\ U_{ij}$.

- **Step 4:** If $R_j \leq N_j$ for every $j \in J$ then the lower bound solution found in Step 3 gives the optimum tool allocations and machining conditions. In this case $W = \sum_{i \in I} C_{ij}^k$ where $(j,\ k)$ pair corresponds to the optimum tool allocation for operation i, as found in Step 3. Otherwise solve the following integer programming (IP) formulation to find the best allocation for every operation that satisfies the tool availability constraints:

$$\text{Minimize} \quad W = \sum_{i \in I} \sum_{j \in J_i} \sum_{k=1}^{n_{ij}} C_{ij}^k \cdot x_{ij}^k$$

$$\text{Subject to:} \quad \sum_{j \in J_i} \sum_{k=1}^{n_{ij}} x_{ij}^k = 1 \qquad \forall\ i \in I$$

$$\sum_{i \in I} \sum_{k=1}^{n_{ij}} k \cdot x_{ij}^k \leq N_j \qquad \forall\ j \in J$$

where x_{ij}^k is a 0–1 binary decision variable which is equal to 1 if the machining of volume i is assigned to tool j at the tool requirement level of k tools.

- **Step 5:** For the optimum tool allocation we compute sum of machining and non-machining times (H) as follows:

$$H = \sum_{i \in I} \left(Q.\frac{\pi.G_i.L_i}{12.v_{ij}^k.f_{ij}^k} + \left((k-1).t_{r_j} + t_{l_j}\right) \right)$$

3.4 Summary

In this chapter, we have given the definition and the underlying assumptions of the joint lot sizing and tool management problem. We presented the traditional two level approach for the solution of this problem and clarified the lot sizing methods that are widely used to determine the lot sizes independent of tool management problems. We also presented the independent tool management problem including tool allocation and machining conditions optimization problems. After giving a mathematical model for the tool management problem, we presented an algorithm to solve the tool allocation and machining conditions optimization problems. In Chapters 4 and 5, we will concentrate on the solution of the joint problem for single and multi period cases, respectively.

Chapter 4

Single Period Model

In view of the high investment and operating costs of computer numerically controlled machines (CNCs) and hence of flexible manufacturing systems (FMSs) attention should be paid to their effective utilization. Most of the existing studies in tool management ignore the lot sizing decision at system level and take it as a predetermined input while deciding on tool allocation and machining parameters. On the other hand, most of the lot sizing algorithms ignore the machine hour and tool availability constraints and treat the production rate either as infinite or given, while this is an important decision variable in practice and significant cost savings can be realized by controlling the production rate. Consequently, by integrating lot sizing and tool management decisions total production cost can be decreased and any infeasibility due to tool and machine hour capacity constraints can be avoided.

In this chapter, we will propose a new solution methodology to find optimal lot sizes, tool allocations and machining parameters by integrating system, machine and tool level decisions for the single period fixed demand case. The remainder of this chapter is organized into six sections as follows. In the next section, we will give the problem definition and additional notations used throughout this chapter. A mathematical model of the problem is introduced in §4.2. The proposed algorithm is described in §4.3. A numerical example and the computational results of an experimental design are presented in §4.4

and §4.5, respectively. Finally, some concluding remarks are provided in §4.6.

4.1 Problem Definition and Notation

In an automated machining environment consisting of a single CNC turning machine, we want to solve the lot sizing and tool management problems simultaneously in order to determine the decision variables defined in §3.1. In addition to the assumptions given in §3.1, it is assumed that production takes place in a single period and for each part demand is known and fixed. The following notation is used in addition to the notation given in the previous chapter.

Parameters :

$\bar{I}_p$: Average inventory level for part p, (part)
d_{ip} : Depth of cut for operation i of part p, (in.)
D_p : Demand for part p, (parts)
F : Set of possible equal lots
G_{ip} : Diameter of the generated surface for operation i of part p, (in.)
h_p : Inventory holding cost of part p, (\$/part/period)
I_p : Set of all operations of part p
J : Set of the available tool types
J_{ip} : Set of the candidate tool types for the operation i of part p
K_p : Set of all alternatives of part p
L_{ip} : Length of the generated surface for operation i of part p, (in.)
M : A very large positive number, i.e. M $\geq$ 100 $max\{D_p\}$ for every $p \in P$
MH_{max} : Maximum available machine hour for production of all parts, (min)
P : Set of all parts
σ_p : Production rate for part p, (parts/period)
r_p : Number of equal lots for part p
SFM_{ip} : Maximum allowable surface roughness for the operation i of part p, (μin.)

S_p : Setup cost for production of part p, (\$/lot)

ts_p : Setup time for production of part p, (min/lot)

y_{ijp} : 0-1 binary indicator which is equal to 1, if tool j is a candidate tool for operation i of part p

Decision Variables :

C^1_{kp}, C^2_{kp} : Total cost of machining, non-machining and tooling for the lot sizes $Q_{1_{kp}}$ and $Q_{1_{kp}}$, respectively for alternative k of part p, (\$/lot)

C_{kp} : Total cost for alternative k of part p (\$/period)

f_{ijp} : Feed rate for operation i of part p using tool j, (ipr)

H^1_{kp}, H^2_{kp} : Total machining and non-machining time required for the lot sizes, $Q_{1_{kp}}$ and $Q_{2_{kp}}$, respectively for alternative k of part p, (min/lot)

H_{kp} : Total time required for alternative k of part p (min/period)

n_{ijp} : Number of tool type j required for completion of operation i of part p

q_{ijp} : Number of times that an operation i of part p can be performed by a tool type j

$Q_{1_{kp}}$: Size of equal lots for alternative k of part p, (parts)

$Q_{2_{kp}}$: Size of last lot for alternative k of part p, (parts)

Q_{1_p} : Size of equal lots for part p, (parts)

Q_{2_p} : Size of last lot for part p, (parts)

R^1_{jkp}, R^2_{jkp} : Tool type j requirement for the lot sizes, $Q_{1_{kp}}$ and $Q_{2_{kp}}$, respectively for alternative k of part p

R_{jkp} : Total tool type j requirement for alternative k of part p

r_{kp} : Number of equal lots for alternative k of part p

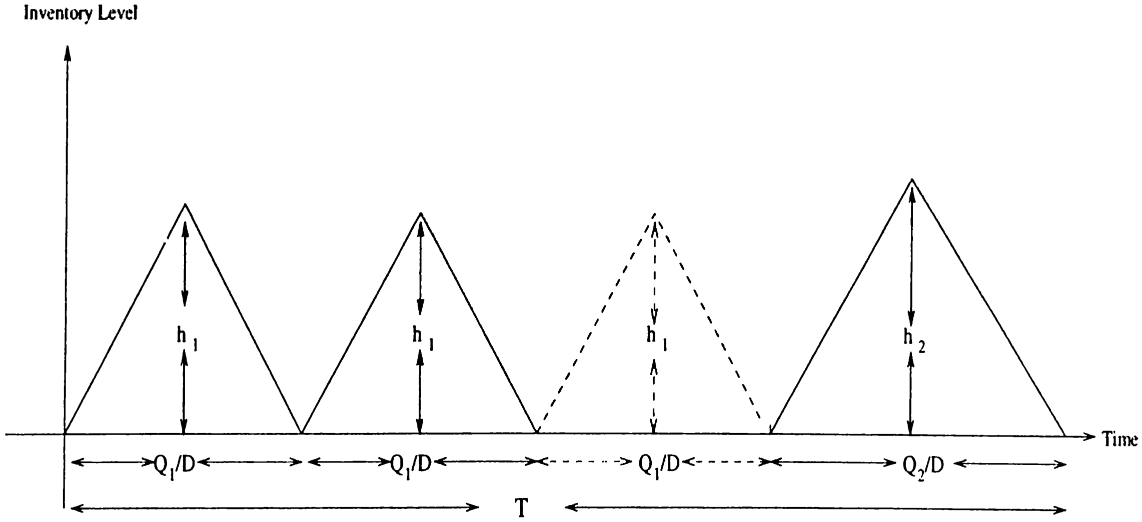
r_p : Number of equal lots for part p

s_p : 0-1 binary decision variable which is equal to 1, if $Q_{2_p} > 0$

v_{ijp} : Cutting speed for operation i of part p using tool j, (fpm)

x_{ijp} : 0-1 binary decision variable which is equal to 1, if tool j is assigned to operation i of part p

z_{kp} : 0-1 binary decision variable which is equal to 1, if alternative k of part p is selected



$h_1 = Q_1.(1 - \frac{D}{\sigma})$, $h_2 = Q_2.(1 - \frac{D}{\sigma})$ and $T = r.(\frac{Q_1}{D}) + \frac{Q_2}{D} = 1$

Figure 4.1: Inventory Level versus Time.

4.2 Mathematical Model

A mathematical formulation of the single period joint lot sizing and tool management problem can be obtained by incorporating some additional components to the objective function and constraints of the tool management problem formulation presented in §3.3. Before giving the mathematical formulation of the problem, it will be helpful to give some remarks about the average inventory, $\bar{I}$, equation used in the mathematical model.

After dropping the part indices p for clarity, $\bar{I}$ is derived as follows. We produce r lots of size Q_1 and one lot of size Q_2, such that $D = r.Q_1 + Q_2$, in a given period T as shown in Figure 4.1. If we denote the total inventory by A then

$$\begin{aligned}\bar{I} &= \tfrac{A}{T} = \left(\tfrac{r.Q_1^2}{2D}(1 - \tfrac{D}{\sigma}) + \tfrac{Q_2^2}{2D}(1 - \tfrac{D}{\sigma}) \right) / \left(r.(\tfrac{Q_1}{D}) + \tfrac{Q_2}{D} \right) \\ &= (1 - \tfrac{D}{\sigma})(r.Q_1^2 + Q_2^2)(\tfrac{1}{2D})\end{aligned}$$

As we can easily verify that

$$\bar{I} = \tfrac{Q}{2}.(1 - \tfrac{D}{\sigma}) \text{ when } Q_1 = Q_2 = Q.$$

A mathematical formulation of the problem can be as follows:

$$\text{Minimize} \sum_{p\in P} S_p \,.\, (r_p + s_p) + \sum_{p\in P} \frac{h_p}{2D_p} \,.\, (r_p \,.\, Q_{1_p}^2 + Q_{2_p}^2) \,.\, (1 - \frac{D_p}{\sigma_p})$$

$$+\sum_{p\in P} D_p \,.\, C_o \sum_{i\in I_p} \sum_{j\in J} x_{ijp} \,.\, t_{m_{ijp}} + \sum_{p\in P}(r_p + s_p) \,.\, C_o \sum_{i\in I_p} \sum_{j\in J} x_{ijp}((n_{ijp} - 1) \,.\, t_{r_j} + t_{l_j})$$

$$+\sum_{p\in P} r_p \sum_{i\in I_p} \sum_{j\in J} x_{ijp} \,.\, D_p \,.\, U_{ijp} \,.\, C_{t_j}$$

Subject to:

- Demand Satisfaction Constraints:

$r_p.Q_{1_p} + Q_{2_p} = D_p$, for every $p \in P$

$Q_{2_p} \leq M.s_p$, for every $p \in P$

- Machine Hour Availability Constraint:

$$\sum_{p\in P} D_p \sum_{i\in I_p} \sum_{j\in J} x_{ijp}.t_{m_{ijp}} + \sum_{p\in P}(r_p + s_p) \sum_{i\in I_p} \sum_{j\in J} x_{ijp}((n_{ijp} - 1)t_{r_j} + t_{l_j}) +$$

$$\sum_{p\in P} ts_p \,.\, (r_p + s_p) \leq MH_{max}$$

- Tool Assignment Constraints:

$\sum_{j\in J} x_{ijp} = 1$, for every $i \in I_p$, $p \in P$

$\sum_{i\in I_p} \sum_{j\in J} (1 - y_{ijp}) \,.\, x_{ijp} = 0$, for every $p \in P$

- Tool Availability Constraints:

$\sum_{p\in P} \sum_{i\in I_p} x_{ijp} \,.\, D_p \,.\, U_{ijp} \leq N_j$, for every $j \in J$

- Tool Life Constraints:

$x_{ijp} \,.\, U_{ijp} \,.\, q_{ijp} \leq 1$, for every $i \in I_p$, $j \in J$, $p \in P$

- Machine Power Constraints:

$x_{ijp} \,.\, C_m \,.\, v_{ijp}^b \,.\, f_{ijp}^c \,.\, d_{ip}^e \leq HP_{max}$, for every $i \in I_p$, $j \in J$, $p \in P$

- Surface Roughness Constraints:

$x_{ijp} \, . \, C_s \, . \, v^g_{ijp} \, . \, f^h_{ijp} \, . \, d^l_{ip} \leq SFM_{ip}$, for every $i \in I_p$, $j \in J$, $p \in P$

- Nonnegativity and Integrality Constraints:

$v_{ijp} > 0$, $f_{ijp} > 0$ for every $p \in P$, $i \in I_p$, $j \in J$

x_{ijp}, s_p binary integers and n_{ijp}, r_p integers for every $p \in P, i \in I_p, j \in J$

In the above nonlinear MIP formulation, the objective function is composed of setup, inventory holding, machining, non-machining and tooling costs, respectively. The first set of constraints ensures that demand for each part is satisfied by r_p equal lots of size Q_{1_p} and a separate lot of size Q_{2_p} if any demand is left unsatisfied. The second constraint ensures that total time required, which is composed of machining, non-machining and set up time components, does not exceed available machine hour. The third set of constraints represents the operational constraints which guarantee that each operation is assigned to a single tool type of its candidate tools set. The fourth set of constraints ensures that total tool requirement does not exceed the amount of tools on hand. The fifth set of constraints guarantees that machining time for an operation does not exceed available tool life and finally the last two sets of constraints represent usual machining operation constraints. The surface roughness presents the quality requirement on the operation and the machine power constraint ensures that machine tool operates without being subject to any damage.

