ENERGY OPERATIONS MANAGEMENT FOR RENEWABLE POWER PRODUCERS IN ELECTRICITY MARKETS

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Ece Çiğdem Karakoyun

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We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

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Renewable energy generation has grown dramatically around the world in recent years, and policies targeted at reducing greenhouse gas emissions that cause global warming are expected to ensure a consistent expansion of renewable power generation in the electricity sector. With the increasing contribution of renewable sources to the overall energy supply, renewable power producers participate in electricity markets where they are imposed to make advance commitment decisions for energy delivery and purchase. Making advance commitments, however, is a complex task due to the inherent intermittency of renewable sources, increasingly volatile electricity prices, and penalties incurred for possible energy imbalances in electricity markets. Integrating renewable sources with energy storage units is among the most effective methods to address this challenging task.

Motivated by the recent trends of paired renewable energy generators and storage units, we study the energy commitment, generation and storage problem of a wind power producer who owns a battery and participates in a spot market operating with hourly commitments and settlements. In each time period, the producer decides how much energy to commit to selling to or purchasing from the market in the next time period, how much energy to generate in the wind power plant, and how much energy to charge into or discharge from the battery. The existence of the battery not only helps smooth out imbalances caused by the fluctuating wind output but also enables the producer to respond to price changes in the market. We formulate the wind power producer's problem as a Markov decision process by taking into account the uncertainties in wind speed and electricity price.

In the first part of this dissertation, we consider two different problem settings: In the first setting, the producer may choose to deviate from her commitments based on the latest available information, using the battery to support such deviations. In the second setting, the producer is required to fulfill her commitments, using the battery as a back-up source. We numerically examine the effects of system components, imbalance pricing parameters, and negative prices on the producer's profits, curtailment decisions, and imbalance tendencies in each problem setting. We provide managerial insights to renewable power producers in their assessment of energy storage adoption decisions and to power system operators in their understanding of the producers' behavior in the market with their storage capabilities.

In the second part of this dissertation, we establish several multi-dimensional structural properties of the optimal profit function such as supermodularity and joint concavity. This enables us to prove the optimality of a state-dependent threshold policy for the storage and commitment decisions under the assumptions of a perfectly efficient system and positive electricity prices. Leveraging this policy structure, we construct two heuristic solution methods for solving the more general problem in which the battery and transmission line can be imperfectly efficient and the price can also be negative. Numerical experiments with data-calibrated instances have revealed the high efficiency and scalability of our solution procedure. In the third part of this dissertation, we characterize the optimal policy structure by taking into account the battery and transmission line efficiency losses and showing the joint concavity of the optimal profit function. In the last part of this dissertation, we consider an alternative problem setting that allows for real-time trading without making any advance commitment. We analytically compare the total cash flows of this setting to those of our original problem setting. We conclude with a numerical investigation of the effect of advance commitment decisions on the producer's energy storage and generation decisions.

Keywords: Renewable energy, energy storage, electricity markets, energy deviation, imbalance pricing mechanism, negative electricity prices, Markov decision processes.

ÖZET

ELEKTRİK PİYASALARINDA YENİLENEBİLİR ENERJİ ÜRETİCİLERİ İÇİN ENERJİ OPERASYONLARI YÖNETİMİ

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Yenilenebilir enerji üretimi son yıllarda dünya genelinde önemli ölçüde artmıştır. Küresel ısınmaya neden olan sera gazı emisyonlarının azaltılması amacıyla oluşturulan politikaların, elektrik sektöründe yenilenebilir enerji üretiminin istikrarlı bir şekilde yaygınlaşmasını sağlaması beklenmektedir. Yenilenebilir enerji kaynaklarının toplam enerji arzına olan katkısının artmasıyla birlikte yenilenebilir enerji üreticileri, enerji dağıtımı ve satın alımı için belirli bir zaman öncesinden taahhüt kararları vermeleri gereken elektrik piyasalarına katılmaktadır. Ancak, belirli bir zaman öncesinden taahhüt vermek yenilenebilir kaynakların doğasındaki aralıklı üretim, giderek artan değişken elektrik fiyatları ve elektrik piyasalarındaki olası enerji dengesizlikleri kaynaklı ortaya çıkan cezalar nedeniyle karmaşık bir görevdir. Yenilenebilir enerji kaynaklarını enerji depolama üniteleri ile entegre etmek, bu zorlu görevi ele almak için bilinen en etkili yöntemlerden biridir.

Son yıllarda yenilenebilir enerji santralleri ve enerji depolama ünitelerinin birlikte kullanımının artmasından hareketle bu tezde, bataryaya sahip olan bir rüzgâr enerjisi üreticisinin enerji taahhüdü, üretimi ve depolama problemi ele alınmıştır. Bu üretici, saatlik taahhütler ve uzlaşmalar ile işleyen bir spot elektrik piyasasına katılmaktadır. Her zaman diliminde, üretici, bir sonraki zaman diliminde piyasaya satmak veya piyasadan satın almak için ne kadar enerji taahhüt edeceğine, rüzgâr enerjisi santralinde ne kadar enerji üreteceğine ve bataryada ne kadar depolayacağına veya bataryadan ne kadar enerji kullanacağına karar vermektedir. Bataryanın varlığı, değişken rüzgâr çıkışından kaynaklanan dengesizlikleri düzeltmeye yardımcı olmakta ve aynı zamanda üreticinin piyasadaki fiyat değişikliklerine yanıt vermesini sağlamaktadır. Rüzgâr enerjisi üreticisinin problemi, rüzgâr hızındaki belirsizlikler ve elektrik fiyatları da dikkate alınarak

bir Markov karar süreci olarak formüle edilmiştir.

Tezin ilk bölümünde, iki farklı durum ele alınmıştır: İlk durumda, üretici, son kullanılabilir bilgilere dayanarak taahhütlerinden sapmayı seçebilir ve bu sapmaları desteklemek amacıyla bataryayı kullanabilir. İkinci durumda ise üretici, bataryayı taahhütlerini yerine getirmek için yedek bir kaynak olarak kullanır. Her durum için sistem bileşenlerinin, dengesizlik fiyatlandırma parametrelerinin ve negatif fiyatların, üreticinin kârlılığı, rüzgâr enerjisi kesilme kararları ve üreticinin dengesizlik eğilimleri üzerindeki etkileri sayısal olarak incelenmiştir. Bu çalışma, yenilenebilir enerji üreticilerine, enerji depolama kullanım kararlarını değerlendirmelerinde ve güç sistemi operatörlerine, üreticilerin depolama kapasiteleri ile piyasadaki davranışlarını anlamarında yönetimsel bir bakış açısı sunmaktadır.

Tezin ikinci bölümünde, eniyi kâr fonksiyonunun süpermodülerlik ve ortak içbükeylik gibi çok boyutlu yapısal özellikleri gösterilmiştir. Böylece, mükemmel verimli sistem ve pozitif elektrik fiyatları varsayımları altında depolama ve taahhüt kararları için duruma bağlı eşik politikasının eniyi olduğu kanıtlanmıştır. Bu politika yapısı kullanılarak, batarya ve iletim hattının mükemmel verimli olmayabileceği ve fiyatın negatif olabileceği daha genel bir problemi çözmek için iki sezgisel çözüm yöntemi geliştirilmiştir. Veri kalibreli örneklerle yapılan sayısal deneyler, geliştirilen çözüm yönteminin yüksek verimliliğini ve ölçeklenebilirliğini ortaya koymaktadır. Tezin üçüncü bölümünde, eniyi kâr fonksiyonunun ortak içbükeylik özelliği gösterilerek batarya ve iletim hattı verimlilik kayıplarını hesaba katan eniyi enerji taahhüt, üretim ve depolama politika yapısı sunulmuştur. Tezin son bölümünde, belirli bir zaman önceden taahhüt verilmeksizin gerçek zamanlı elektrik ticaretine izin veren alternatif bir durum ele alınmıştır. Bu durumun toplam nakit akışları ile orijinal problemin nakit akışları analitik olarak karşılaştırılmıştır. Üreticinin önceden verdiği taahhüt kararlarının, enerji depolama ve üretim kararları üzerindeki etkileri sayısal olarak incelenmiştir.

Anahtar sözcükler: Yenilenebilir enerji, enerji depolama, elektrik piyasaları, enerji sapması, dengesizlik fiyatlandırma mekanizması, negatif elektrik fiyatları, Markov karar süreci.

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Chapter 1

Introduction

Over the last few decades, renewable energy sources such as wind and solar have received much attention in the development of power systems [2]. With the ultimate goal of complete independence from fossil fuels by a certain deadline, many countries have set ambitious target levels for renewable energy generation [3]. For example, the United States (U.S.) Energy Information Administration projects that wind and solar energy will account for 16% of the U.S. electricity generation in 2023 and 18% in 2023, rising up from 14% in 2022 and 8% in 2018 [4]. Government incentives and improvements in the cost and performance of renewable power technologies have sped up renewable energy capacity additions, resulting in low-priced renewable energy [5]. Thanks to such initiatives in various countries, renewable energy sources are anticipated to meet 60% of the world's total energy needs by 2050 [6].

While these sources contribute more to the overall energy supply, the renewable power producers participate in electricity markets where they are imposed to make advance commitment decisions for energy delivery and purchase [7]. Making advance commitments, however, is a complex task due to the inherent intermittency of renewable sources, increasingly volatile electricity prices, and penalties incurred for possible energy imbalances in electricity markets.

Integrating renewable sources with energy storage units, particularly batteries, is among the most well-known methods to effectively address this challenging task. The batteries can be used as a back-up source that serves as a hedge against the cost of imbalance due to fluctuating renewable outputs. They can also help respond effectively to price changes in the electricity markets. The producer can purchase energy from the market when the electricity price is low (or sometimes even negative[1]) and can sell the stored energy to the market when the electricity price is high. Moreover, the producer may choose to shift the existing generation/storage capacities from one period to another by deviating from her commitments when they are due. The latest available information may reveal that the existing capacities can be better utilized in future periods with high/low electricity prices. Consequently, the number of renewable generation sites co-located with batteries has grown in the U.S. from 19 paired sites in 2016 to 53 paired sites in 2019 [10]. This trend will likely continue into the foreseeable future thanks to reductions in installed costs and improvements in battery storage technology [11]. One recent example is the Gemini solar project in Nevada. Gemini is expected to contribute more than 1 GW of combined solar and battery energy capacities [10].

Motivated by the recent trends of paired renewable energy generators and storage units, in this thesis, we study the energy commitment, generation and storage problem of a wind power producer who owns a battery and participates in a spot market operating with hourly commitments and settlements. In each time period, the producer answers the following questions:
(i) How much energy should be committed to selling to or purchasing from the market in the next time period?
(ii) How much energy should be generated in the wind power plant?
(iii) How much energy should be charged into or discharged from the battery?
The existence of the battery not only helps smooth out imbalances caused by the

[1] Negative electricity prices were observed in the markets run by the New York Independent System Operator, Electric Reliability Council of Texas, California Independent System Operator, and the European Energy Exchange [8]. For example, the proportion of negative-priced hours in the zones of California Independent System Operator has grown from 1.7%-2.3% to 6.3%-8.3% over the period 2013/14-2016/17 [9].

fluctuating wind output but also enables price arbitrage. The producer is penalized for any positive (negative) imbalance, which arises when the actual amount of energy provided in real-time is greater (less) than the committed amount.

This thesis formulates the wind power producer's decision-making problem as a Markov decision process (MDP) by taking into account the uncertainties associated with wind speed and electricity price. MDPs offer an elegant mathematical framework to represent the decision-making process of dynamic systems when the outcomes are either random or controlled by a decision-maker who makes challenging, sequential decisions over time. MDPs determine the best course of action for the decision-maker based on the current state and system environment. While MDPs can capture complex systems, they still provide clean analytical formulations that enable optimal policy characterization.

Our knowledge of the energy commitment problems of renewable power generators is largely based on stochastic optimization models that include such uncertainties by building scenario trees and performing sensitivity analysis. MDPs have not been widely used as a modeling approach in this research stream; their use has become more popular only in recent years. From a general point of view, this thesis aims to optimize the operations of renewable power producers with energy storage systems in electricity markets and contribute to the literature in the following ways. First, we develop an MDP framework to distinguish between the following two roles of an energy storage unit: when it is used to support either intentional deviations from commitments or fulfillment of commitments. We investigate the effects of system components and electricity market characteristics on the system operations and profits in different environments. Second, we establish multi-dimensional structural properties of the optimal profit function and leverage these properties to characterize optimal policy structures for renewable power producers who jointly optimize their energy generation, storage, and commitment decisions. Third, we employ our structural results to develop heuristic solution procedures and assess their effectiveness against well-known alternative solution methods. Finally, we establish a theoretical upper bound on the total cash flow difference between two different market settings.

The rest of the thesis is organized as follows. Chapter 2 provides a general overview of electricity markets and summarizes the related literature. In Chapter 3, we study the energy commitment, generation and storage problem for a wind power producer who can own and operate a battery for different purposes. We consider two main problem settings: In the first setting, the producer may choose to deviate from her commitments based on the latest available information, using the battery to support such deviations. In the second setting, the producer is required to fulfill her commitments, using the battery as a back-up source. We also consider the special cases of these settings with no battery. We analytically compare the total profits of the two main settings. We then conduct data-calibrated numerical experiments to examine the effects of system components, imbalance pricing parameters, negative prices, and wind availability on the system operations and profits.

In Chapter 4, we prove the optimality of a state-dependent threshold policy for the wind power producer's energy commitment, generation and storage problem. This policy is valid under perfectly efficient battery and transmission line and positive electricity prices. In our structural analysis, we first formulate the optimal amount of wind energy that should be generated in any period as a function of the state variables. We then partition the state space into two disjoint domains that correspond to the optimal decisions of 'positive imbalance' and 'negative imbalance,' respectively: it is optimal to bring the storage and commitment levels to a different state-dependent threshold pair in each domain. We establish supermodularity of the optimal profit function, revealing the complementarity effect between the commitment and storage levels. We also prove that the optimal target levels for the commitment and storage decisions are higher in the case of positive imbalance than in the case of negative imbalance. We employ our structural results to develop a heuristic solution procedure in a more general problem where the battery and transmission line need not be perfectly efficient. We also consider the case where the price can be negative. Our numerical experiments with data-calibrated instances have revealed the high efficiency and scalability of our solution procedure. Our numerical experiments have also revealed the poor performance of simpler heuristic approaches – purely myopic policies, fixed threshold policies, and a deterministic reoptimization heuristic – with respect to objective value.

In Chapter 5, we extend the optimal policy characterization presented in Chapter 4 in the more general case when the battery and transmission line can be imperfectly efficient. The state space of the problem is partitioned into several disjoint domains that correspond to the optimal decisions of 'positive imbalance' and 'negative imbalance' as well as to the optimal decisions of 'charge and purchase,' 'charge and sell,' and 'discharge and sell,' respectively. We also prove that the optimal target levels for the storage decisions are higher in the case of discharge and sell and lower in the case of charge and purchase.

In Chapter 6, we consider an alternative problem setting that allows for real-time trading without making any advance commitment. We analytically compare the total cash flows of this setting to our original problem setting. After establishing a theoretical upper bound for the difference between total cash flows in these two settings, we conducted numerical experiments to gain further insights into the impact of commitment decisions.

We conclude the thesis with a summary of results and future research directions in Chapter 7.

Chapter 2

Electricity Markets and Related Literature

2.1 General Overview of Electricity Markets

2.1.1 Evolution of electricity markets

The electricity supply chain involves multiple stages, from generation to consumption by end-users. Firstly, electricity is generated by power plants. The generated electricity is transmitted via grids to a distribution center. Then, the distribution center is required to send electricity to a retailer. The retailer is in charge of transferring electricity to end-users. Historically, the electricity market structure was built on vertically integrated utilities that were either government-owned or privately owned. Each of the four main parts of the vertically integrated structure (generation, transmission, distribution, and retailing) had its own management, but they all depended on each other and were monitored by a central authority. This monopolistic structure led to a central dispatch of generation which resulted in low electricity quality and low economic efficiency.

With technological and industrial developments, people consumed more electricity, and it became difficult for governments to manage power plants efficiently. This situation encouraged the formation of a new market structure in which generation was carried out by Independent Power Producers (IPPs), private utilities that owned power plants to generate electricity. The integration of IPPs into the market structure facilitated privatization and competition while decreasing central planning [12]. Nevertheless, the distribution center remained under government control as the sole buyer of generated electricity. This structure demonstrates a primitive version of a liberalized market since there is competition among generators; however, transmission still follows a monopolistic structure.

The aforementioned market structures led to a lack of competition among companies, resulting in high operating costs, high retail prices, and low supply security [13]. To overcome these inefficiencies, a liberalization process was initiated in the electricity sector during the late 1990s, featuring multiple buyers (e.g., regional transmission, distribution, or retail companies) and multiple sellers (i.e., IPPs). This new configuration fostered a deregulated, competitive, and transparent market environment in generation, transmission, distribution, and retailing.

2.1.2 Electricity markets

Electricity trading can be classified into two categories based on their transaction periods: long-term transactions and short-term transactions. Power purchase agreements (PPAs), also known as bilateral contracts, are typically used for long-term transactions (though some may be made for short-term transactions depending on the needs of the parties involved). These contracts are often formed between two parties who agree that one will purchase (usually a utility or a consumer) and the other will sell energy for a fixed price, a variable price, or a combination of both. Merchant/quasi-merchant agreements, on the other hand, are a type of PPA, but they differ in that generated energy is not sold directly to a utility or a customer; instead, it is sold into the spot market at current market prices [14].

Short-term electricity markets, also known as spot markets, are markets where electricity is traded for delivery, usually within a day or an hour time frame (i.e., trading on short-term horizons). Electricity trading is conducted through offers and bids (i.e., commitments) made by market participants (i.e., producers/sellers and consumers/buyers) for selling energy to the market and/or purchasing energy from the market. Short-term electricity markets include the day-ahead market, intraday market, balancing market, and ancillary services. Participants in the day-ahead market submit their commitments to purchase and sell electricity for each hour of the following day at the beginning of each day. Shortly after the market closes, the market price for each hour of the following day is published, and participants know which hours they are obligated to produce or consume.

Participants in the intraday market make transactions closer to the physical delivery of electricity. Trading occurs after the day-ahead market gate closure every day until one or a few hours before the real-time delivery. The main difference between intraday and day-ahead markets is the method for price calculation. The day-ahead market uses a uniform pricing method, with all market participants selling and paying at the same price. The intraday market, on the other hand, is generally run based on the continuous trading principle (although some countries, such as Italy, Spain, and Portugal, operate with auction-based intraday trading as in the day-ahead market) [15]. Prices in this market are determined on a first-come, first-served basis in continuous trading; in other words, offers and bids are continuously matched without any auction. The balancing market, also called the real-time market, is the last market opportunity for market participants to participate [7]. The gate closure of this market is typically half an hour before actual energy delivery. This market, like the day-ahead market, adopts a uniform pricing method, with the same price applied to all market participants.

During the transaction process, ancillary services may be required to balance supply and demand instantly. Regulation services monitor variations in supply and demand at all times, whereas reserve services control changes in load and supply on an hourly basis. In cases where supply and demand cannot be matched in real-time or where there is an urgent need for energy, these services (frequency and reserves) are utilized. In this market, producers (who operate very fast and flexible generators such as small combined heat and power units or pumped-hydro facilities) get paid by offering their availability to increase or decrease their power output based on the system requirements [16].

In order to keep the electricity market operating without any disruption, there should be a control mechanism. The market or system operators (Independent System Operator and Regional Transmission Organizations (ISO/RTO) or Transmission System Operator (TSO)) are important agents in the market that have the responsibility to ensure that the market is operated effectively. These entities are tasked with operating, monitoring, controlling, and coordinating the operation of the power system, as well as providing open and non-discriminatory transmission access to all market participants [17].

2.1.3 Deviations from commitments

The difference between the actual energy selling or purchasing amount and the commitment amount in an electricity market is referred to as an energy imbalance (i.e., an energy deviation). Energy deviations can occur for specific reasons. For example, predictions made by a market participant committed to meeting customers' demands may not always be accurate, as they are often based on seasonal conditions and system constraints. In addition, a market participant may intentionally engage in uninstructed deviations from generation schedules by strategically bidding at high prices or withholding their energy capacity [18, 19]. This situation may occur in markets controlled by the California Independent System Operator (CAISO), New York Independent System Operator (NYISO), Midcontinent Independent System Operator (MISO), and ISO New England [20]. Examples for the European-type markets include those in the Netherlands and Belgium [21, 22].

Although producers may see financial benefits from deviations, they can negatively impact the reliability of grid operations. To address this issue, although specifics vary from one country to another, various penalty schemes are implemented to decrease deviations caused by excess or insufficient supply or consumption of electricity [23]. This means that participants in the market who deviate from their committed production or consumption levels are penalized through an imbalance pricing mechanism.

The purpose of imbalance pricing mechanisms is to force market participants to make accurate predictions about their generation or consumption and to report their true forecast information, thereby reducing overall imbalance rates. Depending on the market design, imbalance pricing mechanisms can be classified into two main categories: single pricing and dual pricing [24]. In single pricing, the imbalance price is often determined by the balancing (real-time) price and the same price is applied to both negative imbalances (i.e., if her commitment amount is greater than her real-time generation or consumption) and positive imbalances (i.e., if her commitment amount is less than her real-time generation or consumption). In dual pricing, the imbalance price depends on the outcome of sequential markets and the direction of imbalance [25].

Imbalance pricing mechanisms may differ in Europe and the U.S. In European electricity markets, imbalances are mostly settled according to the dual pricing mechanism (e.g., the Nordic countries (for generation units), France, and the U.K.) [26]. Single pricing mechanism is also applied in countries such as Germany, the Netherlands, and Belgium [27]. On the other hand, U.S. electricity markets typically rely on the single pricing mechanism to address imbalances [28, 29, 30]. Although single pricing scheme is adopted in the U.S., the rules regarding penalties can change according to the system operators. For example, Electric Reliability Council of Texas (ERCOT) and NYISO penalizes only negative imbalances. However, in CAISO and MISO, a participant is penalized for both her negative and positive imbalances [30, 31, 32, 33].

2.2 Related Literature

This thesis focuses on joint optimization of the operations of renewable power plants with energy storage systems in electricity markets; hence, it is closely related to two streams of literature: (i) the energy commitment problem in electricity markets for renewable power generators and (ii) the joint operation of a renewable power generation and energy storage. We present a review of the related studies from each stream.

Several papers study the energy commitment problem of a renewable power producer: Pinson et al. [34] introduce a trading strategy based on short-term probabilistic forecasts of wind power and use a stochastic optimization model to identify the optimal trading strategy for a wind power producer in the day-ahead market. Morales et al. [35] propose a multi-stage stochastic optimization model that incorporates risk control on profit variability to determine the optimal trading strategy for a wind power producer in day-ahead and real-time markets. Dent et al. [36] derive analytical wind power bidding strategies based on the assumption that wind power has a continuous probability distribution. They also investigate scenarios in which the real-time pricing is determined by wind generation and employ Conditional Value at Risk to reflect risk-averse trading behavior. Botterud et al. [37] develop a stochastic optimization model and generate scenarios of wind power and prices to estimate the distribution of possible profits for each potential trading strategy in the day-ahead market. Bitar et al. [38] explicitly derive the optimal contract size for a wind power producer participating in the day-ahead market. Baringo et al. [39] consider the offering strategies of wind power producers participating in day-ahead and balancing markets using a multi-stage stochastic optimization model with risk constraints. They model the uncertainty of wind, price, bids, and offers using a set of scenarios.

Several other papers study the energy commitment problem of an energy storage operator: Löhndorf et al. [40] optimize the commitment and storage decisions for a pumped hydro energy storage (PHES) facility participating in the day-ahead market. They formulate a multi-stage stochastic program for the intraday decisions and an MDP for the interday decisions. They combine stochastic dual dynamic programming and approximate dynamic programming (ADP) methods to develop a solution approach. Boomsma et al. [25] formulate a multi-stage stochastic programming model to optimize the commitment decision of a hydropower plant with two serially connected reservoirs in the day-ahead market and the balancing market sequentially.

Paine et al. [41] study the commitment and operational decisions for a PHES facility under the regulations of two different system operators. The PHES operator aims to determine how to participate in day-ahead and ancillary services markets in order to maximize her profit. They develop a dynamic programming formulation for their problem and examine the impact of market rules on optimal policies. Jiang and Powell [33] consider a battery storage operator who makes hourly commitments in a real-time market with five-minute settlements. This means that a clearing price is set every five minutes based on the submitted bids and offers. They formulate their problem as an MDP and establish a monotonicity result for the optimal value function. They derive a suboptimal commitment policy by employing a novel ADP technique.

Berrada et al. [42] formulate a mixed-integer linear programming model to analyze the value of a storage unit operating in day-ahead, balancing, and ancillary services markets simultaneously. In their analysis, the operator can charge energy from or discharge it into the day-ahead market or the balancing market. She can also sell capacity to the ancillary services market. Karhinen and Huuki [43] propose an operating strategy for a PHES operator who participates in day-ahead, intraday, and balancing markets. In their problem, the PHES operator first commits to the day-ahead market and acts according to a price arbitrage strategy. If she cannot fulfill her day-ahead commitment during the operation time, she participates in the intraday market to adjust herself. The operator can then sell her available capacity in the balancing market, with the bidding strategy determined through a discrete-time dynamic programming model. Löhndorf and Wozabal [44] develop a multi-stage stochastic optimization model for day-ahead bidding and hourly intraday trading of a storage operator, assessing the value of a coordinated trading strategy.

There are also papers that study the energy commitment problem of a renewable power producer having an energy storage unit: Garcia-Gonzalez et al. [45] develop a two-stage stochastic optimization model for the commitment decisions in the day-ahead market for an energy system consisting of a wind farm and a PHES facility. They build a scenario tree to incorporate the uncertainties in wind speed and electricity price. Löhndorf and Minner [46] formulate the bidding strategy of a renewable power producer with a storage option in the day-ahead market as an MDP. They develop an ADP model that approximates the optimal value function using a set of basis functions.

Kim and Powell [32] focus on optimizing commitments submitted by a wind power producer that utilizes a storage unit in a balancing market with hour-ahead adjustments. They formulate their problem as an MDP and derive a closed-form solution of the optimal commitment amount of the producer. Mauch et al. [47] investigate the profitability of committing in the day-ahead market for an energy system consisting of a wind farm and a compressed air energy storage (CAES) system. They assume that the producer can either sell the generated energy directly or store it in the CAES system for later dispatch. They use Monte Carlo simulation to calculate the expected profit by generating possible wind generation scenarios from various empirical probability distributions. They formulate an MDP to find the optimal dispatch amount with and without CAES.

Ding et al. [48] analyze the coordinated operation of a wind power producer with a PHES facility in day-ahead and intraday markets. They propose a chance-constrained formulation to consider the possibility of wind power generation within a given time interval and scenario while ensuring that the forecasted wind power amount can only exceed the actual amount to a certain extent. Castronuovo et al. [49] consider three utilization of a wind-PHES energy system: (i) PHES facility is used for price arbitrage purposes; (ii) PHES facility is used as a back-up source to decrease any energy deviations in wind generation; and (iii) PHES facility is not included in the commitment decisions; therefore, the wind power producer can only sell energy to the market. It is only included in the operational phase to minimize imbalance costs. They formulate these problems as a stochastic optimization model and incorporate a chance constraint in which the wind power generated is greater than both the amount consumed by the PHES facility and the amount sent to the power grid with a certain probability.

Ding et al. [50] consider the commitment decisions and operational strategies for a wind farm with a storage unit in the day-ahead market. They develop a mixed integer nonlinear programming model to determine the optimal commitments and reserve capacity that should be available before the real-time operation. Gönsch and Hassler [51] formulate the commitment problem of a wind power producer having a storage unit in an intraday market and develop an ADP algorithm that combines approximate policy iteration with classical backward induction to solve the problem. Hassler [52] extends the work of [51] by constructing more straightforward yet effective heuristic decision rules. Finnah and Gönsch [53] extend the work of [51] and [52] by allowing for two storage units (a battery and a hydrogen-based storage unit) and solving the problem with a backward ADP algorithm.

Gomes et al. [54] study the commitment decisions in the day-ahead electricity market for a wind and photovoltaic power producer with battery storage. They formulate a two-stage stochastic optimization model by generating scenarios to include wind speed, solar radiation, and electricity price uncertainties. Al-Swaiti et al. [55] consider a thermal power plant along with a PHES facility in day-ahead and ancillary services markets. They develop a stochastic optimization model to study the producers' coordinated bidding problem by building scenario trees to include the uncertainties in wind speed and electricity price. They perform their analysis for different risk aversion attitudes of the producer. Considering the uncertainties in wind speed and electricity prices, Diaz et al. [56] formulate the energy commitment of a wind power producer operating a storage unit as an MDP for the Spanish day-ahead, intraday, and balancing markets. They assume that the producer can submit her commitments to various markets operating in the same dispatch hour.

Finally, several papers study the optimal operation of renewable power plants together with energy storage systems without explicitly focusing on commitment decisions: Castronuovo and Lopes [57] present a stochastic optimization model for identifying the optimal daily operation schedule to be followed by a wind power plant and a PHES facility. The wind uncertainty is incorporated into the model with scenario generation. Brown et al. [58] optimize the pumping and generation decisions of a PHES facility to fully exploit the wind potential of an island using a linear programming method. The authors employ scenarios generated through fuzzy clustering to account for the stochastic nature of load and renewable production.

Duque et al. [59] propose a statistical method to find the optimal operation schedule of a wind power plant and a PHES facility consisting of a single reservoir. They generate scenarios to represent the uncertainty in wind speed. Vespucci et al. [60] present a stochastic optimization model for daily operation scheduling of an energy system consisting of a wind power plant and a PHES facility. The hourly wind power production uncertainty is modeled using scenario trees. They also evaluate the value of stochastic solution. Harsha and Dahleh [61] formulate the energy storage and sizing problem for a wind power plant as an MDP. They assume that any excess energy is lost or sold back into the grid at zero price. They prove that the optimal storage policy is a price-dependent dual-threshold policy.

Xi et al. [62] evaluate the use of battery storage for multiple applications, such as arbitrage, back-up, and ancillary services. In each time period, the goal is to find the amount of energy that should be charged/discharged and the amount of energy that should be sold to the ancillary services market. They formulate the problem as an MDP and provide a suboptimal operation policy with an ADP technique. Tang et al. [63] present a comparative analysis of several types of energy storage commonly used in real-world scenarios, including lead acid, sodium sulphur, and fuel cell, in the context of an optimal operation problem with stochastic wind power generation and electricity price. They formulate the problem as an MDP and utilize a monotone adaptive dynamic programming method to simulate the optimal policy.

Table 2.1: Classification based on the optimization techniques.

Research Topic	Optimization Technique	References
(i) Energy commitment problem for renewable power generators	Stochastic Optimization	[25], [34], [35], [36], [37], [38], [39], [40], [44], [45], [48], [49], [54], [55]
	Markov Decision Process	[32], [33], [46], [47], [51], [52], [53], [56]
	Deterministic Optimization	[41], [42], [43], [50]
(ii) Joint operation of renewable power generation and energy storage	Stochastic Optimization	[57], [59], [60]
	Markov Decision Process	[8], [61], [62], [63], [64], [65]
	Deterministic Optimization	[58]

Zhou et al. [8] propose an MDP for the operations of a wind farm co-located with an industrial battery. Avci et al. [64] propose an MDP for the operations of a wind farm co-located with a pumped hydro energy storage facility having two connected reservoirs at different altitudes. Both [8] and [64] establish the optimality of state-dependent threshold generation and storage policies under positive electricity prices and limited transmission capacity. They develop time-series models for wind speed and electricity price and incorporate these models into their MDP. They evaluate the use of several heuristic methods aimed at reducing the computational burden of the problem.

Peng et al. [65] focus on optimizing the joint operations of three distinct components within an energy system, namely a renewable source, a flexible source (natural gas), and a storage unit. They identify the optimal structure for operating policies, focusing on the storage control policy. They also examine the investment costs associated with these resources and analyze their interrelationships to identify the most efficient investment strategies.

Table 2.1 summarizes the optimization techniques used in the studies for both research streams (i) and (ii). It can be seen that the majority of papers that study energy commitment problems employ stochastic optimization models in

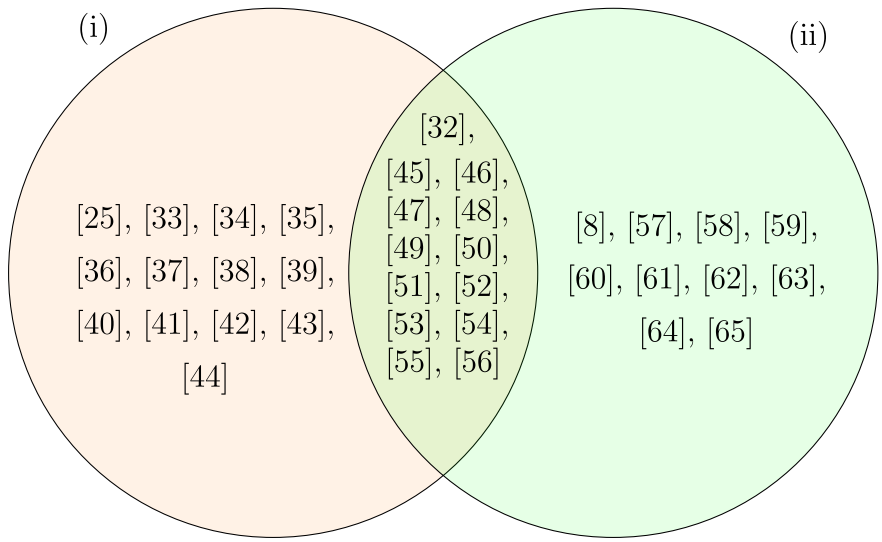

Figure 2.1: Classification based on the research topics: (i) Energy commitment problem for renewable power generators and (ii) Joint operation of renewable power generation and energy storage.

which they capture uncertainty with scenario generation. MDPs have rarely been used as a modeling approach in this stream, but they have gained attention in recent years due to their ability to resolve uncertainties as the decision-maker advances in time and make adaptive decisions based on real-time realizations of uncertainties. In addition, MDPs provide a precise mathematical framework for deriving structural results analytically by capturing the multi-dimensional dynamics and uncertainty inherent in the problem. Figure 2.1 shows papers that present a Venn diagram of studies that fall into the two research streams described in (i) and (ii). It can be seen from Figure 2.1 that out of the 36 reviewed studies, 13 of them examine the joint operation of renewable power plants with energy storage systems while considering commitment decisions. Among these studies, [32], [46], [47], [51], [52], [53], and [56] formulate their problem as an MDP.

This thesis makes several contributions to the research stream of joint optimization of the operations of renewable power plants with energy storage systems in electricity markets. These contributions are discussed as follows:

- Chapter 3 is the first attempt to develop an MDP framework to distinguish between the following two roles of a battery: when it is used to support either intentional deviations from commitments or fulfillment of commitments. We present and compare two different MDP formulations for these different roles of the battery under uncertainty. We investigate the effects of system components (battery and transmission line) and spot market characteristics (negative electricity prices, two-way system-market transactions, and asymmetric imbalance prices) on the system operations and profits in different environments.

- Chapters 4 and 5 are the first attempts to provide multi-dimensional structural properties, including supermodularity and joint concavity, for the optimal profit function. These properties are leveraged to prove the optimality of a state-dependent threshold policy for a renewable power producer who jointly optimizes her energy commitment, generation, and storage decisions. In Chapter 4, we employ the optimal policy structure to construct efficient and scalable heuristic solution methods.

- Chapter 6 is the first attempt to establish a theoretical upper bound on the total cash flow difference between two problem settings: selling energy to or purchasing energy from the market in real-time (i) with making advance commitment decisions and (ii) without making advance commitment decisions. We examine the impact of advance commitment decisions on the producer's storage and generation decisions.

Chapter 3

Deviations from Commitments: Markov Decision Process Formulations for the Role of Energy Storage

3.1 Introduction

Motivated by the increasing trend of renewable power generators co-located with batteries, we study the operations of a wind power plant paired with a battery. This energy system is connected via a transmission line to a spot market that operates with hourly commitments and settlements. We focus on detailed representations of the battery (e.g., energy and power capacities as well as asymmetric and imperfect efficiencies for storing/generating energy) and the transmission line (e.g., selling/purchasing capacity and imperfect efficiency). In this chapter, we examine various roles of the batteries by considering two possible problem settings. In the first setting, the producer may intentionally deviate from her commitments for a better profit. In the second setting, however, the producer

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needs to fulfill her commitments as much as possible, storing any excess wind energy as long as the battery capacity allows. We also consider two other settings that are the special cases of the above settings with no battery. In these settings, the producer decides how much energy to commit to the market, to generate in the wind power plant, and to charge into or discharge from the battery. Since the transmission line has a finite capacity and negative electricity prices may occur in our model, the producer also decides how much energy to curtail in the wind power plant.

Deviations from commitments can be desirable for the wind power producer in our problem setting: If the generated wind energy falls short of meeting the commitment, the producer is expected to discharge the battery to compensate for the shortfall and pay the penalty for the remainder (if there is any). If the generated wind energy exceeds the commitment, the producer is expected to charge the battery and curtail the excess amount. However, if the future price is anticipated to be very low (or high) based on the latest available information, the opportunity cost of charging (or discharging) the battery to meet the commitment may be higher than the corresponding penalty for not meeting the commitment. Hence, the producer may choose to stay in a positive (or negative) imbalance, observed when the committed amount is less (or greater) than the actual amount provided in real-time, without charging (or discharging) the battery as per the commitment, motivating us to study the first problem setting. Such use of the battery to intentionally deviate from commitments is observed in some electricity markets in the U.S. (e.g., markets controlled by CAISO, NYISO, and MISO) and in Europe (e.g., the Netherlands and Belgium) [19, 20, 21, 22].

While the intentional deviations can be financially attractive for the producers, they may lower the reliability of the grid operations. Therefore, some countries explicitly implement some regulations to prevent this practice. For example, the intentional deviations were allowed in the German electricity market from 2003 to 2009, but these deviations gradually disappeared after 2011 when this practice was not allowed by the German system operator [22, 53, 67]. Strategically causing an energy imbalance is also not allowed in Italy [68]. Under such regulations, the producers must fulfill their commitments as much as possible even when it is financially unattractive to do so, motivating us to study the second problem setting.

We formulate the real-time decision-making process of the wind power producer as an MDP. The intentional deviations in our study can arise due to the price and wind uncertainties that are resolved over time. (The intentional deviations could also arise from any potential strategic behavior of the power producers to manipulate the energy dispatch/demand amounts and electricity prices in the market and/or to enter multiple markets to profit from the price differences between markets. This possibility is beyond the scope of this chapter.) Therefore, investigating the different roles of the battery (i.e., a strategic tool to intentionally deviate from commitments in the first setting or a back-up source to fulfill commitments in the second setting) becomes critical in the presence of price and wind uncertainties. The two settings would be identical if there were no such uncertainties (under a mild condition in the initial period of our MDP). Our MDP framework enables us to effectively capture these uncertainties in the hour-ahead market structure by allowing future uncertainties to be resolved only when the producer moves forward in time. The producer can thus make adaptive decisions based on the realizations of uncertainties in real-time. In addition, MDPs provide clean analytical formulations by capturing the complex dynamics inherent in many real-life problems like ours. Focusing on the hour-ahead market structure with our MDP framework, we are able to analytically compare the total profits of the two settings and numerically solve the realistic-size problem instances to optimality.

We conduct an extensive numerical study by focusing on the historical electricity price and wind speed data from a market where negative prices were observed in the past. We construct time series models that can represent the historical data with acceptable accuracy levels. Our time series models capture the mean-reversion, seasonality, and spike components of electricity prices, as well as the significant seasonal patterns of wind speeds. We incorporate these models into our MDP with the help of exogenous state variables. We then solve the problem with a backward dynamic programming algorithm for various realistic configurations of our energy system.

In the literature on joint operation of renewable power generation and energy storage in electricity markets, papers that study MDP representations of a similar problem are those of Kim and Powell [32], Löhndorf et al. [40], Löhndorf and Minner [46], Gönsch and Hassler [51], Hassler [52], Diaz et al. [56], and Finnah and Gönsch [53]. See Chapter 2 for detailed discussions of these studies.

The above studies do not examine the potential supporting role of the storage unit to intentionally deviate from commitments. Specifically, in these studies, the renewable power producer can utilize the storage unit only to reduce the energy imbalance as much as possible. Therefore, there are no generation and storage decisions in their MDP formulations. In addition, none of these studies investigate the effect of negative electricity prices. However, as the renewable power generation grows in electricity markets, the negative prices are unavoidably observed more frequently, increasing the need for this investigation [69, 70, 71]. These studies, except Finnah and Gönsch [53], also assume that electricity can only be transmitted from the energy system to the market (i.e., the renewable power producer can only sell energy to the market). However, when the electricity price is too low or negative, the producer with a storage unit may want to benefit from these prices by purchasing energy from the market. In Kim and Powell [32] and Gönsch and Hassler [51], the imbalance price applies only to the case of negative imbalance (i.e., when the committed amount exceeds the actual amount provided). Löhndorf et al. [40] assume that when the producer cannot fulfill her commitment in the day-ahead market, the imbalance can be cleared in the intraday market. Therefore, they do not explicitly model the penalty costs that can be observed in real-time. Lastly, Hassler [52] and Finnah and Gönsch [53] assume that the imbalance prices are the same regardless of the direction of imbalance (negative or positive). We relax all of the above assumptions in this study.

To sum up, we contribute to the energy literature as follows: We present MDP formulations for the energy commitment, generation, and storage problem of a producer who uses the battery (i) as a strategic tool for supporting intentional deviations from her commitments or (ii) as a back-up source for fulfilling her commitments. We analytically show that the role of the battery in (i) brings an additional profit to the producer, while this role of the battery is likely to be undesirable for the market. We numerically investigate the impacts of the system components, imbalance pricing parameters, negative electricity prices, and wind availability in these settings when the battery is available and when it is not. We summarize below the key findings from our numerical study that may have policy implications for both regulatory authorities and wind power producers in electricity markets:

- If the producer cannot make intentional deviations, the existence of a battery reduces the energy imbalance. However, if she is allowed to make intentional deviations, the existence of a battery has an opposite effect, inducing the negative imbalance more than the positive imbalance.
- When the intentional deviations are allowed, the imbalance amounts can be reduced by properly choosing the imbalance pricing parameters. However, the imbalance amounts in the absence of a battery are not affected by the symmetric imbalance pricing parameters.
- When the intentional deviations are not allowed, the imbalance pricing parameters substantially affect the imbalance amounts of the producer with no battery. However, the existence of a battery makes the producer relatively insensitive to the imbalance pricing parameters.
- When the intentional deviations are not allowed, a higher battery capacity tends to induce a lower imbalance amount. When the intentional deviations are allowed, a higher battery capacity may induce a lower imbalance amount under a low transmission line capacity, whereas it induces a higher imbalance amount under a high transmission line capacity.
- Although the existence of a battery provides the opportunity to purchase energy when the electricity prices are negative, the total profits in all settings suffer from an increment in the negative price occurrence frequency. The supporting role of the battery for intentional deviations becomes more valuable as this frequency grows.

The remainder of this chapter is organized as follows. Chapter 3.2 formulates the energy commitment, generation, and storage problem for each of the two settings. Chapter 3.3 presents our time series models for the electricity price and wind speed as well as their discrete-state representations that we require for our numerical experiments. Chapter 3.4 presents our numerical results and Chapter 3.5 concludes. Additional numerical results for Chapter 3.4.4 are contained in Appendix A.

3.2 Problem Formulation

We consider an energy system that consists of a wind power plant and a battery. See Figure 3.1 for an illustration of this system. The producer can generate energy from the wind power plant and the battery. The producer can also store energy by charging the battery. The producer participates in a single spot market that involves hour-ahead commitments and hourly settlements. The single-market assumption has been made in many papers that focus on the operations of renewable power producers in electricity markets; see, for example, [32, 33, 46, 51, 52, 53] and [72]. Considering the uncertainties in electricity price and wind speed, the producer should make several challenging decisions in each time period: how much energy to generate in the wind power plant, to charge/discharge into/from the battery, and to commit to selling/purchasing to/from the market in the next period. The system makes only a very limited contribution to the overall energy supply in the market; therefore, the producer can be viewed as a price-taker. This assumption has also been made in many papers that focus on the operations of renewable power producers in electricity markets; see, for example, [32, 34, 46, 50, 51, 52] and [56].

The battery has finite energy and power capacities. The energy capacity of the battery is the maximum amount of energy that can be stored in the battery. The power capacity is the maximum amount of energy that can be generated (by discharging the battery) or stored (by charging the battery) in a single time period. We denote the energy capacity of the battery by C_S. We denote the

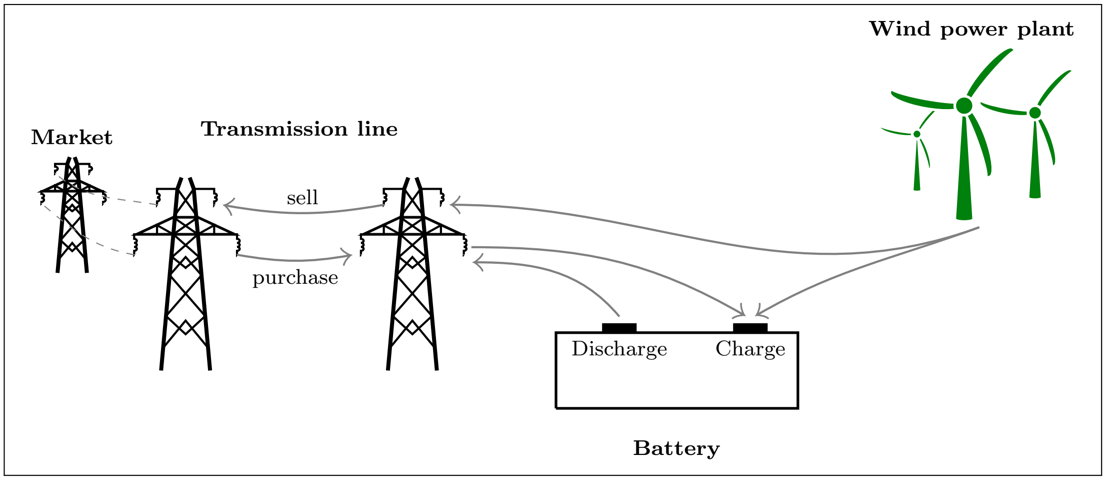


Figure 3.1: Illustration of the energy system.

generation and storage capacities by K_D and K_C, respectively, in power units. The transmission line also has a finite power capacity. We denote this capacity by K_T in power units. For notational convenience, we define $C_C = K_C\Delta t$, $C_D = K_D\Delta t$, and $C_T = K_T\Delta t$, where Δt is the length of one time period. For the battery, we denote by $\gamma \in (0,1]$ and $\theta \in (0,1]$ the efficiency parameters in the discharging and charging modes, respectively. For the transmission line, we denote by $\tau \in (0,1]$ the efficiency parameter in both the selling and purchasing modes. These parameters represent the ratio of energy output to energy input.

We study the energy commitment, generation, and storage problem in this system via a dynamic model over a finite planning horizon of T periods. Let $\mathcal{T} := \{1, 2, \ldots, T\}$ denote the set of periods. We define $S_t \in [0, C_S]$ as the amount of energy accumulated in the battery at the beginning of period t. The producer can either sell or purchase energy in any particular period. We define $Q_t \in \mathbb{R}$ as the amount of energy that the producer is obligated to sell if $Q_t \geq 0$ and to purchase if $Q_t < 0$ in period t, in order to fulfill the commitment that she has made in period $t-1$. We include S_t and Q_t in our state description. We define $W_t \in \mathbb{R}_+$ as the wind speed in period t and $f(W_t)$ as the maximum amount of wind energy that can be generated in period t. We derive $f(W_t)$ from the multiplication of the power output of a wind turbine when the wind speed is W_t, the number of turbines in the wind power plant, and the period length Δt. Lastly, we define $P_t \in \mathbb{R}$ as the electricity price in period t. We also include the

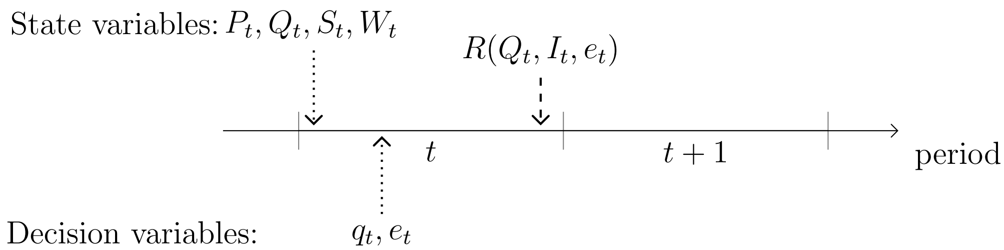


Figure 3.2: Sequence of events in each period.

tuple $I_t := (P_\kappa, W_\kappa)_{\kappa \leq t}$ in our state description. The state tuple I_t evolves over time according to an exogenous stochastic process.

In any period $t \in \mathcal{T}$, the producer first observes the exogenous state variables (P_t and W_t) as well as the endogenous state variables (Q_t and S_t). The producer then determines the amount of energy to be committed to selling or purchasing in period $t+1$ by signing the contract in period t, which we denote by $q_t \in \mathbb{R}$, and the actual amount of energy to be sold or purchased in period t, which we denote by $e_t \in \mathbb{R}$. The transmission line capacity implies that $e_t \leq \tau C_T$ if $e_t \geq 0$ (the energy is sold) and $-C_T \leq e_t$ if $e_t < 0$ (the energy is purchased). The market pays the purchasing cost for the amount of energy it actually receives, whereas the producer pays the purchasing cost for the amount of energy supplied on the other end of the transmission line. Although the maximum amount of energy that can be supplied to the transmission line (on either end) is C_T, the actual amount of energy received by the market (and thus sold by the producer) cannot exceed τC_T due to the efficiency loss during transmission. An alternative interpretation of such market transactions is that the producer is obligated to pay the grid usage fee per unit energy sold or purchased. Finally, we note that the actual amount of energy sold or purchased in period t may be different from the state variable Q_t. See Figure 3.2 for an illustration of the sequence of events.

The producer faces the challenge of ensuring that the energy committed to selling/purchasing is sold/purchased in real-time. If the producer does not fulfill her contractual commitment in real-time (i.e., $Q_t \neq e_t$), she pays a penalty cost that varies with her deviation (i.e., the difference between the committed and realized amounts). There are two possible cases: (i) The commitment is less than the amount that the producer indeed provides in real-time (i.e., $Q_t < e_t$); the producer experiences a 'positive imbalance.' (ii) The commitment is greater than the amount that the producer indeed provides in real-time (i.e., $Q_t > e_t$); the producer experiences a 'negative imbalance.'

In our study, we implement a market-based imbalance pricing mechanism that is mostly related to the U.S. type markets. See Chapter 2.1.3 for detailed discussions on imbalance pricing mechanisms. Specifically, we take the imbalance prices as linear functions of the market price: the imbalance price in the case of positive imbalance is always $K_p^+(K_p^-)$ times the market price and the imbalance price in the case of negative imbalance is always $K_n^+(K_n^-)$ times the market price. This assumption is in line with the literature (e.g., [32, 33, 46, 51, 52, 53, 67]). To approach the problem in a more general sense, we ensure that the producer is settled with a penalized price in the cases of positive and negative imbalances. This principle has been adopted in many related papers (e.g., [25, 34, 46]). The payoff in period t is given by

$$R(Q_t, I_t, e_t) = \begin{cases} Q_t P_t + K_p^+ P_t (e_t - Q_t) & \text{if } P_t \geq 0 \text{ and } Q_t < e_t, \\ Q_t P_t - K_n^+ P_t (Q_t - e_t) & \text{if } P_t \geq 0 \text{ and } Q_t \geq e_t, \\ Q_t P_t + K_p^- P_t (e_t - Q_t) & \text{if } P_t < 0 \text{ and } Q_t < e_t, \\ Q_t P_t - K_n^- P_t (Q_t - e_t) & \text{if } P_t < 0 \text{ and } Q_t \geq e_t, \end{cases}$$

where $0 \leq K_p^+ < 1 < K_n^+$ and $0 \leq K_n^- < 1 < K_p^-$. In this formulation, $K_p^+ P_t$ and $K_n^+ P_t$ denote the imbalance prices when the electricity price is nonnegative in the cases of positive and negative imbalances, respectively. Likewise, $K_p^- P_t$ and $K_n^- P_t$ denote the imbalance prices when the electricity price is negative in the cases of positive and negative imbalances, respectively. The first term of the payoff function corresponds to the instant revenue $Q_t P_t$ in period t, which may be negative due to negative electricity prices or negative commitment levels. The second term of the payoff function captures the penalty payments in period t.

