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A genetic game of trade, growth and externalities

Nedim M. Alemdar, Süheyla Özyıldırım*

Department of Economics, Bilkent University, 06533 Bilkent, Ankara, Turkey

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Abstract

A genetic algorithm is introduced to search for optimal policies in the presence of knowledge spillovers and local pollution in a dynamic North/South trade game. Noncooperative trade compounds inefficiencies stemming from externalities. Cooperative trade policies are efficient and yet not credible. Short of a joint maximization of the global welfare, transfer of knowledge remains as a viable route to improve world welfare.

JEL classification: C61; C73; F42

Keywords: North/South dynamic trade game; Genetic algorithms; Externalities

1. Introduction

This paper aims to contribute to the recent literature exploring linkages between international trade, environmental degradation, and growth by bringing to the fore the dynamic gaming aspects of these issues.[1] The framework we adopt is a dynamic trade game between North and South.

An extensive literature exists studying various aspects of the North/South trade. Galor (1986), an early precursor to this study, emphasizes the dynamic inefficiency of a noncooperative North/South trade wherein North is the engine of growth and South has a comparative advantage in resource production. Noncooperative resource pricing chokes up growth in the North. Labor market imperfections in the South, the surplus labor concomitant with positive

* Corresponding author. E-mail:suheyla@bilkent.edu.tr

[1] See among others Markusen (1975), Clemhout and Wan (1985), Levhari and Mirman (1980), and Dockner and Long (1993).

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subsistence wages, are the main culprit for the inefficiency. These market imperfections may, however, be a blessing in disguise if resource extraction pollutes the Southern environment; diminished Northern growth will check the pollution level in the South about which the North is indifferent.

In a series of papers, Chichilnisky (1993, 1994) draws attention to the salient feature of resource extraction in the South; (domestic) open access to Southern resources results in overexploitation and excessive pollution. Copeland and Taylor (1994) study the volume and the composition of trade between the North and South. Asymmetric endowment of human capital leads to trade between otherwise similar regions. Starting with a low level of human capital, South specializes in low-skill/pollution-intensive goods so that in the aftermath of trade, along with income the level of pollution rises. However, the long-run effects of trade on growth and pollution are unclear since accumulation of human capital is not allowed.

Neither Chichilnisky nor Copeland and Taylor consider the likely consequences of trade on knowledge spillovers. To the extent, trade also gives rise to knowledge spillovers, its undesirable impact on the pollution level will be mitigated. The beneficial effects of trade on capital accumulation or research and development are discussed by Grossman and Helpman (1991).

Our paper then addresses the issues related to trade and environment in a growth-cum-externalities setup. We intend to capture the impact of the Northern growth on the quality of the Southern environment in the presence of transboundary knowledge spillovers. Knowledge accumulation and spillovers bring about additional growth/pollution tradeoffs, and thus, help us identify sources of inefficiencies that have been hitherto overlooked.

The North/South trade in our model is specialized as in Galor (1986). In addition, resource extraction causes environmental pollution in the South. Also, knowledge accumulates in the North and diffuses costlessly to the South highlighting the public good nature of investment. We let the authorities in the South levy an export tax (or an import tariff) to internalize the local social cost of pollution and also to exploit their resource monopoly.

An additional contribution of the paper is in its methodology: We introduce a general purpose *genetic algorithm* (GA) to solve open-loop differential games of infinite duration. The lack of attention paid to the development of computational techniques to solve such problems was first addressed by Pau (1975a,b). We implement the algorithm *GA* to solve numerically the North/South dynamic trade game.

In the GA search for optimal regional policies noncooperative and cooperative modes of behavior are considered. In the noncooperative Nash search, each region is represented by an artificially intelligent player, a *GA*, to adopt policies taking the rival's as given. Choices are evaluated in terms of their impact on the respective preferences (fitness functions) ignoring the *side effects* on the rival's fitness. Policies are then iteratively improved upon using a synchronous Darwinian search mechanism. Fittest policies are found if no improvement in 'lifetime' fitnesses is possible. South chooses resource prices with a view to maximize her own fitness. Resource prices also affect the rate of knowledge accumulation in the North, and hence, the Northern welfare which, however, does not enter into South's calculus of price determination. Likewise, in its search for optimal resource/knowledge mix, the Northern intertemporal calculus discounts the fact that knowledge accumulation attenuates the detrimental side effects of the resource extraction to the Southern environment. As such, from the vantage point of global efficiency, noncooperative regional policies ultimaletly lead to underinvestment in knowledge capital.

In the cooperative search, the world fitness is represented as a weighted sum of each region's respective fitness. All externalities are thus internalized. Obviously, the resulting price and resource/knowledge paths are Pareto-efficient relative to a global fitness. It is worth noting here that though environmental pollution is local in nature, it has global ramifications calling for an international approach to appropriately internalize it.

Cooperative North/South trade agreements may fail to materialize for well-known reasons such as the absence of enforcement mechanisms and high monitoring costs. Nevertheless, there is still scope to improve the world welfare even when regions adopt inefficient noncooperative policies. We show that 'enhancement' of knowledge diffusion has the potential to generate substantial welfare gains for both regions. For instance, if North, acting unilaterally, is to improve the Southern access to the stock of knowledge (related to the pollution abatement), this will improve not only the Southern, but via initially lower resource prices, North's own welfare as well.

The balance of the paper is organized as follows: Section 2 discusses the dynamic trade game between North and South. Section 3 introduces the Genetic Algorithm to solve open-loop dynamic Nash games. Section 4 contains numerical solution of the model and interpretations. A brief conclusion and further extensions follow in Section 5.

2. The model

2.1. Non-cooperative North/South trade

Consider a global economy comprised of two regions, namely, North and South. Employing a concave production technology $Y = F(K, R, u)$, North produces manufactured goods which are consumed and invested in the North or exported to the South at a fixed world price of unity. K stands for *broad capital* measuring the current state of technical knowledge in the North (Griliches, 1979), R is the raw material imported at a monopoly price determined by the South and u captures all other uncounted determinants of output.

The stock of knowledge accumulates in tandem with the rate of investment,

$$\dot{K}_t = Y_t - p_t R_t - \delta K_t - C_t^{\mathscr{N}}, \tag{1}$$

where p_t is the relative price of resources (Southern terms of trade), $0 < \delta < 1$ is the rate of depreciation of the broad capital.[2] Henceforth, a dot over a variable denotes its time derivative while superscripts $\mathscr{N}$ and $\mathscr{S}$ stand for North and South, respectively. Eq. (1) indicates that the rate of knowledge accumulation will be set not only by the North's desired consumption profile, but also the South's. No investment takes place in the South so that the proceeds from the resource sale are totally consumed. Nonetheless, South indirectly affects the pace of knowledge accumulation in the North via its desired trajectory of terms of trade.

