FORECASTING INTEREST RATES USING SHIFTING ENDPOINTS IN SMALL OPEN ECONOMIES: EVIDENCE FROM CANADA AND THE UK

A Master's Thesis

by

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Ankara
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To my family.

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By İbrahim Ata

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

FORECASTING INTEREST RATES USING SHIFTING ENDPOINTS IN SMALL OPEN ECONOMIES: EVIDENCE FROM CANADA AND THE UK

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This thesis examines the forecasting performance of widely used interest rate forecasting methods for small open economies such as Canada and the UK. In particular, I run a horse race between standard models and the models using shifting endpoints to see whether results for the US extends to small open economies. In this setup, three time-varying parameters, interpreted as factors corresponding to level, slope and curvature, are allowed to have shifting long-run means rather than a constant mean. The shifting endpoints are introduced by exponential smoothing of factors, and the connection of factors with certain macroeconomic variables and inflation expectations. In comparison to the random walk benchmark, allowing for shifting endpoints in yield curve factors offers significant gains in out-of-sample forecasting accuracy. Moreover, results suggest that there is a strong evidence that the US as a global economy has a spillover effect on the term structure of interest rates of Canada and the UK.

Keywords: Yield Curve, Forecasting, Shifting endpoints

ÖZET

KÜÇÜK AÇIK EKONOMİLERDE HAREKETLİ UÇ YÖNTEMİ İLE GETİRİ EĞRİSİ TAHMİNİ: KANADA VE İNGİLTERE ÖRNEKLERİ

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Bu tez, Kanada ve İngiltere gibi küçük açık ekonomilerde faizler için kullanılan tahmin yöntemlerinin tahmin performansını incelemektedir. Özellikle, ABD için elde edilen sonuçların küçük açık ekonomileri kapsayıp kapsamadığını görmek için standart modeller ile değişken uç noktaları kullanan modeller arasında tahmin performansı açısından bir karşılaştırma yapılmaktadır. Değişken uç noktalar modeli altında, seviye, eğim ve içbükeylik olarak tabir edilen ve zamanla değişen bu üç faktörün, sabit bir ortalamaya değil de hareketli uçlu uzun dönem ortalamasına sahip olmasına izin verilmiştir. Hareketli uç yöntemi, faktörlerin üstsel düzeltmesi, faktörlerin belirli makroekonomik göstergeler ve enflasyon beklentileri ile ilişkisi üzerine uygulanmıştır. Tahminde temel ölçüt olan rastgele yürüyüş yöntemine kıyasla, getiri eğrisi faktörleri için hareketli uç yöntemine izin verilmesi örneklem dışı tahmin doğruluğunda önemli kazanımlar sunmuştur. Aynı zamanda, ABD ekonomisinin Kanada ve İngiltere getiri eğrileri üzerinde bir yayılma etkisi olduğuna dair güçlü kanıtlar bulunmuştur.

Anahtar Kelimeler: Getiri eğrisi, Tahmin, Hareketli uç yöntemi

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My older brother deserves my heartfelt gratitude for his presence and unwavering support throughout my life. Thank you all for your support and the strength you gave me.

The main motivation for my studies has always been to make my family proud. Therefore, the completion of this thesis is dedicated to my family, who have sacrificed their own lives to support my education.

Last but not least, I am grateful for the unconditional love and never-ending support of my girlfriend, Sena Kocaman. We succeeded together.

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CHAPTER 1

INTRODUCTION

Forecasting the term structure of interest rates is an important topic for investors who manage their portfolios and risk, as well as monetary policymakers who want to infer expectations from yields. The literature for estimating the term structure of interest rates is divided into two: statistical approach and equilibrium approach. The equilibrium approach, including affine equilibrium models, employs economic theory in order to characterize the relationship between asset prices and macro fundamentals. Cox, Ingersoll, and Ross (1985), and Duffie and Kan (1996) are the most prominent papers of the equilibrium approach. The statistical approach, on the other hand, uses statistical models to explain asset prices. To do so, the observed data is smoothed with parametric or non-parametric models in order to obtain the yield curve. Nelson and Siegel (1987) parametric and parsimonious model is one of the most well known models since it explains the yield curve with only three parameters.

Forecasting the yield curve is as important as understanding underlying dynamics. According to Duffee (2002), affine models produced poor forecasts of future changes in treasury yields and the benchmark random walk model gave better forecasts. Thereafter, Diebold and Li (2006) proposed the Dynamic

Nelson-Siegel (hereafter DNS) model, which forecasts the yield curve using Nelson and Siegel parsimonious model's parameters evolving dynamically. Accordingly, the DNS performed better accuracy relative to the random walk and some other models.

Van Dijk, Koopman, Van der Wel, and Wright (2014) built on the DNS model by using the shifting endpoints specification, which was first proposed by Kozicki and Tinsley (2001) in order to link historical shifts in market perceptions of the policy target to inflation. The dynamic Nelson-Siegel approach suggested by Diebold and Li (2006) modeled the factor dynamics as a stationary process around a constant unconditional mean. However, in Van Dijk et al. (2014), long-run means of factors were allowed to shift instead of having constant means.

Van Dijk et al. (2014) applied the shifting endpoint with different specifications. They allowed shifting endpoints with exponential smoothing, inflation expectation and realized macro variables and found that shifting endpoints specification improved the forecast accuracy relative to the DNS and random walk methods with statistically significant results.

The US Treasury yield data is commonly used in the relevant empirical literature for yield curve estimation and forecasting. My paper will differ from the literature since I will check whether the shifting endpoints specification and other benchmark models' predictive accuracy differs for small open economies such as Canada and the UK. Furthermore, since the shifting endpoint specification is also used with realized measures and inflation expectations, I will be able to see whether the US as a global economy and its realized macro variables have an impact on the yield curve shape of these small open economies.

I find that shifting endpoints models with the US-based inflation expectations and macroeconomic indicators are superior models relative to the random walk. Additionally, shifting endpoints with exponential smoothing applied to all three factors outperforms the random walk benchmark. Finally, there are promising results suggesting that there is a spillover effect of the US economy on the term structure of interest rates of Canada and the UK.

In Section 2, I will describe the dynamic Nelson–Siegel model as well as benchmark models that have been agreed upon in the literature, such as random walk, AR(1), and VAR. In Section 3, Canada and UK Bond Yield data will be introduced and represented by the Nelson and Siegel (1987) parametric model. In section 4, the shifting endpoints method will be used with different specifications. In section 5, the results, findings and forecast efficiency will be discussed after briefly showing the results for the Diebold and Mariano test. Finally, in section 6, I will conclude.

CHAPTER 2

SETUP OF BENCHMARK MODELS

2.1 The Dynamic Nelson-Siegel Model

Continuously compounded yield to maturity on a zero-coupon bond with maturity τ periods at time t is denoted as $y_t(\tau)$. Following Nelson and Siegel (1987) and Diebold and Li (2006), the three-factor model for the yield curve is

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}\right) + \varepsilon_t(\tau) \qquad (1)$$

As shown in Diebold and Li (2006), the three time-varying parameters β_{1t}, β_{2t} and β_{3t} may be interpreted as dynamic factors corresponding to level, slope and curvature. The continuously compounded yield is calculated with these factors and their multiplications with corresponding loadings. The loading on β_{1t} is 1 and unlike other loadings on β_{2t} and β_{3t}, it does not converge to zero as τ increases. Hence it may be considered to be random walk, or simply the level factor. The loading on β_{2t} is a function of maturity starting at 1 but monotonically decays to 0; therefore, it may be considered to be a short-term factor, or the slope factor. Finally, the loading on β_{3t} is zero at maturities zero, then rises, and eventually decays to zero as the maturity increases. Hence, it is called a medium-term factor, or curvature of the yield curve. The maturity at which the loading on β_{2t} decays to zero and the maturity at which the loading on β_{3t} reaches its highest value are both determined by the parameter λ_t.[1] The decay parameter is set to $\lambda = 0.0609$ in both Diebold and Li (2006) and Van Dijk et al. (2014). As two- or three-year maturities are typically regarded as medium-term, this value means that the loading on curvature factor reaches its maximum at the maturity $\tau = 30$ months (simply the average of two and three years). This specification is valid for U.S. Treasury yields; however, the decay parameters in Canada and the United Kingdom will be different. For both countries, I used decay parameters that ensure the yield curves have the lowest in-sample fitting error. Accordingly, $\lambda_{Canada} = 0.068$ and $\lambda_{UK} = 0.0739$ imply that the loading on β_3 achieve its maximum at maturity τ of 26 and 24 months, respectively. Nonetheless, adjusting the decay parameters for both countries to $\lambda = 0.0609$ marginally changes my results.

The term $\varepsilon_t(\tau)$ is the measurement error that stems from the different estimation methodologies of countries the US, the UK and Canada. The Nelson and Siegel model is used to generate the US zero-coupon bond yield data. On the other hand, the UK and Canada datasets extracted from the central banks are constructed by using spline-based methods which will be detailed further below. The error term is included because the Nelson and Siegel method will be used to represent the UK and Canada datasets, which are actually created based on distinct methodologies. As a result, while Diebold and Li (2006) does not include the error term in equation (1), this work will. The residuals are assumed to have a zero mean and constant variance σ^2.

Given that the parameter λ is fixed at country-specific values and corresponding loadings along with the desired maturity, the estimates of β_{1t}, β_{2t} and β_{3t} can be estimated by fitting the equation (1) for each month t using ordinary

[1] Even though there are several other estimation methods that contain time-varying λ_t, the decay parameter will be treated as constant.

least squares. Time series of estimates of factors $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$ and $\hat{\beta}_{3t}$ and corresponding residuals are obtained by applying OLS to the yield data for each month. Having the estimates of factors, the model is amended by adding dynamics of level, slope and curvature factors. Following Diebold and Li (2006), for each factor, the univariate first-order autoregressive process given by

$$\beta_{j,t+1} = \mu_j + \phi_j(\beta_{jt} - \mu_j) + \eta_{j,t+1} \tag{2}$$

enables level, slope and curvature factors to vary dynamically for $j = 1, 2, 3$ respectively, where μ_j is the mean of estimated factors. The residuals $\eta_{j,t+1}$ have a mean zero and variance σ_j^2 and are assumed to be independent over time and across maturities.

In order to improve the efficiency of the models, the factors and model parameters could have been estimated using either the Kalman filter or maximum likelihood estimation methods. Since these alternative methods estimate the model parameters and factors in one step, the efficiency of the model would be higher compared to two-step OLS estimation. However, since the primary goal of this paper is to compare the prediction accuracy of forecasting methods to existing literature, I will use the two-step OLS estimation method and leave the results of one-step estimation for future research.

Briefly, the dynamic Nelson-Siegel model has four steps to forecast future yields. First, having the loadings and the panel data of each month's yields, the factors are estimated by using ordinary least squares. Second, estimated factors are used in the separate univariate first-order autoregressive process to estimate the model parameter ϕ_j. Third, by taking the last factor value of in-sample data, equation (2) is iterated to generate the forecast of each factor up to desired forecast horizon of h periods $\beta_{j,t+h}$. Finally, having the out-of-sample factors, interest rates of the desired horizon for each maturity can be calculated via equation (1).

2.2 Random Walk Model

The random walk process is a variant of dynamic Nelson-Siegel method. The estimated factors obtained from OLS estimation are used to create random walk forecasts of level, slope and curvature factors via equation (2). However, for this process, $\mu_j = 0$ and $\phi_j = 1$ for all j. Therefore the forecast of desired horizons of h periods will be exactly the same as the factors of the last month of in-sample data. This model implies that in each period, $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$ and $\hat{\beta}_{3t}$ takes a random step away from its previous value, and the steps are independently and identically distributed. Having the forecast of three factors, equation (1) is used to calculate the yields.

2.3 Vector Autoregression (VAR) of Factors

The VAR(1) approach is applied to estimated factors. The structure is that each factor is a linear function of past lags of itself and past lags of other factors.

$$\begin{pmatrix} \hat{\beta}_{1,t+1} \\ \hat{\beta}_{2,t+1} \\ \hat{\beta}_{3,t+1} \end{pmatrix} = \hat{C} + \hat{\Gamma} \begin{pmatrix} \hat{\beta}_{1t} \\ \hat{\beta}_{2t} \\ \hat{\beta}_{3t} \end{pmatrix} \tag{3}$$

The coefficients and constant terms are estimated according to the vector autoregression model above. Having estimated the model matrix by using in-sample factors, the out-of-sample factors can be obtained for desired forecast horizon h by

$$\begin{pmatrix} \hat{\beta}_{1,t+h} \\ \hat{\beta}_{2,t+h} \\ \hat{\beta}_{3,t+h} \end{pmatrix} = \sum_{j=0}^{h-1} \hat{\Gamma}^j \hat{C} + \hat{\Gamma}^h \begin{pmatrix} \hat{\beta}_{1t} \\ \hat{\beta}_{2t} \\ \hat{\beta}_{3t} \end{pmatrix}$$

where $[\hat{\beta}_{1t} \ \ \hat{\beta}_{2t} \ \ \hat{\beta}_{3t}]^T$ is the end of in-sample factors' values.

2.4 AR(1) Process of Yields

The first-order autoregressive process is applied to the realized interest rates of in-sample data. Having estimated the coefficient of the first-order autoregressive process, the out-of-sample interest rates are calculated by iterating the last month's interest rates of in-sample data.

$$\hat{y}_{t+1}(\tau) = \hat{\Gamma}\hat{y}_t(\tau) \tag{4}$$

Having estimated the model parameter Γ by using the in-sample data, the out-of-sample interest rates for desired horizon h can be forecasted by

$$\hat{y}_{t+h}(\tau) = \hat{\Gamma}^h\hat{y}_t(\tau)$$

where $\hat{y}_t(\tau)$ is the interest rates of the last month in the in-sample data. This process does not depend on equation (1) and equation (2). Therefore, it is an alternative forecasting method to compare the results.

CHAPTER 3

THE DYNAMIC NELSON-SIEGEL MODEL FOR CANADA AND UK ZERO-COUPON YIELD CURVE

Since the primary goal of this study is to see whether shifting endpoint models enhance forecast accuracy for small open economies like Canada and the United Kingdom, I will use two different databases for zero-coupon yields of selected maturities for both countries.

As mentioned above, the estimates of β_{1t}, β_{2t}, and β_{3t} can be obtained by fitting the model equation (1) for each month t using ordinary least squares with country-specific fixed parameter λ, the formula can be written as

$$\begin{pmatrix} \hat{\beta}_{1t} \\ \hat{\beta}_{2t} \\ \hat{\beta}_{3t} \end{pmatrix} = \left(\sum_{i=1}^{N} x_i x_i' \right)^{-1} \sum_{i=1}^{N} x_i y_t(\tau_i)$$

where

$$x_i = \begin{pmatrix} 1 \\ \frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} \\ \frac{1-e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i} \end{pmatrix}$$

for the N yields in dataset with maturities τ_i, i=1,...,N.

3.1 Canada

The risk-free government zero-coupon term structure is not directly observable and needs to be generated from the prices of marketable coupon-bearing bonds. As a result, based on the observed market prices of a collection of coupon-bearing bonds, several estimation methods can be used to derive a zero-coupon yield curve. The Merrill Lynch Exponential Spline (MLES) model, as introduced by Li, DeWetering, Lucas, Brenner, and Shapiro (2001), is the estimation algorithm used by the Bank of Canada to build the historical database of zero-coupon yields. A description of the methodology used to derive the yield curves is provided in Bolder, Johnson, and Metzler (2004). They used principal component analysis in the article to show that the uncorrelated first three principal components account for 99.6 per cent of the correlation between zero-coupon rates, and those principal components are related to the level, slope and curvature factors of the Nelson and Siegel (1987) paper.

The end-of-month yields of Government of Canada bonds and treasury bills are taken from the Bank of Canada data described above. The yields of maturities 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, and 120 months are included in the dataset, which spans the period from January 1986 to May 2022. For the purpose of my study, I have used this dataset starting from January 1986 up to December 2009 to estimate yield curve factors

The time series of a yield in the dataset is plotted in Figure 1. Up until 1991, the trend for all four maturities was upward; however, the rest of the time up through 2009 shows a persistent fall in the trend, despite minor upward movements along the way.

Figure 2 illustrates the proxies directly obtained from the raw data, as well as the resulting factor estimates from the end-of-month dataset for Government of Canada bonds and treasury bills.