We consider two possible problem settings: In the first setting, the producer can intentionally deviate from her commitments in real-time transactions (Chapter 3.2.1). In the second setting, the producer should always fulfill her commitments as much as possible while keeping the curtailment at the minimum level (Chapter 3.2.2).

3.2.1 The use of battery for intentional deviation

In this setting, the producer may choose to deviate from her commitments in real-time transactions by jointly optimizing the available energy sources (wind power plant and battery). Specifically, she determines the amount of wind energy that will be generated in period t, which we denote by $w_t \in \mathbb{R}_+$, and the amount of energy that will be discharged or charged in period t, which we denote by $s_t \in \mathbb{R}$. The battery is discharged if $s_t \geq 0$ and charged if $s_t < 0$. Let $\mathbb{U}(Q_t, S_t, I_t)$ denote the set of action triplets (q_t, s_t, w_t) that are admissible in state (Q_t, S_t, I_t). For any action triplet $(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)$, the following conditions must hold: The observed wind speed limits the amount of wind energy that can be generated in the form of $0 \leq w_t \leq f(W_t)$. The energy and power capacities of the battery imply that $-\min\{C_S - S_t, C_C\} \leq s_t \leq \min\{S_t, C_D\}$. The amount of energy that can be charged/discharged into/from the battery and the amount of wind energy that can be generated are restricted together by the transmission line capacity (see [73] and [74]). Thus, the power capacity of the transmission line implies that $\gamma s_t + w_t \leq C_T$ if $s_t \geq 0$ and $-\tau C_T \leq s_t/\theta + w_t \leq C_T$ if $s_t < 0$. The state variables S_t and Q_t evolve over time as follows: $S_{t+1} = S_t - s_t$ and $Q_t = q_{t-1}$.

There are three different types of decisions that we need to consider in order to formulate the amount of energy sold or purchased in any period t: (i) A certain amount of energy is generated by discharging the battery ($s_t \geq 0$). The resulting energy together with the generated wind energy is sold in the market. (ii) A certain amount of energy is stored by charging the battery ($s_t < 0$). If the generated wind energy is sufficient to charge the battery ($s_t/\theta \geq -w_t$), the excess wind energy is sold to the market. (iii) If the generated wind energy is not sufficient to charge the battery ($s_t/\theta < -w_t$), the required additional energy is purchased from the market. Hence, the amount of energy sold or purchased in period t is defined as a function of actions s_t and w_t:

$$e_t = E(s_t, w_t) := \begin{cases} (\gamma s_t + w_t)\tau & \text{if } s_t \geq 0, \\ (s_t/\theta + w_t)\tau & \text{if } -w_t \leq s_t/\theta < 0, \\ (s_t/\theta + w_t)/\tau & \text{if } s_t/\theta < -w_t \leq 0. \end{cases}$$

Since the decision variable s_t is not constrained by the commitment level Q_t, the producer has the flexibility to use the battery as a strategic tool to intentionally deviate from her commitments. When the battery has sufficient energy to fulfill the commitment, the producer may choose to stay in negative imbalance in order to keep the battery full enough to benefit from high electricity prices in the future. The producer may also choose to stay in positive imbalance in order to keep the battery empty enough in anticipation of lower prices in the future.

Without loss of optimality, the amount of energy committed to selling (i.e., $q_t \geq 0$) can be restricted to take values no larger than the maximum amount of energy that can be sold (bounded by the transmission line capacity, i.e., $q_t \leq \tau C_T$). In addition, the amount of energy committed to purchasing (i.e., $q_t < 0$) can be restricted to take values no larger than the maximum amount of energy that can be stored (bounded by the available storage capacity $\frac{C_S-(S_t-s_t)}{\theta\tau}$, the charging capacity $\frac{C_C}{\theta\tau}$, and the transmission line capacity C_T). Therefore, at optimality, $-\min\left\{\frac{C_S-(S_t-s_t)}{\theta\tau}, \frac{C_C}{\theta\tau}, C_T\right\} \leq q_t \leq \tau C_T$, $\forall t \in \mathcal{T}$, and $-\min\left\{\frac{C_S-S_t}{\theta\tau}, \frac{C_C}{\theta\tau}, C_T\right\} \leq Q_t \leq \tau C_T$, $\forall t \in \mathcal{T}\backslash\{1\}$.

A control policy π is the sequence of decision rules $(\eta_t^\pi(Q_t^\pi, S_t^\pi, I_t))_{t\in\mathcal{T}}$, where $\eta_t^\pi(Q_t^\pi, S_t^\pi, I_t) := (q_t^\pi(Q_t^\pi, S_t^\pi, I_t), s_t^\pi(Q_t^\pi, S_t^\pi, I_t), w_t^\pi(Q_t^\pi, S_t^\pi, I_t))$, and Q_t^π and S_t^π denote the random state variables governed by policy π, $\forall t \in \mathcal{T}\backslash\{1\}$. We denote the set of all admissible control policies by Π. For any initial state (Q_1, S_1, I_1), the optimal expected total cash flow over the finite horizon is given by

$$\max_{\pi\in\Pi} \mathbb{E}\left[\sum_{t\in\mathcal{T}} R(Q_t^\pi, I_t, E(s_t^\pi(Q_t^\pi, S_t^\pi, I_t), w_t^\pi(Q_t^\pi, S_t^\pi, I_t)))\middle| Q_1, S_1, I_1\right].$$

For each period $t \in \mathcal{T}$ and each state (Q_t, S_t, I_t), the optimal profit function $v_t^*(Q_t, S_t, I_t)$ can be calculated with the following dynamic programming recursion:

$$v_t^*(Q_t, S_t, I_t) = \max_{(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, E(s_t, w_t)) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \tag{3.1}$$

where $v_T^*(Q_T, S_T, I_T) = 0$. Note that $v_1^*(Q_1, S_1, I_1)$ is the optimal expected total cash flow for the initial state (Q_1, S_1, I_1) over the finite horizon.

3.2.2 The use of battery for commitment fulfillment

In this setting, the producer is not allowed to intentionally deviate from her commitments in real-time transactions. The producer must utilize the available energy sources to fulfill her commitments as much as possible, storing any excess wind energy as long as the battery capacity allows. In other words, the amount of energy to be sold/purchased in period t should be as close to the state variable Q_t as possible with the minimum curtailment level. Therefore, we choose the actions s_t and w_t in this setting as follows:

$$\widehat{s}_t = \begin{cases} \min\{S_t, C_D, (Q_t/\tau - f(W_t))/\gamma\} & \text{if } Q_t/\tau \geq f(W_t) \geq 0, \\ -\min\{C_S - S_t, C_C, (f(W_t) - Q_t/\tau)\theta\} & \text{if } f(W_t) > Q_t/\tau \geq 0, \\ -\min\{C_S - S_t, C_C, (f(W_t) - \tau Q_t)\theta\} & \text{if } f(W_t) \geq 0 > \tau Q_t, \end{cases}$$

and

$$\widehat{w}_t = \begin{cases} f(W_t) & \text{if } Q_t/\tau \geq f(W_t) \geq 0, \\ Q_t/\tau + \min\{(C_S - S_t)/\theta, C_C/\theta, f(W_t) - Q_t/\tau\} & \text{if } f(W_t) > Q_t/\tau \geq 0, \\ \tau Q_t + \min\{(C_S - S_t)/\theta, C_C/\theta, f(W_t) - \tau Q_t\} & \text{if } f(W_t) \geq 0 > \tau Q_t. \end{cases}$$

Note that in this setting the producer determines only the commitment amount $q_t \in \mathbb{R}$.

The above formulation implies that the battery is charged only when the wind energy potential exceeds the commitment, and it is discharged only when the wind energy potential falls short of the commitment. More specifically, if the commitment exceeds the maximum amount of wind energy that can be generated, the required energy $(Q_t/\tau - f(W_t))$ is discharged from the battery with efficiency factor γ to meet the commitment. However, the amount of energy discharged is restricted by the storage level S_t and the discharging capacity C_D. If the producer committed to selling energy to the market and the maximum amount of wind energy that can be generated exceeds the commitment, the excess energy $(f(W_t)-Q_t/\tau)$ is charged into the battery with efficiency factor θ. If the producer committed to purchasing energy from the market, the maximum amount of wind energy that can be generated along with the energy purchased from the market $(f(W_t) - \tau Q_t)$ is charged into the battery with efficiency factor θ. In both cases, the amount of energy charged must respect the battery's energy capacity C_S and charging capacity C_C; these capacity constraints induce curtailment of some wind energy if the wind energy potential is too high. Since the producer makes her commitment decisions in any period by taking into account the state of the battery in the next period, the amount of energy committed to purchasing from the market can indeed be purchased and stored in real-time at optimality.

The amount of energy sold/purchased in real-time can be formulated as a function of Q_t, S_t, and I_t:

$$\widehat{e}_t = E(Q_t, S_t, I_t) := \begin{cases} \big(\min\{\gamma S_t, \gamma C_D, Q_t/\tau - f(W_t)\} & \\ +f(W_t)\big)\tau & \text{if } Q_t/\tau \geq f(W_t) \geq 0, \\ Q_t & \text{if } f(W_t) > Q_t/\tau \geq 0, \\ Q_t & \text{if } f(W_t) \geq 0 > \tau Q_t. \end{cases}$$

If the maximum amount of wind energy that can be generated is greater than the commitment level, then $E(Q_t, S_t, I_t) = Q_t$. Therefore, the only possible direction of imbalance that the producer can experience is the negative imbalance in which the producer sells energy to the market in real-time less than her commitment. This scenario may arise due to the limited discharging capacity and the wind energy potential less than expected. The shortfall in this case is compensated at a penalized price.

Let $\widehat{\mathbb{U}}(Q_t, S_t, I_t)$ denote the set of actions q_t that are admissible in state (Q_t, S_t, I_t). Recall that, at optimality, $-\min\left\{\frac{C_S-(S_t-\widehat{s}_t)}{\theta\tau}, \frac{C_C}{\theta\tau}, C_T\right\} \leq q_t \leq \tau C_T$, $\forall t \in \mathcal{T}$, and $-\min\left\{\frac{C_S-S_t}{\theta\tau}, \frac{C_C}{\theta\tau}, C_T\right\} \leq Q_t \leq \tau C_T$, $\forall t \in \mathcal{T}\backslash\{1\}$. A control policy π is the sequence of decision rules $(q_t^\pi(Q_t^\pi, S_t^\pi, I_t))_{t\in\mathcal{T}}$, where Q_t^π and S_t^π denote the random state variables governed by policy π, $\forall t \in \mathcal{T}\backslash\{1\}$. We denote the set of all admissible control policies by $\widehat{\Pi}$. For any initial state (Q_1, S_1, I_1), the optimal expected total cash flow over the finite horizon is given by

$$\max_{\pi\in\widehat{\Pi}} \mathbb{E}\left[\sum_{t\in\mathcal{T}} R(Q_t^\pi, I_t, E(Q_t^\pi, S_t^\pi, I_t))\middle| Q_1, S_1, I_1\right].$$

For each period $t \in \mathcal{T}$ and each state (Q_t, S_t, I_t), the optimal profit function $\widehat{v}_t^*(Q_t, S_t, I_t)$ can be calculated with the following dynamic programming recursion:

$$\widehat{v}_t^*(Q_t, S_t, I_t) = \max_{q_t\in\widehat{\mathbb{U}}(Q_t,S_t,I_t)} \left\{R(Q_t, I_t, E(Q_t, S_t, I_t)) + \mathbb{E}_{I_{t+1}|I_t}\left[\widehat{v}_{t+1}^*(q_t, S_{t+1}, I_{t+1})\right]\right\} \tag{3.2}$$

where $\widehat{v}_T^*(Q_T, S_T, I_T) = 0$. Note that $\widehat{v}_1^*(Q_1, S_1, I_1)$ is the optimal expected total cash flow for the initial state (Q_1, S_1, I_1) over the finite horizon.

The following proposition compares the total cash flows of the two problem settings above.

Proposition 3.2.1. $v_1^*(Q_1, S_1, I_1) \geq \widehat{v}_1^*(Q_1, S_1, I_1)$.

Proof. Note that $v_T^*(Q_T, S_T, I_T) = \widehat{v}_T^*(Q_T, S_T, I_T) = 0$. Assuming $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \geq \widehat{v}_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1})$, we show $v_t^*(Q_t, S_t, I_t) \geq \widehat{v}_t^*(Q_t, S_t, I_t)$. Let $\widehat{\mathbb{U}}^F(Q_t, S_t, I_t) = \{(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t) : s_t = \widehat{s}_t, w_t = \widehat{w}_t$, and $q_t \in \widehat{\mathbb{U}}(Q_t, S_t, I_t)\}$. Since $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \geq \widehat{v}_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1})$

and $\widehat{\mathbb{U}}^F(Q_t, S_t, I_t) \subseteq \mathbb{U}(Q_t, S_t, I_t)$,

$$\begin{aligned}
&v_t^*(Q_t, S_t, I_t) \\
&= \max_{(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, E(s_t, w_t)) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \\
&\geq \max_{(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, E(s_t, w_t)) + \mathbb{E}_{I_{t+1}|I_t} \left[\widehat{v}_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \\
&\geq \max_{(q_t, s_t, w_t) \in \widehat{\mathbb{U}}^F(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, E(s_t, w_t)) + \mathbb{E}_{I_{t+1}|I_t} \left[\widehat{v}_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \\
&= \max_{q_t \in \widehat{\mathbb{U}}(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, E(Q_t, S_t, I_t)) + \mathbb{E}_{I_{t+1}|I_t} \left[\widehat{v}_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\} \\
&= \widehat{v}_t^*(Q_t, S_t, I_t).
\end{aligned}$$

Hence $v_t^*(Q_t, S_t, I_t) \geq \widehat{v}_t^*(Q_t, S_t, I_t)$, $\forall t$, implying that $v_1^*(Q_1, S_1, I_1) \geq \widehat{v}_1^*(Q_1, S_1, I_1)$. $\square$

Proposition 3.2.1 implies that the option of deviating from the commitments brings an additional profit to the producer. In Chapter 3.4, we will numerically compare the total cash flows of the two problem settings with data-calibrated instances.

3.3 Experimental Setup for the Numerical Study

As mentioned in the Introduction, the hour-ahead market structure allows for a computationally tractable MDP formulation that enables us to solve the realistic-size problem instances to optimality under uncertainty. This market structure can be arguably viewed as a real-time market structure in the U.S. [32, 33]. Also, recall that the intentional deviations in our study may be financially attractive thanks to the price and wind uncertainties that can only be resolved over time. Since the hour-ahead uncertainties can be thought of as less significant than the uncertainties over longer time frames, the day-ahead and intraday market structures would likely amplify the numerical insights derived in this study. In our numerical study, we use the real-time electricity prices obtained from NYISO, which is one of the largest and most liquid electricity markets where negative prices are observed [8].

In this section, using the historical data available from the State of New York, we develop two distinct time series models for the electricity price and wind speed, respectively (Chapters 3.3.1–3.3.2). We incorporate these parametric models into our MDP by utilizing the exogenous state variables. We then discretize the continuous space of the exogenous state variables for our numerical calculations (Chapter 3.3.3). We conduct numerical experiments for various configurations of our energy system in Chapter 3.4. We set the period length to be one hour in our experiments.

3.3.1 Time series model for the electricity price

We consider the real-time market electricity price data available for Albany, in the State of New York, between the years 2007 and 2019 in which the price is set every five minutes. Since the real-time transactions in Albany are regulated by the NYISO, we retrieve the price data from NYISO [75]. The average, median, minimum, and maximum values of the price are \$45.14, \$34.09, –\$3678.02, and \$3393.33, respectively. As we assume hourly periods, we include the price value of every hour of the day in our time series model. Our time series model consists of the components of seasonality (ψ'_t), mean reversion (ρ_t), and spike (J_t). We model the seasonality component via linear regression, the mean-reversion component via an autoregressive of order one process, and the spike component via an empirical distribution. The spike component of the price represents sudden and large moves in the price that are independent across periods.

We take the following steps to construct our time series model: We first deseasonalize the price data to eliminate the effect of seasonal variation on our spike identification. Following Zhou et al. [8], we fit a linear regression to the price data to obtain a seasonality model $\{\hat{\psi}_t\}_{t\in\mathcal{T}}$:

$$\psi_t = \gamma_1^{(p)} + \sum_{i=1}^{11} \gamma_{2i}^{(p)} D_t^{2i} + \sum_{j=1}^{6} \gamma_{3j}^{(p)} D_t^{3j}$$

where $\gamma_1^{(p)}$ is a constant, and $\gamma_{2i}^{(p)}$ and $\gamma_{3j}^{(p)}$ are the coefficients of the dummy variables D_t^{2i} and D_t^{3j}, that are equal to one if period t is in month i and week day j, respectively. We calculate the deseasonalized prices $\{d_t\}_{t\in\mathcal{T}}$ by removing the seasonal effect from the observed prices $\{P_t\}_{t\in\mathcal{T}}$ (i.e., $d_t = P_t - \hat{\psi}_t$). We model the spikes $\{J_t\}_{t\in\mathcal{T}}$ as a compound Bernoulli process; a spike occurs with probability λ and its size follows an empirical distribution. In order to identify the spikes, we consider the highest five percent and the lowest five percent of the deseasonalized prices as outliers (see Janczura et al. [76] for a detailed discussion on spike identification). The spikes $\{J_t\}_{t\in\mathcal{T}}$ are determined by the differences between these outliers and the mean of the remaining deseasonalized prices after these outliers are removed. We take $J_t = 0$ if the price in period t is not an outlier. We then calculate the despiked prices $\{P_t'\}_{t\in\mathcal{T}}$ by subtracting the spikes from the observed prices (i.e., $P_t' = P_t - J_t$). Since the seasonal variation can be identified more accurately after elimination of the spikes, we now fit the above linear regression to the despiked prices and obtain a more refined seasonality model $\{\hat{\psi}_t'\}_{t\in\mathcal{T}}$. We calculate the despiked and deseasonalized prices $\{\rho_t\}_{t\in\mathcal{T}}$ by removing the refined seasonal effect from the despiked prices $\{P_t'\}_{t\in\mathcal{T}}$ (i.e., $\rho_t = P_t' - \hat{\psi}_t'$).

Following Lucia and Schwartz [77], we model the despiked and deseasonalized price as a stochastic mean-reverting process. We capture the mean-reverting behavior via an autoregressive of order one, AR(1), model. Assuming that error terms $\{\epsilon_t\}_{t\in\mathcal{T}}$ are independent standard normal random variables, we formulate the AR(1) process as follows:

$$\rho_t = \left(1 - \kappa^{(p)}\right) \rho_{t-1} + \sigma^{(p)} \epsilon_t, \ \forall t,$$

where $\kappa^{(p)}$ is the speed of mean-reversion and $\sigma^{(p)}$ is the volatility of white noise. We have found that the mean absolute error (MAE) of this calibration is \$10.02 for the despiked prices. The parameter estimates of the AR(1) process are $\hat{\kappa}^{(p)} = 0.357$ and $\hat{\sigma}^{(p)} = 15.281$. Table 3.1 exhibits the parameter estimates of the seasonality model. Figure 3.3 illustrates the empirical distribution of the spikes. The estimate $\hat{\lambda}$ of the spike occurrence probability is ten percent.

Table 3.1: Parameter estimates of the despiked price seasonality model.

$\hat{\gamma}_1^{(p)}$	39.7689										
i	1	2	3	4	5	6	7	8	9	10	11
$\hat{\gamma}_{2i}^{(p)}$	19.12	11.37	-2.17	-6.50	-9.76	-10.10	-5.27	-6.00	-8.85	-9.83	-7.11
j	1	2	3	4	5	6					
$\hat{\gamma}_{3j}^{(p)}$	3.74	3.82	4.05	3.87	3.55	1.54					

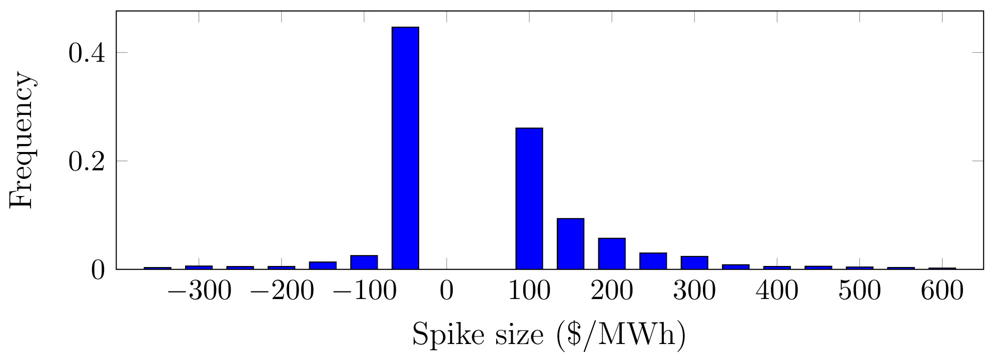


Figure 3.3: Empirical distribution of the spikes. The spike range is indeed [-2600, 2500]; the spikes outside the displayed range are omitted due to their low frequencies.

3.3.2 Time series model for the wind speed

We consider the wind speed data available for Albany, in the State of New York, between the years 2007 and 2012 in which the wind speed is recorded every five minutes. We retrieve this data from NOAA [78]. The average, median, minimum, and maximum values of the wind speed are 8.52, 8.21, 0.05, and 28.67, in m/s, respectively. As we assume hourly periods, we include the wind speed value of every hour of the day in our time series model. Our spectral analysis of the time series indicates the existence of two significant seasonal factors: hourly and daily patterns.

We calibrate our hourly wind speed model via the dynamic harmonic regression with autoregressive integrated moving average (DHR+ARIMA) model [8, 79]: We first fit a linear regression with Fourier terms to our wind speed data to obtain a seasonality model $\{\hat{f}_t\}_{t\in\mathcal{T}}$:

$$f_t = \gamma_0^{(w)} + \gamma_1^{(w)} \cos\left(\frac{2\pi\left(t+\omega_1^{(w)}\right)}{24}\right) + \gamma_2^{(w)} \cos\left(\frac{2\pi\left(\lceil t/24\rceil+\omega_2^{(w)}\right)}{365}\right)$$

where $\lceil . \rceil$ is the ceiling function, $\gamma_0^{(w)}$ is a constant, $\gamma_1^{(w)}$ and $\omega_1^{(w)}$ are the hourly magnitude and phase-shift parameters, and $\gamma_2^{(w)}$ and $\omega_2^{(w)}$ are the daily magnitude and phase-shift parameters, respectively. We calculate the deseasonalized wind speeds $\{\xi_t\}_{t\in\mathcal{T}}$ by removing the seasonal effect from the observed wind speeds $\{w_t\}_{t\in\mathcal{T}}$ (i.e., $\xi_t = w_t - \hat{f}_t$). We then model the deseasonalized wind speed as an AR(1) process as follows:

$$\xi_t = \phi^{(w)}\xi_{t-1} + \sigma^{(w)}\epsilon_t,\ \forall t,$$

where $\phi^{(w)}$ is the autoregressive coefficient and $\sigma^{(w)}$ is the volatility of white noise. We have found that the MAE of our DHR+AR(1) model is only 1.08 m/s. The parameter estimates of the DHR+AR(1) model are $\hat{\gamma}_0^{(w)} = 8.519$, $\hat{\gamma}_1^{(w)} = 1.126$, $\hat{\gamma}_2^{(w)} = 1.74$, $\hat{\omega}_1^{(w)} = 0.002$, $\hat{\omega}_2^{(w)} = -32.431$, $\hat{\phi}^{(w)} = 0.931$, and $\hat{\sigma}^{(w)} = 1.558$.

Finally, we investigate whether there exists any significant correlation between the data sets that we use to model the electricity price and wind speed. We have found that the Pearson correlation coefficient is only 0.031. Hence, constructing independent time series models seems to be benign.

3.3.3 Discretization for the numerical study

Our time series models in Chapters 3.3.1–3.3.2 formulate the random component of each exogenous state variable in our MDP as an AR(1) process. This allows us to reduce the computational burden of our MDP: we can redefine the exogenous state tuple in each period t as $I_t = (\rho_t, J_t, \xi_t)$, without requiring the entire historical data in this tuple. We include the spike component of the price in the tuple I_t for calculation of the effective price in period t. For our numerical study, we now provide discrete-state approximations for the continuous-state AR(1) processes embedded in the tuple I_t.

For the electricity price, we characterize the AR(1) process of the random component (i.e., the despiked and deseasonalized price) as a finite-state Markov chain by transforming it into a lattice. A lattice can be defined as a tree with discrete time steps that specifies attainable price levels and their probabilities for each time step. With the estimated parameters $\hat{\kappa}^{(p)}$ and $\hat{\sigma}^{(p)}$, we employ the trinomial lattice method of Hull and White [80]. Specifically, assuming hourly time steps, the price levels can only be multiples of $\sqrt{3}\hat{\sigma}^{(p)}$ and each price level in each period transitions into three possible price levels in the next period. The transition probabilities are chosen to match the first two moments of the continuous distribution of the original AR(1) process (i.e., normal distribution with a mean of zero and a variance of $\hat{\sigma}^{(p)}$). Regarding the number of time steps that should be iterated, we follow the suggestions of Hull and White [80] and Jaillet et al. [81], and construct a three-hour trinomial lattice for our AR(1) process. The Markov chain obtained from this lattice has the state space $\mathcal{P} := \{-52.93, -26.47, 0, 26.47, 52.93\}$. The transition matrix of this Markov chain is

$$P^{(p)} = \begin{array}{c} \\ \text{-52.93} \\ \text{-26.47} \\ \text{0.0} \\ \text{26.47} \\ \text{52.93} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{-52.93} & \text{-26.47} & \text{0.0} & \text{26.47} & \text{52.93} \end{array} \\ \begin{bmatrix} .351 & .585 & .065 & 0 & 0 \\ .052 & .539 & .409 & 0 & 0 \\ 0 & .167 & .667 & .167 & 0 \\ 0 & 0 & .409 & .539 & .052 \\ 0 & 0 & .065 & .585 & .351 \end{bmatrix} \end{array}.$$

For the electricity price, we also restrict the spikes to take values from the set $\mathcal{J} := \{-350, -300, \ldots, 550, 600\}$. We compute the probability mass function by approximating the spikes with the closest values in $\mathcal{J}$ and using the empirical distribution of these approximate values. Recall that the spike occurrence probability is ten percent in each period.

Since the maximum wind speed observed between 2007 and 2012 is 28.67 m/s, we characterize the AR(1) process of the random component of the wind speed as a Markov chain with the state space $\{0, 1, \ldots, 28\}$. For this state space, we calculate the transition probabilities by following the procedure in Tauchen [82]. In this procedure, we first partition the state space into the evenly spaced intervals $\{(-\infty, 0.5), [0.5, 1.5), \ldots, [26.5, 27.5), (27.5, \infty)\}$ (except the initial and last intervals). We then take the transition probability from one specific state to another state as the probability that the original AR(1) process moves from this specific state to a point in the corresponding next-state interval. In our experiments, we consider a wind power plant with General Electric (GE) 1.5–77 turbines; the power output of each such turbine is determined by the power curve depicted in Figure 3.4. The values of the random component greater than nine, combined with the seasonality component that we have found to be always larger than five for our data, yield the same power output. Hence, for computational purposes, we reformulate our Markov chain by reducing its state space to the set $\mathcal{W} := \{0, 1, \ldots, 10\}$ via the state reduction algorithm of Sheskin [83]. The transition matrix of this Markov chain is

$$P^{(w)} = \begin{array}{cc} & \begin{array}{ccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{array} & \left[\begin{array}{ccccccccccc} .626 & .206 & .114 & .042 & .010 & .002 & 0 & 0 & 0 & 0 & 0 \\ .391 & .252 & .201 & .107 & .039 & .009 & .002 & 0 & 0 & 0 & 0 \\ .191 & .217 & .251 & .195 & .101 & .035 & .008 & .001 & 0 & 0 & 0 \\ .071 & .133 & .222 & .250 & .188 & .095 & .032 & .007 & .001 & 0 & 0 \\ .019 & .057 & .139 & .227 & .248 & .182 & .090 & .030 & .007 & .001 & 0 \\ .004 & .018 & .062 & .146 & .231 & .246 & .176 & .084 & .027 & .006 & .001 \\ .001 & .004 & .019 & .067 & .153 & .235 & .243 & .169 & .079 & .025 & .006 \\ 0 & .001 & .004 & .021 & .071 & .159 & .239 & .241 & .163 & .075 & .025 \\ 0 & 0 & .001 & .005 & .024 & .076 & .166 & .243 & .241 & .164 & .081 \\ 0 & 0 & 0 & .001 & .006 & .026 & .083 & .177 & .257 & .260 & .191 \\ 0 & 0 & 0 & 0 & .001 & .007 & .031 & .097 & .210 & .317 & .338 \end{array} \right] \end{array}.$$

With the above modifications, we obtain $I_t = (\rho_t, J_t, \xi_t) \in \mathcal{I} := \mathcal{P} \times \mathcal{J} \times \mathcal{W}$.

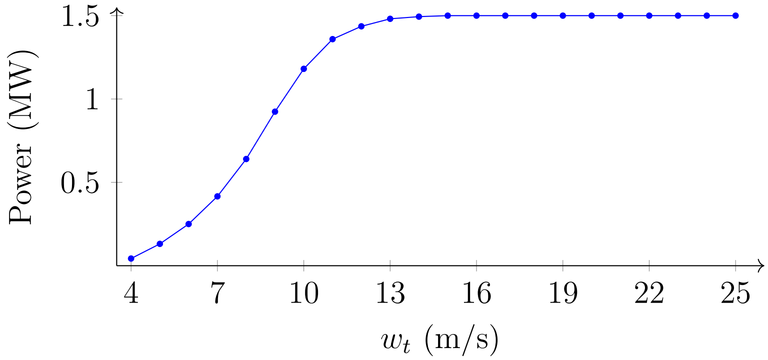


Figure 3.4: Power curve of a single GE 1.5-77 turbine [1].

In addition, we restrict the amount of energy accumulated in the battery and the amount of energy committed to selling/purchasing to take values from the sets $\mathcal{S} := \{n\zeta_a \in [0, C_S] : n \in \mathbb{Z}\}$ and $\mathcal{Q} := \{k\zeta_a \in [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T] : k \in \mathbb{Z}\}$, respectively, where ζ_a is a prespecified constant. For the discrete-state version of our MDP in Chapter 3.2.1, let $\mathbb{U}^D(Q_t, S_t, I_t)$ denote the set of action triplets (q_t, s_t, w_t) that are admissible in state $(Q_t, S_t, I_t) \in \mathcal{Q} \times \mathcal{S} \times \mathcal{I}$. The set $\mathbb{U}^D(Q_t, S_t, I_t)$ consists of the set $\{(k\zeta_a, n\zeta_a, m\zeta_a) \in \mathbb{U}(Q_t, S_t, I_t) : k \in \mathbb{Z}, n \in \mathbb{Z}, m \in \mathbb{Z}_+\}$, as well as the extreme points of $\mathbb{U}(Q_t, S_t, I_t)$. Similarly, for our MDP in Chapter 3.2.2, let $\widehat{\mathbb{U}}^D(Q_t, S_t, I_t)$ denote the set of actions q_t that are admissible in state $(Q_t, S_t, I_t) \in \mathcal{Q} \times \mathcal{S} \times \mathcal{I}$. The set $\widehat{\mathbb{U}}^D(Q_t, S_t, I_t)$ consists of the set $\{k\zeta_a \in \widehat{\mathbb{U}}(Q_t, S_t, I_t) : k \in \mathbb{Z}\}$, as well as the extreme points of $\widehat{\mathbb{U}}(Q_t, S_t, I_t)$.

3.4 Discussion of the Numerical Results

We consider instances in which the planning horizon spans the first week of August ($T = 168$ hours), the number of GE 1.5-77 turbines is 100, the roundtrip efficiency is 0.85, and the transmission line efficiency is 0.97 [8]. The negative price occurrence frequency (NPF) takes values from the set $\{0\%, 4.02\%, 7.66\%, 10.96\%, 13.98\%\}$, $C_C = C_D \in \{50, 75, 100\}$ MWh, $C_S \in \{200, 400, 600, 800\}$ MWh, $C_T \in \{100, 200\}$ MWh, $K_p^+ = K_n^- \in \{0.6, 0.7, 0.8, 0.9\}$, and $K_n^+ = K_p^- \in \{1.1, 1.2, 1.3, 1.4\}$. The observed NPF is 4.02% in our time series model for the price. We obtain the other four values of NPF by multiplying the numbers of negative spike occurrences with certain constants. In all instances, the discretization parameter ζ_a is 25 MWh, the initial storage level S_1 is the closest state to $C_S/2$, the initial commitment level Q_1 is zero, and the initial exogenous state $I_1 = (\rho_1, J_1, \xi_1)$ is (0,0,5). We solved the recursion of our MDP to optimality in each instance, calculating the following metrics:

- The expected total cash flow in million dollars (TCF),
- The expected total amount of wind energy curtailed in MWh (WEC),
- The expected total negative imbalance in MWh (NI),
- The expected total positive imbalance in MWh (PI), and
- The expected total imbalance (deviation) in MWh (ED).

We label the problem setting in Chapter 3.2.1 as 'ID' (the initials of 'intentional deviation') and the problem setting in Chapter 3.2.2 as 'UD' (the initials of 'unintentional deviation'). We also consider two other settings that are the special cases of ID and UD with no battery. We label the problem setting in Chapter 3.2.1 with $C_S = 0$ as 'ID-NB' (ID and the initials of 'no battery') and the problem setting in Chapter 3.2.2 with $C_S = 0$ as 'UD-NB' (UD and the initials of 'no battery'). In this section, we examine the effects of system components and market characteristics on TCF, WEC, NI, PI, and ED. In Chapter 3.4.1, we vary only the values of $C_C = C_D$, C_S, and C_T. In Chapter 3.4.2, we vary only the values of $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$. In Chapter 3.4.3, we vary only the value of NPF.

3.4.1 The impact of the size of system components

In this section, we examine the effects of the battery's energy capacity (C_S) and charging/discharging capacity ($C_C = C_D$), and the transmission line capacity

(C_T) on the values of TCF, WEC, NI, PI, and ED when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, and NPF= 4.02%. See Table 3.2 and Figure 3.5.

We observe from Table 3.2 that TCF increases with C_S, $C_C = C_D$, and C_T for both ID and UD. The benefit of an additional battery capacity diminishes as the battery becomes larger in capacity and thus the transmission line capacity becomes more binding. We also observe that TCFs are higher in ID than in UD, as expected from Proposition 3.2.1. The capacity levels have a greater impact on TCF in ID than in UD because the battery in ID can be used strategically to make more profit. In addition, we note from Table 3.2 that NIs are higher than PIs for ID. When the producer committed to selling energy to the market, selling less than her commitment in real-time may provide a greater return than selling more than her commitment, since selling more is likely to reduce the battery level more. When the producer committed to purchasing energy from the market to benefit from the negative/low prices, purchasing more than her commitment in real-time provides a greater return than purchasing less than her commitment. Therefore, the producer tends to experience negative imbalances more than positive imbalances. Finally, both NIs and PIs increase with the transmission line capacity.

Table 3.2 also shows for ID that an increase in C_S may lead to a slight decrease in ED when C_T and $C_C = C_D$ are low, whereas it leads to an increase in ED when C_T or $C_C = C_D$ is high. The maximum amount of energy committed to selling or purchasing is constrained by the transmission line and battery charging/discharging capacities. Having a large storage capacity gives the flexibility to better meet commitments and also deviate from commitments more substantially. When C_T and $C_C = C_D$ are low, the producer better meets her commitments with an increase in C_S, thanks to the constrained commitment levels. On the other hand, when C_T or $C_C = C_D$ is high, the producer can deviate from her commitments more substantially with an increase in C_S, due to the unconstrained commitment levels. For UD, however, increasing C_S reduces ED in many cases and only slightly raises ED in the other cases. This is because the producer in UD should meet her commitments as much as possible; she may only experience

Table 3.2: Numerical results when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, and NPF = 4.02%.

Setting	C_T	$C_C = C_D$	C_S	TCF	WEC	NI	PI	ED
ID	100	50	200	0.605	2460	2643	395	3038
			400	0.631	2077	2797	313	3109
			600	0.643	1847	2784	279	3062
			800	0.651	1670	2709	261	2970
		75	200	0.616	2520	2931	359	3290
			400	0.648	2162	3220	242	3462
			600	0.665	1949	3304	189	3493
			800	0.676	1777	3283	160	3443
		100	200	0.619	2495	2995	350	3345
			400	0.653	2134	3303	232	3535
			600	0.671	1937	3388	179	3567
			800	0.682	1786	3363	150	3514
	200	50	200	0.746	613	4365	1201	5566
			400	0.774	614	4585	1183	5768
			600	0.788	614	4742	1244	5986
			800	0.797	614	4743	1252	5995
		75	200	0.775	614	5263	951	6214
			400	0.817	616	5906	1234	7140
			600	0.840	619	6113	1304	7418
			800	0.855	618	6290	1339	7628
		100	200	0.795	615	6231	1070	7301
			400	0.848	616	7204	1235	8439
			600	0.879	619	7469	1327	8796
			800	0.899	620	7653	1380	9033
UD	100	50	200	0.540	1769	844	0	844
			400	0.562	1318	611	0	611
			600	0.575	1034	535	0	535
			800	0.584	836	512	0	512
		75	200	0.542	1777	660	0	660
			400	0.567	1313	386	0	386
			600	0.582	1017	275	0	275
			800	0.592	811	224	0	224
		100	200	0.543	1779	646	0	646
			400	0.569	1312	357	0	357
			600	0.584	1011	237	0	237
			800	0.595	799	175	0	175
	200	50	200	0.623	258	683	0	683
			400	0.637	300	611	0	611
			600	0.646	320	634	0	634
			800	0.653	328	665	0	665
		75	200	0.630	156	502	0	502
			400	0.651	181	446	0	446
			600	0.664	204	465	0	465
			800	0.673	220	493	0	493
		100	200	0.633	113	496	0	496
			400	0.658	120	374	0	374
			600	0.675	135	366	0	366
			800	0.687	150	378	0	378

negative imbalances, possibly due to the battery's capacity constraints. The decrease in ED is more significant when the transmission line capacity is low, that is, when the commitment decisions are more restricted.

We observe from Figure 3.5 that when the transmission line capacity is low (the left plots in Figure 3.5), an increase in C_S leads to a decrease in WEC in both ID and UD. Since the low transmission line capacity limits the amount of energy that can be sold in real-time, a battery with larger energy capacity helps the producer store more of the excess wind energy, reducing the curtailment amount. When the transmission line capacity is high (the right plots in Figure 3.5), on the other hand, WECs in ID and UD are not affected much by an increase in C_S. Since the high transmission line capacity allows the producer to sell a large amount of the available wind energy, increasing the battery's energy capacity has no significant impact on the curtailment amounts.

3.4.2 The impact of imbalance pricing parameters

In this section, we examine the effects of the imbalance pricing parameters on the values of TCF, WEC, NI, PI, and ED when $C_S = 400$ MWh (for ID and UD), $C_C = C_D = 50$ MWh (for ID and UD), $C_T = 200$ MWh, and NPF = 4.02%. See Tables 3.3-3.4 and Figures 3.6-3.7.

We first note that a decrease in $K_p^+(K_n^-)$ and an increase in $K_n^+(K_p^-)$ improve the effectiveness of the imbalance pricing mechanism, while draining the producer's profit. We observe from Tables 3.3 and 3.4 that having a battery brings a financial benefit to the producer and this benefit is lower under smaller penalties (e.g., $K_p^+ = 0.9$ and $K_n^+ = 1.1$). We also observe that TCF decreases in ID and ID-NB, but only slightly in UD and UD-NB, as $K_p^+(K_n^-)$ decreases. The impact of K_p^+ in UD and UD-NB is not significant since the positive imbalance is not observed in UD and UD-NB. Another observation is that TCF decreases in ID, ID-NB, and UD-NB, yet only slightly in UD, as $K_n^+(K_p^-)$ increases. Since the producers in ID and ID-NB intentionally cause an energy imbalance, their profits are affected by the changes in the imbalance pricing parameters. The producers

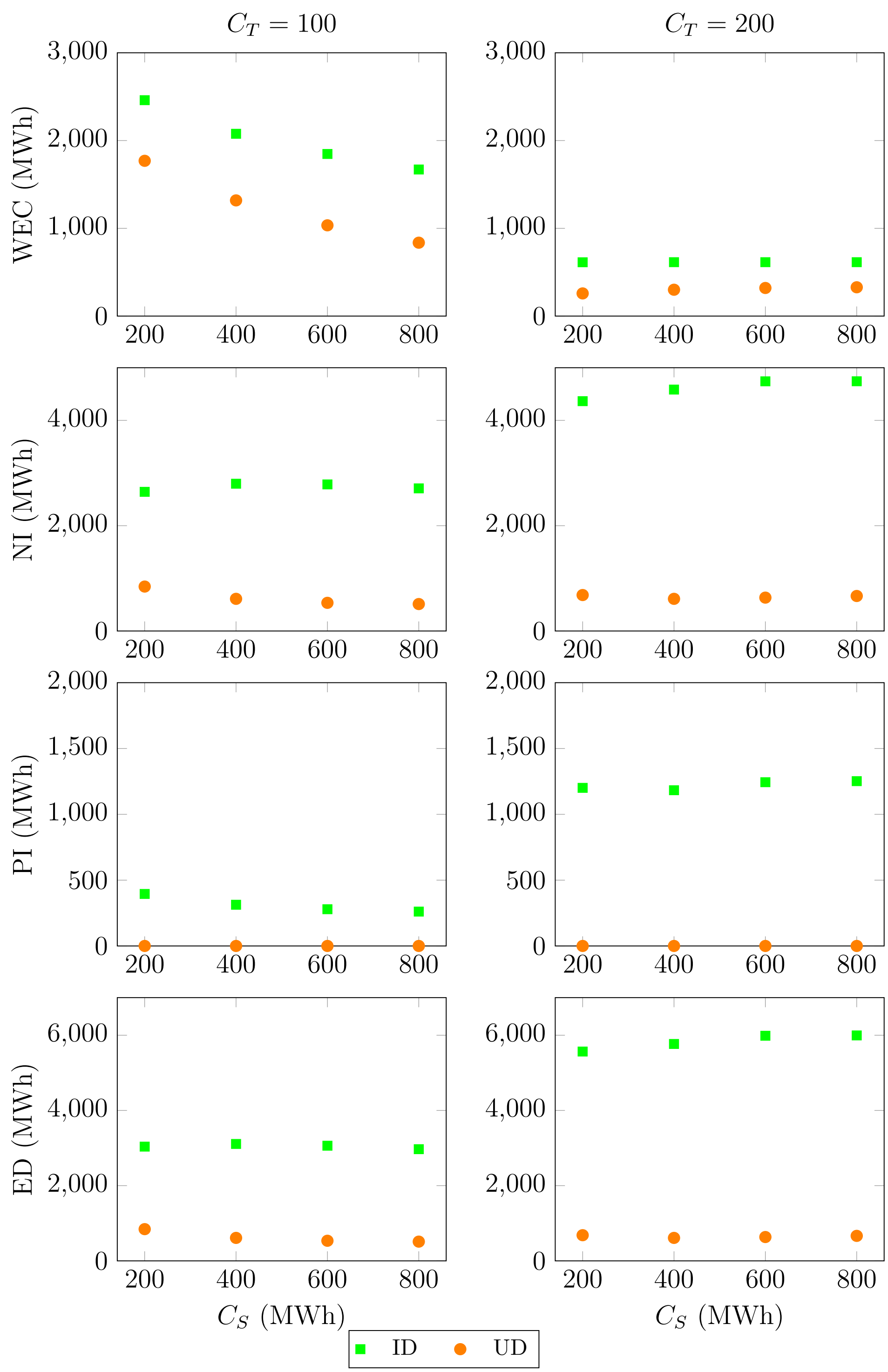


Figure 3.5: WEC, NI, PI, and ED vs. C_S when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, $C_C = C_D = 50$ MWh, and NPF = 4.02%.

Table 3.3: Numerical results for ID and ID-NB when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

Setting	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF	WEC	NI	PI	ED
ID	0.6	1.1	0.747	612	7934	199	8133
		1.2	0.734	612	5202	328	5529
		1.3	0.727	612	3966	414	4380
		1.4	0.723	613	3436	472	3908
	0.7	1.1	0.754	612	7345	256	7601
		1.2	0.743	612	4718	434	5152
		1.3	0.737	613	3647	562	4209
		1.4	0.734	613	3239	647	3886
	0.8	1.1	0.763	612	6407	479	6886
		1.2	0.754	613	4051	843	4895
		1.3	0.750	613	3221	1044	4265
		1.4	0.747	614	2926	1196	4121
	0.9	1.1	0.774	614	4585	1183	5768
		1.2	0.769	615	3126	1755	4881
		1.3	0.766	616	2612	2085	4697
		1.4	0.764	616	2327	2370	4697
ID-NB	0.6	1.1	0.600	612	4161	617	4779
		1.2	0.588	612	2874	1121	3996
		1.3	0.578	612	2399	1428	3827
		1.4	0.571	612	1973	1807	3780
	0.7	1.1	0.606	612	3687	778	4465
		1.2	0.595	612	2551	1317	3869
		1.3	0.588	612	1973	1807	3780
		1.4	0.582	612	1644	2177	3821
	0.8	1.1	0.612	612	2874	1121	3996
		1.2	0.604	612	1973	1807	3780
		1.3	0.599	612	1511	2359	3870
		1.4	0.595	612	1299	2715	4013
	0.9	1.1	0.621	612	1973	1807	3780
		1.2	0.616	612	1299	2715	4013
		1.3	0.613	612	938	3531	4469
		1.4	0.612	612	750	4113	4862

in UD and UD-NB are vulnerable to negative imbalances, but the profit of the producer in UD is quite robust to the changes in $K_n^+(K_p^-)$ as she uses the battery to minimize deviations from her commitments.

We observe from Figure 3.6 that WEC remains similar in UD, but increases in UD-NB, as $K_n^+(K_p^-)$ increases. When $K_n^+(K_p^-)$ is large, the producer in UD-NB prefers to commit to selling small amounts of energy to avoid any negative imbalance, leading to large curtailment amounts. However, the producer in UD can still commit to selling significant amounts of energy as she uses the battery as a back-up source. We also observe that WECs in ID and ID-NB are not affected by the change in the imbalance pricing parameters. This is because the producers in ID and ID-NB can always sell the excess wind energy as long as the transmission line capacity allows.

The effect of imbalance pricing parameters on ED in ID and ID-NB changes according to the producer's imbalance tendency. First, we note from Table 3.3 that ED in ID decreases as K_n^+ increases, while it may increase for small values of K_n^+ as K_p^+ decreases. A higher penalty for negative (positive) imbalance encourages the producer in ID to stay in positive (negative) imbalance more than in negative (positive) imbalance. Since the producer in ID is more inclined to cause negative imbalances, an increase in K_n^+ leads to a more significant drop in NI (compared to the increase in PI), and thus her total deviation decreases. However, for small values of K_n^+, she experiences a greater increase in NI than a decrease in PI as K_p^+ decreases, leading to an increase in ED. Second, we note from Table 3.3 that ED in ID-NB tends to decrease for small values of K_p^+, while it tends to increase for large values of K_p^+, as K_n^+ increases. The producer in ID-NB is more conservative in her commitment decisions, causing positive imbalances more as she can still make a profit if the available wind energy is large enough. A higher penalty for negative imbalance makes the producer much more conservative in her commitment decisions. The decrease in NI dominates the increase in PI for small values of K_p^+, while the reverse is true for large values of K_p^+.

Table 3.4: Numerical results for UD and UD-NB when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

Setting	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF	WEC	NI	PI	ED
UD	0.6	1.1	0.637	300	590	0	590
		1.2	0.635	294	387	0	387
		1.3	0.633	288	302	0	302
		1.4	0.631	283	254	0	254
	0.7	1.1	0.637	300	596	0	596
		1.2	0.635	294	389	0	389
		1.3	0.633	288	303	0	303
		1.4	0.631	283	255	0	255
	0.8	1.1	0.637	300	604	0	604
		1.2	0.635	294	392	0	392
		1.3	0.633	288	305	0	305
		1.4	0.631	283	257	0	257
	0.9	1.1	0.637	300	611	0	611
		1.2	0.635	294	394	0	394
		1.3	0.633	288	306	0	306
		1.4	0.631	283	258	0	258
UD-NB	0.6	1.1	0.576	300	5485	0	5485
		1.2	0.556	557	3976	0	3976
		1.3	0.540	698	3486	0	3486
		1.4	0.526	975	2796	0	2796
	0.7	1.1	0.577	291	5552	0	5552
		1.2	0.557	546	4022	0	4022
		1.3	0.540	689	3512	0	3512
		1.4	0.526	962	2825	0	2825
	0.8	1.1	0.578	284	5611	0	5611
		1.2	0.557	542	4039	0	4039
		1.3	0.541	682	3531	0	3531
		1.4	0.527	948	2857	0	2857
	0.9	1.1	0.579	277	5664	0	5664
		1.2	0.558	539	4050	0	4050
		1.3	0.541	678	3543	0	3543
		1.4	0.527	938	2878	0	2878

We observe from Tables 3.3 and 3.4 that ED in ID is greater than in UD, UD-NB, and ID-NB. Having a battery reduces the energy imbalance in the market if the producer uses it to better fulfill her commitments (from UD-NB to UD). Having a battery increases the energy imbalance if the producer uses it to support her intentional deviations (from UD to ID or from ID-NB to ID). Since the producer in UD aims to fulfill her commitments, ED in UD is not affected much by the changes in the imbalance pricing parameters, compared to ID and UD-NB. Finally, we observe from Figure 3.7 that ED in ID-NB is not affected by the changes in the imbalance pricing parameters as long as they are symmetric, while TCF in ID-NB is affected significantly by these changes. The setting in ID-NB under symmetric imbalance pricing parameters can be viewed as a newsvendor-type inventory problem where the unit costs of overstocking and understocking are equal to each other so that the critical fractile is always 0.5.

3.4.3 The impact of negative electricity prices

In this section, we examine the effects of NPF on the values of TCF, WEC, NI, PI, and ED when $C_S = 400$ MWh (for ID and UD), $C_C = C_D = 50$ MWh (for ID and UD), $C_T = 200$ MWh, $K_p^+ = K_n^- = 0.9$, and $K_n^+ = K_p^- = 1.1$. See Figure 3.8.