Northern optimal consumption plan maximizes the discounted Northern lifelong welfare, namely,

$$\max_{C_t^{\mathscr{N}}, R_t} J_{\mathscr{N}} = \int_0^{\infty} \mathrm{e}^{-\rho_{\mathscr{N}} t}\, U(C_t^{\mathscr{N}})\,\mathrm{d}t \qquad 0 < \rho_{\mathscr{N}} < 1$$

subject to Eq. (1) and $K(0) = K_0$ given, $C_t^{\mathscr{N}} \geq 0$ for all t. $U(C_t^{\mathscr{N}})$ is a strictly concave instantaneous utility function and $\rho_{\mathscr{N}}$ denotes the Northern time preference rate.

We assume endowment asymmetry and let the primary resource be produced only in the South by a constant returns to scale production function which is assumed, for simplicity, to be a fixed coefficient type. That is, $R_t = bL_t, b > 0$ where L_t is the labor employed at time t.[3]

Resource extraction causes pollution. Also knowledge, broad capital accumulated in the North, diffuses, albeit at a diminishing rate, to check the damage done to the Southern environment from resource extraction. Thus, patterns of trade and growth are further complicated by the presence of local and transboundary externalities.[4]

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[2] Griliches (1979) discusses extensively various interpretation of depreciation in the context of broad capital.

[3] If it is assumed that the supply of labor in the South is perfectly elastic at a fixed real wage w in terms of the manufacturing goods, the nature of the labor force coupled with the CRS production function would then determine labor income per unit of raw material as w/b. Competitive firms in the South will charge a price equal to the private marginal cost of resource extraction w/b. The assumed social planner in the South levies an export tax, not only to internalize the social cost of pollution, but also to extract monopoly profit from the North.

[4] In a dynamic game setup with transboundary pollution, Dockner and Long (1993) show that Pareto-efficient steady-state pollution level can be sustained with nonlinear Markov-Perfect strategies if discount rates are sufficiently low. Baç (1996) analyzes incentives to free-ride on transboundary pollution abatements when there are informational asymmetries.

The type of pollution we consider has a high natural decay rate so that the cumulative effects are underplayed. With this specification the magnitude of the stock becomes proportional to the size of the flow defined as in Keeler et al. (1971), Markusen (1975). We specify the pollution $\mathscr{P}$ as

$$\mathscr{P} = \frac{1}{\gamma}\frac{R_t^{\gamma}}{K_t^{\phi}}, \tag{2}$$

where $\gamma > 1$, $0 < \phi < 1$. γ measures the exponential order of environmental damage due to extraction and ϕ is a knowledge diffusion (spillover) parameter, signifying the degree of applicability of knowledge to pollution reduction.

$\mathscr{P}$ enters into the Southern utility as a flow with a negative marginal utility. Given the Northern demand for resources, South chooses the terms of trade to maximize lifetime utility, i.e.,

$$\max_{p_t} J_{\mathscr{S}} = \int_0^{\infty} e^{-\rho_{\mathscr{S}} t}\, U(C_t^{\mathscr{S}}, \mathscr{P}_t)\,dt \quad 0 < \rho_{\mathscr{S}} < 1$$

subject to Eq. (1), (2) and

$$C_t^{\mathscr{S}} = p_t R_t,$$

$$K(0) = K_0 \quad \text{given}, \quad C_t^{\mathscr{S}} \geq 0 \quad \text{for all } t,$$

where $\rho_{\mathscr{S}}$ is the Southern rate of time preference. Instantaneous utility is assumed separable in consumption $C_t^{\mathscr{S}}$, and pollution $\mathscr{P}_t$ so that $U(C_t^{\mathscr{S}}, \mathscr{P}_t) = U(C_t^{\mathscr{S}}) - D(\mathscr{P}_t)$. $U(C_t^{\mathscr{S}})$ is strictly concave and $D(\mathscr{P}_t)$ is strictly increasing in R_t and decreasing in K_t.

2.2. Cooperative North/South trade

In the design of cooperative strategies, the participants have to agree in advance upon how to distribute the potential gains from cooperation. The distributive outcome depends on the weights, ω, that are put on the respective fitnesses. The determination of the value of ω most likely to prevail in a cooperative agreement requires a bargaining framework which recognizes the relative power of the participants. This is outside the scope of our inquiry. Instead, we consider an egalitarian allocation and assume exogenously given equal weights.

Let $\rho = \omega\rho_{\mathscr{N}} + (1-\omega)\rho_{\mathscr{S}}$ be the weighted time preference term. The Pareto-efficient solution is found by

$$\max_{C^{\mathscr{N}}_t, R_t, p_t} J = \int_0^{\infty} e^{-\rho t}\left\{\omega U(C_t^{\mathscr{N}}) + (1-\omega)\left[U(C_t^{\mathscr{S}}) - D(\mathscr{P}_t)\right]\right\} dt$$

$$0 < \rho < 1$$

s.t.

$$\dot{K}_t = Y_t - p_t R_t - \delta K_t - C_t^{\mathcal{N}},$$

$$K(0) = K_0 \quad \text{given} \quad C_t^{\mathcal{N}}, C_t^{\mathcal{S}} \geq 0 \tag{3}$$

Cooperation takes place on the premise that North and South can enter into binding commitments. Precommitment is difficult in the absence of suitable institutions which can enforce global decisions. Still, cooperative solutions, though lacking credibility, are important in so far as they establish an efficiency benchmark against which other solutions can be compared.

3. Solution methods

In the open-loop Nash solution of the game, each player faces a standard optimal control problem which is arrived at by fixing the other player's policies at some arbitrary functions. Hence, each such optimal control problem is parameterized in terms of some open-loop control policies which, however, do not alter the structure of the underlying optimization problems because of their open-loop character. Therefore, in principle, the necessary and/or sufficient conditions for open-loop Nash equilibria can be obtained by listing down the conditions required by each optimal control problem (via minimum principle) and then requiring that these all be satisfied simultaneously (Başar, 1986). Because of the couplings that exists between these various conditions, each one corresponding to the optimal control problem faced by one player, solving analytically for the Nash equilibria of our game poses a formidable task.

Recently, there has emerged a growing interest among economists in the computational aspects of complex dynamic structures which cannot be easily handled with traditional analytical methods. One search technique that has been successfully applied to such complex problems is the genetic algorithm. Genetic algorithm is a globally robust search mechanism which combines a Darwinian survival-of-the-fittest strategy to eliminate unfit characteristics and uses random information exchange, with exploitation of the knowledge contained in the previous solutions. Grefenstette (1986), Michalewicz (1992) and Krishnakumar and Goldberg (1992) used GA to optimize control problems with a single controller. Özyıldırım(1996) extended GA to solve open-loop difference games of finite horizon. In this paper we develop and implement *GA* to solve open-loop differential games of infinite duration. Given the concave–convex structure of the model, a non*GA* algorithm such as a gradient procedure could have performed equally well for numerical experimentation. However, since the application of *GA* to differential games is quite new for the researchers, experimenting with such regular functional forms should be considered a start. Otherwise, the solution procedure is general and independent of the assumed functional forms. One aim of our paper is to propose it as a general purpose alternative game algorithm.

