KANTIAN EQUILIBRIA OF A CLASS OF NASH BARGAINING GAMES

A Master's Thesis

by

ATAKAN DİZARLAR

Department of Economics
İhsan Doğramacı Bilkent University
Ankara
August 2021

In loving memory of Tansel Hoca.

KANTIAN EQUILIBRIA OF A CLASS OF NASH BARGAINING GAMES

The Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

ATAKAN DİZARLAR

In Partial Fulfillment of the Requirements for the Degree of
MASTER OF ARTS IN ECONOMICS

THE DEPARTMENT OF ECONOMICS
İHSAN DOĞRAMACI BİLKENT UNIVERSITY
ANKARA

August 2021

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assoc. Prof. Dr. Emin Karagözoğlu
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assoc. Prof. Dr. Hüseyin Çağrı Sağlam
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Prof. Dr. İsmail Sağlam
Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

Prof. Dr. Refet S. Gürkaynak
Director

ABSTRACT

KANTIAN EQUILIBRIA OF A CLASS OF NASH BARGAINING GAMES

Dizarlar, Atakan

M.A., Department of Economics

Supervisor: Assoc. Prof. Dr. Emin Karagözoğlu

August 2021

This thesis studies Kantian equilibria (Roemer, 2010) of an n-player bargaining game, which is a modified version of the well-known divide-the-dollar game. It starts with introducing the fundamental concepts of Kantian morality and how Kantian moral theory is captured in economic theory. Then, we first show that the Kantian equilibrium exists under fairly minimal assumptions. Second, if the bankruptcy rule used satisfies equal treatment of equals, and is almost nowhere proportional, then only equal division can prevail in any Kantian equilibrium. On the other hand, we show that an 'anything goes' type result emerges only under the proportional rule. Furthermore, using hybrid bankruptcy rules that we construct in a novel fashion, we can characterize the whole equilibrium set. Lastly, we analyse what happens to the equilibrium behavior and the axiomatic properties of the bankruptcy rules under the additive definition of Kantian equilibrium. Our results highlight the interactions between institutions (axiomatic properties of division rules) and agents' equilibrium behavior.

Keywords: Axiomatic Approach, Bankruptcy Games, Kantian Equilibrium, Divide-the-Dollar Game, Equal Division, Kantian Morality, Bargaining, Equal Division.

ÖZET

BIR NASH PAZARLIK OYUNUNDA KANTIYEN DENGELER

Dizarlar, Atakan
Yüksek Lisans, İktisat Bölümü
Tez Danışmanı: Doç. Dr. Emin Karagözoğlu

Ağustos 2021

Bu tez "Doları Bölüştür" oyununun değiştirilmiş bir biçimine tekabül eden, n oyunculu bir pazarlık oyununun Kantiyen dengelerini (Roemer, 2010) incelemektedir. Başlangıçta Kantiyen etik kuramının temel kavramlarını tanıtıyor ve Kantiyen yaklaşımın iktisat disiplini içerisinde nasıl yansıtıldığını ve kuramlaştırıldığını inceliyoruz. Sonrasında, ilk olarak, Kantiyen dengenin oldukça minimal varsayımlar altında varlığının garantilendiğini gösteriyoruz. İkinci olarak, kullanılan herhangi bir iflas kuralının neredeyse hiçbir yerde orantısal kuralla örtüşmemesi ve *eşitlere eşit muamele* aksiyomunu sağlaması halinde, tüm Kantiyen dengelerin sadece eşit paylaşım verdiğini gösteriyoruz. Öte yandan, her paylaşım halinin Kantiyen denge olabilmesinin sadece orantısal kural altında mümkün olduğunu belirtiyoruz. Bunun yanında, özgün bir biçimde oluşturduğumuz hibrit iflas kurallarını kullanarak bütün denge kümesini karakterize edebileceğimizi gösteriyoruz. Son olarak eğer Kantiyen denge tanımını çarpımsal bir temel yerine toplamsal bir temel üzerine kurgularsak, Kantiyen denge durumlarının ve iflas kurallarının sağlamaları gereken aksiyomatik özelliklerin hangi yönlerde değişeceklerini inceliyoruz. Bulgularımız, iflas kurallarının aksiyomatik özellikleri (ki bunlar üzerinde çalıştığımız oyun ortamının kurumları olarak da algılanabilir) ve aktörlerin denge davranışları arasındaki etkileşimin altını çizmektedir.

Anahtar Kelimeler: Aksiyomatik Yaklaşım, İflas Oyunları, Kantiyen Denge, Doları Bölüştür Oyunu, Kantiyen Etik, Pazarlık, Eşit Bölüşüm.

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor Assoc. Prof. Dr. Emin Karagözoğlu for his guidance and support. I will always be thankful to him for nourishing my curiosity and interest in ways for philosophy and economics to intersect. He generously gave his time to our conversations thanks to which I have learned and laughed a lot.

I would like to thank my examining committee members Assoc. Prof. Dr. Hüseyin Çağrı Sağlam and Prof. Dr. İsmail Sağlam for their valuable comments on the study, suggestions for further research and time spared.

A special thanks to my fellow classmates Sıla Erikçi, Halilcan Kola, Nazlıcan Eroğlu, Berrin Özcan, Klajdi Hoxha and Loris Gockaj. Without their friendship, emotional presence and fascinating insights, it would be very hard to concentrate and complete the master's program during the days we had to live solely indoors.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Immanuel Kant's moral philosophy is structured around his notion of 'Categorical Imperative' (CI), which is an unconditional and absolute ethical principle for all rational beings. Interestingly, CI not only is a moral law that rational beings freely place on themselves; but it also aims to model the ethical decision-making procedure of any agent. This helps to explain the significant impact of Kantian morality on philosophy and social sciences: It can offer insights for both positive and normative theories. While it can be a building block of a social narrative, justifying the usage of certain constraints and assumptions[1], it can also explain why human beings choose to act, or not to act, in certain ways.

In this introductory chapter of the thesis, we focus on the fundamental concepts of Kantian morality with a particular emphasis on CI and how Kantian moral theory is comprehended and formalized in economic theory. This chapter is divided into 6 sections. In the first section, we are going to introduce the building blocks of Kantian morality, concentrating on *will*, *maxim* and *duty*. The second

[1]An interesting case for this can be found in political philosophy. Both Rawls and Nozick, who construct significant but competing theories on the role of government and fairness, appeal to Kantian morality as a justifying object in their theories. Rawls (1971) asserts that his principles of justice should be placed within the Kantian realm of political and moral philosophy since his concept of justice as fairness prioritizes what is right like the Kantian morality. Nozick (1974) claims that his side constraints, which help him justify the idea of minimal state, are designed to capture Kantian principles which emphasize the importance of not behaving individuals as mere means to achieve various objectives that undermine their humanity. Both philosophers engage with Kantian principles of morality while forming their theories, even if Rawls' theory is liberal-egalitarian in nature and Nozick's theory has a right-libertarian perspective.

section deals with CI and its first formulation. The third section discusses how the formal economic theory interprets Kantian moral reasoning and introduces it into economic modelling. The forth section motivates the main application of Kantian equilibrium in this thesis. The fifth section summarizes our important findings. The sixth section presents the organization of the rest of the thesis.

1.1 Fundamental Concepts of Kantian Morality

Kant begins his analysis by emphasizing that there is nothing absolutely good without limitation except a good will (Kant & Korsgaard, 1998) [2]. The utmost aim of the first chapter of *Groundwork of Metaphysics of Morals* is to derive CI from an analysis of good will. Will is the source of duty which directs moral attention to the principles -upon which an agent acts- consistent with the moral law, instead of principles moulded by other factors such as satisfying inclinations and achieving certain ends. Besides to his factual observation about goodness of good will, Kant and Korsgaard (1998) mention that one cannot even imagine anything good without limitation except good will. Although other qualities such as courage, persistence, power, and wealth can be praised and encouraged, these traits may become evil if the will that employs them is not good.

For instance, loyalty to a certain group or set of beliefs is ostensibly a positive trait: It strengthens the bond between individuals; and it can help a person to improve his character by building beneficial habits. However, it can also foster corruption, sports cheating, and cultures of crime when it is followed blindly and with unclear goals solely focusing on interests of the group, or on devotion to the fulfilment of a belief system. So, loyalty's goodness is not only derived

[2] The analysis of Kantian morality offered in this thesis relies on Kant's *Groundwork of Metaphysics of Morals*. Instead of presenting a detailed summary of Kant's moral philosophy, this and the following sections aim to provide a conceptual toolbox to help any reader to discern the basics of Kantian morality and derivation of the first formulation of CI; the formulation through which most theoretical economists engaged with Kant.

from the conditions in which loyalty is embraced, but also from the will of the individual who applies it.

This underlines a crucial point in Kantian morality: Any quality which is a candidate for being good without limitation cannot depend on the circumstances whose bearer faces and cannot be good through the quality's competence in realizing a particular and calculated end. Instead, a good will is good through its volition and this is the source of its absolute goodness (Allison, 2011).

Before moving forward with good will and its principles of volition, it is important to discuss what the phrase 'good will' refers to. Allison (2011) argues that there are two objects which can be identified with good will: One is the intentions of an agent in a certain action, which underlines the relationship with good will and varying occurrences; the other is the agent's character which perceives good will as a dispositional trait. Allison (2011, p. 79) mentions that Kant describes the concept of character as "a general orientation of the will with regard to the moral law, which is based on freely adopted principles rather than sentiment". In this sense, character is an orientation which needs to be obtained by the agents acting on principles for their own sake. Then, if a character is good, it is committed to morally good principles.

This gives us two different ways to approach the identification of good will as a dispositional trait (Allison, 2011). One is that an agent with a good character is someone who adapts her preferences, with freely embraced determination, to view the obligatory nature of an action as a sufficient to execute it even though her interests and incentives are opposing to it and its results. The other case is where an agent motivates her action by their consequences and his preferences on them. If she has good character, she also checks whether the action she plans to take is morally permissible. This characterization of good will with good character makes it easier to comprehend the role of good will in the concepts of maxim and duty.

Kant and Korsgaard (1998, p. 31) define maxims as "subjective principles of acting", meaning that these are the principles upon which an agent really acts. These are different from the objective principles, or practical laws, that characterize the perfect rationality in Kantian sense; those are the ones the agent should act upon. The difference between the two will be useful in the analysis of duty. Herissone-Kelly (2018) emphasizes that, due to their role as principles, maxims are general policies that determine action-types under particular circumstances. So, they give agents the justificatory reasons for an action. Moreover, as products of practical reason, maxims are chosen in a reflexively aware manner, and freely adopted by agents (Allison, 2011). In other words, an agent needs to have some consciousness of his engagement with what he is doing and his reasons for it.

To illustrate maxims in the propositional form, Herissone-Kelly (2018, p. 39-40) points out that a maxim may be represented with propositions expressing "an agent's personal determination to act in a certain sort of way in a certain sort of situation". This personal determination to act in a particular way is inevitably connected with achieving personal ends, which can be reflected by incentives and interests. Suppose that I adopt the maxim to maintain my health by means of having balanced diet and exercising regularly. So, in situations in which I am hungry, or exercise is a viable activity within my daily schedule, my actions of eating and exercising is determined by the maxim I adopted. Note that my maxim does not really specify which foods I need to eat and the type of exercises that I can engage with. I can eat a salad with the mixture of both tomatoes and peppers, and beans and spinach; I can go jogging for half an hour or jump rope for 20 minutes. Maintaining my health is the end involved in my maxim which indicates the incentive to maintain my health. Having an incentive to perform some action is to have a reason to perform that action. At this point, it is natural to ask: What is the source of moral incentives that can be incorporated in maxims?

Recall the distinction between the subjective principles of acting, maxims, and the objective ones, practical laws. A maxim is descriptive; it consists of principles on which an agent actually acts. Since reasons of agents to act is naturally affected from concerns for realizing their inclinations and interests in achieving an end, agents can have different sources of motivation while adopting a maxim. This suggests that all the maxims cannot be identified as subjective principles carrying moral content. Then, in some sense, the subjective principles, which prescribe actions, need to be constrained to approach to the objective principles that reflect the commands of morality (Allison, 2011). The constraints on the maxims limit the influence of tastes and preferences of agents in their choice of action. These constraints take the form of duties.

Kant and Korsgaard (1998, p. 13) define duty as "the necessity of an action from respect for the law". Here, the law refers to the moral law, and it will become clear when we focus on the formulation of Categorical Imperative (CI). Allison (2011) explains that "acting from respect for the law" includes adopting a maxim which adheres to the moral law although the actions it prescribes oppose to the suggestions of a maxim based on the agent's preferences. This struggle to respect the moral law in adopting a maxim is the motive of duty.

Given the understanding of character and the concept of maxim, it is possible to update the Allison's previous interpretation of Kant's description of character: "A general orientation of the will, which guides agents in their selection of the more specific maxims on which they actually act" (Allison, 2011, p. 98). This makes character, and so will, a crucial element in adoption of maxims. We noted that, while adopting a maxim, the agent can have different sources of motivation such as inclinations, interests in achieving an end and respect for the moral law. Character guides the agent in basing her maxims to her varying motivations and incentives.

Consequently, good character, or good will, becomes the source of moral incentive because an agent with such a character would act with the sole motivation of duty, or at least check whether his action is morally permissible. Then, an agent with good character would automatically favour moral incentives in the selection of his maxims. This makes good character a necessary condition for an action to possess moral worth because it leads one to accommodate moral incentives into his maxims, giving them moral content, and to act upon them. Since good character is identified with good will, this clarifies the emphasis of Kant on good will's absolute goodness: Good will is the source of a morally valuable action.

1.2 Categorical Imperative and Its Derivation

Categorical Imperative is an unconditional and absolute ethical principle for all rational beings. In order to derive it, we should start with discussing Kant's rational agency. Kant and Korsgaard (1998, p. 24) state that

> Everything in nature works according to the laws. Only a rational being has the capacity to act according to the representation of laws, that is, according to principles, or a will. Since reason is required for the derivation of actions from the laws, the will is nothing other than practical reason.

Before discussing what it means to "act according to the representation of laws", let us find the correspondence of "laws" in the context of human nature and philosophy of morals. Kant frequently compares practical laws as objective principles with maxims. The objective nature of these practical principles underline that they are valid for any rational agent. The validity of these principles characterizes the perfect rationality in Kantian sense, but they still apply to both perfectly rational agents, who are only motivated by moral incentives in the choice of their maxims, and imperfectly rational agents, who can have moral and non-moral incentives while adopting their maxims. While these laws describe how a perfectly rational agent acts; in the case of imperfectly rational agents, the same laws have normative appeal, prescribing how imperfectly agents should act.

Unlike them, maxims, as subjective principles, are valid only for the person that adopts them since they depend on the incentives and motivations, moral or non-moral, of a specific person. So, both types of rational agents do act upon principles, either subjective or objective. However, since the particular principle that the imperfectly rational agent adopted may reflect non-moral incentives as well, the principle does not ensure obedience to the laws. Thus, we can conclude that "acting according to the representation of laws" is acting on objective or subjective principles (Allison, 2011).

Relating this with will, a perfectly rational agent's will is only subject to moral incentives and objective principles. For such an agent, the actions perceived as "objectively necessary" are also "subjectively necessary" due to their relationships with laws (Allison, 2011). On the other hand, an imperfectly rational agent's will is subject to numerous incentives, some of which may contradict with the moral incentives. Hence, such a will comprehends what is "objectively necessary" as "subjectively contingent" (Allison, 2011). This imposes a constraint on a not completely good will, which relates the normative nature of objective principles with that will, and it takes the form of an imperative (Kant & Korsgaard, 1998).

Kant and Korsgaard (1998) distinguish two different types of imperatives: hypothetical and categorical. For an imperative to command in a hypothetical way, the command applies under a specific condition. For example, "If I want to progress with my Italian, I should find more enjoyable ways to practice it" is a hypothetical imperative. The fundamental aspect of hypothetical imperative is that the condition that the command applies is some possible or actual end of an agent (Kant & Korsgaard, 1998). Categorical imperatives' commands are independent of such ends. Since any imperative presupposes a law (i.e. objectively valid practical principle) to establish its necessity, and any imperative is addressed to the maxim of an agent via will, the necessity of a categorical imperative would become absolute (Darwall, 1998). This is because that imperative holds independent of the ends agents set for themselves, like my desire to improve my Italian skills. So, in some sense, the normative appeal of the law which is reflected to the maxim via a categorical imperative is not a function of an end and its desirability (Allison, 2011).

This removal of any pre-given content from the law does not nullify the concept of categorical imperative, but it reduces it to *the* categorical imperative. Its content could only involve something valid for all imperfectly rational agents with varying ends, and it is "the thought of conformity to universal law as such, that is, to the idea of lawfulness regarded as an unconditioned norm" (Allison, 2011, p. 175). Then, the necessity that the categorical imperative derives comes from conformity to universality and lawfulness. By definition of maxim, any maxim of an imperfectly rational agent conforms to such a universal law if the agent can will its maxim, which captures the agent's interests and incentives, as a universal principle that prescribe certain actions (Kant & Korsgaard, 1998). This formulates the Categorical Imperative (CI), which corresponds to the aforementioned moral law: "Act only in accordance with that maxim through which you can at the same time will that it become a universal law" (Kant & Korsgaard, 1998, p. 31).

The motive of duty is to respect CI in adopting a maxim. When a maxim does not conform to CI, and thus is rejected by it, one has a duty not to act on that maxim. These characterize the perfect duties which function as a sufficient condition to act or not to act in certain ways (Allison, 2011). For instance, the maxim to lie whenever it is possible to enhance my well-being would be denied by CI. The reason is that when it is adopted as a universal principle prescribing actions, it would destroy all the relations and activities based on trust and would make any such lie useless.