We observe that TCF decreases as NPF grows. This is because the average electricity price is lower when NPF is larger. An important observation is that having a battery for intentional deviations becomes much more valuable as NPF grows. We also note that for small NPF values, TCF in UD is larger than TCF in ID-NB. For large NPF values, however, TCF in UD is smaller than TCF in ID-NB. Under nonnegative electricity prices, the producers in ID and ID-NB do not curtail the excess wind energy (as long as the transmission line capacity allows) since there is still a positive economic return when they sell the excess energy to the market at a lower price than the market price. Although the producers in UD and UD-NB could be better off by selling the excess energy under nonnegative prices, they curtail the excess wind energy to reduce deviations

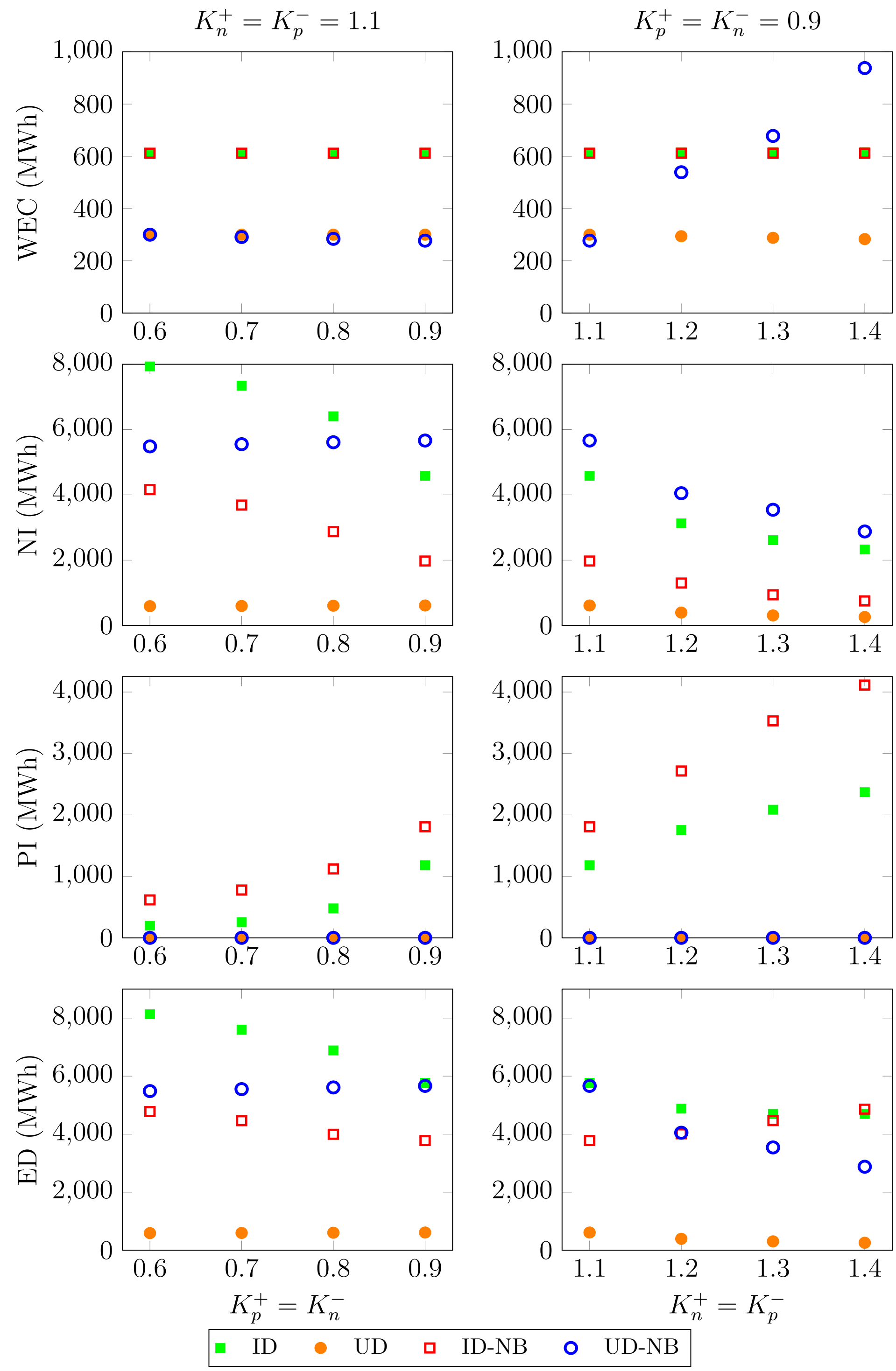


Figure 3.6: WEC, NI, PI, and ED vs. $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$ when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

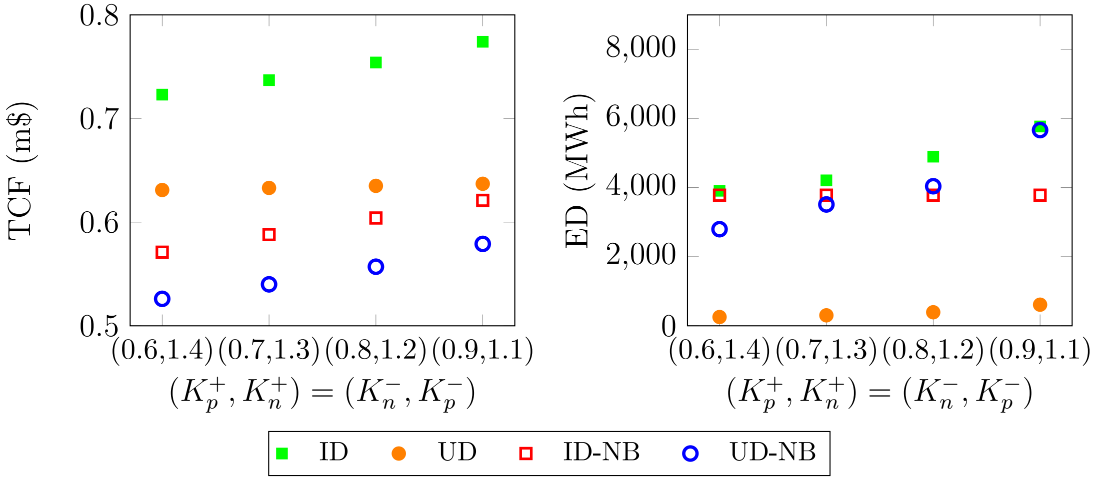


Figure 3.7: The effects of $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$ on ED when $(K_p^+, K_n^+) = (K_n^-, K_p^-)$, $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

from their commitments, as imposed by the problem setting in Chapter 3.2.2. As NPF grows, WEC increases in ID and ID-NB more significantly than in UD and UD-NB. For the producer in ID, the incentive to keep the battery level low to benefit from the negative prices dominates the incentive to store the excess wind energy in the battery. The producer in ID-NB can benefit from the negative prices by selling energy less than her commitment, leading to large curtailment amounts. This also explains why ID-NB can be more profitable than UD when NPF is large.

We observe that ED increases in ID and ID-NB, decreases in UD-NB, and remains similar in UD, as NPF grows. The producer in ID has an incentive to purchase energy at negative prices to sell it in future periods with high prices by deviating from her commitments (leading to greater NIs). She also has an incentive to discharge the battery by selling energy more than her commitment at nonnegative prices to purchase energy in future periods with negative prices (leading to greater PIs). The producer in ID-NB has an incentive to sell energy less than her commitment at negative prices (leading to greater NIs). She also has an incentive to commit to selling less energy when NPF is larger so that the positive imbalances are larger in periods with positive prices (leading to greater

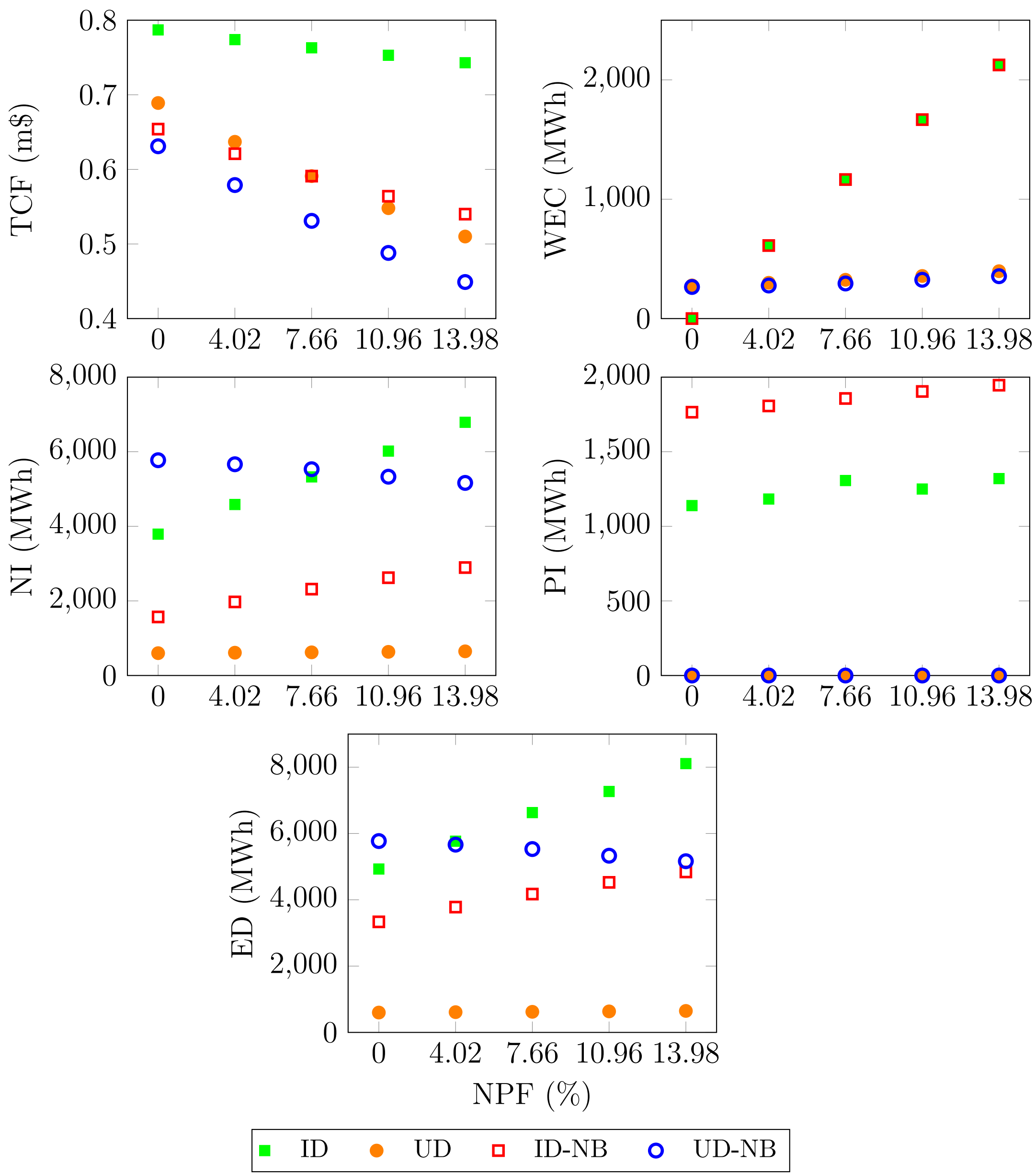


Figure 3.8: TCF, WEC, NI, PI, and ED vs. NPF when $C_S = 400$ MWh (for ID and UD), $C_C = C_D = 50$ MWh (for ID and UD), $C_T = 200$ MWh, $K_p^+ = K_n^- = 0.9$, and $K_n^+ = K_p^- = 1.1$.

PIs). The producer in UD-NB becomes more conservative in her commitment decisions to suffer less from a large NPF, thereby meeting her commitments better. Finally, although the producer in UD wants to take advantage of the negative prices by committing to purchasing more energy, the increase in NPF does not have a significant effect on her deviations since her primary goal is to fulfill her commitments as much as possible with the help of the battery.

3.4.4 The impact of wind availability

Finally, we examine the effects of season and location on our key observations in Chapters 3.4.1-3.4.3, by extending all of our experiments in Chapters 3.4.1-3.4.3 to the month of January in the city of Albany and to the city of Buffalo in the month of August. Recall that we performed our earlier experiments for the month of August in the city of Albany.

Our time series model for the city of Albany implies that the months of August and January exhibit distinct features in terms of wind speed. The average, median, minimum, and maximum values of the wind speed in August are 6.76, 6.54, 0.06, and 27.97, in m/s, respectively. The average, median, minimum, and maximum values of the wind speed in January are 9.98, 9.94, 0.05, and 27.01, in m/s, respectively. We have found that our key insights related to the impacts of imbalance pricing parameters and negative electricity prices remain valid in January (see Tables A.1-A.6 and Figures A.1-A.8 in Appendix A). While our key insights related to the impact of the size of system components remain valid when transmission line capacity is high ($C_T = 200$), we observe a few notable differences when transmission line capacity is low ($C_T = 100$), as can be seen from Figure 3.9. Since there is a better wind availability in January than in August, the transmission line capacity becomes even more binding in January when $C_T = 100$ so that TCF and ED are unaffected by changes in the storage capacity for ID and UD. For the same reason, WEC declines more slowly in January than in August as C_S grows. We also note that the producer in UD can always fulfill her commitments. This is because the producer always commits to selling the

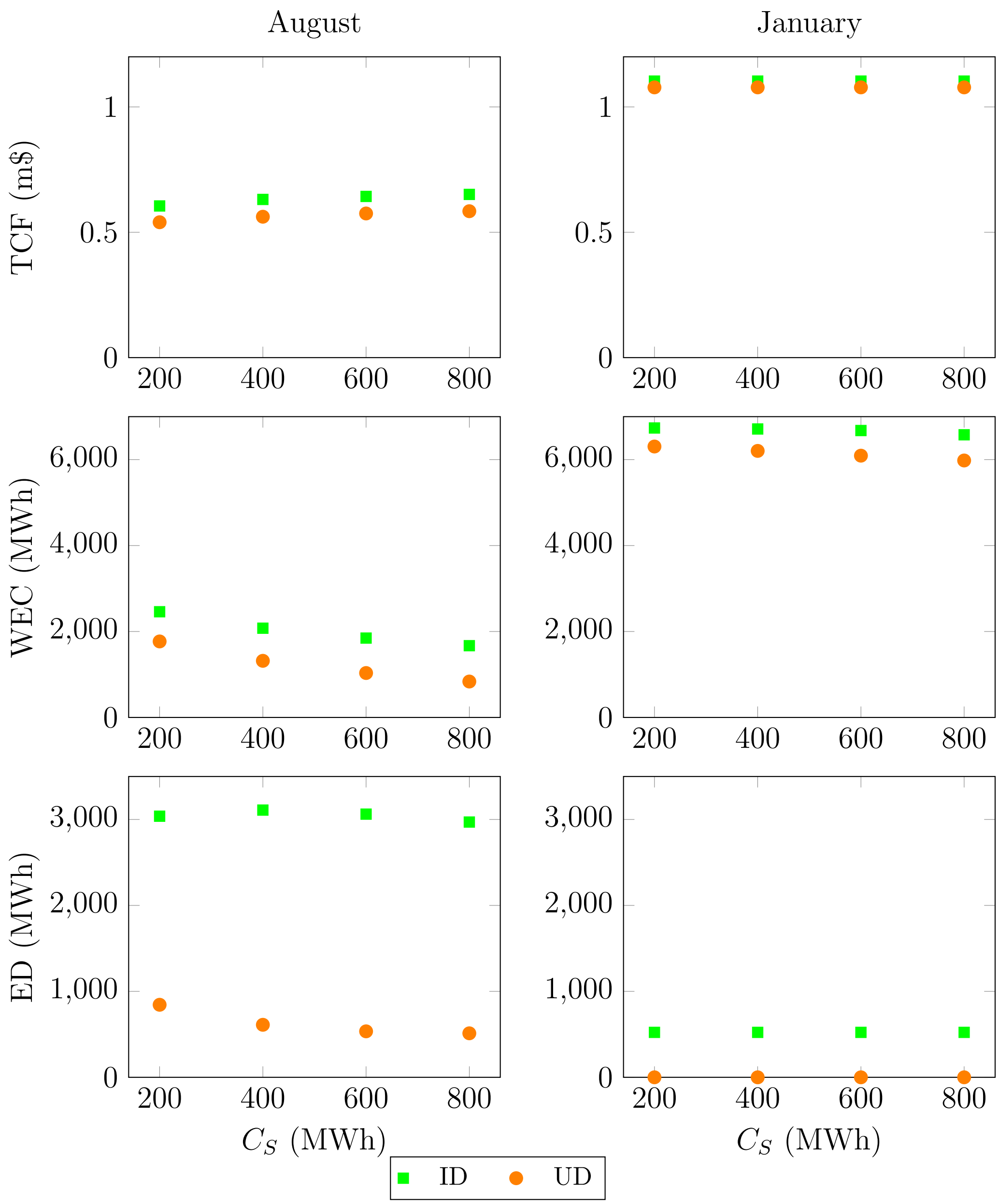


Figure 3.9: TCF, WEC, and ED vs. C_S when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, $C_C = C_D = 50$ MWh, $C_T = 100$ MWh, NPF = 4.02% in August, and NPF = 1.61% in January.

maximum amount of energy that the transmission line capacity allows and is able to supply this amount of energy in real-time thanks to high wind availability.

We next consider the wind speed data available for Buffalo, in the State of New York, between the years 2007 and 2012 in which the wind speed is recorded every five minutes. We retrieve this data from [78]. The average, median, minimum, and maximum values of the wind speed in August are 6.01, 5.65, 0.08, and 20.10, in m/s, respectively. We have found that the MAE of our DHR+AR(1) model is only 0.86 m/s. The parameter estimates of the DHR+AR(1) model are $\hat{\gamma}_0^{(w)} = 7.587$, $\hat{\gamma}_1^{(w)} = 0.917$, $\hat{\gamma}_2^{(w)} = 1.485$, $\hat{\omega}_1^{(w)} = -0.832$, $\hat{\omega}_2^{(w)} = -20.627$, $\hat{\phi}^{(w)} = 0.938$, and $\hat{\sigma}^{(w)} = 1.200$. Our key insights related to the impacts of imbalance pricing parameters and negative electricity prices continue to hold in Buffalo. Since there is a lower wind availability in Buffalo than in Albany, the transmission line capacity becomes less binding in Buffalo when $C_T = 100$ so that our key observations for Buffalo when $C_T = 100$ are similar to those for Albany when $C_T = 200$.

3.5 Concluding Remarks

In this chapter, we study the energy commitment, generation, and storage problem for an energy system that consists of a wind power plant and a battery. We consider the following two possible settings: (i) The battery can be used to support intentional deviations from commitments or (ii) it should be used to minimize such deviations. We model these problems as an MDP by taking into account the electricity price and wind uncertainties. We construct data-calibrated time series models for the electricity price and wind speed, which we incorporate into our MDP formulations. We numerically examine the effects of system components, imbalance pricing parameters, and negative prices on the producer's profits, curtailment decisions, and imbalance tendencies for each problem setting.

Our numerical results for Albany in August show that behaving strategically in commitment fulfillment decisions improves the wind power producer's profit by

9.5%, while increasing the imbalance amount by 7.3%, on average, in the absence of a battery (UD-NB vs. ID-NB). Using the battery as a strategic tool to intentionally deviate from commitments rather than restricting its use to fulfillment of commitments improves the producer's profit by 20.1%, while increasing the imbalance amount by ten times (UD vs. ID). The existence of a battery increases the profit and imbalance amount of the producer making intentional deviations by 26.1% and 24.3%, respectively (ID vs. ID-NB). Although the existence of a battery increases the profit of the producer aiming to fulfill her commitments by 14.4%, it reduces her imbalance amount by 90.0% (UD vs. UD-NB). Our results may guide the wind power producers in their assessment of the battery adoption decisions as well as the system operators in their understanding of the producers' behavior in different environments.

Future research may extend our analysis by examining the roles of different balancing rules, such as dual-pricing and imbalance subsidies, in the producer's profits and operations. In the next chapter, we evaluate the general problem stated in Chapter 3.2.1 and characterize the optimal operating policy for the wind power producer.

Chapter 4

Commitment and Storage Problem of Wind Power Producers: Optimal Policy Characterization under Perfect Efficiency

4.1 Introduction

In Chapter 3, we provide managerial insights into the market participants' behavior by studying the energy commitment problem for a wind power producer in two different market settings. Taking a different route in this chapter, we characterize the optimal policy structure and develop heuristic solution methods for the more general problem setting presented in Chapter 3.2.1. Focusing on structural results and their use for algorithmic efficiency, this chapter complements the insightful discussions in Chapter 3.

Several papers dealing with joint optimization of renewable power generation and energy storage in electricity markets have characterized the optimal policy structure. See Kim and Powell [32], Zhou et al. [8], and Avci et al. [64]. See Chapter 2 for detailed discussions of these studies.

Kim and Powell [32] derive the optimal hour-ahead commitment of a wind power producer in closed form when the wind speed is uniformly distributed. The only decision variable in their setting is the amount of energy committed to selling to the market. We depart from Kim and Powell [32] by considering a more general setting that is flexible enough to allow for energy storage and generation decisions (in addition to commitment decisions). In their setting, Kim and Powell [32] also assume that there is no power capacity constraint, the transmission line is perfectly efficient, the producer can only commit to selling, and the producer never induces any positive imbalance. We relax all these assumptions and require no specific probability distribution for the wind speed in our optimal policy characterization.

Zhou et al. [8] and Avci et al. [64] establish the optimality of state-dependent threshold policies. Zhou et al. [8] include the battery storage level as the only endogenous state variable in their MDP while Avci et al. [64] include the water levels in the upper and lower reservoirs as the two endogenous state variables. These papers present the optimal policy structure in electricity markets that are free of advance commitment decisions. However, the existence of advance commitment decisions makes the problem more challenging.

In this study, unlike Zhou et al. [8], we include not only the storage level of the battery but also the commitment decision of the previous period as our endogenous state variables. Therefore, we characterize the optimal policy structure by establishing several multi-dimensional properties of the optimal profit function (supermodularity and joint concavity in two dimensions) that are not available in [8]. The endogenous state variables in [64] display different interactions than ours in this study. Avci et al. [64] establish *submodularity* of their profit function, revealing the substitutability effect between the water levels in their pumped hydro energy storage facility (i.e., the gain from having more water in the upper reservoir is smaller if there is more water in the lower reservoir, and vice versa). However, we establish *supermodularity* of our profit function under perfect efficiency, revealing the complementarity effect between the commitment and storage levels (i.e., the gain from a higher commitment level is larger if the storage level is higher, and vice versa). While Avci et al. [64] show concavity in single dimension, we prove joint concavity in two dimensions. Consequently, the optimal policy structure in [64] involves only one target level for the water level in the upper reservoir, whereas ours involves two target levels for the storage and commitment decisions that should be jointly optimized.

In our structural analysis, assuming positive electricity prices, we first prove that the system becomes more profitable as the battery storage level grows for a fixed commitment level. We then formulate the optimal amount of wind energy that should be generated in any period as a function of the state variables in that period. When the battery and transmission line are perfectly efficient, we characterize the optimal policy structure by partitioning the state space of the problem into two disjoint domains that correspond to the optimal decisions of 'positive imbalance' and 'negative imbalance,' respectively: it is optimal to bring the storage and commitment levels to a different state-dependent threshold pair in each domain. The optimal threshold levels are higher in the case of positive imbalance than in the case of negative imbalance.

Our structural results can be usefully employed to develop an efficient heuristic solution procedure in a more general problem where the electricity price can also be negative. In this procedure, we implement the optimal policy structure that we found into a backward induction algorithm, in order to calculate the state-dependent threshold pairs for the storage and commitment levels in each period. The storage and commitment actions in states with positive prices are determined by taking into account the computed threshold pairs as well as the system inefficiencies. The storage and commitment actions in states with negative prices are determined by the myopically optimal storage decisions. We call this solution method HC (Heuristic via Complete state space). We also consider a variant of the method HC that uses the output of the method HC executed in a simpler problem, which we obtain by ignoring the spike component of the price and thus reducing the state space drastically. We call this variant HR (Heuristic via Reduced state space). The method HR adopts the storage and commitment decisions found via the method HC executed in the reduced state-space problem if the spike is zero and the price is positive in the current state, and takes a myopic approach to determine these decisions otherwise.

The experimental setup used in this chapter (time series models for the electricity market and wind speed and their discretization procedures) is based on the one presented in Chapter 3.3. We have compiled realistic instances with imperfectly efficient battery and transmission line. Our method HC provides near-optimal solutions in these instances (with an average distance of 0.58% and a maximum distance of 1.63% from the optimal profit), outperforming the standard dynamic programming (DP) algorithm with respect to computation times by two orders of magnitude. Our method HR yields high-quality solutions (with an average distance of 0.84% and a maximum distance of 2.31% from the optimal profit) ten times faster than our method HC. The solution time of our method HR is less than one minute in each instance, while the standard DP algorithm has an average solution time of 237.6 minutes. Finally, when the battery and transmission line are perfectly efficient, our method HC yields the optimal solution in each instance. All these findings highlight the practical importance of our structural results.

The rest of this chapter is organized as follows. Chapter 4.2 briefly restates the energy commitment, generation and storage problem presented in Chapter 3.2.1. Chapter 4.3 establishes the optimal policy structure. Chapter 4.4 constructs the heuristic solution methods based on our structural results. Chapter 4.5 presents numerical results for our heuristic methods. Chapter 4.6 concludes. Proofs of the analytical results are contained in Appendix B.

4.2 Problem Formulation

We briefly restate the problem formulation in Chapter 3.2.1:

Parameters

- C_S: Energy capacity of the battery.
- C_C, C_D: Charging and discharging capacities of the battery, respectively.
- C_T: Transmission line capacity.
- γ, θ: Discharging and charging efficiencies of the battery, respectively.
- τ: Transmission line efficiency.

State Variables

- S_t: Storage level at the beginning of period t; $S_t \in [0, C_S]$.
- Q_t: Commitment level in period t; $Q_t \in \mathbb{R}$. The producer is obliged to sell if $Q_t \geq 0$ and she is obligated to purchase if $Q_t < 0$.
- W_t: Wind speed in period t; $W_t \in \mathbb{R}_+$. This is limited by $f(W_t)$, the maximum amount of wind energy that can be generated in period t.
- P_t: Electricity price in period t; $P_t \in \mathbb{R}$.

We include the tuple $I_t := (P_\kappa, W_\kappa)_{\kappa \leq t}$ in our state description. The state tuple I_t evolves over time according to an exogenous stochastic process.

Decision Variables

- q_t: Amount of energy to be committed to selling or purchasing in period $t+1$ by signing the contract in period t; $q_t \in \mathbb{R}$.
- s_t: Amount of energy to be generated or stored in the battery in period t; $s_t \in \mathbb{R}$. The battery is discharged if $s_t \geq 0$ and charged if $s_t < 0$.
- w_t: Amount of wind energy to be generated in period t; $w_t \in \mathbb{R}_+$.

In any period $t \in \mathcal{T}$, the producer first observes P_t and W_t, as well as Q_t and S_t. She then determines the commitment, storage, and wind generation triplet (q_t, s_t, w_t). $\mathbb{U}(Q_t, S_t, I_t)$ is the set of action triplets (q_t, w_t, s_t) that are admissible in state (Q_t, S_t, I_t). For any action triplet $(q_t, w_t, s_t) \in \mathbb{U}(Q_t, S_t, I_t)$, the following conditions must hold:

$$
\begin{gathered}
0 \le w_t \le f(W_t), \\
-\min\{C_S - S_t, C_C\} \le s_t \le \min\{S_t, C_D\}, \\
\gamma s_t + w_t \le C_T \text{ if } s_t \ge 0, \\
-\tau C_T \le s_t/\theta + w_t \le C_T \text{ if } s_t < 0.
\end{gathered}
$$

The state variables S_t and Q_t evolve over time as follows: $S_{t+1} = S_t - s_t$ and $Q_t = q_{t-1}$.

The amount of energy sold or purchased in period t can be defined as a function of actions s_t and w_t:

$$
E(s_t, w_t) := \begin{cases} (\gamma s_t + w_t)\tau & \text{if } s_t \ge 0, \\ (s_t/\theta + w_t)\tau & \text{if } -w_t \le s_t/\theta < 0, \\ (s_t/\theta + w_t)/\tau & \text{if } s_t/\theta < -w_t \le 0. \end{cases}
$$

Note that $E(s_t, w_t) = \min\{(\gamma s_t + w_t)\tau, (s_t/\theta + w_t)\tau, (s_t/\theta + w_t)/\tau\}$. Since the minimum of affine functions is concave, $E(\cdot,\cdot)$ is jointly concave. For any action triplet $(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)$, the transmission line capacity implies that $E(s_t, w_t) \le \tau C_T$ if $E(s_t, w_t) \ge 0$ (the energy is sold) and $-C_T \le E(s_t, w_t)$ if $E(s_t, w_t) < 0$ (the energy is purchased). In addition, the energy and power capacities of the battery imply that $-\min\{C_S - S_t, C_C\}/(\theta\tau) \le E(s_t, w_t)$. Hence $-\min\{(C_S - S_t)/(\theta\tau), C_C/(\theta\tau), C_T\} \le E(s_t, w_t) \le \tau C_T$.

If the producer does not fulfill her contractual commitment in real-time, she pays a penalty cost that varies with her deviation. There are two decision types that we need to consider in our payoff formulation in any period t: (i) 'positive imbalance;' we call this decision type pi. (ii) 'negative imbalance;' we call this decision type ni.

The payoff in period t:

$$R(Q_t, I_t, s_t, w_t) = \begin{cases} Q_tP_t + K_p^+P_t(E(s_t, w_t) - Q_t) & \text{if } P_t \geq 0 \text{ and} \\ & Q_t < E(s_t, w_t) \quad \textsf{(pi)}, \\ Q_tP_t - K_n^+P_t(Q_t - E(s_t, w_t)) & \text{if } P_t \geq 0 \text{ and} \\ & Q_t \geq E(s_t, w_t) \quad \textsf{(ni)}, \\ Q_tP_t + K_p^-P_t(E(s_t, w_t) - Q_t) & \text{if } P_t < 0 \text{ and} \\ & Q_t < E(s_t, w_t) \quad \textsf{(pi)}, \\ Q_tP_t - K_n^-P_t(Q_t - E(s_t, w_t)) & \text{if } P_t < 0 \text{ and} \\ & Q_t \geq E(s_t, w_t) \quad \textsf{(ni)}, \end{cases}$$

where $0 \leq K_p^+ < 1 < K_n^+$ and $0 \leq K_n^- < 1 < K_p^-$.

A control policy π is the sequence of decision rules $(\eta_t^\pi(Q_t^\pi, S_t^\pi, I_t))_{t \in \mathcal{T}}$, where $\eta_t^\pi(Q_t^\pi, S_t^\pi, I_t) := (q_t^\pi(Q_t^\pi, S_t^\pi, I_t), s_t^\pi(Q_t^\pi, S_t^\pi, I_t), w_t^\pi(Q_t^\pi, S_t^\pi, I_t))$, and Q_t^π and S_t^π denote the random state variables governed by policy π, $\forall t \in \mathcal{T} \backslash \{1\}$. We denote the set of all admissible control policies by Π. For any initial state (Q_1, S_1, I_1), the optimal expected total cash flow over the finite horizon can be written as

$$\max_{\pi \in \Pi} \mathbb{E} \left[\sum_{t \in \mathcal{T}} R(Q_t^\pi, I_t, s_t^\pi(Q_t^\pi, S_t^\pi, I_t), w_t^\pi(Q_t^\pi, S_t^\pi, I_t)) \middle| Q_1, S_1, I_1 \right].$$

For each period $t \in \mathcal{T}$ and each state (Q_t, S_t, I_t), the optimal profit function $v_t^*(Q_t, S_t, I_t)$ can be calculated with the following DP recursion:

$$v_t^*(Q_t, S_t, I_t) = \max_{(q_t, s_t, w_t) \in \mathbb{U}(Q_t, S_t, I_t)} \left\{ R(Q_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \left[v_{t+1}^*(q_t, S_t - s_t, I_{t+1}) \right] \right\} \tag{4.1}$$

where $v_T(Q_T, S_T, I_T) = 0$. Note that $v_1^*(Q_1, S_1, I_1)$ is the optimal expected total cash flow for the initial state (Q_1, S_1, I_1) over the finite horizon.

4.3 Characterization of the Optimal Policy

In this section, we first establish several structural properties of our optimal profit function. We then use these properties to characterize the structure of the optimal energy commitment, generation and storage policy.

4.3.1 Structural results

We first introduce several bounds on the optimal energy commitment decision.

Lemma 4.3.1. *Without loss of optimality, the commitment levels can be constrained as* $-\min\left\{\frac{C_S-(S_t-s_t)}{\theta\tau}, \frac{C_C}{\theta\tau}, C_T\right\} \leq q_t \leq \tau C_T$, $\forall t \geq 1$, *and* $-\min\left\{\frac{C_S-S_t}{\theta\tau}, \frac{C_C}{\theta\tau}, C_T\right\} \leq Q_t \leq \tau C_T$, $\forall t > 1$.

Proof. See Appendix B. □

Lemma 4.3.1 states that the optimal amount of energy that the producer commits to selling must be no larger than the maximum amount of energy that can be sold (bounded by the transmission line capacity), and the optimal amount of energy that the producer commits to purchasing must be no larger than the maximum amount of energy that can be stored (bounded by the available storage capacity, charging capacity, and transmission line capacity).

For our structural analysis, we assume that the electricity price is always non-negative:

Assumption 4.3.1. $P_t \geq 0$, $\forall t \in \mathcal{T}$.

Under Assumption 4.3.1, note that $R(Q_t, I_t, s_t, w_t) = \min\{Q_tP_t + K_p^+P_t(E(s_t,w_t) - Q_t), Q_tP_t + K_n^+P_t(E(s_t,w_t) - Q_t)\}$. Since the minimum of affine functions is concave, $R(Q_t, I_t, s_t, w_t)$ is jointly concave in Q_t and $E(s_t, w_t)$. Furthermore, since $E(s_t, w_t)$ is jointly concave and increasing in s_t and w_t,

$R(Q_t, I_t, s_t, w_t)$ is jointly concave and increasing in s_t and w_t as well. We now establish the following structural property of our optimal profit function.

Lemma 4.3.2. *Under Assumption 4.3.1, $v_t^*(Q_t, S_t, I_t) \leq v_t^*(Q_t, S_t + \alpha, I_t)$, where $\alpha > 0$, $\forall t \in \mathcal{T}$.*

Proof. See Appendix B. □

Lemma 4.3.2 states that the system becomes more profitable as the amount of energy accumulated in the battery grows. This is because if the stored energy is higher, the producer can sell more energy from the battery by charging it less in the long run. Using Lemma 4.3.2, we formulate the optimal amount of wind energy that should be generated in any period.

Lemma 4.3.3. *Under Assumption 4.3.1, $w_t^*(Q_t, S_t, I_t) = \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}/\theta\}$. Moreover, if $w_t^*(Q_t, S_t, I_t) = C_T + \min\{C_S - S_t, C_C\}/\theta$, then $s_t^*(Q_t, S_t, I_t) = -\min\{C_S - S_t, C_C\}$.*

Proof. See Appendix B. □

Lemma 4.3.3 states that it is optimal to generate as much wind energy as possible. If the wind energy potential is large enough, it is optimal to sell and store as much energy as possible. The curtailed amount of wind energy is given by $f(W_t) - (C_T + \min\{C_S - S_t, C_C\}/\theta)$ if $w_t^*(Q_t, S_t, I_t) < f(W_t)$. This lemma allows us to restrict our optimal policy characterization to the energy commitment and storage decisions.

For the rest of our structural analysis, we assume that the battery and transmission line are perfectly efficient. We will relax this assumption in subsequent sections for heuristic solution development and in Chapter 5 for optimal policy characterization.

Assumption 4.3.2. $\gamma = \theta = \tau = 1$.

Under Assumptions 4.3.1 and 4.3.2, using Lemmas 4.3.2 and 4.3.3, we establish several second-order properties of our optimal profit function.

Proposition 4.3.1. *Under Assumptions 4.3.1 and 4.3.2, the following properties hold for $\alpha > 0$ and $\beta > 0$:*

(a) $v_t^*(Q_t+\alpha, S_t, I_t) - v_t^*(Q_t, S_t, I_t) \leq v_t^*(Q_t+\alpha, S_t+\beta, I_t) - v_t^*(Q_t, S_t+\beta, I_t)$, $\forall t$.

(b) $v_t^*(Q_t + \alpha, S_t + \alpha + \beta, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \leq v_t^*(Q_t, S_t + \beta, I_t) - v_t^*(Q_t, S_t, I_t)$, $\forall t$.

(c) $v_t^*(Q_t + \alpha + \beta, S_t + \alpha, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \leq v_t^*(Q_t + \beta, S_t, I_t) - v_t^*(Q_t, S_t, I_t)$, $\forall t$.

Proof. See Appendix B. □

Proposition 4.3.1 says that a larger commitment level is more profitable or less costly when the storage level is larger (point a) or when the commitment and storage levels are smaller by an equal amount (point c). It also says that a larger storage level is more profitable when the commitment level is larger (point a) or when the commitment and storage levels are smaller by an equal amount (point b). The property in point (a) can be viewed as Topkis' [84] supermodularity property in Q_t and S_t, indicating the complementarity effect between the commitment and storage levels.

The summation of properties in points (a) and (c) implies the concavity of $v_t^*(\cdot, S_t, I_t)$, i.e., $v_t^*(Q_t, S_t, I_t) - v_t^*(Q_t + \alpha, S_t, I_t) \leq v_t^*(Q_t + \beta, S_t, I_t) - v_t^*(Q_t + \alpha + \beta, S_t, I_t)$. Hence, increasing the commitment level exhibits diminishing returns: A larger commitment level increases the risk of paying the penalty cost since it is more difficult to meet. It may also induce the producer to consume more of the available energy in the current period and thus suffer from the limited energy availability in the future. A smaller commitment level, on the other hand, increases the risk of missing the opportunity to sell more energy at the market price. Likewise, the summation of properties in points (a) and (b) implies the concavity of $v_t^*(Q_t, \cdot, I_t)$, i.e., $v_t^*(Q_t, S_t, I_t) - v_t(Q_t, S_t + \alpha, I_t) \le v_t^*(Q_t, S_t + \beta, I_t) - v_t(Q_t, S_t + \alpha + \beta, I_t)$. Hence, increasing the storage level also exhibits diminishing returns: Holding some amount of energy in the battery allows the producer to sell energy in future periods with high prices. On the other hand, holding a large amount of energy in the battery curbs the capability of purchasing energy in future periods with low prices.

More critically, the summation of properties in points (b) and (c) implies that $v_t^*(Q_t, S_t, I_t)$ is jointly concave in (Q_t, S_t), i.e., $v_t^*(Q_t + \alpha + \beta, S_t + \alpha + \beta, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \le v_t^*(Q_t + \beta, S_t + \beta, I_t) - v_t^*(Q_t, S_t, I_t)$. This property can be interpreted as follows: Increasing the commitment and storage levels by an equal amount allows the operator to raise the energy delivered to the market with an amount that incurs no additional penalty payment. Such an improvement in the energy delivery exhibits diminishing returns.

4.3.2 Optimal commitment and storage policy

An important implication of Proposition 4.3.1 is that the optimal energy commitment and storage policy can be characterized as following a threshold policy. The joint concavity of the optimal profit function enables us to introduce the optimal state-dependent target levels for the commitment and storage decisions, which we respectively denote by $Y_t^{(\nu)}(I_t)$ and $Z_t^{(\nu)}(I_t)$, in an unconstrained problem free of certain capacity limits and implement these target levels into the optimal policy structure in the original constrained problem. These target levels are also dependent on the decision type. Specifically, for $\nu \in \{\mathsf{pi}, \mathsf{ni}\}$,

$$\left(Y_t^{(\nu)}(I_t), Z_t^{(\nu)}(I_t)\right) := \underset{(q_t, z_t) \in [-\min\{C_C, C_T\}, C_T] \times [0, C_S]}{\arg\max} \left\{ \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right] + R^{(\nu)}(z_t, I_t) \right\} \tag{4.2}$$

where

$$R^{(\nu)}(z_t, I_t) = \begin{cases} -K_p^+ P_t z_t & \text{if } \nu = \mathsf{pi}, \\ -K_n^+ P_t z_t & \text{if } \nu = \mathsf{ni}, \end{cases}$$

and $z_t := S_t - s_t$ is the storage level at the end of period t if the action s_t is taken in period t. Since $Z_t^{(\nu)}(I_t)$ may be inaccessible when the omitted capacity limits are reconsidered, the optimal storage level at the end of period t may be different from $Z_t^{(\nu)}(I_t)$ so that $Y_t^{(\nu)}(I_t)$ may no longer be optimal at this storage level. Therefore, we also introduce the optimal state-dependent target level for the commitment decision after the storage decision is made in the constrained problem:

$$Y_t(S_{t+1}, I_t) := \underset{q_t \in [-\min\{C_C, C_T\}, C_T]}{\arg\max} \left\{ \mathbb{E}\left[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \right] \right\}. \tag{4.3}$$

Note that $Y_t(Z_t^{(\nu)}, I_t) = Y_t^{(\nu)}(I_t)$ for each $\nu \in \{\mathsf{pi}, \mathsf{ni}\}$. Finally, we introduce an auxiliary state-dependent target level for the storage decision, which we denote by $Z_t(Q_t, S_t, I_t)$, that can take the values of $Z_t^{(\mathsf{pi})}(I_t)$ and $Z_t^{(\mathsf{ni})}(I_t)$ depending on the system state; see Theorem 4.3.1.

Let Ω denote the domain of (Q_t, S_t, W_t), i.e., $\Omega := [-\min\{C_C, C_T\}, C_T] \times [0, C_S] \times [0, \infty)$. For the optimal storage policy characterization, we partition this domain into four disjoint subdomains: We define the set $\Psi^0 := \{(Q_t, S_t, W_t) \in \Omega : f(W_t) \geq C_T + \min\{C_S - S_t, C_C\}\}$ as the subdomain where the maximum amount of wind energy that can be generated in period t is greater than the maximum total amount of energy that can be used for selling and storing in period t. We define the set $\Psi^1 := \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\} > f(W_t) \geq Q_t + \min\{C_S - S_t, C_C\}\}$ as the subdomain where the maximum amount of wind energy that can be generated in period t is less than the maximum total amount of energy that can be used for selling and storing in period t, but greater than the maximum total amount of energy that can be used for meeting the commitment and storing in period t. We define the set $\Psi^2 := \{(Q_t, S_t, W_t) \in \Omega : Q_t + \min\{C_S - S_t, C_C\} > f(W_t) \geq Q_t - \min\{S_t, C_D\}\}$ as the subdomain where the maximum amount of wind energy that can be generated in period t is less than the maximum total amount of energy that can be used for meeting the commitment and storing in period t, but greater than the amount of energy required to meet the commitment after the battery is discharged as much as possible in period t. We define the set $\Psi^3 := \{(Q_t, S_t, W_t) \in \Omega : Q_t - \min\{S_t, C_D\} > f(W_t)\}$ as the last subdomain.

The type of imbalance induced by the producer is likely to change from positive to negative as the wind power potential decreases. Hence, the target storage level $Z_t(Q_t, S_t, I_t)$ is likely to switch from $Z_t^{(\mathsf{pi})}(I_t)$ to $Z_t^{(\mathsf{ni})}(I_t)$ as the system moves from Ψ^0 to Ψ^3. Incorporating these subdomains and leveraging the results of Lemmas 4.3.2-4.3.3 and Proposition 4.3.1, we are now ready to formally state the main result of this section:

Theorem 4.3.1. *Under Assumptions 4.3.1 and 4.3.2, the optimal policy structure follows a state-dependent threshold policy with state-dependent target levels for the storage and commitment decisions. The optimal state-dependent target storage levels can be calculated as follows.*

(i) If $(Q_t, S_t, W_t) \in \Psi^0$, $Z_t(Q_t, S_t, I_t) = C_S$.

(ii) If $(Q_t, S_t, W_t) \in \Psi^1$, $Z_t(Q_t, S_t, I_t) = Z_t^{(\mathsf{pi})}(I_t)$.

(iii) If $(Q_t, S_t, W_t) \in \Psi^2$,

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{ni})}(I_t) & \text{if } S_t \leq Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t, \\ S_t + f(W_t) - Q_t & \text{if } Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t < S_t \\ & \quad \text{and } S_t \leq Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t, \\ Z_t^{(\mathsf{pi})}(I_t) & \text{if } Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t < S_t. \end{cases}$$

(iv) If $(Q_t, S_t, W_t) \in \Psi^3$, $Z_t(Q_t, S_t, I_t) = Z_t^{(\mathsf{ni})}(I_t)$.

The optimal state-dependent target storage levels obey $Z_t^{(\mathsf{ni})}(I_t) \leq Z_t^{(\mathsf{pi})}(I_t)$. *Letting* $w = w_t^*(Q_t, S_t, I_t)$, *the optimal energy storage action is*

$$
s_t^*(Q_t,S_t,I_t) = \begin{cases}
-\min\{Z_t(Q_t,S_t,I_t)-S_t, & \textit{if } C_T \le w \\
C_T+w, C_C\} & \textit{and } S_t+w-C_T < Z_t(Q_t,S_t,I_t), \\
C_T-w & \textit{if } C_T \le w \\
 & \textit{and } Z_t(Q_t,S_t,I_t) \le S_t+w-C_T, \\
-\min\{Z_t(Q_t,S_t,I_t)-S_t, & \textit{if } C_T > w \\
C_T+w, C_C\} & \textit{and } S_t < Z_t(Q_t,S_t,I_t), \\
\min\{S_t-Z_t(Q_t,S_t,I_t), & \textit{if } C_T > w \\
C_T-w, C_D\} & \textit{and } Z_t(Q_t,S_t,I_t) \le S_t.
\end{cases}
$$

The optimal energy commitment action is $q_t^*(Q_t,S_t,I_t) = Y_t(S_t - s_t^*(Q_t,S_t,I_t),I_t)$. *Furthermore, the optimal state-dependent target commitment levels obey* $Y_t^{(\mathsf{ni})}(I_t) \le Y_t^{(\mathsf{pi})}(I_t)$.

Proof. See Appendix B. □

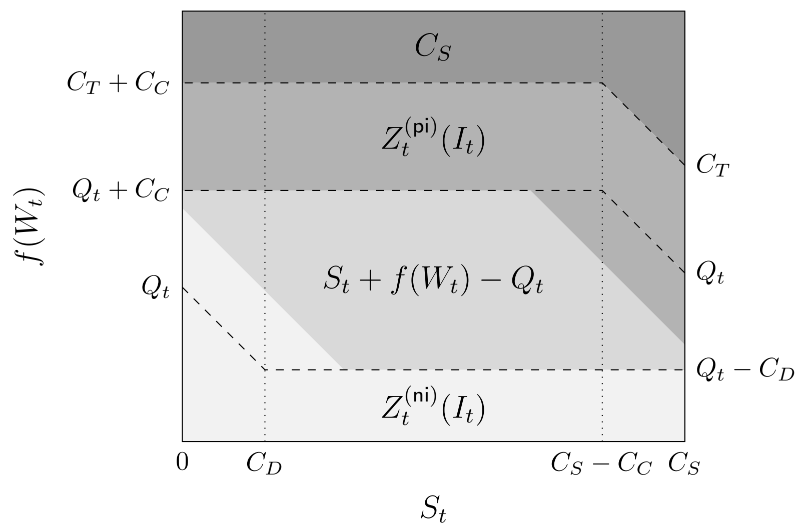
Theorem 4.3.1 explicitly formulates the optimal target storage level $Z_t(Q_t,S_t,I_t)$ in terms of $Z_t^{(\mathsf{pi})}(I_t)$ and $Z_t^{(\mathsf{ni})}(I_t)$, conditional on the subdomains defined earlier. Figure 4.1 provides an illustration of this formulation:

(i) Suppose that the maximum amount of wind energy that can be generated in period t is extremely large (i.e., $(Q_t,S_t,W_t) \in \Psi^0$). See the top region in Figure 4.1. Then it is optimal to increase the storage level of the battery as much as possible.

(ii) Suppose that the maximum amount of wind energy that can be generated in period t is large enough to only meet the commitment and charge the battery as much as possible (i.e., $(Q_t,S_t,W_t) \in \Psi^1$). See the second top region in Figure 4.1.

(iii) Suppose that the maximum amount of wind energy that can be generated in period t is not large enough to meet the commitment and charge the

Figure 4.1: Illustration of $Z_t(Q_t, S_t, I_t)$ for a fixed Q_t. Regions separated by dashed lines correspond to different subdomains of Ω (Ψ^0 to Ψ^3 from top to bottom). Different colors indicate different target levels.



battery as much as possible (i.e., $(Q_t, S_t, W_t) \in \Psi^2$). See the third top region in Figure 4.1. If the storage level of the battery is low (i.e., $S_t \leq Z_t^{(\mathrm{ni})}(I_t) - f(W_t) + Q_t$), it is optimal to bring it as close to $Z_t^{(\mathrm{ni})}(I_t)$ as possible. If the storage level is in a medium range (i.e., $Z_t^{(\mathrm{ni})}(I_t) - f(W_t) + Q_t < S_t \leq Z_t^{(\mathrm{pi})}(I_t) - f(W_t) + Q_t$), it is optimal to meet the commitment as much as possible. If the storage level is high (i.e., $Z_t^{(\mathrm{pi})}(I_t) - f(W_t) + Q_t < S_t$), it is optimal to bring it as close to $Z_t^{(\mathrm{pi})}(I_t)$ as possible.

(iv) Suppose that the maximum amount of wind energy that can be generated in period t is extremely small (i.e., $(Q_t, S_t, W_t) \in \Psi^3$). See the bottom region in Figure 4.1. Then it is optimal to bring the storage level as close to $Z_t^{(\mathrm{ni})}(I_t)$ as possible.

After providing the above formulation of $Z_t(Q_t, S_t, I_t)$, Theorem 4.3.1 derives the optimal action for wind power generation from Lemma 4.3.3, the optimal action for energy storage from the target level $Z_t(Q_t, S_t, I_t)$ by taking into account the charging/discharging capacity levels C_C and C_D, and the optimal action for energy commitment from equation 4.3. Theorem 4.3.1 also states that the optimal target storage level is higher when the optimal storage action leads to a positive imbalance than when it leads to a negative imbalance (i.e., $Z_t^{\text{(ni)}}(I_t) \leq Z_t^{\text{(pi)}}(I_t)$). This result is expected because the optimality of positive (negative) imbalance is a consequence of excess (limited) energy availability. Consequently, the optimal target storage level either increases or stays the same as the current storage level grows for a fixed wind power potential; see Figure 4.1. Notice that the optimal commitment action is based on the optimal target storage level. A larger target storage level in period t provides a better energy availability in period $t+1$, making it optimal to choose a larger commitment level in period t (i.e., $Y_t^{\text{(ni)}}(I_t) \leq Y_t^{\text{(pi)}}(I_t)$). This result is an indication of the complementarity effect revealed in Proposition 4.3.1.

4.4 Heuristic Solution Approach

The structural knowledge in the problem domain can be used to construct effective heuristic solution methods [8, 64, 85]. Theorem 4.3.1 characterizes the optimal policy structure under positive electricity prices and perfect efficiency. We now implement this policy structure into a heuristic solution procedure for the more general problem where the price can also be negative and the system can be imperfectly efficient ($\gamma \leq 1$, $\theta \leq 1$, $\tau \leq 1$). Our heuristic procedure calculates the state-dependent target levels for the storage and commitment decisions in each period with a backward induction algorithm. The storage and commitment actions in states with positive prices are determined by the computed target levels adjusted for the system inefficiencies, while those in states with negative prices are determined by the myopically optimal storage decisions again adjusted for the system inefficiencies. We present below two variants of this heuristic procedure for the discrete-state and discrete-action version of our MDP. Time series models and discretization approach employed here are based on the ones presented in Chapter 3. We define $\mathcal{Q}$ as the discrete space of the commitment level, $\mathcal{S}$ as the discrete space of the storage level, and $\mathcal{I}$ as the discrete space of the exogenous state variables (electricity price and wind speed). See Chapter 3.3.3 for our discretization procedure.

4.4.1 Solution approach via complete state space

Our time series models in Chapter 3.3 imply that it is sufficient to include the mean-reverting component of the price ρ_t, the spike component of the price J_t, and the deseasonalized wind speed ξ_t as our exogenous state variables. The spike component of the price represents sudden and large moves in the price that are independent across periods. These jumps may arise for several reasons such as unexpected power plant or transmission line outages and extreme weather events [86]. We thus redefine $I_t = (\rho_t, J_t, \xi_t) \in \mathcal{I}$ as the exogenous state tuple in period t. We define $\overline{I}_t = (\rho_t, \xi_t)$ as the exogenous state tuple when the spike component is omitted in period t, and $\overline{\mathcal{I}}$ as the discrete space of the exogenous state variables without the spike. We require this notation for our heuristic construction. See Chapter 3.3 for details on the notation.