3.1. Genetic algorithm

Genetic algorithm initiated by Holland (1975) and further extended by De Jong is best viewed in terms of optimizing a sequential decision process involving uncertainty in the form of lack of a priori knowledge, noisy feedback and time varying payoff function. It is a highly parallel mathematical algorithm that transforms a set of (population) individual mathematical objects (typically fixed-length character strings patterned after chromosome strings), each with an associated fitness value, into a new population (i.e., the next generation) using operations patterned after Darwinian principles of reproduction and survival of the fittest after naturally occurring genetic operations (De Jong, 1993).

A *GA* performs a multi-directional search by maintaining a population of individuals, $P(t) = \{x_1, \ldots, x_n\}$ where $x_i = \{x_{i1}, \ldots, x_{iT}\}$; each individual, x_i represents a potential solution vector to the problem at hand. An objective function (fitness) plays the role of an environment to discriminate between 'fit' and 'unfit' solutions. The population experiences a simulated evolution: at each generation the relatively 'fit' solutions *reproduce* while the relatively 'unfit' solutions die. During a single reproductive cycle fit individuals are selected to form a pool of candidates some of which undergo *crossover* and *mutation* in order to generate a new population.

Crossover combines the features of two parent chromosomes to form two similar offsprings by swapping corresponding segments of the parents. The intuition behind the applicability of the crossover operator is the information exchange between different potential solutions. Mutation arbitrarily alters one or more genes of a selected chromosome by a random change with a probability equal to the mutation rate pmut. The mutation operator introduces additional variability into the population. After some number of generations, the program converges. The best individuals represent the optimum solutions.[5]

3.1.1. Genetic algorithm for noncooperative open-loop dynamic games

Considering the fact that GA is a highly parallel mathematical algorithm, we offer a new solution procedure using GA to visualize situations or problems in which there are more than one performance measure and more than one intelligent controller (player) operating with or without coordination with others. We use both the optimization and the learning property of the *GA* to solve the problems of multiple criteria optimization. Since the open-loop n-person Nash equilibria can be obtained as the joint solution to n optimal control

[5] For further details, see Goldberg (1989), Michalewicz (1992), and Arifovic (1994).

problems (Başar and Oldser, 1982), then we can use n parallel *GA*s to optimize the control system.

In this setting, there are n artificially intelligent players (controllers) who update their strategies through *GA* and a referee, or a fictive player, who administers the parallel implementation of the algorithm and acts as an intermediary for the exchange of best responses. This fictive player (*shared memory*) has no decisive role but provides the best strategies in each iteration to the requested parties *synchronously*. In making his decisions, each player has certain expectations as to what the other players will do. These expectations are shaped through the information received from the shared memory in each iteration.

The following figure shows the general outline of the algorithm we use for the two-region dynamic trade game:

```
procedure North GA;
begin
  initialize PN(0)
  randomly initialize
  shared memory;
  synchronize;
  evaluate PN(0);
  t = 1;
  repeat
    select PN(t) from PN(t-1);
    copy best to shared memory;
    synchronize;
    crossover and mutate PN(t);
    evaluate PN(t);
    t=t+1;
  until(termination condition);
end;
```

```
procedure South GA;
begin
  initialize PS(0);
  randomly initialize
  shared memory;
  synchronize;
  evaluate PS(0);
  t = 1;
  repeat
    select PS(t) from PS(t-1);
    copy best to shared memory;
    synchronize;
    crossover and mutate PS(t);
    evaluate PS(t);
    t=t+1;
  until(termination condition);
end;
```

In the above algorithm, each side waits for the presence of the previous best structure of the other side in the synchronize statement.

In each step of this algorithm, two *GAs* are solved. In order to reduce the time complexity, the two *GAs* are solved for one generation while continuously sharing the best responses. This approach has the advantage that while reducing the time complexity it ensures that the convergence is to the global extremum.

3.1.2. Genetic algorithm for cooperative games

In a cooperative game, the strategic rivalry that exists in noncooperative games is eliminated via an 'arbitration' whereby the 'total fitness' as the weighted sum of each player's respective fitness is maximized. This is a typical control problem which can be solved by standard *GA* techniques (Krishnakumar and Goldberg, 1992; Michalewicz, 1992).

In general, controls may involve constraints so that, either penalty functions or substitution may be used to transform the original problem to an unconstrained optimization problem for *GA* implementation.[6] For n control variables, T periods, and k potential solutions, a *GA* performs the following steps to optimize a control problem: (1) randomly generate an initial potential solution set, (2) evaluate the fitness value for a solution set of nTk, (3) apply selection, crossover, and mutation operations to each set of solutions to reproduce a new population, (4) repeat steps (1)–(3) until computation is terminated according to a convergence criterion, (5) choose the solution set nT based on the best fitness value from the current generations as the optimal solution set.

4. Numerical experiments

We need discrete reformulation of our model for numerical computation. Mercenier and Michel (1994) propose time aggregation to transform continuous-time infinite horizon optimal control problems into discrete-time approximations with the *same* steady state. This approach imposes consistency constraints on the joint formulation of preferences and accumulation equations. It is shown that this consistency is achieved by a simple restriction on the choice of discount factor. In the appendix we show that their results extend to open-loop dynamic Nash games. Then we exploit the inherent parallelism in *GA* to solve the time-aggregated North/South dynamic trade game.

The discrete-time approximation of *infinite horizon* North/South trade model with steady-state invariance is as follows:[7]

$$\max \mathcal{J}^i = \sum_{m=0}^{M-1} \theta^i_m \, \Delta_m \, U^i(t_m) + \theta^i_{M-1} \, G^i(K(t_M)), \tag{4}$$

$$\text{s.t. } K(t_{m+1}) - K(t_m) = \Delta_m \, [Y(t_m) - p(t_m)R(t_m) - C^{\mathcal{N}}(t_m) - \delta K(t_m)],$$

$$K(t_0) = K_0 \text{ given}, C^i \geq 0 \; i = \mathcal{N}, \mathcal{S}, \tag{5}$$

where M is the assumed terminal time when the stationary state is reached, Δ_m a scalar factor that converts the continuous flow into stock increments,

[6] We have linear constraints both as equalities and inequalities. The equalities are eliminated at the start by substitution. The constrained problem is then transformed to an unconstrained problem by associating penalties with all constraint violations which are included in the fitness functions. We used arbitrarily large negative numbers to penalize constraint violations. See Michalewicz (1992) for various *GA* approaches to handle linear constraints.

[7] See appendix for derivation.

