Anisotropy of Critical Fields in MgB_2: Two-Band Ginzburg–Landau Theory for Layered Superconductors

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Abstract The temperature dependence of the anisotropy parameter of upper critical field $\gamma_{H_{c2}}(T) = H_{c2}^{\parallel}(T)/H_{c2}^{\perp}(T)$ and London penetration depth $\gamma_\lambda(T) = \lambda_k(T)/\lambda_\perp(T)$ are calculated using two-band Ginzburg–Landau theory for layered superconductors. It is shown that, with decreasing temperature the anisotropy parameter $\gamma_{H_{c2}}(T)$ is increased, while the London penetration depth anisotropy $\gamma_\lambda(T)$ reveals an opposite behavior. Results of our calculations are in agreement with experimental data for single crystal MgB_2 and with other calculations. Results of an analysis of magnetic field H_{c1} in a single vortex between superconducting layers are also presented.

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1 Introduction

Apart from the high transition temperature of 40 K, two-band superconductivity is another unexpected feature of MgB_2 compound, which attracts continuing attention of researchers.[1] In MgB_2 the electron system consists of two types of carriers — derived from boron π and σ orbitals.[2] The origin of superconductivity in this compound can be explained in the framework of ordinary electron-phonon (e-ph) mechnanism. Some of the established facts are as follows. The material shows a pronounced isotope effect.[3] Measurement of the nuclear spin-lattice relaxation rate also indicates that MgB_2 is a phonon mediated superconductor.[4] Unusual superconductivity in this compound is related to two distinct energy gaps assosiated with different parts of the Fermi surface. The larger gap (Δ_σ =7 meV) originates from holelike carriers residing on two cylindrical Fermi surface sheets, derived from σ bonding of the p_{xy} boron orbital (σ-band). The smaller gap (Δ_π =2 meV) originates from the two three-dimensional sheets of electrons and holes derived from π bonding of the p_z orbitals (π-band).[5–7]

The existence of two gaps[8,9] with different anisotropies leads to peculiar physical properties.[10,11] The two-band nature of superconductivity in MgB_2 has been verified by tunneling experiments,[12,13] heat capacity measurements,[14] and point contact spectroscopy.[15] Theoretically, two-band superconductivity has been investigated within the Bardeen–Cooper–Schrieffer approach by Suhl et al.[16] and Moskalenko.[17] Two-band Eliashberg theory was proposed for rare earth nickel borocarbides, RNi_2B_2C,[18] MgB_2,[19] and more recently, for the $MgNi_3C$ compound.[20] One of the salient predictions associated with pronounced two-band effects is the difference between the anisotropy of upper critical field[21] $\gamma_{H_{c2}} = H_{c2}^{\parallel}(T)/H_{c2}^{\perp}(T$) and the penetration depth anisotropy[22] $\gamma_\lambda = \lambda_k(T)/\lambda_\perp(T)$, both of which become temperature-dependent with opposite tendencies. For MgB_2 a strong decrease of $\gamma_{H_{c2}}$ from $H_{c2}(0)$ to $H_{c2}(0)/H_{c2}(T) \sim 2$ is found experimentally,[23,24] although controversy remains[25] about the temperature dependence of $\gamma_\lambda(T)$.

A pronounced temperature dependence of the anisotropy parameter $\gamma_{H_{c2}}$ of the upper critical field was calculated based on the microscopic two-band (TB) model.[26–29] It is well known that Ginzburg–Landau (GL) theory remains a powerful instrument for the study of magnetic phase diagram of superconductors. Isotropic GL theory with two s-wave order parameters was used for the calculation of H_{c2},[30] H_{c1}[31] and other superconducting state parameters,[32] and provided good agreement for bulk MgB_2 samples. However, it is still a matter of discussion whether the two-band Ginzburg–Landau theory can be applied to describe the two-band superconductor MgB_2.[27] In this study we present calculations of the upper critical field and lower critical field using TB GL theory for layered superconductors in the clean limit. In the calculations we present, it is shown that, in contrast to single-band (SB) layered superconductors, TB superconductors reveal temperature-dependent anisotropy of the upper critical field, lower critical field, and London penetration depth. As a result, we argue that the anisotropic two-band GL theory, when properly treated, can be applied to the superconductor compound MgB_2.

The rest of this paper is organized as follows. In the next section we outline the two-band Ginzburg–Landau theory for layered superconductors and derive an expression for the upper critical field $H_{c2}(T)$. In Sec. 3 we derive an expression for the lower critical field $H_{c1}(T)$. Results of our calculations for anisotropy parameters of upper critical field and London penetration depth for MgB_2 are presented in Sec. 4 and analyzed in the light of available experimental data. We conclude in Sec. 5 with a brief summary.