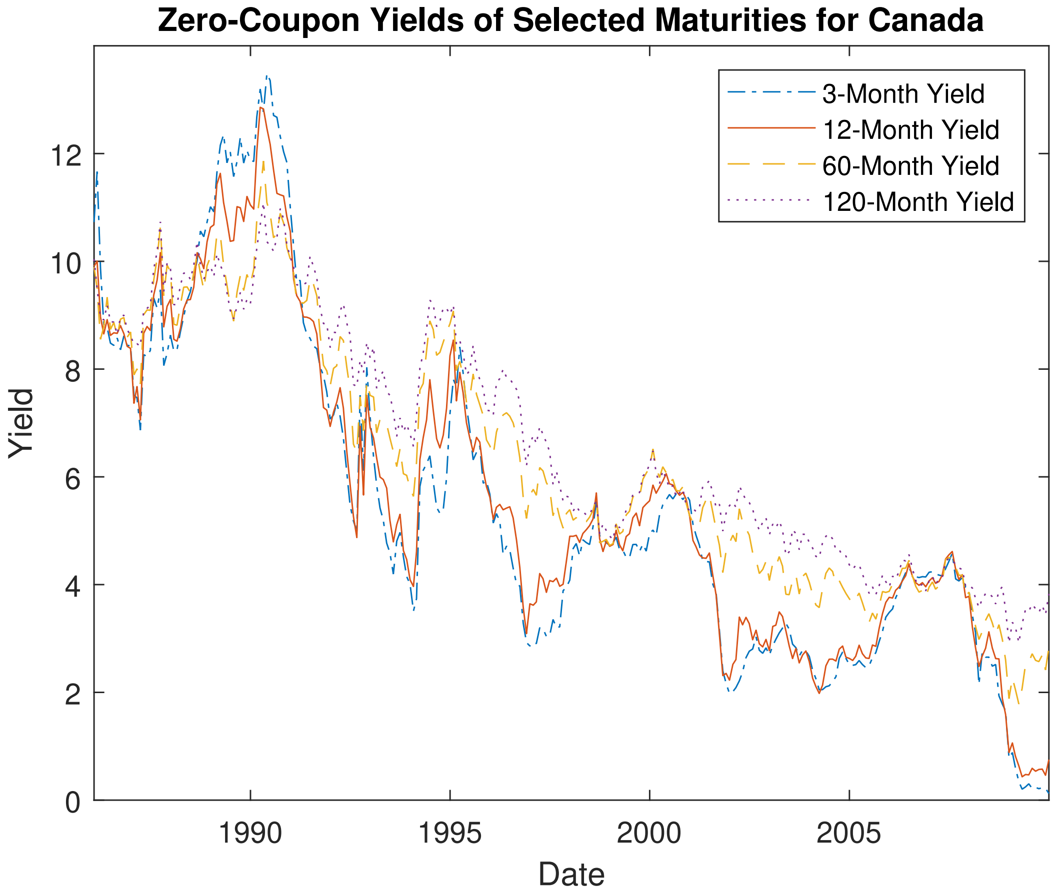

Figure 1: Yields of Government of Canada Bonds Over the Period January 1986 to December 2009. The selected realized yields of Government of Canada bonds over the period January 1986 to December 2009 are presented. The 3-month yield is represented by a dash-dotted line, the 12-month yield is represented by a solid line, the 60-month yield is represented by a dashed line, and the 120-month yield is represented by a dotted line.
Source: Bank of Canada

3.2 The United Kingdom

The zero-coupon yield curve for the UK is estimated using Waggoner (1997) method by the Bank of England. The details of the estimation technique are available in the working paper of Anderson and Sleath (2001).

The dataset consists of the same maturities 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108 and 120 of zero-coupon interest rates. From 1970 to 2022, the original dataset provides daily estimated yield curves for the United Kingdom. In order to be consistent with Canadian data, the monthly yields for the same period starting January 1986 to December 2009 are taken as in-

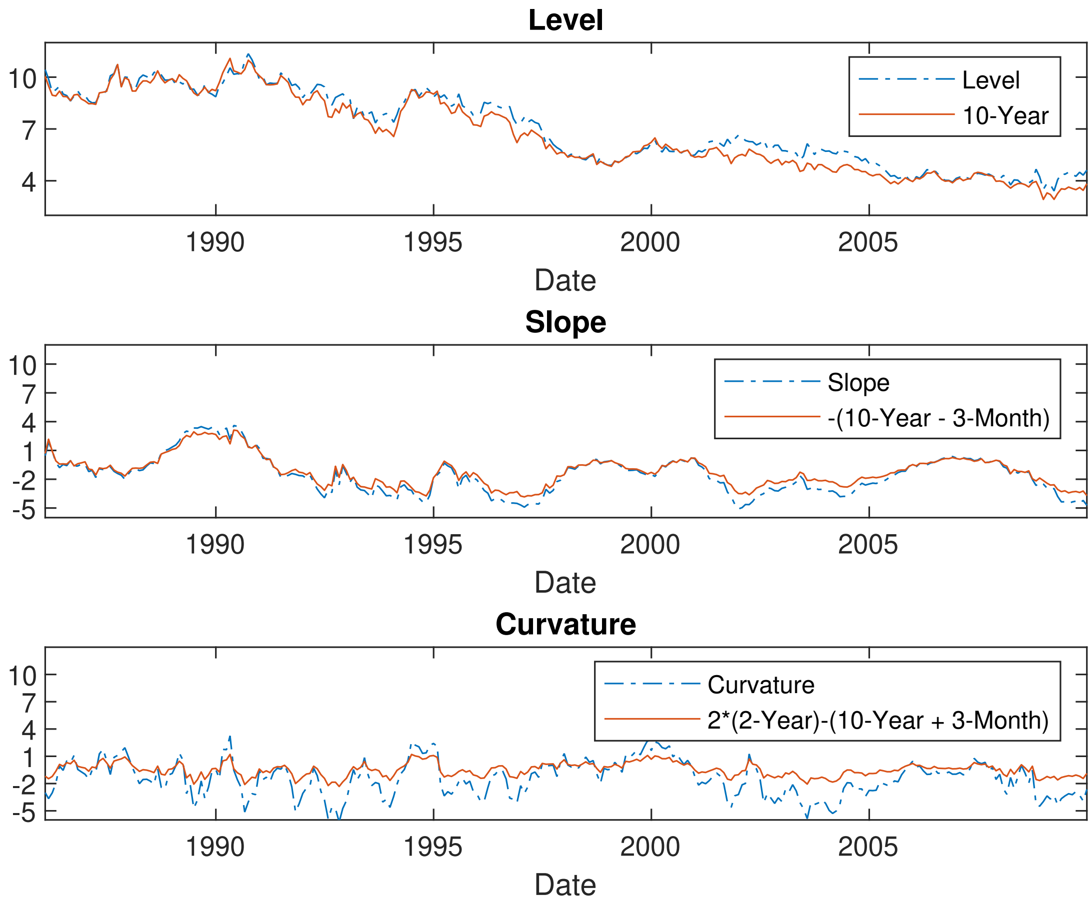

Figure 2: Estimated Level, Slope and Curvature for Canada. The estimated factors β_{1t}, β_{2t}, and β_{3t} are dashed lines calculated from the Nelson and Siegel (1987) model. The data comes from end-of-month yields curves for zero-coupon bonds, generated using pricing data for Government of Canada bonds and treasury bills. The proxies for level, slope and curvature factors are directly extracted from raw data and plotted with a solid line. The longest (120-month) maturity yield, the longest maturity yield minus the shortest (3-month) maturity yield, and two times 24-month yield minus the sum of 3-month and 120-month yield are used for the level factor, slope factor, and curvature factor, respectively.

sample data. The extracted dataset is also expanded by ten years to provide out-of-sample data, which is used to compare predictive accuracy.

The zero-coupon yields for specified maturities are shown in figure 3. After 1991, the yields on England's bonds and treasury bills fell sharply, similar to the realized yields on Government of Canada bonds and treasury bills. The UK's data, on the other hand, is fluctuating less around the declining trend after 1991.

Figure 4 shows the estimated factors for level, slope, and curvature factor de-

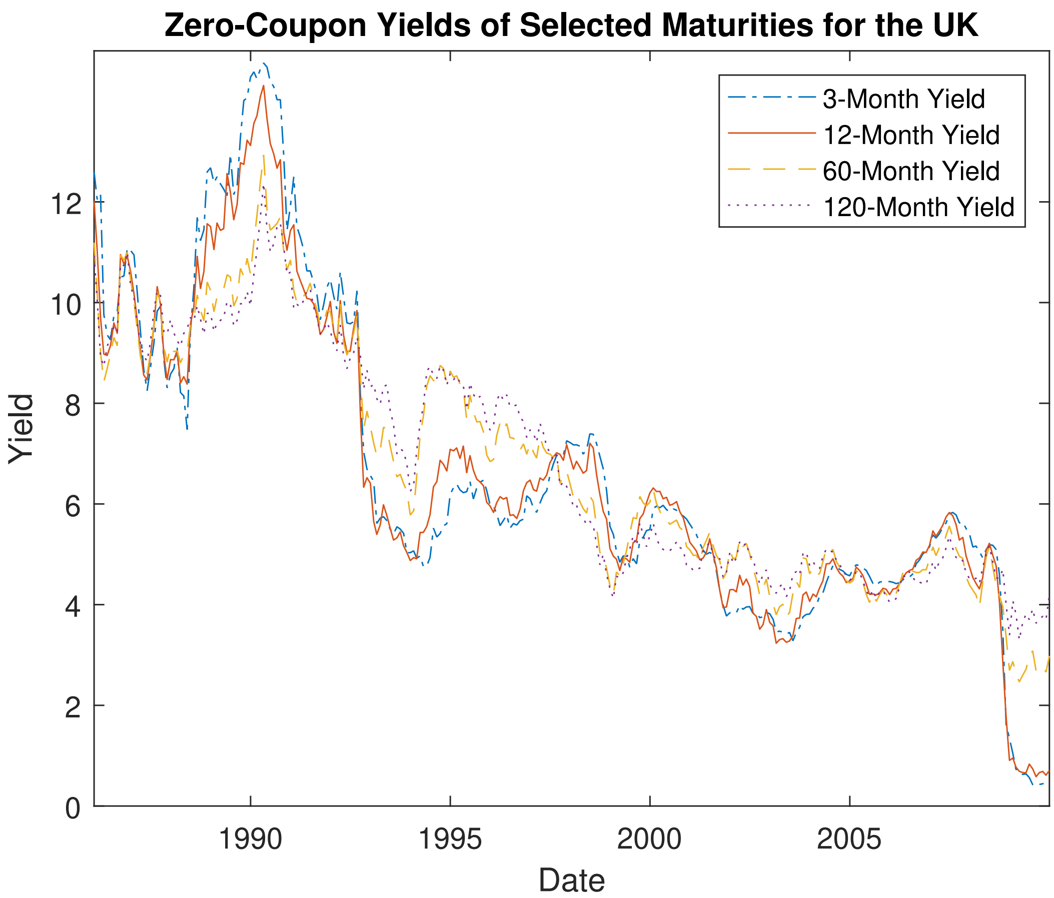

Figure 3: Yields of Bank of England Bonds Over the Period January 1986 to December 2009. The selected realized yields of Bank of England bonds over the period January 1986 to December 2009 are presented. The dash-dotted line represents the 3-month yield, The solid line represents the 12-month yield, The dashed line represents the 60-month yield, and The dotted line represents the 120-month yield.
Source: Bank of England

rived from the UK in-sample data as a result of the OLS method in equation (1) and proxies derived from raw data.

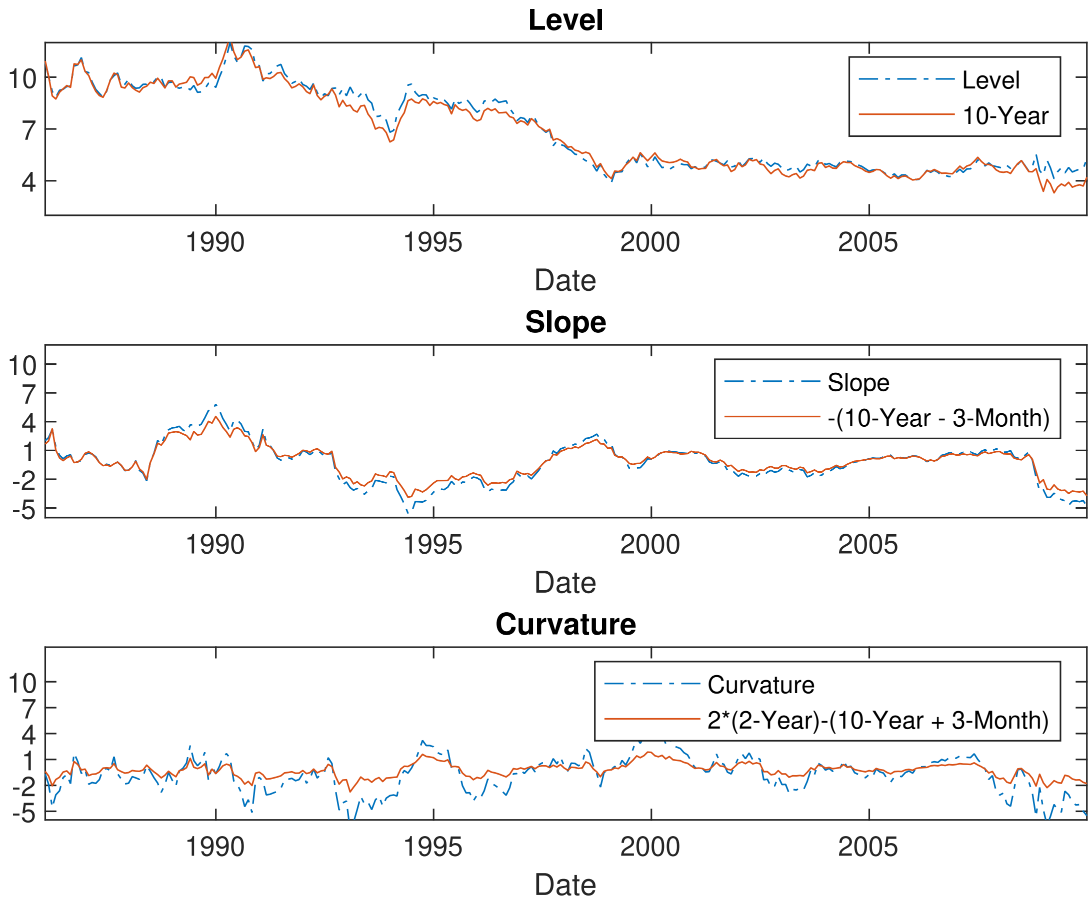

Figure 4: Estimated Level, Slope and Curvature for the UK. The estimated factors β_{1t}, β_{2t}, and β_{3t} are dashed lines calculated from the Nelson and Siegel (1987) model. The dataset is created from Bank of England data. The proxies are the following yields: Level=10-year, Slope= - (10-year - 3-month) and Curvature=2×(2-year) - (10-year + 3-month).

CHAPTER 4

SHIFTING ENDPOINTS

The autoregressive process in dynamic Nelson-Siegel model implies the factor β_{jt} is stationary. This stationary process, by definition, is mean-reverting with constant unconditional mean μ_j. On the other hand, the representative factor plots reveal that factors did not showed a mean-reverting process around a constant mean. In comparison to the slope and curvature components, the level factor follows a smoother random walk. According to Van Dijk et al. (2014), this evidence suggests that the dynamic Nelson-Siegel model lacks a proper stationary process assumption, and that the model can be improved by adding transitory and permanent components into equation (2). This extension will allow long-run means to shift instead of being constant. The following equation having a time-varying unconditional mean is called as shifting endpoint model:

$$\beta_{j,t+1} = \mu_{j,t+1} + \phi_j(\beta_{jt} - \mu_{jt}) + \eta_{j,t+1} \tag{5}$$

In the rest of this section, the shifting endpoint model will be used in several models to investigate forecasting accuracy.

4.1 Shifting Endpoints from Exponential Smoothing

Exponential smoothing is a univariate time series forecasting method that can be extended to data with a systematic trend or seasonal component. The model explicitly uses an exponentially decreasing weight for past observations. Van Dijk et al. (2014) argued that exponential smoothing can be adapted into the unconditional mean $\mu_{j,t+1}$ to allow shifts in the long-run mean, which will be used in equation (5). Accordingly, exponential smoothing recursion is given by

$$\mu_{j,t+1} = \alpha\beta_{jt} + (1-\alpha)\mu_{jt} \tag{6}$$

where $\alpha = 0.1$ as in Van Dijk et al. (2014). The initial value of the unconditional mean μ_{j1} can be taken as the initial value of the estimated factor β_{j1}. Recursively, the unconditional mean for desired period can be found via

$$\mu_{j,t+1} = \alpha\sum_{h=0}^{t-2}(1-\alpha)^h\beta_{j,t-h} + (1-\alpha)^{t-1}\beta_{j1}$$

As a result, at time $t+1$, the unconditional mean is an exponentially weighted moving average of previous factors. In order to use this exponentially smoothed unconditional means, equation (6) can be substituted into equation (5),

$$\beta_{j,t+1} = \omega_j\beta_{jt} + (1-\omega_j)\mu_{jt} + \eta_{j,t+1}, \qquad \text{with } \omega_j = \phi_j + \alpha \tag{7}$$

such that the factor's conditional expectation at time $t+1$ is a weighted average of the factor's unconditional expectation and the unconditional mean at time t. Therefore, a multi-step forecast of β_{jt} can be found via

$$\hat{\beta}_{j,t+h|t} = \omega_j\hat{\beta}_{j,t+h-1|t} + (1-\omega_j)\hat{\mu}_{j,t+h-1|t}\ , \qquad h = 1, 2...$$

where

$$\hat{\mu}_{j,t+h-1|t} = \alpha\hat{\beta}_{j,t+h-2|t} + (1-\alpha)\hat{\mu}_{j,t+h-2|t}$$

Implementation of exponential smoothing to shifting endpoint is simpler than the formulas above. Having the time series of the unconditional means for each month, first the model parameter ϕ_j is estimated. Next, having the model parameter, by taking the last factor value and unconditional mean, exponential smoothing and factor iterations are applied for the out-of-sample data up to horizon $h = 120$ as in equation (5).

Two variants of this approach have been implemented. In the first variant, exponential smoothing is applied only to the unconditional mean of the level factor, β_{1t}, while the slope and curvature factor are forecasted using a first-order autoregressive process as in equation (2). Second, the exponential smoothing to allow shifting endpoints is applied to all three factors. Both variants have the same final step in which equation (1) is used to obtain forecasted yields for each maturity using the out-of-sample factors and a country-specific decay parameter.

