Fair allocation of personal protective equipment to health centers during early phases of a pandemic

Zehranaz Dönmez, Serkan Turhan, Özlem Karsu, Bahar Y. Kara, Oya Karaşan

Abstract

We consider the problem of allocating personal protective equipment, namely surgical and respiratory masks, to health centers under extremely limited supply. We formulate a multi-objective multi-period non-linear resource allocation model for this problem with the objectives of minimizing the number of infected health workers, the number of infected patients and minimizing a deprivation cost function defined over shortages. We solve the resulting problem using the \( \varepsilon \)-constraint algorithm so as to obtain the exact Pareto set. We also develop a customized genetic algorithm to obtain an approximate Pareto frontier in reasonable time for larger instances. We provide a comparative analysis of the exact and heuristic methods under various scenarios and give insights on how the suggested allocations outperform the ones obtained through a set of rule-of-thumb policies, policies that are implemented owing to their simplicity and ease-of-implementation. Our comparative analysis shows that as the circumstances get worse, the trade-off between the deprivation cost and the ratio of infections deepens and that the proposed heuristic algorithm gives very close solutions to the exact Pareto frontier, especially under pessimistic scenarios. We also observed that while some rule-of-thumb policies such as a last-in-first-receives type policy work well in terms of deprivation costs in optimistic scenarios, others like split policies perform well in terms of number of infections under neutral or pessimistic settings. While favoring one of the objectives, these policies typically fail to provide good solutions in terms of the other objective; hence if such policies are to be implemented the choice would depend on the problem characteristics and the priorities of the policy makers. Overall, the solutions obtained by the proposed methods imply that more complicated distribution schemes that are not induced by these policies would be needed for best results.

1. Introduction

Pandemics and epidemics are acknowledged among the most challenging biological natural disasters by the International Federation of Red Cross and Red Crescent Societies (2020). Before 2019, the most recent viral diseases are severe acute respiratory syndrome (SARS) and H1N1. Centers for Disease Control and Prevention (CDC) stated that 26 countries suffered from SARS epidemic, which resulted in more than 8000 infections and 770 deaths during 2003. (CDC, 2017). H1N1 flu virus emerged in 2009 and caused a worldwide pandemic. It is estimated that 151,700–575,400 people died due to H1N1 as stated by CDC (2019a). Most recently, at the end of 2019, COVID-19 virus was identified in Wuhan, China. On March 11, 2020, World Health Organization (WHO) declared pandemic for COVID-19 virus, which has killed more than 3.5 million people over the past year (WHO, 2020c).

According to a framework by WHO (2017), pandemics can be described by four phases based on a global risk assessment procedure: inter-pandemic phase, alert phase, pandemic phase and transition phase. The inter-pandemic phase is the preparation phase between pandemics. If a virus is identified in humans for the first time, alert phase starts where a detailed capacity planning and risk assessment are required in both global and local levels. If the virus is globally spreading, pandemic phase starts. Alert phase and pandemic phase are response phases. As the global risk level reduces transition phase (recovery) takes place, where the process is evaluated. During a pandemic the main purposes of the policy makers are as follows: preventing the spread of the disease, monitoring, evaluating and preparing the health systems, planning the future actions for governments and other stakeholders and implementing the corresponding measures both at individual and social levels. Hierarchy of Controls is implemented to
diminish the spread of the disease as well as protecting the frontline workers such as health care personnel (CDC, 2015). Engineering controls such as vaccination and air-borne isolation rooms are more favorable than administrative controls and using protective equipment since they prevent the spread at the source level. When these controls are not applicable, protective equipment becomes a primary measure. Transmission precautions should be implemented in health centers based on the ways pathogens are transmitted between individuals. For an air-borne disease, precautions include providing masks to the patients as source control and protecting healthcare personnel with adequate equipment (CDC, 2016).

Distribution strategies for the necessary medical equipment should be analyzed in inter-pandemic phases as well as the response (alert and pandemic) phases (WHO, 2017). These distribution strategies are mostly applied for healthcare workers who are exposed to high risk of infection during Aerosol Generating Procedures (AGP) and should be equipped with Personal Protective Equipment (PPE). PPEs are wearable tools, which include masks, gloves, gown and face shields to prevent infection between individuals (CDC, 2020c). The term “mask” includes various types based on leakage and filtering qualities, re-usability and test requirements. Surgical masks are generally worn by the patients whereas healthcare personnel should be equipped with respirators. Although respirators can be categorized as filtering face-piece respirators, powered air-purifying respirators, elastomeric respirators, supplied-air respirators, self-contained breathing apparatus and combination respirators; N95 filtering face-piece respirators are recommended for healthcare settings by CDC and therefore used as a PPE for medical staff. (CDC, 2016, 2019b). Throughout the paper, the terms “respiratory mask” and “respirator” are used to refer to single use N95 filtering face-piece respirators.

During the pandemic phase, PPE shortages can be experienced at the highest rate due to rapidly rising demand. Therefore, defining and solving the PPE allocation problems for this phase is more crucial compared to other phases.

One of the aims of this paper is to define PPE distribution problem accurately. Choosing the most challenging phase in terms of resource allocation, the problem is modeled considering the main actions, known and uncertain parameters, and the limiting constraints resulting from the operational dynamics.

If the disease is an air-borne disease like COVID-19, PPEs are most effective when they are used alongside the social distancing measures. If such measures are difficult to implement, PPE usage becomes extremely important. For instance, medical procedures usually require close contact between health workers and patients therefore, the responsible organizations should provide both parties with the necessary PPE. Especially during AGP, health workers are exposed to high level of infection risk and they are more likely to infect other people. Given that protecting health workers is a public health issue (WHO, 2013), allocation and distribution strategies for health institutions should be determined beforehand and these strategies should be customized considering the dynamics of each phase.

Distribution strategy depends on the item’s shelf life. If the item is for single use the inventory of institutions should be regularly replenished as the demand occurs frequently and limited supply is available each day. If the shelf life of the item is sufficiently long, PPEs can be distributed to institutions beforehand to be used later. In general, inadequate supply of PPEs are more of a limiting factor for stocking compared to shelf life. Especially for surgical masks, supply amounts are quite insufficient in the early phases of the pandemic. WHO (2020b) declared that the production capacities for surgical masks should be increased by 40% to meet the demand of 89 million/month. To tackle the scarcity of PPEs during early phases of a pandemic, various solutions are discussed including extended use (Zhu et al., 2014) and re-use of equipment after they are decontaminated (Bergman et al., 2012; Rowan and Laffey, 2020), improving production capacities and establishing distribution strategies (Patel et al., 2017) and contingency capacity strategies (CDC, 2020b). Equitable allocation strategies for PPEs as one of the most scarce resources during a pandemic, should be investigated considering this supply and demand imbalance (Binkley and Kemp, 2020).

A fair allocation of PPEs is crucial in terms of ethical guidelines as well (Emanuel et al., 2020). To this end, policies that would save most lives should be implemented and the ones who play a role in fighting the pandemic should be prioritized. Hence, healthcare workers should be equipped with proper PPEs and equity should be ensured among them. While total utility should be maximized to ensure efficiency, fairness should be established by providing similar level of service to institutions with similar needs (e.g. hospitals).

In this study, our aim is to establish a centralized fair and efficient allocation of surgical and respiratory masks to a set of hospitals given an extremely limited supply during the pandemic phase. Total number of infected workers and patients are tried to be minimized along with an inequity function on shortages. To this end, a novel multi-objective multi-period non-linear resource allocation model is formulated and optimally solved using $\epsilon$-constraint algorithm. A customized genetic algorithm is proposed to obtain the approximate Pareto frontier. Additionally, rule-of-thumb policies are developed to be used by the decision makers and comparative analysis between these policies, heuristic and optimal solutions are provided. To the best of our knowledge such a resource allocation model has not been studied in the infectious disease literature. Our methodology includes not only constructing such a complex bi-objective non-linear model but also solving it to optimality for small-sized instances. Although our problem tackles a pandemic situation, the adopted methodology provides a great level of insight on how to approach such allocation problems from any similar field.

The rest of the paper is organized as follows. In Section 2, we review the related literature. The allocation problem is defined in Section 3. We introduce the non-linear bi-objective multi-period mathematical model in Section 4. The solution methodology is explained in Section 5. In Section 6, the problem is solved using the $\epsilon$-constraint method, the customized genetic algorithm and the set of rule-of-thumb policies followed by the discussion of results. Finally, Section 7 provides concluding remarks and some future research directions.

2. Related literature

2.1. OR in infectious disease management

Pandemics and epidemics are studied in the literature mostly from two aspects: (i) modeling the disease spread, (ii) improving the preventive and treatment activities (slowing the disease spread) (Liu et al., 2020). For modeling the disease spread, mathematical models such as SIR (susceptible-infectious-recovered) type models are used frequently (Choewell et al., 2016; Lee et al., 2012). These models divide the population into subgroups, namely compartments, to model the transmission of the disease. Complex network models and agent-based models are also used in the literature. The interested reader can refer to Duan et al. (2015) for a detailed literature review for epidemiology progression models.