When a maxim conforms to CI, meaning that it can be willed as a universal principle, one has an imperfect duty to act upon that maxim. These are different from perfect duties in the sense that they do not suggest taking, or not taking, specific actions. Rather, they encourage pursuing certain ends which can be fulfilled in various ways with varying actions (Kant & Korsgaard, 1998), such as developing our own natural talents by practising harmonica and Italian; or helping others in need by donating money to charities and going to meetings of addiction for emotional support. These duties function as necessary conditions for adopting maxims with moral content since they ensure some commitment to follow the requirements of morality (Allison, 2011).

Perfect and imperfect duties are the only ways to include the idea of dutifulness in a maxim. As a result, they give the maxims that incorporate them moral content, which guarantees the moral worth of the actions prescribed by these maxims (Kant & Korsgaard, 1998).

1.3 Kantian Economics

Even though there is only one CI, Kant argues that it has multiple formulations, which involve the Formula of Humanity, Formula of Autonomy and Formula of the Realm of Ends besides to the first formulation (the Formula of Universal Law) that we presented (Allison, 2011). The theoretical economists have mostly engaged with Kantian morality via the first formulation of CI which states that one is to "act only in accordance with that maxim through which you can at the same time will that it become a universal law" (Kant & Korsgaard, 1998, p. 31). This choice seems surprising since the first formulation of CI is also the most demanding one for understanding the gist of Kant's moral theory. Although its emphasis on the concepts of "will" and "maxim" makes it challenging to comprehend which actions can be classified as moral, there is a clear reference to the notion which attracts theoretical economists the most: universalization.

By equating maxims with actions[3], most models aim to construct a Kantian agent who decides to take some particular action after considering the counterfactual: What would happen if all rational agents would also take the same action? If one would rationally will themselves to take the action in a world where every rational agent is ready to implement the same action after her, then taking this action becomes morally acceptable. This approach to Kantian morality operationalizes the universalization idea since it captures a practical mathematical representation of what universalization is with a usual game theoretic notion of strategy. It also eliminates the need of incorporating unfamiliar concepts as variables into the model such as maxim or duty.

The limited application of Kantian reasoning has been mainly in the study of preferences and cooperative behaviour. Laffont (1975) articulates the idea of a model which is composed of agents who strictly follow Kantian morality and expect others to behave like them, in the sense described above, and informally discusses what would be different if some macroeconomic models had Kantian agents instead of selfish agents. Laffont (1975) also highlights the lack of explanation for the 'good outcomes' observed in the tragedy of commons type situations, especially in situations where the individual contribution towards the collectively rational outcome is costly.

[3] While it is possible to consider cases where different *maxims* can lead to the same action, or a specific *maxim* suggesting different actions, identifying *maxims* with actions has been a building block of the economic models with agents who embrace Kantian morality either as a preference (such as Alger and Weibull (2013)) or as an optimization protocol based on John Roemer's account (2019). For a detailed discussion of distinguishing *maxims* and actions, and a way of bringing Kantian optimization (in the Roemerian sense) closer to Kantian ethics, see Braham and van Hees (2020)

Roemer (2019)) argues that one needs to incorporate the *social cooperation* aspect in agents' optimization processes to explain individual behavior that appears to be irrational, instead of just adjusting their preferences. He distinguishes between altruism and cooperation: While altruism is a type of preference which emphasizes the positive effect of caring about others on an agent's well-being, cooperation is a way of acting which can involve self-interested as well as altruistic agents (Roemer, 2019). He emphasizes that optimizing à la Nash, where each agent considers the counterfactual "What would happen if solely I changed my strategy while the other agents kept theirs fixed?", is not designed to explain cooperative behaviour (Roemer, 2019).

A seminal experimental work, which reports more cooperation than what the mainstream models predict is Ostrom (1990). Consider a small fishing community and a lake owned by that community. Each fisherman has different preferences over the number of fish he catches and the labour activities that fishing requires with, possibly, varying skill in fishing. Given that each local chooses how much time she devotes to fishing, the lake generates fish with decreasing returns in respect to the time spent on fishing due to congestion. So, as the whole community's total fishing time increases, the lake will produce less fish and the fish caught per unit of time will decline. In the Nash equilibrium of this game, there is over-fishing, and the equilibrium strategy profile is not Pareto efficient. However, Ostrom (1990) shows that most of such societies avoid the inefficient (Nash equilibrium) outcome by cooperating against the negative externalities of congestion. Roemer (2010) and Roemer (2015) suggest developing a new equilibrium concept based on Kantian morality that can offer an explanation with proper micro foundations for such observations.[4]

There are the two main ways to introduce Kantian reasoning into economic modelling: Incorporating agents who have preferences in compliance with

[4] See Sher (2020) for a critical discussion on the foundations of the Kantian equilibrium.

Kantian morality and incorporating agents who follow an optimization protocol based on Kantian morality. For the former, an influential line of literature has been cultivated by Alger and Weibull (2013). These authors introduce an alternative agent type, *homo moralis*, whose preferences lie in between maximizing her own payoff and choosing an action which would lead to the greatest possible payoff if every agent copies her action. By adding an exogenous parameter κ that takes values between 0 and 1 into the utility function, they come up with a model that can host heterogeneous agents whose preferences are torn between two different goals. When κ, the degree of morality, is 1, the agent is defined to be *homo kantiensis* who is motivated to "do the right thing" by only considering what happens if every agent copies her action (Alger & Weibull, 2013). Furthermore, Alger and Weibull (2016) show that evolution, via the process of natural selection, favours the preferences of *homo moralis*, under random matching that could be assortative. They note that an evolutionary stable variety of *homo moralis* contributes more than an agent who solely tries to maximize her utility in public-goods games and situations, which require cooperation (Alger & Weibull, 2016). Finally, Alger and Weibull (2017) also underline that *homo moralis* preferences improve the outcome in public-goods games and can possibly rule out socially inefficient equilibria in coordination games, while this is not the case for self-interested or altruistic preferences.

While this type of a preference strongly captures the idea of universalization, it also contains a problem regarding the traditional rational choice models: Although the agent is perfectly able to and actually do come up with the best decision available given his preferences and constraints, can he always perform in accordance with that decision? Kantian choice theory offers an environment where it is possible for an agent to identify an action, which comes from some maxim which complies with CI, as a moral one but, still, the agent could fail to perform this action as a part of his duty due to his weakness of *will* (White, 2011). The simplification of Kantian moral theory eliminates such a case and thus, the models including Kantian preferences are identical with models built on purely selfish agents in assuming that the agents cannot fail to act suggested by 'morality' or 'rationality'. From this perspective, it seems that the economic models that incorporate Kantian reasoning in preferences can be criticised by modelling the "imperfect" Kantian agent in a restricted way.

Another way of incorporating Kantian reasoning into economic modelling is to assume that the agents follow an optimization protocol based on Kantian morality. While formalizing such an optimization, it is crucial to specify what universalization of this action means for the rest of the agents. Roemer (2010; 2015; 2019) develops an equilibrium concept called the 'Kantian equilibrium' in the following sense: Given an action profile, when an agent thinks about whether there is a profitable/beneficial deviation for her in a Kantian manner, she evaluates the profile of actions that would occur if everyone deviated like her. Here, a deviation materializes by changing the action to some other, which can be seen as a certain multiple of the original action. In the more commonly used version of Kantian equilibrium, for a given action profile where an action is actually a number (such as effort levels to be exerted or bids to be made), an agent can deviate from his action by multiplying it with some factor $\alpha > 0$ (e.g., initially exerting an effort level of 10, and deviating to 30, that is three times 10). The Kantian counterfactual requires him to consider what happens if the other agents also change their actions in the same way. Then, an action profile is said to be a Kantian equilibrium if no agent prefers that everyone changes his/her action by the same factor $\alpha > 0$.

Some real life behaviors that can be explained with Kantian optimization are voting (Roemer, 2019), not littering (Laffont, 1975), paying taxes, or joining collective movements like strikes, etc. There have been even slogans in public campaigns, which assert the importance of personal duty in crisis periods such as war or economic depression. For example, the expression "Alın Verin, Ekonomiye

Can Verin," which can be loosely translated as "Exchange to Revive the Economy," was a frequently aired Turkish television public spot during the Great Recession. Its main goal was to emphasize that even buying something as simple (or cheap) as a chewing gum could help the economy to recover via the multiplier effect if everybody does so. It appears to have tried to address Kantian tendencies within individuals: "I will spend some money, even though I am more inclined to save for the uncertain days ahead, since; if everybody also spends, it would be easier for the aggregate demand to recover."

These efforts to conjoin Kantian morality with economic modelling take preferences and constraints as exogenously given. These objects are not in the agent's control in the decision-making framework. However, in Kantian agency, character guides the agent in basing her maxims to her varying motivations and incentives. Besides, the motive of duty is to respect the Categorical Imperative in adopting a maxim. So, which incentives to involve while adopting a maxim is subject to conscious control of the agent. White (2019) argues that if we also model preferences and constraints as objects upon which agents have some control of, it is possible to have agents who choose to adhere to Kantian duties autonomously by incorporating perfect duties, as constraints on acting, and imperfect duties, as preferences, into the decision-making framework. Like resource constraints determining the feasibility of an action, perfect duties can be easily represented as constraints of CI on action space because of their (mostly) negative character. Imperfect duties can accord with an agent's preferences since they encourage pursuing certain ends whenever possible (White, 2019). However, as White (2019) recognizes, the agent still needs to decide her duties and *how* to include them in her decision-making process.

This again underlines the importance of concepts unfamiliar to economics such as *judgment*, *maxim* and *will* in Kantian morality. Therefore, capturing the essence of Kantian reasoning in economic modelling might require to define, formalize and operationalize new concepts and objects, instead of revising certain concepts like preferences, constraints and optimization protocols.

1.4 Motivation

Incorporating the concept of Kantian equilibrium into economic models is appealing in that it has the potential to explain the cooperative behaviour in cases where experimental/empirical work reports more cooperation than what the standard models predict. For instance, the behaviour of the observed fishing communities in Ostrom (1990) matches with the Kantian equilibrium of the game (Roemer, 2015). More broadly, as Roemer (2019) argues, Kantian equilibrium may offer valuable insight about *how we cooperate* in various strategic environments.

As also noted by Sher (2020), constant-sum games are seen as poor candidates for Kantian equilibrium since competition rather than coordination is emphasized in such games. The literature on the application of Kantian equilibrium confirms this insight. More precisely, it has been almost exclusively on contributions to the public-goods games (see Roemer (2010); Roemer (2015); Ghosh and Van Long (2015); Van Long (2017); Grafton, Kompas, and Van Long (2017); Eichner and Pethig (2019)) and tragedy of commons scenarios in environmental problems (see Grafton et al. (2017); Bezin and Ponthière (2019); Planas (2018)).[5] [6] Here, we study the Kantian equilibrium of a bargaining game, which has a constant-sum nature. The game we study is a modified version of the divide-the-dollar game, which itself is a simple Nash bargaining game. Our results show, in

[5]Planas (2018) uses Kantian reasoning but not the Kantian equilibrium.

[6]We found two exceptions to this: Studtmann and Suresh (2021) study a prisoners' dilemma game where players derive psychic utility from acting in line with Kantian morality, and show that their material payoffs are Pareto improvement over the Nash equilibrium payoffs. Alger and Laslier (2020) study a *Condorcet jury* setting that contains an information aggregation aspect in addition to coordination.

contrast with the commonly held view, that the Kantian equilibrium may also have a promise in games that have a constant-sum nature.

At this point, some background information on the game we study is in order. In an attempt to provide a strategic justification for the axiomatic Nash bargaining solution (Nash, 1950), Nash (1953) introduced what was later called the *Nash Demand Game (NDG)*. The *Divide the Dollar (DD)* game is a simplified version of the *NDG*, where bargaining frontier is linear and the bargaining set is symmetric. In the *DD* game, n agents simultaneously declare their demands on a dollar. If the sum of demands is less than or equal to one, then everyone receives his demand, whereas if the sum of demands is larger than one, then everyone receives zero. This simple game is frequently used in economics, political science, and international relations, likely because it carries the two defining characteristics of a canonical bargaining situation: (i) joint interest in reaching an agreement and (ii) conflict of interest over which agreement to reach (see Binmore (1998) and Kilgour (2003)). However, the Nash equilibrium set of the *DD* game may cause disappointment for those who use this game to make sharp predictions: any strategy profile where the demands add up to one (i.e., the whole bargaining frontier) constitutes a Nash equilibrium. In other words, there are infinitely many Nash equilibria. Among them, the one that induces an equal division of the dollar is arguably the most reasonable one. Some scholars provided arguments in favor of equal division, referring to its normative appeal, fairness, focality, symmetry, or evolutionary stability (see Nash (1953); Schelling (1960); Young (1993); Skyrms (1996); Bolton (1997)). There is also a strong experimental support for equal division in symmetric bargaining games (see Nydegger and Owen (1975); Roth and Malouf (1979); Van Huyck, Battalio, Mathur, Van Huyck, and Ortmann (1995) and Karagözoğlu and Riedl (2015)). Starting with Brams and Taylor (1994), some scholars attempt to modify the rules of the *DD* game so as to match the equilibrium prediction with the common sense prediction (i.e., equal division).

We will describe, in detail, the earlier literature, which is aimed at obtaining modified versions of the *DD* game, where the equal division outcome is implemented in equilibrium in Chapter 2. That said, it is worthwhile explaining why one would expect the Kantian equilibrium to provide new results and insights in this context. As we mentioned above, in the simple *DD* game or its variants (as in almost any bargaining game), in addition to the competitive aspect, there is also a joint interest in reaching an agreement, which requires some coordination and cooperation. For instance, in the *DD*, for collective benefit, strategy profile must be on the bargaining frontier. If the sum of demands is more than one, the players end up with an extremely inefficient outcome where no one receives a positive payoff. This shares a flavor similar to free-riding incentives and the resulting inefficient outcome in public goods games. Therefore, we expect the Kantian equilibrium to offer new results and insights.

1.5 Summary of Results and Contribution

In this thesis, we propose modifying the rules of the DD game by applying a bankruptcy rule when the players' demands are not jointly feasible (also see Ashlagi, Karagözoğlu, and Klaus (2012)). Our framework is different than other modifications in terms of the optimization concept that the players employ: they are assumed to be Kantian in the sense formulated in Roemer (2010). Accordingly, we focus on the Kantian equilibria of the modified game by bringing the axiomatic properties of different bankruptcy rules into the picture.[7] We, first, show the existence of Kantian equilibrium under a fairly weak assumption (i.e., equal treatment of equals) on the bankruptcy rule used in the game.[8]

[7]To avoid confusion, it would be good to emphasize that Kantian individual behavior and Kantian equilibrium are distinct objects. More precisely, people may behave in a Kantian manner, which does not necessitate an emergence of an equilibrium. We utilize the latter in this thesis.

[8]For a recent discussion on the existence of Kantian equilibrium in general, the reader is referred to Sher (2020).

Any division rule which satisfies *equal treatment of equals* induces a Kantian equilibrium, where equal division is the equilibrium outcome. Second, we show that the use of the proportional rule, arguably the most prominent bankruptcy rule among all, leads to an *anything goes* type result: any efficient division can be supported in Kantian equilibrium. Importantly, we show that there also exist division rules, other than the proportional rule, which satisfy equal treatment of equals, but still induce unequal division in equilibrium. We introduce two properties which separately eliminate these cases. Finally, we construct a family of bankruptcy rules in a novel fashion, with the help of which we span the set of all possible efficient divisions in Kantian equilibria. Our analysis shows how the moral reasoning embraced by the agents affects the strategic interaction and the axiomatic properties of bankruptcy rules, which can be interpreted as *institutions*, influence agents behavior and equilibrium outcomes.

This thesis contributes to three different lines of work: (i) applications of Kantian equilibrium in strategic games, (ii) the Nash program in bargaining games/problems (see Serrano (2021)), and (iii) strategic bankruptcy games. First, to the best of our knowledge, this is the first study to utilize Kantian equilibrium in a bargaining game. Second, it contributes to the *Nash Program*, which - in this context - aims to establish noncooperative foundations for equal division as the equilibrium outcome in DD game, in a novel fashion: We achieve a reconciliation of cooperative (Nash, 1950) and noncooperative (Nash, 1953) aspects of the DD game without resorting to the Nash equilibrium or its refinements. This reassures the cooperative side of Kantian reasoning and optimization. Finally, since our game addresses bankruptcy situations that can arise, it contributes to a relatively small literature on strategic bankruptcy games.

1.6 Organization of the Thesis

The organization of the thesis is as follows: Chapter 2 reviews the relevant literature with a special emphasis on the divide-the-dollar game and its modified versions. Chapter 3 introduces the model and necessary definitions. Chapter 4 presents equilibrium analyses and results. Chapter 5 presents an equilibrium analysis under the alternative, additive definition of the Kantian equilibrium. Chapter 6 ends the thesis with concluding remarks.

CHAPTER 2

LITERATURE REVIEW

Our study falls into two strands of literature in bargaining and distribution games: (i) divide-the-dollar game and its modified versions and (ii) bankruptcy/claims games. We focus on the former in this section since it is the closest one to our work among the two.[1]

As we mentioned in the introduction, despite its appealing characteristics, the *DD* game suffers from the multiplicity of Nash equilibria. In order to overcome this problem and induce equal division as the unique equilibrium outcome, researchers apply different methods to modify the game, by changing its rules in a *reasonable* fashion. In his seminal contribution, Nash (1953) initially suggests to introduce perturbations to the probability function, which decides whether a pair of demands is feasible or not. He informally discusses that the limit of each perturbed game's equilibrium converges to equal division as the perturbations to the probability function approaches to the original probability function. Later, Abreu and Pearce (2015) formalize this idea and specify the conditions for this convergence result to hold.