$\Delta_m = t_{m+1} - t_m$ and θ^i_m the sequence of discount factors of the region i for which the stationary solution of the discrete-time problem is equivalent to the corresponding continuous-time problem. These sequences are given by the following recursions:

$$\theta^{\mathcal{N}}_m = \frac{\theta^{\mathcal{N}}_{m-1}}{1 + \rho_{\mathcal{N}}\Delta_m}, \theta^{\mathcal{N}}_0 > 0 \quad \text{and} \quad \theta^{\mathcal{S}}_m = \frac{\theta^{\mathcal{S}}_{m-1}}{1 + \rho_{\mathcal{S}}\Delta_m}, \theta^{\mathcal{S}}_0 > 0.$$

The functions $G^i(\cdot)$ denote the terminal values.

For numerical experiments, we adopt the following particular functional forms:

$$U(C^i_t) = \begin{cases} \dfrac{C^{i^{1-\sigma}}_t}{1-\sigma} & \text{for } \sigma > 0, \quad \sigma \neq 1, \\ \log\ C^i_t & \text{for } \sigma = 1, \end{cases}$$

and

$$D(\mathscr{P}_t) = \frac{d}{\gamma}\frac{R^\gamma_t}{K^\phi_t}, \quad d > 0$$

where d converts pollution to utility. Also,

$$Y_t = aK^\alpha_t\ R^\beta_t, \quad \alpha + \beta < 1 \quad \text{and} \quad a > 0.$$

All uncounted inputs u, are normalized to one for simplicity. The following set of parameter values are assumed:

$$\alpha = 0.80, \quad \beta = 0.15, \quad \gamma = 2, \quad a = 1, \quad b = 1, \quad d = 0.00001,$$

$$\sigma = 1.50, \quad \delta = 0.08, \quad \rho_{\mathcal{N}} = 0.02, \quad \rho_{\mathcal{S}} = 0.02, \quad \omega = 0.50, \quad \phi = 0.15.$$

These parameter values are assumed for the purposes of illustration, however, they are not totally unjustified. Similar values of $\alpha, \sigma, \delta, \rho_i$ and a are used by Auerbach and Kotlikoff (1987) in a different context. d is so chosen to conform with the assumed utility function. ϕ parameterizes the importance of the effects of knowledge spillovers in the North/South trade game. To highlight the significance of the knowledge spillover, we run the experiment with $\phi = 0.30$ as well. β, γ, and b are chosen to satisfy parameter restrictions and are inconsequential to our arguments about knowledge spillovers. [8]

[8] The genetic operators in this paper were done using the public domain GENESIS package (Grefenstette, 1990) on a SUN SPAC-1000 running Solaris 2.4. A typical run uses *population size*, $j =$ 50, runs 15 million generations for noncooperative game and 30 million generations for cooperative game, *crossover rate* is 0.60 and *mutation rate* is 0.03. None of the results depends on the values of genetic operators other than run time by the choice of number of generations. For each parameter set, we have to implement three separate GAs. Hence, we are limited by the increased computational costs in our scope for a complete sensitivity analysis.

In the time-aggregated model, we assume 21 periods $(M = 20)$ with a dense *equally* spaced gridding of the time horizon T $(t(M) = 200)$, which is sufficient to capture the convergence over time.

As mentioned earlier, we simultaneously run two separate genetic programs, $GA^{\mathcal{N}}$ and $GA^{\mathcal{S}}$, to solve the *noncooperative game*. $GA^{\mathcal{N}}$ generates a population of candidate solutions (chromosomes), $K(t)$ representing the Northern accumulated knowledge. $GA^{\mathcal{S}}$ produces the population of chromosomes $p(t)$ denoting the set of Southern price strategies. Structures K_j, p_j in each population $(j = 1,2,\ldots,50)$ are represented as binary strings $(\{0\ 1\})$ of length l. For string j of length l $(=10)$, decoding works as follows:

$$K_j(t) = \sum_{h=1}^{l} a_j^h(t)\,2^{h-1}, \quad p_j(t) = \sum_{h=1}^{l} a_j^h(t)\,2^{h-1},$$

where $a_j^h(t)$ is the value $\{0\ 1\}$ taken at the hth position in the string. After strings are decoded, integers $K_j(t)$ and $p_j(t)$ are normalized in order to obtain a real number value.

Since K_0 is given and p_0 is free, in each iteration (generation), $GA^{\mathcal{N}}$ computes M while $GA^{\mathcal{S}}$ finds $M + 1$ structures each with a domain, $D_i = [\underline{d}, \bar{d}] \subseteq \Re; i = p, K$. D_i is cut into $(\bar{d} - \underline{d})2^{10}$ equal size ranges. Thus, the noncooperative game has the minimal search domain of $2^{410} = 2.64423\text{E} + 123$.

Cooperative solutions are computationally much more complex than noncooperative ones. In the latter case, the search for the optimum consists of two one-dimensional problems, while the former represents one two-dimensional problem. In the cooperative experiment, three chromosomes, p_t, K_t and R_t, (62 structure) are searched in the minimal domain of 2^{620}.

Regional decisions are updated using genetic operators, selection, crossover, and mutation. The selection strategy is *elitist* so that the best performing strategy in the population of survivors is retained. This selection rule is a natural candidate in noncooperative Nash games. Therefore, it is especially crucial for the dynamic noncooperative game algorithm as it requires best responses be mutually exchanged. Were it not for the elitist selection, the best structures may disappear making for a nonconvergence.

Since *GAs* work with constant-size populations of candidate solutions, *GA* searches are initialized from a number of points. Initialization routines may vary. We, however, start from randomly generated populations so as not to prejudice the convergence of the populations on the initial ones. Therefore, a randomly initialized *GA* is less prone to numerical instability that may be caused by initialization. For the *GA* parameters which might cause instability, we used the parameters chosen and studied on various optimization experiments by Grefenstette (1986). From the result of the experiments in the paper, the convergence is self-evident.

The termination conditions are specified beforehand as a certain number of iterations. We gradually increase the number of iterations until no further improvements are observed.

4.1. The results

Fig. 1 and Table 1 summarize our numerical findings based on the assumed parameter values. First, from Table 1 note that North/South cooperation generates considerable welfare improvements for both regions. Moreover, South has more to gain from such a regime switch indicating the severity of the Northern noncooperation.

Also to be observed from Table 1 is the increase in regional welfares attendant with stronger knowledge diffusion. More significantly, comparing the welfares under the noncooperative regime with augmented knowledge dissemination ($\phi = 0.30$) and cooperation with restricted knowledge spillover ($\phi = 0.15$), we see substantial gains materialize even with uncoordinated trading policies attesting to the importance of access to knowledge.