For each period $t \in \mathcal{T}$ and each state (Q_t, S_t, I_t), we define $v_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ as the profit function of our heuristic method, and $Z_t^{(\nu),\mathsf{HC}}(I_t)$ and $Y_t^{(\nu),\mathsf{HC}}(I_t)$ as the state-dependent target levels under our heuristic method. Following the procedure in Theorem 4.3.1, we compute these profit functions and target levels, as well as the corresponding action triplets $(q_t^{\mathsf{HC}}, s_t^{\mathsf{HC}}, w_t^{\mathsf{HC}})$, by backward induction. See Algorithm 1. We call this method HC. We consider two main scenarios in this method:

- Suppose that the price is positive in the current state. We first calculate w_t^{HC} from Lemma 4.3.3. We then incorporate the battery and transmission line inefficiencies by introducing upper and lower bounds on s_t^{HC}. Specifically, $\underline{s}_t^{\mathsf{HC}} \le s_t^{\mathsf{HC}} \le \overline{s}_t^{\mathsf{HC}}$ where $\underline{s}_t^{\mathsf{HC}} := -\min\{C_S - S_t, C_C, (\tau C_T + w_t^{\mathsf{HC}})\theta\}$ and $\overline{s}_t^{\mathsf{HC}} := \min\{S_t, C_D, \max\{(C_T - w_t^{\mathsf{HC}})\theta, (C_T - w_t^{\mathsf{HC}})/\gamma\}\}$. We slightly modify the optimal storage action formula in Theorem 4.3.1 by taking into account these bounds. We compute s_t^{HC} from the modified formula:

$$s_t^{\mathsf{HC}} = \begin{cases} \underline{s}_t^{\mathsf{HC}} & \text{if } S_t - Z_t^{\mathsf{HC}}(Q_t, S_t, I_t) < \underline{s}_t^{\mathsf{HC}}, \\ S_t - Z_t^{\mathsf{HC}}(Q_t, S_t, I_t) & \text{if } \underline{s}_t^{\mathsf{HC}} \le S_t - Z_t^{\mathsf{HC}}(Q_t, S_t, I_t) \le \overline{s}_t^{\mathsf{HC}}, \\ \overline{s}_t^{\mathsf{HC}} & \text{if } \overline{s}_t^{\mathsf{HC}} < S_t - Z_t^{\mathsf{HC}}(Q_t, S_t, I_t). \end{cases} \quad (4.4)$$

If $Z_t^{(\nu),\mathsf{HC}}(I_t)$ is accessible, $q_t^{\mathsf{HC}} = Y_t^{(\nu),\mathsf{HC}}(I_t)$. Otherwise, we calculate q_t^{HC} from equation (4.3).

- Suppose that the price is negative in the current state. We obtain $w_t^{\mathsf{HC}} = 0$ (i.e., generate no wind energy) and $s_t^{\mathsf{HC}} = -\min\{C_S - S_t, C_C, \theta\tau C_T\}$ (i.e., purchase as much energy as possible) from the myopically optimal solution, but we calculate q_t^{HC} again from equation (4.3).

The number of states in which we need to compute the target levels in each period is of order $O(|\mathcal{I}|)$ and the number of feasible action triplets that we need to consider for target level computation in each state is of order $O(|\mathcal{Q}||\mathcal{S}|)$: the total number of operations required to exhaustively search the action triplets is of order $O(T|\mathcal{Q}||\mathcal{S}||\mathcal{I}|)$. To speed up computation and save memory, we calculate first the profit function $\bar{v}_{t+1}^{\mathsf{HC}}(S_{t+1}, Q_{t+1}, \overline{I}_{t+1}) := \mathbb{E}_{J_{t+1}}\left[v_{t+1}^{\mathsf{HC}}(Q_{t+1}, S_{t+1}, I_{t+1})\right]$ in state $(S_{t+1}, Q_{t+1}, \overline{I}_{t+1})$ in period $t+1$ and then the action triplet in state (Q_t, S_t, I_t) in period t, using the target levels calculated with $\bar{v}_{t+1}^{\mathsf{HC}}(S_{t+1}, Q_{t+1}, \overline{I}_{t+1})$ and the expectation taken with respect to $\overline{I}_{t+1}|\overline{I}_t$. This heuristic method accelerates the standard DP algorithm of our problem, leading to no loss of optimality for perfectly efficient systems with positive prices.

4.4.2 Solution approach via reduced state space

Our method HC calculates the target levels in period t for each exogenous state tuple I_t in the set $\mathcal{I}$; see step 3 of Algorithm 1. If the spike component of the price is assumed to be zero, the set $\mathcal{I}$ can be reduced to the set $\overline{\mathcal{I}}$ in Algorithm 1. Since $|\overline{\mathcal{I}}| = |\mathcal{I}|/|\mathcal{J}|$ where $\mathcal{J}$ is the discrete set of the spike component, the zero-spike assumption significantly reduces the computations of Algorithm 1. We thus consider a reduced state-space version of our method HC that calculates the target levels, which we denote by $Z_t^{\mathsf{HR}}(Q_t, S_t, \overline{I}_t)$ and $Y_t^{\mathsf{HR}}(S_{t+1}, \overline{I}_t)$, by executing Algorithm 1 with $\mathcal{J}$ replaced by $\mathcal{J}^{\mathsf{HR}} := \{0\}$. This variant of our method HC determines the storage action in each state with a zero spike value and a positive price via the target level $Z_t^{\mathsf{HR}}(Q_t, S_t, \overline{I}_t)$, but takes a myopic approach in other states. It also myopically determines the wind power generation action in each

Algorithm 1 Solution approach via complete state space.

$\bar{v}_T^{\mathsf{HC}}(Q_T, S_T, \bar{I}_T) \leftarrow 0$, $\forall (Q_T, S_T, \bar{I}_T) \in \mathcal{Q} \times \mathcal{S} \times \bar{\mathcal{I}}$.
for $t = T-1, \ldots, 1$ **do**
  for $I_t \in \mathcal{I}$ such that $P_t \geq 0$ **do**
    for $\nu \in \{\mathsf{pi}, \mathsf{ni}\}$ **do**
      $\left(Y_t^{(\nu),\mathsf{HC}}(I_t), Z_t^{(\nu),\mathsf{HC}}(I_t)\right) \leftarrow \arg\max_{(q_t,z_t)\in\mathcal{Q}\times\mathcal{S}} \left\{\mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HC}}(q_t, z_t, \bar{I}_{t+1})\right] + R^{(\nu)}(z_t, I_t)\right\}$.
    end for
  end for
  for $(Q_t, S_t, I_t) \in \mathcal{Q} \times \mathcal{S} \times \mathcal{I}$ **do**
    if $P_t \geq 0$ **then**
      Compute w_t^{HC} from Lemma 4.3.3.
      Compute $Z_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ from Theorem 4.3.1 with $Z_t^{(\nu)}(I_t)$ replaced by $Z_t^{(\nu),\mathsf{HC}}(I_t)$, $\forall \nu$.
      Compute s_t^{HC} from equation (4.4).
    else
      $s_t^{\mathsf{HC}} \leftarrow -\min\{C_S - S_t, C_C, \theta\tau C_T\}$ and $w_t^{\mathsf{HC}} \leftarrow 0$.
    end if
    if $P_t \geq 0$ and $S_t - s_t^{\mathsf{HC}} = Z_t^{(\nu),\mathsf{HC}}(I_t)$ for some $\nu \in \{\mathsf{pi}, \mathsf{ni}\}$ **then**
      $Y_t^{\mathsf{HC}}(S_t - s_t^{\mathsf{HC}}, I_t) \leftarrow Y_t^{(\nu),\mathsf{HC}}(I_t)$
    else
      $Y_t^{\mathsf{HC}}(S_t - s_t^{\mathsf{HC}}, I_t) \leftarrow \arg\max_{q_t\in\mathcal{Q}} \left\{\mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HC}}(q_t, S_t - s_t^{\mathsf{HC}}, \bar{I}_{t+1})\right]\right\}$.
    end if
    $q_t^{\mathsf{HC}} \leftarrow Y_t^{\mathsf{HC}}(S_t - s_t^{\mathsf{HC}}, I_t)$
    $v_t^{\mathsf{HC}}(Q_t, S_t, I_t) \leftarrow R(Q_t, I_t, s_t^{\mathsf{HC}}, w_t^{\mathsf{HC}}) + \mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HC}}(q_t^{\mathsf{HC}}, S_t - s_t^{\mathsf{HC}}, \bar{I}_{t+1})\right]$.
  end for
  for $(Q_t, S_t, \bar{I}_t) \in \mathcal{Q} \times \mathcal{S} \times \bar{\mathcal{I}}$ **do**
    $\bar{v}_t^{\mathsf{HC}}(Q_t, S_t, \bar{I}_t) \leftarrow \mathbb{E}_{J_t}\left[v_t^{\mathsf{HC}}(Q_t, S_t, I_t)\right]$.
  end for
end for

state. Finally, it determines the commitment action in each state via the target level $Y_t^{\mathsf{HR}}(S_{t+1}, \bar{I}_t)$. We call this method HR. We construct Algorithm 2 to calculate the expected total cash flow of the resulting heuristic policy with action triplets $(q_t^{\mathsf{HR}}, s_t^{\mathsf{HR}}, w_t^{\mathsf{HR}})$.

In methods HC and HR, we choose to impose the myopic actions in negative-price states since the optimal policy structure is not available under negative prices, and the myopic actions are immediately obtained and are expected to be

Algorithm 2 Cash flow calculation for the solution approach via reduced state space.

$\bar{v}_T^{\mathsf{HR}}(Q_T, S_T, \overline{I}_T) \leftarrow 0$, $\forall (Q_T, S_T, \overline{I}_T) \in \mathcal{Q} \times \mathcal{S} \times \overline{\mathcal{I}}$.
for $t = T-1, \ldots, 1$ **do**
  for $(Q_t, S_t, I_t) \in \mathcal{Q} \times \mathcal{S} \times \mathcal{I}$ **do**
    if $P_t \geq 0$ **then**
      Compute w_t^{HR} from Lemma 4.3.3.
      Compute

$$Z_t \leftarrow \begin{cases} Z_t^{\mathsf{HR}}(Q_t, S_t, \overline{I}_t) & \text{if } J_t = 0, \\ 0 & \text{if } J_t > 0, \\ C_S & \text{if } J_t < 0. \end{cases}$$

      Compute s_t^{HR} from equation (4.4) with $Z_t^{\mathsf{HC}}(Q_t, S_t, I_t)$ and w_t^{HC} replaced by Z_t and w_t^{HR}.
      Compute

$$q_t^{\mathsf{HR}} \leftarrow \begin{cases} Y_t^{\mathsf{HR}}(S_t - s_t^{\mathsf{HR}}, \overline{I}_t) & \text{if } J_t = 0, \\ Y_t^{\mathsf{HR}}(S_t - \overline{s}_t^{\mathsf{HR}}, \overline{I}_t) & \text{if } J_t > 0, \\ Y_t^{\mathsf{HR}}(S_t - \underline{s}_t^{\mathsf{HR}}, \overline{I}_t) & \text{if } J_t < 0. \end{cases}$$

    else
      $s_t^{\mathsf{HR}} \leftarrow -\min\{C_S - S_t, C_C, \theta\tau C_T\}$, $w_t^{\mathsf{HR}} \leftarrow 0$, and $q_t^{\mathsf{HR}} \leftarrow Y_t^{\mathsf{HR}}(S_t + \min\{C_S - S_t, C_C, \theta\tau C_T\}, \overline{I}_t)$.
    end if
    $v_t^{\mathsf{HR}}(Q_t, S_t, I_t) \leftarrow R(Q_t, I_t, s_t^{\mathsf{HR}}, w_t^{\mathsf{HR}}) + \mathbb{E}_{\overline{I}_{t+1}|\overline{I}_t}\left[\bar{v}_{t+1}^{\mathsf{HR}}(q_t^{\mathsf{HR}}, S_t - s_t^{\mathsf{HR}}, \overline{I}_{t+1})\right]$.
  end for
  for $(Q_t, S_t, \overline{I}_t) \in \mathcal{Q} \times \mathcal{S} \times \overline{\mathcal{I}}$ **do**
    $\bar{v}_t^{\mathsf{HR}}(Q_t, S_t, \overline{I}_t) \leftarrow \mathbb{E}_{J_t}\left[v_t^{\mathsf{HR}}(Q_t, S_t, I_t)\right]$.
  end for
end for

often optimal in negative-price states for our data-calibrated instances in Chapter 3.3. The number of positive-price states is much greater than the number of negative-price states in each of our instances. This has two implications: First, the myopic action in a negative-price state – charging the battery as much as possible – is also sensible from a forward-looking perspective since the price will very likely be positive in the next period. Second, the myopic actions in negative-price states, if suboptimal, can only slightly drain the total profit since the negative-price states have only a limited contribution to the total profit. In method HR, we choose to impose the myopic actions in nonzero-spike states since the existence of a spike in any period leads to an extremely high or extremely low price so that the forward-looking perspective is less critical and the myopic action is likely to be optimal. We provide below detailed numerical experiments that verify our intuition and justify our use of myopic actions.

4.4.3 Numerical investigation of the impact of myopic actions

Our method HC takes a myopic approach when the price is negative. Our method HR takes a myopic approach when the price is positive and the spike is nonzero, or when the price is negative. On our experimental test bed in Chapter 4.5, on average, 5.7% of the positive-price states have nonzero spikes, 91.9% of the positive-price nonzero-spike states have positive spikes, and all of the negative-price states have negative spikes. In our experiments, since the negative prices can only arise due to negative spikes, our method HR indeed imposes the myopic actions in all nonzero-spike states, while taking the forward-looking actions in all zero-spike states. We intuitively expect the myopic actions of HR to be often optimal. We performed numerical experiments to test our intuition: We define MD as the percentage of the nonzero-spike states in which the myopic action is indeed optimal according to the exact solution algorithm (i.e., the percentage of the myopic actions of HR that are indeed optimal). We also define FS-TCF as the percentage of the total cash flow that comes from the revenues collected in the nonzero-spike states in which the optimal decision is forward-looking (as opposed to myopic in HR). See Tables 4.1 and 4.2 for our results. We have found that MD is 92.55% and FS-TCF is only 3.49% on average. These results verify our intuition and show that the myopic decisions of HR, if not optimal, can only slightly drain the profits.

We also numerically examined the contribution of the nonzero-spike states to the total cash flow according to the exact solution algorithm: We define MS-TCF as the percentage of the total cash flow that comes from the revenues collected in the nonzero-spike states in which the optimal decision is myopic. We define

Table 4.1: Numerical results when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, $C_S = 500$, and $C_T = 200$.

$C_C = C_D$	NPF	τ	r	MD	MS-TCF	FS-TCF	NS-TCF
40	0	0.95	0.7	100.00%	26.99%	0.00%	73.01%
			0.8	100.00%	27.08%	0.00%	72.92%
			0.9	100.00%	27.09%	0.00%	72.91%
			1	100.00%	27.06%	0.00%	72.94%
		1	0.7	100.00%	26.99%	0.00%	73.01%
			0.8	100.00%	27.08%	0.00%	72.92%
			0.9	100.00%	27.08%	0.00%	72.92%
			1	100.00%	27.05%	0.00%	72.95%
	4.02%	0.95	0.7	99.30%	27.48%	0.04%	72.48%
			0.8	99.40%	27.45%	0.04%	72.51%
			0.9	99.49%	27.42%	0.03%	72.55%
			1	99.57%	27.36%	0.02%	72.61%
		1	0.7	99.33%	27.37%	0.04%	72.58%
			0.8	99.43%	27.35%	0.04%	72.62%
			0.9	99.51%	27.33%	0.03%	72.64%
			1	99.60%	27.27%	0.02%	72.70%
60	0	0.95	0.7	69.80%	16.55%	11.93%	71.52%
			0.8	71.53%	17.26%	11.19%	71.55%
			0.9	74.91%	18.67%	9.68%	71.65%
			1	78.14%	19.84%	8.40%	71.76%
		1	0.7	69.67%	16.88%	12.27%	70.86%
			0.8	71.40%	17.67%	11.53%	70.80%
			0.9	73.18%	18.99%	10.09%	70.92%
			1	79.83%	20.27%	8.65%	71.08%
	4.02%	0.95	0.7	81.42%	17.82%	11.93%	70.25%
			0.8	82.60%	18.41%	11.19%	70.40%
			0.9	84.48%	19.56%	9.88%	70.56%
			1	86.81%	20.86%	8.39%	70.75%
		1	0.7	81.43%	17.62%	11.99%	70.39%
			0.8	82.60%	18.18%	11.24%	70.58%
			0.9	83.77%	18.85%	10.45%	70.70%
			1	87.63%	21.30%	7.82%	70.87%

Table 4.2: Numerical results when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, NPF $= 4.07\%$, $\tau = 0.95$, and $r = 0.8$.

$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	MD	MS-TCF	FS-TCF	NS-TCF
0.6	1.1	99.37%	25.95%	0.04%	74.01%
	1.2	99.23%	25.81%	0.05%	74.14%
	1.3	99.12%	25.68%	0.06%	74.26%
	1.4	99.05%	25.61%	0.07%	74.32%
0.7	1.1	99.38%	26.50%	0.04%	73.46%
	1.2	99.24%	26.41%	0.05%	73.54%
	1.3	99.13%	26.34%	0.06%	73.60%
	1.4	99.06%	26.33%	0.07%	73.60%
0.8	1.1	99.39%	27.01%	0.04%	72.95%
	1.2	99.26%	26.96%	0.05%	72.99%
	1.3	99.17%	26.94%	0.05%	73.01%
	1.4	99.09%	26.95%	0.06%	72.99%
0.9	1.1	99.40%	27.45%	0.04%	72.51%
	1.2	99.28%	27.47%	0.04%	72.49%
	1.3	99.20%	27.49%	0.05%	72.46%
	1.4	99.12%	27.49%	0.06%	72.45%

NS-TCF as the percentage of the total cash flow that comes from the revenues collected in the zero-spike states (in which HR takes the forward-looking perspective). Note that the sum of MS-TCF, FS-TCF, and NS-TCF equals one in each instance. See again Tables 4.2 and 4.1 for our results. We observe that the percentage of the total cash flow that comes from the revenues collected in the nonzero-spike states (MS-TCF plus FS-TCF) is 27.40% on average: the zero-spike states have a much greater impact on the total cash flow than the nonzero-spike states so that the forward-looking perspective is critical in our experiments. The myopic decisions of HR (in the nonzero-spike states) alone cannot guarantee a near-optimal performance. The higher level of sophistication that HR involves via forward-looking decisions (in the zero-spike states) is clearly useful.

Finally, we extended our experiments to instances in which $C_S \in \{0, 250, 500\}$ (in MWh) to examine the impact of storage capacity. See Table 4.3 for our results. We observe that the myopic decisions are optimal in all nonzero-spike states when there is no battery in the system. The myopic decisions are indeed optimal in all states (with or without spikes) in the absence of battery, because

Table 4.3: Numerical results when $C_C = C_D = 40$, $C_T = 200$, NPF = 4.07%, $\tau = 0.95$, and $r = 0.8$.

C_S	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	MD	MS-TCF	FS-TCF	NS-TCF
0	0.6	1.1	100.00%	21.55%	0.00%	78.45%
		1.2	100.00%	21.64%	0.00%	78.36%
		1.3	100.00%	21.71%	0.00%	78.29%
		1.4	100.00%	21.74%	0.00%	78.26%
	0.7	1.1	100.00%	21.94%	0.00%	78.06%
		1.2	100.00%	22.01%	0.00%	77.99%
		1.3	100.00%	22.05%	0.00%	77.95%
		1.4	100.00%	22.08%	0.00%	77.92%
	0.8	1.1	100.00%	22.30%	0.00%	77.70%
		1.2	100.00%	22.35%	0.00%	77.65%
		1.3	100.00%	22.38%	0.00%	77.62%
		1.4	100.00%	22.40%	0.00%	77.60%
	0.9	1.1	100.00%	22.63%	0.00%	77.37%
		1.2	100.00%	22.66%	0.00%	77.34%
		1.3	100.00%	22.67%	0.00%	77.33%
		1.4	100.00%	22.68%	0.00%	77.32%
250	0.6	1.1	99.69%	26.03%	0.02%	73.95%
		1.2	99.49%	26.00%	0.03%	73.97%
		1.3	99.33%	25.81%	0.04%	74.14%
		1.4	99.20%	25.79%	0.06%	74.16%
	0.7	1.1	99.69%	26.61%	0.02%	73.37%
		1.2	99.49%	26.63%	0.03%	73.33%
		1.3	99.31%	26.54%	0.04%	73.42%
		1.4	99.19%	26.55%	0.05%	73.39%
	0.8	1.1	99.69%	27.20%	0.02%	72.78%
		1.2	99.52%	27.25%	0.03%	72.72%
		1.3	99.35%	27.21%	0.04%	72.76%
		1.4	99.22%	27.26%	0.05%	72.69%
	0.9	1.1	99.72%	27.71%	0.01%	72.27%
		1.2	99.59%	27.75%	0.02%	72.23%
		1.3	99.50%	27.75%	0.02%	72.22%
		1.4	99.27%	27.79%	0.04%	72.17%
500	0.6	1.1	99.37%	25.95%	0.04%	74.01%
		1.2	99.23%	25.81%	0.05%	74.14%
		1.3	99.12%	25.68%	0.06%	74.26%
		1.4	99.05%	25.61%	0.07%	74.32%
	0.7	1.1	99.38%	26.50%	0.04%	73.46%
		1.2	99.24%	26.41%	0.05%	73.54%
		1.3	99.13%	26.34%	0.06%	73.60%
		1.4	99.06%	26.33%	0.07%	73.60%
	0.8	1.1	99.39%	27.01%	0.04%	72.95%
		1.2	99.26%	26.96%	0.05%	72.99%
		1.3	99.17%	26.94%	0.05%	73.01%
		1.4	99.09%	26.95%	0.06%	72.99%
	0.9	1.1	99.40%	27.45%	0.04%	72.51%
		1.2	99.28%	27.47%	0.04%	72.49%
		1.3	99.20%	27.49%	0.05%	72.46%
		1.4	99.12%	27.49%	0.06%	72.45%

our commitment problem in this case can be shown to reduce to a newsvendor model where the unit costs of overstocking and understocking are determined by the imbalance pricing parameters. We observe that MD tends to decrease as the size of the battery grows: The forward-looking approach is more critical in the presence of a larger battery that enables a better arbitrage opportunity in the long run. We also note that MD tends to decrease as the imbalance penalty grows: The forward-looking approach is more critical when the intentional deviations from commitments are less profitable in the current period. However, MS-TCF may increase as MD decreases.

4.5 Numerical Results

We now conduct numerical experiments to evaluate the use of our heuristic methods HC and HR, comparing them to the standard DP algorithm (yielding the optimal solution), with respect to objective value and computation time. We consider instances in which the planning horizon spans the first week of August ($T = 168$ hours). The number of GE 1.5–77 wind turbines (N) is 100. Since each such turbine has a power capacity of 1.5 MW, the wind power plant has a power capacity of 150 MW. The energy capacity of the battery (C_S) is 500 MWh. We restrict the charging and discharging capacities (C_C and C_D) to be 40 or 60 MWh; the battery can be fully charged or discharged in about ten hours [8, 87].

The battery round-trip efficiency ($r = \theta\gamma$) varies between 0.60 and 0.70 for a nickel-based battery, 0.70 and 0.80 for a lead-acid battery, between 0.75 and 0.85 for a flow battery (such as zinc-bromine), between 0.75 and 0.90 for a sodium-based battery, and between 0.90 and 0.95 for a lithium-ion battery [88]. The earliest large-scale battery storage installations in the U.S. used nickel-based and sodium-based batteries [89]. However, since 2011, most installations have opted for lithium-ion batteries. For example, Duke Energy added 36 MW of lead-acid battery storage to its Notrees wind power facility in West Texas in 2012, but they replaced the original lead-acid batteries with better performing lithium-ion batteries in 2016 [89]. We thus restrict r to take values from the set $\{0.7, 0.8, 0.9, 1\}$

(including the perfect efficiency).

The charging and discharging efficiencies of the battery are the same and equal to the square-root of the round-trip efficiency. This assumption is common in the literature [88, 90, 91, 92]. The energy capacity of the transmission line (C_T) is 200 MWh. According to the U.S. Energy Information Administration (EIA), the electricity transmission and distribution losses are about 5% of the electricity transmitted and distributed in the U.S. [93]. Therefore, we take transmission line efficiency (τ) as 0.95 or 1. NPF is 4.02% in our time series model. We consider this original setting as well as a hypothetical one where the price is assumed to be always nonnegative. We restrict the imbalance pricing parameters to take values from the set $\{0.6, 0.7, ..., 1.4\}$; our range choice is consistent with many values observed in practice and in the literature [46, 52, 94, 95, 96].

In all instances, the discretization level ζ_a is 20 MWh, the initial storage level S_1 is the closest state to $C_S/2$, the initial commitment level Q_1 is the maximum amount of energy that can be committed to selling, and the initial exogenous state $I_1 = (\rho_1, J_1, \xi_1)$ is $(0, 0, 5)$. We evaluate the performance of our solution approaches in a total of 48 instances with the above specifications. All computations were executed on a dual 3.7 GHz Intel Xeon W-2255 CPU server with 96 GB of RAM. Tables 4.4 and 4.5 exhibit the optimality gaps and computation times of our solution approaches.

Our method HC yields the optimal solution when the battery and transmission line are perfectly efficient and the price is always positive (as shown in Theorem 4.3.1). We also observe that its optimal performance extends to instances where the price can also be negative. When the battery and transmission line are not perfectly efficient, our method HC provides near-optimal solutions with a maximum distance of 1.63% and an average distance of 0.58% from the optimal profit. Our method HC reduces the computation time of the standard DP algorithm by two orders of magnitude.

Our method HR performs only slightly worse than our method HC with respect to objective value: it yields solutions with a maximum distance of 2.31% and an

Table 4.4: Numerical results when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, $C_S = 500$, and $C_T = 200$.

$C_C = C_D$	NPF	τ	r	Optimality gaps		Computation times (minutes)		
				HC	HR	Optimal policy	HC	HR
40	0	0.95	0.7	0.92%	1.28%	205.4	3.2	0.4
			0.8	0.47%	0.84%	203.7	3.2	0.4
			0.9	0.19%	0.49%	204.3	3.2	0.4
			1	0.12%	0.40%	203.1	3.2	0.4
		1	0.7	1.06%	1.04%	202.2	3.2	0.4
			0.8	0.39%	0.59%	201.9	3.2	0.4
			0.9	0.09%	0.30%	202.7	3.2	0.4
			1	0.00%	0.24%	157.5	2.9	0.4
	4.02%	0.95	0.7	0.84%	1.03%	204.5	3.2	0.4
			0.8	0.44%	0.63%	204.5	3.2	0.4
			0.9	0.18%	0.33%	204.2	3.2	0.4
			1	0.11%	0.27%	203.1	3.3	0.4
		1	0.7	0.79%	0.87%	202.6	3.2	0.4
			0.8	0.36%	0.47%	202.9	3.2	0.4
			0.9	0.08%	0.19%	202.3	3.2	0.4
			1	0.00%	0.13%	157.1	2.9	0.4
60	0	0.95	0.7	1.36%	2.31%	318.8	4.1	0.5
			0.8	0.64%	1.30%	319.5	4.5	0.5
			0.9	0.24%	0.76%	319.3	4.7	0.5
			1	0.13%	0.62%	317.3	4.8	0.5
		1	0.7	1.47%	1.56%	319.2	4.1	0.5
			0.8	0.53%	0.86%	317.3	4.5	0.5
			0.9	0.10%	0.47%	321.4	4.7	0.5
			1	0.00%	0.40%	248.7	4.2	0.5
	4.02%	0.95	0.7	1.25%	1.76%	318.8	4.1	0.5
			0.8	0.66%	0.95%	319.4	4.5	0.5
			0.9	0.23%	0.50%	319.3	4.7	0.5
			1	0.13%	0.40%	318.0	4.8	0.5
		1	0.7	1.21%	1.28%	322.1	4.1	0.5
			0.8	0.47%	0.65%	317.5	4.5	0.5
			0.9	0.10%	0.29%	317.1	4.7	0.5
			1	0.00%	0.22%	249.1	4.2	0.5

Table 4.5: Numerical results when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, NPF $= 4.02\%$, $\tau = 0.95$, and $r = 0.8$.

$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	Optimality gaps		Computation times (minutes)		
		HC	HR	Optimal policy	HC	HR
0.6	1.1	0.96%	1.46%	204.8	3.3	0.4
	1.2	1.05%	1.28%	204.4	3.3	0.4
	1.3	1.26%	1.38%	205.0	3.3	0.4
	1.4	1.63%	1.70%	205.1	3.3	0.4
0.7	1.1	0.71%	1.10%	205.6	3.3	0.4
	1.2	0.79%	1.00%	204.7	3.3	0.4
	1.3	0.94%	1.06%	204.0	3.3	0.4
	1.4	1.20%	1.31%	204.2	3.3	0.4
0.8	1.1	0.55%	0.87%	205.9	3.2	0.4
	1.2	0.56%	0.90%	204.4	3.2	0.4
	1.3	0.67%	0.96%	204.5	3.2	0.4
	1.4	0.87%	1.12%	205.3	3.3	0.4
0.9	1.1	0.44%	0.63%	206.5	3.2	0.4
	1.2	0.43%	0.68%	204.0	3.2	0.4
	1.3	0.53%	0.79%	205.2	3.3	0.4
	1.4	0.66%	0.88%	205.8	3.2	0.4

average distance of 0.84% from the optimal profit. Our method HR, however, provides a further significant advantage in computations: the execution of our method HR takes only half a minute while that of the standard DP algorithm takes several hours. All these results highlight the high efficiency and scalability of our solution methods constructed with structural knowledge.

We observe from Table 4.4 that our methods HC and HR induce lower optimality gaps when the battery's charging/discharging capacities are smaller and the negative prices are observed more frequently. Our explanation for this result is that the desire for energy arbitrage becomes more dominant than the efficiency losses during energy transactions in this case, and thus the optimal policy structure under perfect efficiency becomes less restrictive. Consequently, our methods HC and HR that are based on this optimal policy structure perform better.

We observe from Table 4.5 that our methods HC and HR induce larger optimality gaps when the penalty for energy imbalances is higher (when $K_p^+ = K_n^-$

is lower and $K_n^+ = K_p^-$ is higher). Our explanation for this result is as follows: Since the efficiency losses restrict the use of battery for arbitrage purposes, the target storage levels are expected to be more stable under imperfect efficiency than under perfect efficiency. When the penalty for energy imbalances is higher, the target storage levels in the cases of negative and positive imbalances deviate more from each other according to our methods HC and HR, resulting in larger optimality gaps.

4.5.1 Alternative solution methods

Recall that our method HC takes a myopic approach in negative-price states, and our method HR extends this approach to nonzero-spike states. We now evaluate the use of a myopically optimal policy that adopts the optimal solution of the two-period problem in each state (even when the price is positive and the spike component is zero) as an alternative heuristic approach for our problem. Since the commitment decision in the current period can only affect the payoff in the next period, we consider the two-period problem for the myopic policy calculation. We have found that the myopic policy yields solutions with an average distance of 8.88%, a maximum distance of 11.38%, and a minimum distance of 7.26% from the optimal profit (on the same test bed). Comparing these results with our earlier results for HC and HR, we may argue that a forward-looking approach is more suitable in states with positive prices that are not dominantly large.

We also evaluate the use of fixed threshold policies as another heuristic approach for our problem. For the state-dependent threshold policy in Theorem 4.3.1, the target storage and commitment levels vary with the exogenous state variables. For the fixed threshold policy, however, the target levels remain constant within each period but vary from one period to another. We consider two variants of the fixed threshold policy, which we call F1 and F2, respectively. In F1, we calculate the target levels in period t by restricting the exogenous state tuple I_t to its prediction $\hat{I}_t$ based on the initial state tuple I_1 in the backward algorithm of our method HC. The prediction for k periods later is found by first raising the transition probability matrices in Chapter 3.3.3 to the kth power and then taking the expectations via the resulting distributions. In F2, we calculate the target levels in period t by taking the expectation over the exogenous state tuple I_t conditional on the initial state tuple I_1. Specifically, we change step 5 in the backward algorithm of our method HC to

$$\left(Y_t^{(\nu),\mathsf{HC}}, Z_t^{(\nu),\mathsf{HC}}\right) \leftarrow \underset{\left(Y_t^{(\nu)}, Z_t^{(\nu)}\right) \in \mathcal{Q} \times \mathcal{S}}{\arg\max} \left\{ \mathbb{E}_{I_t|I_1} \left[\mathbb{E}_{\bar{I}_{t+1}|\bar{I}_t} \left[\bar{v}_{t+1}^{\mathsf{HC}}(Y_t^{(\nu)}, Z_t^{(\nu)}, \bar{I}_{t+1}) \right] \right. \right.$$
$$\left. \left. + R^{(\nu)}(Z_t^{(\nu)}, I_t) \right] \right\}.$$

Like our method HR, both F1 and F2 use the target levels to determine the storage and commitment decisions in states with a zero spike value and a positive price, and take the myopic approach in all other states. We have found that F1 yields solutions with an average distance of 6.36%, a maximum distance of 14.90%, and a minimum distance of 3.50% from the optimal profit (on the same test bed). F2 yields solutions with an average distance of 7.45%, a maximum distance of 14.89%, and a minimum distance of 3.95% from the optimal profit (again on the same test bed). These results imply that ignoring the state information in target level calculation causes a significant loss of optimality, demonstrating the usefulness of state-dependent policies in our problem.

Finally, we consider a deterministic reoptimization heuristic that solves a simpler version of our problem in each period obtained by replacing the random components with their expected values conditional on the current state. The deterministic problem in state (Q_t, S_t, I_t) is given by

$$\max_{\left\{(q_\eta, s_\eta, w_\eta, S'_\eta)\right\}_{\eta \in \mathcal{T}: \eta \geq t}} \sum_{\eta \in \mathcal{T}: \eta \geq t} R(q_{\eta-1}, I_{t,\eta}, s_\eta, w_\eta)$$

where $S'_t = S_t$, $S'_{\eta+1} = S'_\eta - s_\eta$, $\forall \eta \geq t$, $q_{t-1} = Q_t$, $(q_\eta, s_\eta, w_\eta) \in \mathbb{U}(q_{\eta-1}, S'_\eta, I_{t,\eta})$, $\forall \eta \geq t$, and $I_{t,\eta} := (P_{t,\eta}, W_{t,\eta})$ is the expected exogenous state in period η conditional on the exogenous state I_t in period t. The objective is to maximize the total cash flow in periods from t through T. The objective function can be linearized when $P_{t,\eta} > 0$, $\forall \eta \geq t$, since the payoff function in each period can be shown to be the minimum of affine functions in this case. This heuristic restricts all price expectations to be nonnegative and solves a linear program when $P_t > 0$: the actions in period t are given by the optimal actions of period t obtained from the linear program. It takes a myopic approach when $P_t \leq 0$: the actions in period t are given by the optimal actions of the two-period problem with states $I_{t,t}$ and $I_{t,t+1}$. This heuristic yields solutions with an average distance of 3.12%, a maximum distance of 4.41%, and a minimum distance of 2.10% from the optimal profit (on the same test bed), performing worse than HC and HR by more than one percent on average. The precise modeling of uncertainties as in HC and HR thus seems useful in our problem.

4.6 Concluding Remarks

In this chapter, we establish the multi-dimensional structural properties of the optimal profit function for the energy commitment, generation and storage problem of a wind power producer. We prove the optimality of a state-dependent threshold policy for the storage and commitment decisions under the assumptions of perfectly efficient battery and transmission line and positive electricity prices. Leveraging this policy structure, we construct two heuristic solution methods (HC and HR) for solving the more general problem in which the battery and transmission line can be imperfectly efficient and the price can also be negative. Using data-calibrated time series models of the wind speed and electricity price, we numerically test the performance of these solution methods in the general problem.

Our method HC yields near-optimal solutions with an average distance of 0.58% from the optimal profit in our experiments. It has an average solution time of 3.6 minutes, while the standard DP algorithm has an average solution time of almost 4 hours. Moreover, our method HR provides high-quality solutions (only slightly worse than those of our method HC) with an average distance of 0.84% from the optimal profit within half a minute. These results in the general problem imply that our methods HC and HR enable large computational savings with little loss of optimality. Our experiments have also revealed the poor performance of simpler alternative solution methods (purely myopic solution approach, fixed threshold policies, and deterministic reoptimization heuristic) with respect to objective value.

In the next chapter, we characterize the optimal policy structure, with a different proof technique, in a more general case where the battery and transmission line need not be perfectly efficient.

Chapter 5

Commitment and Storage Problem of Wind Power Producers: Optimal Policy Characterization in the Presence of Efficiency Losses

5.1 Introduction

In this chapter, we characterize the optimal policy structure for the energy commitment, generation and storage problem in Chapter 4 when the battery and transmission line can be imperfectly efficient. We first show that the optimal profit function is jointly concave in the two endogenous state variables without any reliance on perfect efficiency. We then partition the state space of the problem into several disjoint domains that correspond to the optimal decisions of 'positive imbalance' and 'negative imbalance' as well as to the optimal decisions of 'charge and purchase,' 'charge and sell,' and 'discharge and sell,' respectively: it is optimal to bring the storage and commitment levels to a different state-dependent threshold pair in each domain.

The remainder of the chapter is organized as follows. Chapter 5.2 establishes the optimal policy structure and discusses its implications. Chapter 5.3 concludes. The proof of Theorem 5.2.1 is contained in Appendix C.

5.2 Characterization of the Optimal Policy

5.2.1 Structural results

We assume that $\mathbb{E}_{P_\kappa|I_t}[|P_\kappa|] < \infty, \forall \kappa \geq t$. Note that Lemmas 4.3.1-4.3.3 of Chapter 4 still hold when the battery and transmission line are imperfectly efficient.

Proposition 5.2.1. *Under Assumption 4.3.1, $v_t^*(Q_t, S_t, I_t)$ is jointly concave in (Q_t, S_t), $\forall t \in \mathcal{T}$.*

Proof. Note that $v_T^*(\cdot, \cdot, I_T)$ is jointly concave, $\forall I_T$. Pick an arbitrary $t < T$ and fix I_t. Assuming $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ is jointly concave, $\forall I_{t+1}$, we will prove $v_t^*(\cdot, \cdot, I_t)$ is also jointly concave. First, taking a similar path to that in Zhou et al. [8], we transform our problem into an equivalent one with linear constraints. We define the following decision variables:

- s_t^{CG}: The amount of energy charged into the battery from the wind energy generated;
- s_t^{CP}: The amount of energy charged into the battery from the energy purchased;
- s_t^{D}: The amount of energy discharged from the battery;
- w_t^{C}: The amount of wind energy generated and charged into the battery; and
- w_t^{S}: The amount of wind energy generated and sold in the market.

We define $\Gamma(Q_t, S_t, I_t)$ as the set of decision variable tuples $(q_t, s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S) \in \mathbb{R} \times \mathbb{R}_+^5$ that satisfy

$$
\begin{aligned}
& -\min\{C_C/(\theta\tau), C_T\} \le q_t \le \tau C_T, && (5.1)\\
& w_t^S + \gamma s_t^D \le C_T, && (5.2)\\
& s_t^{CP}/(\theta\tau) \le C_T, && (5.3)\\
& s_t^{CG} = \theta w_t^C, && (5.4)\\
& w_t^C + w_t^S \le f(W_t), && (5.5)\\
& s_t^D \le \min\{S_t, C_D\}, && (5.6)\\
& s_t^{CG} + s_t^{CP} \le \min\{C_S - S_t, C_C\}, && (5.7)\\
& s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S \ge 0. && (5.8)
\end{aligned}
$$

Constraint (5.1) uses Lemma 4.3.1 to set bounds on the commitment amount without loss of optimality. Constraints (5.2) and (5.3) are the transmission capacity constraints for selling and purchasing energy, respectively. Constraint (5.4) relates the decision w_t^C to the decision s_t^{CG}. Constraint (5.5) says the wind energy generated is bounded by the available wind potential. Constraints (5.6) and (5.7) are the battery capacity constraints for discharging and charging energy, respectively. Finally, constraint (5.8) says all decision variables except the commitment amount are nonnegative. We now consider the following problem:

$$
\begin{aligned}
\max_{(q_t, s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S) \in \Gamma(Q_t, S_t, I_t)} & \Big\{ R(Q_t, I_t, (w_t^S + \gamma s_t^D)\tau - s_t^{CP}/(\theta\tau)) && (5.9)\\
& + \mathbb{E}\Big[v_{t+1}^*(q_t, S_t + s_t^{CG} + s_t^{CP} - s_t^D, I_{t+1})\Big]\Big\}
\end{aligned}
$$

where

$$
R(Q_t, I_t, e) = \begin{cases} Q_t P_t + K_p^+ P_t (e - Q_t) & \text{if } Q_t < e, \\ Q_t P_t - K_n^+ P_t (Q_t - e) & \text{if } Q_t \ge e. \end{cases}
$$

We show that the above problem is equivalent to ours by constructing an optimal solution to (5.9) that satisfies $s_t^D = 0$ or $s_t^{CG} + s_t^{CP} = 0$. Let

$(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^{D}, \hat{w}^{C}, \hat{w}^{S}) \in \Gamma(Q_t, S_t, I_t)$ denote a feasible solution to (5.9). We consider the following two cases:

(1) Suppose that $\hat{s}^D > 0$ and $\hat{s}^{CP} > 0$: We define $\Delta_1 = \min\{\hat{s}^{CP}, \hat{s}^D\} > 0$. Note that $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP} - \Delta_1, \hat{s}^D - \Delta_1, \hat{w}^C, \hat{w}^S) \in \Gamma(Q_t, S_t, I_t)$ and $(\hat{w}^S + \gamma(\hat{s}^D - \Delta_1))\tau - (\hat{s}^{CP} - \Delta_1)/(\theta\tau) > (\hat{w}^S + \gamma\hat{s}^D)\tau - \hat{s}^{CP}/(\theta\tau)$. Since $R(Q_t, I_t, \cdot)$ is an increasing function, $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP} - \Delta_1, \hat{s}^D - \Delta_1, \hat{w}^C, \hat{w}^S)$ yields a larger objective value to (5.9) than $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^D, \hat{w}^C, \hat{w}^S)$. Thus, $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^D, \hat{w}^C, \hat{w}^S)$ cannot be an optimal solution to (5.9) if $\hat{s}^D > 0$ and $\hat{s}^{CP} > 0$.

(2) Suppose that $\hat{s}^D > 0$, $\hat{s}^{CP} = 0$, and $\hat{s}^{CG} > 0$: We define $\Delta_2 = \min\{\hat{s}^{CG}, \hat{s}^D\} > 0$. Note that $(\hat{q}, \hat{s}^{CG} - \Delta_2, \hat{s}^{CP}, \hat{s}^D - \Delta_2, \hat{w}^C - \Delta_2/\theta, \hat{w}^S + \gamma\Delta_2) \in \Gamma(Q_t, S_t, I_t)$ and $(\hat{w}^S + \gamma\Delta_2 + \gamma(\hat{s}^D - \Delta_2))\tau - \hat{s}^{CP}/(\theta\tau) = (\hat{w}^S + \gamma\hat{s}^D)\tau - \hat{s}^{CP}/(\theta\tau)$. Thus, if $(\hat{q}, \hat{s}^{CG}, \hat{s}^{CP}, \hat{s}^D, \hat{w}^C, \hat{w}^S)$ is an optimal solution to (5.9), then $(\hat{q}, \hat{s}^{CG} - \Delta_2, \hat{s}^{CP}, \hat{s}^D - \Delta_2, \hat{w}^C - \Delta_2/\theta, \hat{w}^S + \gamma\Delta_2)$ is also optimal with $\hat{s}^{CP} + \hat{s}^{CG} - \Delta_2 = 0$ or $\hat{s}^D - \Delta_2 = 0$ depending on the value of Δ_2.

Thus $v_t^*(Q_t, S_t, I_t)$ equals the optimal objective value of (5.9). Next, we show that $|v_t^*(Q_t, S_t, I_t)| < \infty$. Since $-C_T \le Q_t \le \tau C_T$ from Lemma 4.3.1 and $-C_T \le E(s_t, w_t) \le \tau C_T$, note that $|R(Q_t, I_t, s_t, w_t)| \le |P_t| C_T$. Hence, $|v_t^*(Q_t, S_t, I_t)| \le \sum_{\kappa=t}^{T-1} |\mathbb{E}_{P_\kappa|I_t}[P_\kappa]| C_T \le \sum_{\kappa=t}^{T-1} \mathbb{E}_{P_\kappa|I_t}[|P_\kappa|] C_T < \infty$ since $\mathbb{E}_{P_\kappa|I_t}[|P_\kappa|] < \infty, \forall \kappa \ge t$.

Finally, we define $\mathcal{C} := \{(Q_t, S_t, q_t, s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S) \mid (Q_t, S_t) \in \Theta, (q_t, s_t^{CG}, s_t^{CP}, s_t^D, w_t^C, w_t^S) \in \Gamma(Q_t, S_t, I_t)\}$ where $\Theta := \{(Q_t, S_t) \mid -\min\{C_C/(\theta\tau), C_T\} \le Q_t \le \tau C_T, 0 \le S_t \le C_S, (S_t - C_S)/(\theta\tau) \le Q_t\}$. Note that $\mathcal{C}$ is a convex set since Θ and $\Gamma(Q_t, S_t, I_t)$ are polyhedral and thus convex sets. Also, note that the objective function of problem (5.9) is a concave function on $\mathcal{C}$ since $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ is jointly concave and $R(Q_t, I_t, \cdot)$ is concave. Since $v_t^*(Q_t, S_t, I_t) < \infty$, Theorem A.4 in Porteus [97] implies that $v_t^*(Q_t, S_t, I_t)$ is a concave function on Θ. $\square$

5.2.2 Optimal commitment and storage policy

When the battery and transmission line need not be perfectly efficient, the optimal policy structure again involves state-dependent target levels for the commitment and storage decisions.

We need to consider three types of sales/purchase decisions in any period t: (i) A certain amount of energy is generated by discharging the battery ($s_t > 0$). The resulting energy together with the generated wind energy is sold in the market. We label this type of decision DS (the initials of 'discharge' and 'sell'). (ii) A certain amount of energy is stored by charging the battery ($s_t < 0$). If the generated wind energy is sufficient to charge the battery ($s_t/\theta \geq -w_t$), the excess wind energy is sold in the market. We label this type of decision CS (the initials of 'charge' and 'sell'). (iii) If the generated wind energy is not sufficient to charge the battery ($s_t/\theta < -w_t$), the required additional energy is purchased from the market. We label this type of decision CP (the initials of 'charge' and 'purchase'). We also need to consider two types of commitment decisions: (i) 'positive imbalance' (pi) and (ii) 'negative imbalance' (ni).

Hence, we need to consider a total of six decision types to formulate the state-dependent target levels: charge and purchase leading to positive imbalance (piCP) and negative imbalance (niCP), charge and sell leading to positive imbalance (piCS) and negative imbalance (niCS), and discharge and sell leading to positive imbalance (piDS) and negative imbalance (niDS). For $\nu \in \{\mathsf{niCP}, \mathsf{niCS}, \mathsf{niDS}, \mathsf{piCP}, \mathsf{piCS}, \mathsf{piDS}\}$,

$$\left(Y_t^{(\nu)}(I_t), Z_t^{(\nu)}(I_t)\right) := \underset{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[0,C_S]}{\arg\max} \Big\{\mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right] + R^{(\nu)}(z_t, I_t)\Big\}$$

where

$$R^{(\nu)}(z_t, I_t) = \begin{cases} -K_n^+ P_t z_t/(\theta\tau) & \text{if } \nu = \mathsf{niCP}, \\ -K_n^+ P_t \tau z_t/\theta & \text{if } \nu = \mathsf{niCS}, \\ -K_n^+ P_t \gamma\tau z_t & \text{if } \nu = \mathsf{niDS}, \\ -K_p^+ P_t z_t/(\theta\tau) & \text{if } \nu = \mathsf{piCP}, \\ -K_p^+ P_t \tau z_t/\theta & \text{if } \nu = \mathsf{piCS}, \\ -K_p^+ P_t \gamma\tau z_t & \text{if } \nu = \mathsf{piDS}, \end{cases}$$

and $z_t := S_t - s_t$ is the storage level at the end of period t if the action s_t is taken in period t. Since $Z_t^{(\nu)}(I_t)$ may be inaccessible when the capacity limits are taken into account, the optimal storage level at the end of period t may be different from $Z_t^{(\nu)}(I_t)$ so that $Y_t^{(\nu)}(I_t)$ may no longer be optimal at this storage level. Therefore, we also introduce the optimal state-dependent target level for the commitment decision after the storage decision is made in the constrained problem:

$$Y_t(S_{t+1}, I_t) := \underset{q_t \in [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T]}{\arg\max} \left\{ \mathbb{E}\left[v_{t+1}^*(q_t, S_t, I_{t+1}) \right] \right\}.$$

Note that $Y_t(Z_t^{(\nu)}, I_t) = Y_t^{(\nu)}(I_t)$ for each $\nu \in \{\mathsf{niCP}, \mathsf{niCS}, \mathsf{niDS}, \mathsf{piCP}, \mathsf{piCS}, \mathsf{piDS}\}$. Finally, we introduce an auxiliary state-dependent target level for the storage decision, which we denote by $Z_t(Q_t, S_t, I_t)$, that can take the values of $Z_t^{(\mathsf{piCP})}(I_t)$, $Z_t^{(\mathsf{piCS})}(I_t)$, $Z_t^{(\mathsf{piDS})}(I_t)$, $Z_t^{(\mathsf{niCP})}(I_t)$, $Z_t^{(\mathsf{niCS})}(I_t)$, and $Z_t^{(\mathsf{niDS})}(I_t)$ depending on the system state.

Let Ω denote the domain of (Q_t, S_t, W_t), i.e., $\Omega := [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T] \times [0, C_S] \times [0, \infty)$. We define the following disjoint subdomains of Ω:

$$
\begin{aligned}
\Psi_0 &:= \{(Q_t, S_t, W_t) \in \Omega : f(W_t) \geq C_T + \min\{C_S - S_t, C_C\}/\theta\}, \\
\Psi_1^+ &:= \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T, \\
&\qquad f(W_t) \geq Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta,\ Q_t \geq 0\}, \\
\Psi_2^+ &:= \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T, \\
&\qquad Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta > f(W_t),\ Q_t \geq 0\}, \\
\Psi_3^+ &:= \{(Q_t, S_t, W_t) \in \Omega : C_T > f(W_t),\ f(W_t) \geq Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta, \\
&\qquad Q_t \geq 0\}, \\
\Psi_4^+ &:= \{(Q_t, S_t, W_t) \in \Omega : C_T > f(W_t),\ Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \\
&\qquad \geq Q_t/\tau,\ Q_t \geq 0\}, \\
\Psi_5 &:= \{(Q_t, S_t, W_t) \in \Omega : C_T > f(W_t),\ Q_t/\tau > f(W_t)\}, \\
\Psi_1^- &:= \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T, \\
&\qquad f(W_t) \geq \tau Q_t + \min\{C_S - S_t, C_C\}/\theta,\ Q_t < 0\}, \\
\Psi_2^- &:= \{(Q_t, S_t, W_t) \in \Omega : C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T, \\
&\qquad \tau Q_t + \min\{C_S - S_t, C_C\}/\theta > f(W_t),\ Q_t < 0\}, \\
\Psi_3^- &:= \{(Q_t, S_t, W_t) \in \Omega : C_T > f(W_t),\ f(W_t) \geq \tau Q_t + \min\{C_S - S_t, C_C\}/\theta, \\
&\qquad Q_t < 0\}, \text{ and} \\
\Psi_4^- &:= \{(Q_t, S_t, W_t) \in \Omega : C_T > f(W_t),\ \tau Q_t + \min\{C_S - S_t, C_C\}/\theta > f(W_t), \\
&\qquad Q_t < 0\}.
\end{aligned}
$$

Theorem 5.2.1. *Under Assumption 4.3.1, the optimal policy structure follows a state-dependent threshold policy with state-dependent target levels for the storage and commitment decisions. In any period t, if $(Q_t, S_t, W_t) \in \Psi_0$, it is optimal to charge the battery to get as close to C_S as possible.*

If $(Q_t, S_t, W_t) \in \Psi_1^+$, it is optimal to

- *charge to get as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible if $S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$, and*
- *keep unchanged if $Z_t^{(\mathsf{piCS})}(I_t) < S_t$.*

If $(Q_t, S_t, W_t) \in \Psi_1^-$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{piCP})}(I_t)$ *as possible if* $S_t \leq Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t)$,
- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$, *and*
- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_2^+$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{niCP})}(I_t)$ *as possible if* $S_t \leq Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t)$,
- *charge to get as close to* $Z_t^{(\mathsf{niCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t) < S_t \leq Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau)$,
- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau) < S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$, *and*
- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_2^-$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{niCP})}(I_t)$ *as possible if* $S_t \leq Z_t^{(\mathsf{niCP})}(I_t) - \theta(f(W_t) - \tau Q_t)$,
- *charge to get as close to* $Z_t^{(\mathsf{piCP})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCP})}(I_t) - \theta(f(W_t) - \tau Q_t) < S_t \leq Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t)$,
- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$, *and*
- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_3^+$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $S_t \le Z_t^{(\mathsf{piCS})}(I_t)$,
- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t \le Z_t^{(\mathsf{piDS})}(I_t)$, *and*
- *discharge to get as close to* $Z_t^{(\mathsf{piDS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piDS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_3^-$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{piCP})}(I_t)$ *as possible if* $S_t \le Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t)$,
- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \le Z_t^{(\mathsf{piCS})}(I_t)$,
- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t \le Z_t^{(\mathsf{piDS})}(I_t)$, *and*
- *discharge to get as close to* $Z_t^{(\mathsf{piDS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piDS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_4^+$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{niCP})}(I_t)$ *as possible if* $S_t \le Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t)$,
- *charge to get as close to* $Z_t^{(\mathsf{niCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t) < S_t \le Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau)$,
- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau) < S_t \le Z_t^{(\mathsf{piCS})}(I_t)$,
- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t \le Z_t^{(\mathsf{piDS})}(I_t)$, *and*
- *discharge to get as close to* $Z_t^{(\mathsf{piDS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piDS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_4^-$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{niCP})}(I_t)$ *as possible if* $S_t \le Z_t^{(\mathsf{niCP})}(I_t) - \theta(f(W_t) - \tau Q_t)$,

- *charge to get as close to* $Z_t^{(\mathsf{piCP})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCP})}(I_t) - \theta(f(W_t) - \tau Q_t) < S_t \leq Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t)$,

- *charge to get as close to* $Z_t^{(\mathsf{piCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$,

- *keep unchanged if* $Z_t^{(\mathsf{piCS})}(I_t) < S_t \leq Z_t^{(\mathsf{piDS})}$, *and*

- *discharge to get as close to* $Z_t^{(\mathsf{piDS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piDS})}(I_t) < S_t$.