For slows down the disease transmission, policy adjustments are made and logistics activities are improved to provide necessary medical resources and treatments. Operations Research (OR) plays a crucial role in controlling pandemics and epidemics by improving the logistics operations such as facility location (Lee et al., 2006, 2009; Bijukchaitakan et al., 2018; Yang et al., 2021; Li et al., 2021), vehicle routing (Gamchi et al., 2021), inventory management (Hashikura and Kizu, 2009; Adida et al., 2011; Paul and Venkateswaran, 2018) and resource allocation. Disease spread is also modeled through networks aiming to minimize the contamination (Ventresca and Aleman, 2014; Nandi and Medal, 2016). A literature survey is conducted by Dasakkis et al. (2012) for logistics operations during epidemics. Studies that concern resource allocation problems for infectious disease management are listed in.
Table 1. It is observed that most of these studies focus on vaccination, drug supply chain and medical staff allocation whereas PPE allocation is mostly neglected. An exception is a recent paper by Göçmen (2020) studying PPE allocation problem with demand uncertainty. They tackled uncertainty using fuzzy parameters with a focus on routing while neglecting the effect of PPE allocation on the number of infections. Scenario analysis consists of various scenarios for demand levels and inventory capacities. Our study differs from that of Göçmen (2020) with consideration of levels for numerous parameters such as reproduction and infection rates, supply and demand as well as the effect of allocation decisions on infections of health care workers and outpatients. Rowan and Laffey (2020) recently studied policies to handle the shortage of PPE during COVID-19 such as reprocessing, however their focus is more on a conceptual perspective rather than constructing and solving the problem mathematically. Common resource allocation modeling approaches in infectious disease management are discussed by Brandeau (2005).

2.2. Equitable resource allocation in infectious disease management

Equity is an important concern when it comes to serving vulnerable groups of people in the context of disaster management. As a branch of this, infectious disease management should include fair allocation strategies to contribute public health respecting the ethical principles of humanitarian operations as stated by Sokat and Altay (2021) and Brandeau (2005). The concern for equity becomes even more pronounced considering the possible scarcity of items of need that should be delivered to populations such as preventive (PPE, vaccine etc.) and remedial (drugs, hospital beds, medical staff etc.) medical resources.

Prioritization of populations based on the infection and/or death risk level is one of the most used approaches in the OR literature while allocating resources in case of an epidemic/pandemic. Preventive approaches for the infectious diseases include prioritization based on risk levels of the populations (Kaplan and Pollack, 1998; Lasry et al., 2007) whereas treatment resources are allocated considering the severity of symptoms of the patients (Rico et al., 2007; Sun et al., 2014; Anparasan and Lejeune, 2019).

Ensuring a minimum service level for all populations (Anparasan and Lejeune, 2019) or maximizing the minimum service level provided to populations (Sun et al., 2014) are also well-known methods where the former can be seen as a variant of the latter. The difference is that the minimum is limited in a constraint if the former is utilized instead of being maximized in the objective function. Deviation between service levels (Yin and Büyüktahtakın, 2021) and Gini coefficient (Enayati and Özaltın, 2020) can also be used to ensure a balanced allocation. The definition of service level depends on the problem as well as the goals set while constructing the problem.

The last two methods are only considering service levels in terms of the allocation amounts if used alone. However, in the context of disaster management, shortage can cause human suffering which increases non-linearly with the waiting time. Therefore, taking the waiting time of populations for the items and/or services to be provided into account makes the problem definitions resemble the real life situations more closely. The concept of “Deprivation Cost Function” (DCF) introduced by Holquí-Veras et al. (2013) is defined as the economic value of the human suffering caused by a lack of access to a good or service and captures the effect of this time aspect of allocations as well as the amounts allocated on the resulting suffering of populations.

Although DCFs has drawn increasing attention in the field of humanitarian logistics and disaster managements as stated by Shao et al. (2020), there are not many researches focusing on infectious disease management while utilizing DCFs. Malmir and Zobel (2021) focus on allocation of medical resources such as hand sanitizers and PPE to affected areas while minimizing transportation cost, inventory holding cost, and deprivation cost in response to COVID-19. The focus of this work is more on the transportation modes ad logistic issues in a multi-echelon structure rather than estimating and preventing transmission of the disease.

The contributions of this study can be listed as follows:

• To the best of our knowledge, there is no study that focuses on PPE allocation in response to a pandemic in a strategical level considering the effect of the allocation decisions on the future demands. This study aims to fill this gap.

• As highlighted by Brandeau (2005), equity is an important objective while allocating the resources during pandemics, which is overlooked in the literature. By utilizing deprivation cost as an objective function, a fair allocation of limited resources to health institutions is established during a pandemic.

• Various critical objectives such as minimizing the number of infections and minimizing deprivation cost are applied in a multi-period setting.

• A disease transmission model is used where mask distribution policies iteratively affect the arriving infected population on future time periods.

• Our model takes into account the close-contact transmission probabilities during medical procedures apart from the transmission rate within the society under normal circumstances.

• A non-linear bi-objective multi-period multi-item mathematical model is developed and solved to optimality for small sized problems indicating various possible scenarios during a pandemic.

• For the cases where the multi-period allocation problem is challenging to solve to optimality, a customized fine-tuned genetic algorithm is developed.

• Bi-objective structure of the problem is handled with ε-constraint algorithm to observe the true Pareto and with a customized genetic algorithm to obtain the approximate Pareto frontiers.

• Different distribution policies, which are easier in application are constructed and analyzed with respect to the optimal and the heuristic solutions under pessimistic, optimistic and neutral scenarios reflecting different levels of severity that can be encountered in real life.

• The proposed method will help the policy makers to make more informed decisions by analyzing the consequences of various allocation options.

3. Problem setting and real life dynamics

This study focuses on a setting where a certain amount of single-use surgical and respiratory masks are distributed to hospitals in a city where aim is a fair allocation. Each day, all supply must be distributed and hospitals may hold inventory of masks to use later as shelf lives of these items are sufficiently long. It is assumed that the city population is divided into subgroups where each subgroup receives service from a single hospital. Therefore, population served by each hospital is known beforehand.

Identical outpatients with same demand (1 surgical mask per person) arrive at each hospital to receive daily services on each day and each hospital has a specific number of doctors to provide this service. To the best of our knowledge, there is no study that focuses on PPE allocation in response to a pandemic in a strategical level considering the effect of the allocation decisions on the future demands. This study aims to fill this gap.

To the best of our knowledge, there is no study that focuses on PPE allocation in response to a pandemic in a strategical level considering the effect of the allocation decisions on the future demands. This study aims to fill this gap.
Table 1
Literature review for resource allocation models for infectious diseases.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Resource type</th>
<th>Objective</th>
<th>Solution method</th>
<th>Multi-objective</th>
<th>Multi-period</th>
<th>Fairness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaplan and Pollack (1998)</td>
<td>Budget</td>
<td>Number of infections</td>
<td>Non-linear dynamic programming</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Zaric and Brandeau (2002)</td>
<td>Budget</td>
<td>Number of infections or number of QALY's</td>
<td>Heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Rico et al. (2007)</td>
<td>Medical staff</td>
<td>Number of patients in queue</td>
<td>Simulation</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Lasry et al. (2007)</td>
<td>Funds</td>
<td>Number of infections</td>
<td>Multi-level resource allocation model</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Sun et al. (2009)</td>
<td>Medicine</td>
<td>Number of infections</td>
<td>Game theoretical approach</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Kouyuncu and Erol (2010)</td>
<td>Medicine, Hospital beds, Ventilators</td>
<td>Number of infections, Number of deaths, Number of morbidity days</td>
<td>Hierarchical multi-objective optimization</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Tanner and Ntiamo (2010)</td>
<td>Vaccine</td>
<td>Vaccine coverage</td>
<td>Branch &amp; cut algorithm</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Araz et al. (2012)</td>
<td>Vaccine</td>
<td>Waiting time for vaccination</td>
<td>Number of expected cases averted</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Teytelman and Larson (2013)</td>
<td>Vaccine</td>
<td>Number of infections</td>
<td>A heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Ren et al. (2013)</td>
<td>Vaccine</td>
<td>Number of deaths</td>
<td>A heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Liu and Liang (2013)</td>
<td>Medical resources</td>
<td>Total cost</td>
<td>A heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Yarmand et al. (2014)</td>
<td>Vaccine</td>
<td>Vaccination cost</td>
<td>A heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Sun et al. (2014)</td>
<td>Medical staff</td>
<td>The total travel distance</td>
<td>Linear programming</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Chen et al. (2015)</td>
<td>Vaccine, Medicine</td>
<td>Number of deaths</td>
<td>Barrier approach with Lagrangian relaxation</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Liu et al. (2015)</td>
<td>Medical resource</td>
<td>Total logistics cost</td>
<td>A heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Rachaniotis et al. (2017)</td>
<td>Medical staff</td>
<td>Number of infections</td>
<td>Heuristic algorithm</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Long et al. (2018)</td>
<td>Hospital beds</td>
<td>Number of infections</td>
<td>Two-stage epidemic model</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Anparasan and Lejeune (2019)</td>
<td>Medical staff, Ambulance</td>
<td>Patient flow</td>
<td>Linear programming with hierarchical constraints and valid inequalities</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Enayati and Özaltun (2020)</td>
<td>Vaccine</td>
<td>Amount of vaccine used</td>
<td>Optimization based on discretization with multi-parametric disaggregation</td>
<td>–</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>Göçmen (2020)</td>
<td>PPE</td>
<td>Total cost</td>
<td>Inventory routing problem with fuzzy parameters</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Malmir and Zobel (2021)</td>
<td>Relief items</td>
<td>Transportation cost</td>
<td>Non-linear programming</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Yin and Büyüktahtakın (2021)</td>
<td>Treatment centers and resources</td>
<td>Number of infections, Number and deaths</td>
<td>Multi-stage stochastic programming</td>
<td>–</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Yang et al. (2021)</td>
<td>Vaccine</td>
<td>Total facility and transportation cost</td>
<td>A sequential disaggregation and merging algorithm</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Li et al. (2021)</td>
<td>Vaccine stations and medical staff</td>
<td>Average distance, number of facilities, total cost</td>
<td>Two-stage heuristic algorithm</td>
<td>+</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Santini (2021)</td>
<td>Swap tests and reagent</td>
<td>The total and the ending inventory of swaps</td>
<td>Hierarchical integer programming</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>This study</td>
<td>PPE</td>
<td>Number of infections</td>
<td>Non-linear programming</td>
<td>+</td>
<td>+</td>
<td>–</td>
</tr>
</tbody>
</table>

8 h. The maximum recommended wearing duration is 4 h according to WHO (2020a). Hence, it is supposed that each unit of supplies consists of 2 respirators.