4.3 Algorithm

The constraints and the decision variables for lot sizing, tool allocation and machining conditions interact with each other. In order to solve these interrelated problems simultaneously, we propose a new solution procedure by relaxing the machine hour availability constraint, which can be called a coupling constraint among the parts. For the reduced problem, we then relax the set of tool availability constraints. In this resource directed decomposition procedure, we first find the optimum machining conditions for every possible operation-tool pair and select the tool that gives the minimum cost by using the single machining operation problem (SMOP) as a key. This will provide a lower bound for the tool allocation and machining conditions optimization problem. Afterwards, we impose the relaxed constraints as illustrated by the flow chart in Figure 4.2. Consequently, the nonlinear MIP formulation with several set of constraints given in the previous section is polynomially transformed to a much simpler integer programming (IP) formulation as outlined below.

The steps of the proposed algorithm can be summarized as follows. In steps 1 and 2, we find a set of lot size values and alternative production schedules that satisfy the demand satisfaction constraint. In step 3 using the exact solution algorithm of Akturk and Avci [1] given in §3.3.4, we determine the optimum machining conditions and tool allocations for each possible lot size found in steps 1 and 2. In step 4, we calculate total cost, total machine hour and total tool requirements for each alternative. We repeat these steps for all parts, and find a lower bound solution and check its feasibility in step 5. In step 6, we preprocess the alternatives to eliminate the dominated and infeasible ones. Finally in step 7, over the set of remaining non-dominated alternatives, we construct and solve an IP formulation to find the optimum solution. A step-by-step illustration of the proposed algorithm is given in the next section on a numerical example.

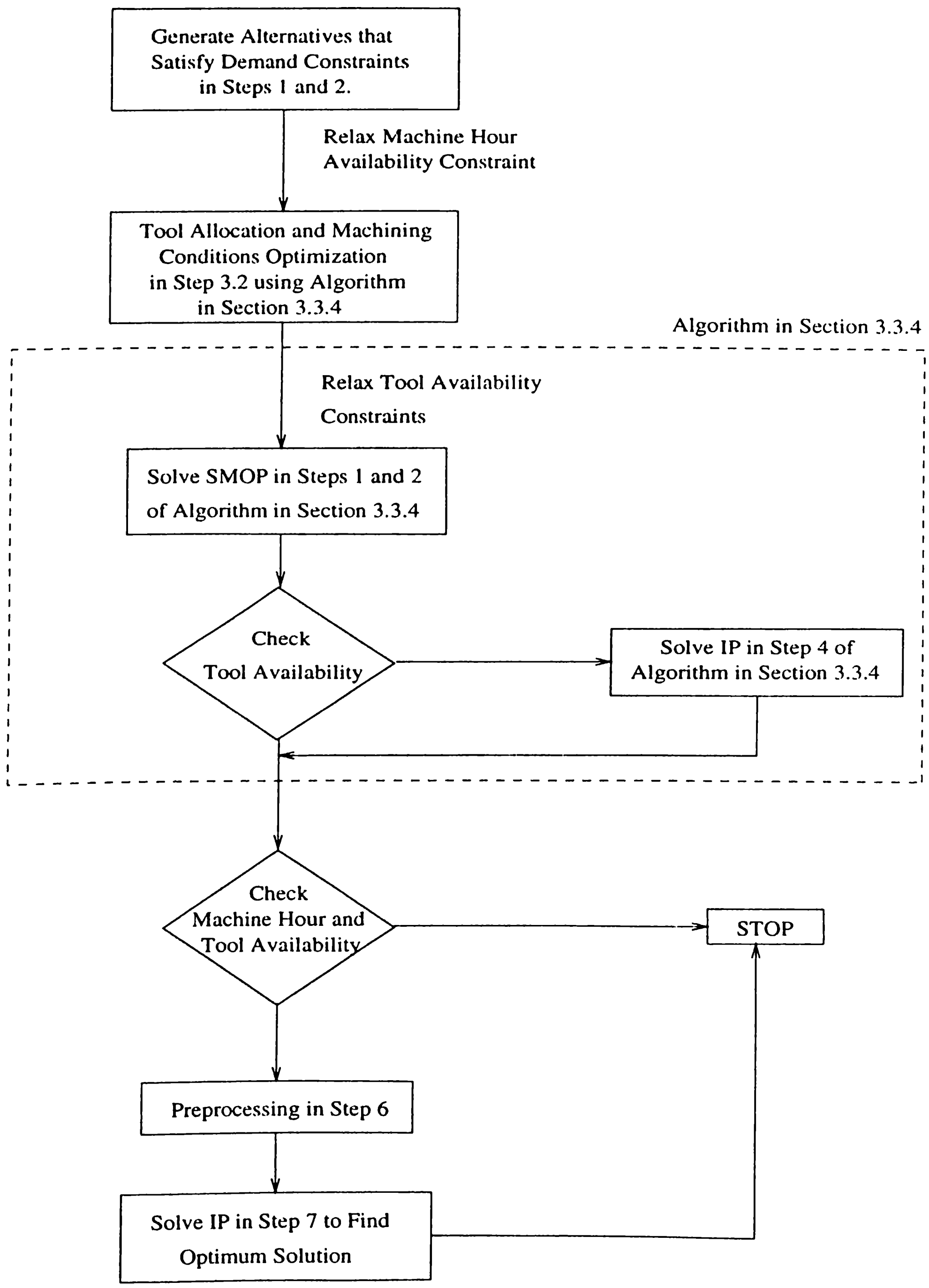


Figure 4.2: Flow Chart of the Algorithm

- **Step 1:** (Determination of Possible Lot Sizes)
  Let $F = \emptyset$ and $r = 1$
  Do following while $r \leq D_p$
  - **Step 1.1:** $B_1 = \lfloor \frac{D_p}{r} \rfloor$ and $F = F \cup \{B_1\}$
  - **Step 1.2:** $r = r + 1$
- **Step 2:** (Determination of Alternative Production Schedules)
  Let $k = 0$, $K_p = \emptyset$
  For every $B_1 \in F$ do the following:
  - **Step 2.1:** $k = k + 1$, $K_p = K_p \cup \{k\}$
  - **Step 2.2:** $Q_{1_{kp}} = B_1$
    If $\frac{D_p}{B_1}$ is integer then $r_{kp} = \frac{D_p}{B_1}$ and $Q_{2_{kp}} = 0$
    Else $B_2 = D_p - \lfloor \frac{D_p}{B_1} \rfloor . B_1$
    If $B_2 \leq B_1/2$ then $r_{kp} = \lfloor \frac{D_p}{B_1} \rfloor - 1$ and $Q_{2_{kp}} = B_1 + B_2$
    Else if $B_2 > B_1/2$ then $r_{kp} = \lfloor \frac{D_p}{B_1} \rfloor$ and $Q_{2_{kp}} = B_2$
- **Step 3:** (Tool Allocation and Machining Conditions Optimization)
  - **Step 3.1:** Determine approximate tool allocations such that $N_{jp} = N_j . \sum_{i \in I_p} y_{ijp} / \sum_{p \in P} \sum_{i \in I_p} y_{ijp}$ for every $j \in J$ and $p \in P$.
  - **Step 3.2:**
    For every $p \in P$
    For every $k \in K_p$
    * Call the algorithm in §3.3.4 with $Q = Q_{1_{kp}}$, $N_j = N_{jp}$ and other necessary input parameters. After execution of the algorithm set the output consequence variables W, R_j and H of the algorithm to C^1_{kp}, R^1_{jkp} and H^1_{kp}, respectively.
    * If $(Q_{2_{kp}} > 0)$ then call the algorithm in §3.3.4 with $Q = Q_{2_{kp}}$, $N_j = N_{jp}$ and other necessary input parameters. After execution of the algorithm set the output consequence variables W, R_j and H of the algorithm to C^2_{kp}, R^2_{jkp} and H^2_{kp}, respectively.

- **Step 4:** (Determination of Parameters for Alternative Production Schedules)
  For every $k \in K_p$ and $p \in P$ find C_{kp}, H_{kp} and R_{jkp} for every $j \in J$ as follows:
  If $Q_{2kp} = 0$ then
  $C_{kp} = r_{kp}.C^1_{kp} + \frac{h_p}{2}.Q_{1kp}.(1 - \frac{D_p}{\sigma_p}) + S_p.r_{kp}$
  $H_{kp} = r_{kp}.H^1_{kp} + r_{kp}.ts_p$
  $R_{jkp} = r_{kp}.R^1_{jkp}$ for all $j \in J$
  Else (If $Q_{2kp} > 0$)
  $C_{kp} = r_{kp}.C^1_{kp} + C^2_{kp} + h_p.\bar{I}_p + S_p.(r_{kp} + 1)$
  $H_{kp} = r_{kp}.H^1_{kp} + H^2_{kp} + ts_p.(r_{kp} + 1)$
  $R_{jkp} = r_{kp}.R^1_{jkp} + R^2_{jkp}$ for all $j \in J$

- **Step 5:** (Lower Bound Check)
  For every part $p \in P$ find alternatives with minimum costs to find the lower bound. If these alternatives satisfy the following machine hour and tool availability constraints, such that $\sum_{p \in P} H_{kp} \leq MH_{max}$, and $\sum_{p \in P} R_{jkp} \leq N_j$ for every $j \in J$ where $k = \arg\min_t\{C_{tp}\}$, then the solution is optimum, STOP.

- **Step 6:** (Preprocessing)
  - **Step 6.1:** (Elimination of Dominated Alternatives)
    Eliminate any dominated alternative $t \in K_p$ for which $\exists k \in K_p$ such that following conditions are satisfied: $C_{tp} \geq C_{kp}$, $H_{tp} \geq H_{kp}$, and $R_{jtp} \geq R_{jkp}$ for every $j \in J$.
  - **Step 6.2:** (Elimination of Infeasible Alternatives)
    Compute $R_{minjp} = \min_{k \in K_p}\{R_{jkp}\}$, $R_{minj} = \sum_{p \in P} R_{minjp}$, $H_{minp} = \min_{k \in K_p}\{H_{kp}\}$ and $H_{min} = \sum_{p \in P} H_{minp}$. If either $R_{minj} > N_j$ or $H_{min} > MH_{max}$ then the part selection problem is infeasible, STOP. Otherwise, eliminate any alternative $t \in K_p$ and $p \in P$ for which either $\exists j \in J$ such that $R_{jtp} > N_j - R_{minj} + R_{minjp}$ or $H_{tp} > MH_{max} - H_{min} + H_{minp}$.