4.2 Shifting Endpoint from Realized Measures

The shifting endpoint μ_{jt} in equation (5) is the permanent component added to equation (2). As a result, it can be considered as the long-run mean, or the steady state of all three factors. According to Diebold, Rudebusch, and Aruoba (2006), there is strong evidence that macro factors influence the yield curve. Therefore, the level factor is linked to inflation, whereas the slope factor is linked to real economic activity as proposed by Van Dijk et al. (2014). Exponentially smoothed realized inflation and growth are proxies for their trends. Let π_t^{ES} be the exponentially smoothed realized inflation and γ_t^{ES} be industrial production growth. As monthly data of inflation and industrial production is noisy, exponential smoothing parameter α is set to 0.01, which extracts the trend from the data as a smooth line. In order to link shifting endpoints and realized measures, Van Dijk et al. (2014) suggested that the unconditional means are derived via the equations

$$\mu_{1t} = \theta_{0,1} + \theta_{1,1}\pi_t^{ES} \tag{8}$$

and

$$\mu_{2t} = \theta_{0,2} + \theta_{1,2}\gamma_t^{ES} \tag{9}$$

The estimates of the coefficients of these equations can be obtained via the following regression

$$\beta_{1t} = \theta_{0,1} + \theta_{1,1}\pi_t^{ES} + \varepsilon_{1t} \tag{10}$$

and

$$\beta_{2t} = \theta_{0,2} + \theta_{1,2}\gamma_t^{ES} + \varepsilon_{2t} \tag{11}$$

Using equation (8) and (9), unconditional means for each and every period is obtained. The interest rates can now be forecasted by first obtaining the level and slope factor using equation (5) and then using the AR(1) technique as in equation (2) to determine the curvature factor. When using this method to forecast the level and slope factors, the future values of unconditional means $\mu_{j,t+h}$ are fixed to its end-of-sample value. In other words, the unconditional means are treated as a random walk in the out-of-sample period. This forecasting method contains level and slope factors while excluding curvature factor.

The second variant of this realized measure, the level factor is again derived using exponentially smoothed monthly inflation, but the slope and curvature factors are not subjected to the shifting endpoint approach and are forecasted using the constant mean AR(1) specification as in equation (2). Finally, the third variant relates the slope factor both with exponentially smoothed inflation and industrial production in order to see whether the shape of the yield curve is changing as a response to Taylor's rule. The unconditional means for desired out of sample dates can be obtained by the following equation

$$\mu_{2t} = \theta_{0,3} + \theta_{1,3}\pi_t^{ES} + \theta_{2,3}\gamma_t^{ES} \tag{12}$$

where the coefficients of this third variant will be obtained by regressing both inflation and industrial production on the slope factor

$$\beta_{2t} = \theta_{0,3} + \theta_{1,3}\pi_t^{ES} + \theta_{2,3}\gamma_t^{ES} + \varepsilon_{3t} \tag{13}$$

The unconditional mean of the level factor is related to inflation as in equation (8), while the curvature factor is again obtained by using equation (2) with a constant mean.

Since I use shifting endpoints to forecast interest rates in Canada and the United Kingdom, realized measures in the United States can be used to see whether there is a spillover effect or not. Because the US inflation and growth can be interpreted as global factors, it is likely that there could be spillovers from the US to small open economies. As a result, monthly CPI and industrial production data for the United States can be utilized to assess whether this approach reduces root mean square errors or not. As a result, the fourth variant uses exponentially smoothed data from the United States to determine the level and slope factor, whereas equation (2) is employed to get the curvature factor. Finally, the fifth variant uses US monthly inflation data for realized measures, and the constant mean AR(1) specification for slope and curvature factors is being used as in equation (2). For each variant, the last step is again to use equation (1) in order to obtain the forecasted yields for each maturity.

4.3 Shifting Endpoints from Expectations

The permanent component μ_{jt} in equation (5) can be considered as the long-run mean, or the steady-state of all three factors as mentioned above. Therefore, long-term expectations and the shifting endpoints can be linked to each other. The expectation of inflation and industrial production data for both Canada and the UK were not accessible; therefore, in order to check the spillover effect, I have used the US data to relate the level factor. The goal is to check whether global economies, such as the United States, have an impact on country-specific bond markets.

To begin, the Tealbook, formerly known as the Federal Reserve Board of Governors' Greenbook, is used to assess inflation expectations that can be linked to the level factor in the model. It is a book prepared by the Federal Reserve Board of Governors staff before each meeting of the Federal Open Market Committee that contains projections of several economic indicators for the U.S. economy.[2] The Tealbook provides forecasts for headline CPI inflation for future quarters (annualized percentage points). Unfortunately, the Tealbook only gives forecast data for the next 9 quarters from each FOMC date, and the dataset provided by the Federal Reserve Bank of Philadelphia for 9 quarters inflation forecasts is mostly lacking. As a result, I use the Greenbook's 5-quarter inflation expectation since the dataset for this horizon is complete.[3] Therefore, I believe it could be important to show that the level factor is not related to the short end of the yield curve. It is expected to see that having five-quarter ahead inflation expectation to link shifting endpoint and level factor does not improve the forecast accuracy.

[2]Each meeting of the Federal Open Market Committee is preceded by the printing of the Greenbook. Since the FOMC meets eight times a year, Greenbook data offers two projections for each quarter and eight for the entire year (usually). The highest value of the quarter is set at the same value for each month in the same quarter in order to provide monthly inflation projection data.

[3]Since the dataset is quarterly, the inflation expectations of each month in the same quarter is set to the same quarterly value.

The next measure of inflation expectation comes from the Federal Reserve Bank of Cleveland, which has created a dataset ranging from one year to thirty years of expected inflation using data on inflation swap rates, nominal yields, TIPS yields and survey forecasts of inflation, explained in Haubrich, Pennacchi, and Ritchken (2012).[4] It is worth noting that data obtained from this inflation expectation model uses financial market data, as opposed to being merely forecasted as Greenbook or Blue Chip Financial Forecasts.

The final measure of inflation expectations can be extracted from again Survey of Professional Forecasters from The Federal Reserve Bank of Philadelphia. The Survey of Professional Forecasters (hereafter SPF) offers both short and long-term mean and median forecasts of all the respondents as well as the individual responses from each economist. Since I am interested in long-term inflation projections to relate them to level factors, the 10-year inflation projection offered by SPF is supposed to be highly informative. The Federal Reserve Bank of Philadelphia took over the SPF from the American Statistical Association and the National Bureau of Economic Research in 1990, hence data for 10-year inflation projections are complete after the fourth quarter of 1991. As a result, the missing data points between 1986 and 1991 are obtained from the combined inflation expectations of the Livingstone Survey and Blue Chip Economic Indicators, both of which are available in The Federal Reserve Bank of Philadelphia.[5]

In a nutshell, the 30-year inflation expectation data was provided by the Federal Reserve Bank of Cleveland, while the 5-quarter inflation projection in Tealbook and 10-year inflation projection in SPF were provided by the Federal Reserve Bank of Philadelphia. In order to relate them to the level factor, the per-

[4]Data includes estimates of the expected rate of inflation over the next 30 years along with the inflation risk premium, the real risk premium, and the real interest rate on a monthly basis.

[5]Quarterly data is once again transformed to monthly data by simply matching the same value for each month within the quarter.

manent component in the equation (5), μ_{1t}, is set equal to the inflation forecast data for each suggested case, as Van Dijk et al. (2014) did for forecasting interest rates of the US. As done in the shifting endpoints method with realized measures, the inflation data is used to obtain unconditional means for each date via the equation

$$\mu_{1t} = \theta_{0,1} + \theta_{1,1}\pi_t \tag{14}$$

The coefficients, again, are estimated by regressing the level factor

$$\beta_{1t} = \theta_{0,1} + \theta_{1,1}\pi_t + \zeta_{1t} \tag{15}$$

Again, the equation (5) is used to forecast the level factor, setting $\mu_{1t} = \theta_{0,1} + \theta_{1,1}\pi_t$ and projecting that μ_{1t} will be constant at its end of the in-sample value, treating the μ_{1t} as a random walk for out-of-sample data. Having the estimate of the level factor as described above, the slope and curvature factors can also be derived via equation (2), which is the constant mean AR(1) model. Finally, the interest rates for desired maturities is obtained from equation (1) for desired forecast horizon h.

CHAPTER 5

RESULTS

In this section, the predictive accuracy of different forecasting methods will be examined. The interest rates on zero-coupon-bearing government bonds for both countries from January 1986 to December 2009 were considered as in-sample data for estimating the model, as described in the previous section. Following estimating models for both countries, the out-of-sample data is forecasted using an expanding window for horizons of $h = 6, 12, 24, 60, 120$ months ahead, which refers to the period from January 2010 to December 2019. Table 1 summarizes the methods and labels them so that they can be reused in upcoming tables. Predictive accuracy will be obtained from the realized and forecasted interest rates of the dates in the out-of-sample data and it is measured by the root mean square prediction errors (RMSPE) between forecast and realized yields in percentage points.

$$RMSPE = \sqrt{\sum_{i=1}^{h} \frac{(\hat{y}_i - y_i)^2}{h}}$$

Having obtained the RMSPEs, the random walk model will be taken as a basis for comparing the predictive accuracy. The reason behind that is the main find-

Table 1: Forecasting Methods and Corresponding Labes

Label	Decription
DL	Dynamic Nelson-Siegel Method
RW	Random walk process for β_1, β_2 and β_3
VAR	Vector autoregression for all three factors.
AR(1)	First-order autoregressive process for realized yields
SEP-Exp1	SEP with exponential smoothing for β_1, β_2 and β_3
SEP-Exp2	SEP with exponential smoothing only for β_1
SEP-Real1	SEP with only realized smoothed CPI for β_1
SEP-Real2	SEP with realized CPI for β_1, and IP for β_2
SEP-Real3	SEP with realized CPI for β_1, and CPI and IP for β_2
SEP-Real-US1	SEP with only US's realized CPI for β_1
SEP-Real-US2	SEP with US's realized CPI and IP for β_1 and β_2
SEP-ExpInf-US1	SEP with 30-year inflation expectation for β_1
SEP-ExpInf-US2	SEP with 5-quarter inflation expectation for β_1
SEP-ExpInf-US3	SEP with SPF 10-year inflation expectation for β_1

Note: The Shifting Endpoint specification is represented by SEP in the table. SPF stands for Survey of Professional Forecasters. CPI and IP stand for consumer price index and industrial production, respectively.

ings of the literature. The random walk model is widely accepted as a benchmark model among several forecasting methods in many papers such as De Pooter (2007), Christensen, Diebold, and Rudebusch (2011) and Exterkate, Dijk, Heij, and Groenen (2013). Therefore, I will also take the random walk as the benchmark and figure out whether any model can improve over the benchmark. To do so, if the RMSPEs ratio is less than 1, the chosen model outperforms the random walk model; however, a ratio greater than 1 indicates that the chosen forecast method does not produce better outcomes than the random walk model.

$$\frac{RMSPE^{Selected}}{RMSPE^{RW}} \lesseqgtr 1$$

where $RMSPE^{Selected}$ specifies the desired model that will be compared to the random walk.

5.1 Statistical Significance of Forecast Results

Diebold and Mariano (1995) test statistic will be used to compare the predictive accuracy of two different forecasting methods. The figures below will also show the significance levels obtained from the Diebold and Mariano test statistics comparing the random walk method and all other forecasting methods. As a result, I will briefly explain the methodology of Diebold and Mariano test statistic before exhibiting the results.

Suppose two forecasting methods given as $f_1, f_2, ..., f_n$ and $g_1, g_2, ..., g_n$. The true state of the world is given as $y_1, y_2, ..., y_n$. As previously stated, the accuracy of forecasting methods is measured using root mean squared prediction errors, with smaller errors indicating better forecasting methods. However,it has to be established whether these errors (differences) are significant for forecasting purposes or just owing to the sample's special selection of data values. To show significance of different forecasting methods, the statistic of Diebold and Mariano (1995) is used.

Let $e_i = f_i - y_i$, and $r_i - g_i - y_i$ be forecast residuals for each month in out-of-sample data and the loss function is chosen as $d_i = e_i^2 - r_i^2$. The time series d_i is called as loss-differential. Now define

$$\bar{d} = \frac{1}{h}\sum_{i=1}^{h} d_i, \qquad \text{where } \mu = E[d_i]$$

For $n > k \geq 1$, define

$$\gamma_k = \frac{1}{h}\sum_{i=k+1}^{h} (d_i - \bar{d})(d_{i-k} - \bar{d})$$

where γ_k is called as auto-correlation function or auto-covariance at lag k. For

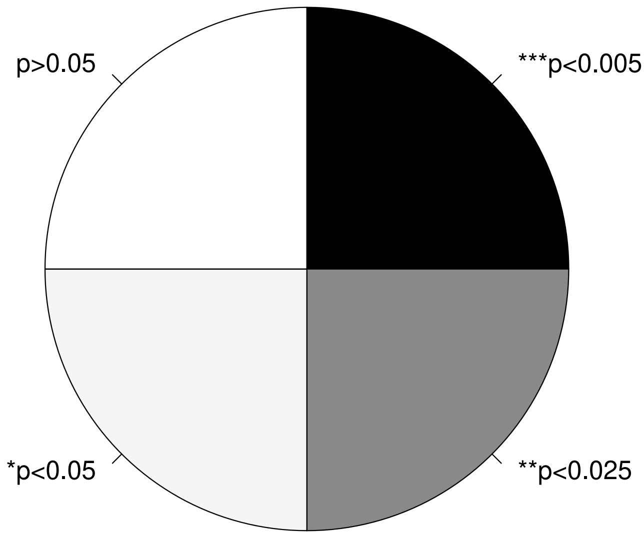

Figure 5: Significance Levels of Diebold-Mariano Test

$h \geq 1$ Diebold and Mariano test statistic is given by

$$DM = \frac{\bar{d}}{\sqrt{(1/n)(\gamma_0 + 2\sum_{k=1}^{h-1} \gamma_k)}}$$

Under the null hypothesis $H_0 : \mu = 0$, DM statistic follows a standard normal distribution:

$$DM \sim N(0, 1)$$

Thus, there is significant difference between two prediction errors if the absolute value of DM test statistic exceeds the critical value. For the statistical significance of the predictive accuracy of different methods, the null hypothesis suggests that there is no difference in the predictive accuracy of these models while the alternative hypothesis proposes that there is a difference between two methods.

When comparing predictive accuracy of different models with respect to the random walk, the filling color of the specific model will be based on the significance level of the model obtained from DM test. Accordingly, figure 5 shows the significance levels of Diebold and Mariano test based on the random walk model. The significance level grows as the fill becomes darker, whereas the white color or no fill indicates that there is no significant difference in predictive accuracy between the specified model and the random walk.

For Canada, the maturities 3-month and 6-month shows various significance levels for $h = 6$ horizons. For example, VAR, SEP-Real-US1 and SEP-ExpInf-US1 models lack of significant differences with respect to the random walk. When the forecast horizon gets longer, the significance levels decrease through horizon $h = 24$ months before reaching their highest significance levels as the horizon extends to $h = 120$ months. Especially when forecast horizon equal to $h = 24$ months, the only highest significance level is observed for the DL model while majority of the models including VAR, AR(1) and realized measures based models are not significantly different from the random walk.

When it comes to 36-, 60- and 120-month maturities, they all are almost significant for horizon $h = 6$ except SEP-Real-US2 and SEP-ExpInf-US1 for 36- and 60-month maturities respectively. Those that are not significant become significantly different as the horizon extends, before there are some more non-significant models at $h = 24$ months.

Turning to the UK bond yields, VAR and AR(1) models do not mostly reach the highest significance level up until the horizon $h = 120$ months. For all horizons except $h = 120$ months, the SEP-Real-US1 and SEP-ExpInf-US1 models are not significantly different from the random walk model for 60-month maturities. Non-significant forecasts are also valid for SEP-ExpInf-US2 and SEP-ExpInf-US3 10-year maturities for all horizons except $h = 120$ months.

Overall, for both countries, most of the models with different maturities are statistically different from the random walk model. For those models lacking significant results at horizon $h = 6$ months lasts non-significant or loses their significance.

5.2 Canada

Figure 6 shows the forecasted yields of bonds with 3-month maturities for all model RMSPEs relative to the random walk's RMSPE. The dashed lines in the graphics are fixed to the 1 so that the relative RMSPEs can be compared to the random walk whether the point forecast is below or under the dashed line. Those over the dashed line indicate that the chosen method has a higher RMSPE than the random walk model, implying that the forecast method is not superior to the random walk model, and those below the dashed line indicate the opposite. The AR(1), SEP-Real-US2 and the RW models produce comparable outcomes; however, the VAR model consistently outperforms the RW model even though medium-term horizons are not significant. The DL model, on the other hand, performs much worse than RW for longer horizons and stays below the dashed line for shorter horizons. SEP-Exp2 and SEP-Real-US1 stay below the dashed line with non-significant performance while SEP-Exp1 is consistently above the dashed line. SEP-Real1 gives better; however, non-significant results for shorter horizons, while SEP-Real2 and SEP-Real3 (Taylor rule model) are consistently drawn below the dashed line. Moreover, inflation expectation-based shifting endpoint models perform better than the RW up to horizon $h = 24$ months, then only SEP-ExpInf-US1 stays below the dashed line even though it is not significant for horizons before two years.