2 Upper Critical Magnetic Field

The free energy functional for two-band layered superconductors can be written as[30–33]

$F[\Psi_{1n},\Psi_{2n}] = XZ\, d^2rF_{1n} + F_{1n,2n} + F_{2n} + F_{1n,1(n+1)}$ n

$+ F_{2n,2(n+1)} + F_{in,j(n+1)} + \frac{1}{8\pi}H^2$, (1)

with

$F_{in} = \frac{\hbar^2}{4m_i}\; \nabla_{2d} - \frac{2\pi iA}{\Phi_0}\; \Psi_{in}{}^2 + \alpha_{i,n}(T)\Psi_{i,n}^2 + \frac{\beta_{i,n}}{2}\Psi^4$ h i,n, (2)

$F_{1n,2n} = \varepsilon(\Psi_{1,n}\Psi^*_{2,n} + \text{c.c.}) + \varepsilon_1\; \nabla_{2d} + \frac{2\pi iA}{\Phi_0}$

$\times\; \Psi^*_{1,n}\; \nabla_{2d} - \frac{2\pi iA}{\Phi_0}\; \Psi_{2,n}$ + c.c.i, (3)

$F_{in,i(n+1)} = \frac{\hbar^2}{4m_i c d^2}\; \Psi_{in} - \Psi_{i,(n\pm1)}\exp -i\frac{2\pi dA_z}{\Phi_0}{}^2$ (4)

$F_{in,j(n+1)} = r\Psi_{in} - \Psi_{j,(n\pm1)}\exp -i\frac{2\pi dA_z}{\Phi_0}{}^2$ (5)

where we choose x,y, and z lying along the a,b, and c crystallographic axes, respectively. Here, m_i denotes the effective mass of the carriers in the plane belonging to band i (i = 1,2). F_{in} is the free energy of separate bands in the plane. The coefficient α is given as $\alpha_{in} = \gamma_i(T - T_{ci})$, which depends on temperature linearly, γ is the proportionality constant, while the coefficient β_{in} is independent of temperature. r is the Josephson coupling term between different order parameters in different planes. $\tilde{H}$ is the external magnetic field and $\tilde{H}$ = curl$\tilde{A}$. The quantities ε and ε_1 describe interband interaction of two order parameters and their gradients, respectively. Due to the identical character of planes we can write $\alpha_{in} = \alpha_i$, $\beta_{in} = \beta_i$. Finally, d is the distance between the planes. Introduction of the term given by Eq. (5) is related to the interlayer interband interaction, while Eq. (4) is related to the interlayer intraband interaction. We believe that introducing such a term comes naturally, if we consider the layered character of superconductors and the presence of two order parameters within the plane. MgB_2 is not so highly anisotropic superconductor as cuprate superconductors, but we think that the present model with Josephson coupling can be used for the explanation of superconducting properties of two-band layered superconductors in general and can be helpful for experimental studies. We note that a similar term and coupling mechanism was also considered by Liu.[34] In our calculation, the numerical value of r = 0.44 (see below) is a parameter describing a best fit to the experimental data.

The choice of the vector potential $\tilde{A}$ as $\tilde{A}$ = (0,Hx,0) corresponds to the perpendicular component of the magnetic field $\tilde{H}$ = (0,0,H). In this case GL equations for TB layered superconductors can be reduced to

$$-\frac{\hbar^2}{4m_1}\frac{d^2}{dx^2} - \frac{x^2}{l_s^4}\Psi_1 + \alpha_1(T)\Psi_1 + \varepsilon^*\Psi_2 + \varepsilon_1\frac{d^2}{dx^2} - \frac{x^2}{l_s^4}\Psi_2 = 0,\quad (6)$$

$$-\frac{\hbar^2}{4m_2}\frac{d^2}{dx^2} - \frac{x^2}{l_s^4}\Psi_2 + \alpha_2(T)\Psi_2 + \varepsilon^*\Psi_1 + \varepsilon_1\frac{d^2}{dx^2} - \frac{x^2}{l_s^4}\Psi_1 = 0,\quad (7)$$

where $l_s^2 = \hbar c/2eH$ is the so-called magnetic length and $\alpha_i^*(T) = \alpha_i(T) + r$, $\varepsilon^* = \varepsilon - r$. Calculation of $H_{c2}^{\perp}$ in a similar manner to that given in our earlier work[32] leads to

$$H_{c2}^{\perp}(T) = \frac{\Phi_0}{2\pi\xi_\perp^2}, \quad (8)$$

where

$$\xi_{\perp}^2 = \frac{\hbar^2}{4D} \quad (9)$$

where the effective coherence lenght ξ_{eff} of two-band superconductors is given by the expression