- **Step 7:** Solve the following 0-1 IP to find the optimum combination of alternatives.

$$\text{Minimize} \quad \sum_{p \in P} \sum_{k \in K_p} C_{kp}.z_{kp}$$

$$\text{Subject to:} \quad \sum_{k \in K_p} z_{kp} = 1 \text{ for every } p \in P$$

$$\sum_{p \in P} \sum_{k \in K_p} R_{jkp}.z_{kp} \leq N_j \text{ for every } j \in J$$

$$\sum_{p \in P} \sum_{k \in K_p} H_{kp}.z_{kp} \leq MH_{max}$$

$$z_{kp} \in \{0, 1\} \text{ for every } p \in P, k \in K_p$$

In the above formulation the first set of constraints ensures that for each part p exactly one alternative is selected. By the second set of constraints, it is guaranteed that tool availability constraint is not violated for any tool type, and finally the third constraint ensures that the solution does not exceed available machine hour.

The first four steps of the above algorithm is executed for every $p \in P$. In step 1, we determine the possible lot sizes for possible setups $r \in \{1, 2, 3, ..., D_p\}$, and keep these lot sizes in a set F. In step 2, we create alternative production schedules using the lot sizes found in step 1. Therefore, for each lot size $B_1 \in F$, we first check if it exactly divides the demand, since in this case we can satisfy the demand by producing $r = D_p/B_1$ lots of size B_1. Otherwise, we determine the remaining unsatisfied demand B_2. If $B_2 \leq B_1/2$, we satisfy demand by producing the remaining amount on the last lot, otherwise we produce a separate lot of size B_2 and according to these decisions, we set the size of equal lots to $Q_{1_{kp}}$, the size of the last lot to $Q_{2_{kp}}$ and the number of equal lots to r_{kp}. In step 3, the available tools are initially divided among parts in accordance to their requirements of each type, and then using the exact solution algorithm of Akturk and Avci [1] given in §3.3.4, we determine optimum tool allocations, machining parameters, machine hour and actual tool requirements and the resulting costs for the lot sizes of $Q_{1_{kp}}$ and $Q_{2_{kp}}$, if $Q_{2_{kp}} > 0$. In step 4, for any alternative k of any part p, using the cost, tool and machine hour requirements for Q_{1kp} and Q_{2kp}, we determine total cost, total tool and machine hour requirements. At the end of first four steps we generate a set of alternatives for all parts. In step 5, we find the lower bound solution by selecting the alternative with minimum cost for every part $p \in P$, and if this solution does not violate machine hour and tool availability constraints, then the solution is optimum, so we stop. In step 6, we preprocess the available alternatives to reduce the search space. For any part p, an alternative $t \in K_p$ is dominated, if there exists another alternative $k \in K_p$ that is no worse than alternative t in terms of cost, machine hour and tool requirements. In step 6.1, we eliminate such dominated alternatives. In step 6.2, we eliminate the alternatives exceeding either tool or machine hour availability limits. Finally, in step 7, we solve the 0-1 IP formulation to find optimum combination of alternatives.

4.4 Numerical Example

In this example problem, there are two parts and they require the first four tool types with technological data presented in Table A.8. The other detailed input data related to the tools and parts are presented in Tables A.1, A.2, A.3, A.4 and A.5.

In the first two steps of the algorithm, the possible lot sizes and alternative production schedules for parts are determined. In step 3, we determine optimum machining conditions and tool allocations for lots that appear in an alternative production schedule of any part. For each alternative, we find total cost, time and requirements of each tool type in step 4.

Data related to all possible alternative production schedules obtained at the end of first four steps are summarized in Table 4.1. The actual tool requirements for alternatives of parts 1 and 2 are given in Tables A.6 and A.7, respectively. The detailed cost and time components for alternatives of part 2 are illustrated in Figures 4.3 and 4.4, respectively.

	Part 1					Part 2				
k	Q_{1kp}	Q_{2kp}	r_{kp}	C_{kp}	H_{kp}	Q_{1kp}	Q_{2kp}	r_{kp}	C_{kp}	H_{kp}
1	50	0	1	178.1	273.1	45	0	1	137.5	152.2
2	25	0	2	132.2	193.3	22	23	1	124.0	163.5
3	16	18	2	133.8	184.8	15	0	3	111.2	176.2
4	12	14	3	126.2	193.2	11	12	3	120.3	189.4
5	10	0	5	132.2	199.0	9	0	5	129.6	204.0
6	9	5	5	142.1	209.2	8	5	5	140.9	218.7
7	8	10	5	142.0	209.2	7	10	5	140.9	218.7
8	7	8	6	151.4	219.4	6	9	6	151.7	233.4
9	6	8	7	161.2	229.7	5	0	9	173.4	262.8
10	5	0	10	180.7	250.1	4	5	10	196.4	292.1
11	4	6	11	201.2	270.5	3	0	15	241.9	350.8
12	3	2	16	252.1	321.6	2	3	21	322.4	452.1
13	2	0	25	334.2	403.4	1	0	45	584.1	778.9
14	1	0	50	590.9	653.6					

Table 4.1: Alternative Production Schedules

We skip step 5 in order to explain the remaining steps. In step 6.1 we eliminate dominated alternatives 5, 6, 7, 8, 9, 10, 11, 12 and 13 for part 1 and 4, 5, 6, 7, 8, 9, 10, 11 and 12 for part 2. Among remaining alternatives, we eliminate alternatives 14 of part 1 and 13 of part 2 due to tool availability in step 6.2. Finally, in step 7, we construct and solve following 0-1 IP formulation to find the best combination of alternatives.

$$\begin{aligned}
\text{Min}\quad & 178.1z_{11} + 132.2z_{21} + 133.8z_{31} + 126.2z_{41} + 137.5z_{12} + 124.0z_{22} + 111.2z_{32} \\
\text{s.t.}\quad & z_{11} + z_{21} + z_{31} + z_{41} = 1 \\
& z_{12} + z_{22} + z_{32} = 1 \\
& 1.0z_{11} + 2.2z_{21} + 3.3z_{31} + 0.3z_{41} + 1.0z_{12} + 0.1z_{22} + 0.1z_{32} \leq 5 \\
& 0.1z_{11} + 0.2z_{21} + 0.3z_{31} + 0.4z_{41} + 2.1z_{12} + 2.1z_{22} + 0.2z_{32} \leq 3 \\
& 2.1z_{12} + 2.1z_{22} + 0.2z_{32} \leq 4 \\
& 0.1z_{11} + 0.1z_{21} + 0.1z_{31} + 0.1z_{41} + 1.1z_{12} + 0.1z_{22} + 0.2z_{32} \leq 3 \\
& 273.1z_{11} + 193.3z_{21} + 184.8z_{31} + 193.2z_{41} + 152.2z_{12} \\
& + 163.5z_{22} + 176.2z_{32} \leq 1000
\end{aligned}$$

The solution of the above problem is as follows: $z_{41} = z_{32} = 1$ giving optimum cost of 237.4. This solution suggests to select alternatives 4 and 3 for parts 1 and 2, respectively. Alternative 4 of part 1 proposes production of 3 lots of size 12 and one lot of size 14, whereas alternative 3 of part 2 corresponds to 3 equal lots of size 15. The detailed machining parameters and tool allocations for parts 1 and 2 are presented in Tables 4.2, 4.3 and 4.4. On the other hand, if we solve the lot sizing and tool management problems separately using a two-level approach, then alternative 2 will be the best solution for both of the parts giving a total cost of 256.2. Thus, we decrease the total production cost by 7.9% by reducing the lot sizes.

Operation#	Tool#	n_{ijp}	v_{ijp}	f_{ijp}	$t_{m_{ijp}}$	U_{ijp}
1	2	1	273.4	0.023	0.44	0.033
2	4	1	229.3	0.021	0.87	0.035
3	2	1	249.5	0.016	1.77	0.068
4	1	1	323.6	0.007	3.00	0.083

Table 4.2: Optimum Tool Allocations for the Equal Lots of Part 1

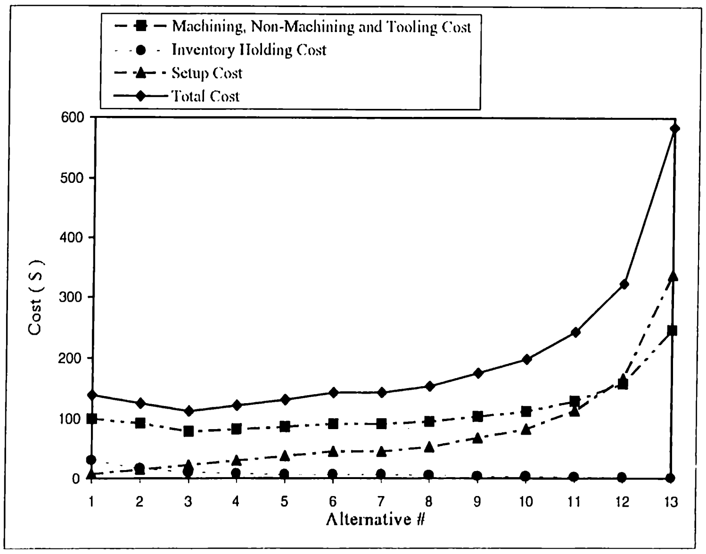


Figure 4.3 : The Detailed Analysis of Cost Components for Part 2.

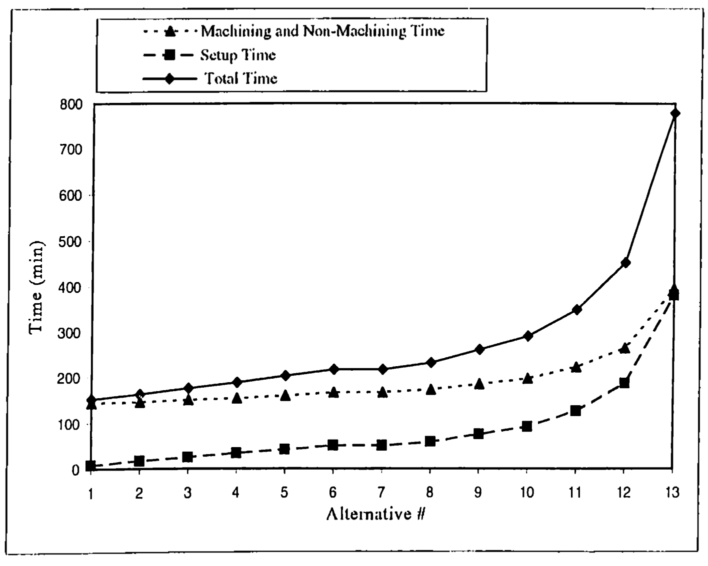


Figure 4.4 : The Detailed Analysis of Time Components for Part 2.

Operation#	Tool#	n_{ijp}	v_{ijp}	f_{ijp}	$t_{m_{ijp}}$	U_{ijp}
1	2	1	273.4	0.023	0.44	0.033
2	4	1	229.3	0.021	0.87	0.035
3	2	1	249.5	0.016	1.77	0.068
4	1	1	310.1	0.006	3.14	0.071

Table 4.3: Optimum Tool Allocations for the Last Lot of Part 1

Operation#	Tool#	n_{ijp}	v_{ijp}	f_{ijp}	$t_{m_{ijp}}$	U_{ijp}
1	4	1	300.4	0.035	0.24	0.020
2	3	1	470.8	0.007	1.06	0.067
3	4	1	230.5	0.019	1.52	0.037
4	1	1	435.4	0.016	2.07	0.041
5	2	1	257.5	0.012	2.93	0.065

Table 4.4: Optimum Tool Allocations for Part 2

4.5 Computational Results

The algorithm presented in the previous section were coded in C language and compiled with Gnu C compiler. The IP formulations in steps 3.3 and 7 were solved by using callable library routines of CPLEX MIP solver on a Sparc station 10 under SunOS 5.4. In this section, the efficiency of the proposed algorithm were tested by comparing the total cost found by the algorithm with the costs found by using a traditional two-level approach. In a two-level approach, lot sizing and machining economics decisions are given

Factors	Definition	Low	High
A	Number of Parts	25	100
B	Demand	UN~[30, 50]	UN~[100, 200]
C	S/I Ratio	3	10
D	Tooling Cost	UN~[3, 4]	UN~[9, 10]
E	Tool Availability	70%	90%
F	Assignment Matrix	Random	Clustered

Table 4.5: Experimental Factors

independently. In the first level lot size is determined by minimizing the sum of setup and inventory holding costs and this lot size is taken as an input by the second level to find the tool management decisions.

