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Performance Analysis of Turbo Codes over Rician Fading Channels with Impulsive Noise

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***Abstract*— The statistical characteristics of impulsive noise differ greatly from those of Gaussian noise. Hence, the performance of conventional decoders, optimized for additive white Gaussian noise (AWGN) channels is not promising in non-Gaussian environments. In order to achieve improved performance in impulsive environments the decoder structure needs to be modified in accordance with the impulsive noise model.**

This paper provides performance analysis of turbo codes over fully interleaved Rician fading channels with Middleton's additive white Class-A impulsive noise (MAWCAIN). Simulation results for the memoryless Rician fading channels using coherent BPSK signaling for both the cases of ideal channel state information (ICSI) and no channel state information (NCSI) at the decoder are provided. An eight state turbo encoder having $(1, 13/15, 13/15)$ generator polynomial is used throughout the analysis. The novelty of this work lies in the fact that this is an initial attempt to provide a detailed analysis of turbo codes over Rician fading channels with impulsive noise rather than AWGN.

***Index Terms*— Channel reliability, ideal channel state information, Middleton's additive white Class-A impulsive noise channel, no channel state information, Rician fading channel, turbo codes.**

I. Introduction

IN various communication environments the Gaussian noise assumption is insufficient to model the true effect of additive noise because of the presence of less probable high amplitude "spikes" in the signal. This non-Gaussian noise [1] which is prevalent because of either man made noise sources or natural phenomena can be momentous in many applications and must be taken into consideration to improve system performance. Automobile ignitions, neon lights and many other electronic devices are the common source of man made noise. On the other hand, lightning discharges, impulsive interference in power line channels or in undersea communication systems noisy aquatic animals or surrounding acoustical noises due to ice cracking in arctic regions are the innate means of the occurrence of impulsive noise [1].

In wireless communication scenarios the performance of communication systems is often degraded by fading in addition to the additive noise. In urban areas, it is often observed that wireless communication systems often contain impulsive noise as the additive noise as opposed to the Gaussian noise due to the abundance of man made noise sources. In such hostile environments, an indispensable solution to improve the performance of the communication system is to employ good forward error correcting codes.

The capability of turbo codes to exhibit excellent performance in AWGN channels close to the channel capacity is provided in detail in [2]–[4]. Similarly, a comprehensive performance analysis of turbo codes via either simulation or analytical means over fading channels is carried out in [5]–[8]. It is well known that the performance of conventional decoders which are designed for AWGN type interference fail to provide good results in impulsive environments [9]. In order to solve this issue, the decoders must be designed to provide optimized performance in non-Gaussian environments. Recently, authors in [10], [11] provided preliminary results for the performance analysis of turbo and LDPC codes over power line channels by modeling the noise component as MAWCAIN [12], [13]. The work presented in [10] and [11] is novel since it is the only known attempt to discuss the performance of turbo and LDPC codes in MAWCAIN environments, despite the existence of an in depth analysis of optimum or sub-optimum receivers for coherent detection in MAWCAIN [14].

This paper extends the analysis of turbo codes to fully interleaved Rician fading channels with MAWCAIN for the cases of ICSI and NCSI. Extensive simulation results for the Rician fading parameter values of $K = 0$ (Rayleigh fading case) and $K = 3$ (mild fading case) are provided to get an understanding on the performance of turbo codes over the Rician fading channels under MAWCAIN. Throughout the simulations, a rate $1/3$ turbo code with a memory of three and an input block size of N bits and an output encoded stream of $FS = 3(K + 3)$ bits is used to study the performance of $(1, 13/15, 13/15)$ code structure. For the second constituent encoder, a random interleaver is chosen to shuffle the input bit sequence and the first encoder is terminated in the zero state.