We can classify the other papers into two groups: The ones that (i) add new

[1] Due to the way we revise the punishment clause in the standard *DD* game and the divisions rules and axioms we utilize, our game can be seen as a bankruptcy game too. Some contributions to this line of work are Chun (1989), Chang and Hu (2008), Atlamaz, Berden, Peters, and Vermeulen (2011), Kıbrıs and Kıbrıs (2013), Karagözoğlu (2014), and Hagiwara and Hanato (2021).

stages to the game and (ii) modify the punishment clause (i.e., for not reaching an agreement) by changing the rule which distributes the dollar.

In the first group, if the sum of demands is larger than the dollar, then the game continues with a possibly different stage (than the first one). Brams and Taylor (1994)'s $DD2$ introduces a second stage in which the players can either continue with their demands or switch to the other player's demand. The rules of first stage, DD game, apply to this second stage as well. They demonstrate that this game is *dominance solvable*, and that the *sophisticated equilibrium* (Moulin, 1986) of this game induces equal division. In Cetemen and Karagözoğlu (2014), when the demands of the players are incompatible, the player with the lower (more agreeable) demand is selected to be a proposer in an ultimatum game in the second period (where the they have to decide how to share the excess they generated). Any accepted proposal in the second stage is deducted from the players' demands in the first stage to finalize the outcome. If the proposal is rejected, they both receive zero. Cetemen and Karagözoğlu (2014) show that the unique subgame perfect Nash equilibrium of this sequential game induces equal division. Karagözoğlu and Rachmilevitch (2018) also introduce a second stage in which the player with the greater demand, the greedier player in some sense, is given the opportunity to alter her demand, say x, such that her revised demand is in $[1-x, x]$. The revised demand gets implemented with some probability λ, which negatively depends on the value of the revised demand (i.e., the closer the revised demand to the initial demand, the lower the chance of its implementation). They propose a condition under which the subgame perfect Nash equilibrium uniquely induces equal division. Rachmilevitch (2020b) also adds continuation stages, but without fixing, in advance, the stage at which the game ends. If the demands are incompatible, the game follows as if the player with the lower demand proposes his demand to the other player. If it is accepted, the player with the lower demand gets his demand and the other player gets a value such that the sum of two awards is equal to the estate; if it is rejected, they play the same game again, and start it by announcing new demands. This formulation is interesting since the result which solves the multiplicity issue is independent of discounting factors (because of the competition to be the less greedy player), even if they are asymmetric.

The second group of papers modifies the punishment clause by changing the division (or the payment) rule. Brams and Taylor (1994)'s $DD1$ changes the division rule so that it prioritizes the lowest demand first. Then, if there is any amount left to share, this amount is distributed among the equivalence class for the second lowest demands, and so on. This process of dividing evenly within equivalence classes continues until the dollar is entirely used up, or every player receives her demand. Anbarcı (2001) modifies the division rule so that each player needs to make sacrifices depending on the other player's demand when the sum of demands exceeds 1. The proportions $\lambda_x, \lambda_y > 0$ are determined in a way, for player x, to equalize the sum of x and $\lambda_x y$ equal to 1, similarly for player y. Although the determination of λ_x and λ_y seems to enable a player to manipulate her opponent's demand so that she receives her entire demand, the division rule assigns the following payoff vectors for player x and y respectively: $\lambda_x x$ and $\lambda_y y.$ This is like a *forced* morality: What you want your opponent to experience will be applied to you. Rachmilevitch (2020a) extends this mechanism to a more general case. Ashlagi et al. (2012) propose modifying the rules of the game by applying a bankruptcy rule $-$simply a division rule that associates any strategy profile whose demands are incompatible with a payoff vector. They study an estate division problem modified as such, by bringing into the picture the axiomatic properties of different bankruptcy rules, and establish a link between these axioms and the equilibrium of the associated estate division game. In particular, they come up with classes of bankruptcy rules satisfying certain axioms, which lead all Nash equilibria of the game to implement equal division. We follow a similar approach in this thesis, but we use the Kantian equilibrium as the equilibrium concept. Rachmilevitch (2017) points out a drawback in the $DD1$ game in Brams and Taylor (1994). In that game, if each player demands approximately the entire dollar, if the least greedy player is unique, then he gets almost the entire dollar even if he is only marginally less greedy than the others. To overcome this problem, the author introduces a parametric family of $DD1$ games. He shows that if the game is *reasonable* (as defined in Brams and Taylor (1994), then there is a unique Nash equilibrium in which equal division prevails.

Finally, Andreozzi (2010) and Rasmusen (2019) are two papers, in which the authors modify the divide-the-dollar game but -maybe- not necessarily to implement equal division in equilibrium but to increase its realism and check the robustness of its results. In Andreozzi (2010), the new game is called "Produce and Divide the Dollar" (*PAD*). This is also a two-stage game where, in the first stage, only one player is able to invest in the production of a pie by incurring a cost (if he does not choose to do so, the game ends and both players receive zero); the second stage is standard DD. The author argues that the weakness of the Nash Demand Game (or Divide-the-Dollar game) is that it ignores incentives by fixing the size of the pie, and then goes on to show that endogenizing the pie (as in *PAD*) leads the social convention to move away from equal division and towards more asymmetric divisions reflecting the costs incurred by the player who invested in the pie. Rasmusen (2019) introduces a costly choice of a *toughness level* and the probability of bargaining failure, which is an increasing function of toughness levels. In the case of a bargaining failure, players get nothing. In the standard *DD* game, players choose their demands and, coordinating these demands on the Pareto frontier is crucial; in Rasmusen (2019), players choose their toughness levels, and coordinating them at a critical level that avoids bargaining failure is crucial. Despite the presence of the same type of need for coordination, this model has a unique Nash equilibrium in pure strategies inducing equal division $-$a result quite different than the one in the *DD* game.

CHAPTER 3

THE MODEL

In this section, we present the model and the necessary definitions in three subsections. The first subsection describes the bargaining game that we study. The second subsection presents the definitions of bankruptcy problems, bankruptcy rules, and the axioms we employ in the equilibrium analysis. Finally, the third subsection presents the definition of Kantian equilibrium, which we use throughout the thesis.

3.1 Bargaining Game

In a bargaining game, denoted by Γ, a finite set of agents $N = \{1, 2, \ldots, n\}$ try to divide a finite, real-valued estate $E > 0$ among themselves. The value of the estate, E, and the set of agents, N, are fixed. Agents have strictly monotonic preferences over the amounts of the estate they receive. Every agent $i \in N$ claims $c_i \in C_i = \mathbb{R}_{++}$, over the estate as a strategy, where C_i denotes his strategy set. The set of strategy profiles, or claim (or demand) vectors, is denoted by $C = C_1 \times C_2 \times \cdots \times C_n$.

The payoff structure of Γ is determined by an estate division rule, $F : C \to \mathbb{R}^N_+$. It associates every strategy profile $c \in C$ with an awards vector $F(c) \in \mathbb{R}^N_+$: $\sum_{i=1}^{n} F_i(c) \leq E$ and $\forall i \in N : F_i(c) \leq c_i$, where $F_i(c)$ denotes the amount of the estate the agent i receives under the strategy profile c. If $\sum_{i=1}^{n} c_i \leq E$, then $F_i(c) = c_i$ for every i $\in$ N. Note that the whole estate does not have to be distributed in the case of strict inequality. Until this point, our game coincides with the divide-the-dollar game except that we do not restrict the value of E to 1. The main modification we make is related to the punishment clause. If $\sum_{i=1}^{n} c_i \geq E$, then we treat this situation as a bankruptcy problem (see Ashlagi et al. (2012) for a similar treatment). In this case, the whole estate will be distributed (i.e. $\sum_{i=1}^{n} F_i(c) = E$). Naturally, in this case, F will behave as a bankruptcy rule.[1] Finally, we assume that the number of players, their preferences, the value of the estate, and the estate division rule are all common knowledge among the players.

In the next subsection, we formally define bankruptcy problems and provide details on division rules and axioms applied in such problems.[2]

3.2 Bankruptcy Problems, Division Rules, and Axioms

In a divide-the-dollar game (DD), if the sum of claims (or demands) is larger than 1, then every agent receives zero. In this thesis, in line with the literature on the modifications of the divide-the-dollar game reviewed in Section 2, we change this punishment clause. In particular, even if $\sum_{i=1}^{n} c_i > E$, we still allocate the whole estate, using a bankruptcy rule. Now, we present the definitions of a bankruptcy problem and a bankruptcy rule. First, some notation: in a bankruptcy (or claims) problem, a finite, real-valued estate $E > 0$ has to be

[1] F denotes the division rule, which is defined for all possible claims vectors independent of whether the sum of claims is less than or greater than E. Later, to avoid confusion, we denote a generic bankruptcy rule that will be used when $\sum_{i=1}^{n} c_i \geq E$, with R. Hence, we can say that R is embedded in F.

[2] Axioms that were not defined in earlier work and are introduced in this thesis will appear later in Chapter 4, when the need for them arises.

distributed among a set, N, of agents who have claims over E, where N is taken to be a finite and subset of natural numbers $\mathbb{N}$, generally $\{1, \ldots, n\}$. The claim of an agent $i \in N$ is denoted by $c_i \in \mathbb{R}_+$.

Definition 1 (Bankruptcy Problem) *A bankruptcy problem is a pair* $(c, E) \in \mathbb{R}_+^N \times \mathbb{R}_+$*, where* $c \equiv (c_i)_{i \in N}$ *is the claims vector and* $\sum_{i=1}^{n} c_i \geq E$*. We denote the set of all such problems with* ζ^N [3].

Definition 2 (Bankruptcy Rule) *A bankruptcy rule is a function that associates each bankruptcy problem* $(c, E) \in \zeta^N$ *with an awards vector* $R(c, E) \in \mathbb{R}_+^N$*, such that* $\sum_{i=1}^{n} R_i(c, E) = E$ *and for all* $i \in N$*,* $0 \leq R_i(c, E) \leq c_i$*.*

For simplicity, in the rest of the thesis, the summations across all agents will be denoted by $\sum$, instead of $\sum_{i=1}^{n}$. For instance, $\sum_{i=1}^{n} c_i$ will be denoted by $\sum c_i$. Note that two properties are embedded in this definition. First, any bankruptcy rule satisfies *efficiency* (i.e. $\sum R_i(c, E) = E$). Second, any bankruptcy satisfies *zero lower bound* and *claims boundedness* (i.e. $0 \leq R_i(c, E) \leq c_i$). Hence, we will not explicitly list them in the inventory of axioms. In our proofs, these defining properties of bankruptcy rules will be implicitly used if need be.

3.2.1 Inventory of Bankruptcy Rules

Here, we present the definitions of some prominent bankruptcy rules that also appear in our equilibrium analysis.

The *proportional rule* is possibly the most prominent bankruptcy rule. The idea of proportionality as a criterion for justice dates back to Aristotle.[4] It distributes

[3] Bankruptcy problems were first formally studied in O'Neill (1982). For excellent reviews of this literature, the reader is referred to Moulin (2002), Thomson (2003), Thomson (2015) and Thomson (2019).

[4] In *Nicomachean Ethics*, Aristotle establishes a close connection between justice and proportionality: "... the just is – the proportional; the unjust is what violates the proportion."

the endowment proportionally with respect to claims.

Definition 3 (Proportional Rule (P)) *For each* $(c, E) \in \zeta^N$ *the Proportional Rule distributes the endowment,* E*, as* $P_i(c, E) = \lambda_p c_i$*, where* $\lambda_p = \frac{E}{\sum c_i}$.

The *constrained equal awards rule* distributes the endowment as equally as possible subject to a constraint, which is “no one should receive more than what he claimed."

Definition 4 (Constrained Equal Awards Rule (CEA)) *For each* $(c, E) \in \zeta^N$ *the Constrained Equal Awards Rule distributes the endowment,* E*, as* $CEA_i(c, E) = min\{c_i, \lambda_{cea}\}$ *where* $\lambda_{cea} \in \mathbb{R}_+$ *is such that* $\sum min\{c_i, \lambda_{cea}\} = E$.

The *constrained equal losses rule* distributes the loss (i.e., the discrepancy between the sum of claims and the endowment) as equally as possible subject to a constraint, which is “no one should receive a negative amount."

Definition 5 (Constrained Equal Losses Rule (CEL)) *For each* $(c, E) \in \zeta^N$ *the Constrained Equal Losses Rule distributes the endowment,* E*, as* $CEL_i(c, E) = max\{c_i - \lambda_{cel}, 0\}$ *where* $\lambda_{cel} \in \mathbb{R}_+$ *is such that* $\sum max\{c_i - \lambda_{cel}, 0\} = E$.

The *Talmud rule* (Aumann & Maschler, 1985) applies a *hybrid* method. If the sum of claims is larger than $2E$, it distributes the endowment in a CEA fashion based on half-claims. If the sum of claims is smaller than $2E$, then it distributes the endowment in a CEL fashion based on half-claims.

Definition 6 (Talmud Rule (T)) *For each* $(c, E) \in \zeta^N$ *and* $\forall i \in N$*, the Talmud Rule distributes the endowment,* E, *as*

$$T_i(c, E) = \begin{cases} min\{\frac{c_i}{2}, \lambda_t\}, \text{ if } E \leq \sum \frac{c_j}{2}, \\ c_i - min\{\frac{c_i}{2}, \lambda_t\}, \text{ otherwise,} \end{cases}$$

where in each case, $\lambda_t \in \mathbb{R}_+$ *is such that* $\sum\limits_{i \in N} T_i(c, E) = E$.

3.2.2 Inventory of Axioms

Here, we provide the definitions of the axioms we use in our equilibrium analysis. *Equal treatment of equals* is a very primitive fairness axiom, which stipulates that any two agents with equal claims should receive equal awards.

Definition 7 (Equal Treatment of Equals (ETE)) *For each* $(c, E) \in \zeta^N$ *and all* $i, j \in N$ *such that* $c_i = c_j$, $R_i(c, E) = R_j(c, E)$.

The next two axioms are concerned about the way awards vector reacts to certain types of changes in claims vector.[5] *Proportional-increase-in-claims invariance* (Marchant, 2008) requires that a proportional increase in all claims should not lead to any change in awards.

Definition 8 (Proportional-increase-in-claims invariance (PICI)) *For each* $(c, E) \in \zeta^N$ *and for each* $\alpha > 0$, $R(\alpha c, E) = R(c, E)$.

Uniform-increase-in-claims invariance (Marchant, 2008) requires that a uniform increase in all claims should not lead to any change in awards.

Definition 9 (Uniform-increase-in-claims invariance (UICI)) *For each* (c, E) $\in \zeta^N$ *and for each* $\alpha > 0$, $R(c + \alpha, E) = R(c, E)$.

Claims monotonicity requires that an increase in an agent's claim –*ceteris paribus*– should not make him worse off.

Definition 10 (Claims Monotonicity (CMON)) *For each* $(c, E) \in \zeta^N$*, each* $i \in N$*, and each* $c_i' > c_i$, $R_i((c_i', c_{-i}), E) \geq R_i(c, E)$.

Order preservation of awards (Aumann & Maschler, 1985) requires that the ordering of awards should conform with the ordering of claims.

[5] Marchant (2008) labels these properties as *multiplicative invariance* and *additive invariance 2*. Here, we follow the terminology introduced by Thomson (2019).

Definition 11 (Order Preservation of Awards (OPA)) *For each* $(c, E) \in \zeta^N$ *and all* $i, j \in N$ *such that* $c_i \leq c_j$*,* $R_i(c, E) \leq R_j(c, E)$.

Nonbossiness (Ashlagi et al., 2012) requires that if an agent, by changing his claim, cannot change his own award, then it must be that he cannot change anyone else's award with this change either.[6]

Definition 12 (Nonbossiness (NB)) *For each* $(c, E) \in \zeta^N$*, each* $i \in N$*, and* c_i' *such that* $R_i(c, E) = R_i((c_i', c_{-i}), E)$*,* $R_j(c, E) = R_j((c_i', c_{-i}), E)$ *for all* $j \neq i$.

3.2.3 Kantian Equilibrium

Here, we present a generic definition of the Kantian equilibrium as well as a specific definition using the notation of our bargaining game.

Definition 13 (Kantian Equilibrium (KE), Roemer (2010)) *Consider the normal form game* $G = \langle N, (A_i), (u_i) \rangle$ *in which every player* $i \in N = \{1, 2, \ldots, n\}$ *chooses a strategy from a common strategy set, which is the set of positive real numbers (i.e.,* $\forall i, j \in N : S_i = S_j = \mathbb{R}_{++}$*). A strategy profile* $s = (s_1, s_2, \ldots, s_n)$ *is a Kantian equilibrium of* G *if* $\forall i \in N$*,* $\arg\max_{\alpha \in \mathbb{R}_+} u_i(\alpha s) = 1$.[7]

Definition 14 (Kantian Equilibrium of the Bargaining Game) *In the bargaining game* Γ *with endowment value* E*, a strategy profile (or a claims vector)* c *is*

[6] Nonbossiness was first introduced by Satterthwhite and Sonnenschein (1981) in the context of implementation and social choice theory. The definition we provide here is an adapted version and belongs to Ashlagi et al. (2012).