There are six experimental factors that can affect the efficiency of our algorithm, which are listed in Table 4.5. Both the number of parts and demand level are most likely to affect the computation times and production costs. The third factor is taken as S/I ratio such that the setup cost for each part is equal to the S/I ratio times the inventory holding cost. The fourth and fifth factors specify the cutting tool cost for each tool type and the tightness of the tool availability constraints, respectively. The number of available tools on hand is taken as 70% and 90% of the available tools for each tool type at low and high levels, respectively. The sixth factor determines the assignment matrix, i.e. random or clustered. At the random level, each cutting tool type can be assigned to a candidate tool set of each operation with an equal probability. But in the clustered case the last operation of each part is taken to be finishing operation whereas the remaining operations to be roughing operations. Since there are six factors and two levels, our experiment is 2^6 full-factorial design, corresponding to 64 combinations. The number of replications for each combination is taken as 5, giving 320 different randomly generated runs.

Other variables were treated as fixed parameters and generated as follows:

- System related parameters, C_o = \$0.5/min, HP_{max} = 5 h.p., and MH_{max} = 60000 min.

- Operation related parameters, G_{ip} and L_{ip} were selected randomly from the interval UN~[1.5, 2.5] and UN~[5, 7] respectively, where UN stands for the uniform distribution.

- Number of operations per part UN~[3, 5].

- An upper bound on the available number of tools for each tool type were taken as a function of the factors A and B, namely part number and demand level. In the low part number case, tool availability was 50 and 200 for low and high demand levels, respectively, and similarly in high part number case, it was 150 and 600 for low and high demand levels, respectively.

- The values of SFM_{ip} and d_{ip} were related with the assignment matrix. For random assignment matrix, SFM_{ip}=UN~[30, 500] and d_{ip}=UN~[0.025, 0.3]. In the clustered case, there were two types of operations, namely roughing and finishing. For roughing operations, SFM_{ip}=UN~[300, 500] and d_{ip}=UN~[0.2, 0.3], and for the finishing operation, SFM_{ip}=UN~[30, 70] and d_{ip}=UN~[0.025, 0.075].

- There are 10 different cutting tool types with technological coefficients given in Table A.8, and other related parameters t_{r_j}=UN~[0.75, 1] and t_{l_j}=UN~[1, 1.5].

- Inventory holding cost, h_p, was selected randomly from the interval UN~[1, 2]. Furthermore, the setup time, ts_p = (S/I ratio) ·UN~[1, 2] and setup cost, S_p = (S/I ratio) · h_p.

- Production rate for each part P_p was found by dividing the available machine hour to the total processing time for each part that was equal to the number of operations times the average processing time per operation.

In a two-level approach, a decision made at the lot sizing level without considering its impact on the tool management problem can lead either to infeasible or inferior results when we consider both the constraints and parameters of the tool management problem. In fact, in our experimental design 72 infeasible cases were observed among 320 randomly generated problems, that is approximately 22.5% of all problems. Among these 72 cases, two cases were due to the machine hour violation while remaining 70 cases were due to the tool availability restriction. We summarize overall results of the proposed joint approach along with the minimum, average and maximum values for total production costs and computation times in Table 4.6. It should be noted that these cost values include all of the production related costs, namely machining, non-machining, tooling, setup and inventory holding costs. In the same table we also presented percent improvements in cost terms obtained over 248 comparable cases. Among these 248 cases, the maximum improvement occurred for the case (0 1 1 1 1 1), where zero and one correspond to the low and high levels of each factor, respectively. The average computation time to find an optimum solution is approximately one minute for the joint approach. Furthermore, we improve the total cost by an average of 6.79% over the two-level approach. A paired-t test was applied to the total cost terms found by the two methods to test the statistical significance of their difference. We found that t-value was 11.65 and the cost values were different with $p \leq 0.000$ significance. As we pointed out before, the two-level approach resulted in 72 infeasible solutions among 320 problems, however these infeasible cases were the ones that would increase the average improvement beyond 6.79% if the two-level approach has found comparable feasible results. This fact can easily be observed in Table 4.7, where we presented the number of infeasible cases and minimum, average and maximum improvement percentages for the most significant two factors on improvements.

We also applied a two-way analysis of variance (ANOVA) test on the performance measures of total cost, computation time and percent improvements. The significance levels (p) and F values for these performance measures against six factors are given in Table 4.8. As it was expected, all of the factors except the fifth one, tool availability, were significant for the total production cost with $p \leq 0.000$.

	Min.	Avg.	Max.	Out of
Joint Total Cost($)	1798.0	13310.6	49527.9	320
Joint Comp. Time (sec.)	5.66	63.40	226.90	320
Two-Level Comp. Time (sec.)	0.01	0.87	10.36	320
Improvement (%)	0.74	6.79	19.11	248

Table 4.6: Overall Results of the Experimental Design

		S/I Ratio	
		Low (Min., Avg., Max.)	High (Min., Avg., Max.)
Demand Level	Low	(0.74, 1.63, 3.27) No Infeasible Cases	(2.78, 6.68, 12.23) No Infeasible Cases
	High	(7.32, 10.55, 15.57) 28 Infeasible Cases	(5.15, 13.08, 19.11) 44 Infeasible Cases

Table 4.7: Percent Improvements and the Number of Infeasible Cases

	Total Cost		Comp. Time		Improvement	
Factors	F	p	F	p	F	p
A	19013.5	0.000	1580.1	0.000	4.0	0.046
B	15317.6	0.000	3.5	0.059	1048.5	0.000
C	689.2	0.000	0.1	0.755	598.4	0.000
D	439.2	0.000	6.7	0.010	61.8	0.000
E	0.1	0.871	38.3	0.000	8.1	0.005
F	55.7	0.000	166.8	0.000	141.0	0.000

Table 4.8: F Values and Significance Levels (p) for ANOVA Results

Among these factors A and B directly affect the amount to be produced, hence total cost of production whereas the third and fourth factors affect the setup and tooling cost components of the total production cost, respectively. Finally, the sixth factor affects the total cost of production due to the tool allocation and consequently machining conditions decisions.

The ANOVA results for the computation time of our algorithm has shown that the most important factors on computation times were the factors A, B, D, E and F. The factors A and B directly affect the size of the problem and the factor D, affects the tool allocation decisions whereas the factor E constrains the number of tools on hand. The significance of factor F, assignment matrix, depends on the fact that, in the clustered case the machining conditions and tool allocation optimization problem is decomposed into two separate problems for roughing and finishing operations, which reduces the number of possibilities. All of the factors were significant on the percent improvements, which also indicated the advantage of the proposed joint approach over a two-level approach.

For the interaction of the factors, the combinations AB, AF and BF were the most significant ones for total cost, computation time and percent improvement performance measures, respectively.

4.6 Summary

In this chapter, after formulating the mathematical model of the single period joint lot sizing and tool management problem, we presented a new algorithm to solve this problem. The proposed algorithm not only improved the overall solution, but also prevented any infeasibility that might occur for the tool management problem due to the decisions made at the lot sizing level. Thus, we have shown that, the interface between the lot sizing and the tool management problems is critical and these two problems should not be viewed in isolation. Although the computational price of the two-level approach is less than the proposed joint approach, the joint approach dominates and gives much better results than any fixed lot size approach due to the increased solution flexibility.

In the next chapter, we will discuss the multi period model of the joint lot sizing and tool management problem, and after giving the mathematical model of this problem, we will present five alternative algorithms.

Chapter 5

Multi Period Model

In multi period joint lot sizing and tool allocation problem, there is a deterministic, but time-varying demand for every part. Actually, dynamic demand case is more realistic compared to static demand case. In manufacturing industry, due to the complexity of planning problems materials requirement planning (MRP) based systems are used as a framework for managing production. In such environments, lot sizing and tool management decisions are given independently. The lot sizing problem is solved by MRP systems, because this problem is considered as a planning problem and is assumed to be solved at a higher level in an organization than is the tool management problem, whereas the tool management problem is considered a low level, detailed decision problem that should be solved after the lot sizing problem. Consequently, these two problems are solved independently in a two-level approach. However, the interface between lot sizing and tool management is critical and these two problems cannot be viewed in isolation. Since in such two-level approaches lot sizes are predetermined prior to the tool management, this might create empty feasible solution spaces and otherwise unnecessarily limit the number of alternatives possible for the tool management problem.

Furthermore, there are several key weaknesses related to the lot size and capacity calculations in MRP systems. In these systems some uncapacitated dynamic lot sizing procedures such as Wagner-Whitin, least unit cost and least period cost are used to solve the lot sizing problems. In these procedures when lot sizes are determined, the production costs either are not taken into account or treated as fixed parameters. Then the lot sizes proposed by these methods are checked by some capacity modules for capacity constraints assuming fixed resource consumption rates. However, in machining environments unit production rates and resource consumption rates are significant decision variables and are functions of machining parameters. Therefore, a production plan proposed by an MRP system may not only be suboptimal, but also be infeasible.

In this chapter, we will propose five alternative algorithms to find lot sizes, tool allocations and machining parameters by integrating system, machine and tool level decisions for multi period dynamic demand case. The remainder of this chapter is organized into six sections as follows. In the next section, we will discuss the problem definition and the additional notation used throughout this chapter. In §5.2 a mathematical model of the problem is introduced. The proposed algorithms are described in §5.3. A numerical example and the computational results of an experimental design are presented in §5.4 and §5.5, respectively. Finally, this chapter concludes with a summary in §5.6.

5.1 Problem Definition and Notation

In an automated machining environment consisting of a single CNC turning machine, we want to solve lot sizing and tool management problems simultaneously, in order to determine the decision variables defined in §3.1. The following assumptions are made in addition to the assumptions given in §3.1.

- There are multiple periods and deterministic, but time-varying demand for every part in every period.
- If production takes place for part p in a period t, then the entering inventory of part p for period t must be zero.

- End of period inventory levels are considered in inventory holding cost calculations.

The following notation is used in addition to the notation given in §3.3.1.

Parameters :

- d_{ip} : Depth of cut for operation i of part p, (in.)
- D_{pt} : Demand for part p in period t, (parts)
- G_{ip} : Diameter of the generated surface for operation i of part p, (in.)
- h_{pt} : Inventory holding cost of part p in period t, (\$/part/period)
- I_p : Set of all operations of part p
- J : Set of the available tool types
- J_{ip} : Set of the candidate tool types for the operation i of part p
- L_{ip} : Length of the generated surface for operation i of part, (in.)
- M : A very large positive number
- MH_t : Maximum available machine hour in period t, (min)
- N_{jt} : Number of available tools of type j in period t
- P : Set of all parts
- SFM_{ip} : Maximum allowable surface roughness for the operation i of part p, (μin.)
- S_{pt} : Setup cost for production of part p in period t, (\$/lot)
- T : Set of all periods
- ts_{pt} : Setup time for production of part p in period t, (min/lot)
- y_{ijp} : 0-1 binary indicator which is equal to 1, if tool j is a candidate tool for operation i of part p

Decision Variables :

- C_{ptk} : Total cost for alternative k of part p in period t, (\$)
- f_{ijpt} : Feed rate for operation i of part p using tool j in period t, (ipr)
- H_{ptk} : Total machining and non-machining time for alternative k of part p in period t, (min)
- HC_{ptk} : Total inventory holding cost for alternative k of part p in period t, (\$)
- I_{pt} : Inventory level of part p at the end of period t, (parts)
- K_{pt} : Set of feasible alternatives of part p in period t

M_{ptk}: Total machining, non-machining and tooling cost for alternative k of part p in period t, (\$)

n_{ijpt} : Number of tool type j required for completion of operation i of part p in period t

q_{ijpt} : Number of times that an operation i of part p can be performed by a tool type j in period t

Q_{pt} : Lot size for part p in period t

Q_{ptk} : Lot size for alternative k of part p in period t

T_{ptk} : Total time requirement for alternative k of part p in period t, (min)

v_{ijpt} : Cutting speed for operation i of part p using tool j in period t, (fpm)

x_{ijpt} : 0-1 binary decision variable which is equal to 1, if tool j is assigned to operation i of part p in period t

Y_{pt} : 0-1 binary decision variable which is equal to 1 , if $Q_{pt} > 0$

z_{ptk} : 0-1 binary decision variable which is equal to 1 , if alternative k is selected for part p in period t

