Numerical Diffraction Synthesis of 2-D Quasioptical Power Splitter

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Introduction

A new diffraction synthesis method is proposed for computing quasioptical 2-D reflector beam splitters in the *E*-polarization case. It is a combination of a numerical gradient (NG) optimization and an efficient analysis method based on singular integral equations (SIEs) which are discretized using a fast and accurate numerical Nystrom-type method of discrete singularities (MDS). The results of design are shown for a 40-λ quasioptical power splitter obtained from an offset parabolic reflector fed by in-focus beam source.

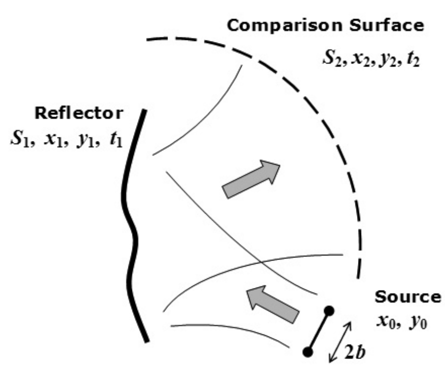

Fig. 1 Geometry of the considered shaped reflector and basic notations.

As known, full-wave tools for the modeling of various scatterers can be built on the SIEs, provided that they are solved with a fast and convergent numerical algorithm having controlled accuracy. Unfortunately, conventional method-of-moments (MoM) discretizations of SIEs for electrically large reflectors need prohibitively large computer resources to provide acceptable accuracy [1]. To overcome this difficulty, fast-convergent and numerically efficient discretization of SIEs is offered by MDS [2]. Accurate MDS solutions to the direct scattering, i.e., the analysis, problems for 2-D quasioptical reflector antennas and beam waveguides have been reported in [2,3].
Due to its flexibility and speed, we have selected the SIE-MDS technique for the direct problem solver used as a full-wave engine for building a numerical synthesis code. Local-search gradient methods are able to provide quicker tuning to the best shape if a good initial guess is used. As we shall demonstrate, the main idea for such synthesis is to derive a separate SIE for the gradient of the objective function and to solve it with MDS as well. This guarantees the accuracy and speeds up the synthesis process, which is expected to be faster than MoM and FDTD and more accurate than GO and PO approaches [4-7].

Essentials of the Analysis and Synthesis Methods

A. Analysis problem formulation

Consider the scattering of a time-harmonic *E*-polarized beam-like wave by a generic perfectly electrically conducting (PEC) zero-thickness cylindrical reflector (Fig. 1). Here the contour of reflector's cross-section is a smooth curve $S_1 = \{(x_1, y_1) \in R^2,\ y_1 = F(x_1)\}$.

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The total field is a sum of the given incident field U_0 and the scattered field U. An electromagnetic boundary-value analysis problem requires U to solve the Helmholtz equation off S_1 and satisfy the Dirichlet condition on S_1, the radiation condition, and the condition of local energy finiteness. This problem is uniquely solvable.

B. Incident field

Keeping in mind the numerical design of a quasioptical beam splitter, we may imagine the primary field provided by a small horn as a feed. In 2-D, such a field can be conveniently simulated with a Hankel function of the first kind and zero order of a complex argument due to complex-valued source point (CSP) – e.g. see [2,3]. This function is a rigorous solution to the Helmholtz equation; it behaves as a Gaussian beam in the near zone of paraxial domain and transforms into cylindrical wave off this domain.

C. MDS analysis: SIE and discretization

The SIE for the surface current function is the well-known electric field integral equation (EFIE). Introducing a C^2-smooth parameterization $x_1(t_1),\ y_1(t_1)$ for S_1, so that $|t_1|\le 1$, and taking into account the current function edge behavior, we can write this EFIE as

$$\frac{i}{4}\int_{-1}^{1} H_0^{(1)}\left(kR\left(t_1,t_0\right)\right) j\left(t_1\right) w\left(t_1\right) dt_1 = -U_0\left(t_0\right), \tag{1}$$