If $(Q_t, S_t, W_t) \in \Psi_5$, *it is optimal to*

- *charge to get as close to* $Z_t^{(\mathsf{niCP})}(I_t)$ *as possible if* $S_t \leq Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t)$,

- *charge to get as close to* $Z_t^{(\mathsf{niCS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t) < S_t \leq Z_t^{(\mathsf{niCS})}(I_t)$,

- *keep unchanged if* $Z_t^{(\mathsf{niCS})}(I_t) < S_t \leq Z_t^{(\mathsf{niDS})}(I_t)$,

- *discharge to get as close to* $Z_t^{(\mathsf{niDS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{niDS})}(I_t) < S_t \leq Z_t^{(\mathsf{piDS})}(I_t) + (Q_t/\tau - f(W_t))/\gamma$, *and*

- *discharge to get as close to* $Z_t^{(\mathsf{piDS})}(I_t)$ *as possible if* $Z_t^{(\mathsf{piDS})}(I_t) + (Q_t/\tau - f(W_t))/\gamma < S_t$.

The optimal commitment action is $q_t^*(Q_t, S_t, I_t) = Y_t(S_t - s)$ *where* s *is the optimal amount of energy generated or stored. Furthermore, the optimal state-dependent target storage levels obey (i)* $Z_t^{(\mathsf{niCP})}(I_t) \leq Z_t^{(\mathsf{niCS})}(I_t) \leq Z_t^{(\mathsf{niDS})}(I_t)$, *(ii)* $Z_t^{(\mathsf{piCP})}(I_t) \leq Z_t^{(\mathsf{piCS})}(I_t) \leq Z_t^{(\mathsf{piDS})}(I_t)$, *(iii)* $Z_t^{(\mathsf{niCP})}(I_t) \leq Z_t^{(\mathsf{piCP})}(I_t)$, *(iv)* $Z_t^{(\mathsf{niCS})}(I_t) \leq Z_t^{(\mathsf{piCS})}(I_t)$, *and (v)* $Z_t^{(\mathsf{niDS})}(I_t) \leq Z_t^{(\mathsf{piDS})}(I_t)$.

Proof. See Appendix C. □

We discuss below the implications of Theorem 5.2.1:

- Suppose that the maximum amount of renewable energy that can be generated is greater than the maximum total amount of energy that can be used for selling and storing (i.e., $(Q_t, S_t, W_t) \in \Psi_0$). Then it is optimal to increase the storage level as much as possible.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the transmission line capacity and the maximum total amount of energy that can be used for meeting the commitment (for selling energy to the market) and storing, but less than the maximum total amount of energy that can be used for selling and storing (i.e., $(Q_t, S_t, W_t) \in \Psi_1^+$). If the storage level is too low (i.e., $S_t \le Z_t^{(\mathsf{piCS})}(I_t)$), it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible. If the storage level is too high (i.e., if $Z_t^{(\mathsf{piCS})}(I_t) < S_t$), it is optimal to keep the storage level unchanged.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the transmission line capacity and the maximum total amount of energy that can be used for meeting the commitment (for purchasing energy from the market) and storing, but less than the maximum total amount of energy that can be used for selling and storing (i.e., $(Q_t, S_t, W_t) \in \Psi_1^-$). If the storage level is too low (i.e., $S_t \le Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t)$), it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCP})}(I_t)$ as possible. If the storage level is in a medium range (i.e., $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \le Z_t^{(\mathsf{piCS})}(I_t)$), it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible. If the storage level is too high (i.e., $Z_t^{(\mathsf{piCS})}(I_t) < S_t$), it is optimal to keep the storage level unchanged.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the transmission line capacity but less than the maximum total amount of energy that can be used for meeting the commitment (for selling energy to the market) and storing, as well as the maximum total amount of energy that can be used for selling and storing (i.e., $(Q_t, S_t, W_t) \in \Psi_2^+$). If the storage level is too low (i.e., $S_t \le Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t)$), it is optimal to bring the storage level as close to $Z_t^{(\mathsf{niCP})}(I_t)$ as possible. If the storage level is in a medium range (i.e.,

$Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t) < S_t \le Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{niCS})}(I_t)$ as possible. If the storage level is high enough (i.e., $Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau) < S_t \le Z_t^{(\mathsf{piCS})}(I_t))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible. If the storage level is too high (i.e., $Z_t^{(\mathsf{piCS})}(I_t) < S_t$), it is optimal to keep the storage level unchanged.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the transmission line capacity but less than the maximum total amount of energy that can be used for meeting the commitment (for purchasing energy from the market) and storing, as well as the maximum total amount of energy that can be used for selling and storing (i.e., $(Q_t, S_t, W_t) \in \Psi_2^-$). If the storage level is too low (i.e., $S_t \le Z_t^{(\mathsf{niCP})}(I_t) - \theta(f(W_t) - \tau Q_t))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{niCP})}(I_t)$ as possible. If the storage level is in medium range (i.e., $Z_t^{(\mathsf{niCP})}(I_t) - \theta(f(W_t) - \tau Q_t) < S_t \le Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCP})}(I_t)$ as possible. If the storage level is high enough (i.e., $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \le Z_t^{(\mathsf{piCS})}(I_t))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible. If the storage level is too high (i.e., $Z_t^{(\mathsf{piCS})}(I_t) < S_t$), it is optimal to keep the storage level unchanged.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the maximum total amount of energy that can be used for meeting the commitment (for selling energy to the market) and storing but less than the transmission line capacity (i.e., $(Q_t, S_t, W_t) \in \Psi_3^+$). If the storage level is too low (i.e., $S_t \le Z_t^{(\mathsf{piCS})}(I_t))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible. If the storage level is in a medium range (i.e., $Z_t^{(\mathsf{piCS})}(I_t) < S_t \le Z_t^{(\mathsf{piDS})}(I_t))$, it is optimal to keep the storage level unchanged. If the storage level is too high (i.e., $S_t > Z_t^{(\mathsf{piDS})}(I_t))$, it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piDS})}(I_t)$ as possible.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the maximum total amount of energy that can be used for meeting the commitment (for purchasing energy from the market) and storing but less than the transmission line capacity (i.e., $(Q_t, S_t, W_t) \in \Psi_3^-$). If the storage level is too low (i.e., $S_t \leq Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t)$), it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCP})}(I_t)$ as possible. If the storage level is in medium range (i.e., $Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$), it is optimal to bring the storage level as close to $Z_t^{(\mathsf{piCS})}(I_t)$ as possible. The optimal actions are similar to those in the scenario $(Q_t, S_t, W_t) \in \Psi_3^+$ if the storage level is high enough or too high, respectively.

- Suppose that the maximum amount of renewable energy that can be generated is greater than the amount of energy that can be used for meeting the commitment but less than the maximum total amount of energy that can be used for meeting the commitment (for selling energy to the market) and storing, as well as the transmission line capacity (i.e., if $(Q_t, S_t, W_t) \in \Psi_4^+$). The optimal actions are similar to those in the scenario $(Q_t, S_t, W_t) \in \Psi_2^+$ if the storage level is too low, in the medium range, or not high enough. The optimal actions are similar to those in the scenario $(Q_t, S_t, W_t) \in \Psi_3^+$ if the storage level is high enough or too high.

- Suppose that the maximum amount of renewable energy that can be generated is less than the maximum total amount of energy that can be used for meeting the commitment (for purchasing energy from the market) and storing, as well as the transmission line capacity (i.e., $(Q_t, S_t, W_t) \in \Psi_4^-$). The optimal actions are similar to those in the scenario $(Q_t, S_t, W_t) \in \Psi_2^-$ if the storage level is too low, in the medium range, or not high enough. The optimal actions are similar to those in the scenario $(Q_t, S_t, W_t) \in \Psi_3^-$ if the storage level is high enough or too high.

- Suppose that the maximum amount of energy that can be generated is less than the transmission line capacity and the amount of energy that can be used for meeting the commitment (i.e., $(Q_t, S_t, W_t) \in \Psi_5$), the optimal actions are similar to those in the scenarios $(Q_t, S_t, W_t) \in \Psi_2^+$ and $(Q_t, S_t, W_t) \in \Psi_4^+$ if the storage level is too low or in the medium range. If the storage level is not high enough (i.e., $Z_t^{(\mathsf{piCS})}(I_t) < S_t \leq Z_t^{(\mathsf{niDS})}(I_t)$), it is optimal to keep the storage level unchanged. If the storage level is high enough (i.e., $Z_t^{(\mathsf{niDS})}(I_t) < S_t \leq Z_t^{(\mathsf{piDS})}(I_t) + (Q_t/\tau - f(W_t))/\gamma$), it is optimal to bring the storage level down as close to $Z_t^{(\mathsf{niDS})}(I_t)$ as possible. If the storage level is too high (i.e., $Z_t^{(\mathsf{piDS})}(I_t) + (Q_t/\tau - f(W_t))/\gamma < S_t$), it is optimal to bring the storage level down as close to $Z_t^{(\mathsf{piDS})}(I_t)$ as possible.

Theorem 5.2.1 also implies that when the storage level is too low, it is optimal to charge the battery and purchase energy from the market. In this case, the operator chooses to bring the storage level up to $Z_t^{(\mathsf{niCP})}(I_t)$ by purchasing energy more than the commitment level if the energy storage is critical, and bring it up to $Z_t^{(\mathsf{piCP})}(I_t)$ by purchasing energy no more than the commitment level otherwise. When the storage level is in the medium range, the operator gains the flexibility to charge the battery and sell energy to the market. In this case, the operator chooses to bring the storage level up to $Z_t^{(\mathsf{niCS})}(I_t)$ by selling energy less than the commitment level if the energy storage is critical, and bring it up to $Z_t^{(\mathsf{piCS})}(I_t)$ by selling energy no less than the commitment level otherwise. When the storage level is high enough, it is optimal to keep the storage level unchanged. However, when the storage level is too high, it is optimal to discharge the battery and sell energy to the market. In this case, the operator chooses to bring the storage level down to $Z_t^{(\mathsf{niDS})}(I_t)$ by selling energy less than the commitment level if the energy storage is critical, and bring it down to $Z_t^{(\mathsf{piDS})}(I_t)$ by selling energy no less than the commitment level otherwise.

The inefficiencies of the battery lead to different marginal payoffs in the energy generation and storage modes. Therefore, $Z_t^{(\mathsf{niCS})}(I_t) \leq Z_t^{(\mathsf{niDS})}(I_t)$ and $Z_t^{(\mathsf{piCS})}(I_t) \leq Z_t^{(\mathsf{piDS})}(I_t)$. Note that $Z_t^{(\mathsf{niCS})}(I_t) = Z_t^{(\mathsf{niDS})}(I_t)$ and $Z_t^{(\mathsf{piCS})}(I_t) = Z_t^{(\mathsf{piDS})}(I_t)$ if $\gamma\theta = 1$. The inefficiency of the transmission line leads to different marginal payoffs in the energy selling and purchasing modes. Therefore, $Z_t^{(\mathsf{niCP})}(I_t) \leq Z_t^{(\mathsf{niCS})}(I_t)$ and $Z_t^{(\mathsf{piCP})}(I_t) \leq Z_t^{(\mathsf{piCS})}(I_t)$. Note that $Z_t^{(\mathsf{niCP})}(I_t) = Z_t^{(\mathsf{niCS})}(I_t)$ and $Z_t^{(\mathsf{piCP})}(I_t) = Z_t^{(\mathsf{piCS})}(I_t)$ if $\tau = 1$.

5.3 Concluding Remarks

In this chapter, we have characterized the structure of the optimal commitment and storage policy for a wind power producer who utilizes a battery that can be imperfectly efficient and may experience transmission losses when selling energy to or purchasing energy from the market. We partition the state space of the problem into several disjoint domains, depending on the wind energy availability, the type and level of the commitment, and the storage level of the battery. Each subdomain corresponds to a set of target storage levels, from which one particular target level is chosen depending on the battery's storage level. We also show that the optimal target levels for the storage decisions are higher in the case of 'discharge and sell' decisions and lower in the case of 'charge and purchase' decisions. Future extensions of this study may consider other storage technologies, such as pumped hydro energy storage and compressed air energy storage. The theory of ADP may be usefully employed in this research direction in order to handle potentially larger state and/or action spaces.

Chapter 6

Commitment and Storage Problem of Wind Power Producers: The Impact of Commitment Decisions

6.1 Introduction

In this chapter, we consider an alternative market setting that allows for real-time trading without making any advance commitment. This type of energy trading can be referred to as merchant agreements (a special type of the power purchase agreements mentioned in Chapter 2) and is widely used as a market setting in the literature; see, for example, [8, 57, 58, 61, 62, 64] and [88]. Under this merchant agreement type, the wind power producer sells her electricity output directly into the market at the prevailing market price [14]. The producer faces the risk of fluctuating electricity prices, but she also has the potential to earn higher profits if the market conditions are favorable and/or if she has an energy storage unit that can be used as a financial hedging instrument (e.g., she can choose to hold back some of her output to sell at a later time if the market conditions are expected to be more favorable). Under this merchant agreement type, the wind power producer is not penalized for not supplying energy in real-time, because these types of agreements typically do not include capacity or availability guarantees and are not based on advance commitments. Making commitments in an electricity market, on the other hand, carries the risk of dispatch uncertainty in real-time. When the wind power producer makes a commitment in an electricity market, she may be unable to meet her contracted energy output requirement in real-time, resulting in financial penalties and lost revenues (i.e., specifically, when she participates in a single market with no option to fix any deviation from her commitments or benefit from price discrepancies that can occur between markets).

This chapter constructs a theoretical upper bound on the possible cost of advance commitments by comparing the total cash flows in our original problem setting and in the above-mentioned alternative problem setting. The key difference between the two problem settings is that the producer in the former setting makes commitment decisions under uncertainty in the electricity price and wind speed of the upcoming time period and is penalized for any possible imbalances that occur in real-time, whereas the producer in the latter setting sells or purchases energy after observing the market price and wind speed in real-time. We also investigate the impact of commitment decisions on the producers' storage and generation decisions.

The remainder of the chapter is organized as follows. Chapter 6.2 compares the total cash flows in the original and alternative problem settings and offers a theoretical bound on the difference in cash flows. Chapter 6.3 presents numerical results and discusses the impact of commitment decisions. Chapter 6.4 concludes. Detailed numerical results are contained in Appendix D.

6.2 Problem Formulation

In spot markets, like the one in the original problem setting (a spot market with hour-ahead commitments and hourly settlements), the market participants are supposed to make advance commitment decisions in the absence of precise knowledge about the future electricity price and renewable energy potential. The profit of a power producer in such markets is potentially lower than in markets that accept dispatch or purchase amounts determined in real-time without any commitment decisions. Recall that the payoff function for the original problem setting is:

$$R(Q_t, I_t, s_t, w_t) = \begin{cases} Q_tP_t + K_p^+P_t(E(s_t,w_t) - Q_t) & \text{if } P_t \geq 0 \text{ and} \\ & Q_t < E(s_t,w_t) \quad (\mathsf{pi}), \\ Q_tP_t - K_n^+P_t(Q_t - E(s_t,w_t)) & \text{if } P_t \geq 0 \text{ and} \\ & Q_t \geq E(s_t,w_t) \quad (\mathsf{ni}), \\ Q_tP_t + K_p^-P_t(E(s_t,w_t) - Q_t) & \text{if } P_t < 0 \text{ and} \\ & Q_t < E(s_t,w_t) \quad (\mathsf{pi}), \\ Q_tP_t - K_n^-P_t(Q_t - E(s_t,w_t)) & \text{if } P_t < 0 \text{ and} \\ & Q_t \geq E(s_t,w_t) \quad (\mathsf{ni}), \end{cases}$$

where $K_p^+P_t$ and $K_n^+P_t$ denote the imbalance prices when the electricity price is positive in the cases of pi and ni, respectively; $K_p^-P_t$ and $K_n^-P_t$ denote the imbalance prices when the electricity price is negative in the cases of pi and ni, respectively; and $0 \leq K_p^+ = K_n^- < 1 < K_n^+ = K_p^-$. Also recall that $v_t^*(Q_t, S_t, I_t)$ denotes the optimal profit function in period t for the producer with hour-ahead commitment decisions:

$$v_t^*(Q_t, S_t, I_t) = \max_{(q_t,s_t,w_t)\in\mathbb{U}(Q_t,S_t,I_t)} \left\{ R(Q_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t}\left[v_{t+1}^*(q_t, S_t - s_t, I_{t+1}) \right] \right\}. \tag{6.1}$$

We now consider an alternative problem setting that enables real-time trading without any advance commitment decisions. In other words, any electricity offered to the market in real-time without any advance commitment, $E(s_t, w_t)$, is always accepted, and the producer makes only real-time trading decisions. This problem setting can be obtained from our original problem setting by restricting all of the imbalance pricing parameters to be 1 ($K_p^+ = K_n^- = K_n^+ = K_p^- = 1$). The payoff function thus becomes $\tilde{R}(I_t, s_t, w_t) = E(s_t, w_t)P_t$; this alternative problem setting is equivalent to the one in Zhou et al. [8]. Recall from Chapter 5 that Zhou et al. [8] investigate the optimal operating policy of a wind power plant co-located with a battery without considering any advance commitment decisions.

A control policy $\tilde{\pi}$ is the sequence of decision rules $(\tilde{\eta}_t^{\tilde{\pi}}(S_t^{\tilde{\pi}}, I_t))_{t\in\mathcal{T}}$, where $\tilde{\eta}_t^{\tilde{\pi}}(S_t^{\tilde{\pi}}, I_t) := (s_t^{\tilde{\pi}}(S_t^{\tilde{\pi}}, I_t), w_t^{\tilde{\pi}}(S_t^{\tilde{\pi}}, I_t))$ and $S_t^{\tilde{\pi}}$ denotes the random state variable governed by policy $\tilde{\pi}$, $\forall t \in \mathcal{T}\backslash\{1\}$. We denote the set of all admissible control policies by $\tilde{\Pi}$. For any initial state (S_1, I_1), the optimal expected total cash flow over the finite horizon can be written as

$$\max_{\tilde{\pi}\in\tilde{\Pi}} \mathbb{E}\left[\sum_{t\in\mathcal{T}} \tilde{R}(I_t, s_t^{\tilde{\pi}}(S_t^{\tilde{\pi}}, I_t), w_t^{\tilde{\pi}}(S_t^{\tilde{\pi}}, I_t)) \middle| S_1, I_1\right].$$

For each period $t \in \mathcal{T}$ and each state (S_t, I_t), the optimal profit function $\tilde{v}_t^*(S_t, I_t)$ can be calculated with the following DP recursion:

$$\tilde{v}_t^*(S_t, I_t) = \max_{(s_t, w_t)\in\tilde{\mathbb{U}}(S_t, I_t)} \left\{\tilde{R}(I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t}[\tilde{v}_{t+1}^*(S_{t+1}, I_{t+1})]\right\} \tag{6.2}$$

where $\tilde{\mathbb{U}}(S_t, I_t)$ denotes the set of admissible action pairs (s_t, w_t) in state (S_t, I_t) and $\tilde{v}_T(S_T, I_T) = 0$. We compare below the total cash flows in the original and alternative settings.

Proposition 6.2.1. *Let* $\Upsilon(I_t) = \max\{(1 - K_p^+)\overline{E}P_t, (K_n^+ - 1)\underline{E}P_t, (1 - K_p^-)\overline{E}P_t, (K_n^- - 1)\underline{E}P_t\}$ *where* $\overline{E} = \tau C_T$ *and* $\underline{E} = \min\{C_C/(\theta\tau), C_T\}$. *Then* $v_t^*(Q_t, S_t, I_t) \leq \tilde{v}_t^*(S_t, I_t) \leq v_t^*(Q_t, S_t, I_t) + \max\{(K_p^+ - 1)Q_tP_t, (K_n^+ - 1)Q_tP_t, (K_p^- - 1)Q_tP_t, (K_n^- - 1)Q_tP_t\} + \Upsilon(I_t) + \sum_{\kappa=t+1}^{T} \mathbb{E}[\Upsilon(I_\kappa)]$, $\forall Q_t$.

Proof. First, we will prove that $v_t^*(Q_t, S_t, I_t) \leq \tilde{v}_t^*(S_t, I_t)$, $\forall Q_t \in \mathbb{R}$, $\forall t \in \mathcal{T}$. Note that $v_T^*(Q_T, S_T, I_T) = \tilde{v}_T^*(S_T, I_T) = 0$, $\forall Q_T \in \mathbb{R}$. Assuming $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \leq \tilde{v}_{t+1}^*(S_{t+1}, I_{t+1})$, we will show $v_t^*(Q_t, S_t, I_t) \leq \tilde{v}_t^*(S_t, I_t)$,

$\forall Q_t \in \mathbb{R}$. Pick an arbitrary Q_t and let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$ and $\tilde{\eta}^*(S_t, I_t) = (\tilde{s}, \tilde{w})$. Also, let $\mathbb{U}(Q_t, S_t, I_t)$ and $\tilde{\mathbb{U}}(S_t, I_t)$ denote the sets of admissible action triplets (q_t, s_t, w_t) and $(\tilde{s}_t, \tilde{w}_t)$ in state (Q_t, S_t, I_t), respectively. Note that $(s, w) \in \tilde{\mathbb{U}}(S_t, I_t)$. Thus:

$$
\begin{aligned}
v_t^*(Q_t, S_t, I_t) &= R(Q_t, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\
&\leq \tilde{R}(I_t, s, w) + \mathbb{E}\Big[\tilde{v}_{t+1}^*(S_t - s, I_{t+1})\Big] \\
&\leq \tilde{v}_t^*(S_t, I_t).
\end{aligned}
$$

The first inequality holds since $\tilde{R}(I_t, s, w) = R(E(s, w), I_t, s, w) = \max_{Q_t} R(Q_t, I_t, s, w)$.

Next, note that $\tilde{v}_T^*(S_T, I_T) = 0 = v_T^*(0, S_T, I_T) \leq v_T^*(0, S_T, I_T) + \Upsilon(I_T)$ since $\Upsilon(I_T) \geq 0$. Assuming $\tilde{v}_{t+1}^*(S_{t+1}, I_{t+1}) \leq v_{t+1}^*(0, S_{t+1}, I_{t+1}) + \Upsilon(I_{t+1}) + \sum_{\kappa=t+2}^{T} \mathbb{E}[\Upsilon(I_\kappa)]$, we will prove $\tilde{v}_t^*(S_t, I_t) \leq v_t^*(0, S_t, I_t) + \Upsilon(I_t) + \sum_{\kappa=t+1}^{T} \mathbb{E}[\Upsilon(I_\kappa)]$, $\forall t \in \mathcal{T}$. Let $\eta^*(0, S_t, I_t) = (q, s, w)$ and $\tilde{\eta}^*(S_t, I_t) = (\tilde{s}, \tilde{w})$. Note that $(0, \tilde{s}, \tilde{w}) \in \mathbb{U}(0, S_t, I_t)$. Thus:

$$
\begin{aligned}
\tilde{v}_t^*(S_t, I_t) - v_t^*(0, S_t, I_t) &\leq \tilde{R}(I_t, \tilde{s}, \tilde{w}) + \mathbb{E}\Big[\tilde{v}_{t+1}^*(S_t - \tilde{s}, I_{t+1})\Big] - R(0, I_t, \tilde{s}, \tilde{w}) \\
&\qquad - \mathbb{E}\Big[v_{t+1}^*(0, S_t - \tilde{s}, I_{t+1})\Big] \\
&\leq \tilde{R}(I_t, \tilde{s}, \tilde{w}) - R(0, I_t, \tilde{s}, \tilde{w}) + \sum_{\kappa=t+1}^{T} \mathbb{E}[\Upsilon(I_\kappa)] \\
&\leq \Upsilon(I_t) + \sum_{\kappa=t+1}^{T} \mathbb{E}[\Upsilon(I_\kappa)].
\end{aligned}
$$

The third inequality holds in each of the following four cases:

(1) Suppose that $P_t \geq 0$ and $0 < E(\tilde{s}, \tilde{w})$: Since $K_p^+ < 1$, $\tilde{R}(I_t, \tilde{s}, \tilde{w}) - R(0, S_t, \tilde{s}, \tilde{w}) = E(\tilde{s}, \tilde{w})(1 - K_p^+)P_t \leq \tau C_T(1 - K_p^+)P_t \leq \Upsilon(I_t)$.

(2) Suppose that $P_t \geq 0$ and $0 \geq E(\tilde{s}, \tilde{w})$: Since $K_n^+ > 1$, $\tilde{R}(I_t, \tilde{s}, \tilde{w}) - R(0, S_t, \tilde{s}, \tilde{w}) = E(\tilde{s}, \tilde{w})(1 - K_n^+)P_t \leq \min\{C_C/(\theta\tau), C_T\}(K_n^+ - 1)P_t \leq \Upsilon(I_t)$.

(3) Suppose that $P_t < 0$ and $0 < E(\tilde{s}, \tilde{w})$: Since $K_p^- > 1$, $\tilde{R}(I_t, \tilde{s}, \tilde{w}) - R(0, S_t, \tilde{s}, \tilde{w}) = E(\tilde{s}, \tilde{w})(1 - K_p^-)P_t \leq \tau C_T(1 - K_p^-)P_t \leq \Upsilon(I_t)$.

(4) Suppose that $P_t < 0$ and $0 \geq E(\tilde{s}, \tilde{w})$: Since $K_n^- < 1$, $\tilde{R}(I_t, \tilde{s}, \tilde{w}) - R(0, S_t, \tilde{s}, \tilde{w}) = E(\tilde{s}, \tilde{w})(1-K_n^-)P_t \leq \min\{C_C/(\theta\tau), C_T\}(K_n^- - 1)P_t \leq \Upsilon(I_t)$.

Finally, we prove that $v_t^*(0, S_t, I_t) - v_t^*(Q_t, S_t, I_t) \leq \max\{(K_p^+ - 1)Q_tP_t, (K_n^+ - 1)Q_tP_t, (K_p^- - 1)Q_tP_t, (K_n^- - 1)Q_tP_t\}$, $\forall t \in \mathcal{T}$.

$$\begin{aligned}
&v_t^*(0, S_t, I_t) - v_t^*(Q_t, S_t, I_t) \\
&\leq R(0, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(q, S_t - s, I_{t+1})\right] - R(Q_t, I_t, s, w) - \mathbb{E}\left[v_{t+1}^*(q, S_t - s, I_{t+1})\right] \\
&\leq \max\{(K_p^+ - 1)Q_tP_t, (K_n^+ - 1)Q_tP_t, (K_p^- - 1)Q_tP_t, (K_n^- - 1)Q_tP_t\}.
\end{aligned}$$

The second inequality holds in each of the following eight cases:

(1) Suppose that $P_t \geq 0$, $Q_t < E(s, w)$, and $0 < E(s, w)$: $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_p^+ - 1)Q_tP_t$.

(2) Suppose that $P_t \geq 0$, $E(s, w) \leq Q_t$, and $E(s, w) \leq 0$: $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_n^+ - 1)Q_tP_t$.

(3) Suppose that $P_t \geq 0$ and $Q_t < E(s, w) \leq 0$: Since $K_n^+ > K_p^+$, $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_n^+ - K_p^+)P_tE(s, w) + (K_p^+ - 1)Q_tP_t \leq (K_p^+ - 1)Q_tP_t$.

(4) Suppose that $P_t \geq 0$ and $0 < E(s, w) \leq Q_t$: Since $K_n^+ > K_p^+$, $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_p^+ - K_n^+)P_tE(s, w) + (K_n^+ - 1)Q_tP_t \leq (K_n^+ - 1)Q_tP_t$.

(5) Suppose that $P_t < 0$, $Q_t < E(s, w)$, and $0 < E(s, w)$: $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_p^- - 1)Q_tP_t$.

(6) Suppose that $P_t < 0$, $E(s, w) \leq Q_t$, and $E(s, w) \leq 0$: $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_n^- - 1)Q_tP_t$.

(7) Suppose that $P_t < 0$ and $Q_t < E(s, w) \leq 0$: Since $K_p^- > K_n^-$, $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_n^- - K_p^-)P_tE(s, w) + (K_p^- - 1)Q_tP_t \leq (K_p^- - 1)Q_tP_t$.

(8) Suppose that $P_t < 0$ and $0 < E(s,w) \leq Q_t$: Since $K_p^- > K_n^-$, $R(0, I_t, s, w) - R(Q_t, I_t, s, w) = (K_p^- - K_n^-)P_t E(s,w) + (K_n^- - 1)Q_t P_t \leq (K_n^- - 1)Q_t P_t$.

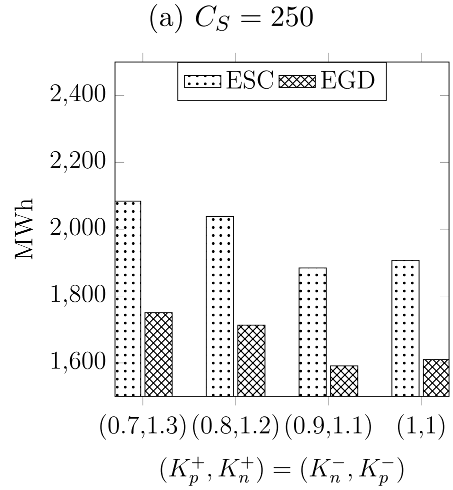
□

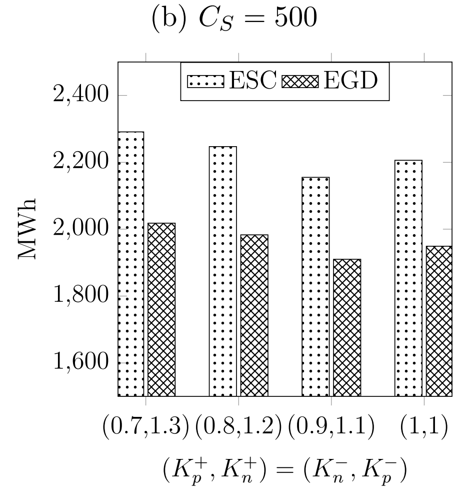
Proposition 6.2.1 states that the optimal total profit in the presence of commitment decisions (i.e., $v_1^*(Q_1, S_1, I_1)$) is lower than the optimal total profit in the absence of commitment decisions (i.e., $\tilde{v}_1^*(S_1, I_1)$). This is because the producer is penalized if she does not meet her commitments in real-time. Proposition 6.2.1 also establishes an upper bound on the loss of total cash flow due to the existence of commitment decisions. This bound can be viewed as the maximum possible penalty incurred due to the maximum possible imbalance amount summed over all future periods. This bound is affected by the market characteristics (electricity prices and imbalance pricing parameters) as well as the energy system characteristics (capacity and efficiency levels). This bound would be tight in energy systems with low capacity levels and in electricity markets with low price volatility and low penalty mechanisms (i.e., high values of $K_p^+ = K_n^-$ and low values of $K_n^+ = K_p^-$).

6.3 Numerical Results

In this section, we perform numerical experiments to gain further insights into the impact of commitment decisions. Note that a decrease in $K_p^+(K_n^-)$ and an increase in $K_n^+(K_p^-)$ increase the penalty for energy imbalances, amplifying the effect of commitment decisions. Using data-calibrated time series models presented in Chapter 3, we consider instances in which the planning horizon spans the first week of August, $N = 100$, $K_p^+ = K_n^- \in \{0.7, 0.8, 0.9\}$ and $K_n^+ = K_p^- \in \{1.1, 1.2, 1.3\}$ (our problem setting) or $K_p^+ = K_n^+ = K_p^- = K_n^- = 1$ (alternative problem setting [8]), $C_S \in \{0, 250, 500\}$ (in MWh), $C_C = C_D \in \{40, 60\}$ (in MWh), $C_T \in \{100, 200\}$ (in MWh), NPF $\in \{0, 4.02\%, 7.66\%, 10.96\%, 13.98\%\}$,

Figure 6.1: ESC and EGD vs. (K_p^+, K_n^+) when $C_C = C_D = 40$, $C_T = 200$, NPF $= 4.02\%$, $\tau = 0.95$, and $r = 0.8$.





$r \in \{0.7, 0.8, 0.9, 1\}$, and $\tau \in \{0.95, 1\}$. We provide our detailed results in Appendix D (Tables D.1–D.6) and present our key results in Figures 6.1–6.3.

We have found that the existence of commitment decisions reduces the total cash flow by 4.91%, on average, on a test bed of 162 instances. This loss in TCF is smaller when the imbalance penalty is lower. This loss is greater under higher price volatility (via higher NPF values) and higher system capacity levels that potentially induce larger strategic deviations. All these results agree with our theoretical upper bound on the TCF difference in Proposition 6.2.1.

While our numerical results verify the insights available from the upper bound in Proposition 6.2.1, we observe that the efficiency levels have a very limited effect on the revenue loss due to commitment decisions. We also examined the impact of commitment decisions on the battery storage and generation decisions: We define ESC as the expected total amount of energy stored by charging the battery, and EGD as the expected total amount of energy generated by discharging the battery. We observe from Figure 6.1 that ESC and EGD increase as the imbalance penalty grows, starting from $(K_p^+, K_n^+) = (0.9, 1.1)$. This increase is smaller when $C_S = 500$. This result can be explained by the different roles of the battery in the presence of commitment decisions: The battery can be used as a strategic tool to support deviations from commitments as well as a backup source to better fulfill

Figure 6.2: ESC and EGD vs. (K_p^+, K_n^+) when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, NPF = 4.02%, and $\tau = 0.95$.

(a) $r = 0.7$ (b) $r = 1$

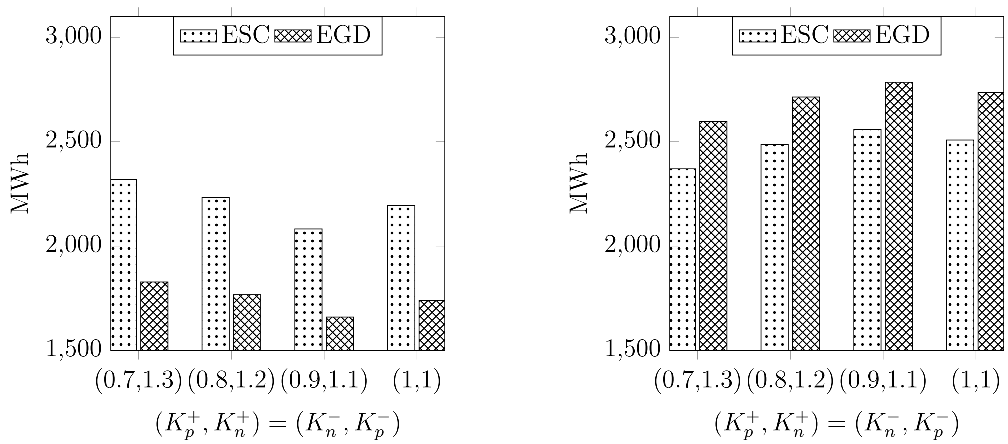
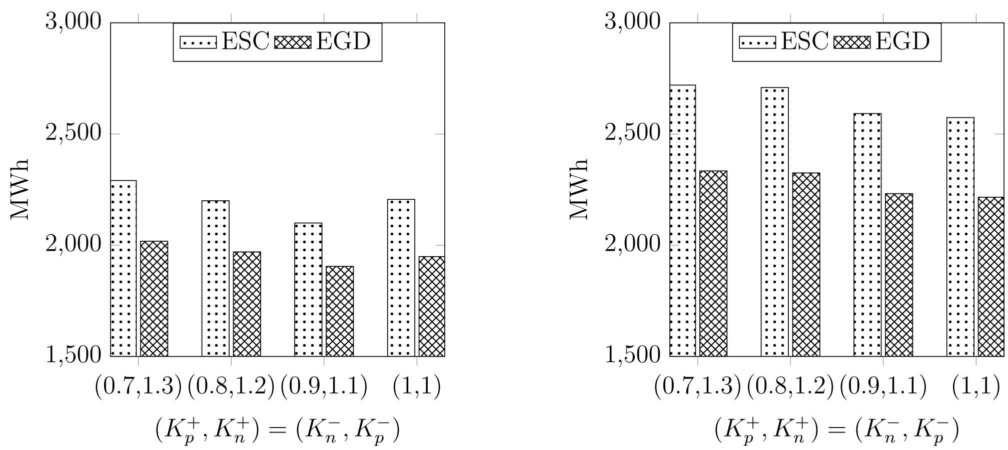


Figure 6.3: ESC and EGD vs. (K_p^+, K_n^+) when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, $\tau = 0.95$, and $r = 0.8$.

(a) NPF = 4.02% (b) NPF = 13.98%



commitments. The producer with a large battery has the potential to greatly deviate from her commitments. Increasing the imbalance penalty reduces the use of the battery for strategic deviations. This effect is thus stronger when the battery is larger, leading to a smaller increase in ESC and EGD when $C_S = 500$. But increasing the imbalance penalty also raises the use of the battery as the backup source. This effect is stronger when the battery is smaller, leading to a larger increase in ESC and EGD when $C_S = 250$.

We observe from Figure 6.2 that, when the roundtrip efficiency is higher, ESC and EGD decrease as the imbalance penalty grows: Under higher efficiencies, the producer may utilize the battery more for strategic deviations since less energy is lost during charging and discharging. Thus increasing the imbalance penalty prevents such utilization of the battery more effectively, reducing the overall utilization of the battery. We note from Figure 6.3 that ESC and EGD increase with NPF. The producer utilizes the battery more when NPF is higher. We again observe that ESC and EGD increase as the imbalance penalty grows. This increase is smaller when NPF is higher: Under higher NPF values, the producer may utilize the battery more for strategic deviations since there is more incentive to purchase energy at negative prices to sell it in future periods with high prices. Thus increasing the imbalance penalty may prevent such utilization of the battery more effectively (although the overall utilization of the battery grows).

6.4 Concluding Remarks

In this chapter, we consider an alternative problem setting in which the producer makes energy trading in real-time without making advance commitment decisions. This problem setting is similar to the one in [8]. We establish a theoretical upper bound on the cost of commitment decisions by comparing our problem setting to this alternative setting. We find that this theoretical bound is dependent on the characteristics of both the energy system, such as capacity and efficiency levels, and the market, such as electricity prices and imbalance pricing parameters. Energy systems with low capacity levels and electricity markets with low price volatility and penalty mechanisms are likely to experience tighter bounds.

We also conducted an extensive numerical study to examine the impact of commitment decisions in different environments, measuring several metrics for the original problem setting and the alternative problem setting. Based on an experimental test bed of 162 instances, we find that the existence of commitment decisions reduces the total cash flow by 4.91%, on average. This loss is larger in energy systems with large capacity levels and in electricity markets with high price volatility and high penalty mechanisms.

Future research may extend our analysis to multi-settlement electricity markets, which may include a combination of the day-ahead market, the intraday market, or the balancing market (the real-time market). In the multi-settlement electricity markets, depending on the relationship between prices in sequential markets, engaging in commitment decisions as well as real-time dispatch/purchase decisions may improve the producer's profit. Therefore, it might be interesting to analytically compare the total cash flows obtained in single-settlement electricity markets to the total cash flows obtained in multi-settlement electricity markets.

Chapter 7

Conclusion

Renewable energy sources such as wind and solar are essential for the development of power systems as countries set ambitious targets for renewable energy generation. While these sources contribute to the overall energy supply, the renewable power producers participate in electricity markets where they need to make advance commitment decisions for energy delivery and purchase, adding a layer of complexity to their operational decisions [7]. The fundamental challenge of intermittency in renewable energy generation is in managing their commitment decisions. Energy storage provides an opportunity to mitigate this challenge by presenting a hedging opportunity against the cost of imbalance. This raises the question of how a renewable power producer could jointly optimize energy generation and storage decisions while making advance commitments in an electricity market.

The energy commitment, generation and storage problem of an energy system that consist of a renewable power plant and an energy storage unit is a challenging problem that requires a thorough understanding of the system dynamics, market characteristics, and uncertainties in energy renewable source and electricity price. Optimizing the operation of renewable power plants together with energy storage systems in electricity markets is a relatively underdeveloped but a promising area for OR/MS scholars [17]. The aim of this thesis is to address this problem by developing mathematical models that can capture the system dynamics and uncertainties inherent in the problem, and providing clean analytical formulations that enable optimal policy characterizations.

This thesis formulates the real-time decision making problem of a wind power producer who owns a battery and participates in a spot market operating with hourly commitments and settlements as an MDP by taking into account the electricity price and wind uncertainties. We provide managerial insights to renewable power producers in their assessment of energy storage adoption decisions, as well as to power system operators in their understanding of the producers' behavior in the market with their storage capabilities. We characterize the structure of the optimal energy commitment, generation and storage policy as a threshold policy. We employ our structural results to develop heuristic solution procedures as an alternative to standard dynamic programming algorithm. Finally, we consider an alternative problem setting that allows for real-time trading without making any advance commitment and evaluate it in comparison to the original problem setting.

Specifically, in Chapter 3, we consider the following two possible settings: (i) The battery can be used to support intentional deviations from commitments or (ii) it should be used to minimize such deviations. We construct data-calibrated time series models for the electricity price and wind speed, which we incorporate into our MDP formulations. We numerically examine the effects of system components, imbalance pricing parameters, and negative prices on the producer's profits, curtailment decisions, and imbalance tendencies for each problem setting. Our findings suggest that the presence of a battery reduces energy imbalance when the producer cannot make intentional deviations. However, if the producer can make intentional deviations, the battery has an opposite effect, leading to a higher negative imbalance than a positive imbalance. Appropriate selection of imbalance pricing parameters can result in a decrease in the imbalance amounts when intentional deviations are allowed. Finally, the supporting role of the battery for intentional deviations becomes more valuable as negative electricity prices are observed more frequently in the market.

In Chapter 4, we consider the same problem presented in Chapter 3 (i.e., the first setting). We use the multi-dimensional structural properties of the optimal profit function to demonstrate the optimality of a state-dependent threshold policy for storage and commitment decisions, under the assumptions of perfectly efficient battery and transmission line, and positive electricity prices. Using this policy structure, we develop two heuristic solution methods (HC and HR) for solving the more general problem, in which the battery and transmission line may not be perfectly efficient and electricity prices may be negative. Our numerical experiments have revealed that both HC and HR outperform the standard dynamic programming algorithm with respect to computation time by two orders of magnitude and yield solutions with an average distance of less than one percent from the optimal profit. Our experiments also show the poor performance of a purely myopic solution approach, simpler fixed threshold policies, and a deterministic reoptimization heuristic with respect to objective value. These results imply that ignoring the current state information, the forward-looking effects, and/or precise modeling of uncertainties in decision-making leads to a significant loss of optimality, demonstrating the usefulness of our state-dependent policies in the energy commitment problem.

In Chapter 5, we characterize the optimal policy structure for the energy commitment, generation and storage problem in Chapter 4 when the battery and transmission line can be imperfectly efficient. The decision space grows to include scenarios such as charging the battery and purchasing energy from the market, charging the battery and selling energy to the market, and discharging the battery and selling energy to the market. These decisions are in addition to the previously considered negative and positive imbalance decisions. We show that the target storage level associated with the charge and purchase decision is the lowest and the target storage level associated with the discharge and sell decision is the highest. In Chapter 6, we consider an alternative problem setting that allows for real-time trading without making any advance commitment. We analytically compare total cash flows of this setting to our original problem setting. We numerically examine the effect of advance commitment decisions on the producer's energy storage and generation decisions. We find that in such single-settlement market setting, the existence of commitment decisions reduces the total cash flow by 4.91%, on average. This loss is larger in electricity markets with high price volatility and high penalty mechanisms.

There are several directions for future research. One would be to extend our analyses in Chapter 3 by examining the effect of different imbalance pricing mechanisms on optimal strategies of renewable power producers. This entails addressing several open questions, such as: (i) What is the impact of different imbalance pricing mechanisms on the commitment decisions of renewable power producers, i.e., whether they tend to make negative or positive imbalances? (ii) How do these pricing mechanisms affect the optimal commitment and storage policy? (iii) Which imbalance pricing mechanism is more effective in integrating a renewable source with an energy storage unit?

Another research direction would be to extend our analyses in Chapters 4-6 to more complex market structures such as a day-ahead market. It is important to note that every electricity market has its own structure. For example, a participant in the day-ahead market should report her hourly commitments one day before her actual production is realized. It may be risky for a renewable power producer to participate in a day-ahead market since the uncertainty level of this market is higher than other markets. This risk may lead to different optimal energy commitment, storage and generation policies. Therefore, it might be interesting to analyze how participating in different markets affects the renewable power producer's profitability and optimal operation policies.

Future research may also extend our models and analyses to multi-settlement electricity markets, which may include a combination of a day-ahead market, an intraday market, or a balancing market. A renewable power producer with a storage unit in multi-settlement electricity markets may co-optimize her commitments to reduce her exposure to risk from the uncertainty of electricity prices, thereby minimizing her imbalances in the market. Additionally, such a producer could take advantage of intertemporal pricing disparities resulting from fluctuating demand and renewable power generation. Such an extension will likely entail the development of multi-stage stochastic programs in addition to MDPs, given that the producer is faced with a multidimensional, multistage decision-making problem under uncertainty.

Finally, future extensions of this study may also consider other storage technologies, such as pumped hydro energy storage and compressed air energy storage. For an energy system with a battery, it is sufficient to keep track of the amount of energy storage via a single endogenous state variable. However, for a pumped hydro energy storage facility, one need to keep track of water levels in upper and lower reservoirs via two endogenous state variables. The theory of ADP may be usefully employed in this research direction in order to handle potentially larger state and/or action spaces.

Bibliography

[1] General Electric, "GE 1.5-77 wind turbine." https://www.ge.com/in/wind-energy/1.5-MW-wind-turbine, 2019. Accessed July 8, 2020.

[2] S. Weitemeyer, D. Kleinhans, T. Vogt, and C. Agert, "Integration of renewable energy sources in future power systems: The role of storage," *Renewable Energy*, vol. 75, pp. 14–20, 2015.

[3] J. Zhong, M. Bollen, and S. Rönnberg, "Towards a 100% renewable energy electricity generation system in Sweden," *Renewable Energy*, vol. 171, pp. 812–824, 2021.

[4] EIA, "Increasing renewables likely to reduce coal and natural gas generation over next two years." https://www.eia.gov/todayinenergy/detail.php?id=55239, 2023. Accessed May 11, 2023.

[5] U.S. Department of Energy, "Wind market reports: 2022 edition." https://www.energy.gov/eere/wind/wind-market-reports-2022-edition#wind, 2023. Accessed May 11, 2023.

[6] IRENA, "Global energy transformation: A roadmap to 2050." https://www.irena.org/-/media/Files/IRENA/Agency/Publication/2018/Apr/IRENA_Report_GET_2018.pdf, 2018. Accessed May 11, 2023.

[7] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, and M. Zugno, *Integrating renewables in electricity markets: Operational problems*, vol. 205. Springer Science & Business Media, 2013.

[8] Y. Zhou, A. Scheller-Wolf, N. Secomandi, and S. Smith, "Managing wind-based electricity generation in the presence of storage and transmission capacity," *Production and Operations Management*, vol. 28, no. 4, pp. 970–989, 2019.

[9] M. Bajwa and J. Cavicchi, "Growing evidence of increased frequency of negative electricity prices in US wholesale electricity markets," in *IAEE Energy Forum*, vol. 37, 2017.

[10] EIA, "Large battery systems are often paired with renewable energy power plants." `https://www.eia.gov/todayinenergy/detail.php?id=43775`, 2020. Accessed May 11, 2023.

[11] IRENA, "Electricity storage and renewables: Costs and markets to 2030." `https://www.irena.org/-/media/Files/IRENA/Agency/Publication/2017/Oct/IRENA_Electricity_Storage_Costs_2017_Summary.pdf`, 2021. Accessed May 11, 2023.

[12] Ç. Çelik, "Electric power market models in developing countries," *ELECO'2003, Bursa, 3-7 December*, 2003.

[13] P. L. Joskow *et al.*, *Lessons Learned from the Electricity Market Liberalization.* Massachusetts Institute of Technology, Center for Energy and Environmental Policy Research, 2008.

[14] J. Bartlett, "Reducing risk in merchant wind and solar projects through financial hedges," *Resources for the future*, vol. 23, 2019.

[15] S. Hagemann and C. Weber, "Trading volumes in intraday markets: Theoretical reference model and empirical observations in selected european markets," tech. rep., EWL working paper, 2015.

[16] D. Perekhodtsev, *Two essays on problems of deregulated electricity markets.* Carnegie Mellon University, 2004.

[17] G. G. Parker, B. Tan, and O. Kazan, "Electric power industry: Operational and public policy challenges and opportunities," *Production and Operations Management*, vol. 28, no. 11, pp. 2738–2777, 2019.

[18] CAISO, "Chapter 4. Real-time Energy Market." https://www.caiso.com/Documents/Chapter4_1998AnnualReport_MarketIssuesandPerformance.pdf, 1998. Accessed May 11, 2023.

[19] CAISO, "CAISO Annual Market Report on Market Issues and Performance." https://www.transmissionhub.com/wp-content/uploads/2018/12/California-ISO-APR-28-2014.pdf, 2013. Accessed May 11, 2023.

[20] CAISO, "Attachment D – Benchmarking against NYISO, PJM, and ISO-NE." https://www.caiso.com/Documents/AttachmentD-BenchmarkingAgainstNYISO_PJMandISO-NE.pdf, 2007. Accessed May 11, 2023.

[21] F. Röben and J. E. De Haan, "Market response for real-time energy balancing–evidence from three countries," in *2019 16th International Conference on the European Energy Market (EEM)*, pp. 1–5, IEEE, 2019.

[22] A. Eicke, O. Ruhnau, and L. Hirth, "Electricity balancing as a market equilibrium: An instrument-based estimation of supply and demand for imbalance energy," *Energy Economics*, vol. 102, p. 105455, 2021.

[23] J. Yang, Z. Y. Dong, F. Wen, Q. Chen, F. Luo, W. Liu, and J. Zhan, "A penalty scheme for mitigating uninstructed deviation of generation outputs from variable renewables in a distribution market," *IEEE Transactions on Smart Grid*, vol. 11, no. 5, pp. 4056–4069, 2020.

[24] S. Clò and E. Fumagalli, "The effect of price regulation on energy imbalances: A difference in differences design," *Energy Economics*, vol. 81, pp. 754–764, 2019.

[25] T. K. Boomsma, N. Juul, and S.-E. Fleten, "Bidding in sequential electricity markets: The Nordic case," *European Journal of Operational Research*, vol. 238, no. 3, pp. 797–809, 2014.