5.2 Mathematical Model

A mathematical formulation of the problem can be as follows:

$$\text{Minimize} \sum_{p\in P}\sum_{t\in T} S_{pt}.Y_{pt} + \sum_{p\in P}\sum_{t\in T} h_{pt}.I_{pt} + \sum_{p\in P}\sum_{t\in T} D_{pt}.C_o \sum_{i\in I_p}\sum_{j\in J} x_{ijpt}.t_{m_{ijpt}} +$$

$$\sum_{p\in P}\sum_{t\in T} C_o.Y_{pt} \sum_{i\in I_p}\sum_{j\in J} x_{ijpt}((n_{ijpt}-1).t_{r_j} + t_{l_j}) +$$

$$\sum_{p\in P}\sum_{t\in T} Y_{pt} \sum_{i\in I_p}\sum_{j\in J} x_{ijpt}.Q_{pt}.U_{ijpt}.C_{t_j}$$

Subject to:

- Production and Inventory Balance Constraints:

$Q_{pt} + I_{p,t-1} - I_{pt} = D_{pt}$, for every $p \in P,\ t \in T$

$Q_{pt} \leq M \cdot Y_{pt}$ for every $p \in P,\ t \in T$

• Machine Hour Availability Constraints:

$$\sum_{p\in P} D_{pt} \sum_{i\in I_p}\sum_{j\in J} x_{ijpt}.t_{m_{ijpt}} + \sum_{p\in P} Y_{pt} \sum_{i\in I_p}\sum_{j\in J} x_{ijpt}.((n_{ijpt}-1).t_{r_j} + t_{l_j}) \; +$$

$$\sum_{p\in P} ts_{pt}.Y_{pt} \leq MH_t, \text{ for every } t \in T$$

• Tool Assignment Constraints:

$$\sum_{j\in J} x_{ijpt} - Y_{pt} = 0, \text{ for every } i \in I_p \;,\; p \in P,\; t \in T$$

$$\sum_{i\in I_p}\sum_{j\in J}(1 - y_{ijp}).x_{ijpt} = 0, \text{ for every } p \in P,\; t \in T$$

• Tool Availability Constraints:

$$\sum_{p\in P}\sum_{i\in I_p} x_{ijpt}.Q_{pt}.U_{ijpt} \leq N_{jt}, \text{ for every } j \in J,\; t \in T$$

• Tool Life Constraints:

$$x_{ijpt}.U_{ijpt}.q_{ijpt} \leq 1, \text{ for every } i \in I_p,\; j \in J,\; p \in P,\; t \in T$$

• Machine Power Constraints:

$$x_{ijpt}.C_m.v_{ijpt}^{b}.f_{ijpt}^{c} \leq HP_{max}, \text{ for every } i \in I_p,\; j \in J,\; p \in P,\; t \in T$$

• Surface Roughness Constraints:

$$x_{ijpt}.C_s.v_{ijpt}^{g}.f_{ijpt}^{h} \leq SFM_{ip}, \text{ for every } i \in I_p,\; j \in J,\; p \in P,\; t \in T$$

• Nonnegativity and Integrality Constraints:

$v_{ijpt} > 0$, $f_{ijpt} > 0$ for every $p \in P$, $i \in I_p$, $j \in J$, $t \in T$

n_{ijpt}, Q_{pt} nonnegative integers for every $p \in P$, $i \in I_p$, $j \in J$, $t \in T$

x_{ijpt}, Y_{pt} binary integers for every $p \in P$, $i \in I_p$, $j \in J$, $t \in T$

In this nonlinear MIP formulation, the objective function is composed of setup, inventory holding, machining, non-machining and tooling costs, respectively. The first set of constraints are production and inventory balance constraints in which both the amount of inventory left in stock at the end of each period and the demand in each period are supplied by either the amount of production in each period or the amount of inventory carried over from the previous period. The second set of constraints ensures that for each period total time required, which is composed of machining, non-machining and setup time components, does not exceed available machine hour. The third set of constraints represents the operational constraints which guarantee that if a certain part is produced in a given period, i.e. $Y_{pt} = 1$, then each operation of this part is assigned to a single tool type of its candidate tools set. The fourth set of constraints ensures that total tool requirement does not exceed the amount of tools on hand. The fifth set of constraints guarantees that machining time for an operation does not exceed available tool life and finally the last two sets of constraints represent usual machining operation constraints. The surface roughness presents the quality requirement on the operation and the machine power constraint ensures that machine tool operates without being subject to any damage.

5.3 Algorithms

The constraints and the decision variables for lot sizing, tool allocation and machining conditions interact with each other. In order to solve these interrelated problems simultaneously, we propose five alternative joint solution algorithms. The first algorithm finds the global optimum solution, whereas the other ones are heuristics and cannot guarantee optimality. The underlying reasoning for all of the algorithms is similar and will be explained on the first algorithm. In this algorithm we first relax the machine hour availability constraint, which can be called a coupling constraint among the parts. For the reduced problem, we then relax the set of tool availability constraints. In this resource directed decomposition procedure, we first find the optimum machining conditions for every possible operation-tool pair and select the tool that gives the minimum cost by using the single machining operation problem (SMOP) as a key. This provides a lower bound for tool allocation and machining conditions optimization problem. Afterwards, we impose the relaxed constraints. consequently, the nonlinear MIP formulation with several set of constraints given in the previous section is polynomially transformed to a much simpler IP formulation.

The steps of the first algorithm can be summarized as follows. In step 1, we determine all alternative lot sizes. An alternative number k for a period t means that, production in period t satisfies cumulative demand of k periods including period t. Therefore, k is in the range of $[1, \cdots, |T| - t + 1]$ for any period t, where $|T|$ denotes the cardinality of the set T. In step 2, for the possible lot sizes found in the first step, we solve tool allocation and machining conditions optimization problem using the algorithm given in §3.3.4. In step 3, we calculate total cost and machine hour requirements for alternative lot sizes, and finally in step 4, we construct and solve an IP formulation to find the optimum solution.

In the second and the third algorithms, we reduce the search space using either least unit cost (LUC) or least period cost (LPC) as a criterion. In these algorithms iteration number of step 2.2 is considerably smaller depending on problem data, which in turn shrinks the size of the IP formulation in final step of these algorithms. In these algorithms, we determine all possible lot sizes similar to the first algorithm, however in the second step of these algorithms we stop solving the tool allocation and machining conditions optimization problem in step 2.2, when we reach a local minimum for cost per unit or cost per period measures. In these algorithms, although we may deviate from global optimum due to the differences in capacity levels between periods, we gain considerably from computation time. The last two algorithms are similar to the second and third algorithms, however in these algorithms unlike the previous ones we present the alternatives with minimum cost per unit and cost per period values as the final solution without solving an IP formulation. A step-by-step execution of all of these algorithms is given on a numerical example in §5.4.

5.3.1 Exact Algorithm

In the first step we determine alternative lot size values for all parts and periods. In step 2.1, the available tools are divided among the parts according to their requirements of each type and then in step 2.2, for the lot sizes found in step 1 we use the algorithm presented in §3.3.4 to determine the optimum tool allocations and machining conditions. When the while loop in step 2.2 is executed for a lot k of a certain (p, t) pair, the algorithm in §3.3.4 may not be able to find a feasible solution due to the insufficient machine capacity. In this case, we do not need to check for larger lot sizes and exit the while loop in order to continue with another (p, t) pair. In the third step, we compute total cost and machine hour requirements for all feasible alternatives, and finally in step 4, we solve an IP formulation to find the optimum combination of alternatives.

- **Step 1:** (Determination of Possible Lot Sizes)
  For every $p \in P$ and $t \in T$
  Set $k = 1$
  While $(k \leq |T| - t + 1)$
  $Q_{ptk} = \sum_{r=t}^{t+k-1} D_{pr}$
  $k = k + 1$

- **Step 2:** (Determination of Tool Allocations and Machining Conditions)
  - **Step 2.1:** Determine approximate tool allocations such that $N_{jpt} = N_{jt}.\sum_{i \in I_p} y_{ijp} / \sum_{p \in P} \sum_{i \in I_p} y_{ijp}$ for every $p \in P$, $t \in T$ and $j \in J$.
  - **Step 2.2:** For every $p \in P$ and $t \in T$
    Set $k = 1$, $K_{pt} = \emptyset$
    While $(k \leq |T| - t + 1)$
    * Call tool allocation and machining conditions optimization algorithm presented in §3.3.4 with $Q = Q_{ptk}$, $N_j = N_{jpt}$ $\forall j \in J$ and other necessary input parameters.
    * If this algorithm finds a feasible solution, then set its output consequence variables W, R_j and H to M_{ptk}, R_{jptk} and H_{ptk},

respectively and set $K_{pt} = K_{pt} \cup \{k\}$, $k = k + 1$.

* Else exit this loop to continue with another $(p,\ t)$ pair.

- **Step 3:** (Determination of Total Cost and Machine Hour Requirements) For every $p \in P$, $t \in T$ and $k \in K_{pt}$ find C_{ptk}, T_{ptk} as follows:

  * $C_{ptk} = M_{ptk} + S_{pt} + HC_{ptk}$, where $HC_{ptk} = \sum_{s=t+1}^{t+k-1} D_{ps} \sum_{r=t}^{s-1} h_{pr}$
  * $T_{ptk} = H_{ptk} + ts_{pt}$

- **Step 4:** Solve the following 0-1 IP formulation to find the optimum combination of alternatives.

$$\text{Minimize} \quad \sum_{p \in P} \sum_{t \in T} \sum_{k \in K_{pt}} C_{ptk} . z_{ptk}$$

$$\text{Subject to:} \quad \sum_{k \in K_{p1}} z_{p1k} = 1 \text{ for every } p \in P$$

$$\sum_{k \in K_{pt}} z_{ptk} - \sum_{r=1}^{t-1} z_{p,r,t-r} \geq 0 \text{ for every } p \in P,\ t = 2, \cdots, |T|$$

$$\sum_{p \in P} \sum_{k \in K_{pt}} R_{jptk} . z_{ptk} \leq N_{jt} \text{ for every } t \in T, j \in J$$

$$\sum_{p \in P} \sum_{k \in K_{pt}} T_{ptk} . z_{ptk} \leq MH_t \text{ for every } t \in T$$

$$z_{ptk} \in \{0, 1\} \text{ for every } p \in P,\ t \in T,\ k \in K_{pt}$$

In the above formulation the first set of constraints ensures that for each part p exactly one alternative is selected for period 1, since we do not allow backlogging. By the second set of constraints, it is guaranteed that for each part demand is satisfied and finally the last two sets of constraints ensure that tool and machine hour availability limits are not exceeded, respectively. In this IP formulation denoting the total number of parts, periods, and tool types by P, T, and J, respectively, it is easy to see that the number of integer variables and the number of constraints are bounded by $P\ T\ (T+1)/2$ and $T\ (1 + P + J)$, respectively.

5.3.2 Look Ahead-LUC (LA-LUC) Algorithm

In this algorithm, initially we determine all alternative lot sizes similar to the previous algorithm. In step 2.1, we divide the available tools among the parts and then in step 2.2 for the lot sizes found in step 1, using the algorithm given in §3.3.4, we determine optimum tool allocations and machining conditions. At each iteration of step 2.2, we compute LUC for any alternative k. If we reach a local optimum for LUC criterion then we stop, and continue with another part and period. At the end of the first two steps, we get a reduced set of alternative lot sizes and finally, in step 3 we solve an IP formulation to find the optimum combination of available alternatives.

- **Step 1:** The same as in Exact algorithm.
- **Step 2:** (Determination of Tool Allocations and Machining Conditions)
  - **Step 2.1:** The same as in Exact algorithm.
  - **Step 2.2:** For every $p \in P$ and $t \in T$
    Set $k = 1$, $K_{pt} = \emptyset$, and $LUC = \infty$
    While $(k \leq |T| - t + 1)$
    * Call the algorithm presented in §3.3.4 with $Q = Q_{ptk}$, $N_j = N_{jpt}$ $\forall j \in J$ and other necessary input parameters.
    * If this algorithm finds a feasible solution, then set its output consequence variables W, R_j and H to M_{ptk}, R_{jptk} and H_{ptk}, respectively and compute C_{ptk} and T_{ptk} as in step 3 of Exact algorithm.
      · If $\frac{C_{ptk}}{Q_{ptk}} < LUC$ then
        set $LUC = \frac{C_{ptk}}{Q_{ptk}}$, $K_{pt} = K_{pt} \cup \{k\}$, and $k = k + 1$.
      · Else exit this loop to continue with another (p, t) pair.
    * Else exit this loop to continue with another (p, t) pair.
- **Step 3:** Solve the IP formulation as in Step 4 of Exact algorithm to find the best combination of alternatives.