where j denotes a smooth function that has to be found, k is the wave number, R is the distance function, $w(t)=(1-t^2)^{-1/2}$ is the Chebychev weight and $|t_0|<1$. The MDS suggests a conversion of a log-singular IE (1) to another SIE with a stronger, Cauchy-type, singularity [2,3]. It is further discretized by using new quadrature formulas of interpolation type with nodes taken at zeros of the Chebyshev polynomials of the first kind, $t_q^n=\cos[(2q-1)\pi/(2n)],\ q=1,2,...n$, and the second kind, $t_{0p}^n=\cos(p\pi/n)$, $p=1,2,...n-1$. This yields a matrix equation, on solving of which we obtain an approximate solution in the form of interpolation polynomial of the order n. The convergence of this method with greater n has been proven - see [2]. Numerical results obtained with MDC have been also validated by the good agreement with the data (e.g., radiation patterns) obtained by other numerical techniques and PO.

D. Numerical gradient synthesis

The synthesis problem is understood as follows. Assuming that the incident field U_0 is given in the whole space, determine a smooth open contour S_1 of the PEC reflector, i.e., the corresponding functions $x_1(t_1),\ y_1(t_1)$, for which the total field, i.e., the function U, differs as little as possible, in definite sense, from the function $\tilde{U}$ that is the prescribed field on a smooth contour S_2 (Fig. 1).

To cast the synthesis problem into a mathematical form, it is convenient to define the objective function on the curve S_2 in terms of the L_2–norm. Keeping in mind MDS, we introduce the appropriate scalar product $\langle\phi,\psi\rangle_S$ and the residual function ρ: $\rho(t_2):=U(t_2)-\tilde{U}(t_2),\ t_2\in S_2$. This enables us to define the objective function I as $I\left[x_1,y_1\right]=\langle\rho,\rho\rangle_{S_2}\to\min$, i.e., the functions x_1,y_1 have to be determined in such a way that I is minimized. By using operator notations, the direct-scattering problem SIE for the surface current function can be compactly written as $(\hat{G}_{11}j)(t_1)=-U_0(t_1)$.

To find the gradient of the objective function, the first variation δI is calculated. To this end, when determining the partial derivatives $\partial I/\partial x_1$, $\partial I/\partial y_1$ of the gradient we require that the following expression is satisfied (and similar for y_1): $\langle \partial I/\partial x_1 \, , \, \delta x_1 \rangle_{S_1} = 2\,\mathrm{Re}\{\langle \rho, \delta_{x_1} \rho \rangle_{S_2}\}$. On performing a number of derivations and using adjoint integral operators, we finally obtain a new log-singular SIE for a certain auxiliary function. Then $\partial I/\partial x_1$ and $\partial I/\partial y_1$ are obtained by using certain smooth integral operators; all mentioned operators have Chebyshev weights. Therefore all of them can be efficiently computed with MDS discretization.

Numerical Results

To test the idea of the proposed synthesis we have considered the problem of designing a quasioptical single-reflector beam splitter. As a distant prototype, we kept in mind [7] for power splitter geometries (Fig. 2) however decided to design a similar system avoiding the sub-reflector and the phaseshifter (Fig. 3).

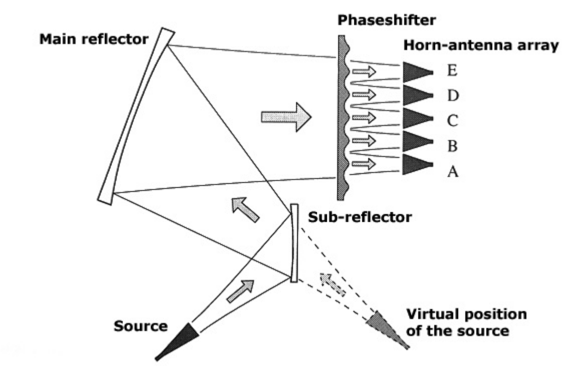

Fig. 2 Double-reflector power splitter geometry. The splitting is done by a phaseshifter (see [7]).

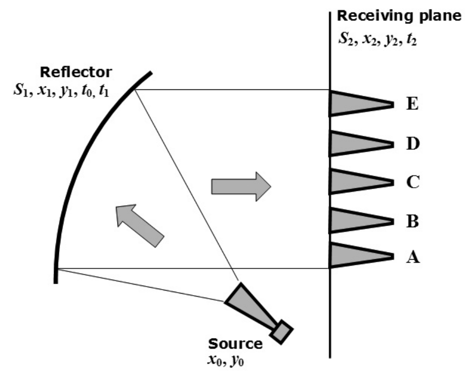

Fig. 3 Considered power splitter geometry avoiding sub-reflector and phaseshifter.