[26] J. P. Chaves-Ávila, R. A. Hakvoort, and A. Ramos, "The impact of European balancing rules on wind power economics and on short-term bidding strategies," *Energy Policy*, vol. 68, pp. 383–393, 2014.

[27] T. Brijs, F. Geth, C. De Jonghe, and R. Belmans, “Quantifying electricity storage arbitrage opportunities in short-term electricity markets in the CWE region,” *Journal of Energy Storage*, vol. 25, p. 100899, 2019.

[28] R. A. van der Veen, A. Abbasy, and R. A. Hakvoort, “Agent-based analysis of the impact of the imbalance pricing mechanism on market behavior in electricity balancing markets,” *Energy Economics*, vol. 34, no. 4, pp. 874–881, 2012.

[29] PJM, “PJM Manual 27: Open access transmission tariff accounting.” `https://www.pjm.com/~/media/documents/manuals/m27.ashx`, 2015. Accessed May 11, 2023.

[30] MISO, “Module C - Energy and Operating Reserve Markets.” `https://docs.misoenergy.org/legalcontent/Module_C_-_Energy_and_Operating_Reserve_Markets.pdf`, 2022. Accessed May 11, 2023.

[31] A. Papavasiliou and S. S. Oren, “Coupling wind generators with deferrable loads,” in *2008 IEEE Energy 2030 Conference*, pp. 1–7, IEEE, 2008.

[32] J. H. Kim and W. B. Powell, “Optimal energy commitments with storage and intermittent supply,” *Operations Research*, vol. 59, no. 6, pp. 1347–1360, 2011.

[33] D. R. Jiang and W. B. Powell, “Optimal hour-ahead bidding in the real-time electricity market with battery storage using approximate dynamic programming,” *INFORMS Journal on Computing*, vol. 27, no. 3, pp. 525–543, 2015.

[34] P. Pinson, C. Chevallier, and G. N. Kariniotakis, “Trading wind generation from short-term probabilistic forecasts of wind power,” *IEEE Transactions on Power Systems*, vol. 22, no. 3, pp. 1148–1156, 2007.

[35] J. M. Morales, A. J. Conejo, and J. Pérez-Ruiz, “Short-term trading for a wind power producer,” *IEEE Transactions on Power Systems*, vol. 25, no. 1, pp. 554–564, 2010.

[36] C. J. Dent, J. W. Bialek, and B. F. Hobbs, “Opportunity cost bidding by wind generators in forward markets: Analytical results,” *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1600–1608, 2011.

[37] A. Botterud, Z. Zhou, J. Wang, R. J. Bessa, H. Keko, J. Sumaili, and V. Miranda, “Wind power trading under uncertainty in LMP markets,” *IEEE Transactions on power systems*, vol. 27, no. 2, pp. 894–903, 2011.

[38] E. Y. Bitar, R. Rajagopal, P. P. Khargonekar, K. Poolla, and P. Varaiya, “Bringing wind energy to market,” *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1225–1235, 2012.

[39] L. Baringo and A. J. Conejo, “Offering strategy of wind-power producer: A multi-stage risk-constrained approach,” *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1420–1429, 2015.

[40] N. Löhndorf, D. Wozabal, and S. Minner, “Optimizing trading decisions for hydro storage systems using approximate dual dynamic programming,” *Operations Research*, vol. 61, no. 4, pp. 810–823, 2013.

[41] N. Paine, F. R. Homans, M. Pollak, J. M. Bielicki, and E. J. Wilson, “Why market rules matter: Optimizing pumped hydroelectric storage when compensation rules differ,” *Energy Economics*, vol. 46, pp. 10–19, 2014.

[42] A. Berrada, K. Loudiyi, and I. Zorkani, “Valuation of energy storage in energy and regulation markets,” *Energy*, vol. 115, pp. 1109–1118, 2016.

[43] S. Karhinen and H. Huuki, “Private and social benefits of a pumped hydro energy storage with increasing amount of wind power,” *Energy Economics*, vol. 81, pp. 942–959, 2019.

[44] N. Löhndorf and D. Wozabal, “The value of coordination in multimarket bidding of grid energy storage,” *Operations Research*, vol. 71, no. 1, pp. 1–22, 2023.

[45] J. Garcia-Gonzalez, R. M. R. de la Muela, L. M. Santos, and A. M. Gonzalez, “Stochastic joint optimization of wind generation and pumped-storage units

in an electricity market," *IEEE Transactions on Power Systems*, vol. 23, no. 2, pp. 460–468, 2008.

[46] N. Löhndorf and S. Minner, "Optimal day-ahead trading and storage of renewable energies–an approximate dynamic programming approach," *Energy Systems*, vol. 1, no. 1, pp. 61–77, 2010.

[47] B. Mauch, P. M. Carvalho, and J. Apt, "Can a wind farm with CAES survive in the day-ahead market?," *Energy Policy*, vol. 48, pp. 584–593, 2012.

[48] H. Ding, Z. Hu, and Y. Song, "Rolling optimization of wind farm and energy storage system in electricity markets," *IEEE Transactions on Power Systems*, vol. 30, no. 5, pp. 2676–2684, 2014.

[49] E. D. Castronuovo, J. Usaola, R. Bessa, M. Matos, I. Costa, L. Bremermann, J. Lugaro, and G. Kariniotakis, "An integrated approach for optimal coordination of wind power and hydro pumping storage," *Wind Energy*, vol. 17, no. 6, pp. 829–852, 2014.

[50] H. Ding, P. Pinson, Z. Hu, and Y. Song, "Integrated bidding and operating strategies for wind-storage systems," *IEEE Transactions on Sustainable Energy*, vol. 7, no. 1, pp. 163–172, 2015.

[51] J. Gönsch and M. Hassler, "Sell or store? An ADP approach to marketing renewable energy," *OR Spectrum*, vol. 38, no. 3, pp. 633–660, 2016.

[52] M. Hassler, "Heuristic decision rules for short-term trading of renewable energy with co-located energy storage," *Computers & Operations Research*, vol. 83, pp. 199–213, 2017.

[53] B. Finnah and J. Gönsch, "Optimizing trading decisions of wind power plants with hybrid energy storage systems using backwards approximate dynamic programming," *International Journal of Production Economics*, vol. 238, p. 108155, 2021.

[54] I. L. Gomes, H. Pousinho, R. Melicio, and V. Mendes, "Stochastic coordination of joint wind and photovoltaic systems with energy storage in day-ahead market," *Energy*, vol. 124, pp. 310–320, 2017.

[55] M. S. Al-Swaiti, A. T. Al-Awami, and M. W. Khalid, "Co-optimized trading of wind-thermal-pumped storage system in energy and regulation markets," *Energy*, vol. 138, pp. 991–1005, 2017.

[56] G. Díaz, J. Coto, and J. Gómez-Aleixandre, "Optimal operation value of combined wind power and energy storage in multi-stage electricity markets," *Applied Energy*, vol. 235, pp. 1153–1168, 2019.

[57] E. D. Castronuovo and J. P. Lopes, "On the optimization of the daily operation of a wind-hydro power plant," *IEEE Transactions on Power Systems*, vol. 19, no. 3, pp. 1599–1606, 2004.

[58] P. D. Brown, J. P. Lopes, and M. A. Matos, "Optimization of pumped storage capacity in an isolated power system with large renewable penetration," *IEEE Transactions on Power systems*, vol. 23, no. 2, pp. 523–531, 2008.

[59] Á. J. Duque, E. D. Castronuovo, I. Sánchez, and J. Usaola, "Optimal operation of a pumped-storage hydro plant that compensates the imbalances of a wind power producer," *Electric Power Systems Research*, vol. 81, no. 9, pp. 1767–1777, 2011.

[60] M. T. Vespucci, F. Maggioni, M. I. Bertocchi, and M. Innorta, "A stochastic model for the daily coordination of pumped storage hydro plants and wind power plants," *Annals of operations research*, vol. 193, no. 1, pp. 91–105, 2012.

[61] P. Harsha and M. Dahleh, "Optimal management and sizing of energy storage under dynamic pricing for the efficient integration of renewable energy," *IEEE Transactions on Power Systems*, vol. 30, no. 3, pp. 1164–1181, 2014.

[62] X. Xi, R. Sioshansi, and V. Marano, "A stochastic dynamic programming model for co-optimization of distributed energy storage," *Energy Systems*, vol. 5, no. 3, pp. 475–505, 2014.

[63] Y. Tang, H. He, Z. Ni, and J. Wen, "Optimal operation for energy storage with wind power generation using adaptive dynamic programming," in *2015 IEEE Power & Energy Society General Meeting*, pp. 1–6, IEEE, 2015.

[64] H. Avci, E. C. Karakoyun, A. S. Kocaman, E. Nadar, and P. Toufani, "Integration of pumped hydro energy storage and wind energy generation: Structural analysis and algorithms.," pp. Working paper, Bilkent University, 2021.

[65] X. Peng, O. Q. Wu, and G. Souza, "Renewable, flexible, and storage capacities: Friends or foes?," pp. Working paper, Indiana University, 2021.

[66] E. C. Karakoyun, H. Avci, A. S. Kocaman, and E. Nadar, "Deviations from commitments: Markov decision process formulations for the role of energy storage," *International Journal of Production Economics*, vol. 255, p. 108711, 2023.

[67] B. Finnah, J. Gönsch, and F. Ziel, "Integrated day-ahead and intraday self-schedule bidding for energy storage systems using approximate dynamic programming," *European Journal of Operational Research*, vol. 301, no. 2, pp. 726–746, 2022.

[68] F. Lisi and E. Edoli, "Analyzing and forecasting zonal imbalance signs in the italian electricity market," *The Energy Journal*, vol. 39, no. 5, 2018.

[69] C. De Jonghe, E. Delarue, R. Belmans, and W. D'haeseleer, "Determining optimal electricity technology mix with high level of wind power penetration," *Applied Energy*, vol. 88, no. 6, pp. 2231–2238, 2011.

[70] M. Vilim and A. Botterud, "Wind power bidding in electricity markets with high wind penetration," *Applied Energy*, vol. 118, pp. 141–155, 2014.

[71] N. Ederer, "The market value and impact of offshore wind on the electricity spot market: Evidence from Germany," *Applied Energy*, vol. 154, pp. 805–814, 2015.

[72] N. Sunar and J. R. Birge, "Strategic commitment to a production schedule with uncertain supply and demand: Renewable energy in day-ahead electricity markets," *Management Science*, vol. 65, no. 2, pp. 714–734, 2019.

[73] D. Lew, L. Bird, M. Milligan, B. Speer, X. Wang, E. M. Carlini, A. Estanqueiro, D. Flynn, E. Gomez-Lazaro, N. Menemenlis, *et al.*, *Wind and solar curtailment*. National Renewable Energy Laboratory, 2013.

[74] L. Bird, D. Lew, M. Milligan, E. M. Carlini, A. Estanqueiro, D. Flynn, E. Gomez-Lazaro, H. Holttinen, N. Menemenlis, A. Orths, *et al.*, "Wind and solar energy curtailment: A review of international experience," *Renewable and Sustainable Energy Reviews*, vol. 65, pp. 577–586, 2016.

[75] NYISO, "Energy market & operational data." `https://www.nyiso.com/energy-market-operational-data`, 2019. Accessed July 8, 2020.

[76] J. Janczura, S. Trück, R. Weron, and R. C. Wolff, "Identifying spikes and seasonal components in electricity spot price data: A guide to robust modeling," *Energy Economics*, vol. 38, pp. 96–110, 2013.

[77] J. J. Lucia and E. S. Schwartz, "Electricity prices and power derivatives: Evidence from the Nordic power exchange," *Review of Derivatives Research*, vol. 5, no. 1, pp. 5–50, 2002.

[78] NOAA, "Integrated surface data, hourly, global." `https://www.ncdc.noaa.gov/data-access`, 2019. Accessed July 8, 2020.

[79] J. Chen and C. Rabiti, "Synthetic wind speed scenarios generation for probabilistic analysis of hybrid energy systems," *Energy*, vol. 120, pp. 507–517, 2017.

[80] J. Hull and A. White, "Numerical procedures for implementing term structure models I: Single-factor models," *Journal of Derivatives*, vol. 2, no. 1, pp. 7–16, 1994.

[81] P. Jaillet, E. I. Ronn, and S. Tompaidis, "Valuation of commodity-based swing options," *Management Science*, vol. 50, pp. 909–921, 2004.

[82] G. Tauchen, "Finite state Markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, vol. 20, no. 2, pp. 177–181, 1986.

[83] T. J. Sheskin, "A Markov chain partitioning algorithm for computing steady state probabilities," *Operations Research*, vol. 33, no. 1, pp. 228–235, 1985.

[84] D. M. Topkis, *Supermodularity and Complementarity.* Princeton University Press, 1998.

[85] E. Nadar, M. Akan, and A. Scheller-Wolf, "Experimental results indicating lattice-dependent policies may be optimal for general assemble-to-order systems," *Production and Operations Management*, vol. 25, no. 4, pp. 647–661, 2016.

[86] J. Seifert and M. Uhrig-Homburg, "Modelling jumps in electricity prices: theory and empirical evidence," *Review of Derivatives Research*, vol. 10, no. 1, pp. 59–85, 2007.

[87] U.S. Department of Energy, "Energy storage market report 2020." `https://www.energy.gov/energy-storage-grand-challenge/downloads/energy-storage-market-report-2020`, 2020. Accessed May 11, 2023.

[88] Y. Zhou, A. Scheller-Wolf, N. Secomandi, and S. Smith, "Electricity trading and negative prices: Storage vs. disposal," *Management Science*, vol. 62, no. 3, pp. 880–898, 2016.

[89] EIA, "Battery storage in the United States: An update on market trends," 2021. Accessed May 11, 2023.

[90] I. Ranaweera and O.-M. Midtgård, "Optimization of operational cost for a grid-supporting PV system with battery storage," *Renewable Energy*, vol. 88, pp. 262–272, 2016.

[91] S. S. Singh and E. Fernandez, "Modeling, size optimization and sensitivity analysis of a remote hybrid renewable energy system," *Energy*, vol. 143, pp. 719–731, 2018.

[92] M. Shabani, E. Dahlquist, F. Wallin, and J. Yan, "Techno-economic impacts of battery performance models and control strategies on optimal design of a grid-connected PV system," *Energy Conversion and Management*, vol. 245, p. 114617, 2021.

[93] EIA, "How much electricity is lost in electricity transmission and distribution in the United States?," 2022. Accessed May 11, 2023.

[94] J. P. Chaves-Ávila, R. A. van der Veen, and R. A. Hakvoort, “The interplay between imbalance pricing mechanisms and network congestions–Analysis of the German electricity market,” *Utilities Policy*, vol. 28, pp. 52–61, 2014.

[95] RTE, “Imbalance settlement price.” `https://www.services-rte.com/en/learn-more-about-our-services/becoming-a-balance-responsible-party/Imbalance-settlement-price.html`, 2020. Accessed May 11, 2023.

[96] PWC, “Overview of the Turkish electricity market.” `https://www.invest.gov.tr/en/library/publications/lists/investpublications/overview-of-turkish-electricity-market.pdf`, 2021. Accessed May 11, 2023.

[97] E. L. Porteus, *Foundations of stochastic inventory theory.* Stanford University Press, 2002.

Appendix A

Additional numerical experiments for the month of January in the city of Albany and for the city of Buffalo in the month of August

Table A.1: Numerical results for Albany in January when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, and NPF = 1.61%.

Setting	C_T	$C_C = C_D$	C_S	TCF	WEC	NI	PI	ED
ID	100	50	200	1.103	6736	426	96	523
			400	1.103	6713	427	96	523
			600	1.103	6677	427	96	523
			800	1.103	6578	427	96	523
		75	200	1.106	6820	508	96	604
			400	1.106	6856	509	96	605
			600	1.106	6896	509	96	605
			800	1.106	6878	509	96	605
		100	200	1.107	6836	532	96	629
			400	1.108	6877	533	96	630
			600	1.108	6913	533	96	630
			800	1.108	6941	533	96	630
	200	50	200	1.613	373	2970	634	3603
			400	1.641	373	3292	815	4107
			600	1.657	373	3319	888	4207
			800	1.668	373	3366	937	4303
		75	200	1.629	374	3402	610	4012
			400	1.667	374	4293	778	5071
			600	1.689	374	4460	872	5332
			800	1.704	373	4528	891	5420
		100	200	1.639	374	3898	597	4495
			400	1.683	374	4939	676	5615
			600	1.710	374	5338	764	6101
			800	1.728	374	5532	800	6331
UD	100	50	200	1.078	6305	0	0	0
			400	1.078	6203	0	0	0
			600	1.078	6092	0	0	0
			800	1.078	5980	0	0	0
		75	200	1.078	6305	0	0	0
			400	1.078	6199	0	0	0
			600	1.078	6089	0	0	0
			800	1.078	5978	0	0	0
		100	200	1.078	6305	0	0	0
			400	1.078	6199	0	0	0
			600	1.078	6089	0	0	0
			800	1.078	5978	0	0	0
	200	50	200	1.507	111	223	0	223
			400	1.524	117	175	0	175
			600	1.535	122	180	0	180
			800	1.544	125	189	0	189
		75	200	1.512	77	247	0	247
			400	1.532	81	131	0	131
			600	1.546	87	109	0	109
			800	1.557	91	104	0	104
		100	200	1.515	47	281	0	281
			400	1.538	50	145	0	145
			600	1.554	53	94	0	94
			800	1.566	57	77	0	77

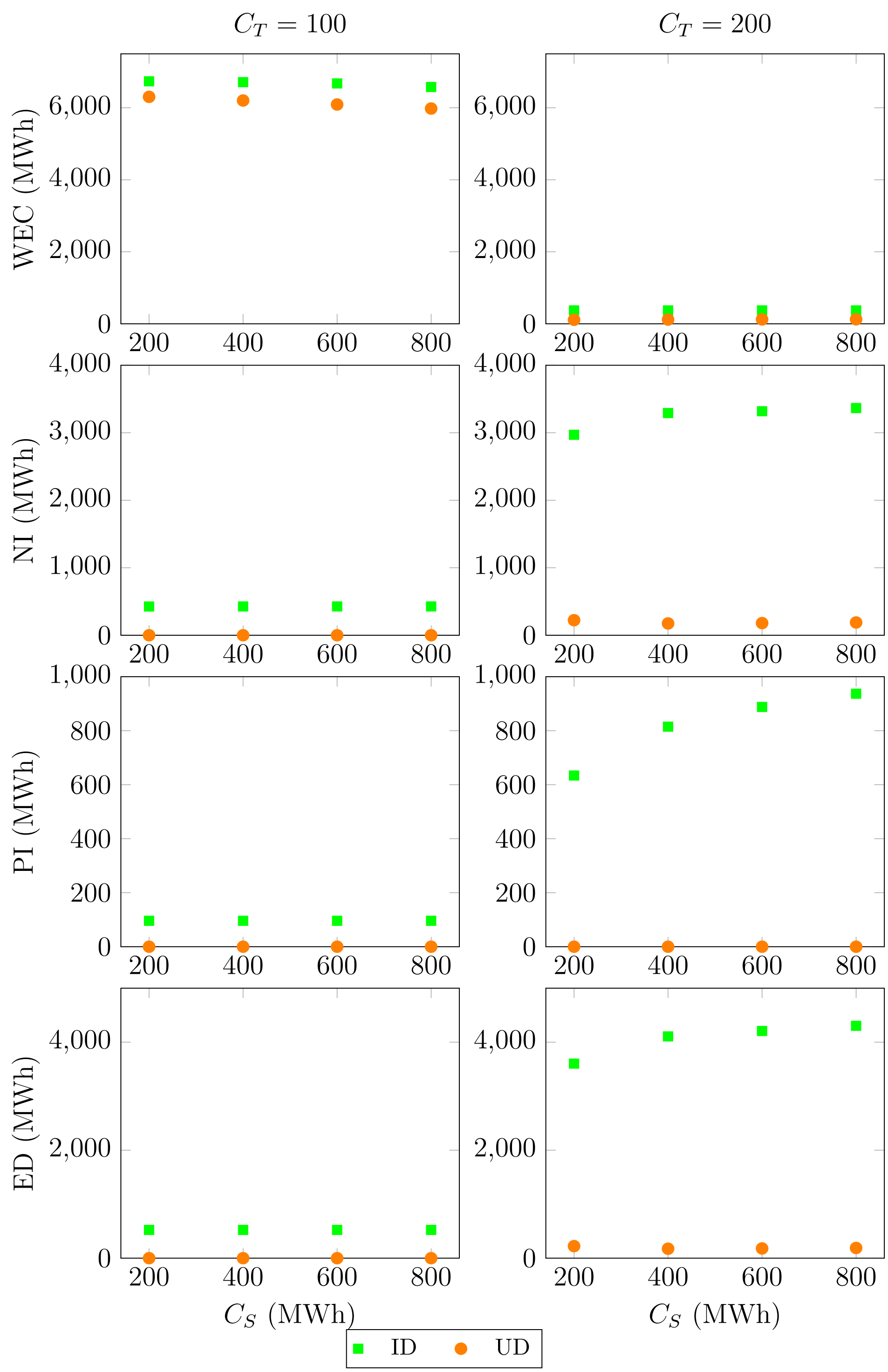


Figure A.1: WEC, NI, PI, and ED vs. C_S for Albany in January when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, $C_C = C_D = 50$ MWh, and NPF = 1.61%.

Table A.2: Numerical results for Albany in January for ID and ID-NB when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 1.61%.

Setting	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF	WEC	NI	PI	ED
ID	0.6	1.1	1.615	373	5867	224	6090
		1.2	1.601	373	2986	369	3354
		1.3	1.594	373	2226	416	2642
		1.4	1.589	373	1879	442	2320
	0.7	1.1	1.622	373	4891	278	5169
		1.2	1.611	373	2694	394	3088
		1.3	1.604	373	2103	438	2540
		1.4	1.600	373	1780	467	2247
	0.8	1.1	1.630	373	3995	348	4343
		1.2	1.621	373	2437	487	2924
		1.3	1.615	373	1968	569	2537
		1.4	1.611	373	1618	640	2258
	0.9	1.1	1.641	373	3292	815	4107
		1.2	1.634	373	1889	1215	3105
		1.3	1.630	373	1533	1464	2997
		1.4	1.627	373	1272	1623	2895
ID-NB	0.6	1.1	1.485	373	2225	261	2487
		1.2	1.474	373	1781	438	2219
		1.3	1.465	373	1569	568	2137
		1.4	1.457	373	1402	714	2116
	0.7	1.1	1.489	373	2130	290	2421
		1.2	1.479	373	1609	540	2149
		1.3	1.471	373	1402	714	2116
		1.4	1.465	373	1236	899	2135
	0.8	1.1	1.493	373	1781	438	2219
		1.2	1.485	373	1402	714	2116
		1.3	1.479	373	1176	984	2159
		1.4	1.474	373	990	1289	2279
	0.9	1.1	1.498	373	1402	714	2116
		1.2	1.493	373	990	1289	2279
		1.3	1.490	373	716	1941	2657
		1.4	1.487	373	584	2376	2960

Table A.3: Numerical results for Albany in January for UD and UD-NB when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 1.61%.

Setting	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF	WEC	NI	PI	ED
UD	0.6	1.1	1.524	117	173	0	173
		1.2	1.522	116	107	0	107
		1.3	1.522	116	77	0	77
		1.4	1.521	116	60	0	60
	0.7	1.1	1.524	117	174	0	174
		1.2	1.522	116	107	0	107
		1.3	1.522	116	77	0	77
		1.4	1.521	116	60	0	60
	0.8	1.1	1.524	117	175	0	175
		1.2	1.522	116	108	0	108
		1.3	1.522	116	77	0	77
		1.4	1.521	116	60	0	60
	0.9	1.1	1.524	117	175	0	175
		1.2	1.522	116	108	0	108
		1.3	1.522	116	77	0	77
		1.4	1.521	116	60	0	60
UD-NB	0.6	1.1	1.467	171	2404	0	2404
		1.2	1.452	228	2056	0	2056
		1.3	1.439	286	1821	0	1821
		1.4	1.427	354	1646	0	1646
	0.7	1.1	1.467	171	2404	0	2404
		1.2	1.452	226	2065	0	2065
		1.3	1.439	286	1821	0	1821
		1.4	1.427	351	1653	0	1653
	0.8	1.1	1.467	171	2405	0	2405
		1.2	1.452	225	2068	0	2068
		1.3	1.439	285	1823	0	1823
		1.4	1.427	350	1655	0	1655
	0.9	1.1	1.467	171	2405	0	2405
		1.2	1.452	223	2076	0	2076
		1.3	1.439	284	1828	0	1828
		1.4	1.427	348	1659	0	1659

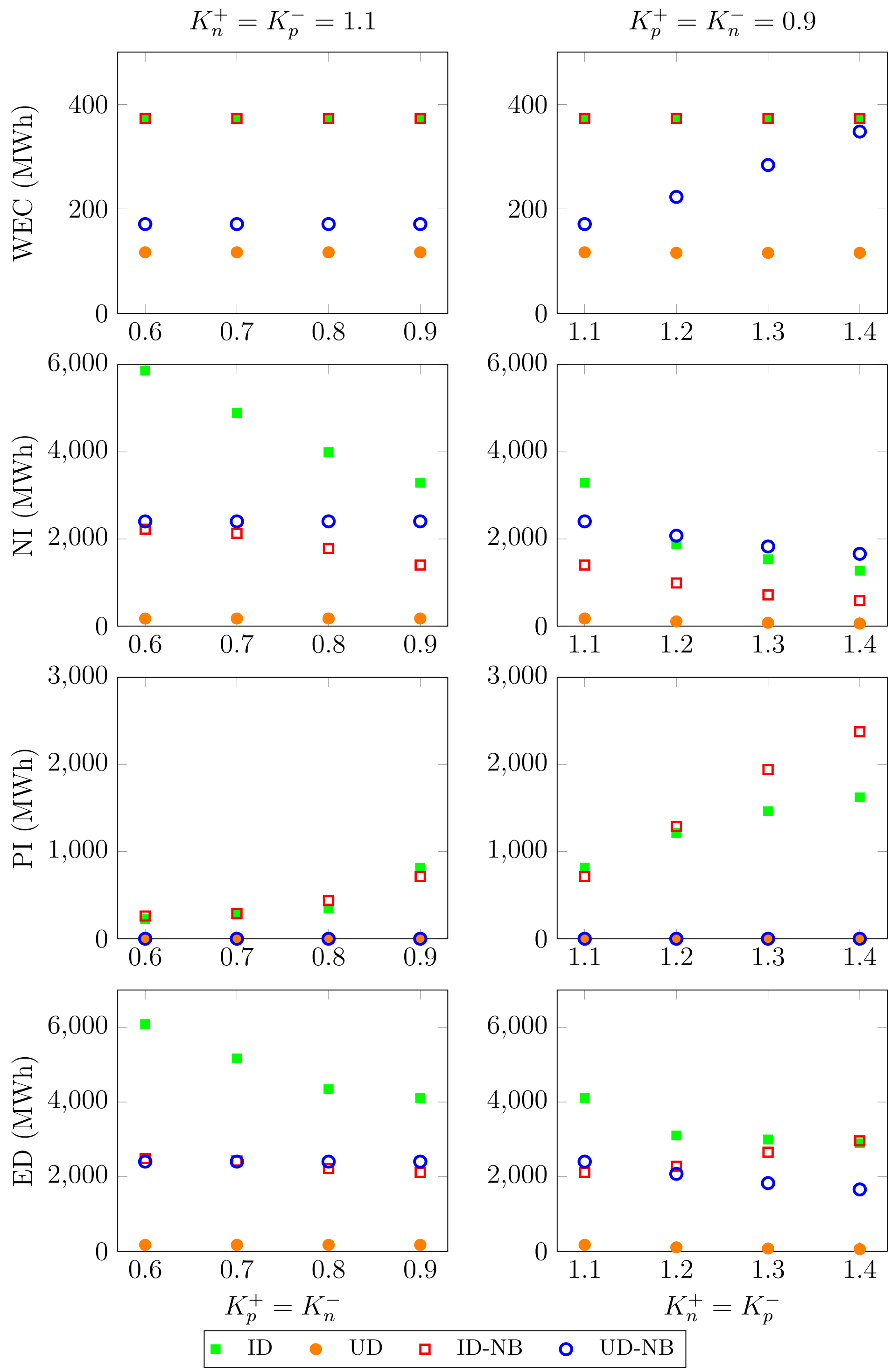


Figure A.2: WEC, NI, PI, and ED vs. $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$ for Albany in January when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 1.61%.

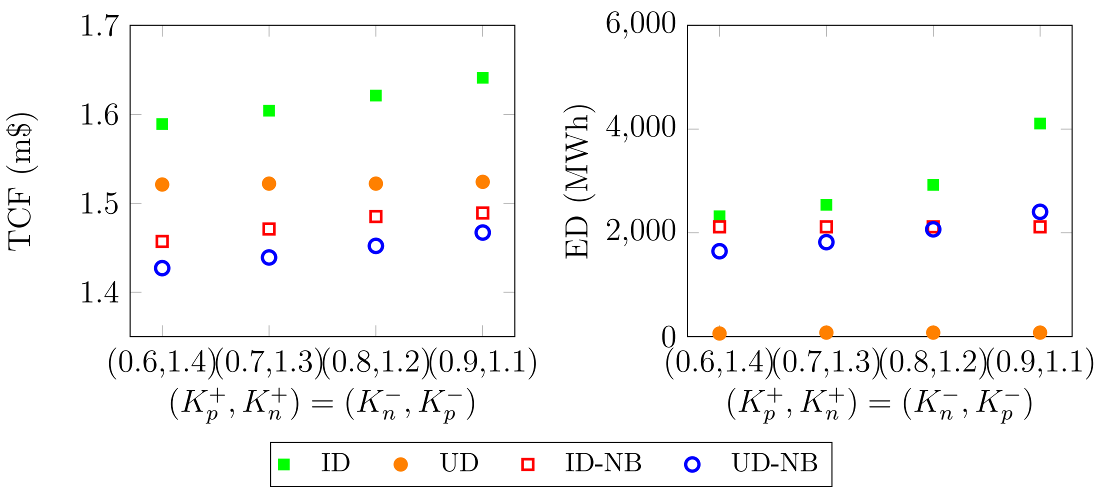


Figure A.3: The effects of $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$ on ED for Albany in January when $(K_p^+, K_n^+) = (K_n^-, K_p^-)$, $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 1.61%.

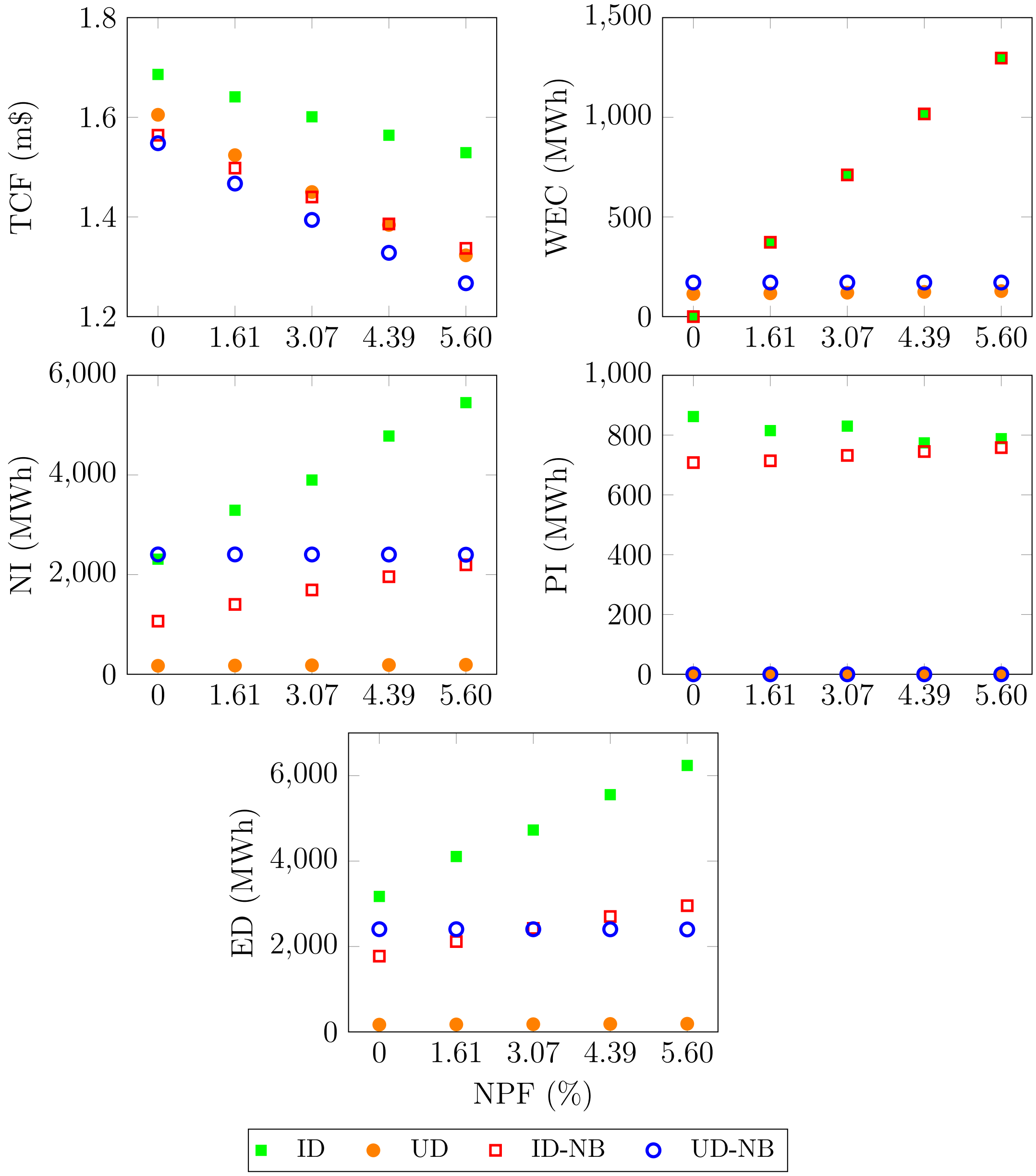


Figure A.4: TCF, WEC, NI, PI, and ED vs. NPF for Albany in January when $C_S = 400$ MWh (for ID and UD), $C_C = C_D = 50$ MWh (for ID and UD), $C_T = 200$ MWh, $K_p^+ = K_n^- = 0.9$, and $K_n^+ = K_p^- = 1.1$.

Table A.4: Numerical results for Buffalo in August when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, and NPF = 4.02%.

Setting	C_T	$C_C = C_D$	C_S	TCF	WEC	NI	PI	ED
ID	100	50	200	0.562	1796	2989	442	3431
			400	0.587	1510	3233	399	3632
			600	0.601	1333	3287	381	3668
			800	0.610	1214	3312	366	3678
		75	200	0.575	1840	3402	400	3802
			400	0.609	1576	3865	314	4179
			600	0.628	1412	4053	272	4325
			800	0.640	1295	4137	242	4379
		100	200	0.578	1803	3505	389	3894
			400	0.615	1539	4021	288	4309
			600	0.636	1385	4258	240	4498
			800	0.649	1263	4378	207	4585
	200	50	200	0.665	534	4162	1096	5258
			400	0.693	536	4365	1126	5491
			600	0.707	536	4530	1190	5721
			800	0.716	536	4543	1197	5740
		75	200	0.695	536	5067	912	5979
			400	0.738	540	5704	1256	6960
			600	0.761	543	5884	1338	7222
			800	0.776	544	6070	1375	7445
		100	200	0.718	537	6064	1060	7124
			400	0.771	537	6992	1305	8298
			600	0.803	543	7232	1427	8658
			800	0.823	544	7443	1494	8937
UD	100	50	200	0.488	1291	581	0	581
			400	0.507	976	422	0	422
			600	0.519	769	400	0	400
			800	0.528	625	407	0	407
		75	200	0.490	1290	472	0	472
			400	0.513	971	247	0	247
			600	0.527	765	181	0	181
			800	0.538	619	154	0	154
		100	200	0.491	1290	480	0	480
			400	0.515	963	240	0	240
			600	0.530	753	160	0	160
			800	0.541	604	121	0	121
	200	50	200	0.546	222	512	0	512
			400	0.561	254	505	0	505
			600	0.570	272	540	0	540
			800	0.576	282	574	0	574
		75	200	0.552	150	421	0	421
			400	0.573	165	388	0	388
			600	0.586	184	414	0	414
			800	0.596	198	444	0	444
		100	200	0.555	114	428	0	428
			400	0.581	115	334	0	334
			600	0.597	127	335	0	335
			800	0.609	139	349	0	349

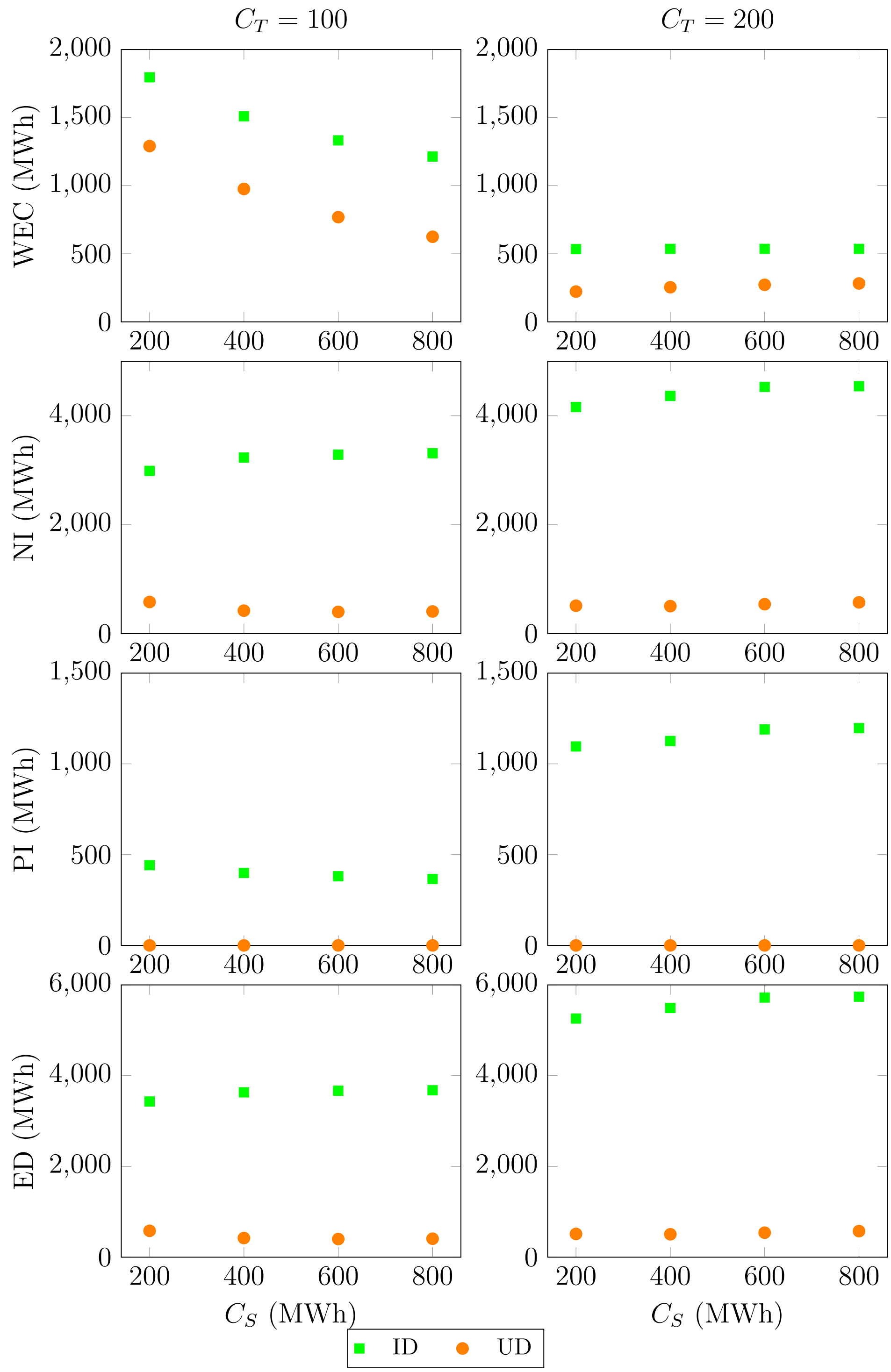


Figure A.5: WEC, NI, PI, and ED vs. C_S for Buffalo in August when $K_p^+ = K_n^- = 0.9$, $K_n^+ = K_p^- = 1.1$, $C_C = C_D = 50$ MWh, and NPF = 4.02%.

Table A.5: Numerical results for Buffalo in August for ID and ID-NB when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

Setting	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF	WEC	NI	PI	ED
ID	0.6	1.1	0.667	533	7775	193	7969
		1.2	0.654	533	5057	317	5374
		1.3	0.648	534	3772	398	4171
		1.4	0.644	535	3269	446	3715
	0.7	1.1	0.674	533	7202	245	7447
		1.2	0.663	534	4516	408	4924
		1.3	0.658	535	3472	520	3991
		1.4	0.654	536	3092	586	3679
	0.8	1.1	0.682	533	6243	450	6693
		1.2	0.674	534	3871	776	4646
		1.3	0.669	536	3078	946	4024
		1.4	0.667	537	2812	1072	3884
	0.9	1.1	0.693	536	4365	1126	5491
		1.2	0.688	538	3020	1627	4647
		1.3	0.685	539	2536	1903	4439
		1.4	0.683	540	2249	2165	4414
ID-NB	0.6	1.1	0.521	533	3522	643	4165
		1.2	0.510	533	2555	1011	3566
		1.3	0.502	533	2090	1307	3397
		1.4	0.496	533	1683	1658	3340
	0.7	1.1	0.526	533	3012	807	3819
		1.2	0.517	533	2226	1207	3433
		1.3	0.511	533	1683	1658	3340
		1.4	0.506	533	1466	1902	3368
	0.8	1.1	0.532	533	2555	1011	3566
		1.2	0.525	533	1683	1658	3340
		1.3	0.521	533	1364	2044	3408
		1.4	0.517	533	1083	2503	3586
	0.9	1.1	0.540	533	1683	1658	3340
		1.2	0.536	533	1083	2503	3586
		1.3	0.534	533	805	3137	3942
		1.4	0.532	533	664	3566	4231

Table A.6: Numerical results for Buffalo in August for UD and UD-NB when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

Setting	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF	WEC	NI	PI	ED
UD	0.6	1.1	0.561	253	491	0	491
		1.2	0.558	248	322	0	322
		1.3	0.557	244	246	0	246
		1.4	0.556	240	201	0	201
	0.7	1.1	0.561	253	495	0	495
		1.2	0.558	248	324	0	324
		1.3	0.557	244	247	0	247
		1.4	0.556	240	203	0	203
	0.8	1.1	0.561	253	500	0	500
		1.2	0.558	248	326	0	326
		1.3	0.557	244	248	0	248
		1.4	0.556	240	203	0	203
	0.9	1.1	0.561	254	505	0	505
		1.2	0.559	248	328	0	328
		1.3	0.557	244	249	0	249
		1.4	0.556	240	204	0	204
UD-NB	0.6	1.1	0.499	341	4671	0	4671
		1.2	0.481	514	3673	0	3673
		1.3	0.466	749	2851	0	2851
		1.4	0.455	923	2396	0	2396
	0.7	1.1	0.499	333	4735	0	4735
		1.2	0.481	506	3706	0	3706
		1.3	0.467	741	2876	0	2876
		1.4	0.455	918	2409	0	2409
	0.8	1.1	0.500	326	4796	0	4796
		1.2	0.482	497	3745	0	3745
		1.3	0.467	734	2895	0	2895
		1.4	0.456	913	2419	0	2419
	0.9	1.1	0.501	317	4867	0	4867
		1.2	0.482	491	3769	0	3769
		1.3	0.468	724	2923	0	2923
		1.4	0.456	909	2429	0	2429

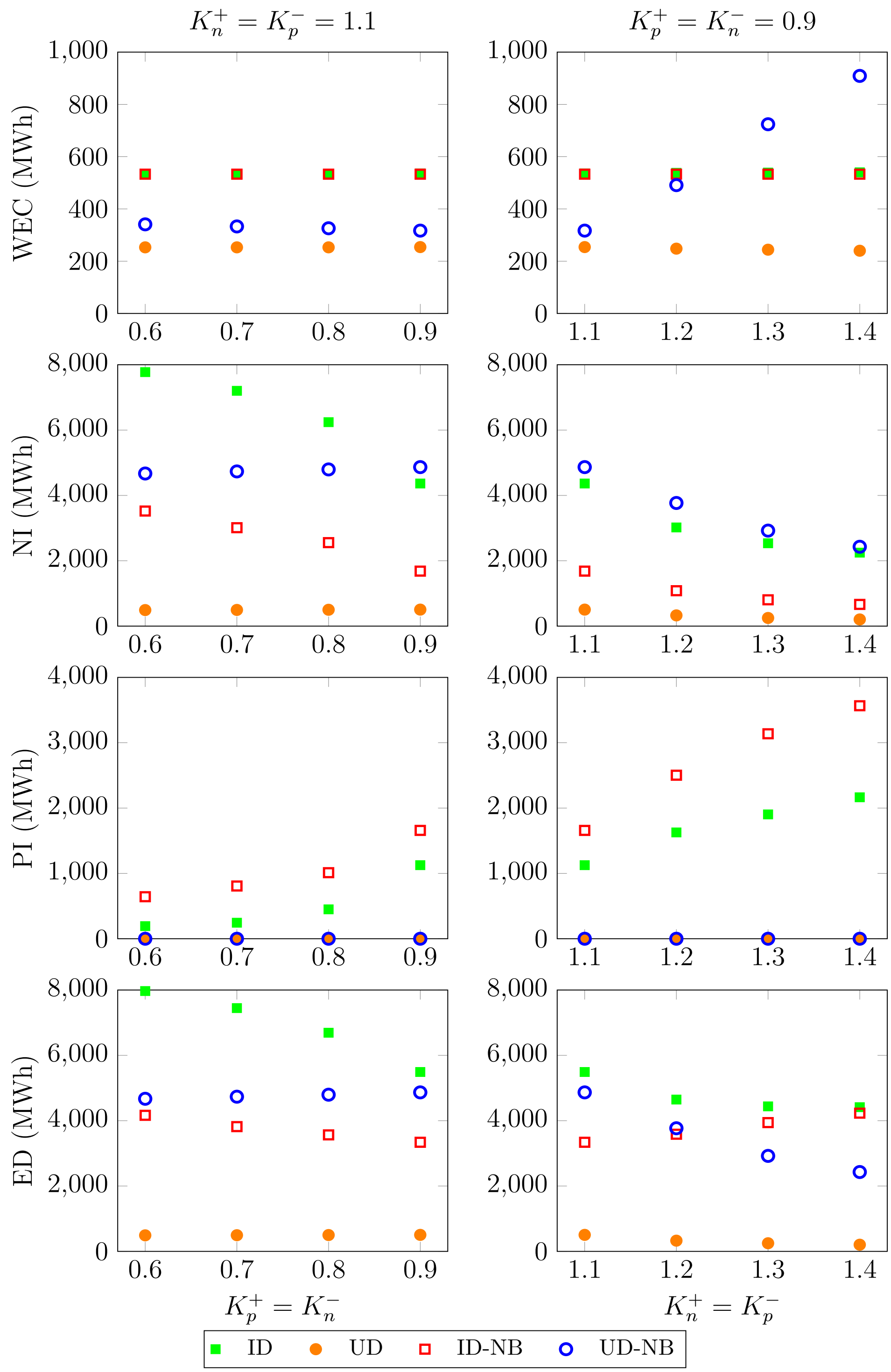


Figure A.6: WEC, NI, PI, and ED vs. $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$ for Buffalo in August when $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF = 4.02%.

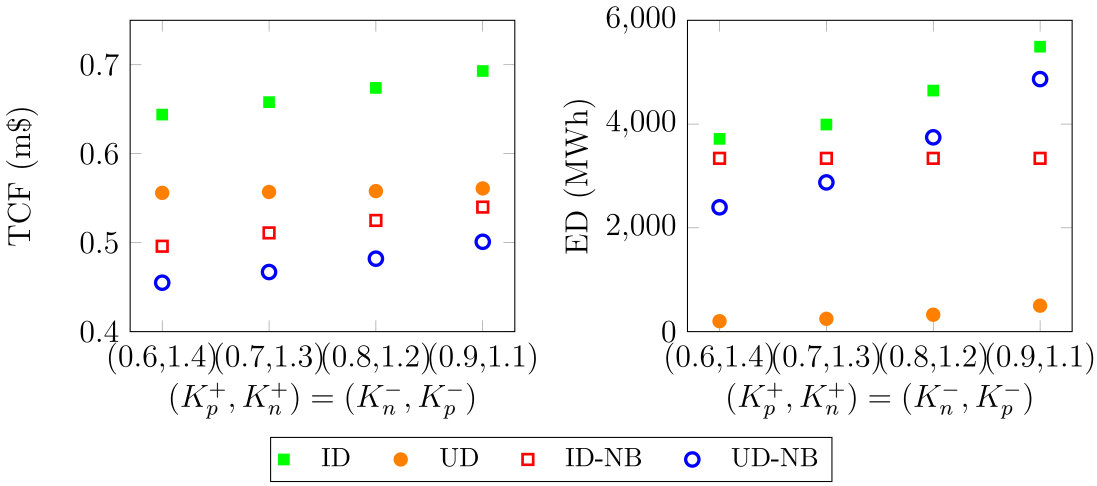


Figure A.7: The effects of $K_p^+ = K_n^-$ and $K_n^+ = K_p^-$ on ED for Buffalo in August when $(K_p^+, K_n^+) = (K_n^-, K_p^-)$, $C_S = 400$ MWh, $C_C = C_D = 50$ MWh, $C_T = 200$ MWh, and NPF $= 4.02\%$.

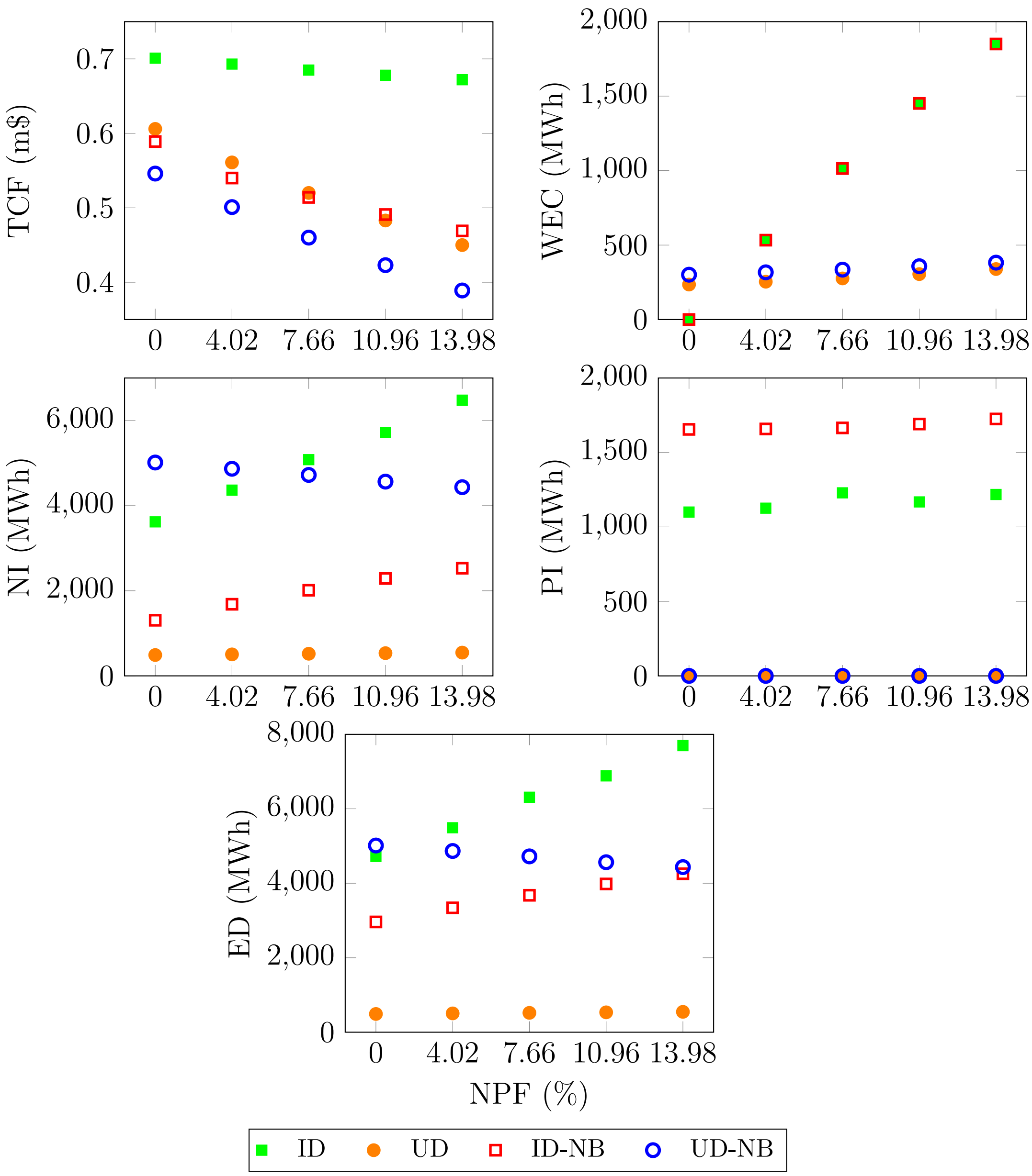


Figure A.8: TCF, WEC, NI, PI, and ED vs. NPF for Buffalo in August when C_S = 400 MWh (for ID and UD), $C_C = C_D$ = 50 MWh (for ID and UD), $C_T = 200$ MWh, $K_p^+ = K_n^- = 0.9$, and $K_n^+ = K_p^- = 1.1$.