D rm⊥ $_1\alpha_1^*(T) + m_2\alpha_2^*(T) + \frac{}{\hbar^2}$ $-4m_1m_2(\alpha_1^*(T)\alpha_2^*(T) - \varepsilon^{*2})$

1 1 2

,

An approximate form for the upper critical field $H_{c2}(T)$ is obtained as

$$_2(T) = -\frac{\hbar c}{2e}\frac{\alpha_1^*(T)\alpha_2^*(T) - \varepsilon^{*2}}{(\hbar^2/4)\left[\alpha_1^*(T)/m_2 + \alpha_2^*(T)/m_1 + 8\varepsilon^*\varepsilon_1/\hbar^2\right]}$$

$H_c^{\perp}$.

(10)

For the calculation of $H_c{}^k{}_2$, we choose H~ = (0,H,0) and A~ = (0,0,−Hx). Then the GL equations for TB superconductors are reduced to the following form:

$$h-\frac{}{\mathrm{d}x^2} + (\alpha_1^* + r) + 2\frac{}{{}^c_1 d^2}\ 1 - \cos\frac{}{}\ \ \Psi_1 + \ \varepsilon + \varepsilon_1 - \frac{}{{}_0}\frac{}{\mathrm{d}x^2}\ \Psi_2 = 0$$

~2 d²

~2

2πdHx2πdHx d²

, (11)

4m1 4m~ Φ_0 i Φ

h

$$-\frac{}{\mathrm{d}x^2} + (\alpha_2^* + r) + 2\frac{}{{}^c_2 d^2}\ 1 - \cos\frac{}{}\ \ \Psi_2 + \ \varepsilon + \varepsilon_1 - \frac{}{{}_0}\frac{}{\mathrm{d}x^2}\ \Psi_1 = 0$$

~2 d2

2

2πdHx2πdHx d²

. (12)

4m2 4m Φ_0 i Φ

By elimination we can get equations for the order parameters Ψ_1 and Ψ_2 from Eqs. (11) and (12), which turn out to be

$$\frac{\hbar^2}{4m_1}\frac{\hbar^2}{4m_2}\frac{\mathrm{d}^4\Psi}{\mathrm{d}x^4} - \ \frac{\hbar^2}{4m_2}\alpha_1 + \frac{\hbar^2}{4m_1}\alpha_2\ \frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + \alpha_1\alpha_2\Psi_1 + \ 1 - \cos\frac{\pi d H x}{}\ \Bigg(2\frac{\hbar^2}{{}^c d^2} - \frac{\hbar^2}{}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \alpha_2$$

identical

1 * * 1 * * 2 *

Φ_0 4m1 h 4m2 i

$$= -\ m_1\alpha_1^*(T) + m_2\alpha_2^*(T) + \frac{1\ \ 1\ \ 2}{\hbar^2}\ +$$ 8εε m m 8εε m m 2

~2 ~2 d2 *

$$+2\frac{}{{}_2^c d^2} - \frac{}{} \frac{}{\mathrm{d}x^2} + \alpha_1 \quad \Psi$$

4m h 4m1 i 1 d² d⁴ 2πdHx 2πdHx 2πdHx d²

$$= \varepsilon^2 + 2\varepsilon\varepsilon_1 \frac{}{\mathrm{d}x^2} + \varepsilon_1^2 \frac{}{\mathrm{d}x^4} - 2\varepsilon r \cos\frac{}{\Phi_0} + r^2\cos^2\frac{}{\Phi_0} - 2\varepsilon_1 r\cos\frac{}{{}_0} \frac{}{\mathrm{d}x^2} \quad \Psi_1 \qquad (13)$$

Φ

Neglecting the higher derivatives of order parameter ($d^4\Psi_1/dx^4$) and small terms, we can obtain the Mathieu equation for the calculation of upper critical field H_{c2}^k:

$$-\left(\frac{}{4m_2}\alpha_1^* + \frac{}{4m_1}\alpha_2^* + 2\varepsilon\varepsilon_1\right)\frac{1}{\mathrm{d}x^2} + 2\left(\frac{}{4m_1^c d^2}\alpha_2^* + \frac{}{4m_2^c d^2}\alpha_1^*\right)\left(1 - \cos\frac{}{\Phi_0}\right)\Psi_1$$