5.3.3 Look Ahead-LPC (LA-LPC) Algorithm

This algorithm is similar to the LA-LUC algorithm, except that, here we use LPC instead of LUC as a criterion to reduce the search space.

- **Step 1:** The same as in Exact algorithm.
- **Step 2:** (Determination of Tool Allocations and Machining Conditions)
  - **Step 2.1:** The same as in Exact algorithm.
  - **Step 2.2:** For every $p \in P$ and $t \in T$
    Set $k = 1$, $K_{pt} = \emptyset$, and $LPC = \infty$
    While $(k \leq |T| - t + 1)$
    * Call the algorithm presented in §3.3.4 with $Q = Q_{ptk}$, $N_j = N_{jpt}$ $\forall j \in J$ and other necessary input parameters.
    * If this algorithm finds a feasible solution, then set its output consequence variables W, R_j and H to M_{ptk}, R_{jptk} and H_{ptk}, respectively and compute C_{ptk} and T_{ptk} as in step 3 of Exact algorithm.
      · If $\frac{C_{ptk}}{k} < LPC$ then
        set $LPC = \frac{C_{ptk}}{k}$, $K_{pt} = K_{pt}$ U $\{k\}$, and $k = k + 1$.
      · Else exit this loop to continue with another (p, t) pair.
    * Else exit this loop to continue with another (p, t) pair.
- **Step 3:** Solve the IP formulation as in Step 4 of Exact algorithm to find the best combination of alternatives.

5.3.4 Single Pass-LUC (SP-LUC) Algorithm

This algorithm is similar to the LA-LUC algorithm, except that instead of solving the IP formulation in step 3 of LA-LUC algorithm to determine the final solution, we present the alternatives with minimum LUC values as the final solution.

- **Step 1:** The same as in Exact algorithm.
- **Step 2:** (Determination of Tool Allocations and Machining Conditions)
  - **Step 2.1:** The same as in Exact algorithm.
  - **Step 2.2:** For every $p \in P$ and $t \in T$
    Set $k = 1$, $K_{pt} = \emptyset$, and $LUC = \infty$
    While $(k \leq |T| - t + 1)$
    * Call the algorithm presented in §3.3.4 with $Q = Q_{ptk}$, $N_j = N_{jpt}$ $\forall j \in J$ and other necessary input parameters.
    * If this algorithm finds a feasible solution, then set its output consequence variables W, R_j and H to M_{ptk}, R_{jptk} and H_{ptk}, respectively and compute C_{ptk} and T_{ptk} as in step 3 of Exact algorithm.
      · If $\frac{C_{ptk}}{Q_{ptk}} < LUC$ then
        set $LUC = \frac{C_{ptk}}{Q_{ptk}}$, $K_{pt} = K_{pt} \cup \{k\}$, and $k = k + 1$.
      · Else exit this loop to continue with another (p, t) pair.
    * Else exit this loop to continue with another (p, t) pair.
- **Step 3:** Instead of solving the IP formulation as in Exact and Look Ahead algorithms, we find the final solution as follows:
  For every $p \in P$
  Set $t = 1$
  While $(t \leq |T|)$
  * Find $r = \text{argmin}_{k \in K_{pt}} \{\frac{C_{ptk}}{Q_{ptk}}\}$
  * Set $z_{ptr} = 1$ and $t = t + r$

5.3.5 Single Pass-LPC (SP-LPC) Algorithm

This algorithm is very similar to the previous algorithm, except that in Step 2.2 instead of LUC, we use LPC as a criterion to add an alternative k to the set K_{pt}.

- **Step 1:** The same as in Exact algorithm.
- **Step 2:** (Determination of Tool Allocations and Machining Conditions)
  - **Step 2.1:** The same as in Exact algorithm.
  - **Step 2.2:** For every $p \in P$ and $t \in T$
    Set $k = 1$, $K_{pt} = \emptyset$, and $LPC = \infty$
    While $(k \leq |T| - t + 1)$
    * Call the algorithm presented in §3.3.4 with $Q = Q_{ptk}$, $N_j = N_{jpt}$ $\forall j \in J$ and other necessary input parameters.
    * If this algorithm finds a feasible solution, then set its output consequence variables W, R_j and H to M_{ptk}, R_{jptk} and H_{ptk}, respectively and compute C_{ptk} and T_{ptk} as in step 3 of Exact algorithm.
      · If $\frac{C_{ptk}}{k} < LPC$ then
        set $LPC = \frac{C_{ptk}}{k}$, $K_{pt} = K_{pt} \cup \{k\}$, and $k = k + 1$.
      · Else exit this loop to continue with another (p, t) pair.
    * Else exit this loop to continue with another (p, t) pair.
- **Step 3:** Instead of solving an IP formulation as in Exact and Look Ahead algorithms, we find the final solution as follows:
  For every $p \in P$
  Set $t = 1$
  While $(t \leq |T|)$
  * Find $r = \text{argmin}_{k \in K_{pt}}\{\frac{C_{ptk}}{Q_{ptk}}\}$
  * Set $z_{ptr} = 1$ and $t = t + r$

5.4 Numerical Example

In this section, we will discuss the detailed execution of all of the algorithms over an example problem. In this problem, there are five parts and they require the first six tool types with technological data given in Table A.8. All of the

	Alternative# (k)							
Period#(t)	1	2	3	4	5	6	7	8
1	40	65	95	125	155	175	215	260
2	25	55	85	115	135	175	220	
3	30	60	90	110	150	195		
4	30	60	80	120	165			
5	30	50	90	135				
6	20	60	105					
7	40	85						
8	45							

Table 5.1: Alternative Lot Sizes (Q_{ptk}) for Part 1

	Alternative# (k)							
Period#(t)	1	2	3	4	5	6	7	8
1	78.6	149.6	264.2	412.2	590.8	726.4	1033.5	Infeasible
2	51.3	152.1	294.5	483.4	632.4	985.5	Infeasible	
3	58.5	133.6	238.6	327.1	541.7	841.0		
4	56.4	136.4	208.9	400.5	682.9			
5	62.2	129.1	324.0	639.3				
6	38.5	133.8	301.0					
7	75.4	211.3						
8	105.7							

Table 5.2: Total Cost (C_{ptk}) Values for Lot Sizes of Part 1

detailed data related to the tools and parts are presented in Tables B.1, B.2, B.3 and B.4. Machine hour availabilities for periods 1 through 8 are $(2600, 2300, 2400, 2400, 2500, 2500, 2400, 2200)$, respectively.

In the first step of Exact algorithm, we find the possible lot sizes for all parts and periods. In step 2, we determine optimum machining conditions and tool allocations for any period of any part as long as a feasible solution is found. In the next step, we determine total cost and machine hour values for the lot sizes for which tool allocation and machining conditions optimization gives a feasible solution. The possible lot sizes (Q_{ptk}) and corresponding cost (C_{ptk}) values for part 1 obtained at the end of first three steps are presented in Tables 5.1 and 5.2, respectively. The machine hour requirements (T_{ptk}) for these lot sizes are given in Table B.5. Finally, in step 4 of Exact algorithm we solve an IP formulation to find the optimum combination of alternatives.

In Look Ahead-LUC and Look ahead-LPC algorithms, we determine possible lot sizes as in Exact algorithm, however unlike the Exact algorithm we stop solving the tool allocation and machining conditions optimization problem when we reach a local optimum for cost per unit and cost per period measures, respectively. As an example, in Look Ahead-LUC algorithm for $p = 1$ and $t = 1$, since $\frac{C_{111}}{Q_{111}} = \frac{78.6}{40} = 1.9$ and $\frac{C_{112}}{Q_{112}} = \frac{149.6}{65} = 2.3$, we no longer iterate in step 2.2 and consequently, K_{11} contains only the first alternative. Similarly, in Look Ahead-LPC algorithm for $p = 1$ and $t = 1$, since $\frac{C_{111}}{1} = \frac{78.6}{1} = 78.6$, $\frac{C_{112}}{2} = \frac{149.6}{2} = 74.8$ and $\frac{C_{113}}{3} = \frac{264.2}{3} = 88.1$, K_{11} contains only the first and the second alternatives. In these algorithms, finally we solve an IP formulation to find the best combination of available alternatives.

In Single Pass-LUC and Single Pass-LPC algorithms similar to the previous two algorithms we determine lot sizes and solve tool allocation and machining conditions optimization problems as described in previous paragraph. However, in these algorithms we do not solve final IP formulation, instead we present the alternatives with minimum cost per unit and cost per period measures as the final solution. Thus, for $p = 1$ and $t = 1$ the proposed lot sizes for these algorithms are $Q_{111} = 40$ and $Q_{112} = 65$, respectively.

We also solved this numerical example using three well-known uncapacitated lot sizing algorithms, namely Wagner-Whitin (WW), Least Unit Cost (LUC) and Least Period Cost (LPC) algorithms. In order to find the solutions proposed by these algorithms we used a two level approach. In the first level, we found the lot sizes that minimized the sum of setup and inventory holding costs and in the second level, the tool allocation and machining conditions optimization problem was solved for the given lot sizes. The lot sizes proposed by all joint and two-level algorithms are presented in Tables from B.6 to B.13. The solutions and corresponding cost values for part 3, proposed by these methods and our algorithms are presented in Table 5.3, as an example. For this numerical example, the total cost values found by these methods and our algorithms, in addition to percent improvements, calculated using the formula below, are given in Table 5.4.

$$Percent\ \ Improvement = 100 \cdot \frac{(TC - JC)}{JC}$$

where JC and TC denote the total cost values found using joint and two-level methods, respectively.

	Period#(t)								
Algorithm	1	2	3	4	5	6	7	8	Cost
Exact	55	0	45	45	45	20	45	50	293.38
LA-LUC	55	0	45	45	45	20	45	50	293.38
LA-LPC	20	35	45	45	45	20	45	50	297.06
SP-LUC	55	0	45	45	45	65	0	50	294.14
SP-LPC	20	35	45	45	65	0	45	50	297.98
WW	55	0	90	0	65	0	95	0	370.41
LUC	55	0	90	0	65	0	95	0	370.41
LPC	55	0	90	0	65	0	95	0	370.41

Table 5.3: Proposed Lot Sizes and Total Cost Values for Part 3

		Exact	LA-LUC	LA-LPC	SP-LUC	SP-LPC
	Cost	2098.1	2099.0	2108.7	2104.2	2188.2
WW	2590.2	23.4 %	23.4 %	22.8 %	23.1 %	18.4 %
LUC	2550.6	21.6 %	21.5 %	20.9 %	21.2 %	16.6 %
LPC	2585.7	23.2 %	23.2 %	22.6 %	22.9 %	18.2 %

Table 5.4: Total Cost Values and Percent Improvements

All of the algorithms proposed in this chapter consider only the production plans having the integrality property, i.e. if production takes place for any part in any period, then the entering inventory for that part in that period should be zero. However, by combining presented single period and multi period algorithms, further cost reductions can be realized. For this numerical example, by giving the production plan proposed by Exact algorithm for period 8 as an input to the single period algorithm, we see that instead of making a single lot of size 45 for part 1 in period 8, we can make two separate lots of sizes 22 and 23 and reduce the cost of production for part 1 in period 8 from 105.7 to 85.3.

5.5 Computational Results

The algorithms presented in §5.3 were coded in C language and compiled with Gnu C compiler. The IP formulations were solved by using callable library routines of CPLEX MIP solver on a Sparc station 10 under SunOS 5.4. In this section, the efficiency of the proposed joint algorithms were tested by comparing the total costs found by these algorithms with the costs found by using two level WW, LUC and LPC algorithms.

