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Trimmed Multilevel Fast Multipole Algorithm for D-Type Volume Integral Equations

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***Abstract*—In this work, we present a trimming scheme for the multilevel tree structure of multilevel fast multipole algorithm (MLFMA), which is applied on D-type volume integral equations. With this approach, the number of iterations and the durations of matrix-vector multiplications are significantly reduced for the solution of multi-scale volumetric problems. The trimming operation is performed on rows and columns of the impedance matrix. In order to eliminate the matrix columns, the current coefficients are estimated via machine learning techniques. The implementation particularly provides significant acceleration for the iterative solutions of electrically large volumetric problems.**

***Index Terms*—MLFMA, volume integral equations, machine learning**

I. Introduction

Great interest has been given to scattering of electromagnetic fields from dielectric objects in the last century. Back scattering from dielectric spheres are used to identify the material of the objects by using the resonances occurred in the dielectric medium [1]. Volume integral equations (VIEs) enable to solve highly inhomogeneous structures in contrary to surface integral equations (SIEs). This inevitable advantage of VIEs, however, cannot be fully utilized, because the matrix equation cannot be solved directly or iteratively due to large number of unknowns. Therefore, to decrease the time and memory complexity of the solution, multilevel fast multipole algorithm (MLFMA) is introduced in [2]. Even with a complexity of $NlogN$, conventional MLFMA applications have challenges for the solution of multi-scale electromagnetic problems. These challenges especially become more significant during the solution of prolonging iterations. To overcome this bottleneck, trimmed MLFMA (T-MLFMA) [3], which is originally developed for SIEs, is extended to VIEs and remarkable acceleration is obtained in its solution time compared to the conventional MLFMA. Moreover, highly accurate results are obtained and comparison with the conventional MLFMA is presented for a homogeneous dielectric sphere as a numerical result.

II. Method and Numerical Results

In this study, the scattering from large-scale dielectric objects formulated by using D-type electric-field volume integral equation (EFVIE) is considered [4]. 3-D objects are discretized with tetrahedral meshes, where the electric flux density in the body is discretized with Saubert-Wilton Glisson (SWG) functions [5] resulting a dense matrix equation to be solved. This resultant matrix equation is solved iteratively by using Flexible Generalized Minimal Residual method (FGMRES) and the matrix-vector multiplications (MVM) are accelerated via MLFMA.

In order to construct the tree structure of MLFMA, the volumetric geometry is placed into a cubic box, which is recursively divided into 8 sub-boxes until the smallest box size of $\lambda/16$ is reached [4]. At the lowest level, one-box buffer is used to determine the near-zone interactions and the rest of the boxes are designated as far-zone or very far-zone interactions to be calculated by MLFMA. In MLFMA, near and far-zone interactions are calculated separately. For the calculation of near-zone interactions, the interactions of basis and testing SWG functions are calculated in an element by element fashion. However, far-zone interactions are calculated in a group by group manner, where the fields due to the basis SWG functions are aggregated in the center of boxes, translated into the center of far boxes and disaggregated into the center of sub-boxes and received by each testing SWG function at the lowest level. Remaining very far-zone interactions are implicitly included at the higher levels. For the translation function calculation, the truncation number is calculated via excess bandwidth formula [6] until the level with $\lambda/4$ box length. After that level, the truncation number is set as 7 to decrease the error in the Green's function calculation for the double precision arithmetic by considering the results in [7].

The near-zone impedance matrix is calculated once by using the near-zone interactions before the iterative solution. The far-zone impedance matrix is obtained by utilizing required MVM at each iteration by considering far-zone interactions in MLFMA. As the iterations proceed, most of the volume current coefficients and the resulting scattered field calculated in the volume rapidly converge and do not contribute to the solution. Therefore, it gives opportunity to trim the basis and testing functions that do not contribute the solution any longer after predefined convergence levels. This trimming process is performed on the far-zone impedance matrix, and the near-zone impedance matrix is untouched. If all basis functions are trimmed in a box, the box is trimmed from the aggregation and translation calculations and if all testing functions are trimmed in a box, the box is also trimmed from the translation and disaggregation calculations in T-MLFMA [3]. This trimming process significantly decreases the time to obtain a solution at each iteration.