[7] Note that this definition employs the Kantian reasoning, which is acting in a way that would be preferred by everybody, by considering deviations that change the given strategy profile in a multiplicative fashion. There is an alternative definition of the Kantian equilibrium, which considers additive deviations too (see Roemer (2010) and Roemer (2015); for a continuum of Kantian equilibria and other Kantian variations, see Roemer (2019)). According to that definition, a strategy profile is a Kantian equilibrium if nobody prefers every player to add, or subtract, the same amount to the given strategy profile. We will also present an equilibrium analysis using this alternative definition in Section 5.

a Kantian equilibrium if $\forall i \in N$*,* $\arg\max_{\alpha \in \mathbb{R}_{++}} F_i(\alpha c, E) = 1$ *or alternatively* $\forall i \in N \wedge \forall \alpha > 0$*,* $F_i(c, E) \geq F_i(\alpha c, E)$*.*

In words, a strategy profile c is a Kantian equilibrium of Γ if no agent prefers that every agent change their claims by the same factor $\alpha > 0$.

Note that in some strategic games, for a given Kantian equilibrium strategy profile c, there may be cases in which when all players' actions are scaled by some $\alpha > 0$, some player's payoff decreases while the others' remain constant. However, in the class of bargaining games studied here, as long as the division rule F satisfies efficiency (i.e. when $\sum c_i \geq E$, $\sum F_i(c, E) = E$), a decrease in some player's awards vector ($F_i(c, E) > F_i(\alpha c, E)$ for some $i \in N$) suggests an increase in some other player's awards vector ($\exists j \in N : F_j(c, E) < F_j(\alpha c, E)$) due to the constant-sum nature of the game.[8] This means that the strategy profile c was not a Kantian equilibrium in the first place since there is at least some agent who prefers every agent to change their claims by α. Therefore, such Kantian equilibrium strategy profiles do not exist in the class of bargaining games we study. Then, we can update the definition of the Kantian equilibrium in the following manner: In the bargaining game Γ with endowment value E, a strategy profile (or a claims vector) c is a *Kantian equilibrium* if $\forall i \in N$, $\arg\max_{\alpha \in \mathbb{R}_{++}} F_i(\alpha c, E) = 1$, which implies only that $\forall i \in N \wedge \forall \alpha > 0$, $F_i(c, E) = F_i(\alpha c, E)$.

[8]Note that this statement would not necessarily hold if there exists an inefficient Kantian equilibrium. However, we prove in Lemma 1 that there is no such Kantian equilibrium.

CHAPTER 4

THE RESULTS

We first present a result that simplifies our equilibrium analysis and implies that we should only be concerned with strategy profiles that create a bankruptcy problem.[1]

Lemma 1 *In a bargaining game* Γ*, if a strategy profile* $c \in C$ *is a Kantian equilibrium under the estate division rule* F *and the estate* $E > 0$*, then it cannot be the case that* $\sum c_i < E$*.*

Proof. Let $E > 0$ be some given estate, and F be the estate division rule used in Γ. Suppose for a contradiction that there exists a Kantian equilibrium strategy profile $c \in C$, such that $\sum c_i < E$. Then, $E - \sum c_i > 0$. Note that since $\sum c_i < E$, it follows that (c, E) is not a bankruptcy problem, and thus $F_i(c) = c_i$. Now, consider the strategy profile αc, where $\alpha = \frac{E}{\sum c_i}$. Hence, the pair $(\alpha c, E)$ is a bankruptcy problem since $\sum \alpha c_i = \alpha \sum c_i = E$. Notice that $\forall i \in N : F_i(\alpha c) = \alpha c_i > c_i = F_i(c)$, which implies that every agent $i \in N$ is strictly better off under αc (compared to c). So, there exists some $\alpha > 0$ for c such that every agent prefers every agent to change their claims by α. Thus, c cannot be a Kantian equilibrium. ■

This result is in line with the efficiency of Kantian equilibrium seen in many

[1] It follows from Lemma 1 that we can replace F (denoting an estate division rule) with R (denoting a bankruptcy rule) in our equilibrium analysis.

other examples (see (Roemer, 2019). It is worthwhile mentioning here that the Kantian equilibrium of a standard divide-the-dollar game leads to the same multiplicity problem as the Nash equilibrium does. In particular, any strategy profile that satisfies $\sum c_i = E$ is clearly a Kantian equilibrium of the DD game.

In the following lemma, we show that if a strategy profile c^* is a Kantian equilibrium of Γ under $E > 0$ and R, then any claims vector, which is parallel to c^* and still generates a bankruptcy problem, is also a Kantian equilibrium of Γ.

Lemma 2 *If $c^* \in C$ is a Kantian equilibrium of Γ, then the strategy profile βc^* for any $\beta > 0$ such that $(\beta c^*, E) \in \zeta^N$ is also a Kantian equilibrium of Γ.*

Proof. Suppose $(c^*, E) \in \zeta^N$ for a given estate $E > 0$, and c^* is a Kantian equilibrium of Γ under some bankruptcy rule R. Now, take any $\beta > 0$ such that $(\beta c^*, E) \in \zeta^N$. We would like to show that the strategy profile βc^* is a Kantian equilibrium of Γ under R (i.e. $\forall \sigma > 0 : (\sigma\beta c^*, E) \in \zeta^N, R(\beta c^*, E) = R(\sigma\beta c^*, E)$). Since c^* is a Kantian equilibrium and $\sigma\beta > 0$, $\forall \sigma > 0 : (\sigma\beta c^*, E) \in \zeta^N, R(c^*, E) = R(\beta c^*, E) = R(\sigma\beta c^*, E)$. Thus, if c^* is a Kantian equilibrium, any $\beta > 0$ such that $(\beta c^*, E) \in \zeta^N$ is also a Kantian equilibrium. ■

In the following lemma, we show that if a claims vector $\bar{c}$ is not a Kantian equilibrium of Γ under some estate $E > 0$ and bankruptcy rule R, then any claims vector which is parallel to $\bar{c}$ and still generates a bankruptcy problem is not a Kantian equilibrium of Γ either.

Lemma 3 *If $\bar{c} \in C$ is not a Kantian equilibrium of Γ, then $\beta\bar{c}$, for any $\beta > 0$ such that $(\beta\bar{c}, E) \in \zeta^N$, is not a Kantian equilibrium of Γ either.*

Proof. Suppose that $(\bar{c}, E) \in \zeta^N$ for some estate E and bankruptcy rule R, $\bar{c}$ is not a Kantian equilibrium. Then, $\exists \alpha > 0 : R(\alpha\bar{c}, E) \neq R(\bar{c}, E)$, where $(\alpha\bar{c}, E) \in \zeta^N$. Now, pick any $\beta > 0$ such that $(\beta\bar{c}, E) \in \zeta^N$ and $\beta \neq \alpha$. We would like to show that $\beta\bar{c}$ is not a Kantian equilibrium of Γ under R either. Suppose to the contrary that the strategy profile $\beta\bar{c}$ is a Kantian equilibrium. Then, $\forall\delta > 0 : (\delta\beta\bar{c}, E) \in \zeta^N, R(\beta\bar{c}, E) = R(\delta\beta\bar{c}, E)$. Let $\delta_1 = 1/\beta$ and $\delta_2 = \alpha/\beta$. This implies that $R(\beta\bar{c}, E) = R(\bar{c}, E)$ and $R(\beta\bar{c}, E) = R(\alpha\bar{c}, E)$. But then, $R(\bar{c}, E) = R(\alpha\bar{c}, E)$, which is a contradiction. So, $\beta\bar{c}$ is not a Kantian equilibrium. Hence, the result follows. ■

Lemma 1 and Lemma 3 together directly imply the following more general result.

Corollary 1 *If $\bar{c} \in C$ is not a Kantian equilibrium of Γ, then $\beta\bar{c}$, for any $\beta > 0$, is not a Kantian equilibrium of Γ either.*

Proof. The proof directly follows from Lemma 1 and Lemma 3. ■

The following lemma shows that there is a tight relationship between the proportional rule and the proportional-increase-in-claims-invariance property. This relationship will be instrumental in our equilibrium analysis.

Lemma 4 *A bankruptcy rule R satisfies proportional-increase-in-claims invariance (PICI) if and only if $R(c, E) = P(c, E)$ for all $(c, E) \in \zeta^N$.*

Proof. First, we show that $PICI \implies P$. Pick any bankruptcy rule R, which satisfies $PICI$. So, for all $\alpha > 0$, $R(\alpha c, E) = R(c, E)$, for any bankruptcy problem (c, E). Now, we would like to show that $\forall i \in N$: $R_i(c, E) = \lambda c_i$ where $\lambda = \frac{E}{\sum c_i}$. From the definition of the bankruptcy problem, we have $\sum c_i \geq E$. Let α^* be the value which gives $\alpha^* \sum c_i = E$. Then, for the claims vector $\alpha^* c$, everyone gets what they claim (i.e. $\forall i \in N$: $R_i(\alpha^* c, E) = \alpha^* ci = \frac{E}{\sum c_i} ci$). By $PICI$, for $\alpha^* > 0$, $R(\alpha^* c, E) = R(c, E)$. So, $\forall i \in N : R_i(c, E) = \frac{E}{\sum c_i} c_i$.

Second, we show that $P \implies PICI$. Pick any $(c, E) \in \zeta^N$. Then, $\forall i \in N$: $R_i(c, E) = \frac{E}{\sum c_i} ci$. Pick any $\alpha > 0$ such that $(\alpha c, E) \in \zeta^N$. Then, $\forall i \in N$: $R_i(\alpha c, E) = \frac{E}{\sum \alpha c_i} \alpha c_i = \frac{E}{\alpha \sum c_i} \alpha c_i = \frac{E}{\sum c_i} c_i = R_i(c, E)$. Thus, $\forall \alpha > 0 : (\alpha c, E)$

$\in \zeta^N$, $R(\alpha c, E) = R(c, E)$. ■

Now, we present one of our main results. Proposition 1 shows that if the proportional rule is used in Γ, then any strategy profile that creates a bankruptcy problem is a Kantian equilibrium.

Proposition 1 *Let $R = P$ if $(c, E) \in \zeta^N$ in Γ. Then, every strategy profile $c \in C$ such that $(c, E) \in \zeta^N$ is a Kantian equilibrium.*

Proof. By Lemma 4, the proportional rule is characterized by $PICI$. So, for each $(c, E) \in \zeta^N$ and for each $\alpha > 0$ such that $(\alpha c, E) \in \zeta^N$, $R(\alpha c, E) = R(c, E)$. Thus, every strategy profile, which creates a bankruptcy problem is a Kantian equilibrium. ■

This result highlights a more serious multiplicity issue than the original one in the *DD* game. In that game, every strategy profile c such that $\sum c_i = E$ is a Nash equilibrium. Here in Γ, due to the nature of the Kantian equilibrium, even those strategy profiles for which $\sum c_i > E$ are Kantian equilibria. This is somewhat surprising given the success of the proportional rule in solving the multiplicity issue in the modified DD game in Ashlagi et al. (2012). These authors show that, under the proportional rule, there exists a unique Nash equilibrium, in which equal division prevails. We show that there are infinitely many Kantian equilibria, and any division of the estate can be supported in equilibrium. These contrasting results highlight the important differences between equilibrium concepts and the differential impact of *institutions* (i.e., bankruptcy rules) in our game. In particular, the Nash equilibrium deals with unilateral deviations, and as such the strong claims monotonicity property satisfied by the proportional rule plays an important role in bringing the unique Nash equilibrium with an equal division. Also of critical importance is the fact that the agents' strategy sets in Ashlagi et al. (2012) are bounded from above. On the other hand, the Kantian equilibrium deals with nonunilateral deviations.

In particular, when considering a deviation, an agent asks the question, "If everyone else also deviates in the same way, would I be better off?" Hence, a property like the claims monotonicity, which allows a change in only one agent's claim, is useless in studying the Kantian Equilibirum of Γ. Instead, a property like the proportional-increase-in-claims invariance is needed, which characterizes the proportional rule alone. It is the proximity (or the alignment) between the nature of Kantian deviations and the proportional-increase-in-claims invariance that leads to the 'anything goes' result here.

Does Γ always admit a Kantian equilibrium? In Proposition 2, we show that the existence is guaranteed under a primitive fairness assumption on R.

Proposition 2 *[Generic Existence Result] Let R be a bankruptcy rule. If R satisfies equal treatment of equals (ETE), then Γ has a Kantian equilibrium.*

Proof. Suppose that a bankruptcy rule R satisfies *ETE*. For a given estate $E > 0$, pick any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$. Clearly, any such (c, E) is a bankruptcy problem since $\sum c_i \geq E$. From the generic efficiency of bankruptcy rules, we have $\sum R_i(c, E) = E$. By *ETE*, since $\forall i, j \in N : c_i = c_j$, $\forall i, j \in N : R_i(c, E) = R_j(c, E)$. Then, $\forall i \in N : R_i(c, E) = \frac{E}{N}$. Now, for any $\alpha > 0$ such that $(\alpha c, E) \in \zeta^N$, $\forall i, j \in N : R_i(\alpha c, E) = R_j(\alpha c, E) = \frac{E}{N}$ by *ETE*. So, there does not exist any α value, which makes someone better off. Thus, any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a Kantian equilibrium. ■

This result shows that the existence of equal-division equilibrium in Γ is also guaranteed under *ETE*. CEA, CEL, and T are some bankruptcy rules that fall in the large family of rules given in this proposition, and these rules have Kantian equilibria *only* in the form described in the generic existence result above.[2]

[2]The Kantian equilibria of these three prominent bankruptcy rules (*CEA, CEL* and *T*) are solved explicitly in the Appendix. See the examples from Example 4 to Example 6 for the corresponding results.

It is also worthwhile mentioning that Proposition 2 provides a sufficient condition for the existence.[3] The following example shows that *ETE* is not a necessary condition.

Example 1 *Consider a game with four players.* $E = 120$*, and the bankruptcy rule* R *distributes* E *as follows:* R *mimics* P *at every claims vector except* $c' = (c_1, c_2, c_3, c_4) = (50, 40, 30, 30)$ *to which it assigns the awards vector,* $(50, 40, 30, 0)$*. Hence, it clearly violates ETE. Pick a strategy profile* $c'' \in \zeta^N$ *which is not parallel to* c' *(i.e.* $\nexists \alpha > 0 : \alpha c' = c''$*). Then,* R *behaves like* P *for* c'' *and any strategy profile parallel to it, while addressing* $(c'', 120)$*. It is easy to show that* c'' *is a Kantian equilibrium. Hence, the result follows.*

The following proposition shows that equal-division Kantian equilibrium cannot be induced by certain types of strategy profiles.

Proposition 3 *Let* R *be a bankruptcy rule. A strategy profile* $c \in C$ *such that not all claims are equal to one another (i.e., there exist at least two agents whose claims are different from each other) cannot induce an equal-division in a Kantian equilibrium of* Γ *under* R*.*

Proof. For a given estate $E > 0$, pick a strategy profile c such that $\exists i, j \in N :$ $c_i \neq c_j$ but still $\forall i, j \in N : R_i(c, E) = R_j(c, E)$. Suppose for a contradiction that this strategy profile is a Kantian equilibrium. Without loss of generality, assume that $c_i < c_j$ and c_i is a minimal claim (i.e. $c_i \in \min_{k \in N} c_k$). Then, $R_i(c, E) = \frac{E}{N} \leq c_i < c_j$. Note that the other possible cases violate claims boundedness. This suggests that $\sum c_i > E$. Since $c_j > c_i$, we have $c_i = \min_{k \in N} c_k < \frac{\sum c_k}{N}$. Multiplying both sides with E and changing the sides of c_i and $\sum c_k$ guarantees

[3] Sher (2020) shows (Proposition 2) that the Kantian equilibrium does not exist in two person zero-sum games and argues that (in Footnote 8) the result can be generalized to n person constant-sum games. Note that the equal treatment of equals property we assume rules out the assumption the author makes to show nonexistence. In Example 7 in the Appendix, we show that the violations of *ETE* can indeed result in nonexistence of Kantian equilibrium.

the existence of some $\alpha > 0$ such that $\frac{E}{\sum c_k} < \alpha < \frac{E}{Nc_i}$. Then, $(\alpha c, E) \in \zeta^N$ because $\frac{E}{\sum c_k} < \alpha$ implies that $E < \alpha \sum c_k \iff E < \sum \alpha c_k$. Besides, $\alpha c_i < \frac{E}{N}$. So, $R_i(\alpha c, E) < \alpha c_i < E/N = R_i(c, E)$. The loss in the payoff of agent i implies that at least one player should get a higher payoff under $R(\alpha c, E)$. So, at least one player prefers everyone to change their claims by factor α. Therefore, such a strategy profile c cannot be a Kantian equilibrium. ■

Is there any *other* Kantian Equilibrim strategy profile (different than the one in the generic existence result) if R satisfies *ETE*? From Proposition 1, we know that under P, any strategy profile that generates a bankruptcy problem is a Kantian equilibrium. Since P satisfies *ETE*, the answer to the question above is affirmative. Then, is there any other Kantian equilibria if we restrict our attention to the set of bankruptcy rules, other than the proportional rule, which satisfies *ETE*? In particular, can we have a Kantian equilibrium strategy profile that induces an unequal division in Γ, if R satisfies *ETE*? The following example shows that the answer is, again, affirmative.