~2 ~2 d²Ψ

~2~2 2πdHx

$$(14) \quad = \varepsilon^2 - \alpha_1^*\alpha_2^* - 2\varepsilon r + 2\varepsilon r \; 1 - \cos\frac{2\pi dHx}{\Phi_0} \quad \Psi_1 \; .$$

At high magnetic fields $H > \Phi_0/2\pi d^2$ upper critical field H_{c2}^k can be defined from the lowest eigenvalue of the Mathieu equation[35] and is given by the following expression,

$$H_{c2}^k = \frac{\Phi_0}{2\pi d}\,\frac{[((\hbar^2/4m_2)\alpha_1^* + (\hbar^2/4m_1)\alpha_2^* + 2\varepsilon\varepsilon_1)(\alpha_2^*(\hbar^2/4m_2^c d^2) + \alpha_1^*(\hbar^2/4m_1^c d^2) - \varepsilon r - (\varepsilon^{*2} - \alpha_1^*\alpha_2^*)/2)]^{1/2}}{\alpha_2^*\hbar^2/4m_1^c d^2 + \alpha_1^*\hbar^2/4m_2^c d^2 - \varepsilon r} \; . \qquad (15)$$

This implies

$$H_{c2}^k \approx \left(\frac{1}{T - T^*}\right)^{1/2} , \qquad (16)$$

where T^* is given by the following expression:

$$T^* = T_c - \frac{\hbar^2}{4m_1^c d^2\gamma_1} - \frac{\hbar^2}{4m_2^c d^2\gamma_2} \; .$$

In our recent work[36] we introduced effective masses in different bands with angular dependence. In layered anisotropic superconductors the effective masses are tensor quantities. Therefore, in the present work we use this latter property. In the vicinity of T_c the expressions for the anisotropy parameter γ_{c2} in the framework of both approaches are similar (see Eq. (45) and Eq. (17) in Ref. [36]). However, in contrast to Ref. [36], here H_{c2}^k tends to ∞ at T^* (see Eq. (16)). Such behavior is a pecularity of low dimensional systems (planes, films, superlattices, see for example Ref. [37]).

3 Lower Critical Magnetic Field

For temperatures close to the critical temperature $T \sim T_c$ and magnetic fields slightly greater than H_{c1}, the influence on the modulus of the order parameters Ψ_{1n} and Ψ_{2n} can be neglected, thus we take $|\Psi_{1n}|$ = const., $|\Psi_{2n}|$ = const. Then, representing Ψ_{in} as $\Psi_{in} = \Psi_{in}\exp(i\phi_i)$, (i = 1,2) the GL free-energy functional presented in Eqs. (1)∼(5), may be rewritten as

$$F[\phi_{1n},\phi_{2n}] = \int d_2 r \sum_n \left(F_{1n} + F_{1n,2n} + F_{2n} + F_{1n,1(n+1)} + F_{2n,2(n+1)} + F_{1n,2n+1} + \frac{1}{8\pi} H^2 \right) \qquad (17)$$

with

~2 dϕ1n 2πA 2 ~2 dϕ2n 2πA 2

$F_{1n} + F_{2n} = n_1(T)8m_1$ dr $- \Phi_0$ $+ n_2(T)8m_2$ dr $- \Phi_0$, (18)

$F_{1n,2n} = n_1(T)n_2(T)^{1/2} \cos(\phi_{1n} - \phi_{2n})\varepsilon + \varepsilon_1 \, d\phi_{1n} - 2\pi A \, d\phi_{2n} - 2\pi A$, (19)

dr Φ_0 dr Φ_0

$\hbar^2 n_1(T)$ $2\pi dA_z$ $\hbar^2 n_2(T)$ $2\pi dA_z$

$F_{1n,1(n+1)} + F_{2n,2(n+1)} = $ $4m_{c}d^2$ h$1 - \cos\phi_{1n} - \phi_{1(n+1)} +$ Φ_0 i$+ 4m_{2c}d^2 \; 1 - \cos\phi_{2n} - \phi_{2(n+1)} +$ Φ_0 ,(20)

1

$F_{1n,2(n+1)} = -r(n_1(T)n_2(T))^{1/2} \cos\phi_{1n} - \phi_{2n+1} + 2\pi dA_z + \cos\phi_{2n} - \phi_{1n+1} + 2\pi dA_z$, (21)