The policy implication is that even if parties fail, say due to enforcement problems, to realize the first best solution, they may still achieve significant improvements in global welfare by strengthening the knowledge flows from North to South. It may be costly to setup global institutions to monitor and enforce North/South cooperations. To the extent that knowledge diffusion can be enhanced relatively cheaply, regions may opt to cooperate on sharing knowledge related to pollution control.

Studying Fig. 1 number of results stand out. To wit, in the long-run cooperation yields sizable increases in knowledge stock, resource use, resource/knowledge mix, pollution level, and consumption irrespective of the extent of knowledge spillovers. Southern terms of trade first deteriorates to recover later on. Furthermore, this recovery is faster with the greater degree of spillovers so that in the long-run coordinated resource prices ultimately surpass the noncooperative ones.

Along the cooperative path Southern terms of trade equate the marginal social benefits of resource use (the marginal utility of manufactured goods times the marginal product of resources) to the marginal pollution costs in the South. Without cooperation, Southern terms of trade depreciates at the margin if the welfare improvement due to the increased marginal export revenue plus the marginal benefit from the accelerated knowledge accumulation in the North (valued at the shadow price of knowledge in the South which reflects also the positive knowledge externality) is greater than the increase in the marginal pollution cost (in terms of Southern disutility).

As such, Southern noncooperation adds to the dynamic inefficiency to the extent her market power limits knowledge growth in the North. This deleterious effect of resource monopoly, however, is mitigated to the degree South

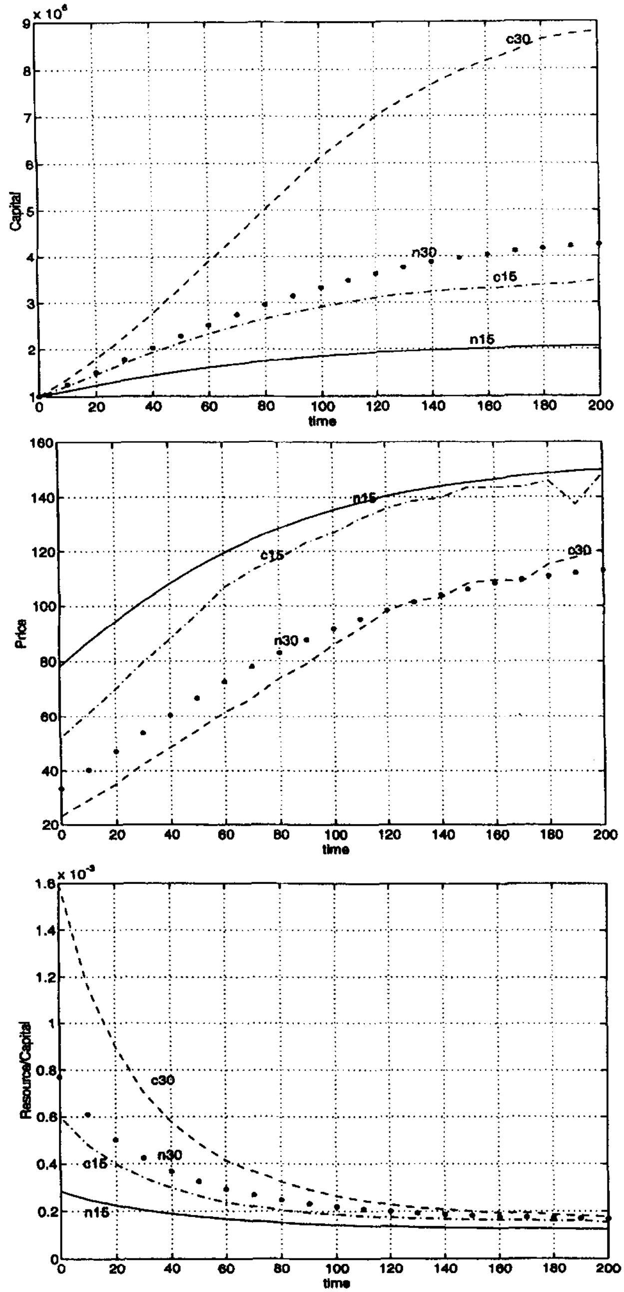

Fig. 1. n15 (c15) stands for noncooperation (cooperation) with $\phi = 0.15$. Likewise for $\phi = 0.30$.

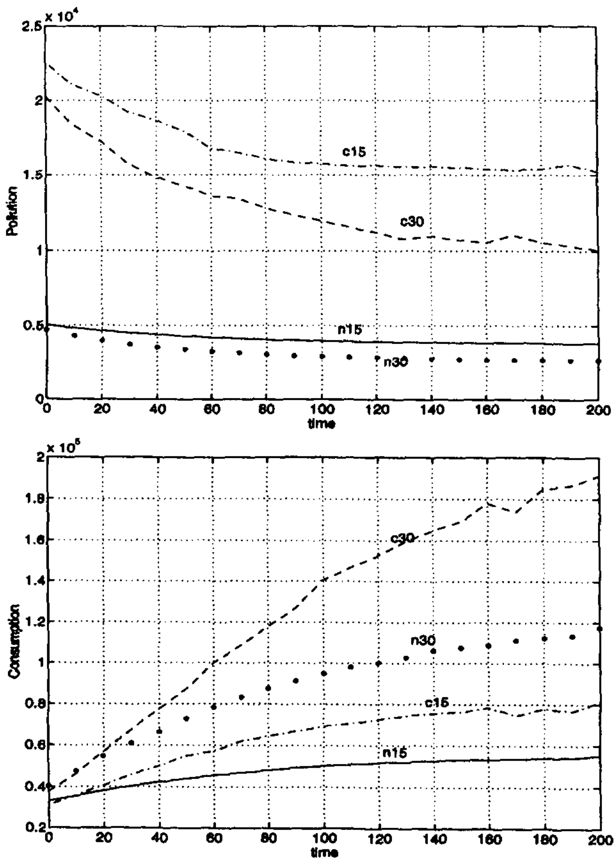

Fig. 1. (Continued).

Table 1
The total discounted welfares

Rate of diffusion (ϕ)	North		South	
	Noncooperative	Cooperative	Noncooperative	Cooperative
0.15	− 0.058138	− 0.054725	− 0.072729	− 0.065882
0.30	− 0.047346	− 0.045009	− 0.060418	− 0.054318

Note: Because of the assumed utility function, values closer to zero indicate higher welfare.

internalizes the knowledge spillovers. On the other hand, noncooperative Northern investment plans are globally inefficient as they understate the true world marginal benefit by the amount of the marginal improvement in the Southern welfare due to the incremental reduction in the pollution level.

Therefore, major gains from cooperation accrue initially when the knowledge stock is so low that a rapid investment plan is called for. Southern terms of trade obliges by shifting down and tilting towards future to accommodate a faster adjustment. Consequently, the knowledge stock accumulates at a more rapid rate; the pollution level starts higher but falls off precipitously; and the regional consumptions rise more swiftly.