The comparison contour S_2 is taken as a straight line, with five isolated intervals, A to E, on which the field of the synthesized reflector is to be focused (Fig. 4). The aim is to match the magnitude (or intensity) of a prescribed field on S_2. The prescribed field function on this contour is taken as a superposition of 5 Gaussian functions. First, as an initial guess, we take an offset parabolic contour S_1 of the fixed size, d=40λ, and fixed coordinates of the end points. The feed field is also fixed – this is the field radiated by a CSP having the aperture parameter kb=8 aimed at the center of the reflector (b is the imaginary part of the source coordinate). It is placed at the parabola's geometrical focal point and provides –10 dB edge illumination. After performing the synthesis as explained above, we have compared the prescribed and optimized field amplitudes on S_2 (Fig. 5). A very close similarity between five curves is

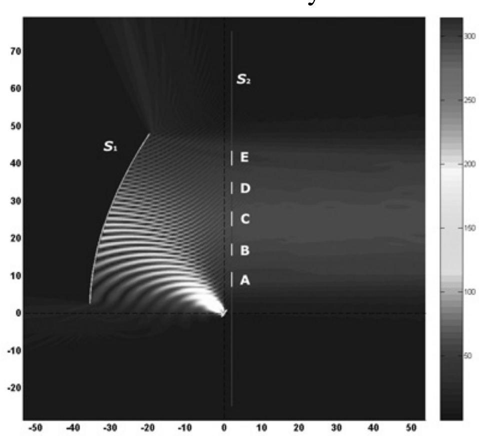

Fig. 4 Near field of an offset parabolic starting surface for the antenna, having d=40λ aperture, kb=7, f=0.9d, β=140^0

observed at the intervals corresponding to the bright spots. The total near field of the synthesized reflector is shown in Fig. 5. A clear splitting of the reflected beam and its focusing on the desired intervals A-E is seen.

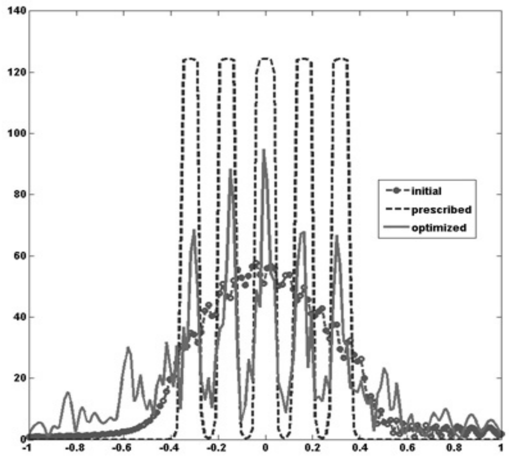

Fig. 3 Initial, prescribed and optimized field amplitudes on the antenna on Fig. 4

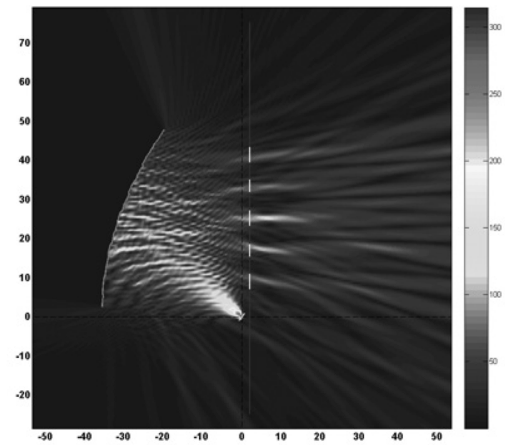

Fig. 5 Near field of an offset optimized surface for the antenna on Fig. 4

The proposed synthesis method has been implemented on a PC in *MATLAB*, and the widely used BFGS-algorithm has been applied to minimize I. Approximately 30 function calls for 700 surface variables x_1, y_1 and 90 minutes of computing time with an Intel Centrino Duo 2.1GHz processor were necessary to generate the shown results. These parameters can be greatly reduced if choosing initial-guess contour S_1 and comparison contour S_2 in optimal way

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