Example 2 *[KE with an unequal division under ETE] Consider the claims problem with $N = 3$ and $E = 90$. The division rule R^* distributes E in the following fashion: If a strategy profile c, given that $(c, E) \in \zeta^N$, is equal to $c^* = (30, 40, 50)$ or it is parallel to c^* (i.e. $\exists \alpha > 0 : \alpha c = c^*$; R^*) behaves like P. For any other strategy profile, it behaves like CEA. This rule satisfies ETE. Hence, under R^*, Γ has a Kantian equilibrium with equal division (from Proposition 2). Moreover, it is clearly different from P. In addition to this, c^* and any c which is parallel to c^* are Kantian equilibria as well. Obviously, they are not equal-division equilibria.*[4]

Example 2 shows that there can be Kantian equilibria that induce an unequal division even if we restrict our attention to the set of bankruptcy rules, other than the proportional rule, which satisfies *ETE*. Inspired by this observation,

[4]Note from Example 4 that there are no other equilibria under CEA.

we now go further than simply excluding the proportional rule, and define a property that completely rules out proportional divisions under any unequal claims vector.

Definition 15 (No Proportionality for Unequal Claims Vectors (NPUC)) *For any* $(c, E) \in \zeta^N$*, a division rule* R *satisfies no proportionality for unequal claims vectors if for any unequal claims vector* c*,* $\exists i \in N : R_i(c, E) \neq P_i(c, E)$*.*

Note that for a bankruptcy rule, R, to be different than P, it is enough to have one bankruptcy problem in which the awards vectors of R and P do not coincide. Hence, if R satisfies *NPUC*, then $R \neq P$; but not vice versa. The next proposition shows that there is no Kantian equilibrium that induces an unequal division, under this strong property.

Proposition 4 *Let* R *be a bankruptcy rule that satisfies ETE and NPUC. Under* R*,* Γ *has no Kantian equilibrium other than the ones described in the generic existence result.*

Proof. Suppose that R satisfies *ETE* and *NPUC*. By Proposition 2, any claims vector c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a KE. So, all equal claims vectors are Kantian equilibria. Take any unequal claims vector c. If R assigns equal division as the awards vector, by Proposition 3, such a c cannot be a Kantian equilibrium. Then, the only remaining possibility for c to be a Kantian equilibrium is that R assigns an awards vector, which is different from equal division. By *NPUC*, $\exists i \in N : R_i(c, E) > P_i(c, E)$. This suggests that $\exists i \in N : R_i(c, E) > \frac{E}{\sum c_i} c_i \iff \frac{R_i(c,E)}{c_i} > \frac{E}{\sum c_i}$. Then, $\exists \alpha > 0 : \frac{R_i(c,E)}{c_i} > \alpha > \frac{E}{\sum c_i} \iff \alpha c_i < R_i(c, E) \wedge \alpha \sum c_i = \sum \alpha c_i > E$. Thus, $(\alpha c, E) \in \zeta^N$. Since $R_i(\alpha c, E) \leq \alpha c_i < R_i(c, E)$, the agent i is worse off under the claims vector αc. Then, there must be another agent who gets more under the strategy profile αc. So, there is at least one agent who prefers every player to change his claim by α. Thus, the strategy profile c is not a Kantian equilibrium, and there is no strategy profile inducing Kantian equilibrium other than the ones described in the generic existence result. ■

As we mentioned earlier, *NPUC* is a strong property. Many non-proportional bankruptcy rules fail to satisfy it since there exists at least one claims vector at which the awards vector they assign coincides with that of the proportional rule. A well-known example is the Talmud rule. When the sum of claims is equal to $2E$, the awards vector assigned by the Talmud rule coincides with that of the proportional rule. Hence, a natural question is: Can *NPUC* be weakened, yet the same result in Proposition 4 still holds? To answer this question, we first present the next property, which is a weakening of *NPUC*.

Definition 16 (Weak No Proportionality for Unequal Claims Vectors) (WNPUC) *For any* $(c, E) \in \zeta^N$*, a division rule* R *satisfies weak no proportionality for unequal claims vectors if for any unequal claims vector* c*,* $\exists i \in N \wedge \exists \alpha^* > 0 : R_i(\alpha^* c, E) \neq P_i(\alpha^* c, E)$ *where* $(\alpha^* c, E) \in \zeta^N$.

WNPUC holds if, for any bankruptcy problem, there exists a claims vector parallel to the original one and this claims vector still generates bankruptcy, and an agent whose award in the new bankruptcy problem is different from what the proportional rule gives him. As such, it is much weaker than *NPUC*. For example, the Talmud Rule, the Reverse Talmud Rule (Chun, Schummer, & Thomson, 2001), all interior members of the TAL-family (Moreno-Ternero & Villar, 2006), and the Reverse-TAL-family of rules (van den Brink, Funaki, & van der Laan, 2013) satisfy *WNPUC* but fail to satisfy *NPUC*. The next proposition shows that the result in Proposition 4 still follows if we replace *NPUC* with *WNPUC*.

Proposition 5 *Let* R *be a bankruptcy rule that satisfies ETE and WNPUC. Under* R*,* Γ *has no Kantian equilibrium other than the ones described in the generic existence result.*

Proof. Suppose that R satisfies *ETE* and *WNPUC*. By Proposition 2, any claims vector c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a KE. So, all equal claims vectors are KE. Now, take any unequal claims vector c. If R assigns equal division as the awards vector, by Proposition 3, such a c cannot be a Kantian equilibrium. Then, only remaining possibility for c to be a Kantian equilibrium is that R assigns an awards vector, which is different from equal division. Suppose to the contrary that c is a Kantian equilibrium. By *WPNUC*, for c, $\exists i \in N \wedge \exists \alpha^* > 0 :$ $R_i(\alpha^* c, E) > P_i(\alpha^* c, E)$ where $(\alpha^* c, E) \in \zeta^N$. Then, by Lemma 2, any $\beta > 0$ such that $(\beta c, E) \in \zeta^N$ is also a Kantian equilibrium. So, $\alpha^* c$ is also a KE (i.e. $\forall \sigma > 0 : (\sigma\alpha^* c, E) \in \zeta^N, R(\alpha^* c, E) = R(\sigma\alpha^* c, E))$. This implies that for $\sigma = \frac{1}{\alpha^*}$, $R_i(\alpha^* c, E) = R_i(c, E) > P_i(\alpha^* c, E) = P_i(c, E) = \frac{Ec_i}{\sum c_i}$. So, $\frac{R_i(c,E)}{c_i} > \frac{E}{\sum c_i}$. Then, $\exists \beta > 0 : \frac{R_i(c,E)}{c_i} > \beta > \frac{E}{\sum c_i} \iff \beta c_i < R_i(c, E) \wedge \beta \sum c_i = \sum \beta c_i > E$. Thus, $(\beta c, E) \in \zeta^N$, and the payoff of agent i is smaller due to the fact that $R_i(\beta c, E) \leq \beta c_i < R_i(c, E)$. Since one agent experiences a loss in her payoff, there must be another agent who gets more under the claims vector βc. So, at least one player prefers everyone to change their claims by factor β, and c cannot be a Kantian equilibrium. We have a contradiction: c is both a Kantian equilibrium and not a Kantian equilibrium. Thus, any strategy profile c that does not induce equal division as the awards vector is not a Kantian equilibrium. ■

Note that we have two extreme cases: On one hand, we have the bankruptcy rules satisfying *ETE* and *NPUC* without any unequal claims vector as KE; on the other hand, we have the proportional rule in which every claims vector (inducing bankruptcy) is a Kantian equilibrium. There are also bankruptcy rules satisfying only *ETE* without having Kantian equilibria that induce equal division. However, we also know from Example 2 that there can be bankruptcy rules which satisfy *ETE* with an unequal claims vector as their Kantian equilibrium, given that the rule behaves like R^*. Now, the question is the following: Is it possible to construct a transition between R^* and P? For this purpose, we first would like to generalize the case in Example 2.

Lemma 5 *Let R be a bankruptcy rule, and (c^1, E) be a bankruptcy problem where $c^1 \in C$ is such that not all claims are equal to one another. For any $(c, E) \in \zeta^N$, R distributes E in the following way: If the strategy profile $c \in C$ is equal to c^1 or it is parallel to c^1 (i.e. $\exists \alpha > 0 : \alpha c = c^1$), then $R(c, E) = P(c, E)$. For any other strategy profile, R allocates the estate in a way that satisfies NPUC and ETE. Under R, Γ has some set of strategy profiles which are Kantian equilibria with unequal division. Particularly, this set only involves the strategy profile c^1 and the claims vectors parallel to it.*

Proof. Since R either behaves like the proportional rule or like any division rule which satisfies *NPUC* and *ETE*, R satisfies *ETE* for any $(c, E) \in \zeta^N$. So, by Proposition 2, any strategy profile c such that $c_1 = c_2 = \cdots = c_n \geq \frac{E}{N}$ is a Kantian equilibrium. Take any strategy profile $c \in C$ which is not equal to c^1 and not parallel to c^1. Then, R behaves like a division rule which satisfies *NPUC* and *ETE*. By Proposition 4, all the possible Kantian equilibria of Γ have already been mentioned. So, c cannot be a Kantian equilibrium. Now, take any strategy profile c', which is equal to c^1 or parallel to c^1. Then, $R(c', E) = P(c', E)$ and $\forall \alpha > 0 :$ $(\alpha c', E) \in \zeta^N, R(\alpha c', E) = P(\alpha c', E)$. As we have seen explicitly in Proposition 1, $P(\alpha c', E) = P(c', E) \forall \alpha > 0$ where $(\alpha c', E)$. So, $R(c', E) = R(\alpha c', E)$ and c' is a Kantian equilibrium. Since the strategy profile c' is either equal or parallel to c^1, and c^1 is a strategy profile such that not all claims are equal to one another, Γ has a set of strategy profiles which are Kantian equilibria with an unequal division. ■

We would like to extend the set of strategy profiles which induces unequal-division Kantian equilibria. To do that, we take any $c^2 \in C$ such that c^1 and c^2 are linearly independent. These strategy profiles are the ones that are not parallel to c^1. Then, we update the bankruptcy rule R as follows: R behaves like the proportional rule when the strategy profile creating the bankruptcy problem is in the span of c^1 and c^2, $span(\{c^1, c^2\})$. The following lemma formalizes this idea.

Lemma 6 *Let R be a bankruptcy rule, and (c^1, E) and (c^2, E) be bankruptcy problems where $c^1, c^2 \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R} : c^1 = \lambda c^2$). For any $(c, E) \in \zeta^N$, R distributes E as follows: If the strategy profile $c \in C$ is in the $span(\{c^1, c^2\})$, then $R(c, E) = P(c, E)$, where $span(\{c^1, c^2\}) = \sum_{i=1}^{2} \lambda_i c^i$ such that $\forall i \in \{1, 2\} : c^i \in \{c^1, c^2\}$ and $\lambda_i \in \mathbb{R}$. For any other strategy profile, R allocates the estate in a way that satisfies NPUC and ETE. Under R, in addition to the ones described in the generic existence result (Proposition 2), the (unequal) strategy profiles in the $span(\{c^1, c^2\})$ are also Kantian equilibria of Γ.*

Proof. Since R either behaves like the proportional rule or like any division rule which satisfies *NPUC* and *ETE*, R satisfies *ETE* for any $(c, E) \in \zeta^N$. So, by Proposition 2, any strategy profile c such that $c_1 = c_2 = \cdots = c_n \geq \frac{E}{N}$ is a Kantian equilibrium. Take any strategy profile $c \in C$ which is not in the $span(\{c^1, c^2\})$. Then, R behaves like any division rule that satisfies *NPUC* and *ETE*. By Proposition 4, all the possible Kantian equilibria of Γ have already been mentioned. So, c cannot be a Kantian equilibrium. Now, take any strategy profile $c' \in span(\{c^1, c^2\})$ and suppose that c' is such that not all claims are equal to one another. Then, $R(c', E) = P(c', E)$. Since $c' \in span(\{c^1, c^2\})$, there exists $\lambda_1^*, \lambda_2^* \in \mathbb{R}_+$ such that $c' = \lambda_1^* c^1 + \lambda_2^* c^2$. For any strategy profile c'' which is parallel to c' (i.e., $\forall \alpha > 0 : (\alpha c', E) \in \zeta^N \wedge c'' = \alpha c'$), we have $c'' = \alpha c' = \alpha(\lambda_1^* c^1 + \lambda_2^* c^2) = \alpha\lambda_1^* c^1 + \alpha\lambda_2^* c^2$. So, $c'' = \lambda_1' c^1 + \alpha\lambda_2' c^2$ where $\lambda_1' = \alpha\lambda_1^*$ and $\lambda_2' = \alpha\lambda_2^*$. This suggests that $c'' \in span(\{c^1, c^2\})$ as well. Then, $R(c'', E) = P(c'', E) = P(c', E) = R(c', E)$. So, $\forall \alpha > 0 : (\alpha c', E) \in \zeta^N, R(\alpha c', E) = R(c', E)$, and c' is a Kantian equilibrium. Thus, any strategy profile which is in the $span(\{c^1, c^2\})$ is a Kantian equilibrium, and Γ has a set of strategy profiles which are Kantian equilibria with unequal division. ■

We can expand the set of strategy profiles which induce unequal division Kantian equilibria by extending the $span(\{c^1, c^2\})$. To do that, we can pick any strategy profile $c^3 \notin span(\{c^1, c^2\})$ and consider the span of c^1, c^2, and c^3. This ensures that c^1, c^2, and c^3 are linearly independent.[5] The next lemma investigates the strategy profiles, inducing unequal division Kantian equilibria under $span(\{c^1, c^2, c^3\})$.

Lemma 7 *Let R be a bankruptcy rule and $(c^1, E), (c^2, E)$, and (c^3, E) be bankruptcy problems where $c^1, c^2, c^3 \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R}^3 - \{0\} : \lambda_1 c^1 + \lambda_2 c^2 + \lambda_3 c^3 = 0$). For any $(c, E) \in \zeta^N$, R distributes the estate E in the following way: If the strategy profile $c \in C$ is in the $span(\{c^1, c^2, c^3\}), R(c, E) = P(c, E)$, where $span(\{c^1, c^2, c^3\}) = \sum_{i=1}^{3} \lambda_i c^i$ such that $\forall i \in \{1, 2, 3\} : c^i \in \{c^1, c^2, c^3\}$ and $\lambda_i \in \mathbb{R}$. For any other strategy profile, R allocates the estate as any division rule which satisfies NPUC and ETE. Under R, besides the ones described in the generic existence result, the unequal strategy profiles in the $span(\{c^1, c^2, c^3\})$ are also Kantian equilibria of Γ.*

Proof. Similar to the proof of Lemma 6. ■

We can iterate this process of adding new strategy profiles to the span $-$such that the existing claims vectors and the recently added ones are linearly independent$-$ and modifying the bankruptcy rule R to behave as the proportional rule for the new span. Since n linearly independent vectors span $\mathbb{R}^n$, R will behave like the proportional rule for all the strategy profiles creating a bankruptcy problem when we have n linearly independent vectors $\{c^1, c^2, \ldots, c^n\}$.

Proposition 6 *Let R be a bankruptcy rule and $(c^1, E), (c^2, E), \ldots, (c^n, E)$ be bankruptcy problems where $c^1, c^2, \ldots, c^n \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R}^n - \{0\} : \lambda_1 c^1 + \lambda_2 c^2 + \cdots + \lambda_n c^n = 0$). For any $(c, E) \in \zeta^N$, R distributes the estate E as follows: If the strategy profile $c \in C$ is in the $span(\{c^1, c^2, \ldots, c^n\})$,*

[5] Since the proportional rule is the only rule under which we have an *anything goes* result, this is the only method which can achieve this purpose.

then $R(c, E) = P(c, E)$*, where* $span(\{c^1, c^2, \ldots, c^n\}) = \sum_{i=1}^{n} \lambda_i c^i$ *such that* $\forall i \in \{1, 2, \ldots, n\} : c^i \in \{c^1, c^2, \ldots, c^n\}$ *and* $\lambda_i \in \mathbb{R}$*. Then, it must be that* $R = P$*.*

Proof. This result mainly depends on the theorem/proposition which states that if n vectors $c^1, c^2, \ldots, c^n \in \mathbb{R}^N$ are linearly independent, these set of vectors span $\mathbb{R}^N$, (i.e. $\mathbb{R}^N = span(\{c^1, c^2, \ldots, c^n\})$). Note that any set of independent vectors can always be extended to be a basis (i.e., independent spanning set) of the vector space. $\mathbb{R}^N$ has dimension n which means that any basis can have no more than n elements (in fact a basis has exactly n elements). Now, if the linearly independent set of n vectors did not span $\mathbb{R}^N$, then we would have a basis consisting of more than n elements by extending the set of linearly independent vectors. This is contradictory. So, given that $c^1, c^2, \ldots, c^n \in \mathbb{R}^N$ are linearly independent, $span(\{c^1, c^2, \ldots, c^n\}) = \mathbb{R}^N$. Since R distributes the estate E as the proportional rule for any $(c, E) \in \zeta^N : c \in span(\{c^1, c^2, \ldots, c^n\}) = \mathbb{R}^N, R(c, E) = P(c, E)$ for any $(c, E) \in \zeta^N$. ■

This result is important in that it shows how we can span the whole space of Kantian equilibrium divisions from 'equal-division only' in one extreme to 'anything goes' in the other by varying the properties of the bankruptcy rule[6]. In particular, if the rule satisfies *ETE*, and is almost nowhere proportional, then there can only be equal-division Kantian equilibria. On the other hand, if the rule is proportional everywhere (i.e., it is the proportional rule), then any division can be sustained in Kantian equilibrium (e.g., anything goes). In between these two extremes, there are bankruptcy rules that we constructed above, which induce only some unequal divisions but not others. By varying the domain on which a rule can induce unequal-division equilibria, we can go from one extreme to the other.

