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Enhanced transmission through a sub-wavelength aperture: resonant approaches employing metamaterials

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Abstract

In this paper, we propose a number of resonant metamaterial-based approaches to enhance the power transmission through sub-wavelength apertures. The extraordinary transmission beyond the diffraction limit is usually obtained using setups that rely on covers supporting proper surface plasmon polaritons or leaky waves. However, the actual implementation in real-life applications of these structures is strongly limited by their large transverse extension. After briefly reviewing the aforementioned limitations, we present in the paper new setups based on resonant approaches, characterized by covers whose transverse dimensions are of the same order of magnitude as the aperture size. The phenomenology of the enhanced transmission, the related theoretical aspects, the results of the numerical simulations and the successful experimental verification using the newly proposed setups are then presented in detail.

Keywords: metamaterials, enhanced transmission, split-ring resonators

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The power transmission enhancement through a sub-wavelength aperture is useful to develop several applications, such as high-resolution spatial filters, high-capacity switches, ultra-diffractive imaging systems, high-precision laser lithography, etc. For this reason, it attracted great interest in the scientific community aiming to find new and ever more effective ways to obtain this phenomenon.

According to the seminal paper by Bethe [1], the power transmission through a sub-wavelength aperture in a flat, indefinite, perfect conducting screen with vanishing thickness is very small and goes as the fourth power of the linear electrical dimension of the aperture. As a first approximation, neglecting the retardation effects due to the finite thickness of the screen and the higher order multi-poles, the transmission through the aperture can be modeled as due to both an equivalent magnetic dipole moment parallel to the screen and an equivalent electric dipole moment normal to the screen [1].

The basic idea is, thus, to put a cover or an artificial structure in close proximity or around the aperture in order to increase the amplitude of these two dipole moments as much as possible. In a well known paper, Ebbesen’s group has shown experimentally that it is possible to enhance by four orders of magnitude the power transmitted through a sub-wavelength aperture by using proper corrugations on the metallic screen around the holes [2]. Since the experiment has been performed at optical frequencies, the metallic screen behaves as a plasmonic material and the natural explanation of the enhanced transmission has been given, in the physics community, in terms of the excitation of proper surface plasmon polaritons propagating along the corrugated screen and capable of coupling the impinging electromagnetic radiation with the hole, increasing, thus, the power transmitted on the other side of the screen.

This interesting phenomenon has been investigated also by the microwave community and it has been elegantly explained by Oliner’s and Jackson’s group in terms of proper leaky waves...
excited by the impinging radiation on the corrugated metallic screen having a negative real part of the permittivity [3, 4]. In their works, Oliner and Jackson have shown that the needed leaky-modes are such to provide the maximum coupling between the electromagnetic field of the incident radiation and the magnetic and/or electric dipole moments introduced by Bethe. In this way, it is possible to enable the maximum achievable transfer of the power carried by the impinging wave on one side of the screen into the radiating waves on the other side.

Especially in the microwave community, this explanation in terms of leaky waves led to several contributions, by different groups worldwide, devoted to proposing new setups based on anomalous leaky-wave excitation to obtain the enhanced transmission [5–7].

However, the setups based on leaky waves, according to the microwave community definition, or surface plasmon polaritons, according to the physics community definition, exhibit intrinsic limitations which are further discussed in the paper. From a practical viewpoint, two main limitations concerning these kinds of cover can be considered: (a) the need of an electrically large transverse extension; (b) the weak robustness versus the variation of the geometrical and electrical parameters.

In order to overcome these limitations, we propose in the paper a number of resonant approaches based on the employment of single-negative or resonant metamaterials. The physical phenomena behind the operation of the proposed setups are presented in detail, as well as the related theoretical aspects. Full-wave numerical simulations and, in the case of the most promising setup, also the experimental results are shown to confirm the validity of the present approach.

The paper is organized as follows. In section 2, we briefly review some of the previous setups based on the excitation of the leaky waves and employing metamaterials, discussing their main limitations. In section 3, we propose new setups based on resonant approaches, presenting the theoretical aspects, the results of the numerical simulations, and the experimental results.

2. Enhanced transmission through sub-wavelength apertures: leaky-wave-based setups

As previously anticipated, after the publication of the papers by Oliner and Jackson [3, 4] giving the explanation in terms of leaky waves of Ebbesen’s group experiments, several different layouts have been suggested in the microwave community. We review here the ones based on the employment of metamaterials [5–7].