This paper concerns only medical procedures that require close contact between the patient and the doctor and usage of protective equipment is possible for both parties (unlike dental and oral procedures). During such procedures, due to impossibility of distancing, mask usage is crucial and probability of transmission differs from the normal spreading behavior of the disease. Such medical procedures require respirator usage for doctors and patients should wear surgical masks (CDC, 2016). A patient who has COVID-19 virus may infect health workers depending on whether they and workers are equipped with mask or not. When structuring the mathematical model, for simplicity, it is assumed that an individual starts transmitting the disease as soon as they are infected, i.e. the time between infection and infectiousness is zero. The extension for the more general case where there is an incubation period is relatively straightforward. Moreover, an infected doctor may cause infection to other patients similarly depending on PPE use. In each appointment, one of the four cases occurs: both parties wear corresponding masks, none wears mask, only patient wears surgical mask or only doctor wears respirator. Overall transmission probability is calculated considering the probability of these cases.

Note that due to the disease spreading, increase in the number of currently infected patients causes an increase in the probability of infection in the patients arriving in the upcoming periods. Therefore, aim of this study is to mitigate the effect of the mask distribution policy of each period on demand levels of upcoming periods. This effect is measured in terms of the number of patients and doctors infected during appointments and the resulting increase in the ratio of infected patients in future periods. It is assumed that due to daily screen tests for health care workers, infected doctors are identified with 100% accuracy and quarantined each day. Therefore they do not contribute to the disease spreading outside of the appointments. Moreover, doctors are
assumed to be uninfected at the beginning of each day since infected doctors are replaced with uninfected ones.

Due to the scarcity of supply, hospitals are likely to experience shortage during the planning horizon, which causes deprivation. Duration of this deprivation is as crucial as the amount deprived. Therefore, a deprivation cost function which measures both of these aspects of shortage is considered.

The aim is to provide corresponding masks (surgical masks for patients, respirators for doctors) to a set of hospitals so that number of infected health workers, number of infected patients and deprivation costs of hospitals are minimized considering a limited supply.

4. Mathematical model

A multi-objective multi-period non-linear programming model is developed which takes the number of doctors in each hospital, number of outpatients arriving each day, probability of infection with/without each mask type and disease spreading rate as input parameters. Outputs are the distributed amounts, inventory and shortage amounts for each mask type for each hospital at the end of each period. Our aim is to determine good mask allocation strategies so as to avoid the negative effect of sub optimal (unbalanced) allocations to the disease prevalence. To that end, we define infection probability for each hospital as a function of the number of masks distributed, ratio of infected doctors and infected patients and find the expected ratio of virus-carrier arriving patients depending on the number of previously infected patients. Given that the set of hospitals is $H$, the set of discrete time periods is $T$ and the set of mask types are $J$ where $|J| = 2$ (the first type of mask is surgical mask, while the second one is respirator); parameters and decision variables are given in Table 2.

Ratio of infectious individuals to the population for each hospital at the beginning is known whereas this ratio for next periods is recursively determined. We consider each appointment as a Bernoulli trial with success probability $\theta_{h,t}$ at period $t$. Since each doctor and patient get infected with a probability $\beta_{h,t}$ and the infection probability during an appointment in hospital $h$ in period $t$ due to an infected doctor is calculated as $1 - \frac{1 - \theta_{h,t}}{1 - \beta_{h,t}}$. Similarly, probability of the doctor not wearing a respirator is calculated as $\frac{U_{h,t}'}{W_{h,t}'}$. Moreover, probability of the scenario where only the patient is equipped with PPE is the multiplication of these two probabilities $\left(1 - \frac{U_{h,t}'}{W_{h,t}'}\right) \frac{U_{h,t}'}{W_{h,t}'}$. In such a case (if the patient is the virus-carrier side of the appointment) the important parameters are the outward infection rate of the surgical mask ($\beta_{j}$) and the inward protection rate without any mask ($\theta$) to calculate the probability of the doctor getting infected during the appointment. In Table 3, infection rates for possible cases and the corresponding probabilities are calculated. Moreover expected transmission probability is calculated by taking the weighted sum of these transmission rates.

There are 3 main iterative calculations in our approach: Calculating the ratio of infected doctors ($Y$), ratio of primary infections ($Z$) and ratio of infected individuals in the population of each hospital ($\alpha$). Primary infection is defined as the number of infected patients during hospital appointments resulting from the mask allocation strategy in each time period. Since each doctor and patient get infected with a probability and multiplication with a probability cannot preserve integrality, infection cases are calculated as ratios of infected individuals to total number of individuals. Therefore, output variables $Z, Y, \alpha$ are continuous variables instead of integers.

Assuming that the daily workload of doctors is equal, each doctor in hospital $h$ sees $\frac{W_{h,t}'}{W_{h,t}'}$ patients on day $t$. Consider the case for a single doctor who is uninfected and has $\frac{W_{h,t}'}{W_{h,t}'}$ appointments. Let $p'$ be the probability of doctor getting infected during an appointment. We consider each appointment as a Bernoulli trial with success probability

### Table 2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Decision variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{h,t}^j$</td>
<td>Amount of supplies of mask type $j$ for period $t$</td>
</tr>
<tr>
<td>$W_{h,t}^j$</td>
<td>Number of outpatients arriving on period $t$ in hospital $h$</td>
</tr>
<tr>
<td>$W_{h,t}^j$</td>
<td>Number of doctors at the beginning of period $t$ in hospital $h$</td>
</tr>
<tr>
<td>$R$</td>
<td>Reproduction rate</td>
</tr>
<tr>
<td>$P_{pop,h}$</td>
<td>Size of the population for which hospital $h$ provides service</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Outward infection rate without mask</td>
</tr>
<tr>
<td>$\beta_{j}$</td>
<td>Outward infection rate with mask type $j$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Inward infection rate without mask</td>
</tr>
<tr>
<td>$\theta_{j}$</td>
<td>Inward infection rate of mask type $j$</td>
</tr>
<tr>
<td>$a_{h,j}$</td>
<td>Ratio of infected arriving patients to hospital $h$ at the beginning</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Service level threshold for deprivation</td>
</tr>
<tr>
<td>$X_{h,t}^j$</td>
<td>Amount of supply for mask type $j$ allocated to hospital $h$ on period $t$</td>
</tr>
<tr>
<td>$Y_{h,t}$</td>
<td>Ratio of infected doctors at the end of period $t$ in hospital $h$</td>
</tr>
<tr>
<td>$\alpha_{h,t}$</td>
<td>Ratio of infected individuals in the environment of hospital $h$ at the beginning of period $t$</td>
</tr>
<tr>
<td>$Z_{h,t}$</td>
<td>Ratio of number of primary infections in period $t$ in hospital $h$ to the daily number of outpatients</td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Possible cases</th>
<th>Probability of occurring</th>
<th>$P_{h,t}^1$</th>
<th>$P_{h,t}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both wearing mask</td>
<td>$(1 - \frac{U_{h,t}'}{W_{h,t}'}) (1 - \frac{U_{h,t}'}{W_{h,t}'})$</td>
<td>$\beta_{h,t}$</td>
<td>$\theta_{h,t}$</td>
</tr>
<tr>
<td>None wearing mask</td>
<td>$\frac{U_{h,t}'}{W_{h,t}'} \frac{U_{h,t}'}{W_{h,t}'}$</td>
<td>$\theta$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Only patient wearing surgical mask</td>
<td>$(1 - \frac{U_{h,t}'}{W_{h,t}'}) \frac{U_{h,t}'}{W_{h,t}'}$</td>
<td>$\theta_{h,t}$</td>
<td>$\theta_{h,t}$</td>
</tr>
<tr>
<td>Only doctor wearing respirator</td>
<td>$\frac{U_{h,t}'}{W_{h,t}'} (1 - \frac{U_{h,t}'}{W_{h,t}'})$</td>
<td>$\theta_{h,t}$</td>
<td>$\theta_{h,t}$</td>
</tr>
</tbody>
</table>

Z. Dönmez et al.
1 − \beta \) (Tuckwell and Williams, 2007). Since the number of patients per doctor may not be an integer, number of trials is taken as \( \frac{W'_{h,i}}{W_{h,i}} \). For the doctor to stay uninfected at the end of the day the outcome of all \( \frac{W'_{h,i}}{W_{h,i}} \) trials should be success.

Assuming that the trials are independent, the number of successes follows a Binomial distribution and it being \( \frac{W'_{h,i}}{W_{h,i}} \) has a probability of \( (1 - \beta)[(1 - \beta)^\alpha] \). The expected number of doctors infected in day \( h \) in hospital \( h \) is given as \( W'_{h,i} \times (1 - (1 - \beta)[(1 - \beta)^\alpha]) \). Therefore, deprivation time can be defined as waiting time for service equal to the threshold, it is said that the service level is good enough. Hence the expected number of primary infections would be \( a_{h,j} = (a_{h,j-1})(1 + \frac{R}{10} + \frac{Z_{h,t-1}W'_{h,i}}{PoP_h}) \) (5).

Given that the supply for masks is quite scarce during a pandemic, they should be allocated to a set of hospitals considering fairness between these hospitals. To this end, a fairness function which takes the form of a DCF is established using shortage values as a metric. In our case, DCF is caused by the lack of access to proper protective equipment which cause probability of infection to increase for individuals. Deprivation time (waiting time for PPE) and distribution vector \( X \) (and hence the shortage amounts) are the main inputs to DCF. The rate of increase in deprivation cost gets larger as the deprivation time increases. Therefore, it is expected that a DCF should be monotonic, non-linear and convex with respect to the deprivation time.