Φ_0 Φ_0

where $n_i(T) = 2|\Psi_i|^2$ are the densities of superconducting electrons for different bands, respectively. The temperature dependences of $n_i(T)$ are defined by the equilibrium values of order parameters Ψ_i (see Eqs. (6a) and (6b) from Ref. [32]). The choice of the vector potential A as A = (0,−Hx,0) corresponds to the perpendicular component of the magnetic field H = (0,0,H). The equation determining the equilibrium values of the magnetic field can be obtained by minimizing the free energy functional with respect to the two-dimensional vector potential A,

curl curlA = $2\pi(X \, \hbar^2 \, n_i(T) d\phi_i - 2\pi A + (\varepsilon_1 n_1(T) n_2(T))^{1/2} \cos(\phi_{1n} - \phi_{2n}) X \, d\phi_i - 2\pi A)$. (22)

2

4π Φ_0 i $4m_i$ dr Φ_0 i dr Φ_0

Equation (22) together with Maxwell equations yields For a single vortex centered at the origin, the solution the well-known London equation of Eq. (23) for distances $r \geq \xi_k$ is given as[37]

$$\lambda_\perp^{-2}(T)\frac{\partial^2 H}{\partial r^2} - H = 0 , \quad (23)$$

$$H_{c1} = \frac{\Phi_0}{4\pi\lambda_\perp^2} \left(\ln \frac{\lambda_\perp}{\xi_k} + \Omega_0 \right), \quad (25)$$

where $\lambda_\perp$ is the London penetration depth along super- The quantity Ω_0 corresponds to the "core" energy of the conducting layer, determined by the expression vortex filament and $\Omega_0 \sim 1$.[37]

$$\lambda_\perp^{-2}(T) = \frac{4\pi e^2}{c^2}\left(\frac{n_1(T)}{m_1} + 2\varepsilon_1 (n_1(T) n_2(T))^{1/2} + \frac{n_2(T)}{m_2}\right). \quad (24)$$

For the magnetic field H = (H,0,0), minimization of the free energy functional gives the following equations

$$\frac{1}{4\pi}\frac{\partial H}{\partial y} = \frac{2ed}{\hbar c} X \frac{\hbar^2}{4m_{ci}d} n_i(T) \sin \phi_{in} - \phi_{in+1} + \frac{2\pi}{\Phi_0} A_z d + r(n_1(T)n_2(T))^{1/2} \sin\phi_{1n} - \phi_{2n+1} + \frac{2\pi dA_z}{\Phi_0}$$

$$+ \sin \phi_{2n} - \phi_{1n+1} + \frac{2\pi dA_z}{\Phi_0}, \quad (26)$$

$$-\frac{1}{4\pi}\frac{\partial H}{\partial z} = \frac{2e}{\hbar c} X \frac{n_i(T)}{m_i}\left(\frac{d\phi_{1n}}{dr} - \frac{2\pi A_y}{\Phi_0}\right) , \quad (27)$$

$\sim^2 \partial^2\phi_{1,n}$ $\sim 2\pi^{1/2}$ $2\pi dA^z$

-4 m ∂r^2 $+ 4m_{c1}d \sin\phi_{1n} - \phi_{1n\pm1} +$ $g\Phi_0 A_z d - r\, n_1(T)n_2(T)\sin\phi_{1n} - \phi_{2n+1} +$

Φ_0

1

$+ \sin\phi_{2n} -$ $-\frac{}{4m_2}\frac{}{\partial r^2} + \frac{}{4m_2^c d}\sin\Big(\phi_{2n} - \phi_{2n\pm1} +$ $\phi_{1n+1} + 2\pi dA^z = 0,$ (28)

Φ_0

$\sim^2 \partial^2\phi_{2,n}\sim^2$ 2π $^{1/2}\sin\phi_{1n} - \phi_{2n+1} + 2\pi dA_z$

gA

Φ_0 $_z d - r(n_1(T)n_2(T))$ Φ_0

$2\pi dA^z$

$+ \sin\phi_{2n} - \phi_{1n+1} + = 0.$ (29)