[6]The set of results marking the transition between R^* and P are established so that the bankruptcy rule at hand satisfies *NPUC*. It is also possible to implement the same set of results with any bankruptcy rule which satisfies *WNPUC*. This is shown explicitly in the Appendix.

CHAPTER 5

THE ADDITIVE DEFINITION OF KANTIAN EQUILIBRIUM

Roemer (2015) provides an alternative definition of the Kantian equilibrium, in which deviations are defined in an *additive* fashion. A natural question is: what happens to the equilibrium behavior and the axiomatic properties of the bankruptcy rules we need to use to induce various types of equilibrium divisions (in particular the equal division) under this alternative definition? In this section, we briefly address these questions. We start by giving the alternative definition of the Kantian equilibrium with additive deviations. To avoid confusion, we label this one as Kantian* equilibrium.

Definition 17 (Kantian Equilibrium – The Additive Version) *In the bargaining game* Γ *with endowment value* E*, a strategy profile (or a claims vector)* c *is a Kantian* equilibrium if* $\forall i \in N \wedge \forall \alpha \in \mathbb{R} : (c + \alpha) \in C$*,* $F_i(c, E) \geq F_i(c + \alpha, E)$.

Lemma 8 *In a bargaining game* Γ*, if a strategy profile* $c \in C$ *is a Kantian* equilibrium under the estate division rule* F *and* $E > 0$*, then it cannot be the case that* $\sum c_i < E$.

Proof. The proof is similar to that of Lemma 1, and as such omitted. ■

This suggests that while agents are engaging with Kantian reasoning in an additive fashion, they should only be considering the strategy profiles which create a bankruptcy problem. In other words, given a strategy profile c, the agents should compare their awards vectors only for the strategy profiles $c + \alpha$ where $\alpha \in \mathbb{R} : (c + \alpha, E) \in \zeta^N$.

Proposition 7 *Let R be a bankruptcy rule. If R satisfies equal treatment of equals (ETE), then Γ has a Kantian* equilibrium.*

Proof. The proof is similar to that of Proposition 2, and as such omitted. ■

The following lemma shows that there is a tight relationship between the constrained equal losses rule and the uniform-increase-in-claims invariance (UICI) property. This relationship will be instrumental in the corresponding equilibrium analysis.

Lemma 9 *A bankruptcy rule R satisfies uniform-increase-in-claims invariance (UICI) if and only if $R(c, E) = CEL(c, E)$ for all $(c, E) \in \zeta^N$.*

Proof. See Marchant (2008) for the proof of this result.[1] ■

It is easy to see that the proportional rule, which we characterized above with *PICI* (see Lemma 4) fails to satisfy *UICI* since $\frac{E}{\sum c_i} c_i$ is clearly not invariant under uniform changes in claims. The following proposition shows that if the constrained equal losses rule is used in Γ, then any strategy profile that creates a bankruptcy problem is a Kantian* equilibrium.

Proposition 8 *Let $R = CEL$ if $(c, E) \in \zeta^N$ in Γ. Then, every strategy profile $c \in C$ such that $(c, E) \in \zeta^N$ is a Kantian* equilibrium.*

Proof. The proof is similar to that of Proposition 1. By Lemma 9, we know that *CEL* is characterized by *UICI*. Thus, for all $(c, E) \in \zeta^N$ and α such that

[1] Marchant (2008) characterizes *CEL* with *UICI* and *Null Claims Consistency*. Since we do not allow the population to vary, and our strategy space does not include the zero vector, we can drop *Null Claims Consistency* here.

$(c+\alpha, E) \in \zeta^N$, $CEL(c, E) = CEL(c+\alpha, E)$. Therefore, every strategy profile, which creates a bankruptcy problem is a Kantian* equilibrium.[2] ■

This result and Proposition 1 (also Theorem 1-3 in Ashlagi et al. (2012)) highlight the importance of the relationship between the equilibrium concept and the bankruptcy rule (or the axioms it satisfies) used in the game. The following proposition shows that if the proportional rule is used in Γ, then we obtain Kantian* equilibria that are identical to the ones in Proposition 2.

Proposition 9 *Let $R = P$ if $(c, E) \in \zeta^N$ in Γ. Then, in every Kantian* equilibrium of Γ, equal division prevails.*

Proof. Let $c^* \in C$ be a strategy profile in which every agent claims kE for some $k > 0$. From Lemma 1, we know that $k \geq \frac{1}{n}$. Hence, we can start with a strategy profile in which every agent claims $\frac{E}{n}$. In this case, a uniform increase in all claims by α does not lead to any change in payoffs since $\frac{\frac{E+\alpha n}{n}}{E+\alpha n}E = \frac{E}{n}$. Thus, any strategy profile $c^* \in C$ in which (i) all agents claim the same amount and (ii) the sum of claims is greater than or equal to E is a Kantian* equilibrium of Γ. Clearly, starting with $\frac{E}{n}$, a uniform decrease in all claims by α is not preferred by any agent. Finally, starting with a strategy profile in which (i) not all claims are equal and (ii) the sum of claims is greater than or equal to E, a uniform change in claims makes some agents better off and some others worse off. Hence, such strategy profiles cannot be Kantian* equilibria. ■

[2] Note that *UICI* employs the definition "for each $(c, E) \in \zeta^N$ and for each $\alpha > 0$, $R(c+\alpha, E) = R(c, E)$" while Kantian* equilibrium is defined by "$\forall \alpha \in \mathbb{R} : (c+\alpha) \in C$, $R_i(c, E) \geq R_i(c+\alpha, E)$." So, it seems as if there is a difference between the two concepts in terms of the possible strategy profiles to which any agent can deviate. For an agent to consider the counterfactual scenario with a negative α value, the sum of claims needs to be strictly greater than the estate value at that particular strategy profile c. Then, there will be some negative real number β which would make the sum of claims equal to the estate under $c - \beta$. Clearly, the set of claims vectors which any agent can potentially deviate to under $c - \beta$ includes the corresponding set under the strategy profile c if we consider only the deviations in which each agent adds some positive amount. However, due to *UICI*, the awards vector of c and $c - \beta$ are equal. So, not only $c - \beta$ being a Kantian* equilibrium implies c being a Kantian* equilibrium, but also if c is a Kantian* equilibrium, $c - \beta$ is a Kantian* equilibrium. Thus, two ways of defining the possible set of deviations are equivalent when the bankruptcy rule satisfies *UICI*.

Is there any *other* Kantian* equilibrium of Γ when R satisfies *ETE*? *CEL* satisfies *ETE*, and from Proposition 8, we know that when $R = CEL$, any strategy profile that creates a bankruptcy problem is a Kantian* equilibrium; so, the answer to the question above is affirmative. Is there any other Kantian* equilibrium if we restrict our attention to the set of bankruptcy rules, other than *CEL*, which satisfies *ETE*? In particular, can we have a Kantian* equilibrium strategy profile that induces an unequal division? The next example shows that the answer is affirmative again.

Example 3 (Kantian* equilibrium inducing unequal division under ETE) *Consider the bankruptcy problem with* $N = 3$ *and* $E = 90$. *The bankruptcy rule* R^+ *distributes* E *in the following fashion: If a strategy profile* c *is equal to* $c^+ = (32, 40, 48)$ *or it is equal to* $c^+ + \alpha$ *for some* $\alpha \in \mathbb{R} : (c^+ + \alpha, E) \in \zeta^N$*, then* R^+ *behaves like CEL. For any other strategy profile, it behaves like P. This rule satisfies ETE since both CEL and P satisfy ETE. Furthermore,* R^+ *is obviously different from CEL. From Proposition 7,* Γ *has a Kantian* equilibrium with equal division under* R^+*, which coincides with the generic Kantian equilibria of* Γ.

Now, we would like to show that c^+ *is a Kantian* equilibrium. Note that the shortfall is* 30 *(i.e.* $\sum c_i^+ - E = 30$*). For any* $\alpha \in \mathbb{R} : (c^+ + \alpha, E) \in \zeta^N$*, the shortfall is equal to* $\sum (c_i^+ + \alpha) - E = \sum c_i^+ + 3\alpha - E = 30 + 3\alpha$*. Then, in strategy profiles* c^+ *and* $c^+ + \alpha$*, after the equal distribution of the shortfall per person, the corresponding awards vectors are* $c^+ - 10$ *and* $c^+ + \alpha - (10 + \alpha)$*. So, in both cases we have* $c^+ - 10$*, and adding* α *does not have any effect on how CEL distributes* E*. Thus, for any* $\alpha \in \mathbb{R} : (c^+ + \alpha, E) \in \zeta^N$*, we have* $CEL(c^+, E) = CEL(c^+ + \alpha, E)$*, and* c^+ *is a Kantian* equilibrium inducing unequal division, particularly* $(24, 30, 36)$.

In what follows, we define a property on how R treats unequal claims vectors, to eliminate the sort of situations in this example. It stipulates that for every unequal claims vector, there must be some agent who receives more under R than what he would receive under *CEL*.

Definition 18 (No Equal Losses for Unequal Claims (NELUC)) *For any* $(c, E) \in \zeta^N$*, a bankruptcy rule* R *satisfies NELUC if for any unequal claims vector* $c, \exists i \in N : R_i(c, E) > c_i - \frac{\sum c_i - E}{N}$.

Note that *CEA* and *P* both satisfy *NELUC*. Any rule defined as a convex combination of *CEA*, *CEL*, or *P* also satisfies it. In the following proposition, we show that assuming *NELUC* rules out all the Kantian* equilibria other than the ones in the generic existence result.

Proposition 10 *Let* R *be a bankruptcy rule which satisfies ETE and NELUC. Under* R*,* Γ *has no Kantian* equilibrium other than the ones described under the corresponding generic existence result.*

Proof. Suppose that R satisfies *ETE* and *NELUC*. By Proposition 7, any strategy profile c such that $c_1 = c_2 = \cdots = c_n \geq \frac{E}{N}$ is a Kantian* equilibrium. So, all equal claims vectors are Kantian* equilibria. Take any unequal claims vector c. By *NELUC*, $\exists i \in N : \frac{\sum c_i - E}{n} > c_i - R_i(c, E)$. This suggests that for the agent $i, \exists \alpha \in \mathbb{R} : \frac{\sum c_i - E}{n} > \alpha > c_i - R_i(c, E) \iff \frac{E - \sum c_i}{n} < -\alpha \wedge c_i - \alpha < R_i(c, E)$. Then, since $\frac{E - \sum c_i}{n} < -\alpha$ implies that $\sum c_i - n\alpha > E, (c - \alpha, E) \in \zeta^N$. Besides, it follows from $c_i - \alpha < R_i(c, E)$ that $R_i(c - \alpha, E) \leq c_i - \alpha < R_i(c, E)$ by *claims boundedness*. Then, the agent i is worse off under the strategy profile $c - \alpha$. So, there must exist another agent who receives more under the strategy profile $c - \alpha$, compared to c, and there must exist someone who prefers every player to change her claim by $-\alpha$ in additive fashion. Thus, the strategy profile c is not a Kantian* equilibrium, and there is no strategy profile inducing Kantian* equilibrium other than the ones described in the generic existence result. ■

CHAPTER 6

CONCLUSION

In this thesis, we studied the Kantian equilibria of a bargaining game, which is a modified version of the well-known divide-the-dollar game. We first showed that the Kantian equilibrium of this game exists under a fairly weak assumption (i.e., equal treatment of equals) on the bankruptcy rule used in the game. Interestingly, there are stark differences between the axiomatic properties of bankruptcy rules, which induce equal division of the dollar under the Nash equilibrium and the Kantian equilibrium. For instance, the proportional rule, which induced equal division in a unique Nash equilibrium, leads to infinitely many Kantian equilibria. This result highlights the importance of institutions and the incentives they provide in driving individual behavior; and shows that Kantian behavior (as operationalized in the Kantian equilibrium) does not necessarily lead to egalitarian outcomes. Furthermore, we offered a partial characterization of the family of bankruptcy rules, which induces (i) equal division in Kantian equilibrium and (ii) an anything goes result. Finally, we provided a novel method to construct hybrid bankruptcy rules that can induce any subset of the space of efficient divisions in Kantian equilibrium.

The literature on Kantian equilibrium and its applications almost exclusively focused on public goods and tragedy of commons type games. However, the joint interest in reaching an agreement, one of the defining characteristics of any bargaining game, makes the study of Kantian behavior in bargaining and/or estate division games a promising venue. To the best of our knowledge, this is the first study to incorporate the Kantian equilibrium into a bargaining game and study its distributive implications. As such, it contributes to the literature on the Kantian equilibrium. Due to the modified divide-the-dollar game it studies and the questions it answers on the set of equal-division equilibria, it also contributes to the Nash program, in a novel fashion, by using an equilibrium concept other than the Nash equilibrium. In fact, to the best of our knowledge, it is the first study that provides support for the Nash bargaining solution using neither Nash equilibrium nor any of its refinements. Finally, since the game we study can be see as a bankruptcy game, it contributes to the literature on the strategic modeling of bankruptcy situations.

Lastly, a few words on potential venues of research on the topic are in order. We focused on a simple, static bargaining game in this thesis. Future research may study Kantian equilibrium in dynamic bargaining games with more complicated strategic interaction, and investigate whether it has the potential to offer new insights on negotiation behavior. We assumed that all agents are Kantian. Future research may study bargaining/distribution games where the society is composed of both Kantians and Nashians and/or there is uncertainty about the player types. Such models may also set the stage for further study of the evolution of norms in bargaining/estate division games with mixed populations (see Laslier (2020) for an example in coordination games).

REFERENCES

Abreu, D., & Pearce, D. (2015). A dynamic reinterpretation of Nash bargaining with endogenous threats. *Econometrica*, *83*, 1641–1655.

Alger, I., & Laslier, J.-F. (2020). *Homo moralis goes to the voting booth: coordination and information aggregation.* (Available at SSRN: https://ssrn.com/abstract=3756263)

Alger, I., & Weibull, J. W. (2013). Homo moralis - preference evolution under incomplete information and assortative matching. *Econometrica*, *81*, 2269–2302.

Alger, I., & Weibull, J. W. (2016). Evolution and Kantian morality. *Games and Economic Behaviour*, *98*, 56–67.

Alger, I., & Weibull, J. W. (2017). Strategic behavior of moralists and altruists. *Games*, *8*, 38.

Allison, H. E. (2011). *Kant's groundwork for the metaphysics of morals: A commentary*. Oxford University Press.

Anbarcı, N. (2001). Divide-the-dollar game revisited. *Theory and Decision*, *50*, 295–304.

Andreozzi, L. (2010). An evolutionary theory of social justice: Choosing the right game. *European Journal of Political Economy*, *26*, 320–329.

Ashlagi, I., Karagözoğlu, E., & Klaus, B. (2012). A non-cooperative support for equal division in estate division problems. *Mathematical Social Sciences*, *63*, 228–233.

Atlamaz, M., Berden, C., Peters, H., & Vermeulen, D. (2011). Non-cooperative solutions to claims problems. *Games and Economic Behaviour*, *73*, 39–51.

Aumann, R. J., & Maschler, M. (1985). Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory*, *36*, 295–313.

Bezin, E., & Ponthière, G. (2019). The tragedy of the commons and socialization: Theory and policy. *Journal of Environmental Economics and Management*, *98*, 102260.

Binmore, K. (1998). *Game theory and the social contract, vol ii*. Cambridge: MIT Press.

Bolton, G. (1997). The rationality of splitting equally. *Journal of Economic Behavior Organization*, *32*(3), 365-381. Retrieved from https://EconPapers.repec.org/RePEc:eee:jeborg:v:32:y:1997:i:3:p:365-381

Braham, M., & van Hees, M. (2020). Kantian Kantian Optimization. *Erasmus Journal for Philosophy and Economics*, *13*(2), 30–42. Retrieved from https://ejpe.org/journal/article/view/513

Brams, S. J., & Taylor, A. D. (1994). Divide the dollar: Three solutions and extensions. *Theory and Decision*, *37*, 211–231.

Cetemen, E., & Karagözoğlu, E. (2014). Implementing equal division with an ultimatum threat. *Theory and Decision*, *77*(2), 223-236.

Chang, C., & Hu, C.-C. (2008). A non-cooperative interpretation of the f-just rules of bankruptcy problems. *Games and Economic Behavior*, *63*(1), 133-144. Retrieved from https://www.sciencedirect.com/science/article/pii/S0899825607001613

Chun, Y. (1989). A noncooperative justification for egalitarian surplus sharing. *Mathematical Social Sciences*, *17*(3), 245-261. Retrieved from https://www.sciencedirect.com/science/article/pii/0165489689900553

Chun, Y., Schummer, J., & Thomson, W. (2001). Constrained egalitarianism: A new solution for claims problems. *Seoul Journal of Economics*, *14*, 269-297.

Darwall, S. (1998). *Philosophical ethics: An historical and contemporary introduction*. Westview Press.

Eichner, T., & Pethig, R. (2019). *Kant-Nash tax competition* (Tech. Rep.). (CESifo Working Paper Series: https://ideas.repec.org/p/ces/ceswps/_7571.html)

Ghosh, A., & Van Long, N. (2015). *Kant's rule of behavior and Kant-Nash equilibria in games of contributions to public goods* (Tech. Rep.). (CESifo Area Conference on Public Sector Economics)

Grafton, R. Q., Kompas, T., & Van Long, N. (2017). A brave new world? Kantian–Nashian interaction and the dynamics of global climate change mitigation. *European Economic Review*, *99*, 31-42. Retrieved from https://www.sciencedirect.com/science/article/pii/S0014292117300727

Hagiwara, M., & Hanato, S. (2021). A strategic justification of the constrained equal awards rule through a procedurally fair multilateral bargaining game. *Theory and Decision*, *90*(2), 233-243.