By using a proper DCF, aim in this study is to minimize the number of consecutive periods for which demand is poorly satisfied. Each period is evaluated to specify a poor service level. In our case, for all hospitals and all time periods, shortage may always occur due to the high scarcity of masks. Therefore rather than taking a binary approach in which any positive shortage amount indicates poor service, we define the service level as the proportion of demand received by the hospital and declare poor service if the demand for a mask type \( j \) is met less than a pre-determined proportion (\( \delta \)) on a period \( t \). Then it is said that the service level is poor for a hospital \( h \); if it is bigger than or equal to the threshold, it is said that the service level is good enough. Therefore, deprivation time can be defined as waiting time for service level to increase to \( \delta \) again. Assuming that the beginning deprivation cost values (\( DC_{h,j}^0 \)) are zero, deprivation (\( DC_{h,j}^t \)) is the accumulated shortage until a good service level as Eq. (6) shows.

The system starts with a given ratio of infected patients who visit the hospitals at the beginning of the first period, \( a_{h,1} \). At the end of each period \( t \), \( Z_{h,t}W'_{h,i} \) new cases arise due to hospital appointments (primary infections). Secondary cases due to primary infections are past days is already included as \( a_{h,j-1} \). While calculating \( a \) for the beginning of the next day, ratio of already infected individuals, \( a_{h,j-1} \), is added as well. Since we investigate the initial phase in which there is scarcity, our planning horizon is shorter than the average recovery time of the virus. Based on this, all infected individuals remain infected in the population throughout the planning horizon.

Proportion of the number of these new infections to the hospital’s share of patients living in the city indicates the increase in the ratio of infectious individuals in the neighborhood of hospital \( h \) at the beginning of period \( t \). Following this idea \( a_{h,j} \), where \( t \geq 2 \), is given in Eq. (5).

\[ a_{h,j} = (a_{h,j-1})(1 + \frac{R}{10} + \frac{Z_{h,t-1}W'_{h,i}}{PoP_h}) \]

A binary variable \( SL_{h,j} \) is defined based on Eq. (7) indicating the quality of the service level (value 1 means poor service whereas 0 means good service).

\[ SL_{h,j} = \begin{cases} 1, & \text{if } U_{h,j}^t \geq \delta W_{h,i}^t, \forall t \in T, \forall h \in H, \forall j \in J \\ 0, & \text{otherwise} \end{cases} \]

An instance for a hospital \( h \) and mask type \( j \) with \( \delta = 20\% \) is shown in Table 4, where \( |T| = 7 \). Demand of hospital \( h \) for mask type \( j \) is assumed to be constant and equals to 50 for all periods. Then, if shortage for a period is less than or equal to 20\% × 50 = 10, required service level is achieved and deprivation does not occur. If shortage is more than or equal to 11 units, deprivation occurs. One of
the objective functions of the allocation inventory model is determined as minimizing the maximum deprivation cost among the hospitals.

In addition to fairness, disease spread is another performance measure that is affected by the mask allocation decisions. Number of infections due to medical procedures conducted in hospitals should be minimized to decelerate the spreading of the disease. Ratio of infected doctors and infected patients can be used as the objective functions for this purpose. Preliminary work shows that these two objective functions are highly correlated as one can expect. Hence one of these objective functions is used in the mathematical model. As health care workers should be prioritized during pandemics and epidemics (Binkley and Kemp, 2020; Emanuel et al., 2020), ratio of infected doctors accumulated during the planning horizon is selected as an objective function for the mathematical model. To further establish equity, all objective functions are formed using the Rawlsian approach.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{t,h}$</td>
<td>35</td>
<td>20</td>
<td>25</td>
<td>43</td>
<td>50</td>
<td>32</td>
<td>42</td>
</tr>
<tr>
<td>$\theta_{t,h}$</td>
<td>15</td>
<td>30</td>
<td>25</td>
<td>7</td>
<td>0</td>
<td>18</td>
<td>8</td>
</tr>
<tr>
<td>$\delta_{k,t,\jmath}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Delta C_{t,\jmath}$</td>
<td>15</td>
<td>15+30</td>
<td>15+30+25</td>
<td>0</td>
<td>0</td>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that due to the trade-off between the two objectives, the multi-objective multi-period non-linear allocation inventory problem can be modeled as in Eqs. (8)–(25) given in Box 1.

Objective function (8), $\text{Obj}_y$, minimizes the maximum number of infected health workers among hospitals during the planning horizon as a metric of disease spread due to inadequate PPE use of hospitals whereas objective function (9), $\text{Obj}_DC$, is to minimize the maximum deprivation cost over hospitals. Constraints (10) ensure that all daily supply is distributed and Constraints (11) are the inventory balance equations. Constraints (12) ensure that shortage and inventory cannot occur together for a specific hospital and a specific time period and Constraints (13) indicate the beginning inventory levels. Constraints (14) and (15) calculate deprivation cost related to each mask type for each hospital for the periods where service level is poor. Otherwise, they assign deprivation cost as zero. The service level is labeled as poor or good in constraints (16) and (17). Constraints (18) are to find infection rate during an appointment due to an infectious patient while constraints (19) calculate the same rate due to an infected doctor. Constraints (20) indicate the ratio of infected doctors, (21) calculates ratio of infected arriving patients and (22) specifies ratio primary infections during appointments. Constraints (23)–(25) are the domain constraints.

5. Solution approaches

In this section we elaborate on the solution approach that we use to solve the bi-objective mask distribution problem discussed in Section 4. Our aim is to determine a set of good quality solutions with respect to the two objectives, to be presented to the decision makers for further comparison.

Note that due to the trade-off between the two objectives, the problem will not have a single solution that performs best with respect to both objectives, rather a set of Pareto solutions that perform to different degrees in the two criteria. A Pareto solution is a (feasible) solution of the problem such that there is no other (feasible) solution that performs at least as good as it in all criteria, while being strictly better with respect to at least one criterion.

Various exact and heuristic approaches for multi-objective optimization problems are used in the literature to determine the set (or a subset of) exact and approximate Pareto solutions, respectively. One common way of generating Pareto solutions is iteratively solving single objective scalarization problems, each time with different parameters so as to generate different Pareto solutions. Weighted sum, $\epsilon$-constraint and Tchebycheff scalarization are widely used examples of these scalarizations.

We utilize $\epsilon$-constraint method to solve the non-linear bi-objective mathematical model. Even for small sized instances it is challenging to obtain all non-dominated solutions for such a model in the exact Pareto front. We therefore utilize two heuristic methods: (i) a set of rule-of-thumb policies, which can be applied easily without any computational effort, (ii) a novel meta-heuristic approach, a genetic algorithm (GA) to establish an approximate Pareto frontier.

In the upcoming sections, we provide the details of the rule-of-thumb policies and the customized genetic algorithm as ways to obtain heuristic solutions for the dimensions where solving the non-linear multi-period mathematical model to optimality will be computationally extremely challenging.

5.1. Solving the mathematical model

Solving our mathematical model even with a single objective is quite challenging as it is a highly non-linear mixed integer programming model. To get the exact Pareto solutions, the $\epsilon$-constraint algorithm necessitates solving a sequence of the following two models:

\[
\text{min} \quad \text{Obj}_y
\]
\[
\text{s.t.} \quad \text{Obj}_DC \geq \sum_{\ell \in T} \sum_{j \in J} \text{DC}_{h,j} \quad \forall h \in H
\]
\[
(10) - (25)
\]

and

\[
\text{min} \quad \text{Obj}_DC
\]
\[
\text{s.t.} \quad \text{Obj}_y \geq \sum_{\ell \in T} \sum_{j \in J} \text{DC}_{h,j} \quad \forall h \in H
\]
\[
(10) - (25)
\]

These models not only have non-convex quadratic constraints as (12), (15), (17), (18) and (19) but also constraints involving high exponent variables such as (20)–(22). Starting with version 9.0, Gurobi Optimization has been offering a novel bilinear solver feature that is capable of handling non-convex quadratic optimization models to global optimality. Gurobi can also handle powers of variables by piecewise-linear approximation with user controlled piece number information — the higher the number of pieces, the better the approximation. In our preliminary experimentation, this option turned out to be computationally ineffective. Thus, we opted to do handle the variables with high integer exponent values by writing them as a sequence of quadratic products at the expense of increasing the number of variables and constraints. In particular, if our power of variable to handle is $x^p$ where $p = 2^k$ for some positive integer $k$ for simplicity, we add $k$ new variables $y_1, \ldots, y_k$ and the following quadratic equations

\[ y_1 = x^2 \quad \text{and} \quad y_i = y_{i-1}^2, \quad \text{for} \quad i = 2, \ldots, k \]

to our model. In this fashion $y_i$ will be equal to $x^p$ and any integer exponent $p$ can be handled with $O(\log_2 p)$ additional variables and quadratic constraints.