The paper is organized as follows. Following the general introduction and the literature survey of section I, section II provides details about the Middleton's additive white Class-A impulsive noise model followed by the Rician fading channel. This section also contains the structure of the turbo encoder and decoder which are used for the performance analysis. Section III focuses on the modified channel reliability expression for both the ICSI and NCSI scenarios in the light of impulsive noise. Section IV provides simulation results and a detailed discussion on them. Finally, the findings of this work are summarized in Section V.

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II. Turbo Decoding over Rician Fading Channels with MAWCAIN Channel

The subsections below give details about the Middleton's additive white Class-A impulsive noise, the Rician fading distribution and the turbo code encoder and decoder structures.

A. Middleton's Additive White Class-A Impulsive Noise Model

Generally, the narrow-band impulsive noise models are established by either modeling the underlying physical mechanism or by using an empirical model [1]. Though, empirical models are mathematically less cumbersome they lack to provide a direct relationship between their parameters and the measurable quantities. Middleton's additive white Class-A impulsive noise model [12], [13] is based on a physical model approach which is derived by directly characterizing the physical mechanisms which account for the impulsive nature. The mathematical representation for this model assumes that the impulsive noise sources are distributed by Poisson distribution and always contain the background Gaussian noise. The MAWCAIN model is given by the following probability density function (pdf)

$$p(x) = \sum_{m=0}^{\infty} \frac{\exp(-A) A^m}{m! \sqrt{2\pi} \sigma_m} \exp\left(\frac{-x^2}{2\sigma_m^2}\right) \quad (1)$$

where m is the number of impulsive noise sources, A is called the impulsive index and σ_m^2 is defined as

$$\sigma_m^2 = \frac{\sigma^2 (m + A\Gamma)}{A(1+\Gamma)}. \quad (2)$$

The Gaussian to impulsive noise power ratio $\Gamma = \sigma_G^2/\sigma_I^2$ with σ_G^2 and σ_I^2 representing the Gaussian and impulsive noise powers respectively. The measure of total noise power σ^2 equals

$$\sigma^2 = \sigma_G^2 + \sigma_I^2. \quad (3)$$

At any time instant the noise at the receiver can be characterized by Gaussian pdf having a variance of

$$\sigma_m^2 = \sigma_G^2 + \frac{m}{A}\sigma_I^2 = \left(\frac{m + A\Gamma}{A\Gamma}\right)\sigma_G^2. \quad (4)$$

For large values of A ($A \geq 10$) the Class-A impulsive noise becomes continuous and its statistical features become similar to Gaussian noise and hence, it can be modeled as a Gaussian channel.

B. Rician Fading Distribution

In many propagation scenarios there exist a line of sight component having constant amplitude and a number of reflected waves. The sum of direct path and the reflected components result in a signal having Rician [15] envelope distribution. The normalized Rician distribution i.e. its second moment is equal to unity ($E[a^2] = 1$) along with its mean and variance for $a \geq 0$ is

$$p(a) = 2a(1+K)e^{-K-(1+K)a^2} I_0\left(2a\sqrt{K(1+K)}\right) \quad (5)$$

$$m_a = \frac{1}{2}\sqrt{\frac{\pi}{1+K}} e^{-\frac{K}{2}} \left[(1+K)I_0\left(\frac{K}{2}\right) + KI_1\left(\frac{K}{2}\right)\right] \quad (6)$$

$$\sigma_a^2 = 1 - m_a^2. \quad (7)$$

In the above expressions, K is the Rician fading parameter which represents the power ratio of the direct and reflected signal components. Additionally, $I_0(\cdot)$ and $I_1(\cdot)$ represent the modified Bessel functions of first kind having zero and first order. Small values of K imply severe fading whereas, large values indicate mild fading. When $K = 0$, the Rician pdf becomes the well known Rayleigh pdf.