Herissone-Kelly, P. (2018). *Kant on maxims and moral motivation: A new interpretation*. Springer Verlag.

Kant, I., & Korsgaard, C. M. (1998). *Kant: Groundwork of the metaphysics of morals* (M. Gregor, Ed.). Cambridge University Press.

Karagözoğlu, E. (2014). A noncooperative approach to bankruptcy problems with an endogenous estate. *Annals of Operations Research*, *217*, 299-318.

Karagözoğlu, E., & Rachmilevitch, S. (2018). Implementing egalitarianism in a class of Nash demand games. *Theory and Decision*, *85*, 495–508.

Karagözoğlu, E., & Riedl, A. (2015). Performance information, production uncertainty, and subjective entitlements in bargaining. *Management Science*, *61*, 2611–2626.

Kilgour, M. (2003). Game models of negotiation and arbitration. *Encyclopedia of Life Support Systems*. (EOLSS Publishers)

Kıbrıs, , & Kıbrıs, A. (2013). On the investment implications of bankruptcy laws.

Games and Economic Behaviour, *80*, 85–99.

Laffont, J.-J. (1975). Macroeconomic constraints, economic efficiency, and ethics: An introduction to Kantian economics. *Economica*, *42*, 430–437.

Laslier, J.-F. (2020). Do Kantians drive others to extinction? *Erasmus Journal for Philosophy and Economics*, *13*, 98–108.

Marchant, T. (2008). Scale invariance and similar invariance conditions for bankruptcy problems. *Social Choice and Welfare*, *31*, 693–707.

Moreno-Ternero, J. D., & Villar, A. (2006). The TAL-family of rules for bankruptcy problems. *Social Choice and Welfare*, *27*, 231–249.

Moulin, H. (1986). *Game theory for the social science*. New York: New York University Press.

Moulin, H. (2002). Chapter 6: Axiomatic cost and surplus sharing. In *Handbook of social choice and welfare* (Vol. 1, p. 289-357). Elsevier. Retrieved from https://www.sciencedirect.com/science/article/pii/S1574011002800108

Nash, J. F. (1950). The bargaining problem. *Econometrica*, *18*, 155–162.

Nash, J. F. (1953). Two person cooperative games. *Econometrica*, *21*, 128–140.

Nozick, R. (1974). *Anarchy, state, and utopia*. New York: Basic Books.

Nydegger, R. V., & Owen, G. (1975). Two person bargaining: An experimental test of Nash axioms. *International Journal of Game Theory*, *3*, 239–249.

O'Neill, B. (1982). A problem of rights arbitration from the Talmud. *Mathematical Social Sciences*, *2*, 345–371.

Ostrom, E. (1990). *Governing the commons: the evolution of institutions for collective action*. Cambridge: Cambridge University Press.

Planas, L. C. (2018). Moving toward greener societies: Moral motivation and green behaviour. *Environmental and Resource Economics*, *70*, 835–860.

Rachmilevitch, S. (2017). Punishing greediness in divide-the-dollar games. *Theory and Decision*, *82*, 341–351.

Rachmilevitch, S. (2020a). An implementation of the Nash bargaining solution by iterated strict dominance. *Economics Letters*, *188*, 108960.

Rachmilevitch, S. (2020b). Rewarding moderate behavior in a dynamic Nash demand game. *International Journal of Game Theory*, *49*, 639–650.

Rasmusen, E. B. (2019). *Back to bargaining basics: A simple 50-50 model for splitting a pie.* (Available at SSRN: https://ssrn.com/abstract=3362093)

Rawls, J. (1971). *A Theory of Justice*. Cambridge, MA: Harvard University Press.

Roemer, J. E. (2010). Kantian equilibrium. *Scandinavian Journal of Economics*, *112*, 1–24.

Roemer, J. E. (2015). Kantian optimization: A microfoundation for cooperation. *Journal of Public Economics*, *125*, 45–57.

Roemer, J. E. (2019). *How We Cooperate: A Theory of Kantian Optimization*. New Haven, CT: Yale University Press.

Roth, A. E., & Malouf, M. W. (1979). Game theoretic models and the role of information in bargaining. *Psychological Review*, *86*, 574–594.

Satterthwhite, M. A., & Sonnenschein, H. (1981). Strategy-proof allocation mechanisms at differentiable points. *Review of Economic Studies*, *48*, 587–597.

Schelling, T. C. (1960). *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

Serrano, R. (2021). Sixty-seven years of the Nash program: time for retirement? *SERIEs*, *12*, 35–48.

Sher, I. (2020). Normative aspects of Kantian equilibrium. *Erasmus Journal for Philosophy and Economics*, *13*, 43–84.

Skyrms, B. (1996). *Evolution of the Social Contract*. Cambridge: Cambridge University Press.

Studtmann, P., & Suresh, S. G. (2021). Universalizing and the we: endogenous game theoretic deontology. *Economics and Philosophy*, *37*(2), 244–259.

Thomson, W. (2003). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey. *Mathematical Social Sciences*, *45*, 249–297.

Thomson, W. (2015). Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: An update. *Mathematical Social Sciences*, *74*, 41–59.

Thomson, W. (2019). *How to Divide when there isn't Enough: from Aristotle, the Talmud, and Maimonides to the Axiomatics of Resource Allocation*. Cambridge University Press.

van den Brink, R., Funaki, Y., & van der Laan, G. (2013). Characterization of the Reverse Talmud bankruptcy rule by Exemption and Exclusion properties. *European Journal of Operational Research*, *228*(2), 413-417.

Van Huyck, J., Battalio, R., Mathur, S., Van Huyck, P., & Ortmann, A. (1995). On the origin of convention: Evidence from symmetric bargaining games. *International Journal of Game Theory*, *24*, 187–212.

Van Long, N. (2017). Mixed-strategy kant-nash equilibrium and private contributions to a public good. In (p. 107-126). (10.1007/978-3-319-49442-5_6)

White, M. D. (2011). *Kantian Ethics and Economics*. Stanford: California: Stanford University Press.

White, M. D. (2019). With All Due Respect: A Kantian Approach to Economics. In *The oxford handbook of ethics and economics* (p. 107-126).

Young, P. H. (1993). An evolutionary model of bargaining. *Journal of Economic Theory*, *59*, 145–168.

APPENDIX

We solve for the Kantian equilibria of Γ under the three prominent bankruptcy rules we defined in Section 3.2.1: CEA, CEL, and T.

Example 4 (Constrained Equal Awards Rule) *For each $(c, E) \in \zeta^N$, a way to calculate the constrained equal awards vector of the given bankruptcy problem is to begin with equal division and adjust the awards vector if an agent's award exceeds her claim. Pick any bankruptcy problem $(c, E) \in \zeta^N$. If c is such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}, (c, E) \in \zeta^N$ since $\sum c_i \geq E$. Note that CEA satisfies ETE. Hence, $\forall i, j \in N : CEA_i(c, E) = CEA_j(c, E)$ and $\forall i \in N : CEA_i(c, E) = \frac{E}{N}$. So, everyone receives equal division at such a c since every agent's awards will be less than or equal to her claim. For any $\alpha > 0 : (\alpha c, E) \in \zeta^N$, the strategy profile αc also exhibits $\alpha c_1 = \alpha c_2 = \cdots = \alpha c_N \geq \frac{E}{N}$. Again, by ETE, $\forall i, j \in N$: $CEA_i(\alpha c, E) = CEA_j(\alpha c, E) = \frac{E}{N}$. So, no agent prefers that every agent change their claims by the same factor $\alpha > 0$. Thus, any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a Kantian equilibrium of Γ under CEA.*

The other case is that c is a strategy profile in which not everyone claims the same amount (i.e. $\exists i \in N : \forall l \in N, c_i \leq c_l$ and $\exists j \in N : \forall l \in N, c_j \geq c_l$ where $c_i < c_j$). If $c_i < \frac{E}{N}, CEA_i(c, E) = c_i$ and $CEA_j(c, E) > \frac{E}{N}$. Then, the agent i, whose claim is a minimal claim among all claims, has an incentive to deviate to some $\alpha > 1$ such that $\alpha c_i > \frac{E}{N}$. Clearly, the strategy profile $(\alpha c, E) \in \zeta^N$. Now, under the strategy profile αc, we have $\forall l \in N : c_l > \frac{E}{N}$. Since CEA starts with giving everyone *equal division and every agent's claim is greater than equal division, there does not exist any other way to distribute the loss. So,* $\forall l \in N : CEA_l(\alpha c, E) = \frac{E}{N} > c_i = CEA_i(c, E)$*. Hence, the agent* i *prefers everyone to change their claims by the factor* $\alpha > 1$*. Therefore, a strategy profile* c *in which the agent with a minimal claim declares a claim less than the equal division cannot be a Kantian equilibrium.*

Next, consider the strategy profile in which the agent i *has a claim greater than the equal division (i.e.* $c_i \geq \frac{E}{N}$*, and* $\forall l \in N : c_l \geq c_i$*). We have argued that every agent receives equal division under that case. Then, the agent* j*, whose claim is a maximal claim among all claims, has an incentive to pick some* $\beta < 1$ *such that* $\beta c_i < \frac{E}{N}$*. By choosing such a* β*, he aims to maintain* $(\beta c, E) \in \zeta^N$ *and keep his claim,* c_j*, above equal division; while bringing the agent* i*'s claim to a level which is lower than the equal division. In other words, agent* j *seeks a* $\beta < 1$ *such that* $\sum_{l \in N} \beta c_l \geq E, \beta c_j > \frac{E}{N}$ *and* $\beta c_i < \frac{E}{N}$*. Note that since* $c_j \geq c_l$ *for any* $l \in N$*, and* $c_j > c_i, \beta c_j \leq \frac{E}{N}$ *implies that* $(\beta c, E) \notin \zeta^N$*. So, for such a* β*, we need* $\sum_{l \in N} \beta c_l \geq E$ *and* $\beta c_i < \frac{E}{N}$*. By the definition of* c_i*, we have* $c_i < \frac{\sum c_l}{N}$*. Then, by multiplying both sides with* E*, we have* $E c_i < E \frac{\sum c_l}{N}$*, which is equivalent to* $\frac{E}{\sum c_l} < \frac{E}{N c_i}$*. Notice that any* β *within this interval (i.e.* $\frac{E}{\sum c_l} < \beta < \frac{E}{N c_i}$*) ensures that* $(\beta c, E) \in \zeta^N$ *and under the claims problem* $(\beta c, E), \beta c_i < \frac{E}{N} = CEA_i(c, E)$*. Now, since* $\beta c_i < \frac{E}{N}, CEA_i(\beta c, E) = \beta c_i < \frac{E}{N} = CEA_i(c, E)$*. So, under the claims problem* $(\beta c, E)$*, there will be a loss to distribute among the other agents. The agent* j *has still a claim* βc_j*, which is greater than the equal division. This suggests that the award of agent j is now greater than the equal division thanks to the loss of the agent* i*. So, the agent* j *prefers everyone to change their claims by the factor* $\beta < 1$*. Therefore, a strategy profile* c *in which the agent with a minimal claim declares a claim greater than or equal to the equal division cannot be a Kantian equilibrium.*

We can conclude that any strategy profile in which at least two agents have different claims is not a Kantian equilibrium under CEA. Hence, in any Kantian equilibrium strategy profile of Γ *under CEA, every agent declares the same claim.*

In Ashlagi et al. (2012), under *CEA*, any strategy profile where every agent claims an amount greater than or equal to the average estate is a Nash equilibrium. Notice here that in Kantian equilibria as well, all claims are greater than equal to the average estate but strategy profiles with unequal claims are not Kantian equilibria.

Example 5 (Constrained Equal Losses Rule) *For each $(c, E) \in \zeta^N$, a way to calculate the constrained equal losses vector of a given bankruptcy problem is to begin with each agent obtaining their claim and distribute the shortfall, $\sum c_l - E$, equally. If, as a result, any agent receives a negative amount, give him zero, and then distribute the remaining amount equally among rest of the agents who have not received negative amounts. Iterate this process until every agent receives a nonnegative award and all the loss is shared.*

Pick any bankruptcy problem $(c, E) \in \zeta^N$. If $c \in C$ is such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}, (c, E) \in \zeta^N$ since $\sum c_i \geq E$. Note that CEL satisfies ETE. Hence, $\forall i, j \in N : CEL_i(c, E) = CEL_j(c, E)$ and $\forall i \in N : CEL_i(c, E) = \frac{E}{N}$. So, everyone receives an equal award at such a c since every agent contributes to the shortfall equally and the shortfall is distributed equally. For any $\alpha > 0 : (\alpha c, E) \in \zeta^N$, the strategy profile αc also exhibits $\alpha c_1 = \alpha c_2 = \cdots = \alpha c_N \geq \frac{E}{N}$. Again, by ETE, $\forall i, j \in N : CEL_i(\alpha c, E) = CEL_j(\alpha c, E) = \frac{E}{N}$. So, no agent prefers that every agent change their claims by the same factor $\alpha > 0$. Thus, any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a Kantian equilibrium of Γ under CEL.

Consider the other case that c is a strategy profile in which every agent does not claim the same amount (i.e. $\exists i \in N : \forall l \in N c_i \leq c_l$ and $\exists j \in N : \forall l \in N c_j \geq c_l$, where $c_i < c_j$). Then, the agent j is always among the set of agents whose claims are greater than the average of the claims, and the agent i is always among the set of agents whose claims are less than the average of the claims.

We have two subcases. One is the case where $CEL_i(c, E) > 0$. This suggests *that* $c_i > \frac{\sum c_l - E}{N}$. *Then, player* j *has an incentive to pick an* $\alpha > 1$ *to increase the shortfall* $\sum \alpha c_l - E$ *at a level so that the difference between the claim of agent* i *and the loss per agent is smaller under* $(\alpha c, E)$ *compared to* (c, E). *In other words,* $\alpha > 1$ *should be such that* $\alpha c_i - (\frac{\sum \alpha c_l - E}{N}) < c_i - (\frac{\sum c_l - E}{N})$. *Then,* $\alpha c_i - (\frac{\sum \alpha c_l - E}{N}) < c_i - (\frac{\sum c_l - E}{N}) \iff c_i(\alpha - 1) < \frac{(\alpha-1)\sum c_l}{N} \iff c_i < \frac{\sum c_l}{N}$. *Recall that the inequality* $c_i < \frac{\sum c_l}{N}$ *is ensured by the hypothesis. So,* $\exists \alpha > 1 : 0 \leq CEL_i(\alpha c, E) = \alpha c_i - (\frac{\sum \alpha c_l - E}{N}) < c_i - (\frac{\sum c_l - E}{N}) = CEL_i(c, E)$. *If we pick an* $\alpha > 1 : CEL_i(\alpha c, E) > 0,$ *the difference* $CEL_i(c, E) - CEL_i(\alpha c, E)$ *will be distributed to the rest of the players whose claims are greater than* αc_i, *under* $(\alpha c, E)$. *So, player* j *will surely experience an increase in his awards vector* $CEL_j(\alpha c, E)$, *compared to* $CEL_j(c, E)$. *This suggests that player* j *prefers that everyone change their claims by the same factor* α. *Therefore, any strategy profile in which the agent with a minimal claim receives some positive award under Constrained Equal Losses is not a KE.*

For the second, consider the case where $CEL_i(c, E) = 0$. *Without loss of generality, suppose that for any player* k *whose claim* c_k *is greater than* c_i, $CEL_k \neq 0$. *This suggests that* $c_i \leq \frac{\sum c_l - E}{N}$. *Then, player* i *has an incentive to pick an* $\beta < 1 : (\beta c, E) \in \zeta^N$ *to decrease the shortfall* $\sum \beta c_l - E$ *at such a level that the difference between the claim of agent* j *and the loss per agent is larger under* $(\beta c, E)$ *compared to* (c, E). *In other words,* $\beta < 1$ *should be such that* $\beta c_j - (\frac{\sum \beta c_l - E}{N}) < c_j - (\frac{\sum c_l - E}{N})$. *Then,* $\beta c_j - (\frac{\sum \beta c_l - E}{N}) < c_j - (\frac{\sum c_l - E}{N}) \iff c_j(\beta - 1) < \frac{(\beta-1)\sum c_l}{N} \iff c_j > \frac{\sum c_l}{N}$*(since* $\beta < 1$). *Recall that the inequality* $c_j > \frac{\sum c_l}{N}$ *is ensured by the hypothesis. So,* $\exists \beta < 1 : (\beta c, E) \in \zeta^N$ *and* $CEL_j(\beta c, E) = \beta c_j - (\frac{\sum \beta c_l - E}{N}) < c_j - (\frac{\sum c_l - E}{N}) = CEL_j(c, E)$. *The difference* $CEL_j(c, E) - CEL_j(\beta c, E)$ *will be distributed to the rest of the players whose claims are less than* βc_j *under* $(\beta c, E)$. *So, player* j *will surely experience a decrease in his awards vector* $CEL_j(\beta c, E)$, *compared to* $CEL_j(c, E)$. *This suggests that there will be some players who prefer everyone to change their claims by the factor* β. *If* $CEL_i(c, E) = 0$, *then* $CEL_i(\beta c, E) > CEL_i(c, E)$. *However, If* $CEL_i(c, E) < 0$, $\beta < 1$ *needs to be small enough for* *player i to also have $CEL_i(\beta c, E) > 0$. Therefore, any strategy profile in which the agent with a minimal claim receives 0 as his award under Constrained Equal Losses is not a Kantian equilibrium.*

Example 6 (Talmud Rule) *Without loss of generality, suppose that $c_1 \leq c_2 \leq \cdots \leq c_n$. Thomson (2019) gives an algorithm to get the awards vector $T(c, E)$ for each $(c, E) \in \zeta^N$. Initially, we distribute the units of E equally until each player gets $c_1/2$. Then, the agent 1 ceases to get any part of E, and the next units of E will be distributed equally among other players until all of them get $c_2/2$. Then, the agent 2 ceases to get any part of E, and the next units will be distributed equally among the other $N-2$ players. We continue in this manner until $E = \sum \frac{c_i}{2}$. If $E > \sum \frac{c_i}{2}$, the next units will be entirely assigned to the agent n until her loss is reduced to $\frac{c_{n-1}}{2}$. If there is still more to allocate, the next units of E will be allocated among the agents n and $n-1$ until both their losses are equal to $\frac{c_{n-2}}{2}$. We proceed by this until E is used up completely.*

Pick any bankruptcy problem $(c, E) \in \zeta^N$. If c is such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$, $(c, E) \in \zeta^N$ since $\sum c_i \geq E$. Note that T satisfies ETE. Hence, $\forall i, j \in N : T_i(c, E) = T_j(c, E)$ and $\forall i \in N : T_i(c, E) = \frac{E}{N}$. So, everyone receives equal division at such a c since every agent's half claim is equal to each other's. For any $\alpha > 0 :$ $(\alpha c, E) \in \zeta^N$, the strategy profile αc also exhibits $\alpha c_1 = \alpha c_2 = \cdots = \alpha c_N \geq \frac{E}{N}$. Again, by ETE, $\forall i, j \in N : T_i(\alpha c, E) = T_j(\alpha c, E) = \frac{E}{N}$. So, no agent prefers that every agent change their claims by the same factor $\alpha > 0$. Thus, any claims vector c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a Kantian equilibrium of Γ under T.