Φ_0

The last system of equations is nonlinear, the elimination of ϕ_{1n} and ϕ_{2n} is carried out after expansion of sine function in Eqs. (26)~(29). Taking into account the discrete character of z variable and procedure of replacing the finite differences by differentations in Eqs. (24)~(27), we can obtain the following system of equations:

$$\frac{\partial^2\phi_{1n}}{\partial y^2} + \frac{m_1}{m_1^c}\frac{\partial^2\phi_{1n}}{\partial z^2} - \frac{4rm_1}{\sim^2}\Big(\frac{n_2(T)}{n_1(T)}\Big)^{1/2}(\phi_{1n} - \phi_{2n}) = 0\,, \quad (31)$$

$$\frac{\partial^2\phi_{2n}}{\partial y^2} + \frac{m_2}{m_2^c}\frac{\partial^2\phi_{2n}}{\partial z^2} - \frac{4rm_2}{\sim^2}\Big(\frac{n_1(T)}{n_2(T)}\Big)^{1/2}(\phi_{2n} - \phi_{1n}) = 0\,, \quad (32)$$

$$\lambda_\parallel^2(T)\frac{\partial^2 H}{\partial y^2} + \lambda_\perp^2(T)\frac{\partial^2 H}{\partial z^2} - H = -\frac{\Phi_0}{2\pi}\frac{n_1(T)}{n_1(T)+n_2(T)}\Big(1 - \frac{\lambda_\parallel^2}{\lambda_\perp^2}\Big)\frac{\partial^2}{\partial z\partial y}(\phi_{1n} - \phi_{2n})\,, \quad (30)$$

where $\lambda_k^2(T)$ is the penetration depth in the direction perpendicular to planes, determined as

$$\lambda_k^{-2}(T) = \frac{4\pi e^2 n_1(T)}{c^2 m_{c1}} + \frac{n_2(T)}{m_{c2}} + \sim^2 \ \ n_1(T)n_2(T) \quad o. \quad (33)$$

$4d^2r$ $1/2$

Equation (30) gives the solution corresponding to a single vortex, directed parallel to the superconducting layer. In this case, boundary condition requiring that the total magnetic flux through the yz plane is equal to the flux quantum Φ_0. As one can see from Eq. (30) in contrast to single-band superconductors, magnetic field in TB superconductors is nonhomogeneous. For the calculation of magnetic distribution in TB superonductors, it is necessary to solve the differential Eqs. (31) and (32) for ϕ_{1n} and ϕ_{2n}. The solutions of Eqs. (31) and (32) in the case of small Josephson couplings ($4rd^2/\sim^2 \ll 1$) are in the form:

$$\phi_{in}(y,z) = \tan^{-1}\Big(\frac{m}{m_{ci}}\Big)^{1/2}\frac{y}{x}\,. \quad (34)$$

Using the transformation $y = \lambda_k\rho\sin\theta$ and $x = \lambda_\perp\rho\cos\theta$ we can rewrite Eqs. (30) as

$$\rho\frac{\partial}{\partial\rho}\Big(\rho\frac{\partial H}{\partial\rho}\Big) + \frac{\partial^2 H}{\partial\theta^2} - \rho^2 H = g(\theta)$$

, (35)

where

$\chi^2 \sin^2\theta - \cos^2\theta$

g(θ) = χ_1 1 2 2 2 2

(cos θ + χ_1 sin θ)

$\chi^2 \sin^2\theta -$ 2 2 2 2 2 $\cos^2\theta$

+ χ_2.

(36)

(cos θ + χ_2 sin θ)

In the last equation we introduced the notation:

χ_i =m___ ___m_{ci} 1/2 λλ⊥k , i = 1,2. (37)

i

Using the formula for Fourier harmonics of the right hand side of Eq. (35),

g(θ) = Xg_n cos(nθ), (38)

where coefficients are defined as

2 π

g_n = _ Z g(θ)cos(nθ)dθ , (39) π 0

solution of the equation for the magnetic field [Eq. (35)] can be written as:

H(ρ,θ) = Xh_n(ρ)cos(nθ). (40)

The equation for $h_n(\rho)$ has the form of nonhomogeneous Bessel equation. After some transformations under ρ 1, we have the following expression for magnetic field H(ρ,θ)

$$H(\rho,\theta) = \frac{\Phi_0}{2\pi\lambda_\perp\lambda_\|} - \ln\rho + \frac{1}{2}\chi\rho^2\cos(2\theta) \text{hi}, \quad (41)$$

where

$$\chi = \sum_{=1,2} \frac{\chi_i}{(1+\chi_i)^2}. \quad (42)$$

i

As follows from Eq. (41) the existence of two order of parameters and their anisotropy leads to an additional angular dependence of magnetic field in a vortex. Nonsymmetric behavior of magnetic field in a vortex in single band layered superconductors was investigated by a number of researchers.[38–40] Transformation to (zy) coordinates yields for magnetic field, under the conditions y λ_k and z $\lambda_\perp$.

$$(y,z) = \frac{\Phi_0}{2\pi\lambda_\perp\lambda_\|}\left\{-\ln\left[\frac{y}{\|}^2 + \frac{z}{}^2\right.\right.$$