Another important set of results has to do with the long-run effects of an increase in the knowledge diffusion parameter ϕ. First, note the rise in the optimal long-run resource/capital ratio. This will be true because, a higher rate of knowledge diffusion will reduce the long-run pollution cost and thereby the supply price of resources, and make the increased use of resources for any given level of knowledge optimal. Also, worthy of notice is the increase in the stationary knowledge stock and the fall in the pollution level. The marginal reduction in the pollution level due to a higher K outweighs the incremental increase due to a higher R so that the overall long-run pollution will fall.

For the dynamic inefficiency of the noncooperative trade regime and the failure of cooperation, we provide the following explanations: In the noncooperative mode the shadow value (the marginal benefit) of the knowledge stock differs for the two regions as the regions have different preferences (fitnesses) leading to conflicting policies and harmful 'policy externalities'. Moreover, when policies are chosen with a view to maximize own fitnesses taking the rival's as given, the 'incentive' effects of the policies are ignored. The South chooses resource prices for any 'given investment policy' of the North, thus, ignoring the fact that a lower price today (lower consumption) may 'induce' the North to invest more today which then leads to higher prices (higher Southern consumption) as the higher knowledge stock shifts the demand for resources tomorrow. The North, on the other hand, ignores the fact that an initially higher investment profile (lower consumption) may induce South to ask for lower resource prices today in return for higher prices tomorrow (as the demand for resources will shift) and also to higher Northern consumption in the future as the amount to be invested will be lower in the future (higher Northern consumption).

Parties ignore the incentive effects for the fact that promises are not credible. If South were to offer cooperative prices, it would not be optimal for the North to invest as much promised as along the cooperative path: North will consume more and invest less. Likewise, if North were to commit itself to the investment plan along the cooperative path, then it would not be optimal for the South to ask for the cooperative prices: South will raise prices and consume more. Failing to cooperate, the parties will revert to their respective Nash strategies.

5. Concluding remarks

This paper has introduced genetic algorithms to search for optimal policies in the presence of knowledge spillovers and local pollution in a dynamic North/South trade game. Cooperative trade policies are efficient but fail to be enforceable. Noncooperative trade policies compound inefficiencies stemming from externalities. Competitive resource production in the South overpollutes whereas 'local' internalization of pollution together with resource monopoly limit growth and trade and result in underpollution.

Because of the spillovers, the stock of knowledge is partially a common property (see Grossman and Helpman, 1991 for this point). The North underinvests because it cannot fully capture the benefits from investment in knowledge. Even though the pollution is local in the South, the North still has an incentive to speed up knowledge diffusion. The South in turn internalizes the benefits from accelerated spillovers in the form of reduced pollution costs, compensating North with initially lower resource prices.

The model can be extended in number of directions. For instance, one obvious modification would be to allow pollution to accumulate which then adds an extra dimension to the intertemporal pollution/growth tradeoff. The transboundary effects of pollution can be considered to further add to the dynamic gaming aspects of international relations. These, however, would come at the expense of increased computational cost as there would be an additional state variable in the system dynamics.

Also, other forms of noncooperative behavior, such as Stackelberg leader/follower setup, could be considered. In this framework one needs to utilize the necessary conditions from the follower's problems as constraints to the leader. In the *GA* game algorithm we develop, it is not obvious how to handle this without having to first analytically derive the necessary conditions for the follower. This, however, would violate the integrity of the GA as a 'blind' algorithm. In order to numerically solve Stackelberg leader/follower model, a new *GA* game algorithm needs to be devised.

Appendix A.

A.1. A general sketch of GA for the solution of dynamic games

Two parallel *GAs* use genetic operators to iterate on constant-size populations, $Pi(t), i = \mathcal{N},\mathcal{S}$ of candidate solutions. During each iteration step, t, called a generation, structures in the current populations are evaluated to reproduce new populations as

```
procedure North GA (South GA);
begin
  initialize PN(0) (PS(0));
  randomly initialize
  shared memory;
  synchronize;
  evaluate PN(0) (PS(0));
  t = 1;
  repeat
    select PN(t) (PS(t)) from PN(t-1) (PS(t-1));
    copy best to shared memory;
    synchronize;
    crossover and mutate PN(t) (PS(t));
    evaluate PN(t) (PS(t));
    t=t+1;
  until(termination condition);
 end;
```

The initial populations are randomly produced and a randomly selected individual from each population is sent to the computer shared memory to be exchanged synchronously. As both *GAs* (North and South) need to reach the shared memory, a priority protocol is required. By synchronization, one *GA* uses the memory if the memory is not currently in use by the other. If the memory is in use, however, the late arriver waits to access the memory. The whole procedure to reach the shared memory is the synchronization process. Upon the exchange of the information, the initial populations $PN(0)$ ($PS(0)$ are evaluated. At $t = 1$, a new population, $Pi(t)$, is formed from the previous, $Pi(t-1)$. We select populations to reproduce on the basis of their relative fitnesses. Best performing individuals in each population are sent (copied) to the shared memory again to be exchanged synchronously. The selected individuals are then recombined using genetic operators, crossover and mutation to form new populations. Crossover is the most important genetic operator. It operates by swapping corresponding segments of a string of parents to produce offsprings. For example, if parents are represented by vectors, $x_1 = (a_1, b_1, c_1, d_1, e_1)$ and $x_2 = (a_2, b_2, c_2, d_2, e_2)$, then crossing the vectors from the second to fifth elements would produce the offsprings $(a_1, b_1, c_2, d_2, e_2)$ and $(a_2, b_2, c_1, d_1, e_1)$. The mutation operator arbitrarily alters one or more components of a selected structure in order to introduce variability in the populations so that the likelihood of getting stuck at a local extremum is reduced. This procedure of creating new populations, exchange of the best individuals and evaluation of the populations in each generation iterate a fixed number of times or until *GAs* find an acceptable approximate solution.

A.2. Discrete-time approximation of the model with steady-state invariance

We generalize the result by Mercenier and Michel (1994) to transform continuous-time infinite horizon control problems to discrete-time approximations for multi-player games. Consider an n-player continuous-time dynamic game with the state vector $x(t) \in \Re^k$ and the control vector $u_i \in \Re^{v_i}$, $i = 1, 2, \ldots, n$:

$$\max J^i = \int_0^\infty e^{-\rho_i t} g_i(x(t), u_1(t), \ldots, u_n(t))\,dt$$

$$\text{s.t.} \quad \dot{x}(t) = f(x(t), u_1(t), \ldots, u_n(t)), \quad x(0) = x_0 \text{ given.}$$

The following relations for $i = 1, 2 \ldots, n$ characterize the stationary open-loop Nash equilibria $(\hat{x}, \hat{u}_1, \ldots, \hat{u}_n, \hat{q}_1, \ldots, \hat{q}_n)$:

$$f(\hat{x}, \hat{u}_1, \ldots, \hat{u}_n) = 0, \rho_i \hat{q}_i = \nabla_x H^i(\hat{x}, \hat{u}_1, \ldots, \hat{u}_n), \text{ and } \nabla_u H^i(\hat{x}, \hat{u}_1, \ldots, \hat{u}_n) = 0, \tag{A.1}$$

where $H^i(x, u_1 \ldots, u_n) = g_i(x, u_1 \ldots, u_n) + q_i' f(x, u_1, \ldots, u_n)$ is the current valued Hamiltonian, $q_i(t)' \in \Re^k$ is the transpose of the costate vector.