Figure 7 shows the relative RMSEs of 12-Month Maturities of the government bonds of Canada. There is no significantly different better model among AR(1), VAR and DL relative to the RW. The DL model becomes inferior as the horizon extends. SEP-Real-US2 is below the dashed line with only one significant horizon at $h = 120$ months while SEP-Exp1 is above the dashed line for all

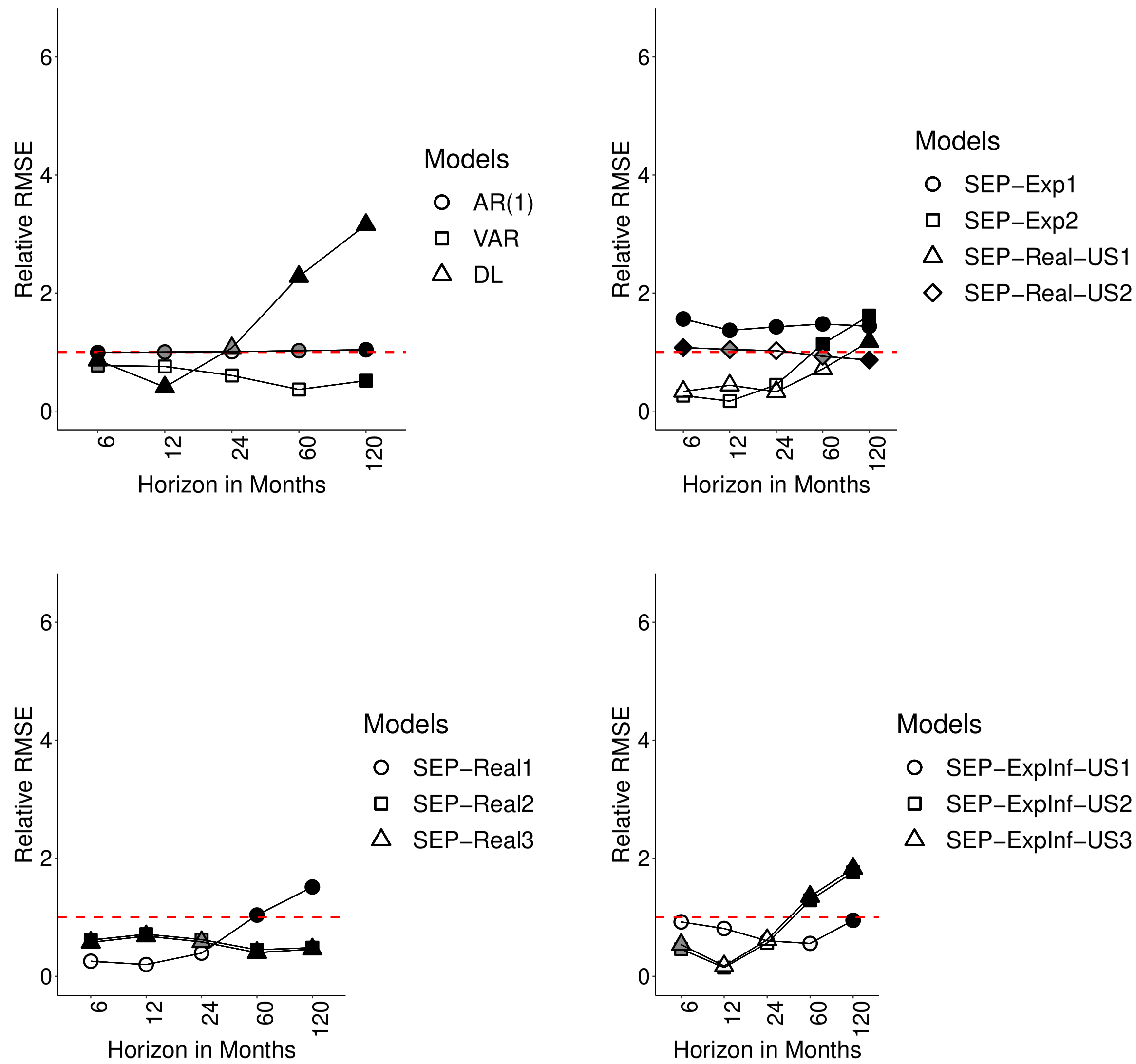

Figure 6: For Canada Bonds with 3-Month Maturity , The Relative RMSEs of All The Models with respect to Random Walk Process

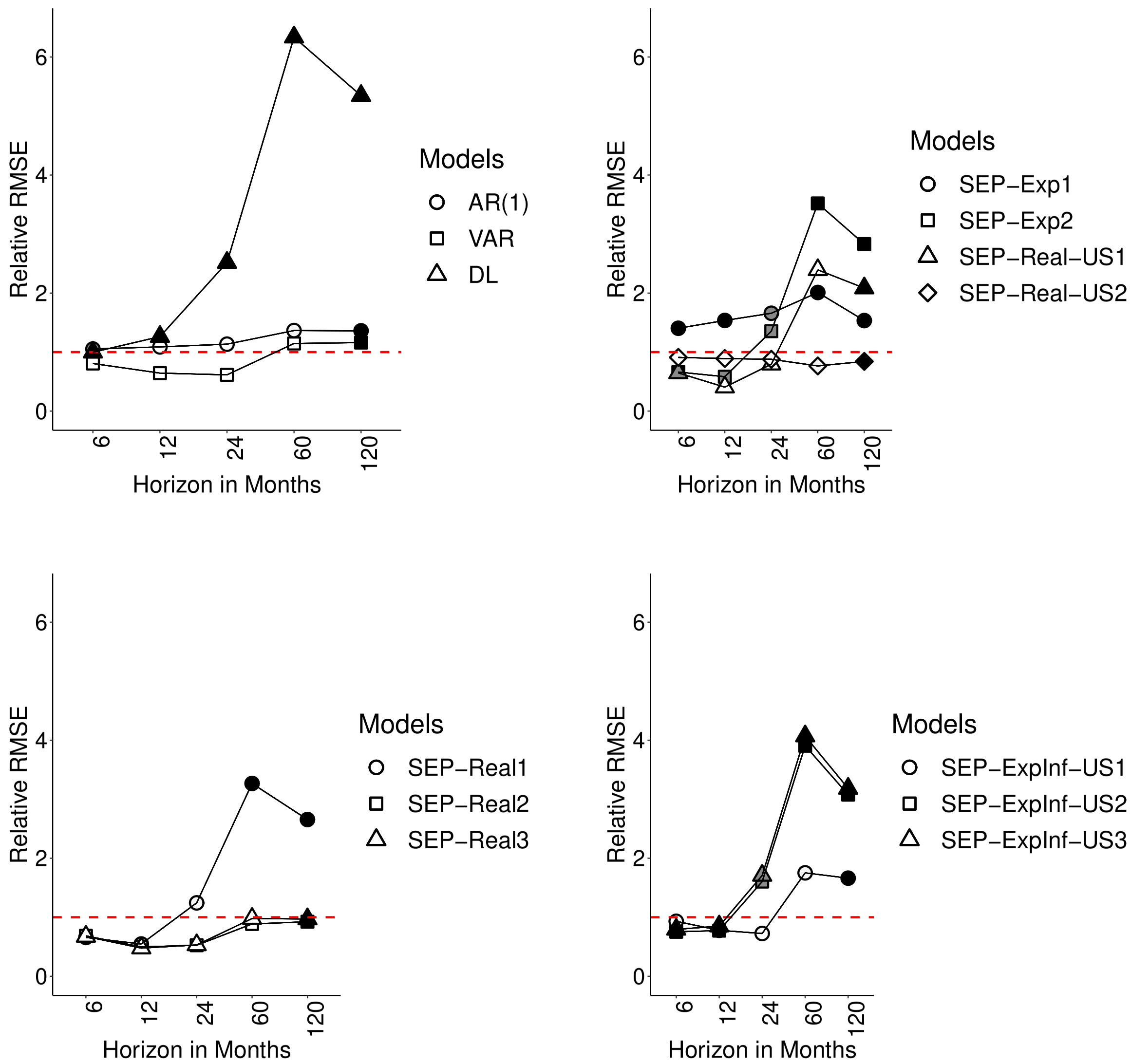

Figure 7: For Canada Bonds with 12-Month Maturity , The Relative RMSEs of All The Models with respect to Random Walk Process

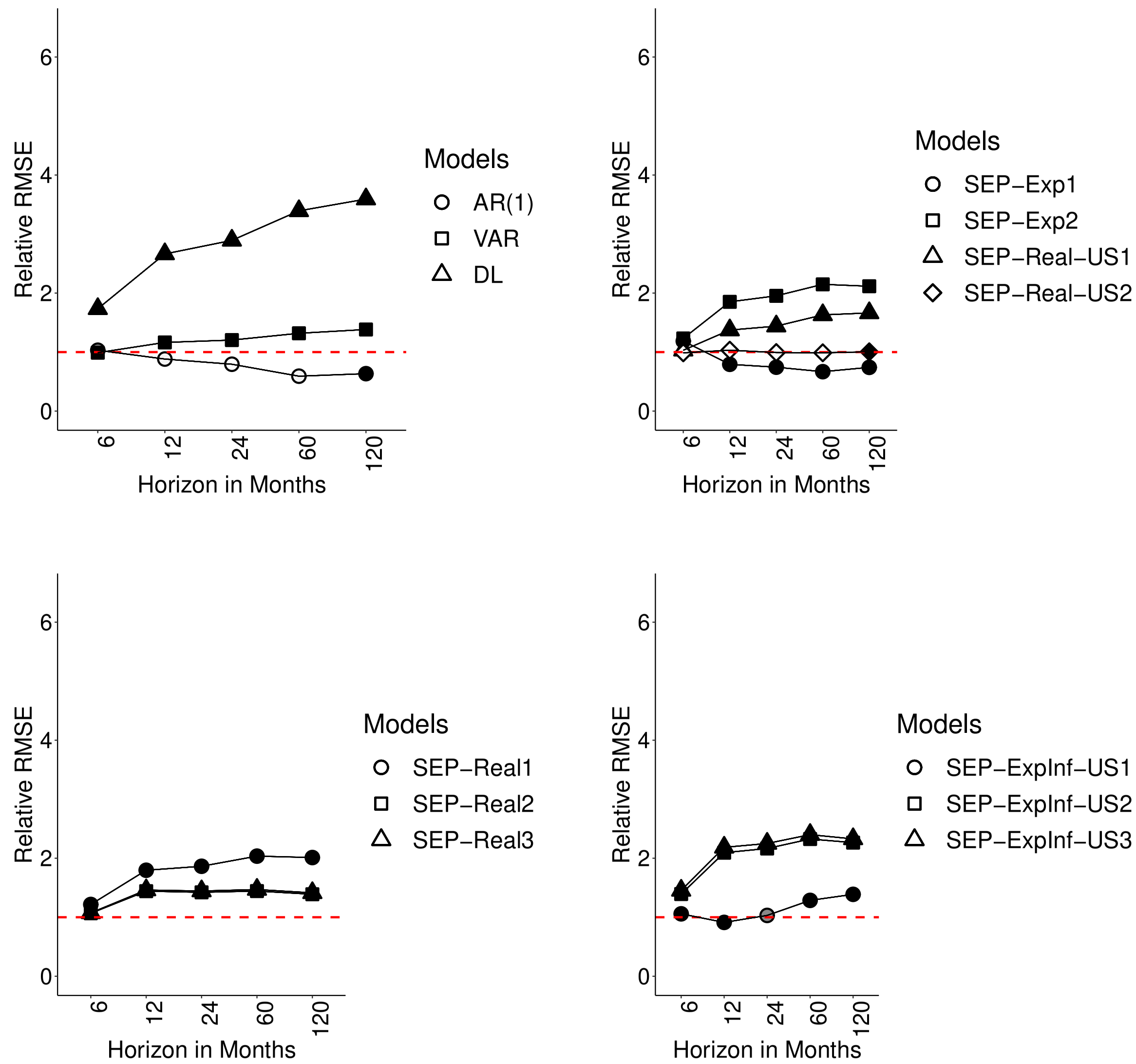

Figure 8: For Canada Bonds with 36-Month Maturity , The Relative RMSEs of All The Models with respect to Random Walk Process

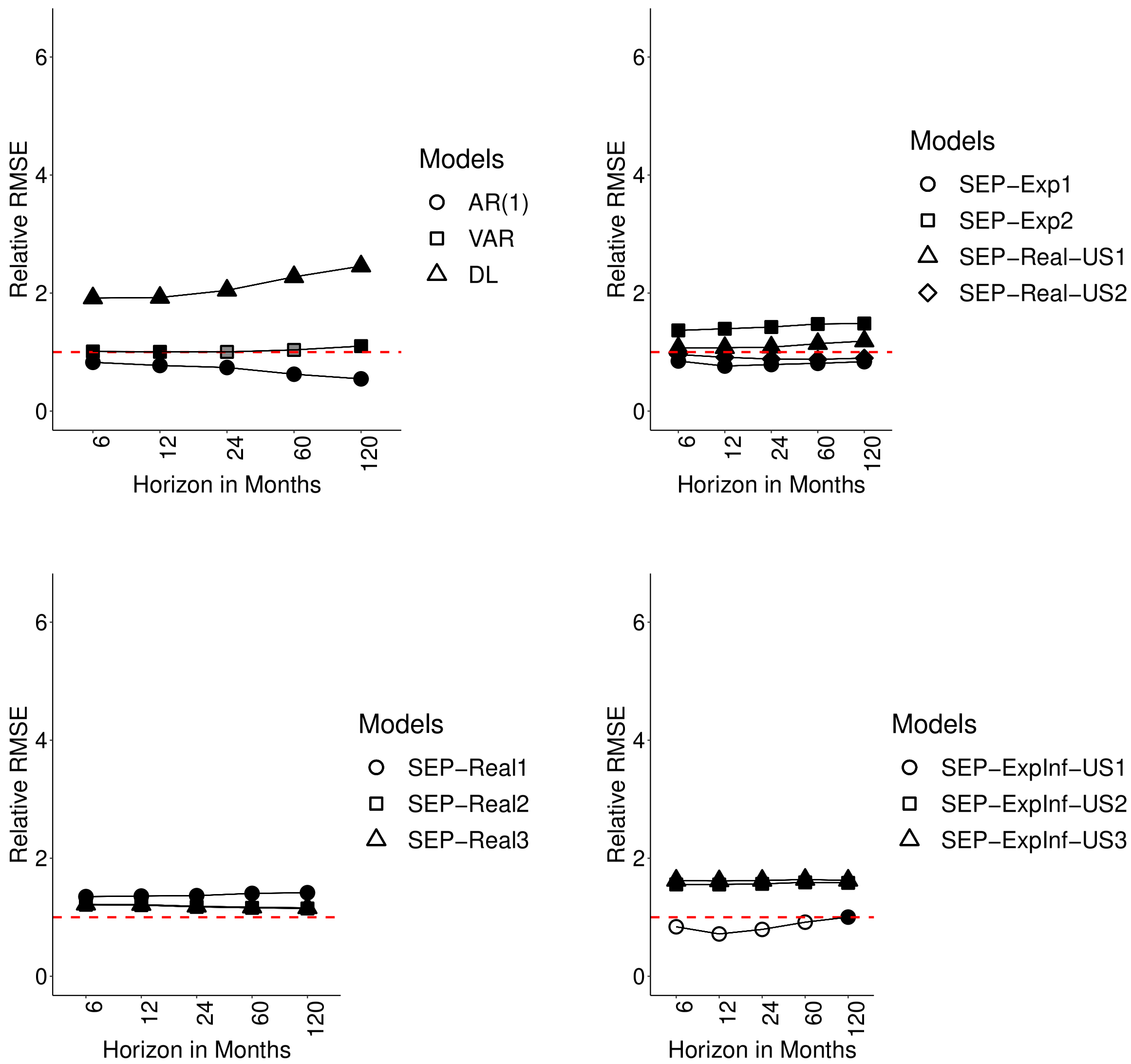

Figure 9: For Canada Bonds with 60-Month Maturity , The Relative RMSEs of All The Models with respect to Random Walk Process