Regarding the basis trimming, the value of coefficients at each iteration should be estimated, since the actual value of

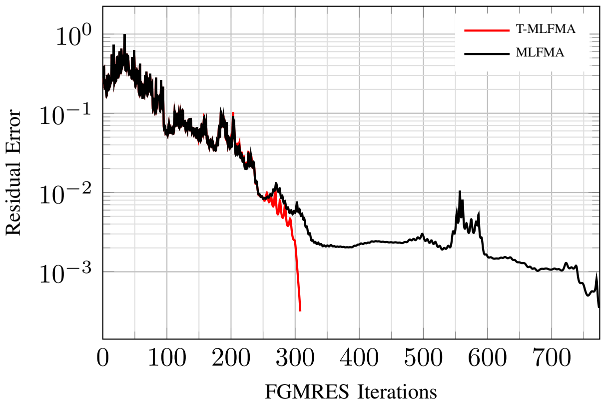

Fig. 1: Residual errors of the iterative solution of the dielectric sphere with diameter 1.5λ and ϵ=6

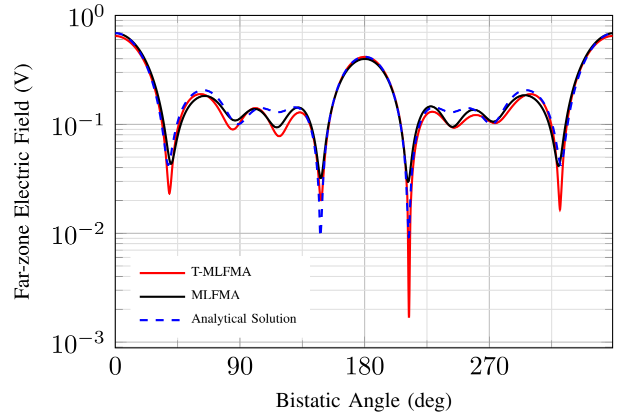

Fig. 2: The scattered electric field of the dielectric sphere with diameter 1.5λ and ϵ=6

coefficients are not present in the solver. For this purpose, to calculate the estimated values of coefficients, a fully connected neural network (FCNN) is created. As a preliminary set of results (in order to test the trimming idea in VIE), we work on dielectric spheres for training which are used to attack generic dielectric problems. Therefore, the data set is generated by using the solutions of full sphere with λ/4 diameter and different dielectric constants 1.5, 2, 3, 4, 6, 8, 10, 15, 20 and 25. Inputs of the network are the real and imaginary parts of the normalized value of coefficients obtained from the most recent 3 iterations. With this proposed method, the training with small size full sphere solutions is sufficient to estimate the coefficients for larger complex inhomogeneous geometries. Different than how T-MLFMA is used for SIEs, the interactions of SWG functions are arbitrary in any volume and a sufficiently large volume contains various interactions of SWG functions with many directions. To indicate this assumption, as a preliminary work, the trained network is used for the solution of full sphere geometries for different permittivity values. On the contrary to basis trimming, to check the convergence of right-hand side vector for testing trimming is more trivial and converged indexed testing functions are trimmed from the calculations accordingly. To check the convergence of basis and testing functions, a threshold error called trimming error to ensure the behavior of the convergence is introduced. In all trials, a block diagonal preconditioner is used to increase the convergence rate [2].

The performance of the method is observed on the scattering from a full sphere with 1.5λ diameter and permittivity as 6. The geometry is discretized with 777,308 SWG functions. In this example, the trimming error is set as 0.03. Moreover, the number of MLFMA iterations is reduced from 774 to 309. Due to the decrease of the number of iterations and time per iteration, total MVM time is also dropped from 9.36 hours to 3.76 hours by the end of the solution. As the result of trimming, the relative norm error of scattered field with respect to conventional MLFMA is obtained as 0.064. Fig. 2 depicts the comparison for scattered electric field for T-MLFMA, conventional MLFMA, and MIE series solutions. Moreover, T-MLFMA and MLFMA results show significant consistency for forward and back scattered fields.

III. Conclusion

In this paper, a T-MLFMA implementation on VIE is considered. The converged coefficients are estimated via an FCNN and those matrix columns are eliminated from proceeding iterations. For the training of FCNN, coefficients that are obtained from the solution of $\lambda/4$ sphere with various dielectric permittivities are used. A similar procedure is followed for testing coefficients. However, ML techniques are not utilized, but testing coefficients are compared with the actual right hand side vector. Therefore, the corresponding matrix rows are eliminated from the calculation in the next iterations. Consequently, solution time of scattering from large complex inhomogeneous objects is decreased significantly.

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