The discrete-time approximation of the above problem is

$$\max \tilde{J}^i = \sum_{m=0}^{\infty} \theta_m^i \Delta_m\, g_i(x(t_m), u_1(t_m), \ldots, u_n(t_m))$$

$$\text{s.t.}\, x(t_{m+1}) - x(t_m) = \Delta_m f(x(t_m), u_1(t_m), \ldots, u_n(t_m)), \quad x(t_0) = x_0 \text{ given,}$$

where Δ_m converts the continuous flows into stock increments, i.e., ($\Delta_m = t_{m+1} - t_m$) and θ_m^i is the sequence of discount factors for which the stationary solution of the discrete-time problem is the same that of the continuous-time problem. The *recurrence* for θ_m^i is generated from the optimality conditions of the discretized game.

The optimality conditions satisfy

$$\Delta_m \nabla_{u_i}\{g_i(x(t_m), u_1(t_m), \ldots, u_n(t_m)) + q_i(t_m)' f(x(t_m), u_1(t_m), \ldots, u_n(t_m))\} = 0, \tag{A.2}$$

$$\theta_m^i \Delta_m \{ \nabla_x \{g_i(x(t_m), u_1(t_m), \ldots, u_n(t_m)) + q_i(t_m)' f(x(t_m), u_1(t_m), \ldots, u_n(t_m))\}\} - \theta_{m-1}^i q_i(t_{m-1}) + \theta_m^i q_i(t_m) = 0. \tag{A.3}$$

Imposing the stationary equivalence of the continuous and discrete-time problems, using Eq. (A.1) in Eq. (A.2) and Eq. (A.3), the following recursions are obtained:

$$\theta^i_{m-1}\hat{q}_i = \theta^i_m\hat{q}_i + \theta^i_m\Delta_m\rho_i\hat{q}_i,$$

for any $\theta^i_0 > 0$.

For the GA application, we truncate the original infinite horizon continuous-time problem. The finite horizon discrete-time approximation becomes

$$\max \tilde{J}^i = \sum_{m=0}^{M-1} \theta^i_m \Delta_m g_i(x(t_m), u_1(t_m), \ldots, u_n(t_m)) + \beta^i_M G^i(x(t_M)) \quad i = 1, \ldots, n$$

$$\text{s.t.} \quad x(t_{m+1}) - x(t_m) = \Delta_m f(x(t_m), u_1(t_m), \ldots, u_n(t_m)), \quad x_0 \text{ given},$$

where it is assumed that stationary solution is reached at T_M.

The steady-state invariance property imposes specific restrictions on the choice of functions $G^i(\cdot)$. The terminal value $G^i(\cdot)$ is

$$G^i(\hat{x}) = \int_0^\infty e^{-\rho_i t} g_i(\hat{x}, \hat{u}_1(x), \ldots, \hat{u}_n(x))\,dt = \frac{1}{\rho_i} g_i(\hat{x}, \hat{u}_1(x), \ldots, \hat{u}_n(x)),$$

so that recursion is terminated at $\beta^i_M = \theta^i_{M-1}$.

A.3. Tables

Table 2
Noncooperative game with $\phi = 0.15$

t	K_t	p_t	R_t	$\mathcal{P}_t$	$C_t^{\mathcal{N}}$	$C_t^{\mathcal{S}}$	R_t/K_t	$\mathcal{P}_t/K_t$
0	1000000.000	77.967	283.125	5045.758	33064.556	22074.356	0.000283	0.005046
1	1120234.604	86.442	279.039	4818.407	35627.665	24120.597	0.000249	0.004301
2	1234604.106	94.252	276.186	4652.048	38184.480	26031.175	0.000224	0.003768
3	1340175.953	101.564	273.253	4498.064	40373.921	27752.721	0.000204	0.003356
4	1436950.147	108.045	271.306	4388.060	42354.554	29313.215	0.000189	0.003054
5	1524926.686	114.027	269.286	4284.599	44088.316	30705.942	0.000177	0.002810
6	1604105.572	119.345	267.680	4201.621	45662.203	31946.254	0.000167	0.002619
7	1674486.804	124.164	266.037	4123.555	46772.060	33032.227	0.000159	0.002463
8	1739002.933	128.485	264.800	4062.196	48103.356	34022.724	0.000152	0.002336
9	1794721.408	132.141	263.921	4016.228	49353.218	34874.649	0.000147	0.002238
10	1841642.229	135.298	263.001	3972.867	50203.064	35583.532	0.000143	0.002157
11	1882697.947	137.957	262.438	3942.815	51027.299	36205.093	0.000139	0.002094
12	1917888.563	140.283	261.849	3914.248	51790.614	36733.103	0.000137	0.002041
13	1947214.076	142.278	261.241	3887.233	52206.763	37168.797	0.000134	0.001996
14	1973607.038	143.939	260.983	3871.732	52931.005	37565.710	0.000132	0.001962
15	1994134.897	145.269	260.702	3857.408	53316.818	37871.848	0.000131	0.001934
16	2011730.205	146.432	260.411	3843.733	53386.591	38132.566	0.000129	0.001911
17	2029325.513	147.595	260.122	3830.200	53746.547	38392.741	0.000128	0.001887
18	2043988.270	148.592	259.824	3817.304	54085.868	38607.874	0.000127	0.001868
19	2055718.475	149.423	259.519	3805.080	54405.853	38778.193	0.000126	0.001851
20	2064516.129	149.922	259.545	3803.405	55337.091	38911.488	0.000126	0.001842