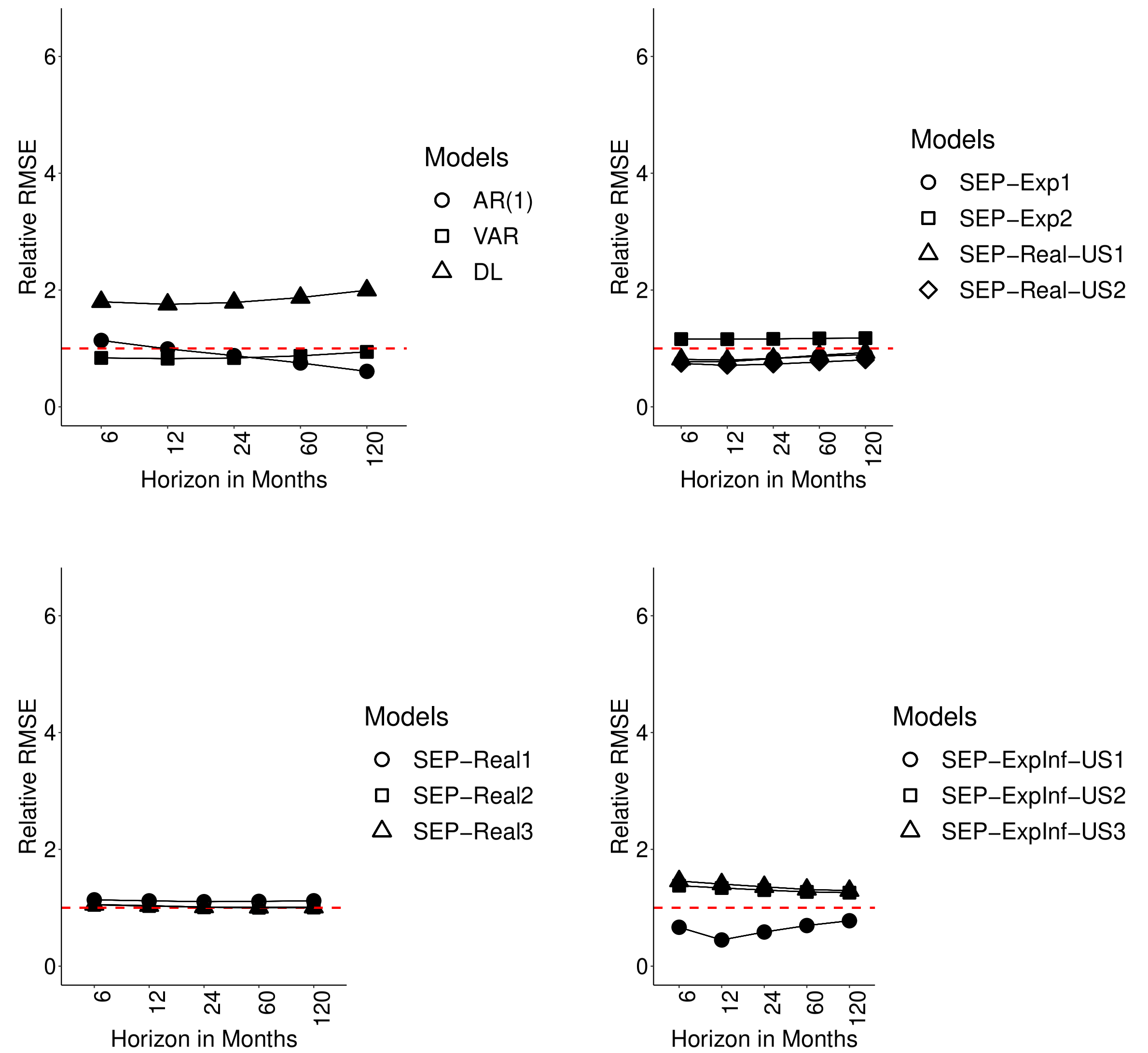

Figure 10: For Canada Bonds with 120-Month Maturity , The Relative RMSEs of All The Models with respect to Random Walk Process

horizons. The SEP-Real2 and SEP-Real3 always showed better performance compared to the RW with mostly insignificant forecasts. The Survey of Professional Forecasters and the Greenbook inflation expectations are not better than the RW for horizons more than $h = 24$ months while all the inflation-based shifting endpoints models stay below the dashed line.

The forecasting results of 36-month maturity bonds are displayed in figure 8. The AR(1) and SEP-Exp1 are the only models better than the RW benchmark. For short horizons, the SEP-ExpInf-US1 model can compete with the RW. Other than these models, there is no better forecasting method relative to the RW. The AR(1) SEP-Exp-1, SEP-Real-US2, and SEP-ExpInf-US1 also give better forecasts in figure 9 in which the 60-month maturity bond's forecast results are displayed. It is worth noting that SEP-ExpInf-US1 only gives a statistically different forecast for horizon $h = 120$ month.

Finally, in figure 10, forecast results of 120-month maturity bonds relative to the random walk are shown. Accordingly, VAR, SEP-Exp1, SEP-Real-US1 and SEP-Real-US2 forecast results have lower RMSPEs compared to the RW model. Moreover, AR(1) yields for higher forecast horizons stay below the dashed line. The realized measures-based shifting endpoint models are similar to each other and close to the dashed line, indicating that they can compete with the RW. The SPF and Greenbook inflation expectations are inferior models relative to the RW. Lastly and most importantly, the 30-year inflation expectation-based model, SEP-ExpInf-US1, is drawn below the dashed line for all horizons with significant results. Therefore, the downward trend in the in-sample data is captured by SEP-ExpInf-US1 for 10-year maturity bonds.

While SEP-Real-US2 is primarily drawn below the dashed line for all maturities, the realized measures of the Canada-based models are generally drawn below the dashed line for short maturities. These findings may corroborate the assumption of a spillover effect from the global economy, particularly the

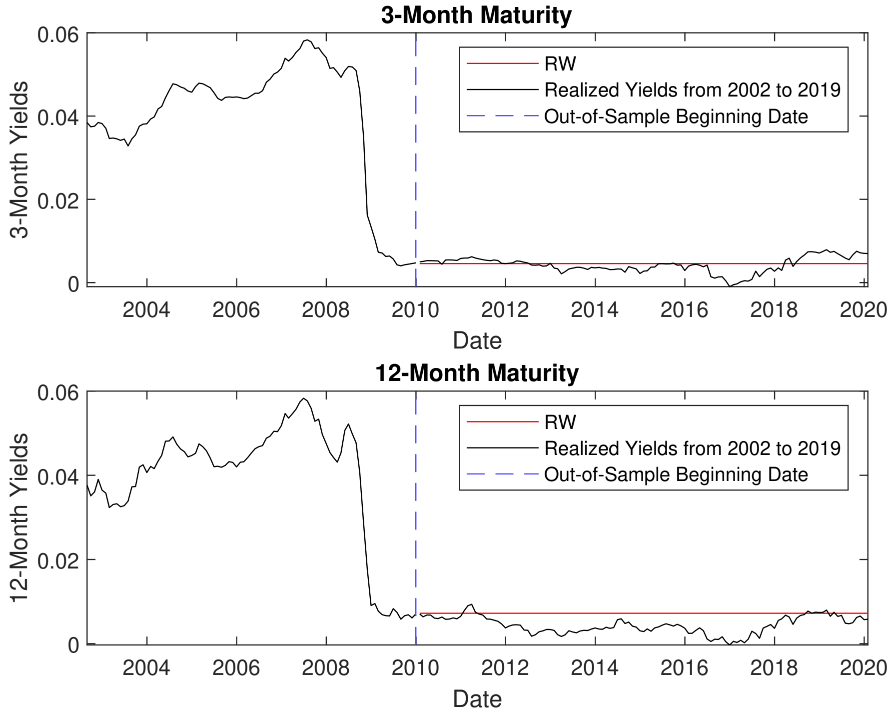

Figure 11: Random Walk Model's Forecast Performance for the UK

United States. The 30-year inflation expectation model consistently stays below the dashed line for longer maturities. This evidence suggests that this model captures the downward trend of yields for longer maturities.

5.3 The United Kingdom

The RMSPEs of the random walk forecasting method for 3- and 12-month maturities in the United Kingdom are extremely low relative to the other methods. Figure 11 plots the random walk forecast with an extended period of the out-of-sample dates for 3- and 12-month maturities. As a result, reporting relative RMSPE with respect to the random walk is ineffective for these two maturities since the ratio for some suggested models becomes too large to portray with figures. Therefore, I will report the relative RMSPEs for 3- and 12-month maturities with respect to the RW in a table form.

As shown in Table 2, almost there is no better forecast compared to the RW for the horizons $h = 6, 60$ months except the AR(1). Furthermore, as the horizon extends, the relative RMSPEs rise dramatically. For 12-month maturities, again

Table 2: 3-Month and 12-Month Maturities Realtive RMSPEs with Respect to the RW for the UK

Forecast Horizon	h=6-Months		h=12-Months		h=24-Months		h=60-Months		h=120-Months	
Maturity	*3*	*12*	*3*	*12*	3	12	3	12	3	12
DL	8.14***	10.04***	12.16***	13.03***	17.42***	13.86***	28.45***	10.80***	22.18***	12.49***
VAR	10.00**	1.00	9.63	0.91	6.62	3.25	11.01	5.07	10.04***	6.47***
AR(1)	0.91***	0.65**	1.06**	0.59	1.22**	0.80	0.79**	0.60*	1.15***	0.64***
SEP-Exp1	4.40***	0.32***	5.63***	0.78***	6.77***	1.60**	6.59***	0.61***	4.19***	0.75***
SEP-Exp2	3.64*	7.33***	6.56	9.32**	10.70	9.81**	21.08***	8.31***	16.23***	9.31***
SEP-Real1	6.91***	9.12	10.27***	11.64***	14.52***	12.06***	23.23***	9.04***	17.56***	10.03***
SEP-Real2	6.49***	8.88***	9.59***	11.27***	13.34***	11.52***	20.70***	8.42***	15.08***	9.06***
SEP-Real3	6.80***	9.05***	10.05***	11.52***	14.01**	11.83***	21.72***	8.68***	15.84***	9.36***
SEP-Real-US1	2.55***	2.01***	1.96***	2.81***	2.24**	3.66***	9.69***	4.26***	9.26***	5.45***
SEP-Real-US2	4.36***	0.99***	4.65***	1.25***	4.05**	1.69***	2.89**	2.17***	2.27***	2.54***
SEP-ExpInf-US1	1.99***	2.42***	1.50***	3.12***	1.97***	3.64***	9.08**	4.06***	8.82***	5.21***
SEP-ExpInf-US2	3.51***	6.52***	5.56**	8.15***	8.46***	8.28***	15.81**	6.51***	12.82***	7.46***
SEP-ExpInf-US3	1.41***	4.86***	3.36***	6.47***	6.89***	7.22***	15.45***	6.34***	12.86***	7.46***

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$.

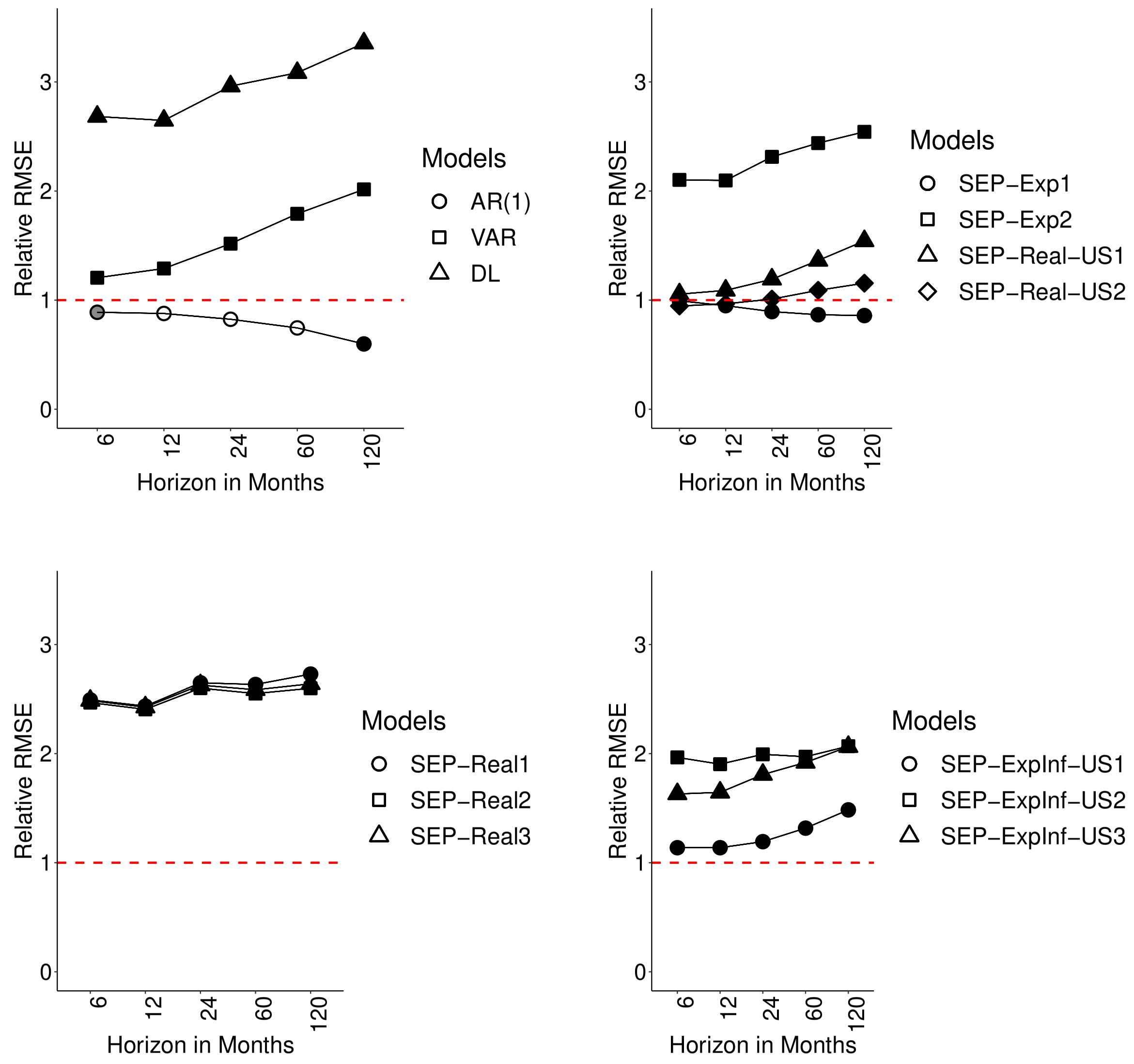

Figure 12: For the UK Bonds with 36-Month Maturity , The Relative RMSEs of All The Models with respect to Random Walk Process

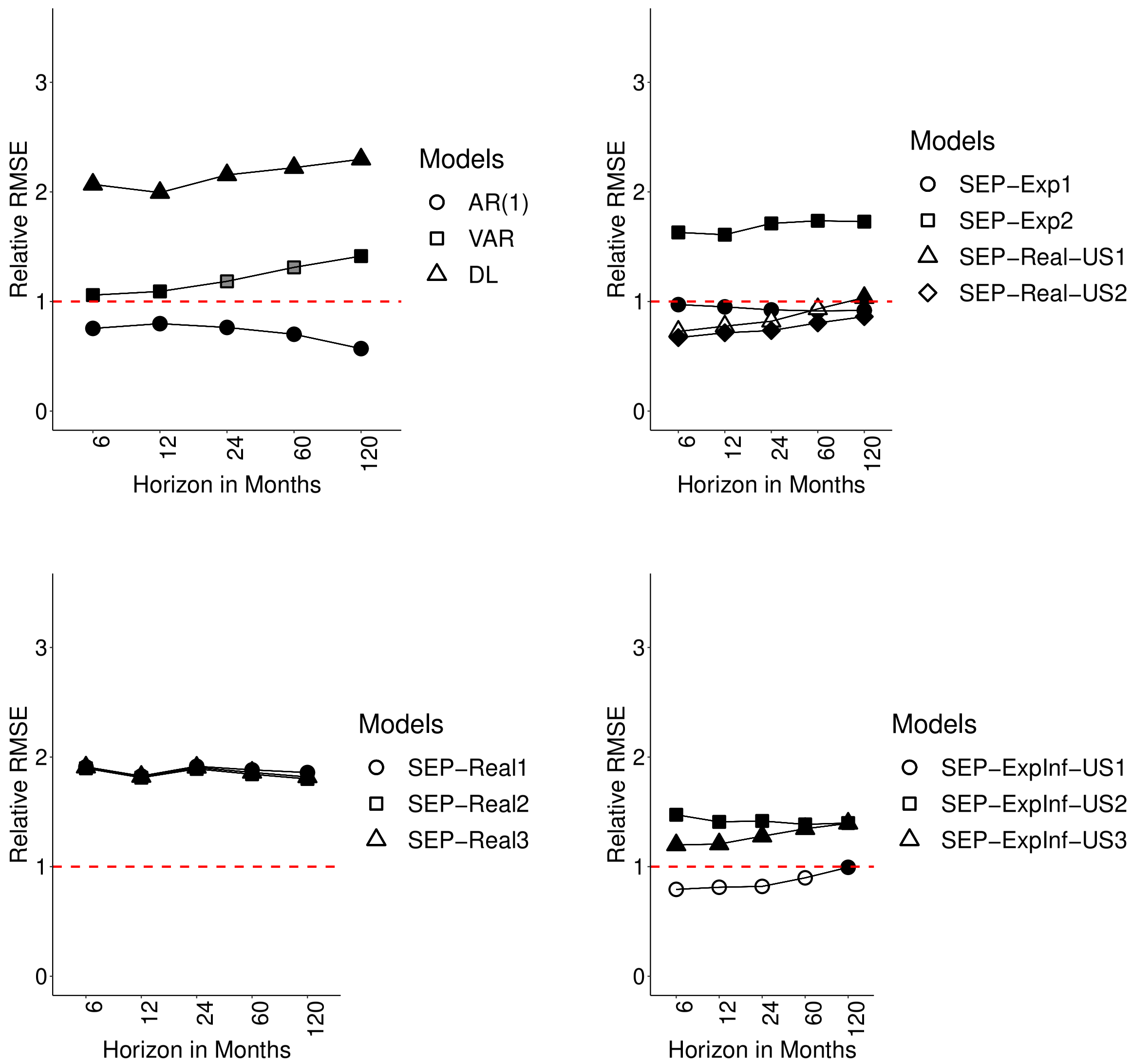

Figure 13: For the UK Bonds with 60-Month Maturity, The Relative RMSEs of All The Models with respect to Random Walk Process