Appendix B

Supplement to optimal policy characterization under perfect efficiency

This chapter includes supplementary material for Chapter 4: Commitment and Storage Problem of Wind Power Producers: Optimal Policy Characterization under Perfect Efficiency.

Proof of Lemma 4.3.1. Let $\underline{Q}_t$ and $\bar{Q}_t$ denote some bounds on $E(s_t, w_t)$ in state (Q_t, S_t, I_t) such that $\underline{Q}_t \leq E(s_t, w_t) \leq \bar{Q}_t$. Note that $R(\cdot, I_t, s_t, w_t)$ is a decreasing function for $Q_t \geq E(s_t, w_t)$ since $P_t(1 - K_n^+) < 0$ for $P_t \geq 0$ and $P_t(1 - K_n^-) < 0$ for $P_t < 0$. Thus $R(\bar{Q}_t + \alpha, I_t, s_t, w_t) < R(\bar{Q}_t, I_t, s_t, w_t)$ for $\alpha > 0$. This implies that

$$
\begin{aligned}
v_t^*(\bar{Q}_t + \alpha, S_t, I_t) &= \max_{(q_t, s_t, w_t) \in \mathbb{U}(\bar{Q}_t + \alpha, S_t, I_t)} \Big\{ R(\bar{Q}_t + \alpha, I_t, s_t, w_t) \qquad \text{(B.1)} \\
&\qquad + \mathbb{E}_{I_{t+1}|I_t} \Big[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \Big] \Big\} \\
&< \max_{(q_t, s_t, w_t) \in \mathbb{U}(\bar{Q}_t, S_t, I_t)} \Big\{ R(\bar{Q}_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \Big[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \Big] \Big\} \\
&= v_t^*(\bar{Q}_t, S_t, I_t). \qquad \text{(B.2)}
\end{aligned}
$$

Also, note that $R(\cdot, I_t, s_t, w_t)$ is an increasing function for $Q_t \le E(s_t, w_t)$ since $P_t(1 - K_p^+) > 0$ for $P_t \ge 0$ and $P_t(1 - K_p^-) > 0$ for $P_t < 0$. Thus $R(\underline{Q}_t - \alpha, I_t, s_t, w_t) < R(\underline{Q}_t, I_t, s_t, w_t)$. This implies that

$$
\begin{aligned}
v_t^*(\underline{Q}_t - \alpha, S_t, I_t) &= \max_{(q_t, s_t, w_t) \in \mathbb{U}(\underline{Q}_t - \alpha, S_t, I_t)} \Big\{ R(\underline{Q}_t - \alpha, I_t, s_t, w_t) && \text{(B.3)} \\
&\qquad + \mathbb{E}_{I_{t+1}|I_t} \Big[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \Big] \Big\} \\
&< \max_{(q_t, s_t, w_t) \in \mathbb{U}(\underline{Q}_t, S_t, I_t)} \Big\{ R(\underline{Q}_t, I_t, s_t, w_t) + \mathbb{E}_{I_{t+1}|I_t} \Big[v_{t+1}^*(q_t, S_{t+1}, I_{t+1}) \Big] \Big\} \\
&= v_t^*(\underline{Q}_t, S_t, I_t), \forall t \in \mathcal{T}. && \text{(B.4)}
\end{aligned}
$$

Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$. Assume to the contrary that $q > \bar{Q}_{t+1}$. Then, $v_t^*(Q_t, S_t, I_t) = R(Q_t, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \ge R(Q_t, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(\bar{Q}_{t+1}, S_t - s, I_{t+1})\Big]$. But this leads to a contradiction since $v_{t+1}^*(q, S_t - s, I_{t+1}) < v_{t+1}^*(\bar{Q}_{t+1}, S_t - s, I_{t+1})$ from (B.2). Hence $q \le \bar{Q}_{t+1}$. Now, assume to the contrary that $q < \underline{Q}_{t+1}$. Then, $v_t^*(Q_t, S_t, I_t) = R(Q_t, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \ge R(Q_t, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(\underline{Q}_{t+1}, S_t - s, I_{t+1})\Big]$. But this leads to a contradiction since $v_{t+1}^*(q, S_t - s, I_{t+1}) < v_{t+1}^*(\underline{Q}_{t+1}, S_t - s, I_{t+1})$ from (B.4). Hence $q \ge \underline{Q}_{t+1}$. We showed that $\underline{Q}_{t+1} \le q_t \le \bar{Q}_{t+1}$. Since $-\min\{(C_S - S_{t+1})/(\theta\tau), C_C/(\theta\tau), C_T\} \le E(s_{t+1}, w_{t+1}) \le \tau C_T$, note that $-\min\{(C_S - (S_t - s_t))/(\theta\tau), C_C/(\theta\tau), C_T\} \le q_t \le \tau C_T$, $\forall t \in \mathcal{T}$. Since $Q_{t+1} = q_t$, we have $-\min\{(C_S - S_t)/(\theta\tau), C_C/(\theta\tau), C_T\} \le Q_t \le \tau C_T$, $\forall t \in \mathcal{T} \backslash \{1\}$. $\square$

Proof of Lemma 4.3.2. For $\alpha > 0$, note that $v_T^*(Q_T, S_T, I_T) = v_T^*(Q_T, S_T + \alpha, I_T) = 0$. Assuming $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \le v_{t+1}^*(Q_{t+1}, S_{t+1} + \alpha, I_{t+1})$, we show $v_t^*(Q_t, S_t, I_t) \le v_t^*(Q_t, S_t + \alpha, I_t)$. Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$. Also, let $\hat{s} = \max\{s, S_t + \alpha - C_S\}$ and

$$
\hat{w} = \begin{cases} w & \text{if } \hat{s} = s, \\ \max\{0, w - (\hat{s} - s)/\theta\} & \text{if } \hat{s} \ne s. \end{cases}
$$

We show that $(q, \hat{s}, \hat{w}) \in \mathbb{U}(Q_t, S_t + \alpha, I_t)$: If $s < S_t + \alpha - C_S$, since $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$, note that $-C_C \leq s < S_t + \alpha - C_S = \hat{s} \leq \min\{S_t + \alpha, C_D\}$. If $s \geq S_t + \alpha - C_S$, since $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$, note that $-\min\{C_S - S_t - \alpha, C_C\} \leq \hat{s} = s \leq \min\{S_t + \alpha, C_D\}$. Thus $-\min\{C_S - S_t - \alpha, C_C\} \leq \hat{s} \leq \min\{S_t + \alpha, C_D\}$. Since $s \leq \hat{s}$, we have $0 \leq \hat{w} \leq w \leq f(W_t)$. If $\hat{s} = s$, then $-C_T \leq E(s, w) = E(\hat{s}, \hat{w}) \leq \tau C_T$. If $s < \hat{s} = S_t + \alpha - C_S \leq 0$, then $\hat{s}/\theta + \hat{w} = \hat{s}/\theta + \max\{0, w - (\hat{s} - s)/\theta\} = \max\{\hat{s}/\theta, s/\theta + w\} \leq C_T$ and $-\tau C_T \leq s/\theta + w \leq \hat{s}/\theta + \hat{w}$. Hence $(q, \hat{s}, \hat{w}) \in \mathbb{U}(Q_t, S_t + \alpha, I_t)$. We consider the following three cases to show that $E(s, w) \leq E(\hat{s}, \hat{w})$:

(1) If $s \geq S_t + \alpha - C_S$, then $\hat{s} = s$ and $\hat{w} = w$. Thus $E(s, w) = E(\hat{s}, \hat{w})$.

(2) If $s < S_t + \alpha - C_S \leq 0$ and $-w \leq s/\theta < 0$, then $\hat{s} \leq 0$ and $\hat{s}/\theta + \hat{w} = \max\{\hat{s}/\theta, s/\theta + w\} = s/\theta + w \geq 0$. Thus $E(s, w) = (s/\theta + w)\tau = (\hat{s}/\theta + \hat{w})\tau = E(\hat{s}, \hat{w})$.

(3) If $s < S_t + \alpha - C_S \leq 0$ and $s/\theta < -w \leq 0$, then $\hat{s} \leq 0$ and $\hat{s}/\theta + \hat{w} = \max\{\hat{s}/\theta, s/\theta + w\} \leq 0$. Thus $E(s, w) = (s/\theta + w)/\tau \leq (\hat{s}/\theta + \hat{w})/\tau = E(\hat{s}, \hat{w})$.

Hence $E(s, w) \leq E(\hat{s}, \hat{w})$. Thus $R(Q_t, I_t, s, w) \leq R(Q_t, I_t, \hat{s}, \hat{w})$. Finally, note that $\hat{s} \leq s + \alpha$ since $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$. Thus:

$$\begin{aligned} v_t^*(Q_t, S_t, I_t) &= R(Q_t, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\ &\leq R(Q_t, I_t, \hat{s}, \hat{w}) + \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha - \hat{s}, I_{t+1})\Big] \\ &\leq v_t^*(Q_t, S_t + \alpha, I_t). \end{aligned}$$

The first inequality above holds as we assume $v_{t+1}^*(Q_{t+1}, S_{t+1}, I_{t+1}) \leq v_{t+1}^*(Q_{t+1}, S_{t+1} + \alpha, I_{t+1})$. □

Proof of Lemma 4.3.3. Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$. Also, let $\bar{w} := \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}/\theta\}$ denote the maximum amount of wind energy that can be generated in state (Q_t, S_t, I_t) and $(q, \hat{s}, \hat{w}) \in \mathbb{U}(Q_t, S_t, I_t)$ denote any feasible action triplet with $\hat{w} < \bar{w}$. We will show that it is possible to construct a feasible action triplet that leads to a larger profit than the one under the action triplet $(q, \hat{s}, \hat{w})$ in each of the following four cases:

(1) Suppose that $E(\hat{s}, \hat{w}) < 0$: We define $\Delta_1 = \min\{-\tau E(\hat{s}, \hat{w}), \bar{w} - \hat{w}\} > 0$. Note that $-C_T \le E(\hat{s}, \hat{w}) < E(\hat{s}, \hat{w}) + \Delta_1/\tau = E(\hat{s}, \hat{w} + \Delta_1) \le 0$ and $0 \le \hat{w} < \hat{w} + \Delta_1 \le \bar{w} \le f(W_t)$. Hence, $(q, \hat{s}, \hat{w} + \Delta_1) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w} + \Delta_1) > R(Q_t, I_t, \hat{s}, \hat{w})$. Thus, the triplet $(q, \hat{s}, \hat{w} + \Delta_1)$ is more profitable than the triplet $(q, \hat{s}, \hat{w})$.

(2) Suppose that $E(\hat{s}, \hat{w}) \ge 0$ and $\hat{s} = -\min\{C_S - S_t, C_C\}$: We define $\Delta_2 = \bar{w} - \hat{w} > 0$. Note that $0 \le E(\hat{s}, \hat{w}) < E(\hat{s}, \hat{w} + \Delta_2) = (\hat{s}/\theta + \hat{w} + \Delta_2)\tau = (\hat{s}/\theta + \bar{w})\tau \le (-\min\{C_S - S_t, C_C\}/\theta + C_T + \min\{C_S - S_t, C_C\}/\theta)\tau = \tau C_T$ and $0 \le \hat{w} < \hat{w} + \Delta_2 = \bar{w} \le f(W_t)$. Hence, $(q, \hat{s}, \hat{w} + \Delta_2) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w} + \Delta_2) > R(Q_t, I_t, \hat{s}, \hat{w})$. Thus, the triplet $(q, \hat{s}, \hat{w} + \Delta_2)$ is more profitable than the triplet $(q, \hat{s}, \hat{w})$.

(3) Suppose that $E(\hat{s}, \hat{w}) \ge 0$ and $-\min\{C_S - S_t, C_C\} < \hat{s} \le 0$: We define $\Delta_3 = \min\{\min\{C_S - S_t, C_C\}/\theta + \hat{s}/\theta, \bar{w} - \hat{w}\} > 0$. Since $0 < \Delta_3 \le \min\{C_S - S_t, C_C\}/\theta + \hat{s}/\theta$, note that $-\min\{C_S - S_t, C_C\} \le \hat{s} - \theta\Delta_3 < \hat{s} \le 0$. Also, note that $E(\hat{s}, \hat{w}) = (\hat{s}/\theta + \hat{w})\tau = ((\hat{s} - \theta\Delta_3)/\theta + \hat{w} + \Delta_3)\tau = E(\hat{s} - \theta\Delta_3, \hat{w} + \Delta_3)$ and $0 \le \hat{w} < \hat{w} + \Delta_3 \le \bar{w} \le f(W_t)$. Hence, $(q, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w}) = R(Q_t, I_t, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3)$. Then, by Lemma 4.3.2,

$$\begin{aligned} R(Q_t, I_t, \hat{s}, \hat{w}) + \mathbb{E}\Big[v^*_{t+1}(q, S_t - \hat{s}, I_{t+1})\Big] < R(Q_t, I_t, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3) \\ + \mathbb{E}\Big[v^*_{t+1}(q, S_t - \hat{s} + \theta\Delta_3, I_{t+1})\Big]. \end{aligned}$$

Thus, the triplet $(q, \hat{s} - \theta\Delta_3, \hat{w} + \Delta_3)$ is more profitable than the triplet $(q, \hat{s}, \hat{w})$.

(4) Suppose that $E(\hat{s}, \hat{w}) \ge 0$ and $\hat{s} > 0$: We define $\Delta_4 = \min\{\gamma\hat{s}, \bar{w} - \hat{w}\} > 0$. Since $0 < \Delta_4 \le \gamma\hat{s}$, note that $0 \le \hat{s} - \Delta_4/\gamma < \hat{s} \le \min\{S_t, C_D\}$. Also, note that $E(\hat{s}, \hat{w}) = (\gamma\hat{s} + \hat{w})\tau = (\gamma(\hat{s} - \Delta_4/\gamma) + \hat{w} + \Delta_4)\tau = E(\hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4)$

and $0 \leq \hat{w} < \hat{w} + \Delta_4 \leq \bar{w} \leq f(W_t)$. Hence, $(q, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4) \in \mathbb{U}(Q_t, S_t, I_t)$ and $R(Q_t, I_t, \hat{s}, \hat{w}) = R(Q_t, I_t, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4)$. Then, by Lemma 4.3.2,

$$R(Q_t, I_t, \hat{s}, \hat{w}) + \mathbb{E}\Big[v^*_{t+1}(q, S_t - \hat{s}, I_{t+1})\Big] < R(Q_t, I_t, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4) + \mathbb{E}\Big[v^*_{t+1}(q, S_t - \hat{s} + \Delta_4/\gamma, I_{t+1})\Big].$$

Thus, the triplet $(q, \hat{s} - \Delta_4/\gamma, \hat{w} + \Delta_4)$ is more profitable than the triplet $(q, \hat{s}, \hat{w})$.

Hence we showed that $w = \bar{w}$. Next, suppose that $w = C_T + \min\{C_S - S_t, C_C\}/\theta$. We will show that $s = -\min\{C_S - S_t, C_C\}$. First, assume to the contrary that $s > 0$. In this case, $w\tau < (\gamma s + w)\tau = E(s, w) \leq \tau C_T$. But this leads to a contradiction since $w \geq C_T$. Thus $s \leq 0$. Note that $-\min\{C_S - S_t, C_C\} \leq s$ since $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$. Also, note that $(s/\theta + C_T + \min\{C_S - S_t, C_C\}/\theta)\tau = E(s, w) \leq \tau C_T$, implying that $-\min\{C_S - S_t, C_C\} \geq s$. Hence, the only feasible action is $s = -\min\{C_S - S_t, C_C\}$. □

Proof of Proposition 4.3.1. Note that $v^*_T(\cdot, \cdot, I_T)$ satisfies properties (a)-(c), $\forall I_T$. Pick an arbitrary $t < T$ and fix I_t. Assuming $v^*_{t+1}(\cdot, \cdot, I_{t+1})$ satisfies properties (a)-(c), $\forall I_{t+1}$, we will prove that $v^*_t(\cdot, \cdot, I_t)$ satisfies properties (a)-(c).

(a) First we prove that $v^*_t(Q_t + \alpha, S_t, I_t) - v^*_t(Q_t, S_t, I_t) \leq v^*_t(Q_t + \alpha, S_t + \beta, I_t) - v^*_t(Q_t, S_t + \beta, I_t)$. Let $\eta^*_t(Q_t + \alpha, S_t, I_t) = (q, s, w)$ and $\eta^*_t(Q_t, S_t + \beta, I_t) = (q', s', w')$. We consider the following four scenarios to prove the statement:

(a1) Suppose that $s' \geq s$. We show that $E(s', w') \geq E(s, w)$: If $w' < C_T + C_S - S_t - \beta$, by Lemma 4.3.3, $w' = \min\{f(W_t), C_T + C_C\} < C_T + C_S - S_t - \beta < C_T + C_S - S_t$. Thus, again by Lemma 4.3.3, $w = \min\{f(W_t), C_T + C_C\} = w'$. Since $s' \geq s$, we have $E(s', w') = s' + w' \geq s + w = E(s, w)$. If $w' = C_T + C_S - S_t - \beta$, by Lemma 4.3.3, $s' = S_t + \beta - C_S$. Thus, and since $(q, s, w) \in \mathbb{U}(Q_t + \alpha, S_t, I_t)$, we have $E(s', w') = C_T \geq E(s, w)$. Hence

$E(s', w') = E(s, w) + \delta$ for some $\delta \geq 0$. Note that $(q, s, w) \in \mathbb{U}(Q_t, S_t, I_t)$ and $(q', s', w') \in \mathbb{U}(Q_t + \alpha, S_t + \beta, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t + \alpha, S_t, I_t) - v_t^*(Q_t, S_t, I_t) \\
& \leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s, w) - \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\
& = R(Q_t + \alpha + \delta, I_t, s', w') - R(Q_t + \delta, I_t, s', w') \\
& \leq R(Q_t + \alpha, I_t, s', w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t + \beta - s', I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t + \beta - s', I_{t+1})\Big] \\
& \leq v_t^*(Q_t + \alpha, S_t + \beta, I_t) - v_t^*(Q_t, S_t + \beta, I_t).
\end{aligned}
$$

The second inequality above holds since $R(\cdot, I_t, s', w')$ is concave.

(a2) Suppose that $s' < s$ and $q' - q > s - s'$. Since $(q', s', w') \in \mathbb{U}(Q_t, S_t + \beta, I_t)$, $(q, s, w) \in \mathbb{U}(Q_t + \alpha, S_t, I_t)$, and $s' < s$, we have $-\min\{C_S - S_t, C_C\} \leq -\min\{C_S - S_t - \beta, C_C\} \leq s' < s \leq \min\{S_t, C_D\} \leq \min\{S_t + \beta, C_D\}$. Hence, $(q + s - s', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$ and $(q' - s + s', s, w) \in \mathbb{U}(Q_t + \alpha, S_t + \beta, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t + \alpha, S_t, I_t) - v_t^*(Q_t, S_t, I_t) \\
& \leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q + s - s', S_t - s', I_{t+1})\Big] \\
& \leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q' - s + s', S_t - s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
& \leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q' - s + s', S_t + \beta - s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t + \beta - s', I_{t+1})\Big] \\
& \leq v_t^*(Q_t + \alpha, S_t + \beta, I_t) - v_t^*(Q_t, S_t + \beta, I_t).
\end{aligned}
$$

The second inequality above holds since $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ satisfies property (c). The third inequality above holds since $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ satisfies property (b).

(a3) Suppose that $s' < s$ and $s - s' \geq q' - q > 0$. Recall from scenario (a2) that $-\min\{C_S - S_t, C_C\} \leq s' < \min\{S_t, C_D\}$ and $-\min\{C_S - S_t - \beta, C_C\} < s \leq \min\{S_t + \beta, C_D\}$. Hence, $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$ and $(q, s, w) \in \mathbb{U}(Q_t + \alpha, S_t + \beta, I_t)$. Thus:

$$\begin{aligned}
&v_t^*(Q_t + \alpha, S_t, I_t) - v_t^*(Q_t + \alpha, S_t + \beta, I_t) \\
&\leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\
&- R(Q_t + \alpha, I_t, s, w) - \mathbb{E}\Big[v_{t+1}^*(q, S_t + \beta - s, I_{t+1})\Big] \\
&\leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t + q - q' - s', I_{t+1})\Big] \\
&- R(Q_t + \alpha, I_t, s, w) - \mathbb{E}\Big[v_{t+1}^*(q, S_t + \beta + q - q' - s', I_{t+1})\Big] \\
&\leq R(Q_t, I_t, s', w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
&- R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t + \beta - s', I_{t+1})\Big] \\
&\leq v_t^*(Q_t, S_t, I_t) - v_t^*(Q_t, S_t + \beta, I_t).
\end{aligned}$$

The second inequality above holds since $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ satisfies properties (a) and (b) (which together imply the concavity of $v_{t+1}^*(q, \cdot, I_{t+1})$). The third inequality above holds since $v_{t+1}^*(\cdot, \cdot, I_{t+1})$ satisfies property (b).

(a4) Suppose that $s' < s$ and $0 \geq q' - q$. Since $\eta_t^*(Q_t + \alpha, S_t, I_t) = (q, s, w)$, we have

$$\begin{aligned}
v_t^*(Q_t + \alpha, S_t, I_t) &= R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] \\
&> R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q', S_t - s, I_{t+1})\Big].
\end{aligned}$$

Since $\eta_t^*(Q_t, S_t + \beta, I_t) = (q', s', w')$, we have

$$\begin{aligned}
v_t^*(Q_t, S_t + \beta, I_t) &= R(Q_t, I_t, s', w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t + \beta - s', I_{t+1})\Big] \\
&> R(Q_t, I_t, s', w') + \mathbb{E}\Big[v_{t+1}^*(q, S_t + \beta - s', I_{t+1})\Big].
\end{aligned}$$

The inequalities above imply that

$$\begin{aligned}
&\mathbb{E}\Big[v_{t+1}^*(q, S_t - s, I_{t+1})\Big] - \mathbb{E}\Big[v_{t+1}^*(q', S_t - s, I_{t+1})\Big] \\
&> \mathbb{E}\Big[v_{t+1}^*(q, S_t + \beta - s', I_{t+1})\Big] - \mathbb{E}\Big[v_{t+1}^*(q', S_t + \beta - s', I_{t+1})\Big].
\end{aligned}$$

But this leads to a contradiction since $v^*_{t+1}(\cdot, \cdot, I_{t+1})$ satisfies property (a). Hence this scenario is not possible.

(b) Next we prove that $v^*_t(Q_t + \alpha, S_t + \alpha + \beta, I_t) - v^*_t(Q_t + \alpha, S_t + \alpha, I_t) \leq v^*_t(Q_t, S_t + \beta, I_t) - v^*_t(Q_t, S_t, I_t)$. Let $\eta^*_t(Q_t + \alpha, S_t + \alpha + \beta, I_t) = (q, s, w)$ and $\eta^*_t(Q_t, S_t, I_t) = (q', s', w')$. We consider the following seven scenarios to prove the statement:

(b1) Suppose that $s \leq s' + \alpha$ and $q - q' > \alpha + \beta - s + s'$. Since $\eta^*_t(Q_t + \alpha, S_t + \alpha + \beta, I_t) = (q, s, w)$, we have

$$\begin{aligned}
&v^*_t(Q_t + \alpha, S_t + \alpha + \beta, I_t) \\
&= R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v^*_{t+1}(q, S_t + \alpha + \beta - s, I_{t+1})\Big] \\
&> R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v^*_{t+1}(q' + \alpha + \beta - s + s', S_t + \alpha + \beta - s, I_{t+1})\Big].
\end{aligned}$$

Since $\eta^*_t(Q_t, S_t, I_t) = (q', s', w')$, we have

$$\begin{aligned}
v^*_t(Q_t, S_t, I_t) &= R(Q_t, I_t, s', w') + \mathbb{E}\Big[v^*_{t+1}(q', S_t - s', I_{t+1})\Big] \\
&> R(Q_t, I_t, s', w') + \mathbb{E}\Big[v^*_{t+1}(q - \alpha - \beta + s - s', S_t - s', I_{t+1})\Big].
\end{aligned}$$

The inequalities above imply that

$$\begin{aligned}
&\mathbb{E}\Big[v^*_{t+1}(q, S_t + \alpha + \beta - s, I_{t+1})\Big] \\
&- \mathbb{E}\Big[v^*_{t+1}(q' + \alpha + \beta - s + s', S_t + \alpha + \beta - s, I_{t+1})\Big] \\
&> \mathbb{E}\Big[v^*_{t+1}(q - \alpha - \beta + s - s', S_t - s', I_{t+1})\Big] - \mathbb{E}\Big[v^*_{t+1}(q', S_t - s', I_{t+1})\Big].
\end{aligned}$$

But this leads to a contradiction since $v^*_{t+1}(\cdot, \cdot, I_{t+1})$ satisfies property (c). Hence this scenario is not possible.

(b2) Suppose that $s \leq s' + \alpha$ and $\alpha + \beta - s + s' \geq q - q' > \alpha - s + s'$. We show that $(q - \alpha + s - s', s', w') \in \mathbb{U}(Q_t, S_t + \beta, I_t)$: Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, note that $-C_C \leq s' \leq C_D$ and $s' \leq S_t < S_t + \beta$. Since

$(q,s,w) \in \mathbb{U}(Q_t+\alpha, S_t+\alpha+\beta, I_t)$, note that $S_t+\beta-C_S \leq s-\alpha \leq s'$. Thus $-\min\{C_S-S_t-\beta, C_C\} \leq s' \leq \min\{S_t+\beta, C_D\}$. Hence $(q-\alpha+s-s', s', w') \in \mathbb{U}(Q_t, S_t+\beta, I_t)$. We also show that $(q'+\alpha-s+s', s, w) \in \mathbb{U}(Q_t+\alpha, S_t+\alpha, I_t)$: Since $(q,s,w) \in \mathbb{U}(Q_t+\alpha, S_t+\alpha+\beta, I_t)$, note that $-C_C \leq s \leq C_D$ and $S_t+\alpha-C_S < S_t+\alpha+\beta-C_S \leq s$. Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, note that $s \leq s'+\alpha \leq S_t+\alpha$. Thus $-\min\{C_S-S_t-\alpha, C_C\} \leq s \leq \min\{S_t+\alpha, C_D\}$. Hence $(q'+\alpha-s+s', s, w) \in \mathbb{U}(Q_t+\alpha, S_t+\alpha, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t+\alpha, S_t+\alpha+\beta, I_t) - v_t^*(Q_t, S_t+\beta, I_t) \\
& \leq R(Q_t+\alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha+\beta-s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q-\alpha+s-s', S_t+\beta-s', I_{t+1})\Big] \\
& \leq R(Q_t+\alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q'+\alpha-s+s', S_t+\alpha+\beta-s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t+\beta-s', I_{t+1})\Big] \\
& \leq R(Q_t+\alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q'+\alpha-s+s', S_t+\alpha-s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t-s', I_{t+1})\Big] \\
& \leq v_t^*(Q_t+\alpha, S_t+\alpha, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}
$$

The second inequality above holds since $v_{t+1}^*(\cdot,\cdot,I_{t+1})$ satisfies property (c). The third inequality above holds since $v_{t+1}^*(\cdot,\cdot,I_{t+1})$ satisfies property (b).

(b3) Suppose that $s \leq s'+\alpha$ and $\alpha-s+s' \geq q-q' > 0$. Recall from scenario (b2) that $-\min\{C_S-S_t-\alpha, C_C\} \leq s \leq \min\{S_t+\alpha, C_D\}$ and $-\min\{C_S-S_t-\beta, C_C\} \leq s' \leq \min\{S_t+\beta, C_D\}$. Hence, $(q,s,w) \in \mathbb{U}(Q_t+\alpha, S_t+\alpha, I_t)$ and $(q', s', w') \in \mathbb{U}(Q_t, S_t+\beta, I_t)$. Thus:

$$
\begin{aligned}
&v_t^*(Q_t+\alpha, S_t+\alpha+\beta, I_t) - v_t^*(Q_t+\alpha, S_t+\alpha, I_t) \\
&\leq R(Q_t+\alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha+\beta-s, I_{t+1})\Big] \\
&- R(Q_t+\alpha, I_t, s, w) - \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha-s, I_{t+1})\Big] \\
&\leq \mathbb{E}\Big[v_{t+1}^*(q, S_t+\beta+q-q'-s', I_{t+1})\Big] - \mathbb{E}\Big[v_{t+1}^*(q, S_t+q-q'-s', I_{t+1})\Big] \\
&\leq R(Q_t, I_t, s', w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t+\beta-s', I_{t+1})\Big] \\
&- R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t-s', I_{t+1})\Big] \\
&\leq v_t^*(Q_t, S_t+\beta, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}
$$

The second inequality above holds since $v_{t+1}^*(\cdot,\cdot,I_{t+1})$ satisfies properties (a) and (b) (which together imply the concavity of $v_{t+1}^*(q,\cdot,I_{t+1})$). The third inequality above holds since $v_{t+1}^*(\cdot,\cdot,I_{t+1})$ satisfies property (b).

(b4) Suppose that $s \leq s'+\alpha$ and $0 \geq q-q'$. Since $\eta_t^*(Q_t,S_t,I_t) = (q',s',w')$, we have

$$
\begin{aligned}
v_t^*(Q_t,S_t,I_t) &= R(Q_t,I_t,s',w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t-s', I_{t+1})\Big] \\
&> R(Q_t,I_t,s',w') + \mathbb{E}\Big[v_{t+1}^*(q, S_t-s', I_{t+1})\Big].
\end{aligned}
$$

Since $\eta_t^*(Q_t+\alpha, S_t+\alpha+\beta, I_t) = (q,s,w)$, we have

$$
\begin{aligned}
&v_t^*(Q_t+\alpha, S_t+\alpha+\beta, I_t) \\
&= R(Q_t+\alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha+\beta-s, I_{t+1})\Big] \\
&> R(Q_t+\alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q', S_t+\alpha+\beta-s, I_{t+1})\Big].
\end{aligned}
$$

The inequalities above imply that

$$
\begin{aligned}
&\mathbb{E}\Big[v_{t+1}^*(q', S_t-s', I_{t+1})\Big] - \mathbb{E}\Big[v_{t+1}^*(q, S_t-s', I_{t+1})\Big] \\
&> \mathbb{E}\Big[v_{t+1}^*(q', S_t+\alpha+\beta-s, I_{t+1})\Big] - \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha+\beta-s, I_{t+1})\Big].
\end{aligned}
$$

But this leads to a contradiction since $v_{t+1}^*(\cdot,\cdot,I_{t+1})$ satisfies property (a). Hence this scenario is not possible.

(b5) Suppose that $s' + \alpha < s$ and $\min\{f(W_t), C_T + C_C\} < C_T + C_S - S_t - \alpha - \beta$. Lemma 4.3.3 implies that $w = \min\{f(W_t), C_T + C_C\} = w'$. We show that $(q', s'+\alpha, w') \in \mathbb{U}(Q_t+\alpha, S_t+\alpha, I_t)$: Since $(q, s, w) \in \mathbb{U}(Q_t+\alpha, S_t+\alpha+\beta, I_t)$, we have $s' + \alpha < s \leq C_D$. Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, we have $-C_C < -C_C + \alpha \leq s' + \alpha$ and $S_t - C_S + \alpha \leq s' + \alpha \leq S_t + \alpha$. Thus $-\min\{C_S - S_t - \alpha, C_C\} \leq s' + \alpha \leq \min\{S_t + \alpha, C_D\}$. Since $E(\cdot, w')$ is an increasing function, $E(s, w) > E(s' + \alpha, w') > E(s', w')$. Hence $(q', s' + \alpha, w') \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$. We also show that $(q, s - \alpha, w) \in \mathbb{U}(Q_t, S_t + \beta, I_t)$: Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, we have $-C_C \leq s' < s - \alpha$. Since $(q, s, w) \in \mathbb{U}(Q_t + \alpha, S_t + \alpha + \beta, I_t)$, we have $S_t + \beta - C_S \leq s - \alpha < s \leq C_D$ and $s - \alpha \leq S_t + \beta$. Thus $-\min\{C_S - S_t - \beta, C_C\} \leq s - \alpha \leq \min\{S_t + \beta, C_D\}$. Since $E(\cdot, w)$ is an increasing function, $E(s, w) > E(s - \alpha, w) > E(s', w')$. Hence $(q, s - \alpha, w) \in \mathbb{U}(Q_t, S_t + \beta, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t + \alpha, S_t + \alpha + \beta, I_t) - v_t^*(Q_t, S_t + \beta, I_t) \\
& \leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha + \beta - s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s - \alpha, w) - \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha + \beta - s, I_{t+1})\Big] \\
& = \alpha P_t \\
& = R(Q_t + \alpha, I_t, s' + \alpha, w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
& \leq v_t^*(Q_t + \alpha, S_t + \alpha, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}
$$

(b6) Suppose that $s' + \alpha < s$ and $C_T + C_S - S_t - \alpha - \beta \leq \min\{f(W_t), C_T + C_C\} < C_T + C_S - S_t$. Lemma 4.3.3 implies that $w = C_T + C_S - S_t - \alpha - \beta$, $w' = \min\{f(W_t), C_T + C_C\}$, $s = S_t + \alpha + \beta - C_S \leq 0$, and $E(s, w) = C_T$. Thus, $s' < s - \alpha < s \leq 0$ and $s' + \alpha < s \leq 0$. Let $w'_a = \min\{w', C_T - (s' + \alpha)\}$ and $w_a = \min\{w', w + \alpha\}$. We show that $(q', s' + \alpha, w'_a) \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$: Recall from scenario (b5) that $-\min\{C_S - S_t - \alpha, C_C\} \leq s' + \alpha \leq \min\{S_t + \alpha, C_D\}$ when $s' + \alpha < s$. If $w'_a = w'$, then $-C_T \leq s' + w' < s' + \alpha + w' = s' + \alpha + w'_a \leq s' + \alpha + C_T - (s' + \alpha) = C_T$ and $0 \leq w'_a = w' \leq f(W_t)$. If $w'_a = C_T - (s' + \alpha)$, then $s' + \alpha + w'_a = C_T$ and $0 \leq C_T < w'_a = C_T - (s' + \alpha) < w' \leq f(W_t)$. Hence $(q', s' + \alpha, w'_a) \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$.

We also show that $(q, s-\alpha, w_a) \in \mathbb{U}(Q_t, S_t+\beta, I_t)$: Recall from scenario (b5) that $-\min\{C_S - S_t - \beta, C_C\} \leq s - \alpha \leq \min\{S_t + \beta, C_D\}$ when $s' + \alpha < s$. Also, note that $s - \alpha + w_a \leq s + w \leq C_T$. If $w_a = w'$, then $-C_T \leq s' + w' < s - \alpha + w_a$ and $0 \leq w_a = w' \leq f(W_t)$. If $w_a = w + \alpha$, then $-C_T \leq s + w = s - \alpha + w_a$ and $0 \leq w < w + \alpha = w_a \leq w' \leq f(W_t)$. Hence $(q, s - \alpha, w_a) \in \mathbb{U}(Q_t, S_t + \beta, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t + \alpha, S_t + \alpha + \beta, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \\
& \leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\left[v_{t+1}^*(q, S_t + \alpha + \beta - s, I_{t+1})\right] \\
& - R(Q_t + \alpha, I_t, s' + \alpha, w_a') - \mathbb{E}\left[v_{t+1}^*(q', S_t - s', I_{t+1})\right] \\
& \leq R(Q_t, I_t, s - \alpha, w_a) + \mathbb{E}\left[v_{t+1}^*(q, S_t + \alpha + \beta - s, I_{t+1})\right] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\left[v_{t+1}^*(q', S_t - s', I_{t+1})\right] \\
& \leq v_t^*(Q_t, S_t + \beta, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}
$$

The second inequality above holds in each of the following four cases:

(1) Suppose that $w_a = w' < w + \alpha$ and $w_a' = w' < C_T - (s' + \alpha)$. Then,

$$
\begin{aligned}
& R(Q_t + \alpha, I_t, s, w) - R(Q_t + \alpha, I_t, s' + \alpha, w_a') \\
& \leq R(Q_t + \alpha, I_t, s, w') - R(Q_t + \alpha, I_t, s' + \alpha, w') \\
& = R(Q_t, I_t, s - \alpha, w_a) - R(Q_t, I_t, s', w').
\end{aligned}
$$

The inequality above holds since $w \leq w'$ and $R(Q_t + \alpha, I_t, s, \cdot)$ is an increasing function.

(2) Suppose that $w_a = w' < w + \alpha$ and $w_a' = C_T - (s' + \alpha) < w'$. Then,

$$
\begin{aligned}
& R(Q_t + \alpha, I_t, s, w) - R(Q_t + \alpha, I_t, s' + \alpha, w_a') = 0 \\
& \leq R(Q_t, I_t, s - \alpha, w_a) - R(Q_t, I_t, s', w').
\end{aligned}
$$

The equality above holds since $E(s, w) = E(s' + \alpha, w_a') = C_T$. The inequality above holds since $s' < s - \alpha$ and $R(Q_t, I_t, \cdot, w')$ is an increasing function.

(3) Suppose that $w_a = w + \alpha < w'$ and $w'_a = w' < C_T - (s' + \alpha)$. Then,

$$\begin{aligned}
&R(Q_t + \alpha, I_t, s, w) - R(Q_t, I_t, s - \alpha, w_a) \\
&= \alpha P_t + R(Q_t, I_t, s - \alpha, w) - R(Q_t, I_t, s, w) \\
&\leq \alpha P_t = R(Q_t + \alpha, I_t, s' + \alpha, w'_a) - R(Q_t, I_t, s', w').
\end{aligned}$$

The inequality above holds since $R(Q_t, I_t, \cdot, w)$ is an increasing function.

(4) Suppose that $w_a = w + \alpha < w'$ and $w'_a = C_T - (s' + \alpha) < w'$. Then,

$$\begin{aligned}
&R(Q_t + \alpha, I_t, s, w) - R(Q_t + \alpha, I_t, s' + \alpha, w'_a) = 0 \\
&\leq R(Q_t, I_t, s - \alpha, w_a) - R(Q_t, I_t, s', w').
\end{aligned}$$

The equality above holds since $E(s, w) = E(s' + \alpha, w'_a) = C_T$. The inequality above holds since $E(s - \alpha, w_a) = C_T \geq E(s', w')$.

(b7) Suppose that $s' + \alpha < s$ and $C_T + C_S - S_t \leq \min\{f(W_t), C_T + C_C\}$. Lemma 4.3.3 implies that $w = C_T + C_S - S_t - \alpha - \beta$, $s = S_t + \alpha + \beta - C_S$, $E(s, w) = C_T$, $w' = C_T + C_S - S_t$, $s' = S_t - C_S$, and $E(s', w') = C_T$. Note that $w' = w + \alpha + \beta$. We show that $(q', s' + \alpha, w' - \alpha) \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$: Recall from scenario (b5) that $-\min\{C_S - S_t - \alpha, C_C\} \leq s' + \alpha \leq \min\{S_t + \alpha, C_D\}$ when $s' + \alpha < s$. Note that $E(s' + \alpha, w' - \alpha) = E(s', w') = C_T$ and $0 \leq w < w' - \alpha < w' \leq f(W_t)$. Hence $(q', s' + \alpha, w' - \alpha) \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$. We also show that $(q, s - \alpha, w + \alpha) \in \mathbb{U}(Q_t, S_t + \beta, I_t)$: Recall from scenario (b5) that $-\min\{C_S - S_t - \beta, C_C\} \leq s - \alpha \leq \min\{S_t + \beta, C_D\}$ when $s' + \alpha < s$. Note that $E(s - \alpha, w + \alpha) = E(s, w) = C_T$ and $0 \leq w \leq w + \alpha < w' \leq f(W_t)$. Hence $(q, s - \alpha, w + \alpha) \in \mathbb{U}(Q_t, S_t + \beta, I_t)$. Thus:

$$\begin{aligned}
&v_t^*(Q_t + \alpha, S_t + \alpha + \beta, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \\
&\leq R(Q_t + \alpha, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha + \beta - s, I_{t+1})\Big] \\
&- R(Q_t + \alpha, I_t, s' + \alpha, w' - \alpha) - \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
&= R(Q_t, I_t, s - \alpha, w + \alpha) + \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha + \beta - s, I_{t+1})\Big] \\
&- R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
&\leq v_t^*(Q_t, S_t + \beta, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}$$

The equality above holds since $E(s, w) = E(s' + \alpha, w' - \alpha) = E(s', w') = E(s - \alpha, w + \alpha) = C_T$.

(c) Finally we prove that $v_t^*(Q_t + \alpha + \beta, S_t + \alpha, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \leq v_t^*(Q_t + \beta, S_t, I_t) - v_t^*(Q_t, S_t, I_t)$. Let $\eta_t^*(Q_t + \alpha + \beta, S_t + \alpha, I_t) = (q, s, w)$ and $\eta_t^*(Q_t, S_t, I_t) = (q', s', w')$. We consider the following three scenarios to prove the statement:

(c1) Suppose that $s' + \alpha \geq s$. Lemma 4.3.3 implies that $w' = \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}\} \geq \min\{f(W_t), C_T + \min\{C_S - S_t - \alpha, C_C\}\} = w$. Hence $w' = w + \delta$ for some $\delta \geq 0$. Note that $(q, s, w) \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$ and $(q', s', w') \in \mathbb{U}(Q_t + \beta, S_t, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t + \alpha + \beta, S_t + \alpha, I_t) - v_t^*(Q_t + \alpha, S_t + \alpha, I_t) \\
& \leq R(Q_t + \alpha + \beta, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha - s, I_{t+1})\Big] \\
& - R(Q_t + \alpha, I_t, s, w) - \mathbb{E}\Big[v_{t+1}^*(q, S_t + \alpha - s, I_{t+1})\Big] \\
& = (\alpha + \beta)P_t + R(Q_t, I_t, s - \alpha - \beta - \delta, w') - \alpha P_t - R(Q_t, I_t, s - \alpha - \delta, w') \\
& \leq \beta P_t + R(Q_t, I_t, s' - \beta, w') - R(Q_t, I_t, s', w') \\
& = R(Q_t + \beta, I_t, s', w') + \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t - s', I_{t+1})\Big] \\
& \leq v_t^*(Q_t + \beta, S_t, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}
$$

The second inequality above holds since $s' \geq s - \alpha - \delta$ and $R(Q_t, I_t, \cdot, w')$ is concave.

(c2) Suppose that $s' + \alpha < s$ and $\min\{f(W_t), C_T + C_C\} \leq C_T + C_S - S_t - \alpha$. Lemma 4.3.3 implies that $w = \min\{f(W_t), C_T + C_C\} = w'$. We show that $(q', s' + \alpha, w') \in \mathbb{U}(Q_t + \alpha, S_t + \alpha, I_t)$: Since $(q, s, w) \in \mathbb{U}(Q_t + \alpha + \beta, S_t + \alpha, I_t)$, we have $s' + \alpha < s \leq C_D$. Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, we have $-C_C < -C_C + \alpha \leq s' + \alpha$ and $S_t - C_S + \alpha \leq s' + \alpha \leq S_t + \alpha$. Thus $-\min\{C_S - S_t - \alpha, C_C\} \leq s' + \alpha \leq \min\{S_t + \alpha, C_D\}$. Since

$E(\cdot, w')$ is an increasing function, $E(s,w) > E(s'+\alpha, w') > E(s', w')$. Hence $(q', s'+\alpha, w') \in \mathbb{U}(Q_t+\alpha, S_t+\alpha, I_t)$. We also show that $(q, s-\alpha, w) \in \mathbb{U}(Q_t+\beta, S_t, I_t)$: Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, we have $-C_C \leq s' < s-\alpha$. Since $(q,s,w) \in \mathbb{U}(Q_t+\alpha+\beta, S_t+\alpha, I_t)$, we have $s-\alpha < s \leq C_D$ and $S_t - C_S \leq s-\alpha \leq S_t$. Thus $-\min\{C_S - S_t, C_C\} \leq s-\alpha \leq \min\{S_t, C_D\}$. Since $E(\cdot, w)$ is an increasing function, $E(s,w) > E(s-\alpha, w) > E(s', w')$. Hence $(q, s-\alpha, w) \in \mathbb{U}(Q_t+\beta, S_t, I_t)$. Thus:

$$
\begin{aligned}
& v_t^*(Q_t+\alpha+\beta, S_t+\alpha, I_t) - v_t^*(Q_t+\alpha, S_t+\alpha, I_t) \\
& \leq R(Q_t+\alpha+\beta, I_t, s, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha-s, I_{t+1})\Big] \\
& - R(Q_t+\alpha, I_t, s'+\alpha, w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t-s', I_{t+1})\Big] \\
& = R(Q_t+\beta, I_t, s-\alpha, w) + \mathbb{E}\Big[v_{t+1}^*(q, S_t+\alpha-s, I_{t+1})\Big] \\
& - R(Q_t, I_t, s', w') - \mathbb{E}\Big[v_{t+1}^*(q', S_t-s', I_{t+1})\Big] \\
& \leq v_t^*(Q_t+\beta, S_t, I_t) - v_t^*(Q_t, S_t, I_t).
\end{aligned}
$$

(c3) Suppose that $s'+\alpha < s$ and $C_T + C_S - S_t - \alpha < \min\{f(W_t), C_T + C_C\}$. Lemma 4.3.3 implies that $w = C_T + C_S - S_t - \alpha$ and $s = S_t + \alpha - C_S$. Since $(q', s', w') \in \mathbb{U}(Q_t, S_t, I_t)$, we have $S_t - C_S \leq s' < s - \alpha = S_t - C_S$. But this leads to a contradiction. Hence this scenario is not possible.

Hence $v_t^*(\cdot, \cdot, I_t)$ satisfies properties (a), (b), and (c). □

Proof of Theorem 4.3.1. Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$ denote the optimal action triplet in state (Q_t, S_t, I_t). We consider the following four scenarios to characterize the optimal state-dependent target storage level:

(i) Suppose that $(Q_t, S_t, W_t) \in \Psi^0$. Thus $f(W_t) \geq C_T + \min\{C_S - S_t, C_C\}$. Lemma 4.3.3 implies that $w = C_T + \min\{C_S - S_t, C_C\}$ and $s = -\min\{C_S - S_t, C_C\}$. Hence $Z_t(Q_t, S_t, I_t) = C_S$.

(ii) Suppose that $(Q_t, S_t, W_t) \in \Psi^1$. Thus $C_T + \min\{C_S - S_t, C_C\} > f(W_t) \geq Q_t + \min\{C_S - S_t, C_C\}$. Lemma 4.3.3 implies that $w = f(W_t)$. Since

$s \geq -\min\{C_S - S_t, C_C\}$, note that $E(s, w) = s + f(W_t) \geq Q_t$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C, C_T\}, C_T] \times [0, C_S]} \left\{ R^{(\mathsf{pi})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1}) \right] \right\}.$$

Note that $\left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{pi})}(I_t)\right)$ yields the maximum value. Hence $Z_t(Q_t, S_t, I_t) = Z_t^{(\mathsf{pi})}(I_t)$.

(iii) Suppose that $(Q_t, S_t, W_t) \in \Psi^2$. Thus $Q_t + \min\{C_S - S_t, C_C\} > f(W_t) \geq Q_t - \min\{S_t, C_D\}$. Lemma 4.3.1 implies that $C_T \geq Q_t$ and Lemma 4.3.3 implies that $w = f(W_t)$. We now consider the following two cases:

- Suppose that $Q_t \leq E(s, w) = s + f(W_t)$. Thus $Q_t - f(W_t) \leq s$. Since $Q_t - f(W_t) > -C_C$, we have $s > -C_C$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C, C_T\}, C_T] \times [0, S_t + f(W_t) - Q_t]} \left\{ R^{(\mathsf{pi})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1}) \right] \right\}.$$

  Since $R^{(\mathsf{pi})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1}) \right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = \min\{Z_t^{(\mathsf{pi})}(I_t), S_t + f(W_t) - Q_t\}$. Hence, $Z_t(Q_t, S_t, I_t) = S_t + f(W_t) - Q_t$ if $S_t + f(W_t) - Q_t \leq Z_t^{(\mathsf{pi})}(I_t)$ and $Z_t(Q_t, S_t, I_t) = Z_t^{(\mathsf{pi})}(I_t)$ otherwise.

- Suppose that $Q_t > E(s, w) = s + f(W_t)$. Thus $s < Q_t - f(W_t)$. Since $Q_t - f(W_t) \leq C_D$, we have $s < C_D$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C, C_T\}, C_T] \times [S_t + f(W_t) - Q_t, C_S]} \left\{ R^{(\mathsf{ni})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1}) \right] \right\}.$$

  Since $R^{(\mathsf{ni})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1}) \right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = \max\{Z_t^{(\mathsf{ni})}(I_t), S_t + f(W_t) - Q_t\}$. Hence, $Z_t(Q_t, S_t, I_t) = Z_t^{(\mathsf{ni})}(I_t)$ if $S_t + f(W_t) - Q_t \leq Z_t^{(\mathsf{ni})}(I_t)$ and $Z_t(Q_t, S_t, I_t) = S_t + f(W_t) - Q_t$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{ni})}(I_t) & \text{if } S_t \le Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t, \\ S_t + f(W_t) - Q_t & \text{if } Z_t^{(\mathsf{ni})}(I_t) - f(W_t) + Q_t < S_t \\ & \text{and } S_t \le Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t, \\ Z_t^{(\mathsf{pi})}(I_t) & \text{if } Z_t^{(\mathsf{pi})}(I_t) - f(W_t) + Q_t < S_t. \end{cases}$$

(iv) Suppose that $(Q_t, S_t, W_t) \in \Psi^3$. Thus $Q_t - \min\{S_t, C_D\} > f(W_t)$. Lemma 4.3.1 implies that $C_T \ge Q_t$ and Lemma 4.3.3 implies that $w = f(W_t)$. Since $s \le \min\{S_t, C_D\}$, note that $E(s, w) = s + f(W_t) < Q_t$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C, C_T\}, C_T] \times [0, C_S]} \left\{ R^{(\mathsf{ni})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right] \right\}.$$

Note that $\left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{ni})}(I_t)\right)$ yields the maximum value. Hence $Z_t(Q_t, S_t, I_t) = Z_t^{(\mathsf{ni})}(I_t)$.