The first metamaterial-based setup to obtain the enhanced transmission working at microwave frequencies has been suggested by the group of the University ‘Roma Tre’ in collaboration with the University of Pennsylvania and makes use of isotropic and homogeneous metamaterials [5]. In that case, the metallic screen has been assumed as a perfect conducting screen, while the cover used to enhance the power transmission is made of a metamaterial with constitutive parameters having a near-zero real part. In [5] it has been shown how it is possible in this way to excite a proper leaky mode, which couples the energy from the impinging wave and tunnels it through the aperture on the other side of the screen. The setup proposed in [5] is sketched in figure 1(a).

According to [5], the design of such a cover is obtained by fixing the near-zero constitutive parameters of the metamaterial and, then, adjusting the thickness of the slab, as a quarter or half of the wavelength in the material, respectively, depending on the incident polarization.

Even though the results presented in [5] have shown for the first time how it is possible to enhance the power transmission through sub-wavelength apertures at microwave frequencies, the setups made of near-zero metamaterials are, nevertheless, rather difficult to implement in real life, since the thickness of the cover is electrically large compared to the free-space wavelength. With the wavelength in the near-zero metamaterial being very large, in fact, the total thickness of the cover gets larger and larger compared to the free-space wavelength. In order to solve this problem, the same authors, then, have proposed the setup sketched in figure 1(b) [6]. In this case, the cover is made of a bi-layer of conjugate matched single-negative metamaterials, according to the definition given in [8]. Namely, the two slabs have the same sub-wavelength thicknesses and oppositely signed constitutive parameters, in such a way that the first slab is made of an epsilon-negative metamaterial and the second one of a mu-negative metamaterial. The characteristic impedances of the two slabs are both imaginary, with the same absolute value, and conjugated. For this reason, the bi-layer has been called in [8] a conjugate matched bi-layer. The interesting result shown in [8] is that this bi-layer may support a compact resonance at the interface (analogous of a surface plasmon resonance in physics notation), whatever the total thickness of the bi-layer is.

3 In [5] the proposed setup is actually different from the one in figure 1(a), since the near-zero metamaterial cover is placed on both sides of the screen. In that case, in fact, the authors were interested not only in the enhanced transmission, but also in beaming the radiation on the exit side of the screen. By putting the same cover on both faces, in fact, it is possible to obtain, by reciprocity, a very directive beam on the other side of the screen.
In this way, thus, it is possible to reduce the dimensions of any resonant component, as has been extensively shown at microwave frequencies [9–13]. The setup to obtain the enhanced transmission proposed in [6] is again based on the excitation of leaky waves which are now supported in a cover with sub-wavelength total thickness.

All of the setups considered so far [2–7] are based on the excitation of highly-directive leaky waves, since they have to couple an impinging plane-wave field with the aperture. Therefore, in order to match the incident radiation with the mode excited in the cover, the imaginary part of the leaky-mode propagation constant is required to be extremely small. This means that, in order to be effective, the reverse extension of the cover should be electrically large, extending, thus, for several wavelengths over the aperture. For this reason, even if the setup in figure 1(b) reduces the thickness of the cover, it is not able to reduce the reverse extension of the cover itself, leading to several problems in the actual implementation for real-life applications. The advantage of having a reasonable transmission through a sub-wavelength aperture on a screen is limited, in fact, by the employment of a cover with electrically large transverse dimensions. Another issue related to these structures, especially if based on metamaterials, is the difficulty found in the excitation of the required leaky-modes in the case of a real-life setup, since the layout is very sensitive to slight variations of the geometrical parameters and to the values of the electrical parameters (especially in the case of near-zero values [5–7]).

3. Enhanced transmission through sub-wavelength apertures: the resonant approaches

In this section, we present new setups based on a different physical mechanism, other than the leaky-wave excitation. In section 3.1 we propose two layouts based on the excitation of compact resonances at the interface between two conjugate matched single-negative metamaterials placed on the two faces of the screen above the aperture. In section 3.2 we present another layout, which is much more robust against the variation of the geometrical parameters, and is based on the material resonance effect. Both the proposed approaches are intended to reduce the reverse extension of the cover so that its dimensions are comparable with those of the aperture.

3.1. Enhanced transmission through sub-wavelength apertures based on the employment of single-negative metamaterials

In order to reduce the transverse extension of the cover above the aperture, the enhanced transmission should be ruled by a different physical mechanism rather than the excitation of surface plasmon polarization or leaky waves on the screen or in the cover. On the other hand, the physical mechanism should be such that the amplitudes of Bethe’s equivalent electric and magnetic dipole moments [1], which are responsible for the enhanced transmission, are enhanced, as well. Of course, the amplitudes of these dipole moments are related to the amplitude of the electromagnetic field at the aperture.