C. Turbo Encoder

This paper utilizes the eight state turbo encoder as shown in Fig. 1. The reason to choose this encoder is due to the fact that this structure is used by 3^{rd} generation cellular mobile communication systems based on 3^{rd} Generation Partnership Project (3GPP) [16]. The given turbo encoder uses two identical recursive systematic convolutional (RSC) encoders with each constituent encoder having a rate of $1/2$. The generator matrix for the given encoder can be expressed as

$$G(D) := \begin{bmatrix} 1 & \frac{1+D+D^3}{1+D^2+D^3} \end{bmatrix} \quad (8)$$

where D is used to represent a unit delay. An alternate representation for the generator polynomials can be written in octal form as $(1, 13/15, 13/15)$.

The classical rate $1/3$ turbo encoder generates three output sequences. The first sequence which is referred to as the systematic bit is composed of the information bits $u = (u_1, u_2, ..., u_N)$. The second output sequence, which corresponds to the first parity bits $p_1 = (p_{1,1}, p_{1,2}, ..., p_{1,N})$ is obtained as a result of encoding the input message sequence $\mathbf{u}$. The third output sequence which provides the second stream of parity bits $p_2 = (p_{2,1}, p_{2,2}, ..., p_{2,N})$ results by encoding the interleaved input message sequence $\mathbf{u}$. As a result the turbo encoder is a rate 1/3 block encoder which has N input bits and $FS = 3(N+3)$ output bits. The term $(N+3)$ appears due to the termination of the first encoder to the all zero state.

This work uses BPSK modulation for the transmitted sequence x (the turbo encoded transmitted sequence). The modulated signal $x \in \{1, -1\}$ and the received bits are represented by $\mathbf{y}$. The AWGN or MAWCAIN noise is assumed to have a variance of $N_0/2$.

D. Turbo Decoder

The Turbo decoder uses two component decoders by sharing information to iteratively decode the received sequence $\mathbf{y}$. The decoder is based on the *Soft Input/Soft Output* (SISO) algorithm which takes as an input *a priori* information and produces *a posteriori* information as an output. The BCJR algorithm [17] which is also known as the forward-backward algorithm is the core behind the turbo decoding algorithm. Although, the BCJR algorithm provides optimal results for estimating the outputs of a Markov process in white noise, it

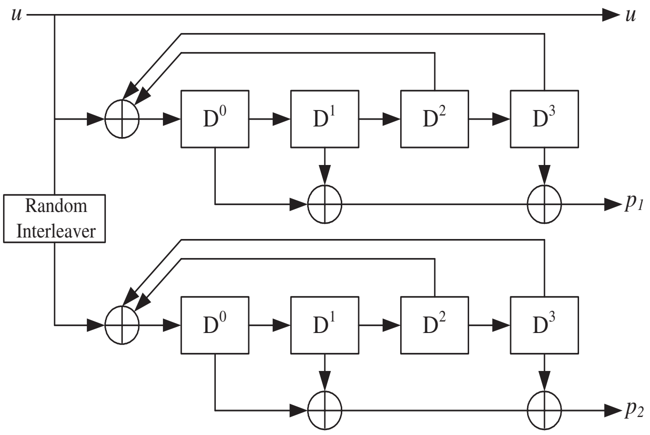

Fig. 1. The structure of the utilized turbo encoder.

suffers from numerical complications due to the use of non-linear functions with mixed multiplication and addition operations. Hence, different derivative of this algorithm such as Log-MAP, Max-Log-MAP or SOVA [18]–[20] are often utilized in practice. This work utilizes the Log-MAP decoding scheme whose details can be found in [20]. The derivation of Log-MAP decoding is based on *a posteriori* log-likelihood ratio (LLR), that is, the logarithm of the ratio of the probabilities of a given bit being "+1" or "-1" given the received/observed output $\mathbf{y}$. The derivation gives rise to the following equation

$$L(\hat{\mathbf{x}}) = L_c(\mathbf{y}) + L(\mathbf{x}) + L_e(\hat{\mathbf{x}}) \tag{9}$$

where $\mathbf{x}$ represents the BPSK modulated sequence and $\hat{\mathbf{x}}$ is the estimated soft value. This LLR gives the soft channel output $L(\hat{x})$ in terms of three quantities *a priori* values $L(\mathbf{x})$, extrinsic information $L_e(\hat{\mathbf{x}})$ and the channel reliability based on the received channel output $\mathbf{y}$, $L_c(\mathbf{y})$ [20]. As mentioned in [10] the turbo decoder over MAWCAIN can be obtained by only adjusting the channel reliability expression according to the impulsive noise channel.