Next, take any strategy profile $c \in C$ such that not all claims are equal to one another. Then, without loss of generality, assume that $\exists i \in N : \forall l \in N c_i \leq c_l$ and $\exists j \in N : \forall l \in N c_j \geq c_l$ where $c_i < c_j$. We have two subcases. The first is when $\frac{c_i}{2} < \frac{E}{N}$. This implies that $T_i(c, E) < \frac{E}{N}$. Then, player i has an incentive to pick an $\alpha > 1$ such that $\frac{\alpha c_i}{2} \geq \frac{E}{N}$. Under $(\alpha c, E)$, the estate E would not be enough to distribute $\frac{\alpha c_i}{2}$ to every agent and everyone would receive equal division. The reason *is that the Talmud algorithm initially tries to divide the estate equally so that every agent receives* $\frac{c_i}{2}$. *Since* $\frac{\alpha c_i}{2} \geq \frac{E}{N}$, *this step leads every agent to get equal division* $\frac{E}{N}$. *So, for some* $\alpha > 1$ *such that* $\alpha \geq \frac{2E}{Nc_i}$, $T_i(\alpha c, E) = \frac{E}{N} > T_i(c, E)$. *Thus, the agent* i *prefers everyone to change their claims by the factor* α, *and such a strategy profile* c *is not a Kantian equilibrium.*

The second is when the strategy profile c *is such that* $\frac{c_i}{2} \geq \frac{E}{N}$. *We have argued above that under such a strategy profile, every agent receives equal division. Now, the agents other than* i *have an incentive to pick some* $\beta < 1 : \frac{\beta c_i}{2} < \frac{E}{N}$, *so that the agent* i *would receive less than equal division while they could receive higher than equal division. Then,* β *should be such that* $(\beta c, E) \in \zeta^N$*(i.e.* $\beta \sum c_l \geq E$) *and* $\frac{\beta c_i}{2} < \frac{E}{N}$. *This implies that* $\frac{E}{\sum c_l} \leq \beta < \frac{2E}{Nc_i}$. *Since* $E < 2E$ *and* $\sum c_l > Nc_i$, *it always holds that* $\frac{E}{\sum c_l} < \frac{2E}{Nc_i}$ *for the strategy profile* c. *This inequality ensures the existence of some* β *which both creates a bankruptcy problem and incentivizes some agent other than* i *to deviate from the strategy profile* c. *So, under* $(\beta c, E), T_i(c, E) = \frac{\beta c_i}{2} < \frac{E}{N} = T_i(c, E)$. *Even if* E *is so large that the loss equalization process of Talmud extends to involve agent* i, *there would be some agent other than* i *who receives more than equal division, like the case under the constrained equal losses rule. So, there exists at least some agent who prefers everyone to change their claims by the factor* β. *Thus, such a strategy profile* c *is not a Kantian equilibrium; any strategy profile such that not all claims are equal to one another is not a Kantian equilibrium.*

The following example shows that Kantian equilibrium may exist even when *ETE* is violated.

Example 7 (Violation of ETE and nonexistence of Kantian equilibrium)
Consider the bankruptcy rule R *defined as follows: For any claims vector* c *in which all the claims are equal, i.e.* $\forall i, j \in N : c_i = c_j$ *where* $i \neq j$, R *distributes* E *in the following manner: (a) If* $\sum c_i = E$, *then* $R_i(c, E) = R_j(c, E) = \frac{E}{n}$ *for all* $i, j \in N$. *(b) If* $\sum c_i > 2E$, *then* R *behaves as a priority-based rule, and distributes* E *in a*

way that the priority ordering is from lower index to higher index. That is, first, it assigns agent 1 his full claim (if feasible). If there is any amount left, then agent 2 is assigned an award (again, his full claim if feasible or the residual), and so on. (c) If $E < \sum c_i \leq 2E$, then R behaves as a priority-based rule, and distributes E in a way that the priority ordering is from higher index to lower index. That is, first, it assigns agent n his full claim (if feasible). If there is any amount left, then agent $n-1$ is assigned an award (again, his full claim if feasible or the residual), and so on. (d) For any other strategy profile, i.e. where all claims are not equal to one another, R behaves like CEA.

This rule clearly violates ETE. Now, we would like to show that R does not have any Kantian equilibrium. From Example 4, we know that any unequal claims vector is not a Kantian equilibrium. Take any strategy profile c in which all claims are equal. If $\sum c_i = E$, then $c_i = c_j = \frac{E}{n} \forall i, j \in N$. In that case, at least the first agent, agent 1, and the last agent, agent n, have incentives to pick $\alpha > 1$. For instance, under the strategy profile $3c$, agent 1 is better-off. So, any strategy profile which makes the sum of claims equal to E is not a Kantian equilibrium. If $\sum c_i > 2E$, like the preceding instance, then agent 1 receives more than equal division and agent n receives 0 since for any $j \in N, c_j > \frac{2E}{n}$. However, the agent n has an incentive to deviate to some $\alpha < 1 : E < \alpha \sum c_i \leq 2E$ so that he receives some positive awards under the strategy profile αc. So, any strategy profile such that $\sum c_i > 2E$ is not a Kantian equilibrium. Lastly, consider the case where $E < \sum c_i \leq 2E$. Then, agent 1 receives less than the equal division. By deviating to a large enough $\alpha > 1$, i.e. any $\alpha > 1 : \alpha c_1 > E$, agent 1 can get the estate E completely. So, there exists some α which would make agent 1 better off under the strategy profile αc; and any c such that $E < \sum c_i \leq 2E$ is not a Kantian equilibrium. Thus, R violates ETE and does not have any Kantian equilibrium strategy profile.

Next, we provide the set of results which shows how we can span the whole space of Kantian equilibrium divisions from 'equal-division only' in one extreme to 'anything goes' in the other by requiring the bankruptcy rule to satisfy *WNPUC*. Notice that what follows will be a reproduction of the corresponding results in the main text using *WNPUC* instead of *NPUC*.

Lemma 10 *Let R be a bankruptcy rule, and (c^1, E) be a bankruptcy problem where $c^1 \in C$ is such that not all claims are equal to one another. For any $(c, E) \in \zeta^N$, R distributes E in the following way: If the strategy profile $c \in C$ is equal to c^1 or it is parallel to c^1 (i.e. $\exists \alpha > 0 : \alpha c = c^1$), then $R(c, E) = P(c, E)$. For any other strategy profile, R allocates the estate in a way that satisfies WNPUC and ETE. Under R, Γ has some set of strategy profiles which are Kantian equilibria with unequal division. Particularly, this set only involves the strategy profile c^1 and the claims vectors parallel to it.*

Proof. Since R either behaves like the proportional rule or like any division rule which satisfies *WNPUC* and *ETE*, R satisfies *ETE* for any $(c, E) \in \zeta^N$. So, by Proposition 2, any strategy profile c such that $c_1 = c_2 = \cdots = c_n \geq \frac{E}{N}$ is a Kantian equilibrium. Take any strategy profile $c \in C$ which is not equal to c^1 and not parallel to c^1. Then, R behaves like a division rule which satisfies *WNPUC* and *ETE*. By Proposition 5, all the possible Kantian equilibria of Γ have already been mentioned. So, c cannot be a Kantian equilibrium. Now, take any strategy profile c', which is equal to c^1 or parallel to c^1. Then, $R(c', E) = P(c', E)$ and $\forall \alpha > 0 :$ $(\alpha c', E) \in \zeta^N, R(\alpha c', E) = P(\alpha c', E)$. As we have seen explicitly in Proposition 1, $P(\alpha c', E) = P(c', E) \forall \alpha > 0$ where $(\alpha c', E)$. So, $R(c', E) = R(\alpha c', E)$ and c' is a Kantian equilibrium. Since the strategy profile c' is either equal or parallel to c^1, and c^1 is a strategy profile such that not all claims are equal to one another, Γ has a set of strategy profiles which are Kantian equilibria with an unequal division. ■

We would like to extend the set of strategy profiles which induces unequal-division Kantian equilibria. To do that, we take any $c^2 \in C$ such that c^1 and c^2 are linearly independent. These strategy profiles are the ones that are not parallel to c^1. Then, we update the bankruptcy rule R as follows: R behaves like the proportional rule when the strategy profile creating the bankruptcy problem is in the span of c^1 and c^2, $span(\{c^1, c^2\})$. The following lemma formalizes this idea.

Lemma 11 *Let R be a bankruptcy rule, and (c^1, E) and (c^2, E) be bankruptcy problems where $c^1, c^2 \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R} : c^1 = \lambda c^2$). For any $(c, E) \in \zeta^N$, R distributes E as follows: If the strategy profile $c \in C$ is in the $span(\{c^1, c^2\})$, then $R(c, E) = P(c, E)$, where $span(\{c^1, c^2\}) = \sum_{i=1}^{2} \lambda_i c^i$ such that $\forall i \in \{1, 2\} : c^i \in \{c^1, c^2\}$ and $\lambda_i \in \mathbb{R}$. For any other strategy profile, R allocates the estate in a way that satisfies WNPUC and ETE. Under R, in addition to the ones described in the generic existence result (Proposition 2), the (unequal) strategy profiles in the $span(\{c^1, c^2\})$ are also Kantian equilibria of Γ.*

Proof. Since R either behaves like the proportional rule or like any division rule which satisfies *WNPUC* and *ETE*, R satisfies *ETE* for any $(c, E) \in \zeta^N$. So, by Proposition 2, any strategy profile c such that $c_1 = c_2 = \cdots = c_n \geq \frac{E}{N}$ is a Kantian equilibrium. Take any strategy profile $c \in C$ which is not in the $span(\{c^1, c^2\})$. Then, R behaves like any division rule that satisfies *WNPUC* and *ETE*. By Proposition 5, all the possible Kantian equilibria of Γ have already been mentioned. So, c cannot be a Kantian equilibrium. Now, take any strategy profile $c' \in span(\{c^1, c^2\})$ and suppose that c' is such that not all claims are equal to one another. Then, $R(c', E) = P(c', E)$. Since $c' \in span(\{c^1, c^2\})$, there exists $\lambda_1^*, \lambda_2^* \in \mathbb{R}_+$ such that $c' = \lambda_1^* c^1 + \lambda_2^* c^2$. For any strategy profile c'' which is parallel to c' (i.e., $\forall \alpha > 0 : (\alpha c', E) \in \zeta^N \wedge c'' = \alpha c'$), we have $c'' = \alpha c' = \alpha(\lambda_1^* c^1 + \lambda_2^* c^2) = \alpha \lambda_1^* c^1 + \alpha \lambda_2^* c^2$. So, $c'' = \lambda_1' c^1 + \alpha \lambda_2' c^2$ where $\lambda_1' = \alpha \lambda_1^*$ and $\lambda_2' = \alpha \lambda_2^*$. This suggests that $c'' \in span(\{c^1, c^2\})$ as well. Then, $R(c'', E) = P(c'', E) = P(c', E) = R(c', E)$. So, $\forall \alpha > 0 : (\alpha c', E) \in \zeta^N, R(\alpha c', E) = R(c', E)$, and c' is a Kantian equilibrium. Thus, any strategy profile which is in the $span(\{c^1, c^2\})$ is a Kantian equilibrium, and Γ has a set of strategy profiles which are Kantian equilibria with unequal division. ■

We can expand the set of strategy profiles which induce unequal division Kantian equilibria by extending the $span(\{c^1, c^2\})$. To do that, we can pick any strategy profile $c^3 \notin span(\{c^1, c^2\})$ and consider the span of c^1, c^2, and c^3. This ensures that c^1, c^2, and c^3 are linearly independent.[1] The next lemma investigates the strategy profiles, inducing unequal division Kantian equilibria under $span(\{c^1, c^2, c^3\})$.

Lemma 12 *Let R be a bankruptcy rule and $(c^1, E), (c^2, E)$, and (c^3, E) be bankruptcy problems where $c^1, c^2, c^3 \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R}^3 - \{0\} : \lambda_1 c^1 + \lambda_2 c^2 + \lambda_3 c^3 = 0$). For any $(c, E) \in \zeta^N$, R distributes the estate E in the following way: If the strategy profile $c \in C$ is in the $span(\{c^1, c^2, c^3\}), R(c, E) = P(c, E)$, where $span(\{c^1, c^2, c^3\}) = \sum_{i=1}^{3} \lambda_i c^i$ such that $\forall i \in \{1, 2, 3\} : c^i \in \{c^1, c^2, c^3\}$ and $\lambda_i \in \mathbb{R}$. For any other strategy profile, R allocates the estate as any division rule which satisfies WNPUC and ETE. Under R, besides the ones described in the generic existence result, the unequal strategy profiles in the $span(\{c^1, c^2, c^3\})$ are also Kantian equilibria of Γ.*

Proof. Similar to the proof of the above Lemma. ■

We can iterate this process of adding new strategy profiles to the span —such that the existing claims vectors and the recently added ones are linearly independent— and modifying the bankruptcy rule R to behave as the proportional rule for the new span. Since n linearly independent vectors span $\mathbb{R}^n, R$ will behave like the proportional rule for all the strategy profiles creating a bankruptcy problem when we have n linearly independent vectors $\{c^1, c^2, \ldots, c^n\}$.

Proposition 11 *Let R be a bankruptcy rule and $(c^1, E), (c^2, E), \ldots, (c^n, E)$ be*

[1] Since the proportional rule is the only rule under which we have an *anything goes* result, this is the only method which can achieve this purpose.

bankruptcy problems where $c^1, c^2, \ldots, c^n \in C$ *are linearly independent (i.e.,* $\nexists \lambda \in \mathbb{R}^n - \{0\} : \lambda_1 c^1 + \lambda_2 c^2 + \cdots + \lambda_n c^n = 0$*). For any* $(c, E) \in \zeta^N$, R *distributes the estate* E *as follows: If the strategy profile* $c \in C$ *is in the* $span(\{c^1, c^2, \ldots, c^n\})$*, then* $R(c, E) = P(c, E)$*, where* $span(\{c^1, c^2, \ldots, c^n\}) = \sum_{i=1}^{n} \lambda_i c^i$ *such that* $\forall i \in \{1, 2, \ldots, n\} : c^i \in \{c^1, c^2, \ldots, c^n\}$ *and* $\lambda_i \in \mathbb{R}$*. Then, it must be that* $R = P$.

Proof. This result mainly depends on the theorem/proposition which states that if n vectors $c^1, c^2, \ldots, c^n \in \mathbb{R}^N$ are linearly independent, these set of vectors span $\mathbb{R}^N$, (i.e. $\mathbb{R}^N = span(\{c^1, c^2, \ldots, c^n\})$). Note that any set of independent vectors can always be extended to be a basis (i.e., independent spanning set) of the vector space. $\mathbb{R}^N$ has dimension n which means that any basis can have no more than n elements (in fact a basis has exactly n elements). Now, if the linearly independent set of n vectors did not span $\mathbb{R}^N$, then we would have a basis consisting of more than n elements by extending the set of linearly independent vectors. This is contradictory. So, given that $c^1, c^2, \ldots, c^n \in \mathbb{R}^N$ are linearly independent, $span(\{c^1, c^2, \ldots, c^n\}) = \mathbb{R}^N$. Since R distributes the estate E as the proportional rule for any $(c, E) \in \zeta^N : c \in span(\{c^1, c^2, \ldots, c^n\}) = \mathbb{R}^N$, $R(c, E) = P(c, E)$ for any $(c, E) \in \zeta^N$. ■