Let us consider the case in which the screen can be assumed as a perfect conductor, which is the typical way of representing metals at microwaves. The idea is that, instead of exciting a leaky mode, whose field distribution is such that the amplitude of the tangential magnetic field is maximum on the screen, we excite a resonance within or very close to the aperture, so that the amplitude of the equivalent dipole moments can be enhanced, as well. In this way, the transmission enhancement is only due to the resonance at the aperture and there is no need to make use of an electrically large extended coupling structure (either cover or corrugations).

One possibility to excite a compact resonance across the aperture is to employ the same conjugate matched bi-layers used in figure 1(b). This time, the two layers are placed across the aperture, one at the input face and the other one at the output face, as depicted in figure 2(a), whatever the order of the layers is. Since the structure is reciprocal, in fact, both the layouts shown in figure 2(a) work in the same way.

While in the setup presented in figure 1(b) the compact resonance arising at the interface of the two conjugate matched materials has been exploited to squeeze the thickness of the cover at the input face of the screen to support a highly-directive leaky mode, in the setup shown in figure 2(a) the same resonance is used as such, just to enhance the amplitude of the equivalent dipoles describing the transmission through the aperture.

In order to test the setup of figure 2(a), full-wave simulations have been performed, using the commercial...
Figure 3. Simulated enhancement of the tangential (a) electric and (b) magnetic field along the axis passing through the center of the hole. The simulation is performed at 3 GHz with the following geometrical parameters: screen extension (10 cm × 10 cm), hole radius (1 mm), single-negative (SNG) cover thicknesses (4 mm) and transverse extension (5 mm × 5 mm). Input face cover: Lorentz dispersion for the permeability ($\mu_r = -1$ at 3 GHz) and constant permittivity ($\epsilon_r = 1$). Output face cover: Drude dispersion for the permittivity ($\epsilon_r = -1$ at 3 GHz) and constant permeability ($\mu_r = 1$).

code CST Microwave Studio based on the finite integration technique and including material losses in the dispersion models. The design frequency was set at 3 GHz and at this frequency the two materials are designed to be conjugate matched (i.e. same thickness and same values but opposite signs of the constitutive parameters). At 3 GHz the free-space wavelength is 10 cm, while the circular aperture has a diameter of 2 mm (1/50 of the wavelength) and the transverse extension of the cover of 5 mm × 5 mm (1/20 × 1/20 of the wavelength). The structure is illuminated by a plane wave, impinging normally on the screen. Figure 3 shows the enhancement of the tangential (to the screen) electric and magnetic field amplitudes at the design frequency along the axis of the circular aperture. Clearly, the employment of the bi-layer leads to a significant enhancement of the field amplitude.

Consequently, the power transmission through the hole is highly enhanced (more than 40 dB), as shown by the transmission coefficient amplitude reported in figure 4.

This result is rather interesting, since the power transmission is obtained with cover dimensions of only 5 mm × 5 mm × 4 mm, with the hole diameter being 2 mm and the wavelength 100 mm. The dimensions of the covers, thus, are much smaller than the wavelength and of the same order of magnitude as the hole diameter.

In addition, the proposed layout is rather robust against the variation of the cover dimensions, the angle of incidence and the polarization of the impinging field. Such features, in fact, come straightforwardly from the electromagnetic behavior of the conjugate matched metamaterial bi-layer [8]. Several simulations have been performed, whose results are not reported here for the sake of brevity, confirming, as expected, the properties of the proposed setup.

So far, we have considered that the bi-layer is made of ideal isotropic and homogeneous metamaterials following a given dispersion law [4]. However, when going to the practical implementation of the structure, the materials would consist of proper arrangements of inclusions and, thus, their electromagnetic behavior would be rather far from the ideal one. In this regard, the main issue is that it is very hard to excite the compact resonant at the interface between the two conjugate matched metamaterials, when they are made of actual inclusions. For instance, the material with a negative real part of the permittivity may be implemented using a wire [14].

Figure 4. Simulated power transmission through the aperture as a function of frequency. The simulation is performed using the same parameters shown in the caption of figure 3. Both plots are normalized to the value of the power transmission at the design frequency (3 GHz) of the structure without any metamaterial cover.