Table 3
Cooperative game with $\phi = 0.15$

t	K_t	p_t	R_t	$\mathscr{P}_t$	$C_t^{\mathscr{N}}$	$C_t^{\mathscr{S}}$	R_t/K_t	$\mathscr{P}_t/K_t$
0	1000000.000	51.818	598.651	22558.876	31053.778	31021.008	0.000599	0.022559
1	1225806.448	60.909	586.921	21031.282	35933.968	35748.814	0.000479	0.017157
2	1460410.552	69.853	584.233	20298.802	40712.731	40810.621	0.000400	0.013899
3	1697947.216	79.384	574.457	19186.562	45916.300	45602.827	0.000338	0.011300
4	1923753.664	87.889	571.281	18622.863	50150.338	50209.026	0.000297	0.009680
5	2140762.468	96.979	564.927	17921.275	54982.562	54786.292	0.000264	0.008371
6	2331378.304	106.657	550.020	16771.980	57228.799	58663.375	0.000236	0.007194
7	2507331.376	112.815	548.553	16501.593	61906.551	61885.174	0.000219	0.006581
8	2653958.944	117.361	543.666	16071.253	64431.200	63804.988	0.000205	0.006056
9	2791788.856	123.079	541.711	15835.157	66743.358	66673.302	0.000194	0.005672
10	2912023.456	126.745	542.199	15763.725	69246.953	68720.993	0.000186	0.005413
11	3017595.304	132.023	540.733	15595.052	70757.598	71389.460	0.000179	0.005168
12	3105571.852	135.689	542.444	15626.375	72579.693	73603.737	0.000175	0.005032
13	3175953.076	138.622	541.711	15531.866	74402.654	75092.852	0.000171	0.004890
14	3228739.000	139.648	542.688	15549.477	75613.083	75785.369	0.000168	0.004816
15	3275659.828	143.460	542.199	15487.926	76374.164	77784.150	0.000166	0.004728
16	3304985.332	143.607	541.222	15411.513	78440.272	77723.273	0.000164	0.004663
17	3319648.096	143.607	540.000	15331.820	74690.809	77547.801	0.000163	0.004619
18	3375366.568	145.806	541.711	15390.638	77901.150	78984.908	0.000160	0.004560
19	3398826.976	137.302	547.087	15681.350	76759.806	75116.168	0.000161	0.004614
20	3483870.964	148.739	540.000	15221.176	81028.038	80319.062	0.000155	0.004369

Table 4
Noncooperative game with $\phi = 0.30$

t	K_t	p_t	R_t	$\mathscr{P}_t$	$C_t^{\mathscr{N}}$	$C_t^{\mathscr{S}}$	R_t/K_t	$\mathscr{P}_t/K_t$
0	1000000.000	33.343	769.100	4687.433	40390.275	25644.170	0.000769	0.004687
1	1249266.862	40.205	760.908	4291.783	47658.693	30592.521	0.000609	0.003435
2	1506842.620	47.067	754.121	3985.032	54830.827	35494.555	0.000500	0.002645
3	1764418.377	53.803	747.500	3734.328	60987.681	40217.422	0.000424	0.002116
4	2021994.135	60.283	743.355	3545.101	66417.730	44812.030	0.000368	0.001753
5	2279569.892	66.637	739.617	3385.541	72826.629	49286.121	0.000324	0.001485
6	2520527.859	72.610	734.888	3243.138	78298.376	53360.227	0.000292	0.001287
7	2744868.035	77.947	732.542	3141.080	83202.928	57099.637	0.000267	0.001144
8	2952590.420	82.903	729.719	3049.443	87493.390	60496.074	0.000247	0.001033
9	3143695.015	87.478	726.686	2967.777	91280.228	63569.031	0.000231	0.000944
10	3318181.818	91.417	725.973	2914.347	94835.891	66366.589	0.000219	0.000878
11	3476050.831	95.103	723.980	2858.231	97954.462	68852.409	0.000208	0.000822
12	3617302.053	98.280	723.134	2817.681	100047.483	71069.273	0.000200	0.000779
13	3750244.379	101.329	721.700	2776.297	102748.345	73129.473	0.000192	0.000740
14	3866568.915	103.998	720.379	2740.904	106069.849	74917.967	0.000186	0.000709
15	3957966.764	106.031	719.810	2717.463	107546.961	76322.329	0.000182	0.000687

Table 4
Continued

t	K_t	p_t	R_t	$\mathscr{P}_t$	$C_t^{\mathscr{N}}$	$C_t^{\mathscr{S}}$	R_t/K_t	$\mathscr{P}_t/K_t$
16	4041055.718	108.192	716.810	2678.120	108703.692	77552.851	0.000177	0.000663
17	4115835.777	109.589	718.356	2674.931	111021.151	78724.278	0.000175	0.000650
18	4173998.045	110.860	718.100	2661.795	112211.082	79608.752	0.000172	0.000638
19	4223851.417	112.004	717.454	2647.559	113297.510	80357.660	0.000170	0.000627
20	4265395.894	113.021	716.437	2632.319	117610.283	80972.110	0.000168	0.000617

Table 5
Cooperative game with $\phi = 0.30$

t	K_t	p_t	R_t	$\mathscr{P}_t$	$C_t^{\mathscr{N}}$	$C_t^{\mathscr{S}}$	R_t/K_t	$\mathscr{P}_t/K_t$
0	1000000.000	23.072	1597.290	20217.970	37433.968	36852.467	0.001597	0.020218
1	1364824.609	29.335	1591.687	18287.835	46345.522	46692.902	0.001166	0.013399
2	1787925.607	35.152	1608.048	17213.262	56806.064	56525.621	0.000899	0.009628
3	2263924.062	42.422	1591.687	15711.831	67700.291	67522.498	0.000703	0.006940
4	2766351.580	48.686	1593.480	14828.288	77747.379	77579.514	0.000576	0.005360
5	3304057.180	54.726	1604.014	14245.315	87214.859	87780.484	0.000485	0.004311
6	3877001.532	61.548	1604.462	13585.639	99967.284	98752.037	0.000414	0.003504
7	4432326.508	66.582	1627.547	13429.120	108431.263	108364.748	0.000367	0.003030
8	5014080.547	73.964	1618.358	12795.634	117888.573	119699.843	0.000323	0.002552
9	5569405.523	79.109	1615.893	12360.970	126898.505	127831.485	0.000290	0.002219
10	6115920.811	85.820	1611.634	11955.405	140779.303	138310.269	0.000264	0.001955
11	6565450.873	91.524	1602.221	11567.393	146735.396	146642.091	0.000244	0.001762
12	6979742.183	98.123	1591.015	11198.678	152487.541	156115.849	0.000228	0.001604
13	7332326.348	101.815	1572.188	10774.731	159340.646	160071.535	0.000214	0.001469
14	7640822.743	102.933	1595.273	10957.160	164925.320	164206.236	0.000209	0.001434
15	7940548.78	108.190	1586.308	10710.010	169193.976	171622.663	0.000200	0.001349
16	8178528.343	109.532	1581.602	10552.658	178158.237	173236.347	0.000193	0.001290
17	8354800.761	108.637	1622.392	11033.182	173812.724	176252.449	0.000194	0.001321
18	8619248.717	115.125	1595.049	10565.188	184793.166	183629.538	0.000185	0.001226
19	8725043.631	117.585	1581.378	10346.920	186627.283	185946.965	0.000181	0.001186
20	8795560.464	120.270	1561.206	10060.310	191094.069	187765.933	0.000177	0.001144

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