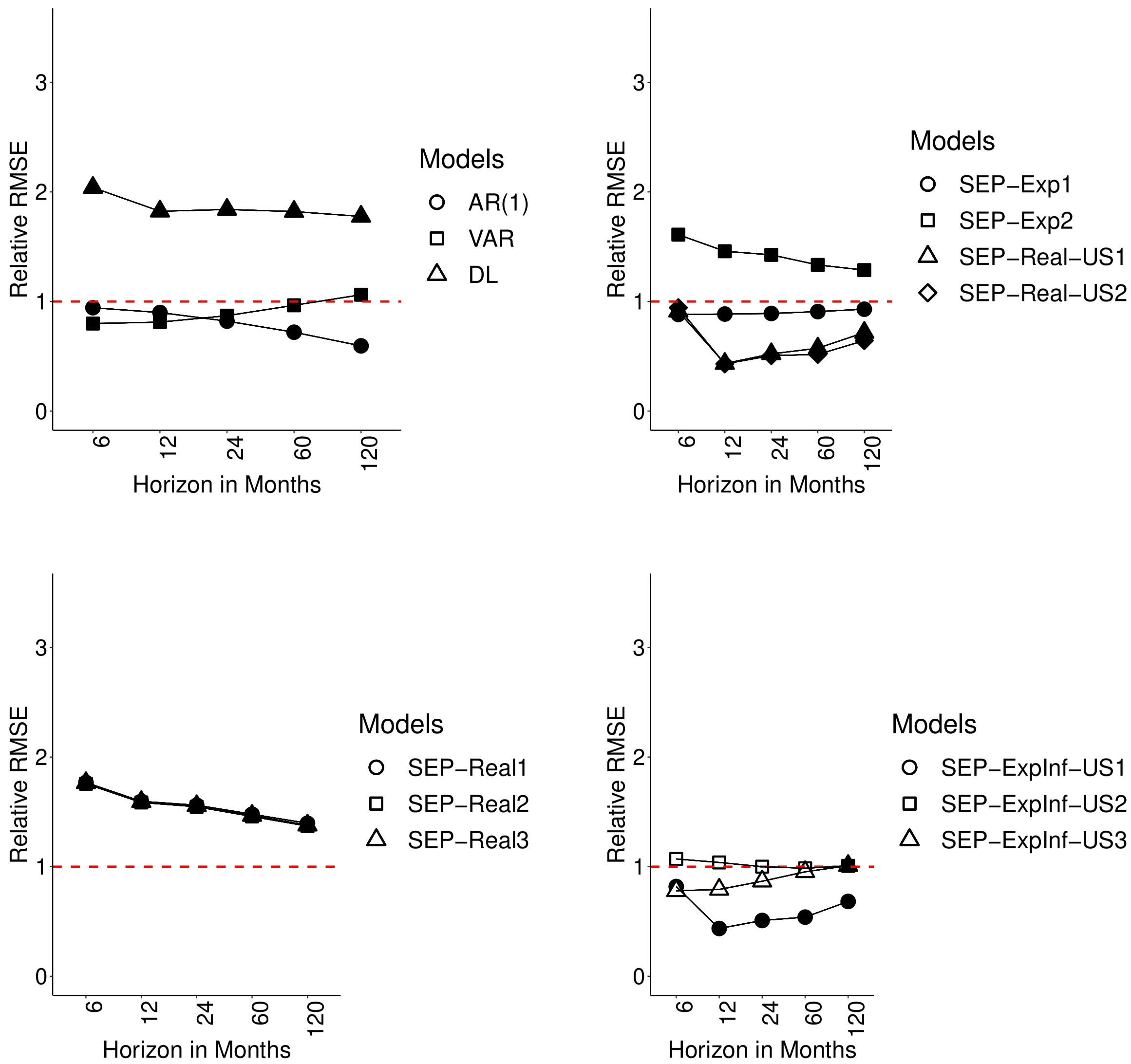

Figure 14: For the UK Bonds with 120-Month Maturity, The Relative RMSEs of All The Models with respect to Random Walk Process

AR(1) model is able to perform better than the RW model even though it loses its significance for horizons $h = 12, 24$ months. Briefly, it can be stated that shifting endpoints-based methods fails to improve the prediction accuracy of the RW for these selected maturities for all horizons.

Turning to figure 12, bonds with 36-month maturity forecasts are improved by AR(1) and SEP-Exp1 for all horizons. The SEP-Real-US1 is close to the dashed line for short horizons while it is located above the dashed line as the horizon increases. The other methods are not able to increase the forecast accuracy relative to the random walk model.

As the maturity increases to 60-month, some shifting endpoints-based models start to stay below the dashed line indicating that the forecast is improved relative to the RW. The AR(1), SEP-Exp1, SEP-Real-US1, SEP-Real-US2 and SEP-ExpInf-US1 stay below the dashed line although SEP-Real-US1 and SEP-ExpInf-US1 are not significantly different than the RW for short horizons. The realized measures of the UK do not improve the forecast accuracy.

Turning to Figure 14, the forecast for bonds with the longest maturity 120-month is significantly improved by the US-based models SEP-Real-US1, SEP-Real-US2, SEP-ExpInf-US1 and SEP-Exp1. Moreover, AR(1) and VAR models also do a better job relative to the RW.

Overall, for short horizons such as 3- and 12-month there is no improvement obtained by using the shifting endpoints models. On the other hand, shifting endpoint with the US realized measures and expected inflation models performed better than the random walk as maturity increases. The UK realized yields-based models fall behind the random walk model for all maturities and horizons.

5.4 Are Forecasts Good Enough?

Although the random walk is selected as the benchmark model to compare the forecasting accuracy of the other models, it is not clear which model does a better job. The key question of this section will be whether or not models with a relative RMSPE (with respect to the random walk) smaller than 1 are the best forecasting models to use. Even though the models outperformed the random walk model on out-of-sample dates, they may not even come close to the realized yields. In order to test forecast efficiency, I will use Mincer and Zarnowitz (1969) regression as explained in the Edge and Gurkaynak (2010). The estimates of the following regression

$$y_t^h = \alpha^h + \Theta^h \hat{y_t}^h + \varepsilon_t^h \tag{16}$$

where h defines the horizon or the dimension of the regression inputs. A good forecast should have zero intercept $\alpha = 0$, unit slope $\Theta = 1$ and high R^2. If the intercept is not zero, the forecast is biased on average while a slope different than 1 indicates that the forecast has consistently underestimated or overestimated deviations from the mean. Moreover, the low R^2 values mean that the variations of the realized yields are not fully captured.

Since the forecasting results are compared to the random walk model, the regression efficiency test is not applicable for the random walk models because of the linear dependency of the random walk coefficient and the intercept. Therefore, I have checked the forecasting efficiency of some of the selected models. The DL, SEP-Exp-1, SEP-Real-US2 and SEP-ExpInf-US1 models are regressed on the realized yields since these models performed better for some maturities and horizons relative to the random walk except for the Diebold-Li method.

Figures A1-A10 in the appendix display the forecasted yields and realized yields in a time series. Starting with Canada, the results in table 3 are not surprising since there was a constant upward trend in the 3-month maturity bond for Canada for the first year after December 2009. Therefore, the DL model has a high R^2 and slope close to 1 for horizons 6 and 12 months. However, as the forecast horizon increases, the DL model constantly overestimates the realized yields with low R^2. For longer maturities experiencing a downward trend, the regression coefficient of the DL model becomes negative since the realized yields and DL model follows opposite directions. The results clearly show that the dynamic Nelson-Siegel model is not able to reflect the trend in the data as a result of the constant mean assumption.

Turning to the SEP-Exp-1 in table 4, the forecasts and the realized yields moved in different directions for 3- and 12-month maturities. However, for longer maturities, the shifting endpoints specification with exponential smoothing was able to capture the downward trend in the data even though it consistently underestimated.

In table 5, the SEP-Real-US2 model's forecast regression does not produce efficient results as well. Shorter horizons of 3-month maturity have a negative and very big slope suggesting that the forecast underestimates the yields with negative sign. However, as maturity increases, the downward trend of the realized yields is captured despite underestimating forecasts. Especially, the 120-month maturity forecast was able to cover the dramatic fall in the realized yields for two years forecast horizon. As the forecast horizon increased, the model was not able to keep its downward sloping trend and underestimated the yields.

For short maturities, the SEP-ExpInf-US1 model, 30-year inflation expectation based, has positive slopes with underestimating forecasts. The R^2 is comparably high for some horizons. As maturity increases, the inflation-based model is able to capture the downward slope and move in the same direction with realized yields for short horizons. Especially, the 120-month maturity is forecasted with reasonable coefficients even though it underestimates the yields. As shown

Table 3: Mincer-Zarnowiz Regression for DL of Canada

Canada	Forecast Horizon				
DL	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	0.741***	0.880***	0.340***	0.131***	0.100***
	(0.084)	(0.044)	(0.062)	(0.019)	(0.026)
Intercept	-0.001	-0.001**	0.003**	0.005***	0.005***
	(0)	(0)	(0.001)	(0)	(0.001)
Adjusted R^2	0.938	0.973	0.554	0.431	0.100
12-Month					
Slope	0.985*	0.549**	0.103	-0.029	0.074*
	(0.418)	(0.136)	(0.083)	(0.022)	(0.035)
Intercept	-0.002	0.002	0.008***	0.011***	0.008***
	(0.005)	(0.002)	(0.001)	(0.001)	(0.001)
Adjusted R^2	0.476	0.581	0.023	0.013	0.028
36-Month					
Slope	0.521	-0.444	-0.551**	-0.412**	-0.179***
	(0.748)	(0.308)	(0.161)	(0.046)	(0.052)
Intercept	0.008	0.031**	0.034***	0.0301***	0.021***
	(0.018)	(0.008)	(0.004)	(0.001)	(0.002)
Adjusted R^2	-0.114	0.088	0.317	0.563	0.081
60-Month					
Slope	0.115	-0.921*	-0.919***	-0.622***	-0.388***
	(0.824)	(0.371)	(0.209)	(0.074)	(0.06)
Intercept	0.023	0.055**	0.056***	0.044***	0.034***
	(0.026)	(0.012)	(0.007)	(0.003)	(0.003)
Adjusted R^2	-0.243	0.319	0.441	0.538	0.250
120-Month					
Slope	-1.312	-1.371**	-1.324***	-0.926***	-0.746***
	(0.803)	(0.356)	(0.244)	(0.112)	(0.006)
Intercept	0.087	0.088	0.087	0.069	0.061
	(0.031)	(0.014)	(0.011)	(0.005)	(0.003)
Adjusted R^2	0.250	0.555	0.552	0.533	0.548

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 4: Mincer-Zarnowiz Regression for SEP-Exp-1 of Canada

Canada	Forecast Horizon				
SEP-Exp-1	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	-2.578**	-3.772***	-2.385***	-1.893***	-1.684***
	(0.322)	(0.243)	(0.277)	(0.143)	(0.444)
Intercept	0.002***	0.002***	0.003***	0.004***	0.004***
	(0)	(0)	(0)	(0)	(0.001)
Adjusted R^2	0.927	0.955	0.760	0.745	0.101
12-Month					
Slope	-3.645	-2.462**	-1.060*	-0.173	-0.449
	(1.42)	(0.572)	(0.47)	(0.246)	(0.6127)
Intercept	0.031*	0.02***	0.016***	0.011	0.012***
	(0.008)	(0.003)	(0.001)	(0.001)	(0.002)
Adjusted R^2	0.527	0.614	0.614	-0.008	-0.004
36-Month					
Slope	-2.089	1.763	2.691*	4.394***	4.750***
	(2.558)	(1.343)	(1.054)	(0.574)	(0.911)
Intercept	0.062	-0.015	-0.065***	-0.065***	-0.072***
	(0.05)	(0.025)	(0.01)	(0.01)	(0.016)
Adjusted R^2	-0.071	0.062	0.493	0.493	0.180
60-Month					
Slope	-0.709	3.647*	4.621**	7.261***	8.684***
	(2.701)	(1.564)	(1.395)	(0.885)	(1.126)
Intercept	0.046	-0.075	-0.102*	-0.174***	-0.213***
	(0.075)	(0.042)	(0.037)	(0.023)	(0.029)
Adjusted R^2	-0.228	0.287	0.302	0.528	0.329
120-Month					
Slope	3.492	5.293**	6.574***	11.520***	15.091***
	(2.496)	(1.323)	(1.654)	(1.414)	(1.481)
Intercept	-0.091	-0.157**	-0.203**	-0.381***	-0.510***
	(0.09)	(0.047)	(0.058)	(0.049)	(0.052)
Adjusted R^2	0.160	0.576	0.391	0.525	0.463

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 5: Mincer-Zarnowiz Regression for SEP-Real-US2 of Canada

Canada	Forecast Horizon				
SEP-Real-US2	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	-18.796**	-33.967***	-2.119	1.228**	0.982*
	(2.415)	(5.083)	(4.045)	(0.363)	(0.379)
Intercept	0.021***	0.036***	0.009*	0.006***	0.007***
	(0.002)	(0.004)	(0.004)	(0)	(0.001)
Adjusted R^2	0.922	0.798	-0.033	0.150	0.045
12-Month					
Slope	17.626*	15.433***	4.729	-0.731*	1.131*
	(6.003)	(3.293)	(2.949)	(0.43)	(0.623)
Intercept	-0.130	-0.113**	-0.027	0.016	0.001
	(0.047)	(0.026)	(0.023)	(0.003)	(0.005)
Adjusted R^2	0.604	0.655	0.063	0.031	0.019
36-Month					
Slope	24.569	15.549	12.453**	15.106***	6.947**
	(14.73)	(10.77)	(3.534)	(2.15)	(2.616)
Intercept	-0.501	-0.310	-0.244**	-0.301***	-0.133*
	(0.313)	(0.228)	(0.074)	(0.045)	(0.055)
Adjusted R^2	0.262	0.089	0.331	0.450	0.048
60-Month					
Slope	1.354	6.850*	5.968***	7.683***	9.329***
	(2.701)	(1.564)	(1.395)	(0.885)	(1.126)
Intercept	-0.011	-0.170*	-0.144**	-0.192***	-0.239***
	(0.233)	(0.072)	(0.04)	(0.02)	(0.029)
Adjusted R^2	-0.241	0.361	0.411	0.651	0.377
120-Month					
Slope	3.154	3.335**	3.505***	5.101***	6.527***
	(1.884)	(0.84)	(0.746)	(0.5)	(0.531)
Intercept	-0.078	0.085*	-0.091**	-0.148***	-0.198***
	(0.068)	(0.03)	(0.026)	(0.017)	(0.018)
Adjusted R^2	0.264	0.572	0.477	0.635	0.556

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 6: Mincer-Zarnowiz Regression for SEP-ExpInf-US1 of Canada

Canada	Forecast Horizon				
SEP-ExpInf-US1	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	6.309**	3.542***	0.707**	0.226***	0.170**
	(1.269)	(0.375)	(0.195)	(0.041)	(0.053)
Intercept	-0.004*	0.000	0.0048***	0.0068***	0.0067***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Adjusted R^2	0.826	0.889	0.345	0.325	0.707
12-Month					
Slope	8.188	2.564**	0.088	-0.101*	0.132
	(5.219)	(0.825)	(2.949)	(0.273)	(0.0501)
Intercept	-0.054	-0.011	0.010**	0.011***	0.008***
	(0.04)	(0.006)	(0.002)	(0.001)	(0.002)
Adjusted R^2	0.226	0.459	-0.040	0.049	0.013
36-Month					
Slope	-6.437	3.484	-5.414***	-1.489***	-0.682***
	(6.299)	(5.575)	(1.233)	(0.202)	(0.197)
Intercept	0.152	-0.051	0.130***	0.048***	0.029***
	(0.128)	(0.012)	(0.025)	(0.004)	(0.004)
Adjusted R^2	0.008	-0.061	0.442	0.473	0.084
60-Month					
Slope	1.354	6.850*	5.968***	7.683***	9.329***
	(8.139)	(2.548)	(1.442)	(0.729)	(1.089)
Intercept	-0.011	-0.170*	-0.144**	-0.192***	-0.239***
	(0.233)	(0.072)	(0.04)	(0.02)	(0.029)
Adjusted R^2	-0.241	0.361	0.411	0.651	0.377
120-Month					
Slope	3.154	3.335**	3.505***	5.101***	6.527***
	(1.884)	(0.84)	(0.746)	(0.501)	(0.531)
Intercept	-0.078	-0.085*	-0.091**	-0.014***	-0.198***
	(0.068)	(0.03)	(0.026)	(0.017)	(0.017)
Adjusted R^2	0.264	0.572	0.477	0.635	0.556

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 7: Mincer-Zarnowiz Regression for DL of the UK

UK	Forecast Horizon				
DL	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	0.033	0.035	0.007	-0.071***	-0.021
	(0.015)	(0.021)	(0.012)	(0.006)	(0.012)
Intercept	0.005***	0.004***	0.005***	0.006***	0.005***
	(0)	(0)	(0)	(0)	(0)
Adjusted R^2	0.434	0.140	-0.028	0.646	0.020
12-Month					
Slope	-0.103	-0.053*	-0.034	-0.111***	-0.046**
	(0.043)	(0.019)	(0.033)	(0.015)	(0.014)
Intercept	0.007***	0.007***	0.007***	0.008***	0.006***
	(0)	(0)	(0)	(0)	(0)
Adjusted R^2	0.486	0.383	0.002	0.459	0.074
36-Month					
Slope	-0.720*	-0.598***	-0.436***	-0.328***	-0.299***
	(0.17)	(0.115)	(0.104)	(0.063)	(0.033)
Intercept	0.036**	0.033***	0.029***	0.024***	0.023***
	(0.004)	(0.003)	(0.003)	(0.002)	(0.001)
Adjusted R^2	0.770	0.700	0.414	0.302	0.399
60-Month					
Slope	-1.102*	-0.991***	-0.768***	-0.578***	-0.650***
	(0.25)	(0.195)	(0.165)	(0.101)	(0.05)
Intercept	0.065**	0.061	0.053***	0.044***	0.047***
	(0.008)	(0.007)	(0.007)	(0.005)	(0.002)
Adjusted R^2	0.786	0.692	0.471	0.347	0.577
120-Month					
Slope	-1.494*	1.501**	-1.315***	-1.101***	-1.410***
	(0.524)	(0.339)	(0.268)	(0.147)	(0.071)
Intercept	0.106*	0.105***	0.098***	0.086	0.102***
	(0.023)	(0.015)	(0.013)	(0.007)	(0.004)
Adjusted R^2	0.587	0.627	0.483	0.483	0.764