5.2. Rule-of-thumb policies

We propose some rule-of-thumb policies as basic approaches having the potential to establish a reasonably fair allocation. Such policies may be the preferred by decision makers in real-life applications, owing to their simplicity, ease of implementation and communication. For each such policy, we determine the solution and include this solution to the initial pool of the genetic algorithm as will be explained in Section 5.3.1. Following this, we observe the improvement genetic algorithm makes on these solutions by comparing them to the Pareto optimal solutions the model yields.
A naive approach namely, Equal Split policy, would be equally splitting the supply to the hospitals for each time period. Proportional Split policy is distributing masks to each hospital in proportion to the population that they give service to \((P_{oh})\). A last-in-first-receives distribution strategy is analyzed as a third policy, namely LIFR, meaning that if a hospital is visited last in the current period, it will be visited first in the next period. Such a strategy resembles cyclic priority queues with dynamically changing priorities of channels i.e. hospitals in our case (Browne and Yechiali, 1989). The priorities for the first period are determined based on the demand levels. Based on this ordering, for each hospital, 80% of the demand \((\{1 - \delta + W_{h,1}^t\}\) is satisfied when the supply is sufficient to avoid deprivation. If the supply is not enough the hospital is given what is available. In the next period, the visiting order is reversed. A similar policy can be applied by distributing all of the supply to the first visited hospital in each period, which will be referred as \(LIFR_{all}\).

Let \(O^t_{h,j}\) be the visiting order of hospital \(h\) in period \(t\) for mask type \(j\) as given in (27).

\[
\hat{H}^t_{h,j} = \{|h' \in H | O^t_{h',j} \leq O^t_{h,j} - 1\}
\]

Distribution amounts with respect to each rule-of-thumb policy are given in Table 5. Allocation vectors for the Equal Split and Proportional Split policies take integer values by rounding down operations and distributing an arbitrary hospital \(|H|h hospital the amount left from the other hospitals. For instance with 100 units of supply and 3 hospitals, Equal Split policy yields the distribution vector (33, 33, 34).

### Table 5

Distribution amounts with respect to rule-of-thumb policies.

<table>
<thead>
<tr>
<th>Policies</th>
<th>Distribution amounts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal Split</td>
<td>(X_{h,j}^t = \left\lfloor \frac{O^t_{h,j} - S^j_t - \sum_{j=1}^{H-1} X_{h,j}^t}{P_{oh}} \right\rfloor ) (\forall h \in H \setminus {H}), (X_{H,j}^t = S^j_t - \sum_{j=1}^{H-1} X_{H,j}^t)</td>
</tr>
<tr>
<td>Proportional Split</td>
<td>(X_{h,j}^t = \left\lfloor \frac{S^j_t \cdot P_{oh} - \sum_{j=1}^{H-1} X_{h,j}^t}{\sum_{j=1}^{H-1} P_{oh}} \right\rfloor ) (\forall h \in H \setminus {H}), (X_{H,j}^t = S^j_t - \sum_{j=1}^{H-1} X_{H,j}^t)</td>
</tr>
<tr>
<td>LIFR_{eq}</td>
<td>(X_{h,j}^t = \max{0, \min</td>
</tr>
<tr>
<td>LIFR_{all}</td>
<td>(X_{h,j}^t = \min{0, S^j_t - \sum_{k=1}^{j} X_{h,k}^t})</td>
</tr>
</tbody>
</table>

5.3. The proposed genetic algorithm

Inspired by the evolutionary processes seen in the nature, the genetic algorithms (GA) fall into the class of hyper-heuristics. Since
they can be applied to any problem as long as a proper solution representation can be found on which the algorithm operators can work, there have been various applications of regarding these algorithms (Mitchell, 1996; Deb et al., 2002). In multi-objective problems, a transformation of the objectives into a single one is required in order for conventional (single objective) GA operators to work. The scalarization can be achieved simply by applying a weighted sum to the objectives or their normalized values (Murata et al., 1996; Qu and Suganthan, 2010). Genetic algorithms are used to obtain good quality heuristic solutions (also referred to as approximate) solutions to problems that are difficult to solve due to various reasons such as being prohibitively large-sized or involving non-linearity. When applied to multi-objective programming problems, the algorithm returns a set of approximate Pareto solutions, i.e. solutions that may not lie on the true (exact) Pareto frontier. The general procedure followed by the proposed genetic algorithm is given in Fig. 1. We provide details of each step in the following sections.

5.3.1. Solution representation and initial population
A matrix $K$ of size $|T| \times |H| \times |J|$ with values $K_{h,j}^i$ between $[0, 1]$ is chosen as the solution representation. The algorithm starts with an initial population, which is the set of randomly generated feasible solutions and solutions obtained by the pre-determined distribution policies including the aforementioned rule-of-thumb strategies. Additionally, alternative orderings are possible. One of such alternatives, which will be referred as Rolling$_i$, adopts the allocation amounts given by the policy LIFR$_i$ with a different ordering scheme. Consider a problem with $|H|$ hospitals having $|H|!$ options for the ordering of the first period. As Equivalence (28) indicates, Rolling$_i$ increases orders by $\delta$ units after each period $t - 1$ where $c \in \{1, \ldots, |H| - 1\}$ leading $|H| - 1$ options for this increment, given that $t \geq 2$.

$$O_{h,j}^{t-1} + \epsilon = O'_{h,j} \pmod{|H|} \tag{28}$$

Number of enumerations is $(|H|!(|H| - 1))^2$ for the planning horizon for a set of $|H|$ hospitals and 2 mask types. Due to the computational effort it requires, such an approach is not considered as one of the rule-of-thumb policies. However, this policy is utilized in addition to the rule-of-thumb policies while generating the initial population for the genetic algorithm.

After solutions to the rule-of-thumb policies and the possible Rolling$_i$ orderings are found, the remaining slots are filled by solutions constructed as follows: $X_{h,j}^t = \bigcup_{i\in J, j \in T} \{X_{h,j}^i\}$ where $\hat{X}$ is a randomly picked solution from the existing solutions, for each $j \in J, i \in T$. Number of solutions obtained by Rolling$_i$ orderings is limited to ensure the initial population includes at least a pre-determined number of randomly generated solutions.

The mask distributions are scaled so that they sum to 1 for each day and mask type, to enable performing mutation and crossover between the solutions. Future populations are generated recursively; each new population is generated using the selected parents and the survivors of the previous population.

The mask distribution for each hospital, period and mask type is calculated by rounding the multiplication of available number of masks per period and the scaled scores as shown below.

$$K_{h,j}^i : \text{score of hospital } h \text{ at period } t \text{ for mask type } j \text{ in the solution.}$$

Fig. 1. Flow chart of the genetic algorithm proposed.
Unlike elites, survivors are selected using stochastic selectors from the next population without undergoing any alteration process. The best five solutions of the current population are transferred to the next population, without undergoing any alteration process. The objective values are obtained and the terminating condition is checked. If the terminating condition is not met, the algorithm continues its iterations from the fitness value calculation step (5.3.2). Otherwise, it returns the Pareto set of solutions that were observed throughout, which represents an approximation for the true Pareto frontier.

6. Computational analysis

6.1. Data generation

In order to simulate various scenarios and analyze the optimal and the heuristic solutions corresponding to these problem instances, parameters of the mathematical model are generated as shown in Table 6. For all parameters, discrete level sets are used as high, medium and low. In each instance, each parameter is generated using one of these options. The instance with worst case indicates high levels for parameters \( \beta, \beta, \gamma, \delta, \theta, \gamma, \delta, \lambda \) and low level for parameters \( \delta \), \( \delta \), \( \delta \), \( \delta \). Similarly, the instance with best circumstances is the one where parameters \( \beta, \beta, \gamma, \delta, \theta, \gamma, \delta, \lambda \) are in the low level and parameters \( \delta \), \( \delta \), \( \delta \) are in high level.

Recall that we consider the allocation problem arising at the beginning of the pandemic where mask scarcity is experienced the most, hence we generated the problem instances accordingly. Inward and outward infection rates of the masks are used as specified by (Eikenberry et al., 2020). Reproduction number is taken from (Wu et al., 2020), which estimates the reproduction number with 95% confidence interval in China in January, 2020. \( \alpha \) is calculated using the number of positive cases in Turkey during the pandemic phase and the population of the country.

In order to generate \( W_{ij} \) values, additional parameters \( \lambda \) (mean of the Poisson distribution for the arriving outpatients) and \( \delta \) (ratio of doctors to the total number of arriving outpatients for a specific hospital) are used. Similarly, for the parameters \( \lambda \) (number of residents in the neighborhood of each hospital), \( \delta \) (ratio of number of doctors to the number of individuals that live in the district for each hospital) is used and the 

\[ K_{ij} = \min \{ 1, \max \{ 0, W_{ij} \} \} \]  

which ensures that each index stays within range of [0, 1]. The mutation operator has two important roles:

- **Exploration of the search space.** The crossover operator results in solutions that are convex and/or dimensional combinations of the existing solutions hence allows limited exploration of the solution space. Mutation increases exploration of the search space by making small random changes on the solutions.
- **Maintaining diversity.** The crossover operator reduces the average distance between the individuals of the population, which will eventually result in convergence of the population into one solution. By making small random changes on the solutions, mutation maintains diversity as it changes the population’s position in the search space and hence prevents premature convergence.

5.3.6. Next population & terminating condition & the output

The next population is built by combining the offspring, the elites and the survivors. The objective values are obtained and the terminating condition is checked. If the terminating condition is not met, the algorithm continues its iterations from the fitness value calculation step (5.3.2). Otherwise, it returns the Pareto set of solutions that were observed throughout, which represents an approximation for the true Pareto frontier.

The genetic algorithms are composed of various different parts with a number of parameters, and there is not an optimal set of parameters that is guaranteed to provide the best performance in every problem. Therefore, in order to fine-tune the genetic algorithm parameters, we utilized another genetic algorithm, whose properties and procedures are described in Appendix B.