There are seven experimental factors that can affect the efficiency of our algorithm, which are listed in Table 5.5. Both the number of parts and demand mean as well as demand variability are likely to affect the computation times

Factors	Definition	Low	High
A	Number of Parts	10	30
B	Demand Mean (μ)	10	30
C	Demand Variability	UN~[0.9μ, 1.1μ]	UN~[0.6μ, 1.4 μ]
D	S/I Ratio	2	6
E	Tooling Cost	UN~[3,4]	UN~[9,10]
F	Tool Availability	Tight	Loose
G	Assignment Matrix	Random	Clustered

Table 5.5: Experimental Factors

and production costs. The fourth factor is taken as S/I ratio such that the setup cost for each part is equal to the S/I ratio times the inventory holding cost. The fifth and sixth factors specify the cutting tool cost for each tool type and the tightness of the tool availability constraints, respectively. Tightness of the tool availability is likely to affect both computation times and production costs, since if the tool availability constraint is violated for any tool type, we need to solve the IP formulation to determine the optimum tool allocations and machining conditions in the algorithm that we call in step 2.2. Furthermore, the convexity of the production cost function is very sensitive to the tightness of the tool availability constraint. The seventh factor determines the assignment matrix, i.e. random or clustered. At the random level, each cutting tool type can be assigned to a candidate tool set of each operation with an equal probability. But in the clustered case the last operation of each part is taken to be finishing operation whereas the remaining operations to be roughing operations. Since there are seven factors and two levels, our experiment is 2^7 full-factorial design corresponding to 128 combinations. The number of replications for each combination is taken as 5, giving 640 different randomly generated runs. Other variables were treated as fixed parameters and generated as follows:

- System related parameters, C_o = \$0.5/min, HP_{max} = 5 h.p.

- Operation related parameters, G_{ip} and L_{ip} were selected randomly from the interval UN~[1.5, 2.0] and UN~[2.5, 3.0] respectively, where UN stands for the uniform distribution.

- Number of operations per part UN~[3, 5].

- There were 12 two-shift weekly periods, and MH_t =UN~[4600, 5000].

- Tool availability for each tool type was taken as a function of the factors A and B, namely part number and demand mean. In low cases of these two factors tool availability was UN~[2, 3] and UN~[10, 15] for tight and loose cases, respectively. In high cases of these two factors tool availability was UN~[9, 12] and UN~[45, 60] for tight and loose cases, respectively. Whenever one of these factors was in low and the other was in high case, tool availability was UN~[4, 5] and UN~[20, 25] for tight and loose cases, respectively.

- The values of SFM_{ip} and d_{ip} were related with the assignment matrix. For random assignment matrix, SFM_{ip}=UN~[30, 500] and d_{ip}=UN~[0.025, 0.3]. In the clustered case, there were two types of operations, namely roughing and finishing. For roughing operations, SFM_{ip}=UN~[300, 500] and d_{ip}=UN~[0.2, 0.3], and for the finishing operation, SFM_{ip}=UN~[30, 70] and d_{ip}=UN~[0.025, 0.075].

- There were 10 different cutting tool types with tecnological data given in Table A.8 and other parameters t_{r_j}=UN~[0.75, 1] and t_{l_j}=UN~[1, 1.5].

- Weekly inventory holding cost for each part in each period, h_{pt}, was selected randomly from the interval UN~[0.06, 0.08]. Furthermore, the setup time, ts_{pt} = (S/I ratio) $\cdot$ UN~[2, 3] and setup cost, S_{pt} = (S/I ratio) $\cdot$ 50 $\cdot$ h_{pt}, where the constant 50 is used to convert weekly inventory holding costs into yearly equivalents.

In two level approaches such as WW, LUC and LPC, lot sizing decision is given without considering its impact on the tool management problem, which can lead to infeasible or inferior results when we consider both the

	Inf. Cases	Exact	LA-LUC	LA-LPC	SP-LUC	SP-LPC
WW	30	6.9 %	6.5 %	5.9 %	5.8 %	4.7 %
LUC	38	7.7 %	7.2 %	6.6 %	6.5 %	5.4 %
LPC	40	7.9 %	7.5 %	6.8 %	6.7 %	5.6 %

Table 5.6: Number of Infeasible Cases and Percent Improvements

	Exact	LA-LUC	LA-LPC	SP-LUC	SP-LPC	WW	LUC	LPC
Min.	21.5	1.2	1.2	1.0	1.0	0.1	0.1	0.1
Avg.	117.2	54.9	53.7	54.3	53.1	5.2	5.4	5.4
Max.	287.6	190.7	184.8	189.3	183.5	21.9	20.6	20.6

Table 5.7: Computation Time (sec.) Results for the Algorithms

constraints and parameters of the tool management problem. In Table 5.6, we presented the number of infeasible cases that we encountered in the experimental design out of 640 runs. In the same table, we also summarized the percent improvements achieved over the cases for which these three algorithms found feasible solutions. Although, there is a danger of infeasibility for the two single pass algorithms, in our experimental design we did not encounter any such cases. The computation time results for joint and two-level algorithms are presented in Table 5.7. Baker [3] has stated that the LPC algorithm is better than the LUC algorithm for the lot sizing problem, however our computational experiments indicate that the LUC-based algorithms perform better than the LPC-based algorithms in machining environments for both joint and two-level methods.

The statistical analysis of cost, time and improvement data obtained from experimental design has shown that there is a 99% correlation between our algorithms. Also, it was observed that on the average WW algorithm gives better solutions compared to LUC and LPC algorithms. Therefore for further statistical analysis we used our Exact algorithm and WW algorithm as representatives of joint and two level approaches. Among the 610 comparable cases, the maximum improvement occurred for the case (0 1 0 1 0 0 0), where zero and one correspond to the low and high levels of each factor, respectively. We applied a paired-t test to the total cost terms found by these two algorithms to check the statistical significance of their difference. We found that t-value was 5.80 and the cost values were different with $p \leq 0.000$ significance.

	Total Cost		Comp. Time		Improvement	
Factors	F	p	F	p	F	p
A	49278.4	0.000	10580.1	0.000	6.2	0.013
B	39093.3	0.000	6.6	0.010	705.1	0.000
C	0.0	0.990	1.1	0.287	7.4	0.007
D	1601.3	0.000	0.0	0.910	425.1	0.000
E	1613.3	0.000	58.8	0.000	495.1	0.000
F	51.2	0.000	44.2	0.000	131.0	0.000
G	181.7	0.000	1029.0	0.000	1.8	0.177

Table 5.8: F Values and Significance Levels (p) for ANOVA Results

We also applied a two-way analysis of variance (ANOVA) test on the performance measures of total cost, computation time and percent improvements. The significance levels (p) and F values for these performance measures against seven factors are given in Table 5.8. As it was expected, all of the factors except the third one, demand variability, were significant for the total production cost with $p \leq 0.000$. Among these factors A and B directly affect the amount to be produced, hence total cost of production whereas the fourth and fifth factors affect the setup and tooling cost components of the total production cost, respectively. The sixth factor, tightness of the tool availability, affects the structure of production cost function and hence the total production cost. Finally, the seventh factor affects the total cost of production due to the tool allocation and consequently machining conditions decisions.

The ANOVA results for the computation time of Exact algorithm has shown that the most important factors on computation times were the factors A, E, F and G with $p \leq 0.000$ significance and the factor B with $p \leq 0.010$. The factors A and B directly affect the size of the problem, whereas the factor F constrains the number of tools on hand. The significance of factor G, assignment matrix, depends on the fact that, in the clustered case the tool allocation and machining conditions optimization problem is decomposed into two separate problems for roughing and finishing operations, which reduces the number of possibilities. Almost all of the factors were statistically significant on the percent improvements, which also indicated the advantage of the Exact algorithm and hence the other proposed joint algorithms, over two level WW, LUC and LPC algorithms.

For the interaction of the factors, the ANOVA results has shown that the combination of the most significant factors were also significant. For example, for the 2-way interaction of the factors, the combinations AB, AG and BF were the most significant ones for total cost, computation time and percent improvement performance measures, respectively.

		S/I Ratio	
		Low (Min., Avg., Max.)	High (Min., Avg., Max.)
Demand Level	Low	(0.70, 1.58, 3.20) No Infeasible Cases	(0.40, 5.38, 14.20) No Infeasible Cases
	High	(0.10, 7.25, 21.50) No Infeasible Cases	(5.80, 14.95, 36.5) 30 Infeasible Cases

Table 5.9: Percent Improvements and the Number of Infeasible Cases

As it can be seen from Table 5.6, WW algorithm resulted in 30 infeasible cases, however these cases were the ones that would increase the average improvement of Exact algorithm over WW algorithm beyond 6.9% if the WW algorithm has found comparable feasible results. This fact can be observed in Table 5.9, where we presented the number of infeasible cases and minimum, average and maximum improvement percentages for the most significant two factors, namely demand mean and S/I ratio, on percent improvements.

5.6 Summary

In this chapter we have shown that there is a close relationship between the lot sizing and tool management decisions. Therefore, these problems cannot be viewed in isolation. Especially, in a two-level approach lot sizing decisions are done prior to the tool management decisions, which unnecessarily restricts the feasible solution space for the tool management problems, consequently we may end up with either infeasible or inferior results. We have proposed five solution procedures for the joint problem. The first one is an exact algorithm which guarantees the global optimality. The second and third ones are equipped with a look ahead mechanism to guarantee at least local optimality. As it can be seen from the previous section, LUC criterion gives better results compared to LPC criterion in such tool management problems both in joint and two level approaches. Among the algorithms especially, LA-LUC algorithm can be used in MRP softwares to determine lot sizes in conjunction with the tool allocation and machining conditions decisions. There are two advantages of the proposed LA-LUC algorithm over the traditional two-level approaches. First of all, we guarantee that the lot sizing decisions will satisfy the tool management related constraints, so that we ensure overall feasibility. Furthermore, it improves the total production cost by 6.5 % on the average compared to WW algorithm.

Chapter 6

Conclusion

This chapter provides a brief summary of the contributions of this thesis and addresses some possible extensions of this study for future research. In this thesis, we have studied joint lot sizing and tool management problem for single and multi period cases. We proposed new solution methodologies to find optimal lot sizes, tool allocations and machining parameters by integrating system, machine and tool level decisions for production of multiple parts consisting of multiple operations in a CNC environment. In the next section, we will make a short summary of the contributions we have made to this problem.

6.1 Contributions

We showed that the interface between the lot sizing and the tool management problems is critical and these two problems cannot be viewed in isolation. Because determining lot sizes prior to the tool management decisions might create empty feasible solution spaces and otherwise unnecessarily limit the number of alternatives possible for the tool management problem.

We have discussed single period joint lot sizing and tool management problem, and proposed a new algorithm to solve this problem. In this algorithm, we considered the interaction components such as tooling costs, production rates, tool and machine hour capacity constraints, between lot sizing and tool management problems. Over a set of 320 randomly generated problems, we tested the performance of our algorithm and have seen that the infeasibilites were prevented and the total production cost was reduced on the average by 6.8% compared to a traditional two-level approach. Although the computational price of the two-level approach was less than the proposed joint approach, the joint approach dominated and resulted in much better solutions than two-level approach due to the increased solution flexibility.

We also considered the multi period version of the joint lot sizing and tool management problem. We proposed five new algorithms to solve this problem. Over a set of 640 randomly generated problems, we tested all of our algorithms and have seen that all of our algorithms reduced the total production cost on the average by approximately 6.0% compared to some popular two-level approaches. Among the algorithms that we presented, especially the second one, look ahead-LUC algorithm, was the most promising one considering both its improvement percentage and computation time. Because, by using this algorithm instead of a two-level approach, we can guarantee that the lot sizing decisions will satisfy the tool management related constraints, so that we ensure overall feasibility. Furthermore, it improves the total production cost on the average by 6.5% compared the Wagner-Whitin algorithm which showed the best performance among all of the two-level approaches that we tested.

In various studies, LPC criterion has been reported to give better results compared to LUC criterion. However, as a result of our experimental designs, we can say that LUC criterion is a better choice than LPC criterion for both two-level and joint approaches, when we consider the impact of lot sizing decisions at the lower levels.

6.2 Future Research Directions

At the end, there are several future research directions emanating from this research study as such:

- The single and multi period algorithms that we presented can be integrated. After solving the multi period problem, the production plan proposed for each period can be given as an input to the single period algorithm and further improvement can be achieved by dividing the batches proposed by the multi period algorithm into smaller transfer batches.
- Backlogging case may be incorporated to all of the algorithms.
- In this study we considered a CNC turning machine, however some other machine types such as milling and drilling, may be considered.
- Integrality property assumption in the multi period model may be relaxed and the convexity of total production cost function may be used in order to extend the possibilities for the lot sizes. In such an approach piecewise linearization of the production cost function may be a good starting point.
- In this study we considered only a single CNC machine, however the scope of the study can be extended by considering multiple CNC machines as well as material handling systems.
- Considering this study as a part of a sophisticated computerized decision making system for automated manufacturing environments, the interfacing of proposed algorithms to such a system can be considered as a research suggestion.