4 The term 'ideal' refers here to materials whose constitutive parameters are described by lossy (either Drude or Lorentz) dispersion models. The term 'ideal', thus, does not refer to the absence of losses, which, indeed, are included in the dispersion models, but just to remark that we are not referring to real-life inclusion-based metamaterials.
or a parallel-plate medium [15], while the one with a negative real part of the permeability using a split-ring [16] or other magnetic resonators [17–19]. The problem is that the interface between the two materials, now, is not well defined as in the case of ideal materials and, thus, it is very difficult to tune all of the geometrical parameters in order to get the resonance, even at the simulation level.

This issue can be overcome considering that the layout shown in figure 2(a) can be simplified. We demonstrate this assumption through an intuitive approach. Let us consider the boundary conditions of the tangential electric and magnetic field at the hole aperture, between the two materials in the case of a plane wave impinging normally on the structure:

\[
\begin{align*}
\frac{1}{\mu_1} \frac{\partial E_y^{[1]}}{\partial z} & \bigg|_{\text{hole}} = \frac{1}{\mu_2} \frac{\partial E_y^{[2]}}{\partial z} \bigg|_{\text{hole}} \\
\frac{1}{\varepsilon_1} \frac{\partial H_z^{[1]}}{\partial z} & \bigg|_{\text{hole}} = \frac{1}{\varepsilon_2} \frac{\partial H_z^{[2]}}{\partial z} \bigg|_{\text{hole}}
\end{align*}
\]

where 1, 2 refer to the different materials and \( z \) is the spatial coordinate along the propagation axis. Since the hole dimensions are very small compared to the wavelength, the boundary conditions forced by the perfect electric conducting (PEC) metallic screen:

\[
\left. \frac{\partial H_z}{\partial z} \right|_{\text{PEC}} = 0, \quad \left. \frac{\partial E_y}{\partial z} \right|_{\text{PEC}} \neq 0
\]

affect also the field within the hole. Therefore, the boundary conditions at the hole can be roughly given as follows:

\[
\begin{align*}
\frac{1}{\mu_1} \frac{\partial E_y^{[1]}}{\partial z} & \bigg|_{\text{hole}} = \frac{1}{\mu_2} \frac{\partial E_y^{[2]}}{\partial z} \bigg|_{\text{hole}} \\
\frac{1}{\varepsilon_1} \frac{\partial H_z^{[1]}}{\partial z} & \bigg|_{\text{hole}} = \frac{1}{\varepsilon_2} \frac{\partial H_z^{[2]}}{\partial z} \bigg|_{\text{hole}}
\end{align*}
\]

From these expressions it is clear that the only constitutive parameter playing a role is the permeability. The permittivities of the two materials, in fact, do not affect the enhanced transmission, due to the boundary condition forced by the screen. What is needed, thus, is only a change of the sign of the permeabilities across the hole. Therefore, the metamaterial characterized by the dispersive behavior of the permittivity can be removed and replaced by any regular non-magnetic material or even the vacuum.

According to the above discussion, thus, the layout of figure 2(a) can be simplified to the one proposed in figure 2(b), using only one cover made of a metamaterial described by a negative permeability of \(-1\) at the design frequency. Also in this case, since the structure is reciprocal, the metamaterial cover can be put on either of the two faces of the screen.

The power transmission of the same layout as in figure 4, but without the metamaterial cover described by the Drude dispersion of the permittivity is reported in figure 5. The power transmission is almost unchanged, but the layout is dramatically simplified.

Again, for the sake of brevity we do not show here the results confirming the robustness of the proposed layout against the angle of incidence, the polarization and the cover dimensions, since the results are easily predictable.

More interestingly, the layout of figure 2(b) can be now easily implemented through a proper arrangement of real-life inclusions returning a Lorentz-like permeability dispersion. In figure 6 we show the layout employing a linear array of split-ring resonators, which we have employed in the simulations to enhance the power transmission through the hole.

The array of inclusions has been optimized such that it exhibits the same reflection/transmission properties as an ideal metamaterial sample with dimensions 6.5 mm \( \times \) 6.5 mm \( \times \) 6.5 mm characterized by a Lorentz-like dispersion and exhibiting a negative permeability equal to \(-1\) at the design frequency of 3 GHz. Since the volume of the sample is electrically small, it is not easy to derive its actual effective parameters. However, we have used the application-oriented approach proposed in [20]. This approach does not return the actual parameters but operative quantities, which are useful to design the inclusions and their arrangement to get the same reflection/transmission properties of an ideal metamaterial sample for a given polarization and for given sample dimensions. In the case of the sample depicted in figure 6, we have retrieved such operative quantities, which we may call \textit{reduced effective parameters}, only for the TEM polarization with the magnetic field aligned along the axis of the rings and for those particular sample dimensions. The reduced permeability of the inclusion arrangement of figure 6 is depicted in figure 7 together with the power transmission properties of the proposed layout. Again, we are able to obtain a significant enhancement of the power transmission at the design frequency, at which the retrieved reduced permeability is equal to \(-1\).
Figure 6. Setup to enhance the power transmission through a sub-wavelength aperture on a metallic perfect conducting screen based on a single-negative metamaterial cover with a Lorentz-like permeability and implemented through real-life split-ring resonators made of copper. The geometry is described by the following data: number of resonators $= 17$, $\ell = 6.475$ mm, $w = 0.185$ mm, $g = 0.37$ mm, $s = 0.185$ mm, $d = 0.46$ mm. The separation between two adjacent resonators is 0.37 mm.