6.2. Application details for the proposed GA

The genetic algorithms are composed of various different parts with a number of parameters, and there is not an optimal set of parameters that is guaranteed to provide the best performance in every problem. Therefore, in order to fine-tune the genetic algorithm parameters, we utilized another genetic algorithm, whose properties and procedures are described in Appendix B.
Table 6
Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>2.86</td>
<td>2.68</td>
<td>2.47</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1</td>
<td>0.995</td>
<td>0.990</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.3</td>
<td>0.15</td>
<td>0.0</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1</td>
<td>0.995</td>
<td>0.990</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.05</td>
<td>0.025</td>
<td>0.0</td>
</tr>
<tr>
<td>$a_{st}$</td>
<td>0.009</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>$s_h$</td>
<td>1000</td>
<td>750</td>
<td>500</td>
</tr>
<tr>
<td>$d_{pat}$</td>
<td>0.1</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>$d_{pop}$</td>
<td>0.002</td>
<td>0.0013</td>
<td>0.0006</td>
</tr>
<tr>
<td>$d_{j}^{y}$</td>
<td>0.8</td>
<td>0.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Table 7
Objective function values obtained by the rule-of-thumb policies and the best points with respect to the two objectives in exact and approximate pareto fronts.

<table>
<thead>
<tr>
<th>Point</th>
<th>Optimistic scenario</th>
<th>Neutral scenario</th>
<th>Pessimistic scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Best^{DC}$</td>
<td>(0.0039,368)</td>
<td>(0.0443,2001)</td>
<td>(0.6172,5670)</td>
</tr>
<tr>
<td>$GBest^{DC}$</td>
<td>(0.0056,422)</td>
<td>(0.0413,2519)</td>
<td>(0.6564,6907)</td>
</tr>
<tr>
<td>$Best^{q}$</td>
<td>(0.0012,1762)</td>
<td>(0.0289,3348)</td>
<td>(0.4295,27463)</td>
</tr>
<tr>
<td>$GBest^{q}$</td>
<td>(0.0017,4525)</td>
<td>(0.0301,8549)</td>
<td>(0.4845,26666)</td>
</tr>
<tr>
<td>Equal split</td>
<td>(0.0053,3149)</td>
<td>(0.0357,12391)</td>
<td>(0.5066,19441)</td>
</tr>
<tr>
<td>Proportional split</td>
<td>(0.0050,2462)</td>
<td>(0.0356,12196)</td>
<td>(0.5602,19329)</td>
</tr>
<tr>
<td>LIFR$_t$</td>
<td>(0.0050,589)</td>
<td>(0.0481,3368)</td>
<td>(0.7052,19185)</td>
</tr>
<tr>
<td>LIFR$_{qf}$</td>
<td>(0.0890,23696)</td>
<td>(0.0850,30583)</td>
<td>(1.1491,31805)</td>
</tr>
</tbody>
</table>

6.3. Results

In this section, using the data generation procedures explained in Section 6.1, results for various scenarios are discussed. These scenarios are selected in a way that represents different situations encountered during the pandemic phase. A pessimistic scenario represents a case where the reproduction rate and beginning infected individual ratios around the hospitals, mask infection rates and number of daily visitors to the hospitals are high. Doctor to patient and doctor to population ratios are low, and supply for PPE is scarce in such a scenario. On the other hand, these parameters take reversed values in an optimistic scenario. All parameters take medium values for a neutral scenario. We first observe the Pareto frontiers obtained in each of the optimistic, neutral and pessimistic scenarios with the set of 3 hospitals and 7 days as the length of the planning horizon. The threshold for deprivation cost remains the same (20%) for all of the scenarios. Approximated Pareto fronts are compared with the true Pareto fronts for all scenarios. Finally, resulting distribution schemes of selected optimal solutions, GA solutions and rule-of-thumb policies are discussed in depth for the pessimistic scenario, which is the most likely and challenging one due to insufficient supply and higher infection rates.

Fig. 3 shows the true Pareto and the approximate Pareto respectively obtained by the mathematical model and GA for the three scenarios along with the points returned by rule-of-thumb policies. Table 7 includes the objective function values corresponding to the solutions with best values of the two objective functions, the respective GA solutions and the rule-of-thumb policies. Let us name the objective function values corresponding to endpoints of the exact Pareto as $Best^{DC}$ and $Best^{q}$ respectively. Note that each such point is a vector $\mathbf{Y} = (\mathbf{Obj}_1, \mathbf{Obj}_2)$ where $\mathbf{Obj}_1$ is the deprivation cost across all hospitals ($Obj_{1,h}$ value) while the second element showing the maximum ratio of infected doctors ($Obj_{2,h}$ value) obtained by the mathematical model and GA for the three hospitals in these instances. These two policies are most appropriate to use when a neutral or pessimistic scenario occurs to obtain acceptable $Obj_{1,h}$ values. However, it should be noted that the resulting $Obj_{1,h}$ values are quite high.

The quality of an approximate Pareto set is assessed by the closeness of its hypervolume to the hypervolume of the exact Pareto front. Given that the solution set for GA is $N_{GA}$ and the Pareto optimal solution set is $N_{Opt}$, the ratio $\frac{H(N_{Opt})}{H(N_{GA})}$ is the area of the region: $\bigcup_{n \in N_{Opt}} \{ \mathbf{y} \in \mathbb{R}^2 : n \leq y \leq \mathbf{Y} \}$, as shown in Fig. 4 for the pessimistic scenario.

As seen in Fig. 3, in an optimistic scenario, GA yields compatible results to the Pareto optimal solutions in terms of $Obj_{1,h}$, while returning less compatible values with respect to $Obj_{2,h}$. When the conditions worsen, GA performs better in terms of objective function $Obj_{1,h}$.

As the circumstances get worse the difference between the two extreme solutions of the Pareto set deepens, i.e. the ranges of the two objective function values increase. As a result, solely minimizing the maximum ratio of infected doctors ($Obj_{2,h}$) leads to very high deprivation cost figures and vice versa. Investigating the whole Pareto frontier provides the decision maker information on the trade-off between the two criteria. For example, it can be seen that a small amount of increase in $Obj_{1,h}$ from its minimum level yields a remarkable decrease in the maximum deprivation cost ($Obj_{1,h}$).

In the optimistic scenario, it is observed that $Obj_{1,h}$ value obtained by applying LIFR$_t$ is significantly closer to the best value in that objective. On the other hand, as the scenario gets worse, the performance of LIFR$_t$ solution with respect to deprivation cost declines.

It is also observed that Equal Split and Proportional Split yield quite similar results. This is expected as the populations are close for the three hospitals in these instances. These two policies are most appropriate to use when a neutral or pessimistic scenario occurs to obtain acceptable $Obj_{1,h}$ values. However, it should be noted that the resulting $Obj_{1,h}$ values are quite high.
In order to compare GA with the mathematical model and observe the trade-off between objective functions, sample solutions for a pessimistic instance are analyzed in depth, which consist of two ends ($Best^{DC}, Best^{Y}, GBest^{DC}, GBest^{Y}$) and middle points ($Mid, GMid$) of the true and approximate Pareto as shown in Fig. 3c.

Table 8 includes the supply information for mask type 1 (surg. mask) and demand vector for a pessimistic instance with 3 hospitals during 7 periods generated using the values given in Table 6. The supply for mask type 2 (resp. mask) is 169 for all periods and the demand vector for this mask type is $(141, 142, 139)$, respectively, for each hospital and constant throughout the planning horizon since the number of doctors remains the same. Tables C.11–C.13 give the distribution matrix $X$ respectively for the solutions resulted from $Best^{DC} - GBest^{DC}$, $Best^{Y} - GBest^{Y}$ and $Mid - GMid$ pairs for the pessimistic scenario.

<table>
<thead>
<tr>
<th>Periods</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_t$</td>
<td>1176</td>
<td>1181</td>
<td>1214</td>
<td>1217</td>
<td>1238</td>
<td>1205</td>
<td>1171</td>
</tr>
<tr>
<td>$W_{t1}$</td>
<td>974</td>
<td>962</td>
<td>995</td>
<td>1054</td>
<td>1032</td>
<td>974</td>
<td>1032</td>
</tr>
<tr>
<td>$W_{t2}$</td>
<td>993</td>
<td>977</td>
<td>1024</td>
<td>1026</td>
<td>1063</td>
<td>1044</td>
<td>938</td>
</tr>
<tr>
<td>$W_{t3}$</td>
<td>971</td>
<td>1012</td>
<td>1016</td>
<td>961</td>
<td>999</td>
<td>993</td>
<td>956</td>
</tr>
</tbody>
</table>

The allocation strategies corresponding to the best solutions with respect to the deprivation cost are quite similar in GA and the mathematical model as seen in Fig. 5. The solutions indicate that starting from a hospital, distributing $(1-\delta)\%$ of the demand and performing the same
activity for a different hospital in each period yields a significant decrease in Obj$_{DC}$ as it prevents consecutive number of periods with high shortage. Therefore, visiting hospitals in a rolling manner throughout the planning horizon and meeting the demand partially based on the service level threshold ($\delta$) is the strategy that should be adopted while minimizing Obj$_{DC}$. It can be also observed that the amounts distributed...
are aligned with the ones given by the policy \( \text{LIFR}_a \) with a different visiting order. There is a regular visiting order followed for surg. mask (each hospital is visited first, second and third in almost same number of time periods).

The percentage of supply allocated to each hospital in each period is shown in Fig. 6 for solutions \( \text{Best}^Y \) and \( \text{GBest}^Y \). Note that \( \text{Obj}_Y \) focuses on equitable protection of the doctors (equity in the outcome) rather than equitable distributing the masks (equity in resource allocation) and it may ensure this through different means. This can be observed in solutions: Almost all of the supply for the surg. mask is allocated to a single hospital (1st hospital in the solutions) whereas the respiratory mask is distributed to another hospital in each period. As a consequence of this strategy, for the doctors in the 1st hospital the infection rate is reduced by equipping the patients with mask rather than the doctors. For the other two hospitals, most of the doctors are provided with masks either at the beginning or at the end of the time horizon. The main difference between two solutions is how the surgical mask is distributed in the second period. While the supply is allocated to 1st hospital in the optimal solution, this amount is given to 2nd hospital in the heuristic solution of GA. Both of the solutions change the strategy in the last periods to make up for the neglected hospitals, i.e., 2nd hospital is given more of surg. mask and 1st hospital is given more of resp. mask in the last periods for both of the solutions. Overall, the distribution schemes are quite similar as Fig. 6 indicates.