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 8: Mincer-Zarnowiz Regression for SEP-Exp-1 of the UK

UK	Forecast Horizon				
SEP-Exp-1	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	-0.092	-0.098	-0.041	0.315***	0.263**
	(0.04)	(0.061)	(0.043)	(0.047)	(0.091)
Intercept	0.005***	0.005***	0.005***	0.005***	0.005***
	(0)	(0)	(0)	(0)	(0)
Adjusted R^2	0.452	0.124	-0.003	0.420	0.057
12-Month					
Slope	0.794*	0.317*	0.182	0.857***	0.711***
	(0.316)	(0.123)	(0.197)	(0.116)	(0.141)
Intercept	0.001	0.004***	0.005***	0.001***	0.002***
	(0.002)	(0)	(0)	(0)	(0)
Adjusted R^2	0.515	0.337	-0.006	0.474	0.170
36-Month					
Slope	20.283*	7.406**	4.351***	4.191***	4.728***
	(5.244)	(2.034)	(1.12)	(0.673)	(0.482)
Intercept	-0.403*	-0.136**	-0.073**	-0.070***	-0.081***
	(0.108)	(0.041)	(0.022)	(0.012)	(0.009)
Adjusted R^2	0.736	0.526	0.380	0.389	0.444
60-Month					
Slope	22.356**	11.346**	7.548***	7.862***	10.309***
	(3.232)	(2.712)	(1.726)	(1.119)	(0.875)
Intercept	-0.654**	-0.319**	-0.203***	-0.213***	-0.286***
	(0.098)	(0.082)	(0.051)	(0.032)	(0.025)
Adjusted R^2	0.903	0.599	0.440	0.450	0.536
120-Month					
Slope	10.127*	9.77**	8.941***	12.816***	19.544***
	(3.32)	(2.157)	(2.016)	(1.459)	(1.606)
Intercept	-0.374	-0.359**	-0.325**	-0.482***	-0.754***
	(0.135)	(0.087)	(0.081)	(0.058)	(0.063)
Adjusted R^2	0.624	0.639	0.447	0.563	0.552

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 9: Mincer-Zarnowiz Regression for SEP-Real-US2 of the UK

UK	Forecast Horizon				
SEP-Real-US2	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	-0.117	-0.129	-0.327**	-0.450***	-0.120***
	(0.042)	(0.107)	(0.109)	(0.028)	(0.058)
Intercept	0.005***	0.005***	0.005***	0.006***	0.004***
	(0)	(0)	(0)	(0)	(0)
Adjusted R^2	0.561	0.038	0.256	0.805	0.026
12-Month					
Slope	-5.870	-0.666	-0.451	-0.593***	-0.246***
	(2.525)	(0.417)	(0.278)	(0.08)	(0.712)
Intercept	0.048	0.011**	0.01***	0.01***	0.007***
	(0.018)	(0.003)	(0.002)	(0.008)	(0.009)
Adjusted R^2	0.468	0.123	0.065	0.478	0.084
36-Month					
Slope	25.410*	30.103*	-9.825***	-2.114**	-2.083***
	(8.145)	(12.818)	(2.502)	(0.639)	(0.281)
Intercept	-0.503*	-0.601*	0.216***	0.055***	0.055***
	(0.166)	(0.262)	(0.517)	(0.138)	(0.006)
Adjusted R^2	0.635	0.291	0.385	0.144	0.311
60-Month					
Slope	3.564*	3.790***	3.419**	5.724***	-0.443
	(0.867)	(0.684)	(0.91)	(1.001)	(1.022)
Intercept	-0.77*	-0.083**	-0.072**	-0.140***	0.024
	(0.025)	(0.019)	(0.025)	(0.027)	(0.028)
Adjusted R^2	0.760	0.729	0.363	0.349	-0.007
120-Month					
Slope	1.357*	1.597***	1.601***	2.733***	3.773***
	(0.508)	(0.342)	(0.386)	(0.337)	(0.473)
Intercept	-0.011	-0.021	-0.020	-0.062***	-0.010***
	(0.019)	(0.012)	(0.013)	(0.011)	(0.015)
Adjusted R^2	0.551	0.653	0.412	0.522	0.344

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

Table 10: Mincer-Zarnowiz Regression for SEP-ExpInf-US1 of the UK

UK	Forecast Horizon				
SEP-ExpInf-US1	*6*	*12*	*24*	*60*	*120*
3-Month					
Slope	-0.638**	0.504	-0.029	-0.164***	-0.032
	(0.118)	(0.268)	(0.054)	(0.011)	(0.0206)
Intercept	0.007***	0.003*	0.005***	0.006***	0.004***
	(0)	(0.001)	(0)	(0)	(0.001)
Adjusted R^2	0.848	0.187	-0.031	0.769	0.013
12-Month					
Slope	-0.618	-0.247*	-0.166	-0.260***	-0.078**
	(0.25)	(0.098)	(0.119)	(0.037)	(0.028)
Intercept	0.011**	0.008***	0.008***	0.008***	0.006***
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
Adjusted R^2	0.504	0.322	0.039	0.045	0.050
36-Month					
Slope	-8.862*	-6.712***	-3.893***	-0.976**	-0.817***
	(2.096)	(1.412)	(0.814)	(0.29)	(0.111)
Intercept	0.205**	0.159***	0.100***	0.033***	0.029***
	(0.045)	(0.03)	(0.018)	(0.007)	(0.003)
Adjusted R^2	0.771	0.662	0.487	0.148	0.305
60-Month					
Slope	5.520*	5.364***	4.575**	2.291*	-2.459***
	(1.262)	(0.99)	(1.293)	(1.125)	(0.358)
Intercept	-0.137*	-0.132***	-0.109**	-0.049	0.086***
	(0.037)	(0.028)	(0.036)	(0.032)	(0.011)
Adjusted R^2	0.783	0.720	0.333	0.051	0.278
120-Month					
Slope	1.553*	1.714***	1.657***	2.817***	2.751***
	(0.562)	(0.372)	(0.388)	(0.33)	(0.551)
Intercept	-0.019	-0.026	-0.023	-0.066***	-0.071***
	(0.021)	(0.013)	(0.013)	(0.011)	(0.017)
Adjusted R^2	0.570	0.647	0.427	0.548	0.167

Note: Significance levels are $***p < 0.01$, $**p < 0.05$, $*p < 0.1$. Numbers in parenthesis are standard errors of coefficients.

in Figure A5, the downward slope moves together with realized yields.

Turning to the UK, for 3-month maturities, the realized yields can be regarded as a stationary process around a constant mean as displayed in figure A6. Therefore, in table 7, the DL model for the UK overestimates the yields for short maturities some with small negative signs. 60- and 120-month maturities are forecasted in the opposite direction with negative slopes close to 1 and high R^2. These results suggest that the Diebold-Li model does not perform well enough.

In table 8, the SEP-Exp-1 model captures the downward trend in the realized yields even though it over-or under-estimates most of the time. For 1-year maturity, the model forecasts the yields with some high R^2 and promising coefficients. Lastly, in tables 9 and 10, the US-based models, SEP-Real-US2 and SEP-ExpInf-US1, give remarkable forecasts for 60- and 120-month maturities with high R^2. Moreover, as shown in Figures A9 and A10, the downward trend in the realized yields is highly captured up to the 24-month horizon.

Overall, the regression results in Tables 3 through 10 reveal that all methods of forecasting were quite poor, even if some of the selected models exhibited have promising relative RMSPEs less than 1. However, the downward trends in the realized yields starting with the out-of-sample date are mostly captured by the shifting endpoints specifications, especially for longer horizons. Particularly, as the yields of bonds going to hit the zero lower bound, the Diebold-Li method is quite poor to capture the downward trend while the shifting endpoint specification is able to follow the trend.

CHAPTER 6

CONCLUSIONS

The dynamically evolving factor dynamics were assumed to be stationary around a constant mean. On the other hand, the shifting endpoints methodology is employed for the factor dynamics with moving long-run mean rather than being constant. This specification allowed to produce forecasts in the same direction as the trend of the out-of-sample yields. The dynamic Nelson-Siegel model was not able to capture the downward trend in historical Canada and the UK bond yields observed after the financial crises of 2008. Due to the shifting long-run mean specification, the downward trend in the yield curve is forecasted better than the benchmark random walk and the dynamic Nelson Siegel models.

In Canada, particularly, the shifting endpoints methodology significantly outperformed the benchmark random walk process for short maturities. Turning to the longer maturities, on the other hand, shifting endpoints with exponential smoothing for all factors and with the US-based realized yields and 30-year inflation expectations improved the forecast accuracy. In the case of the UK, shifting endpoint with exponential smoothing for all factors improved forecast accuracy compared to the random walk for all maturities except 3-month. Additionally, the trend component of inflation and industrial production and 30-

year inflation expectation models increased the forecast accuracy compared to the random walk for 5- and 10-year maturities.

The results suggest that inflation expectations and the trend components of realized measures in the US as a global economy produce a spillover effect on the small open economies' nominal interest rates. Using the country-specific macroeconomic indicators of both Canada and the UK, on the other hand, did not outperform the benchmark random walk model and fell behind the US-based models. This evidence consolidates the spillover effect of the US economy. Nevertheless, these forecasts are not good enough as a result of Mincer and Zarnowitz regression. For shorter horizons of longer maturities, the regression results were promising. However, as the forecast horizon increases, the shifting endpoints-based forecast yields became almost constant as shown in figures in the appendix. The reason behind this can be the fact that the time-varying unconditional means were treated as a random walk for each value of the out-of-sample factors.

To conclude, the shifting endpoint specification improved the forecast results compared to the widely accepted benchmark random walk model. The US inflation and macroeconomic variables-based models were able to capture the downward trend of the yield curve of both Canada and the UK. This evidence supports the idea that the United States has a spillover effect on the yield curves of Canada and the United Kingdom. Besides, shifting endpoints with exponential smoothing applied to all factors was also able to perform better than the random walk model.