Figure 7. Solid line: Simulated power transmission through the aperture as a function of frequency. The plot is normalized to the value of the power transmission at the design frequency (3 GHz) of the structure without any cover. Dotted line: Retrieved reduced relative permeability function (real part) of the inclusion arrangement depicted in figure 6. The simulation is performed with the following geometrical parameters: screen extension (10 cm $\times$ 10 cm), hole radius (1 mm). The cover at the input face is the one depicted in figure 6.

3.2. Enhanced transmission through sub-wavelength apertures based on the employment of resonant magnetic inclusions

The interesting result reported in figure 7 suggested an alternative approach—based on a completely different physical phenomenon—to get the power transmission through a sub-wavelength aperture. From figure 7, in fact, we see that the power transmission, though with different values, is obtained at two different frequencies: one corresponding to an effective reduced permeability equal to $-1$ (the one we have studied in section 3.1) and one corresponding to the resonance of the permeability function, and, thus, to the collective resonance of the split-ring resonators.

Therefore, by using an arrangement of magnetic inclusions as depicted in figure 6, we are able in principle to increase the power transmission through sub-wavelength apertures by using two different resonant approaches, based on two different phenomena. As such, the second resonant approach, i.e. the one based on the resonance of the split-ring resonators, can be easier implemented in real-life fabricated setups. The resonance of the split-rings, in fact, is a phenomenon which can be easily obtained just by fixing properly the dimensions of the rings, while the plasmonic-like resonance, existing at the virtual interface between the arrangement of split-rings with reduced permeability equal to $-1$ and the vacuum on the other side of the screen, creates much more difficulties. In the latter case, the main problems are related to the nature of the virtual interface and to the high sensitivity of the layout to the geometrical dimensions.

For the above reasons, we decided first to test experimentally the layout based on the resonance of the split-rings. Indeed, as we have recently shown in [21], it is possible to use even only one split-ring placed in front of the hole to get the expected enhancement of the transmission. In particular, both of the configurations depicted in figure 8 have been tested. The configuration of figure 8(a) gave better results in terms
of power transmission enhancement, because the impinging field is able to excite a strong parallel to the screen magnetic dipole moment and a strong perpendicular to the screen electric dipole moment. According to Bethe [1], such dipole moments are those ones responsible for the power transmission and, thus, increasing their amplitude would result also in an increased field intensity on the other side of the metallic screen. Differently from the configuration in figure 8(a), the one depicted in figure 8(b) is less efficient since the strong electric dipole moment excited by the impinging field in the split-ring is parallel to the screen and, thus, its contribution is canceled out by its out-of-phase image due to the metallic screen.

Finally, in figure 9 we show the experimental curves of the power transmission for the two configurations reported in figures 8(a) and (b) in the case of a circular split-ring resonating around 3.8 GHz.

4. Conclusions

In this paper, we have presented novel resonant approaches based on metamaterials to enhance the power transmission through sub-wavelength apertures. Differently from the previous setups, the proposed structures are not based on leaky wave and plasmonic modes and, thus, are electrically small both in the transverse dimensions and in thickness. It has been numerically demonstrated that a huge transmission enhancement of the order of 40 dB can be obtained with a double cover made of conjugate matched metamaterials, having dimensions comparable with the aperture size. A similar enhancement has been numerically demonstrated considering a structure made by a single cover having a negative permeability equal to $-1$ at the design frequency. The physical phenomena behind the enhanced transmission have been deeply discussed and a physical implementation of the structure through proper arrangements of split-ring resonators has been proposed and tested numerically. Finally, an alternative layout based on resonating split-rings has also been proposed and investigated both numerically and experimentally. The final result is that, instead of using complex and bulky structures, even a single split-ring resonator, if properly placed in front of the aperture, enables the power transmission enhancement.

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