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Appendix A

TABLES of the SINGLE PERIOD MODEL

Tool#	t_{r_j}	t_{l_j}	N_j	C_{t_j}
1	0.91	1.06	5	4.67
2	0.91	1.18	3	4.05
3	0.82	1.34	4	4.35
4	0.96	1.30	3	4.99

Table A.1: Tooling Information

Tool#(j)	Part 1				Part 2				
1	0	1	0	1	1	0	1	1	0
2	1	0	1	0	0	1	0	1	1
3	0	1	0	0	1	1	1	1	0
4	0	1	0	0	1	0	1	0	0

Table A.2: Possible Operation-Tool Assignments for Parts

Part#(p)	D_p	hp	S_p	ts_p	σ_p
1	50	1.4	7.0	5.5	625
2	45	1.5	7.5	8.5	500

MH_{max} = 1000 min, C_o = \$0.5/min, and HP_{max} = 5 hp.

Table A.3: Data for Numerical Example

Operation#(i)	SFM_{ip}	d_{ip}	G_{ip}	L_{ip}
1	336.0	0.06	1.75	6.60
2	335.0	0.24	1.63	5.20
3	342.0	0.14	2.43	6.10
4	167.0	0.30	2.31	5.10

Table A.4: Operation Data for Part 1

Operation#(i)	SFM_{ip}	d_{ip}	G_{ip}	L_{ip}
1	229.0	0.05	1.60	6.50
2	110.0	0.05	1.60	6.50
3	308.0	0.27	1.58	5.10
4	148.0	0.04	2.44	6.50
5	264.0	0.19	1.93	5.70

Table A.5: Operation Data for Part 2

	Alternative#(k)													
j	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1.04	2.15	3.27	0.32	0.44	0.53	0.53	0.62	0.71	0.88	1.06	1.50	2.21	6.54
2	0.04	0.15	0.28	0.40	0.50	0.61	0.61	0.71	0.81	1.01	1.21	1.72	2.52	5.05
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.02	0.07	0.11	0.14	0.18	0.21	0.21	0.25	0.28	0.35	0.42	0.60	0.88	0.00

Table A.6: Actual Tool Requirements for Alternatives of Part 1

	Alternative#(k)												
j	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1.04	0.08	0.12	0.17	0.21	0.25	0.25	0.29	0.37	0.45	0.62	1.42	4.96
2	2.07	2.13	0.20	0.26	0.33	0.39	0.39	0.46	0.59	0.72	0.98	1.43	6.08
3	2.07	2.14	0.20	0.28	0.35	0.42	0.42	0.49	0.63	0.77	1.06	1.55	0.00
4	1.06	0.11	0.17	0.23	0.28	0.34	0.34	0.40	0.51	0.63	0.85	0.83	0.00

Table A.7: Actual Tool Requirements for Alternatives of Part 2

j	α	β	γ	TC_j	b	c	e	C_m	g	h	l	C_s
1	4.0	1.40	1.16	40960000	0.91	0.78	0.75	2.39	-1.52	1.00	0.25	204620000
2	4.3	1.60	1.20	37015056	0.96	0.70	0.71	1.63	-1.60	1.00	0.30	259500000
3	3.7	1.30	1.10	13767340	0.90	0.75	0.72	2.31	-1.45	1.01	0.25	202010000
4	3.7	1.28	1.05	11001020	0.80	0.75	0.70	2.41	-1.63	1.05	0.30	205740000
5	4.1	1.26	1.05	48724925	0.80	0.77	0.69	2.54	-1.69	1.00	0.40	204500000
6	4.1	1.30	1.10	57225273	0.87	0.77	0.69	2.21	-1.55	1.00	0.25	202220000
7	3.7	1.30	1.05	13767340	0.83	0.75	0.73	2.32	-1.63	1.01	0.30	203500000
8	3.8	1.20	1.05	23451637	0.88	0.83	0.72	2.32	-1.55	1.01	0.18	213570000
9	4.2	1.65	1.20	56158018	0.90	0.78	0.65	1.70	-1.54	1.10	0.32	211825000
10	3.8	1.20	1.05	23451637	0.81	0.75	0.72	2.29	-1.55	1.01	0.18	203500000

Table A.8: Technological Exponents and Coefficients of the Available Tools

Appendix B

TABLES of the MULTI PERIOD MODEL

		Period#(t)							
Part#(p)		1	2	3	4	5	6	7	8
1	D_{pt}	40	25	30	30	30	20	40	45
	S_{pt}	10.3	10.9	11.0	11.0	9.1	10.2	8.7	8.2
	ts_{pt}	10.2	8.7	7.5	9.3	11.1	10.5	6.3	9.3
	h_{pt}	0.14	0.15	0.15	0.15	0.12	0.14	0.12	0.11
2	D_{pt}	30	30	45	25	50	45	20	25
	S_{pt}	9.3	9.5	10.1	9.6	9.5	10.7	8.6	10.3
	ts_{pt}	8.1	10.2	9.3	6.3	6.9	7.8	11.4	10.2
	h_{pt}	0.12	0.13	0.13	0.13	0.13	0.14	0.12	0.14
3	D_{pt}	20	35	45	45	45	20	45	50
	S_{pt}	7.8	9.7	8.6	11.2	11.1	8.1	8.8	8.2
	ts_{pt}	9.9	9.9	10.2	6.9	11.4	11.7	11.1	6.6
	h_{pt}	0.10	0.13	0.11	0.15	0.15	0.11	0.11	0.11
4	D_{pt}	35	35	50	20	50	45	50	30
	S_{pt}	8.4	10.8	11.0	8.7	10.4	10.3	10.9	10.1
	ts_{pt}	6.0	12.0	10.2	10.5	12.0	11.4	11.7	9.9
	h_{pt}	0.11	0.14	0.15	0.12	0.14	0.14	0.15	0.13
5	D_{pt}	40	25	45	45	35	20	50	30
	S_{pt}	10.3	8.8	10.4	8.2	9.2	10.0	10.3	7.9
	ts_{pt}	8.1	6.3	6.0	10.2	6.9	8.7	12.0	8.4
	h_{pt}	0.14	0.12	0.14	0.11	0.12	0.13	0.14	0.10

Table B.1: Cost and Time Data Related to Parts

Part#(p)	Operation#(i)	Tool# (j)						SFM_{ip}	d_{ip}	G_{ip}	L_{ip}
1	1	1	0	0	1	1	0	89.0	0.04	2.00	2.10
	2	0	1	0	0	1	1	451.0	0.10	1.70	2.70
	3	1	0	0	0	0	1	427.0	0.20	3.10	3.50
	4	0	0	0	0	1	0	45.0	0.10	2.80	4.00
	5	1	1	0	0	0	1	204.0	0.11	3.20	2.60
2	1	0	1	0	1	0	0	333.0	0.17	1.60	3.20
	2	1	1	1	0	1	0	57.0	0.05	1.10	2.20
	3	1	1	0	1	1	1	226.0	0.09	2.70	2.10
	4	0	1	0	0	1	0	101.0	0.04	1.20	2.70
	5	0	1	1	1	0	1	57.0	0.26	2.60	3.30
3	1	1	0	0	0	1	0	176.0	0.06	1.20	3.30
	2	0	1	0	1	0	1	412.0	0.03	3.00	3.20
	3	1	0	1	1	1	0	367.0	0.12	1.00	3.50
	4	0	1	1	1	1	0	184.0	0.16	1.50	4.00
	5	0	0	1	0	1	0	236.0	0.28	2.10	2.90
4	1	0	1	0	0	0	0	409.0	0.20	1.10	2.20
	2	1	0	1	0	1	1	363.0	0.06	3.20	2.20
	3	1	0	1	1	1	1	251.0	0.04	3.00	2.40
5	1	1	1	1	0	1	0	117.0	0.30	3.20	3.30
	2	1	1	0	1	1	1	88.0	0.23	3.40	2.70
	3	0	1	1	0	0	0	163.0	0.27	1.30	3.10

Table B.2: Operation-Tool Assignments and Operation Data for Parts

	Period #(t)							
Tool#(j)	1	2	3	4	5	6	7	8
1	8	10	10	10	12	9	10	7
2	11	12	8	10	12	11	8	10
3	12	10	10	12	12	11	10	8
4	10	11	10	11	7	7	11	9
5	7	9	12	9	11	10	9	9
6	11	12	12	9	10	8	9	12

Table B.3: Tool Availability

	Tool#(j)					
	1	2	3	4	5	6
C_{t_j}	3.52	3.28	3.87	3.85	3.15	3.51
t_{l_j}	0.95	0.96	0.96	0.99	0.85	0.84
t_{r_j}	1.44	1.20	1.14	1.34	1.27	1.16

Table B.4: Tooling Information

	Alternative# (k)							
Period#(t)	1	2	3	4	5	6	7	8
1	139.7	253.6	466.2	736.6	1059.2	1302.6	1850.1	Infeasible
2	88.4	260.2	527.8	879.4	1155.0	1805.5	Infeasible	
3	88.0	229.9	401.1	561.6	946.8	1485.6		
4	92.6	230.1	364.6	715.5	1233.7			
5	116.5	231.5	600.9	1198.1				
6	59.5	232.9	523.6					
7	125.4	358.7						
8	196.3							

Table B.5: Total Machine Hour (T_{ptk}) Requirements for Lot Sizes of Part 1

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	40	25	30	30	30	20	40	45
2	30	30	45	25	50	45	45	0
3	55	0	45	45	45	20	45	50
4	70	0	70	0	95	0	80	0
5	40	25	45	45	35	20	50	30

Table B.6: Lot Sizes Proposed by the Exact Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	40	25	30	30	30	20	40	45
2	30	30	45	25	50	45	45	0
3	55	0	45	45	45	20	45	50
4	70	0	70	0	50	45	80	0
5	40	25	45	45	35	20	50	30

Table B.7: Lot Sizes Proposed by the Look Ahead-LUC Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	40	25	30	30	30	20	40	45
2	30	30	45	25	50	45	20	25
3	20	35	45	45	45	20	45	50
4	70	0	70	0	50	45	80	0
5	40	25	45	45	35	20	50	30

Table B.8: Lot Sizes Proposed by the Look Ahead-LPC Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	40	25	30	30	30	20	40	45
2	30	30	45	25	50	45	45	0
3	55	0	45	45	45	65	0	50
4	70	0	70	0	50	95	0	30
5	40	25	45	45	35	20	50	30

Table B.9: Lot Sizes Proposed by the Single Pass-LUC Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	65	0	30	30	30	20	40	45
2	30	30	70	0	50	65	0	25
3	20	35	45	45	65	0	45	50
4	70	0	70	0	95	0	80	0
5	40	25	45	45	55	0	50	30

Table B.10: Lot Sizes Proposed by the Single Pass-LPC Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	65	0	60	0	50	0	95	0
2	60	0	70	0	95	0	45	0
3	55	0	90	0	65	0	95	0
4	70	0	70	0	95	0	80	0
5	65	0	45	100	0	0	80	0

Table B.11: Lot Sizes Proposed by the Wagner-Whitin (WW) Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	65	0	60	0	50	0	95	0
2	60	0	70	0	95	0	45	0
3	55	0	90	0	65	0	95	0
4	70	0	70	0	95	0	80	0
5	65	0	90	0	55	0	80	0

Table B.12: Lot Sizes Proposed by the Least Unit Cost (LUC) Algorithm

	Period#(t)							
Part#(p)	1	2	3	4	5	6	7	8
1	65	0	60	0	50	0	95	0
2	60	0	70	0	115	0	0	25
3	55	0	90	0	65	0	95	0
4	70	0	70	0	95	0	80	0
5	65	0	90	0	55	0	80	0

Table B.13: Lot Sizes Proposed by the Least Period Cost (LPC) Algorithm

VITA

Siraceddin Önen was born on November 27, 1970 in Diyarbakır, Turkey. He attended to the Department of Computer Engineering, Boğaziçi University, in 1990 and later in 1991 he started double major program with the Department of Industrial Engineering and graduated from two departments with honors in July 1994. In October 1994, he joined to the Department of Industrial Engineering at Bilkent University as a research assistant. From that time to the present, he worked with Assist. Prof. M. Selim Aktürk for his graduate study at the same department.