The distribution strategy of both the mathematical model and GA while minimizing \( \text{Obj}_Y \) is aligned with the \( \text{LIFR}_{all} \) in terms of the distributed amounts. Namely, allocating almost all of the supply to a single hospital for each mask type in each period is common for solutions obtained by the model, GA and \( \text{LIFR}_{all} \). Although last-in-first-receives ordering is not utilized in the optimal and the heuristic solutions unlike \( \text{LIFR}_{all} \), the similarity indicates that variants of \( \text{LIFR}_{all} \) with different ordering schemes can be used to improve the objective function \( \text{Obj}_Y \).

The percentage of demand satisfied in each period for each hospital is given as \( (\frac{I_{t} - I_{t-1} + X_{t}}{W_{t}}) \). Fig. 7 shows the relationship between the ratio of infected doctors \( (Y_{t}) \) and demand satisfaction rates for each period in the solution of the point \( \text{Best}^Y \). The most significant observation that can be driven from Fig. 7 is that even though the \( \text{Obj}_Y \) value mostly depends on the demand satisfaction rate of the doctors rather than the patients, it is possible to control the infection rate of the doctors by fully equipping the patients. Although the doctors are not provided with resp. mask in the first 4 periods for the 1st hospital, increase rate for the non-cumulative ratio of the infected doctors is slow since the demand satisfaction rate of the patients is 1. As 2nd and 3rd hospitals take turns being allocated resp. mask and are not provided with surg. mask, the periods where they do not receive resp. mask result in quite high values of \( \text{Obj}_Y \). For this reason, periodic value of the \( \text{Obj}_Y \) fluctuates for 2nd and 3rd hospitals whereas it has a relatively smaller (due to high demand satisfaction rate for patients) but continuous increase for the 1st hospital.

The analysis of two ends gives us the ability to explain the reason for the trade-off between two objective functions: Trying to meet the
demand partially based on a pre-determined service level threshold as a way to decrease $\text{Obj}_{DC}$, prevents fully protecting at least one side of the medical appointments namely, patients or doctors. This results in higher $\text{Obj}_Y$ values. On the other hand, fully covering at least one of these parties for each hospital would yield a rapid increase in the deprivation cost values for the other hospitals since the demand for at least one mask type is unsatisfied. When a pessimistic scenario occurs, supply to demand ratio gets smaller and this trade-off due to unsatisfied demands becomes more significant.

The solutions regarding $\text{Mid}$ and $\text{GMid}$ balance the two objective functions unlike the end points. In Fig. 8, it is observed that GA solution allocates all supply to a single hospital in the majority of periods, similar to $\text{LIFR}_{all}$. On the other hand, the optimal solution resembles a policy that is a combination of $\text{LIFR}_a$ and $\text{LIFR}_{all}$ policies. Overall, the solution $\text{Mid}$ has a more complicated distribution scheme, which is not induced by any of the rule-of-thumb policies.

7. Conclusion

Public facilities continuing their services during pandemics should prioritize the safety of their medical staff as well as the public health. Even though the disease continues to spread, reducing the transmission rates within the facility is possible with precautions and measures such as PPE usage and source control. Since healthcare is one of the essential services and occasionally requires close-contact medical procedures, hospitals and health centers should be prioritized while allocating PPE resources. Considering the inadequacy of such a resource during the early phases of a pandemic, allocation between the healthcare facilities should be done in a fair and efficient manner. The aim of this paper is to establish such a PPE allocation while (i) minimizing the disease transmission within the hospitals (ii) distributing the shortage which is likely to occur due to scarcity equivalently between the hospitals. To this end, a bi-objective non-linear multi-period allocation model is formulated where non-linearity comes from both the spreading scheme of the disease and way of penalizing shortage with an increasing rate using deprivation cost. The non-linear model is solved to optimality for small sized instances whereas a set of simple policies called rule-of-thumb policies and a customized genetic algorithm are proposed for solving larger problems.

In order to resemble possible real life situations during a pandemic, three scenarios are generated using various levels for problem parameters: pessimistic, neutral and optimistic. Both true and approximate Pareto sets are obtained for these scenarios besides the solutions obtained by rule-of-thumb policies. Optimal and heuristic distribution solutions (end points and a middle point from the Pareto sets) are detailed for the pessimistic scenario. The comparison analysis shows that:

- Hypervolume criterion which evaluates the area of two Pareto fronts, shows that the area implied by customized GA is closer to the area of optimal Pareto front and gives more compatible $\text{Obj}_{DC}$ range as the scenario worsens. Additionally, the range between best and worst values of objective functions in Pareto sets gets larger in a pessimistic case.
- Deprivation cost can be improved by meeting demand partially based on deprivation cost threshold for a different hospital in each period while the maximum ratio of infected doctors is decreased by maximizing either patient or doctor demand satisfaction rate for each hospital. The latter requires allocating a single type of mask to a hospital throughout the planning horizon and results in higher values of deprivation cost due to the unsatisfied demand of the other mask type.
- Solutions that balance two objective functions give results such as $\text{Mid}$, and are not implied by any of the rule-of-thumb policies.

To the best of our knowledge, there is no OR study that focuses on protective equipment allocation to a set of hospitals considering the effect of this allocation on infections during medical appointments. Although this study fills this gap, it is not free from some limitations and several directions for future work can be proposed. First, the assumption of daily screen tests for doctors can be relaxed and demand for resp. masks can be considered as a variable for each period depending on the previous demand satisfaction rates. Secondly, it is assumed that an infected person starts infecting other people
immediately. Various values for the time between infection and infectiousness can be considered as a possible direction for future work. Lastly, although the allocation model is constructed as deterministic, a decision maker is likely to encounter uncertainties in such a real-life problem. For instance, infection rates are mostly determined by usage of PPE (e.g. higher when not worn properly). Therefore, methods such as stochastic optimization and robust programming can be used to capture such uncertainties.

CRediT authorship contribution statement

Zehranaz Dönmez: Methodology, Writing – original draft, Formal analysis. Serkan Turhan: Methodology, Software, Writing – original draft. Özlem Karsu: Writing – review & editing, Validation, Formal analysis. Bahar Y. Kara: Conceptualization, Supervision, Writing – review & editing. Oya Karaşan: Software, Supervision, Writing – review & editing.

Acknowledgments

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Appendix A. Generating hospital parameters

Algorithm 1: Generating the hospital related data

<table>
<thead>
<tr>
<th>Input:</th>
<th>Output:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>T</td>
</tr>
</tbody>
</table>

/* incoming patients */

1. for $h \in H, t \in T$ do
2. $W_{h,1} = \text{Poisson}(\lambda_h)$ /* available doctors */
3. for $h \in H$ do
4. $W_{h,2} = \sum_{t \in T} W_{h,1} \times d_{pat}$ /* populations */
5. for $h \in H$ do
6. $Pop_{h} = \frac{W_{h,2}}{d_{pop}}$ /* available number of masks */
7. for $t \in T$ do
8. $S_{1t} = d_{1t} \times \sum_{h \in H} W_{h,1}$ /* surg. mask */
9. $S_{2t} = d_{2t} \times \sum_{h \in H} W_{h,2}$ /* resp. mask */
10. return $S_{1t}, W_{h,1}, W_{h,2}, Pop_{h}$
Appendix B. Fine tuning genetic algorithm

We propose a new genetic algorithm in order to fine tune the parameters of the main algorithm. The details of the fine tuning algorithm are as follows:

B.1. Fine tuning algorithm

The main GA procedures are implemented in the same way as described in Section 5.3. We set population size, offspring count, survivor count and elite count to 100, 50, 20 and 30, respectively. We use Roulette Wheel Selection (Lipowski and Lipowska, 2012) and Tournament Selection with tournament size of 5 (Miller and Goldberg, 1995) as parent and survivor selectors, respectively. The algorithm is terminated if there is no update on the best solution over the last 10 populations. A sample size of 5 and 20 benchmark instances were used to assess the performance of settings, instances can be found in Table B.9. Table B.10 shows the algorithm settings obtained after fine tuning the GA.

Table B.9
Benchmark instances used in the fine-tuning algorithm.