H

λ λ⊥

+ χ ___ − ___ . (43) z 2y 2 λ⊥ λ_k

Using the last expression for magnetic field in a single vortex H(y,z), we can calculate the energy of the vortex. Due to the fact that the vortex lies between the superconducting layers, the lower limit of integration with respect to z must be equal to d. The result can be expressed as

$$H_{c1}^{k} \simeq 4\_\_\_\pi\lambda\Phi_{\perp0}\lambda_k - \ln\frac{\lambda_\perp}{} + 2\;2 - \frac{1}{}\chi\, d \quad \pi. \quad (44)$$

4 Result and Discussion

At low magnetic fields H $\Phi_0/2\pi d^2$ and after expansion of cosines in Eq. (10), we can get the final expression for anisotropy parameter of upper critical field suitable for comparison with experiments.

$$\gamma_{Hc2} = \frac{H^{k}}{H_{c\perp 22}}$$

$$= \left[\frac{\delta_m(T-T_{c1}) + (T-T_{c2}) + 8\varepsilon_2 x\eta T_c}{(\frac{m_2}{m_2^c})x(T-T_{c1}) + (\frac{m_1}{m_1^c})(T-T_{c2})}\right]^{1/2}, \quad (45)$$

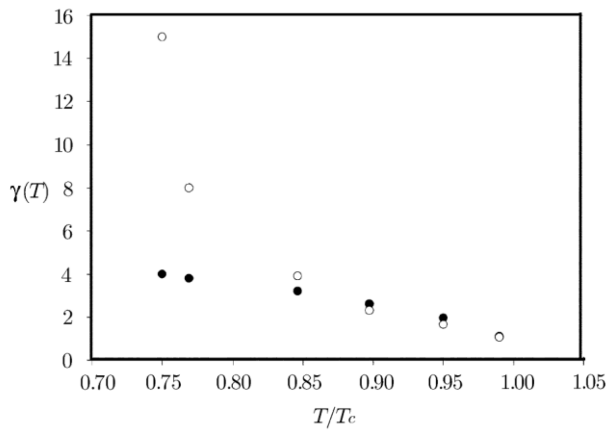

Fig. 1 Temperature dependence of the anisotropy parameter γ_{Hc2}. The full line is the result of TB GL theory for layered superconductors, full symbols are experimental data from Ref. [21].

In Fig. 1 we plot the anisotropy parameter γ as a function of reduced temperature T/T_c. Experimental results of Lyard et al.[21] are given by the full symbols. The full points denote the results of calculations from the present layered TB GL theory. Here we use the following values for various parameters: T_{c1} = 20K, T_{c2} = 10K, $\varepsilon^2 = 3/8$, $\delta_m = m_1/m_2 = 3$, η = −0.16, and r = 0.44. The same parameters were also used in Refs. [30]~[32]

to determine the temperature dependence of superconducting state parameters in the framework of isotropic TB GL theory. Mass anisotropy parameters for single crystals $m_2/m^c_2 = 1.3$ and $m_1/m^c_1 = 0.03$ are the same as in Ref. [34]. As shown in Refs. [30]~[32] isotropic GL theory gives a good description of the temperature dependences of measureable parameters of bulk samples of MgB_2. As can be seen from Eq. (45) influence of σ (strong) band is effectively switched off and anisotropy parameter is mainly defined by π (weak) band. As a consequence, for weak magnetic fields there is good agreement with experimental data of the anisotropy of upper critical field. Enhancement of γ with decreasing temperature was observed experimentally by many groups.[41–43] Thus, there is a general agreement in the temperature behavior of γ_H.

At high magnetic fields, $H_c^k{}_2$ (see Eq. (16)) goes to infinity as $(T - T^*)^{1/2}$. It means that, the orbital depairing effect of a magnetic field parallel to the layers does not destroy superconductivity. This corresponds to the case where the cores of the vortices fit between the superconducting layers and external magnetic field has no effect on superconductivity. In fact, other magnetic mechanisms will limit the divergence. The divergence of $H_c^k{}_2$ at T^* will be removed by taking into account spin-orbit scattering[44] and paramagnetic effect.[45,46] Similar anisotropy of upper critical field was observed for the other possible class of two-band superconductors — nonmagnetic borocarbides $Y(Lu)Ni_2B_2C$.[47,48]

As shown in a number of investigations,[30–32] maximal positive curvature of upper critical field of bulk samples can be achieved by inclusion of an intergradient interaction. In the case of no intergradients of order parameters η = 0, the curvature reaches maximum at the point of $0.5T_c$. Intergradient interaction shifts this maximum to the region close to critical temperature. Such behavior is in good agreement with experimental data for bulk samples. As we can see from Eq. (45), in the case of anisotropic GL equations, the intergradient term also plays a crucial role in determining the temperature dependence of anisotropy parameter γ_{Hc2}.