We thus characterized the optimal state-dependent target storage level. We now show that $Z_t^{(\mathsf{ni})}(I_t) \le Z_t^{(\mathsf{pi})}(I_t)$: For simplicity, let $u_t^*(q_t, z_t) = \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right]$. By definitions of $Z_t^{(\nu)}(I_t)$ and $Y_t^{(\nu)}(I_t)$, the following inequalities hold.

$$u_t^* \left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{ni})}(I_t) \right) - K_n^+ P_t Z_t^{(\mathsf{ni})}(I_t) \ge u_t^* \left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{pi})}(I_t) \right) - K_n^+ P_t Z_t^{(\mathsf{pi})}(I_t),$$

$$u_t^* \left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{pi})}(I_t) \right) - K_p^+ P_t Z_t^{(\mathsf{pi})}(I_t) \ge u_t^* \left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{ni})}(I_t) \right) - K_p^+ P_t Z_t^{(\mathsf{ni})}(I_t).$$

The summation of the above inequalities implies that $-K_n^+ P_t Z_t^{(\mathsf{ni})}(I_t) - K_p^+ P_t Z_t^{(\mathsf{pi})}(I_t) \ge -K_n^+ P_t Z_t^{(\mathsf{pi})}(I_t) - K_p^+ P_t Z_t^{(\mathsf{ni})}(I_t)$. Since $K_n^+ > K_p^+$, $Z_t^{(\mathsf{ni})}(I_t) \le Z_t^{(\mathsf{pi})}(I_t)$.

We next characterize the optimal energy storage action. For notational convenience, we suppress the dependency of Z_t on (Q_t, S_t, I_t). We consider the following three scenarios:

(i) Suppose that $C_T \leq w$. If $s > 0$, then $E(s, w) = s + w > C_T$. But this leads to a contradiction since $E(s, w) \leq C_T$. Thus $s \leq 0$. Since $s + w \leq C_T$, note that $S_t + w - C_T \leq S_t - s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C,C_T\},C_T]\times[S_t+w-C_T,C_S]} \Big\{ R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \max\{Z_t, S_t + w - C_T\}$. We consider the following two cases:

- Suppose that $S_t + w - C_T < Z_t$. Then $z^*_t = Z_t$. Since $-C_T \leq s + w$, note that $s \geq -(C_T + w)$. Hence, taking into account the capacity constraints, we obtain $s = -\min\{Z_t - S_t, C_T + w, C_C\}$.
- Suppose that $Z_t \leq S_t + w - C_T$. Then $z^*_t = S_t + w - C_T$. Recall from Lemma 4.3.3 that $w = \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}\}$. Thus, $w \leq C_T + C_C$, that is, $w - C_T \leq C_C$. Since $s < 0$, taking into account the capacity constraints, we obtain $s = -\min\{w - C_T, C_C\} = C_T - w$.

(ii) Suppose that $C_T > w$ and $s < 0$. We consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C,C_T\},C_T]\times[S_t,C_S]} \Big\{ R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \max\{S_t, Z_t\}$. Since $-C_T \leq s + w$, note that $s \geq -(C_T + w)$. Hence, taking into account the capacity constraints, we obtain $s = -\min\{Z_t - S_t, C_T + w, C_C\}$ if $Z_t > S_t$.

(iii) Suppose that $C_T > w$ and $s \geq 0$. We consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C,C_T\},C_T]\times[0,S_t]} \Big\{ R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t =$

$\min\{Z_t, S_t\}$. Since $s + w \leq C_T$, note that $s \leq C_T - w$. Hence, taking into account the capacity constraints, we obtain $s = \min\{S_t - Z_t, C_T - w, C_D\}$ if $Z_t \leq S_t$.

Lastly, we consider the following problem to obtain the optimal energy commitment action:

$$\max_{q_t \in [-\min\{C_C, C_T\}, C_T]} \left\{ \mathbb{E}\left[v_{t+1}^*(q_t, S_t - s, I_{t+1}) \right] \right\}.$$

Note that $Y_t(S_t - s, I_t)$ yields the maximum value in this problem. Hence $q = Y_t(S_t - s, I_t)$. We now show that $Y_t^{(\mathsf{ni})}(I_t) \leq Y_t^{(\mathsf{pi})}(I_t)$: By definitions of $Z_t^{(\nu)}(I_t)$ and $Y_t^{(\nu)}(I_t)$, the following inequalities hold.

$$u_t^* \left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{ni})}(I_t) \right) - K_n^+ P_t Z_t^{(\mathsf{ni})}(I_t) \geq u_t^* \left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{ni})}(I_t) \right) - K_n^+ P_t Z_t^{(\mathsf{ni})}(I_t),$$
$$u_t^* \left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{pi})}(I_t) \right) - K_p^+ P_t Z_t^{(\mathsf{pi})}(I_t) \geq u_t^* \left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{pi})}(I_t) \right) - K_p^+ P_t Z_t^{(\mathsf{pi})}(I_t).$$

The summation of the above inequalities implies that

$$\begin{aligned} & u_t^* \left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{pi})}(I_t) \right) - u_t^* \left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{pi})}(I_t) \right) \\ & \leq u_t^* \left(Y_t^{(\mathsf{ni})}(I_t), Z_t^{(\mathsf{ni})}(I_t) \right) - u_t^* \left(Y_t^{(\mathsf{pi})}(I_t), Z_t^{(\mathsf{ni})}(I_t) \right). \end{aligned}$$

Since $Z_t^{(\mathsf{ni})}(I_t) \leq Z_t^{(\mathsf{pi})}(I_t)$ from Theorem 4.3.1, $Y_t^{(\mathsf{ni})}(I_t) \leq Y_t^{(\mathsf{pi})}(I_t)$ by property (a) in Proposition 4.3.1. $\square$

Appendix C

Supplement to optimal policy characterization in the presence of efficiency losses

This chapter includes supplementary material for Chapter 5: Commitment and Storage Problem of Wind Power Producers: Optimal Policy Characterization in the Presence of Efficiency Losses.

Proof of Theorem 5.2.1. Let $\eta_t^*(Q_t, S_t, I_t) = (q, s, w)$. Fix Q_t, S_t, and I_t.

(i) Suppose that $(Q_t, S_t, W_t) \in \Psi_0$. Thus $f(W_t) \geq C_T + \min\{C_S - S_t, C_C\}/\theta$. Lemma 4.3.3 implies that $w = C_T + \min\{C_S - S_t, C_C\}/\theta$ and $s = -\min\{C_S - S_t, C_C\}$. Hence $Z_t(Q_t, S_t, I_t) = C_S$.

(ii) Suppose that $(Q_t, S_t, W_t) \in \Psi_1^+$. Thus $C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T$, $f(W_t) \geq Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta$, and $Q_t \geq 0$. Lemma 4.3.3 implies that $w = f(W_t)$. Note that $f(W_t) \geq C_T \geq Q_t/\tau$. If $s > 0$, then $E(s, w) = (\gamma s + f(W_t))\tau > \tau C_T$. But this leads to a contradiction since $E(s, w) \leq \tau C_T$. Thus $s \leq 0$. Since $s \geq -\min\{C_S - S_t, C_C\}$, note that $s/\theta + f(W_t) \geq s/\theta + Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta \geq Q_t/\tau$. Thus

$E(s,w) = \min\{(s/\theta + w)\tau, (s/\theta + w)/\tau\} \geq \min\{Q_t, Q_t/\tau^2\} \geq Q_t \geq 0$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,C_S]} \left\{ R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]\right\}.$$

Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \max\{Z^{(\mathsf{piCS})}_t(I_t), S_t\}$. Hence, $Z_t(Q_t, S_t, I_t) = Z^{(\mathsf{piCS})}_t(I_t)$ if $S_t \leq Z^{(\mathsf{piCS})}_t(I_t)$ and $Z_t(Q_t, S_t, I_t) = S_t$ otherwise.

(iii) Suppose that $(Q_t, S_t, W_t) \in \Psi^-_1$. Thus $C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \geq C_T$, $f(W_t) \geq \tau Q_t + \min\{C_S - S_t, C_C\}/\theta$, and $Q_t < 0$. Recall from the proof of scenario (ii) that $s \leq 0$ when $f(W_t) \geq C_T$. Since $s \geq -\min\{C_S - S_t, C_C\}$, note that $s/\theta + f(W_t) \geq s/\theta + \tau Q_t + \min\{C_S - S_t, C_C\}/\theta \geq \tau Q_t$. Thus $E(s,w) = \min\{(s/\theta + w)\tau, (s/\theta + w)/\tau\} \geq \min\{\tau^2 Q_t, Q_t\} \geq Q_t$. We now consider the following two cases:

- Suppose that $s/\theta + w > 0$. Thus $0 \geq s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,S_t+\theta f(W_t)]} \Big\{ R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]\Big\}.$$

  Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{piCS})}_t(I_t), S_t + \theta f(W_t)\}$ if $S_t \leq Z^{(\mathsf{piCS})}_t(I_t)$ and $z^*_t = S_t$ otherwise.

- Suppose that $0 \geq s/\theta + w$. Thus $-\theta f(W_t) \geq s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),C_S]} \Big\{ R^{(\mathsf{piCP})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]\Big\}.$$

  Since $R^{(\mathsf{piCP})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where

$z_t^* = Z_t^{(\mathsf{piCP})}(I_t)$ if $S_t + \theta f(W_t) \le Z_t^{(\mathsf{piCP})}(I_t)$ and $z_t^* = S_t + \theta f(W_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{piCP})}(I_t) & \text{if } S_t \le Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t), \\ \min\{Z_t^{(\mathsf{piCS})}(I_t), S_t + \theta f(W_t)\} & \text{if } Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) < S_t \\ & \text{and } S_t \le Z_t^{(\mathsf{piCS})}(I_t), \\ S_t & \text{if } Z_t^{(\mathsf{piCS})}(I_t) < S_t. \end{cases}$$

(iv) Suppose that $(Q_t, S_t, W_t) \in \Psi_2^+$. Thus $C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \ge C_T$, $Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta > f(W_t)$, and $Q_t \ge 0$. Recall from the proof of scenario (ii) that $s \le 0$ and $f(W_t) \ge Q_t/\tau$ when $f(W_t) \ge C_T$. We now consider the following three cases:

- Suppose that $s/\theta + w \ge Q_t/\tau > 0$. Thus $Q_t \le E(s, w)$ and $0 \ge s > -\theta(f(W_t) - Q_t/\tau)$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T] \times [S_t, S_t + \theta(f(W_t) - Q_t/\tau)]} \Big\{ R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]\Big\}.$$

  Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = \min\{Z_t^{(\mathsf{piCS})}(I_t), S_t + \theta(f(W_t) - Q_t/\tau)\}$ if $S_t \le Z_t^{(\mathsf{piCS})}(I_t)$ and $z_t^* = S_t$ otherwise.

- Suppose that $Q_t/\tau > s/\theta + w > 0$. Thus $Q_t > E(s, w)$ and $-\theta(f(W_t) - Q_t/\tau) \ge s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T] \times [S_t + \theta(f(W_t) - Q_t/\tau), S_t + \theta f(W_t)]} \Big\{ R^{(\mathsf{niCS})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]\Big\}.$$

  Since $R^{(\mathsf{niCS})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where

$z_t^* = \min\{Z_t^{(\mathsf{niCS})}(I_t), S_t + \theta f(W_t)\}$ if $S_t + \theta(f(W_t) - Q_t/\tau) \le Z_t^{(\mathsf{niCS})}(I_t)$ and $z_t^* = S_t + \theta(f(W_t) - Q_t/\tau)$ otherwise.

- Suppose that $0 \ge s/\theta + w$. Thus $Q_t \ge 0 \ge E(s,w)$ and $-\theta f(W_t) \ge s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),C_S]} \Big\{ R^{(\mathsf{niCP})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niCP})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = Z_t^{(\mathsf{niCP})}(I_t)$ if $S_t + \theta f(W_t) \le Z_t^{(\mathsf{niCP})}(I_t)$ and $z_t^* = S_t + \theta f(W_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{niCP})}(I_t) & \text{if } S_t \le Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t), \\ \min\{Z_t^{(\mathsf{niCS})}(I_t), & \text{if } Z_t^{(\mathsf{niCP})}(I_t) - \theta f(W_t) < S_t \\ S_t + \theta f(W_t)\} & \text{and } S_t \le Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau), \\ \min\{Z_t^{(\mathsf{piCS})}(I_t), & \text{if } Z_t^{(\mathsf{niCS})}(I_t) - \theta(f(W_t) - Q_t/\tau) < S_t \\ S_t + \theta(f(W_t) - Q_t/\tau)\} & \text{and } S_t \le Z_t^{(\mathsf{piCS})}(I_t), \\ S_t & \text{if } Z_t^{(\mathsf{piCS})}(I_t) < S_t. \end{cases}$$

(v) Suppose that $(Q_t, S_t, W_t) \in \Psi_2^-$. Thus $C_T + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \ge C_T$, $\tau Q_t + \min\{C_S - S_t, C_C\}/\theta > f(W_t)$, and $Q_t < 0$. Recall from the proof of scenario (ii) that $s \le 0$ when $f(W_t) \ge C_T$. Note that $f(W_t) \ge C_T \ge 0 > \tau Q_t$. We now consider the following three cases:

- Suppose that $s/\theta + w > 0$. Thus $E(s,w) > 0 \ge Q_t$ and $0 \ge s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,S_t+\theta f(W_t)]} \Big\{ R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{piCS})}_t(I_t), S_t + \theta f(W_t)\}$ if $S_t \leq Z^{(\mathsf{piCS})}_t(I_t)$ and $z^*_t = S_t$ otherwise.

- Suppose that $0 \geq s/\theta + w \geq \tau Q_t$. Thus $Q_t \leq E(s, w)$ and $-\theta f(W_t) \geq s \geq -\theta(f(W_t) - \tau Q_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),S_t+\theta(f(W_t)-\tau Q_t)]} \Big\{R^{(\mathsf{piCP})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{piCP})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{piCP})}_t(I_t), S_t + \theta(f(W_t) - \tau Q_t)\}$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCP})}_t(I_t)$ and $z^*_t = S_t + \theta f(W_t)$ otherwise.

- Suppose that $0 \geq \tau Q_t > s/\theta + w$. Thus $Q_t > E(s, w)$ and $-\theta(f(W_t) - \tau Q_t) > s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta(f(W_t)-\tau Q_t),C_S]} \Big\{R^{(\mathsf{niCP})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niCP})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = Z^{(\mathsf{niCP})}_t(I_t)$ if $S_t + \theta(f(W_t) - \tau Q_t) \leq Z^{(\mathsf{niCP})}_t(I_t)$ and $z^*_t = S_t + \theta(f(W_t) - \tau Q_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z^{(\mathsf{niCP})}_t(I_t) & \text{if } S_t \leq Z^{(\mathsf{niCP})}_t(I_t) - \theta(f(W_t) - \tau Q_t), \\ \min\{Z^{(\mathsf{piCP})}_t(I_t), & \text{if } Z^{(\mathsf{niCP})}_t(I_t) - \theta(f(W_t) - \tau Q_t) < S_t \\ S_t + \theta(f(W_t) - \tau Q_t)\} & \text{and } S_t \leq Z^{(\mathsf{piCP})}_t(I_t) - \theta f(W_t), \\ \min\{Z^{(\mathsf{piCS})}_t(I_t), & \text{if } Z^{(\mathsf{piCP})}_t(I_t) - \theta f(W_t) < S_t \leq Z^{(\mathsf{piCS})}_t(I_t), \\ S_t + \theta f(W_t)\} & \\ S_t & \text{if } Z^{(\mathsf{piCS})}_t(I_t) < S_t. \end{cases}$$

(vi) Suppose that $(Q_t, S_t, W_t) \in \Psi_3^+$. Thus $C_T > f(W_t)$, $f(W_t) \geq Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta$, and $Q_t \geq 0$. Lemma 4.3.3 implies that $w = f(W_t)$. Since $s \geq -\min\{C_S - S_t, C_C\}$, note that $E(s, w) = \min\{(\gamma s + w)\tau, (s/\theta + w)\tau, (s/\theta + w)/\tau\} \geq \min\{Q_t, Q_t/\tau^2\} \geq Q_t \geq 0$. We now consider the following two cases:

- Suppose that $s \geq 0$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[0,S_t]} \left\{R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = S_t$ if $S_t \leq Z_t^{(\mathsf{piDS})}(I_t)$ and $z_t^* = Z_t^{(\mathsf{piDS})}(I_t)$ otherwise.

- Suppose that $s < 0$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,C_S]} \left\{R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right]\right\}.$$

  Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = Z_t^{(\mathsf{piCS})}(I_t)$ if $S_t \leq Z_t^{(\mathsf{piCS})}(I_t)$ and $z_t^* = S_t$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{piCS})}(I_t) & \text{if } S_t \leq Z_t^{(\mathsf{piCS})}(I_t), \\ S_t & \text{if } Z_t^{(\mathsf{piCS})}(I_t) < S_t \leq Z_t^{(\mathsf{piDS})}(I_t) \\ Z_t^{(\mathsf{piDS})}(I_t) & \text{if } Z_t^{(\mathsf{piDS})}(I_t) < S_t. \end{cases}$$

(vii) Suppose that $(Q_t, S_t, W_t) \in \Psi_3^-$. Thus $C_T > f(W_t)$, $f(W_t) \geq \tau Q_t + \min\{C_S - S_t, C_C\}/\theta$, and $Q_t < 0$. Lemma 4.3.3 implies that $w = f(W_t)$. Since $s \geq -\min\{C_S - S_t, C_C\}$, note that $E(s, w) = \min\{(\gamma s + w)\tau, (s/\theta + w)\tau, (s/\theta + w)/\tau\} \geq \min\{\tau^2 Q_t, Q_t\} \geq Q_t$. We now consider the following three cases:

- Suppose that $s \geq 0$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[0,S_t]} \left\{ R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\right\}.$$

  Since $R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = S_t$ if $S_t \leq Z^{(\mathsf{piDS})}_t(I_t)$ and $z^*_t = Z^{(\mathsf{piDS})}_t(I_t)$ otherwise.

- Suppose that $s < 0$ and $s/\theta + w > 0$. Thus $0 > s \geq -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,S_t+\theta f(W_t)]} \Big\{ R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

  Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{piCS})}_t(I_t), S_t + \theta f(W_t)\}$ if $S_t \leq Z^{(\mathsf{piCS})}_t(I_t)$ and $z^*_t = S_t$ otherwise.

- Suppose that $s < 0$ and $0 \geq s/\theta + w$. Thus $-\theta f(W_t) \geq s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),C_S]} \Big\{ R^{(\mathsf{piCP})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

  Since $R^{(\mathsf{piCP})}(z_t, I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = Z^{(\mathsf{piCP})}_t(I_t)$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCP})}_t(I_t)$ and $z^*_t = S_t + \theta f(W_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z_t^{(\mathsf{piCP})}(I_t) & \text{if } S_t \le Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t), \\ \min\{Z_t^{(\mathsf{piCS})}(I_t), S_t + \theta f(W_t)\} & \text{if } Z_t^{(\mathsf{piCP})}(I_t) - \theta f(W_t) \le S_t \\ & \text{and } S_t \le Z_t^{(\mathsf{piCS})}(I_t), \\ S_t & \text{if } Z_t^{(\mathsf{piCS})}(I_t) < S_t \le Z_t^{(\mathsf{piDS})}(I_t), \\ Z_t^{(\mathsf{piDS})}(I_t) & \text{if } Z_t^{(\mathsf{piDS})}(I_t) < S_t. \end{cases}$$

(viii) Suppose that $(Q_t, S_t, W_t) \in \Psi_4^+$. Thus $C_T > f(W_t)$, $Q_t/\tau + \min\{C_S - S_t, C_C\}/\theta > f(W_t) \ge Q_t/\tau$, and $Q_t \ge 0$. Lemma 4.3.3 implies that $w = f(W_t)$. We now consider the following four cases:

- Suppose that $s \ge 0$. Thus $Q_t \le E(s, w)$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T] \times [0, S_t]} \left\{ R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right] \right\}.$$

  Since $R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = S_t$ if $S_t \le Z_t^{(\mathsf{piDS})}(I_t)$ and $z_t^* = Z_t^{(\mathsf{piDS})}(I_t)$ otherwise.

- Suppose that $s < 0$ and $s/\theta + w \ge Q_t/\tau \ge 0$. Thus $Q_t \le E(s, w)$ and $0 > s \ge -\theta(f(W_t) - Q_t/\tau)$. We then consider the following problem:

$$\max_{(q_t, z_t) \in [-\min\{C_C/(\theta\tau), C_T\}, \tau C_T] \times [S_t, S_t + \theta(f(W_t) - Q_t/\tau)]} \Big\{ R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right] \Big\}.$$

  Since $R^{(\mathsf{piCS})}(z_t, I_t) + \mathbb{E}\left[v_{t+1}^*(q_t, z_t, I_{t+1}) \right]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = \min\{Z_t^{(\mathsf{piCS})}(I_t), S_t + \theta(f(W_t) - Q_t/\tau)\}$ if $S_t \le Z_t^{(\mathsf{piCS})}(I_t)$ and $z_t^* = S_t$ otherwise.

- Suppose that $s < 0$ and $Q_t/\tau > s/\theta + w \ge 0$. Thus $Q_t > E(s, w)$ and $-\theta(f(W_t) - Q_t/\tau) > s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta(f(W_t)-Q_t/\tau),S_t+\theta f(W_t)]}\Big\{R^{(\mathsf{niCS})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niCS})}(z_t,I_t)+\mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t=\min\{Z^{(\mathsf{niCS})}_t(I_t),S_t+\theta f(W_t)\}$ if $S_t+\theta(f(W_t)-Q_t/\tau)\le Z^{(\mathsf{niCS})}_t(I_t)$ and $z^*_t=S_t+\theta(f(W_t)-Q_t/\tau)$ otherwise.

- Suppose that $s<0$ and $0>s/\theta+w$. Thus $Q_t\ge 0>E(s,w)$ and $-\theta f(W_t)\ge s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),C_S]}\Big\{R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niCP})}(z_t,I_t)+\mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t=Z^{(\mathsf{niCP})}_t(I_t)$ if $S_t+\theta f(W_t)\le Z^{(\mathsf{niCP})}_t(I_t)$ and $z^*_t=S_t+\theta f(W_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t,S_t,I_t)=\begin{cases} Z^{(\mathsf{niCP})}_t(I_t) & \text{if } S_t\le Z^{(\mathsf{niCP})}_t(I_t)-\theta f(W_t),\\ \min\{Z^{(\mathsf{niCS})}_t(I_t), & \text{if } Z^{(\mathsf{niCP})}_t(I_t)-\theta f(W_t)<S_t\\ S_t+\theta f(W_t)\} & \text{and } S_t\le Z^{(\mathsf{niCS})}_t(I_t)-\theta(f(W_t)-Q_t/\tau),\\ \min\{Z^{(\mathsf{piCS})}_t(I_t), & \text{if } Z^{(\mathsf{niCS})}_t(I_t)-\theta(f(W_t)-Q_t/\tau)<S_t\\ S_t+\theta(f(W_t)-Q_t/\tau)\} & \text{and } S_t\le Z^{(\mathsf{piCS})}_t(I_t),\\ S_t & \text{if } Z^{(\mathsf{piCS})}_t(I_t)<S_t\le Z^{(\mathsf{piDS})}_t(I_t),\\ Z^{(\mathsf{piDS})}_t(I_t) & \text{if } Z^{(\mathsf{piDS})}_t(I_t)<S_t.\end{cases}$$

(ix) Suppose that $(Q_t,S_t,W_t)\in\Psi^-_4$. Thus $C_T>f(W_t)$, $\tau Q_t+\min\{C_S-S_t,C_C\}/\theta>f(W_t)$, and $Q_t<0$. Note that $f(W_t)\ge 0>Q_t\tau$. Lemma 4.3.3 implies that $w=f(W_t)$. We now consider the following four cases:

- Suppose that $s \geq 0$. Thus $Q_t < 0 \leq E(s,w)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[0,S_t]} \left\{ R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big] \right\}.$$

Since $R^{(\mathsf{piDS})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t = S_t$ if $S_t \leq Z^{(\mathsf{piDS})}_t(I_t)$ and $z^*_t = Z^{(\mathsf{piDS})}_t(I_t)$ otherwise.

- Suppose that $s < 0$ and $s/\theta + w \geq 0$. Thus $Q_t < 0 \leq E(s,w)$ and $0 \geq s > -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,S_t+\theta f(W_t)]} \Big\{ R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big] \Big\}.$$

Since $R^{(\mathsf{piCS})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{piCS})}_t(I_t), S_t + \theta f(W_t)\}$ if $S_t \leq Z^{(\mathsf{piCS})}_t(I_t)$ and $z^*_t = S_t$ otherwise.

- Suppose that $s < 0$ and $0 > s/\theta + w \geq \tau Q_t$. Thus $Q_t \leq E(s,w)$ and $-\theta f(W_t) \geq s \geq -\theta(f(W_t) - \tau Q_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),S_t+\theta(f(W_t)-\tau Q_t)]} \Big\{ R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big] \Big\}.$$

Since $R^{(\mathsf{piCP})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{piCP})}_t(I_t), S_t + \theta(f(W_t) - \tau Q_t)\}$ if $S_t + \theta f(W_t) \leq Z^{(\mathsf{piCP})}_t(I_t)$ and $z^*_t = S_t + \theta f(W_t)$ otherwise.

- Suppose that $s < 0$ and $0 > \tau Q_t > s/\theta + w$. Thus $Q_t > E(s,w)$ and $-\theta(f(W_t) - \tau Q_t) > s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta(f(W_t)-\tau Q_t),C_S]} \Big\{ R^{(\mathsf{niCP})}(z_t,I_t) + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big] \Big\}.$$

Since $R^{(\mathsf{niCP})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = Z^{(\mathsf{niCP})}_t(I_t)$ if $S_t + \theta(f(W_t) - \tau Q_t) \le Z^{(\mathsf{niCP})}_t(I_t)$ and $z^*_t = S_t + \theta(f(W_t) - \tau Q_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t, S_t, I_t) = \begin{cases} Z^{(\mathsf{niCP})}_t(I_t) & \text{if } S_t \le Z^{(\mathsf{niCP})}_t(I_t) - \theta(f(W_t) - \tau Q_t), \\ \min\{Z^{(\mathsf{piCP})}_t(I_t), & \text{if } Z^{(\mathsf{niCP})}_t(I_t) - \theta(f(W_t) - \tau Q_t) < S_t \\ S_t + \theta(f(W_t) - \tau Q_t)\} & \text{and } S_t \le Z^{(\mathsf{piCP})}_t(I_t) - \theta f(W_t), \\ \min\{Z^{(\mathsf{piCS})}_t(I_t), & \text{if } Z^{(\mathsf{piCP})}_t(I_t) - \theta f(W_t) < S_t \le Z^{(\mathsf{piCS})}_t(I_t), \\ S_t + \theta f(W_t)\} & \\ S_t & \text{if } Z^{(\mathsf{piCS})}_t(I_t) < S_t \le Z^{(\mathsf{piDS})}_t(I_t), \\ Z^{(\mathsf{piDS})}_t(I_t) & \text{if } Z^{(\mathsf{piDS})}_t(I_t) < S_t. \end{cases}$$

(x) Suppose that $(Q_t, S_t, W_t) \in \Psi_5$. Thus $C_T > f(W_t)$ and $Q_t/\tau > f(W_t)$. Note that $Q_t > \tau f(W_t) \ge 0$. Lemma 4.3.3 implies that $w = f(W_t)$. We now consider the following four cases:

- Suppose that $s \ge 0$ and $\gamma s + w \ge Q_t/\tau$. Thus $Q_t \le E(s, w)$ and $s \ge (Q_t/\tau - f(W_t))/\gamma$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[0,S_t-(Q_t/\tau-f(W_t))/\gamma]} \Big\{ R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]\Big\}.$$

  Since $R^{(\mathsf{piDS})}(z_t, I_t) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = S_t - (Q_t/\tau - f(W_t))/\gamma$ if $S_t - (Q_t/\tau - f(W_t))/\gamma \le Z^{(\mathsf{piDS})}_t(I_t)$ and $z^*_t = Z^{(\mathsf{piDS})}_t(I_t)$ otherwise.

- Suppose that $s \ge 0$ and $\gamma s + w < Q_t/\tau$. Thus $Q_t > E(s, w)$ and $(Q_t/\tau - f(W_t))/\gamma > s \ge 0$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t-(Q_t/\tau-f(W_t))/\gamma,S_t]} \Big\{R^{(\mathsf{niDS})}(z_t,I_t) \\ + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niDS})}(z_t,I_t)+\mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t = S_t$ if $S_t \le Z^{(\mathsf{niDS})}_t(I_t)$ and $z^*_t = \max\{Z^{(\mathsf{niDS})}_t(I_t), S_t-(Q_t/\tau-f(W_t))/\gamma\}$ otherwise.

- Suppose that $s<0$ and $s/\theta+w\ge 0$. Thus, $Q_t > (s/\theta+f(W_t))\tau = E(s,w)$ since $f(W_t)<Q_t/\tau$, and $0>s\ge -\theta f(W_t)$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,S_t+\theta f(W_t)]} \Big\{R^{(\mathsf{niCS})}(z_t,I_t) \\ + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niCS})}(z_t,I_t)+\mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z^{(\mathsf{niCS})}_t(I_t), S_t+\theta f(W_t)\}$ if $S_t \le Z^{(\mathsf{niCS})}_t(I_t)$ and $z^*_t = S_t$ otherwise.

- Suppose that $s<0$ and $0>s/\theta+w$. Thus $Q_t\ge 0>E(s,w)$ and $-\theta f(W_t)>s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta f(W_t),C_S]} \Big\{R^{(\mathsf{niCP})}(z_t,I_t) \\ + \mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]\Big\}.$$

Since $R^{(\mathsf{niCP})}(z_t,I_t)+\mathbb{E}\Big[v^*_{t+1}(q_t,z_t,I_{t+1})\Big]$ is jointly concave in (q_t,z_t), note that (q^*_t,z^*_t) yields the maximum value in this problem where $z^*_t = Z^{(\mathsf{niCP})}_t(I_t)$ if $S_t+\theta f(W_t) \le Z^{(\mathsf{niCP})}_t(I_t)$ and $z^*_t = S_t+\theta f(W_t)$ otherwise.

Combining all of the above observations, we obtain

$$Z_t(Q_t,S_t,I_t)=\begin{cases} Z_t^{(\mathsf{niCP})}(I_t) & \text{if } S_t \le Z_t^{(\mathsf{niCP})}(I_t)-\theta f(W_t),\\ \min\{Z_t^{(\mathsf{niCS})}(I_t), & \text{if } Z_t^{(\mathsf{niCP})}(I_t)-\theta f(W_t) < S_t \le Z_t^{(\mathsf{niCS})}(I_t),\\ S_t+\theta f(W_t)\} & \\ S_t & \text{if } Z_t^{(\mathsf{niCS})}(I_t) < S_t \le Z_t^{(\mathsf{niDS})}(I_t),\\ \max\{Z_t^{(\mathsf{niDS})}(I_t), & \text{if } Z_t^{(\mathsf{niDS})}(I_t) < S_t\\ S_t-(Q_t/\tau-f(W_t))/\gamma\} & \text{and } S_t \le Z_t^{(\mathsf{piDS})}(I_t)+(Q_t/\tau-f(W_t))/\gamma,\\ Z_t^{(\mathsf{piDS})}(I_t) & \text{if } Z_t^{(\mathsf{piDS})}(I_t)+(Q_t/\tau-f(W_t))/\gamma < S_t. \end{cases}$$

We next characterize the optimal energy storage action. For notational convenience, we suppress the dependency of Z_t on (Q_t, S_t, I_t). We consider the following three scenarios:

(i) Suppose that $C_T \le w$. If $s > 0$, then $E(s,w) = (\gamma s + w)\tau > \tau C_T$. But this leads to a contradiction since $E(s,w) \le \tau C_T$. Thus $s \le 0$. Since $s/\theta + w \le C_T$, note that $S_t + \theta(w - C_T) \le S_t - s$. We then consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t+\theta(w-C_T),C_S]} \Big\{ R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]\Big\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\Big[v^*_{t+1}(q_t, z_t, I_{t+1})\Big]$ is jointly concave in (q_t, z_t), note that (q_t^*, z_t^*) yields the maximum value in this problem where $z_t^* = \max\{Z_t, S_t + \theta(w - C_T)\}$. We consider the following two cases:

- Suppose that $S_t + \theta(w - C_T) < Z_t$. Then $z_t^* = Z_t$. Since $-\tau C_T \le s/\theta + w$, note that $s \ge -\theta(\tau C_T + w)$. Hence, taking into account the capacity constraints, we obtain $s = -\min\{Z_t - S_t, \theta(\tau C_T + w), C_C\}$.
- Suppose that $Z_t \le S_t + \theta(w - C_T)$. Then $z_t^* = S_t + \theta(w - C_T)$. Recall from Lemma 4.3.3 that $w = \min\{f(W_t), C_T + \min\{C_S - S_t, C_C\}/\theta\}$. Thus, $w \le C_T + C_C/\theta$, that is, $\theta(w - C_T) \le C_C$. Since $s < 0$, taking

into account the capacity constraints, we obtain $s = -\min\{\theta(w - C_T), C_C\} = -\theta(w - C_T)$.

(ii) Suppose that $C_T > w$ and $s < 0$. We consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[S_t,C_S]} \left\{R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]\right\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \max\{S_t, Z_t\}$. Since $-\tau C_T \le s/\theta + w$, note that $s \ge -\theta(\tau C_T + w)$. Hence, taking into account the capacity constraints, we obtain $s = -\min\{Z_t - S_t, \theta(\tau C_T + w), C_C\}$.

(iii) Suppose that $C_T > w$ and $s \ge 0$. We consider the following problem:

$$\max_{(q_t,z_t)\in[-\min\{C_C/(\theta\tau),C_T\},\tau C_T]\times[0,S_t]} \left\{R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]\right\}.$$

Since $R(Q_t, I_t, S_t - z_t, w) + \mathbb{E}\left[v^*_{t+1}(q_t, z_t, I_{t+1})\right]$ is jointly concave in (q_t, z_t), note that (q^*_t, z^*_t) yields the maximum value in this problem where $z^*_t = \min\{Z_t, S_t\}$. Since $\gamma s + w \le C_T$, note that $s \le (C_T - w)/\gamma$. Hence, taking into account the capacity constraints, we obtain $s = \min\{S_t - Z_t, (C_T - w)/\gamma, C_D\}$.

Combining all of the above observations, we obtain

$$s = \begin{cases} -\min\{Z_t - S_t, \theta(\tau C_T + w), C_C\} & \text{if } C_T \le w \text{ and } S_t + \theta(w - C_T) < Z_t, \\ -\theta(w - C_T) & \text{if } C_T \le w \text{ and } Z_t \le S_t + \theta(w - C_T), \\ -\min\{Z_t - S_t, \theta(\tau C_T + w), C_C\} & \text{if } C_T > w \text{ and } S_t < Z_t, \\ \min\{S_t - Z_t, (C_T - w)/\gamma, C_D\} & \text{if } C_T > w \text{ and } Z_t \le S_t. \end{cases}$$

We now show that $Z^{(\mathsf{niCP})}_t(I_t) \le Z^{(\mathsf{niCS})}_t(I_t) \le Z^{(\mathsf{niDS})}_t(I_t)$ and $Z^{(\mathsf{piCP})}_t(I_t) \le Z^{(\mathsf{piCS})}_t(I_t) \le Z^{(\mathsf{piDS})}_t(I_t)$: We fix I_t and suppress the dependencies of $Y^{(\nu)}_t$ and $Z^{(\nu)}_t$

on I_t for each $\nu \in \{\mathsf{niCP}, \mathsf{niCS}, \mathsf{niDS}, \mathsf{piCP}, \mathsf{piCS}, \mathsf{piDS}\}$ for notational convenience. For simplicity, let $u_t^*(q_t, z_t) = \mathbb{E}\Big[v_{t+1}^*(q_t, z_t, I_{t+1})\Big]$. By definitions of $Z_t^{(\nu)}$ and $Y_t^{(\nu)}$, the following inequalities hold.

$$u_t^*(Y_t^{(\mathsf{niCP})}, Z_t^{(\mathsf{niCP})}) - K_n^+ P_t Z_t^{(\mathsf{niCP})}/(\theta\tau) \geq u_t^*(Y_t^{(\mathsf{niCS})}, Z_t^{(\mathsf{niCS})}) - K_n^+ P_t Z_t^{(\mathsf{niCS})}/(\theta\tau),$$
$$u_t^*(Y_t^{(\mathsf{niCS})}, Z_t^{(\mathsf{niCS})}) - \tau K_n^+ P_t Z_t^{(\mathsf{niCS})}/\theta \geq u_t^*(Y_t^{(\mathsf{niCP})}, Z_t^{(\mathsf{niCP})}) - \tau K_n^+ P_t Z_t^{(\mathsf{niCP})}/\theta.$$

The summation of the above inequalities implies that $K_n^+(\tau - 1/\tau)Z_t^{(\mathsf{niCP})}/\theta \geq K_n^+(\tau - 1/\tau)Z_t^{(\mathsf{niCS})}/\theta$. Since $K_n^+ > 0$, $0 < \theta \leq 1$, and $0 < \tau \leq 1$, $Z_t^{(\mathsf{niCP})} \leq Z_t^{(\mathsf{niCS})}$. Again, by definitions of $Z_t^{(\nu)}$ and $Y_t^{(\nu)}$, the following inequalities hold.

$$u_t^*(Y_t^{(\mathsf{niCS})}, Z_t^{(\mathsf{niCS})}) - \tau K_n^+ P_t Z_t^{(\mathsf{niCS})}/\theta \geq u_t^*(Y_t^{(\mathsf{niDS})}, Z_t^{(\mathsf{niDS})}) - \tau K_n^+ P_t Z_t^{(\mathsf{niDS})}/\theta,$$
$$u_t^*(Y_t^{(\mathsf{niDS})}, Z_t^{(\mathsf{niDS})}) - \tau\gamma K_n^+ P_t Z_t^{(\mathsf{niDS})} \geq u_t^*(Y_t^{(\mathsf{niCS})}, Z_t^{(\mathsf{niCS})}) - \tau\gamma K_n^+ P_t Z_t^{(\mathsf{niCS})}.$$

The summation of the above inequalities implies that $K_n^+\tau(\gamma - 1/\theta)Z_t^{(\mathsf{niCS})} \geq K_n^+\tau(\gamma - 1/\theta)Z_t^{(\mathsf{niDS})}$. Since $K_n^+ > 0$, $0 < \theta \leq 1$, and $0 < \tau \leq 1$, $Z_t^{(\mathsf{niCS})} \leq Z_t^{(\mathsf{niDS})}$. Following similar steps, it can be shown that $Z_t^{(\mathsf{piCP})} \leq Z_t^{(\mathsf{piCS})} \leq Z_t^{(\mathsf{piDS})}$.

We next show that $Z_t^{(\mathsf{niCP})} \leq Z_t^{(\mathsf{piCP})}$, $Z_t^{(\mathsf{niCS})} \leq Z_t^{(\mathsf{piCS})}$, and $Z_t^{(\mathsf{niDS})} \leq Z_t^{(\mathsf{piDS})}$: By definitions of $Z_t^{(\nu)}$ and $Y_t^{(\nu)}$, the following inequalities hold.

$$u_t^*(Y_t^{(\mathsf{niCP})}, Z_t^{(\mathsf{niCP})}) - K_n^+ P_t Z_t^{(\mathsf{niCP})}/(\theta\tau) \geq u_t^*(Y_t^{(\mathsf{piCP})}, Z_t^{(\mathsf{piCP})}) - K_n^+ P_t Z_t^{(\mathsf{piCP})}/(\theta\tau),$$
$$u_t^*(Y_t^{(\mathsf{piCP})}, Z_t^{(\mathsf{piCP})}) - K_p^+ P_t Z_t^{(\mathsf{piCP})}/(\theta\tau) \geq u_t^*(Y_t^{(\mathsf{niCP})}, Z_t^{(\mathsf{niCP})}) - K_p^+ P_t Z_t^{(\mathsf{niCP})}/(\theta\tau).$$

The summation of the above inequalities implies that $(-K_n^+ + K_p^+)Z_t^{(\mathsf{niCP})} \geq (-K_n^+ + K_p^+)Z_t^{(\mathsf{piCP})}$. Since $K_n^+ > K_p^+$, $Z_t^{(\mathsf{niCP})} \leq Z_t^{(\mathsf{piCP})}$. Following similar steps, it can be shown that $Z_t^{(\mathsf{niCS})} \leq Z_t^{(\mathsf{piCS})}$ and $Z_t^{(\mathsf{niDS})} \leq Z_t^{(\mathsf{piDS})}$.

$\square$

Appendix D

Numerical results for the impact of commitment decisions

We consider instances in which the planning horizon spans the first week of August, $N = 100$, $K_p^+ = K_n^- \in \{0.7, 0.8, 0.9\}$ and $K_n^+ = K_p^- \in \{1.1, 1.2, 1.3\}$ (our problem setting) or $K_p^+ = K_n^+ = K_p^- = K_n^- = 1$ (alternative problem setting [8]), $C_S \in \{0, 250, 500\}$ (in MWh), $C_C = C_D \in \{40, 60\}$ (in MWh), $C_T \in \{100, 200\}$ (in MWh), NPF $\in \{0, 4.02\%, 7.66\%, 10.96\%, 13.98\%\}$, $r \in \{0.7, 0.8, 0.9, 1\}$, and $\tau \in \{0.95, 1\}$. We solve the recursion of our MDP to optimality in each instance, calculating the following metrics:

- The percentage loss in the expected total cash flow due to the existence of commitment decisions (TCF-Loss),
- The expected total amount of energy purchased in MWh (EP),
- The expected total amount of energy sold in MWh (ES),
- The expected total amount of energy stored by charging the battery in MWh (ESC), and
- The expected total amount of energy generated by discharging the battery in MWh (EGD).

Table D.1: Numerical results when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, NPF $= 4.02\%$, and $r = 0.8$.

τ	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF-Loss	ES	EP	ESC	EGD
0.95	0.7	1.1	5.04%	14014	437	2125	1886
		1.2	6.37%	14001	435	2185	1933
		1.3	7.15%	13977	431	2291	2018
	0.8	1.1	4.06%	14010	435	2142	1900
		1.2	5.13%	13988	434	2247	1983
		1.3	5.71%	13962	431	2363	2075
	0.9	1.1	2.76%	14006	435	2155	1910
		1.2	3.40%	13978	432	2289	2016
		1.3	3.75%	13958	429	2371	2081
	1	1	0.00%	13998	437	2206	1949
1	0.7	1.1	4.97%	14764	430	2140	1898
		1.2	6.31%	14739	418	2199	1945
		1.3	7.09%	14711	413	2315	2036
	0.8	1.1	4.01%	14759	428	2154	1909
		1.2	5.09%	14725	417	2261	1994
		1.3	5.68%	14695	413	2389	2095
	0.9	1.1	2.74%	14751	424	2167	1919
		1.2	3.39%	14713	415	2314	2036
		1.3	3.74%	14697	414	2382	2089
	1	1	0.00%	14764	446	2214	1956

Table D.2: Numerical results when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, NPF = 4.02%, and $\tau = 0.95$.

r	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF-Loss	ES	EP	ESC	EGD
0.7	0.7	1.1	5.02%	13870	472	2093	1668
		1.2	6.40%	13832	470	2222	1760
		1.3	7.23%	13805	469	2319	1828
	0.8	1.1	4.03%	13878	473	2067	1650
		1.2	5.16%	13828	469	2233	1767
		1.3	5.81%	13798	470	2347	1848
	0.9	1.1	2.72%	13873	472	2082	1660
		1.2	3.42%	13850	468	2150	1709
		1.3	3.84%	13825	469	2242	1774
	1	1	0.00%	13844	475	2194	1740
0.8	0.7	1.1	5.04%	14014	437	2125	1886
		1.2	6.37%	14001	435	2185	1933
		1.3	7.15%	13977	431	2291	2018
	0.8	1.1	4.06%	14010	435	2142	1900
		1.2	5.13%	13988	434	2247	1983
		1.3	5.71%	13962	431	2363	2075
	0.9	1.1	2.76%	14006	435	2155	1910
		1.2	3.40%	13978	432	2289	2016
		1.3	3.75%	13958	429	2371	2081
	1	1	0.00%	13998	437	2206	1949
0.9	0.7	1.1	5.03%	14214	402	2175	2179
		1.2	6.37%	14210	400	2196	2199
		1.3	7.06%	14196	396	2305	2297
	0.8	1.1	4.05%	14204	401	2269	2264
		1.2	5.08%	14198	398	2305	2296
		1.3	5.60%	14188	394	2379	2363
	0.9	1.1	2.72%	14193	398	2356	2342
		1.2	3.29%	14182	396	2448	2425
		1.3	3.60%	14179	394	2466	2441
	1	1	0.00%	14209	406	2260	2255
1	0.7	1.1	5.02%	14404	378	2456	2683
		1.2	6.37%	14400	373	2369	2596
		1.3	7.01%	14397	370	2370	2597
	0.8	1.1	4.01%	14403	377	2496	2723
		1.2	5.03%	14397	370	2487	2714
		1.3	5.52%	14395	368	2445	2672
	0.9	1.1	2.65%	14402	377	2558	2785
		1.2	3.19%	14396	370	2543	2769
		1.3	3.50%	14397	370	2511	2737
	1	1	0.00%	14407	382	2508	2735

Table D.3: Numerical results when $C_S = 500$, $C_C = C_D = 40$, $C_T = 200$, $r = 0.8$, and $\tau = 0.95$.

NPF	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF-Loss	ES	EP	ESC	EGD
0	0.7	1.1	3.57%	14359	127	1960	1767
		1.2	5.10%	14359	127	1959	1766
		1.3	5.95%	14331	126	2093	1872
	0.8	1.1	3.16%	14359	127	1962	1767
		1.2	4.35%	14346	126	2023	1816
		1.3	4.99%	14313	125	2180	1940
	0.9	1.1	2.39%	14346	126	2021	1814
		1.2	3.08%	14317	125	2161	1925
		1.3	3.43%	14293	123	2276	2017
	1	1	0.00%	14337	135	2109	1884
4.02%	0.7	1.1	5.04%	14014	437	2125	1886
		1.2	6.37%	14001	435	2185	1933
		1.3	7.15%	13977	431	2291	2018
	0.8	1.1	4.06%	14010	435	2142	1900
		1.2	5.13%	13988	434	2247	1983
		1.3	5.71%	13962	431	2363	2075
	0.9	1.1	2.76%	14006	435	2155	1910
		1.2	3.40%	13978	432	2289	2016
		1.3	3.75%	13958	429	2371	2081
	1	1	0.00%	13998	437	2206	1949
7.66%	0.7	1.1	6.42%	13705	722	2292	2009
		1.2	7.59%	13688	721	2375	2076
		1.3	8.30%	13672	720	2452	2137
	0.8	1.1	4.91%	13703	723	2307	2021
		1.2	5.87%	13680	721	2416	2108
		1.3	6.41%	13664	720	2490	2166
	0.9	1.1	3.12%	13710	724	2271	1993
		1.2	3.71%	13681	722	2410	2103
		1.3	4.05%	13670	719	2457	2140
	1	1	0.00%	13709	726	2287	2004
10.96%	0.7	1.1	7.72%	13422	983	2469	2142
		1.2	8.77%	13402	980	2554	2210
		1.3	9.44%	13391	977	2597	2244
	0.8	1.1	5.72%	13421	985	2480	2151
		1.2	6.59%	13399	982	2575	2226
		1.3	7.12%	13390	979	2611	2255
	0.9	1.1	3.46%	13430	986	2435	2115
		1.2	4.01%	13407	982	2534	2193
		1.3	4.35%	13403	981	2551	2207
	1	1	0.00%	13425	985	2451	2127
13.98%	0.7	1.1	8.94%	13154	1217	2653	2280
		1.2	9.91%	13139	1212	2709	2325
		1.3	10.56%	13133	1208	2720	2334
	0.8	1.1	6.48%	13155	1217	2648	2277
		1.2	7.29%	13140	1214	2709	2325
		1.3	7.81%	13135	1210	2719	2333
	0.9	1.1	3.78%	13166	1217	2592	2232
		1.2	4.31%	13154	1215	2644	2273
		1.3	4.64%	13149	1212	2654	2281
	1	1	0.00%	13169	1218	2574	2216

Table D.4: Numerical results when $C_S = 500$, $C_T = 200$, NPF = 4.02%, $\tau = 0.95$, and $r = 0.8$.

$C_C = C_D$	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF-Loss	ES	EP	ESC	EGD
40	0.7	1.1	5.04%	14014	437	2125	1886
		1.2	6.37%	14001	435	2185	1933
		1.3	7.15%	13977	431	2291	2018
	0.8	1.1	4.06%	14010	435	2142	1900
		1.2	5.13%	13988	434	2247	1983
		1.3	5.71%	13962	431	2363	2075
	0.9	1.1	2.76%	14006	435	2155	1910
		1.2	3.40%	13978	432	2289	2016
		1.3	3.75%	13958	429	2371	2081
	1	1	0.00%	13998	437	2206	1949
60	0.7	1.1	5.54%	14164	802	3020	2592
		1.2	6.99%	14162	791	2983	2563
		1.3	7.77%	14134	787	3109	2663
	0.8	1.1	4.50%	14171	799	2969	2551
		1.2	5.65%	14151	790	3031	2600
		1.3	6.25%	14119	783	3165	2707
	0.9	1.1	3.10%	14171	793	2945	2532
		1.2	3.81%	14128	784	3117	2668
		1.3	4.21%	14101	777	3220	2750
	1	1	0.00%	14127	793	3169	2709

Table D.5: Numerical results when $C_S = 500$, $C_C = C_D = 40$, NPF = 4.02%, $\tau = 0.95$, and $r = 0.8$.

C_T	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF-Loss	ES	EP	ESC	EGD
100	0.7	1.1	2.85%	12564	374	2394	1946
		1.2	3.57%	12570	369	2387	1940
		1.3	4.10%	12574	365	2383	1937
	0.8	1.1	2.19%	12563	376	2386	1939
		1.2	2.80%	12571	372	2380	1934
		1.3	3.22%	12575	367	2372	1927
	0.9	1.1	1.43%	12563	379	2375	1930
		1.2	1.84%	12572	374	2362	1916
		1.3	2.11%	12577	369	2353	1912
	1	1	0.00%	12556	386	2371	1929
200	0.7	1.1	5.04%	14014	437	2125	1886
		1.2	6.37%	14001	435	2185	1933
		1.3	7.15%	13977	431	2291	2018
	0.8	1.1	4.06%	14010	435	2142	1900
		1.2	5.13%	13988	434	2247	1983
		1.3	5.71%	13962	431	2363	2075
	0.9	1.1	2.76%	14006	435	2155	1910
		1.2	3.40%	13978	432	2289	2016
		1.3	3.75%	13958	429	2371	2081
	1	1	0.00%	13998	437	2206	1949

Table D.6: Numerical results when $C_C = C_D = 40$, $C_T = 200$, NPF = 4.02%, $\tau = 0.95$, and $r = 0.8$.

C_S	$K_p^+ = K_n^-$	$K_n^+ = K_p^-$	TCF-Loss	ES	EP	ESC	EGD
0	0.7	1.1	4.57%	13847	0	0	0
		1.2	6.17%	13847	0	0	0
		1.3	7.27%	13847	0	0	0
	0.8	1.1	3.64%	13847	0	0	0
		1.2	4.85%	13847	0	0	0
		1.3	5.68%	13847	0	0	0
	0.9	1.1	2.42%	13847	0	0	0
		1.2	3.14%	13847	0	0	0
		1.3	3.58%	13847	0	0	0
	1	1	0.00%	13847	0	0	0
250	0.7	1.1	4.97%	13922	404	1943	1638
		1.2	6.22%	13892	390	2032	1709
		1.3	6.97%	13877	385	2084	1750
	0.8	1.1	4.02%	13924	399	1913	1615
		1.2	5.03%	13890	389	2038	1713
		1.3	5.60%	13874	385	2098	1761
	0.9	1.1	2.75%	13929	398	1884	1591
		1.2	3.40%	13895	389	2012	1693
		1.3	3.74%	13881	387	2073	1741
	1	1	0.00%	13918	392	1907	1610
500	0.7	1.1	5.04%	14014	437	2125	1886
		1.2	6.37%	14001	435	2185	1933
		1.3	7.15%	13977	431	2291	2018
	0.8	1.1	4.06%	14010	435	2142	1900
		1.2	5.13%	13988	434	2247	1983
		1.3	5.71%	13962	431	2363	2075
	0.9	1.1	2.76%	14006	435	2155	1910
		1.2	3.40%	13978	432	2289	2016
		1.3	3.75%	13958	429	2371	2081
	1	1	0.00%	13998	437	2206	1949