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APPENDIX

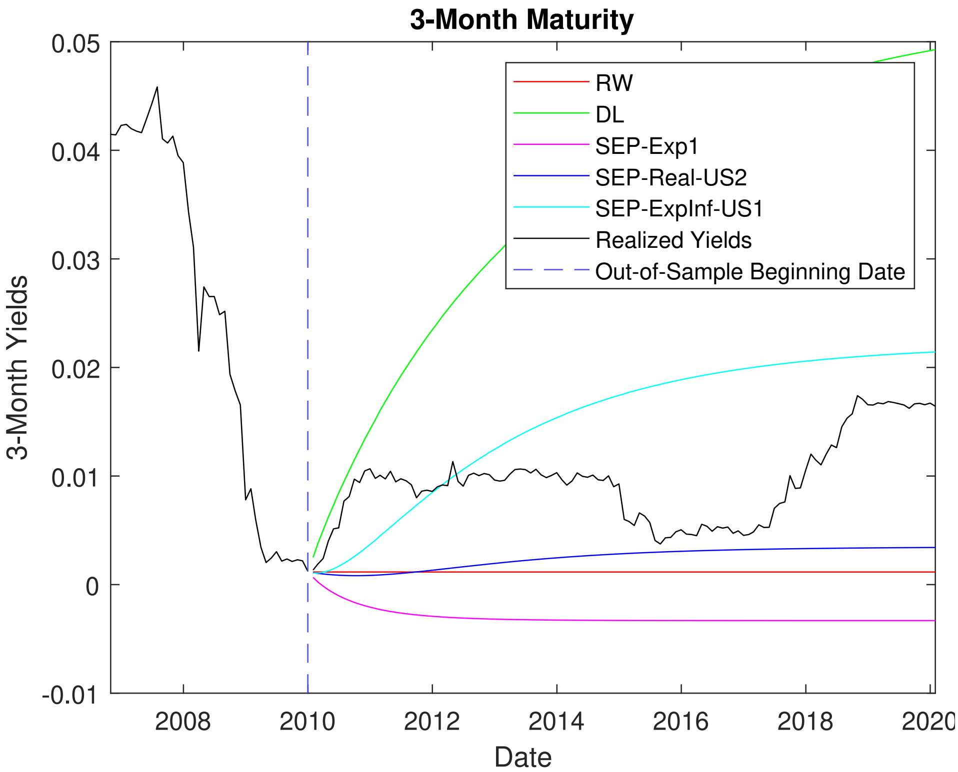

Figure A1: Forecast of Yields of Selected Models for 3-Month Maturity in Canada

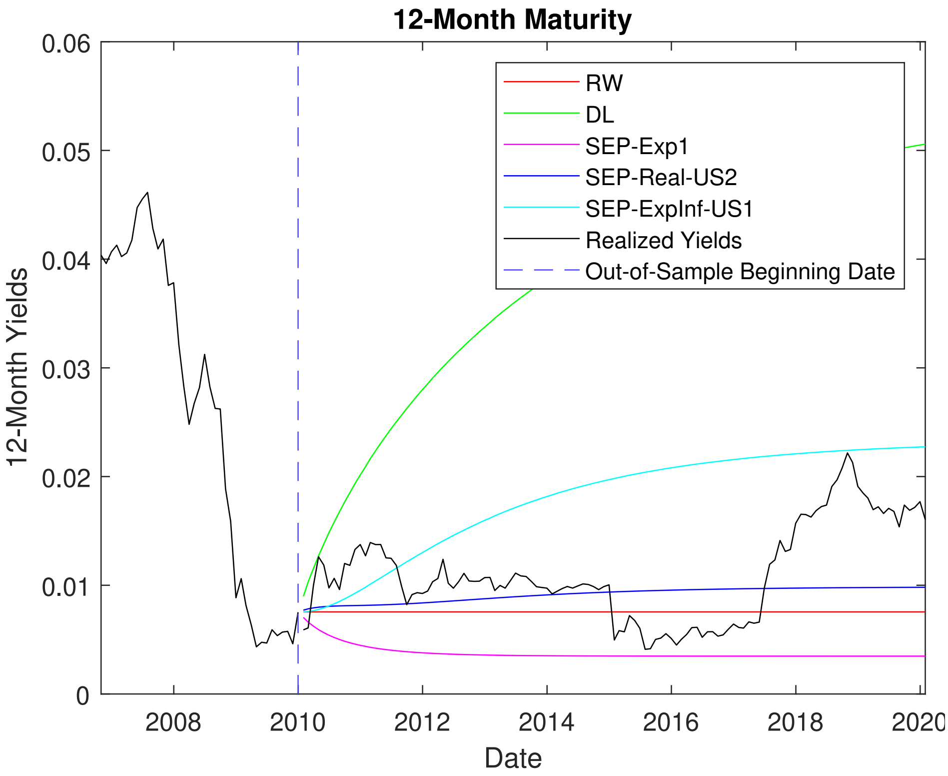

Figure A2: Forecast of Yields of Selected Models for 12-Month Maturity in Canada

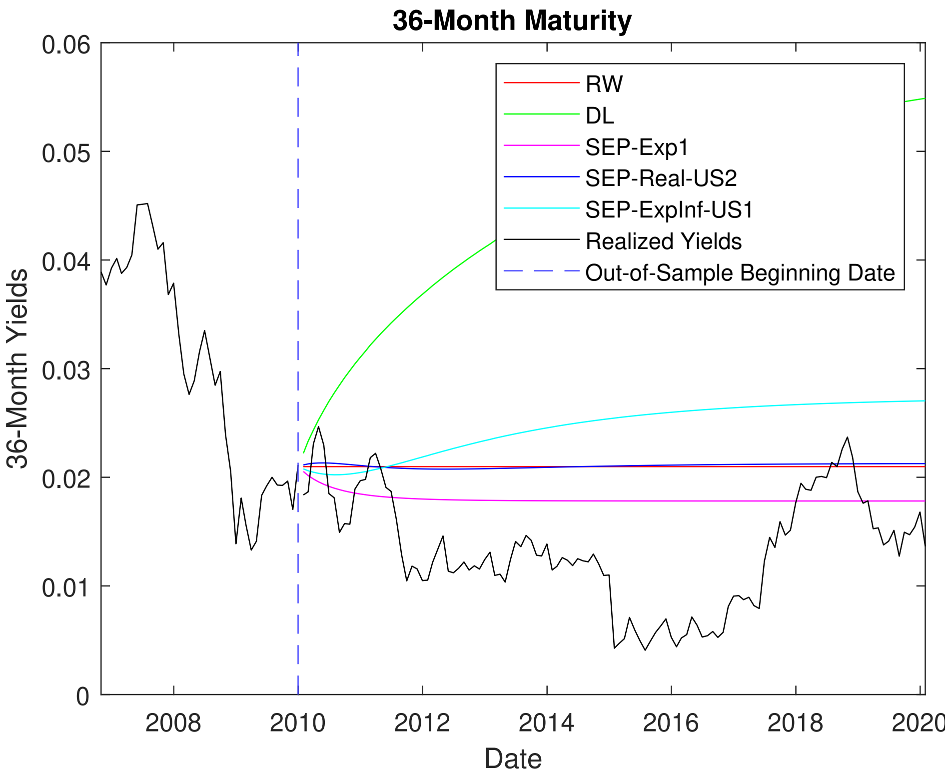

Figure A3: Forecast of Yields of Selected Models for 36-Month Maturity in Canada

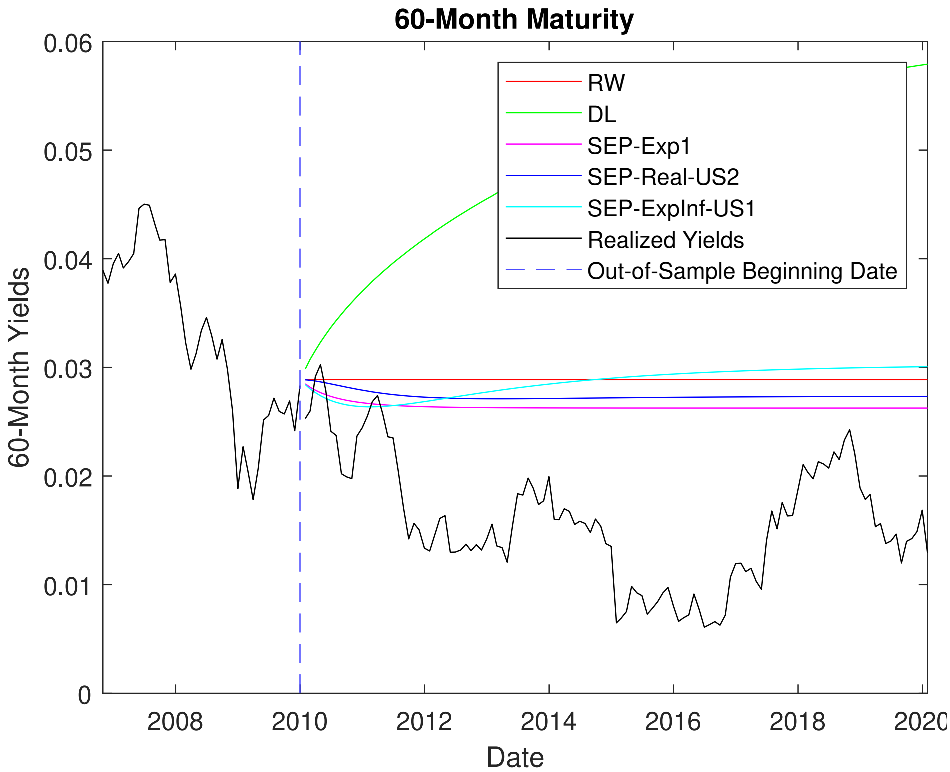

Figure A4: Forecast of Yields of Selected Models for 60-Month Maturity in Canada

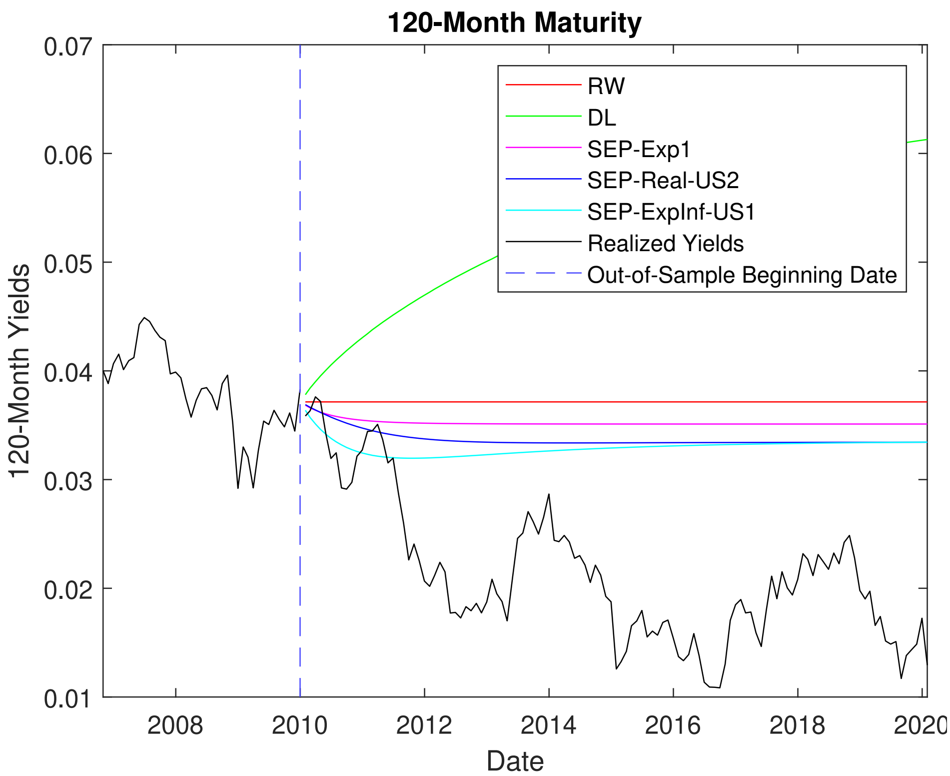

Figure A5: Forecast of Yields of Selected Models for 120-Month Maturity in Canada

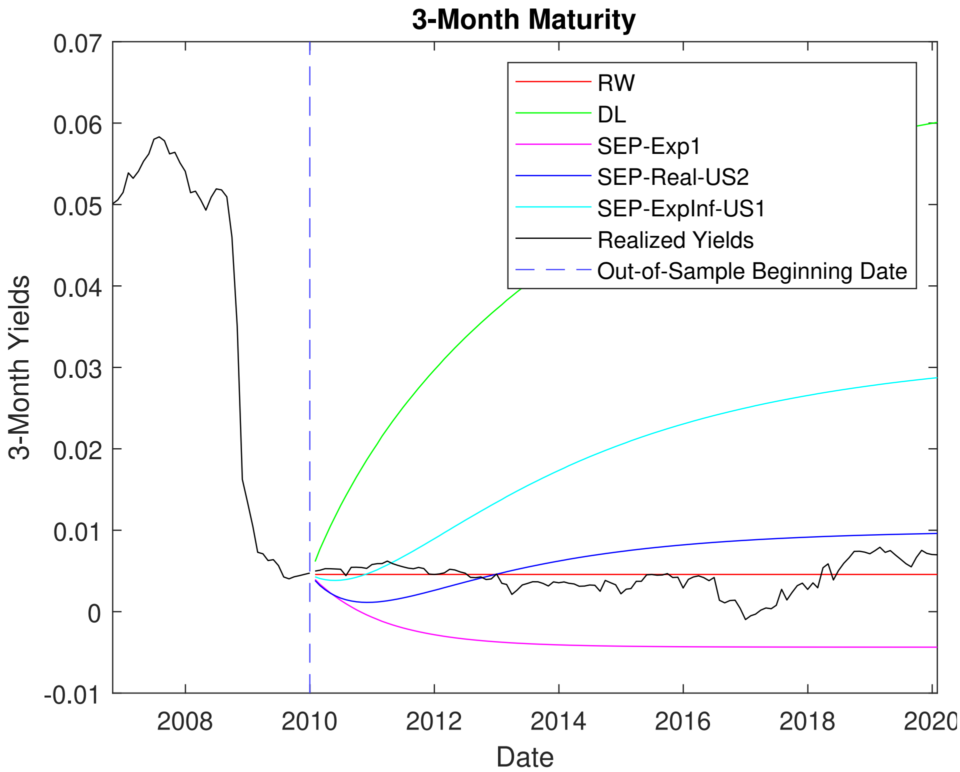

Figure A6: Forecast of Yields of Selected Models for 3-Month Maturity in the UK

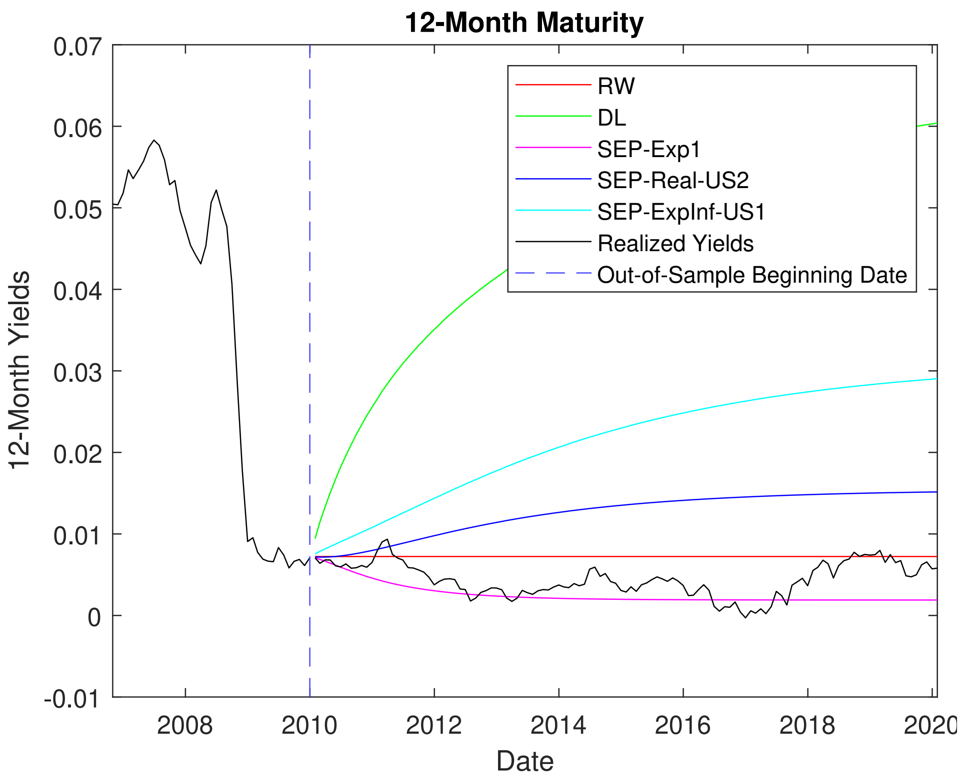

Figure A7: Forecast of Yields of Selected Models for 12-Month Maturity in the UK

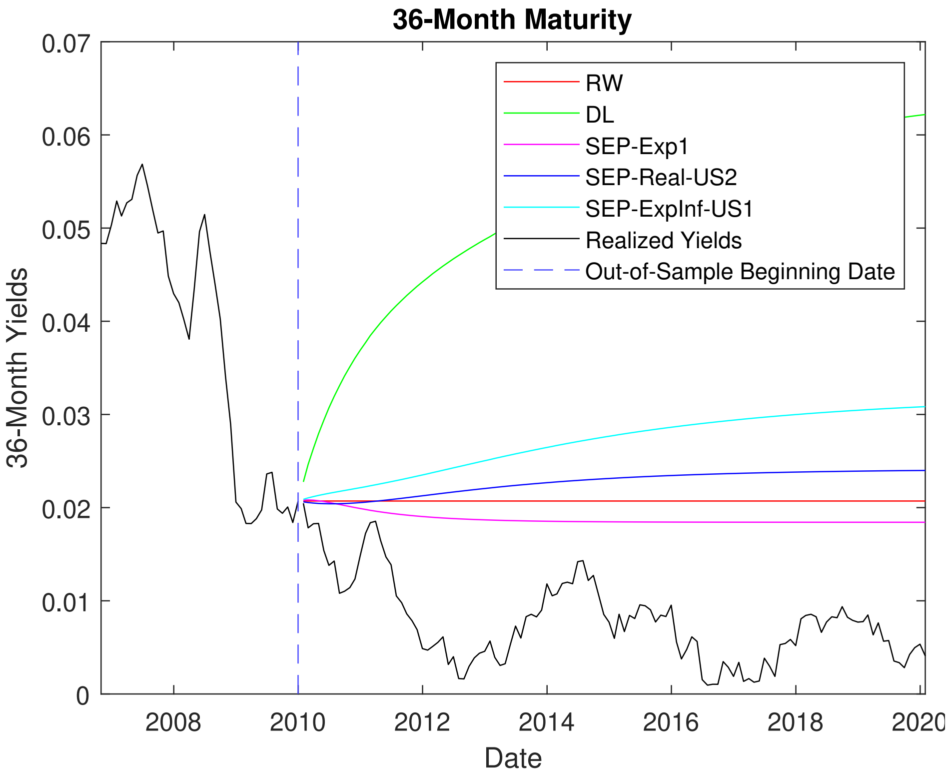

Figure A8: Forecast of Yields of Selected Models for 36-Month Maturity in the UK

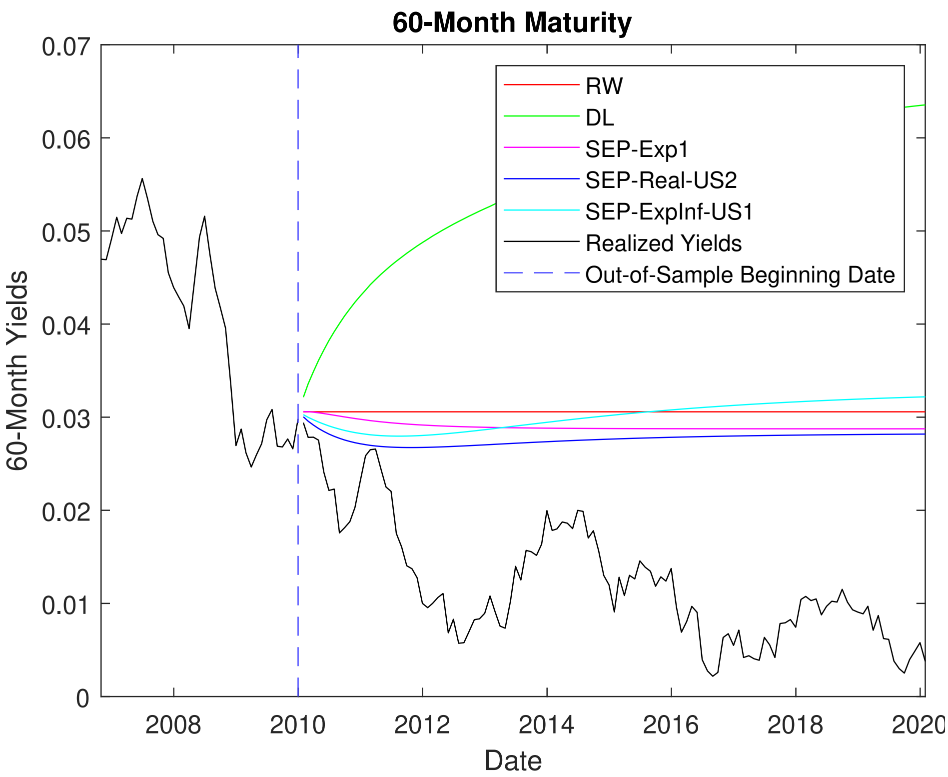

Figure A9: Forecast of Yields of Selected Models for 60-Month Maturity in the UK

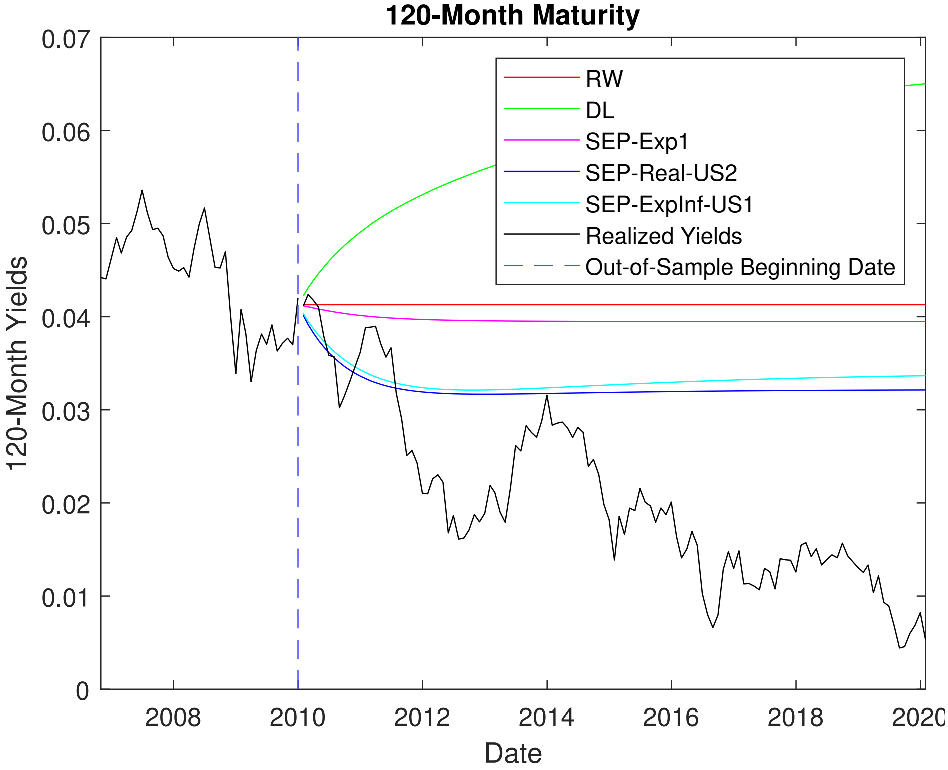

Figure A10: Forecast of Yields of Selected Models for 120-Month Maturity in the UK