Using Eqs. (24) and (33), for the anisotropy parameter of London penetration depth $\gamma_\lambda = \lambda_k/\lambda_\perp$, we obtain the following expression:

γλ =

n1(T)/mn (1T+ 2)/mεc1(+n1n(2T()Tn)2/m(Tc2))+ 41/2rd+2n/2~(2T)/m2 . (46)

1 1

In Fig. 2 we display the anisotropy parameter γ_λ as a function of reduced temperature T/T_c. Experimental data from Lyard et al.[49] are shown by diamonds. The squares denote the results of calculations using Eqs. (6a) and (6b) from Ref. [49] and Eq. (24) and (33). Due to the negative sign of intergradient interaction η, with decreasing temperature, anisotropy factor of the London penetration depth γ_λ also decreases. Similar experimental results were obtained also by Cubitt[25] and Zehetmayer.[50]

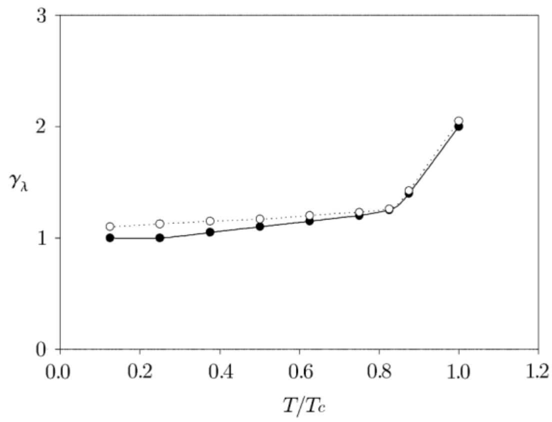

Fig. 2 Temperature dependence of the anisotropy parameter γ_λ. The full line is the result of TB GL theory for layered superconductors, full symbols are experimental data from Ref. [49].

In studies[26,27] within the weak-coupling TB anisotropic BCS model anisotropy parameters of H_{c2} and λ were calculated introducing average parameters. Results of these calculations are also in agreement with the above presented TB GL theory calculations. The anisotropy parameter of London penetration depth γ_λ evaluated for two-band superconductors with arbitrary interband and intraband scattering times using Eilenberger theory was given by Kogan and Zhelezina.[51]

As shown by Bulaevskii[52] in the case of SB layered superconductors, upper critical field is defined by the expressions: $H_c^k{}_2 = \Phi_0/2\pi\xi_\perp\xi_k$ and $H_c^\perp = \Phi_0/2\pi\xi_2{}_k{}^2$. Note that in this case the anisotropy parameter γ_{Hc2} and γ_λ are temperature-independent. As stated earlier, the coefficients α and β in the GL model are field-dependent. It should be possible to generalize the present model introducing field-dependent parameters α and β. We remark that a very recent paper taking into account field independent TB GL theory without intergradient interaction term has appeared.[53]

Another version of the GL approximation was presented by Golubov and Koshelev,[27] which corresponds to an effective single-band GL theory. In the context of this approach[27] the ratio of order parameters is temperature and field independent. It means that the two-band GL theory is equivalent to an effective single band approximation. In contrast to the Golubov and Koshelev[27] approach, in our consideration the ratio of order parameters is temperature-and field-dependent[30–32] (see also Eqs. (6), (7), (11), and (12)). Structure of a single tilted vortex in layered two-band superconductors also seems to be an interesting problem as discussed by Bulaevskii et al.[54] using the above presented two-band GinzburgLandau equations.

5 Conclusion

In summary, we have shown that the available experimental data on the anisotropy parameter $\gamma_{H_{c2}}(T)$ and $\gamma_\lambda(T)$ for MgB_2 can be described in the framework of TB layered GL theory at temperatures close to T_c. In contrast, for SB layered superconductors, the anisotropy parameter is temperature dependent. Presence of two order parameters with different dimensionalities play a significant role in determining the temperature dependence of anisotropy parameter of upper critical field $\gamma_{H_{c2}}(T)$ and London penetration depth $\gamma_\lambda(T)$. We have also shown that two order parameters and their anisotropy lead to additional angular dependence of magnetic field in a single vortex between the planes.

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