<table>
<thead>
<tr>
<th>Instance name</th>
<th>T</th>
<th>H</th>
<th>(d_{x_1}^i)</th>
<th>(d_{x_2}^i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instance 1</td>
<td>7</td>
<td>3</td>
<td>89.96%</td>
<td>82.14%</td>
</tr>
<tr>
<td>Instance 2</td>
<td>7</td>
<td>3</td>
<td>84.8%</td>
<td>64.29%</td>
</tr>
<tr>
<td>Instance 3</td>
<td>7</td>
<td>4</td>
<td>65.38%</td>
<td>74.92%</td>
</tr>
<tr>
<td>Instance 4</td>
<td>7</td>
<td>4</td>
<td>74.88%</td>
<td>71.5%</td>
</tr>
<tr>
<td>Instance 5</td>
<td>7</td>
<td>5</td>
<td>75.27%</td>
<td>78.57%</td>
</tr>
<tr>
<td>Instance 6</td>
<td>7</td>
<td>5</td>
<td>60.22%</td>
<td>64.26%</td>
</tr>
<tr>
<td>Instance 7</td>
<td>7</td>
<td>5</td>
<td>57.15%</td>
<td>82.68%</td>
</tr>
<tr>
<td>Instance 8</td>
<td>7</td>
<td>4</td>
<td>57.15%</td>
<td>82.68%</td>
</tr>
<tr>
<td>Instance 9</td>
<td>8</td>
<td>3</td>
<td>58.07%</td>
<td>62.36%</td>
</tr>
<tr>
<td>Instance 10</td>
<td>8</td>
<td>3</td>
<td>43.94%</td>
<td>80%</td>
</tr>
<tr>
<td>Instance 11</td>
<td>8</td>
<td>3</td>
<td>65.38%</td>
<td>74.92%</td>
</tr>
</tbody>
</table>

Table B.10
Fine-tuned genetic algorithm properties.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>361</td>
</tr>
<tr>
<td>Time limit</td>
<td>3600 s</td>
</tr>
<tr>
<td>Survivor count</td>
<td>171</td>
</tr>
<tr>
<td>Elite count</td>
<td>90</td>
</tr>
<tr>
<td>Offspring count</td>
<td>100</td>
</tr>
<tr>
<td>Parent selection</td>
<td>Tournament selector with a size of 73</td>
</tr>
<tr>
<td>Survivor selection</td>
<td>Truncation selector (Crow and Kimura, 1979)</td>
</tr>
<tr>
<td>(\sigma_{mut}) (Mutator Variance)</td>
<td>0.63</td>
</tr>
<tr>
<td>Crossover operator</td>
<td>Convex combination</td>
</tr>
</tbody>
</table>

B.1.1. Initial population

The initial pool of algorithm settings is generated using the following random distributions and set of candidate parameters:

- **Population Size**: Random integer between 10 and 1000.
- **Offspring, survivor and elite count**: Population size is randomly distributed to each parameter using Algorithm 2.
- **Mutation parameter** \(\sigma_{mut}\): U(0, 1).
- **Crossover probability**: Randomly drawn from (0, U(0, 1), 1). By setting the mating chance as zero, only the mutation operator alters the population, which may produce better results by saving processing time.
- **Crossover operator**: Randomly selected as one of the following: Single Point, Two Point, Uniform with swap probability drawn from U(0,1), and convex combination crossover (Hasançebi and Erbatur, 2000; Hamid et al., 2019).
- **Parent and survivor selectors**: Randomly selected as one of the following:
  - Roulette Wheel Selection (Lipowski and Lipowska, 2012).
  - Tournament Selection (Miller and Goldberg, 1995). Tournament size = \(\lfloor\text{population}\_\text{size} \times \text{ratio}\rfloor\), where ratio: U(0, 1).
- **Linear Rank Selection** (Shukla et al., 2015). Parameter: U(0, 2).
- **Exponential Rank Selection** (Shukla et al., 2015). Smoothing coefficient: U(0, 1).
- **Truncation Selection** (Crow and Kimura, 1979).
- **Weight Policy**: The fine-tuning algorithm considers different fitness value calculation techniques in the form of weighted sum functions of the normalized objective function values. The fine tuning algorithm checks the performance of the following weight selection strategies:
  - Pre-determined Weights: \((\omega, 1 - \omega)\), where \(\omega = \frac{1}{\Delta}x \in [1, 2, \ldots, \Delta]\) with \(\Delta\) randomly selected from \(\{1, 2, \ldots, 10\}\). Weights are updated following the same order, with each set having equal amount of run time.
  - Random Weights: \((\omega, 1 - \omega)\) where \(\omega \sim U(0, 1)\). Weights are randomized after every N populations. N is randomly selected from \(\{10, 20, 50, 75, 100, 125, 150, 200, 300, 400\}\) for a setting.

We implemented the fine-tuning algorithm for a month and observed that best results are obtained under the pre-determined weights policy with \(\Delta = 2\). Hence we use this policy in our main experiments. This policy implies that the algorithm will focus on the first objective in the first half, while switching to the other objective in the second half in fitness value calculations.

B.1.2. Fitness value calculation

Fitness values indicate the quality of the Pareto front found by each setup compared to the other settings in the population. The calculation is shown in Algorithm 3.

B.1.3. Crossover and mutation

After the selection process, algorithm follows the same rules described in Section 5.3. The algorithm settings are explored by crossover and mutation operators as shown in Algorithms 4 and 5. Due to the immense amount of sub-problems needed to be solved to run this algorithm, it was parallel processed in 20 threads, evaluation of the initial population took 20.8 h and the remaining populations took 10.4 h, because only the offspring were evaluated. Each algorithm setting was run for 150 s for each problem and sample to obtain results in a reasonable time.

B.2.

Algorithm 2: Generating uniformly distributed population properties using a fixed population size

**Input**: population\_size

**Output**: offspring\_count, survivor\_count, elite\_count

/* constrained uniform-random weight set generation, with set summing up to 1 */
1 weights = (0, U(0,1), U(0,1), 1)
2 sorted_weights = sort(weights)
/* generating the uniformly distributed values, summing up to population\_size */
3 offspring\_count = \(\lfloor\text{sorted_weights}_1 - \text{sorted_weights}_0 \times \text{population}\_\text{size}\rfloor\)
4 survivor\_count = \(\lfloor\text{sorted_weights}_2 - \text{sorted_weights}_0 \times \text{population}\_\text{size}\rfloor\)
5 elite\_count = population\_size - offspring\_count - survivor\_count
6 return offspring\_count, survivor\_count, elite\_count
Algorithm 3: Fitness value generator of the fine-tuning algorithm

```
Input: population, sample_size, benchmark_instances
Output: fitness_values

/* Initialization */
fitness_values = {0 ∀ solution ∈ population}
/* Evaluation */
for benchmark_instance ∈ benchmark_instances do
    /* Sample each solution and calculate the Pareto front */
    for solution ∈ population do
        if fitness_value(solution) ∈ benchmark_instance then
            sampled_Pareto_front = solve_instance_with_settings(solution, benchmark_instance)
            sampled_Pareto_front = sampled_Pareto_front ∪ Pareto_frontsolution
            Pareto_frontsolution = efficient_solutions(Pareto_frontsolution)
            /* Increment the fitness values by the number of efficient solutions in the Pareto front of solutions obtained by every solution for this benchmark instance */
            population_Pareto_front = efficient_solutions(Pareto_frontsolution)
            for solution ∈ population, efficient_solution ∈ Pareto_frontsolution do
                if fitness_value(solution) ∈ population_Pareto_front then
                    fitness_values = fitness_values + 1
    return fitness_values
```

B.4.

See Algorithm 4.

B.5.

See Algorithm 5.

Appendix C. Solutions obtained by end points and middle points of the optimal and approximate pareto fronts for the pessimistic scenario

C.1.

See Table C.11.

C.2.

See Table C.12.

Algorithm 4: Crossover operator used in the fine-tuning algorithm

```
/* Initialization */
Offspring1 = Parent1
Offspring2 = Parent2
/* Re-combine the population size properties */
for parameter ∈ {offspring_count, survivor_count, elite_count} do
    ϕ = U(0, 1)
    Offspring1.parameter = ϕ × Parent1.parameter + (1 − ϕ) × Parent2.parameter
    Offspring2.parameter = (1 − ϕ) × Parent1.parameter + ϕ × Parent2.parameter
/* Re-adjust the population size parameters */
Offspring1.population_size = ⌈floor(Offspring1.population_size × (1 − ϕ))⌉
Offspring2.population_size = ⌈floor(Offspring2.population_size × (1 − ϕ))⌉
/* Recombine the crossover and selection operators */
for operator ∈ {crossover_operator, parent_selection_operator, survivor_selection_operator} do
    if Offspring1.operator = Offspring2.operator then
        /* Recombine the parameters of the operators, if they are the same type */
        ϕ = U(0, 1)
        Offspring1.operator.parameter = ϕ × Parent1.operator.parameter + (1 − ϕ) × Parent2.operator.parameter
        Offspring2.operator.parameter = (1 − ϕ) × Parent1.operator.parameter + ϕ × Parent2.operator.parameter
    else
        /* Swap operators with a probability of 25% */
        if U(0, 1) ≤ 0.25 then
            Offspring1.operator = Parent2.operator
            Offspring2.operator = Parent1.operator
        else
            /* Weight policy */
            if U(0, 1) ≤ 0.1 then
                Offspring1.weight_policy = Parent2.weight_policy
                Offspring2.weight_policy = Parent1.weight_policy
            /* Recombine the parameters of the operators, if they are the same type */
            Offspring1.weight_policy = Parent2.weight_policy
            Offspring2.weight_policy = Parent1.weight_policy
        return Offspring1, Offspring2
```
Algorithm 5: Mutation operator used in the fine-tuning algorithm

Input: Offspring
Output: Offspring
1 // Initialize */
2 Offspring = Offspring
3 // Mutate the population size while holding the offspring, survivor and elite ratio the same */
4 if U(0, 1) ≤ 0.10 then
5 Offspring.population_size = [U(10, 1000)]
6 for parameter ∈ [offspring_count, survivor_count] do
7 Offspring.parameter = Offspring.population_size
8 Offspring.elite_count = Offspring.population_size – Offspring.offspring_count – Offspring.survivor_count
9 // Mutate the offspring, survivor and elite ratios */
10 if U(0, 1) ≤ 0.20 then
11 // Call Algorithm 2 */
12 Offspring.offspring_count, Offspring.survivor_count, Offspring.elite_count = Algorithm 2(Offspring.population_size)
13 // Mutation parameter */
14 if U(0, 1) ≤ 0.10 then
15 // Crossover probability */
16 if U(0, 1) ≤ 0.20 then
17 // Crossover and selection operators */
18 for operator ∈ {crossover_operator, parent_selection_operator, survivor_selection_operator} do
19 if U(0, 1) ≤ 0.05 then
20 // Weight policy */
21 if U(0, 1) ≤ 0.1 then
22 return Offspring

References