III. Channel Reliability

The performance of the turbo decoding principle depends on the sharing of information between the constituent decoders. The computation of the LLR gives rise to a variable called the channel reliability [18]. The channel reliability value is also based on the LLR for a particular channel and can be written as follows

$$L_c(y_n) = \ln \left[\frac{P(y_n|x_n = 1)}{P(y_n|x_n = -1)} \right]. \tag{10}$$

In order to obtain good turbo code performance over a particular channel the above expression needs to be adopted according to the underlying channel. Subsections below provide the corresponding channel reliability expressions for channels with AWGN or MAWCAIN.

A. Channel Reliability for the AWGN Channel

The channel reliability value for a BPSK modulated data over AWGN channel with fading can be expressed as

$$L_c(y_n) = \ln \left[\frac{\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y_n - a_n)^2}{2\sigma^2}\right)}{\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(y_n + a_n)^2}{2\sigma^2}\right)} \right] \tag{11}$$

a simplified version after mathematical manipulations takes the form of

$$L_c(y_n) = 4 y_n a_n \frac{E_s}{N_0}. \tag{12}$$

In equation (12) y_n represents the received bit through the channel and a_n is the value of the fading coefficient. In the case of no fading, $a_n = 1$. In the case of fading with no channel state information the fading value becomes the expected value $a_n = E[\mathbf{a}]$ of the underlying fading channel. When we have ideal channel state information available at the decoder than each a_n is the exact fading value.

B. Channel Reliability for the MAWCAIN Channel

Similarly, for the MAWCAIN channel model, the channel reliability becomes

$$L_c(y_n) = \ln \left[\frac{\sum_{m=0}^{\infty} \frac{A^m}{m!\sqrt{2\pi}\sigma_m} \exp\left(-\frac{(y_n - a_n)^2}{2\sigma_m^2}\right)}{\sum_{m=0}^{\infty} \frac{A^m}{m!\sqrt{2\pi}\sigma_m} \exp\left(-\frac{(y_n + a_n)^2}{2\sigma_m^2}\right)} \right]. \tag{13}$$

Again, identical interpretation follows as for the case of AWGN channel. Other than this, since the MAWCAIN channel consists of an infinite series the above equation is too cumbersome to compute. A simple rule as suggested by [10] is to truncate the series to $m = 0, 1, ..., L$ where, $L = max\{2, \lceil 10A \rceil\}$. A basic interpretation of the above rule is that for small index values of m only the first few terms in the summation are significant due to $A^m/m!$. Hence, higher index values can be ignored. As the value of A is increased the number of significant terms in the summation also increases and hence more terms are need to obtain a better reliability value.

Fig. 2 shows the graphical representation of the channel reliability for various cases of Γ and A in MAWCAIN channel when both σ and $\mathbf{a}$ are unity. Additionally, the channel reliability expression for AWGN channel is also provided in the plot. One simple observation from Fig. 2 is that the $L_c(y_n)$ in MAWCAIN channel has a nonlinear behavior in contrast to the Gaussian channel case. Also the channel reliability value at $y_n = +1$ for MAWCAIN channel is higher than the AWGN case. Small values of Γ for same values of A gives rise to a peak whereas, different values A for similar values of Γ shifts the local minimum in the vicinity of $y_n = +2$ and has a wider spread for small A values. Note that small values of A correspond to more impulsive channel whereas, smaller values of Γ points to the fact that power in the impulsive noise component is greater than the smaller values of Γ. From these observation one expects the turbo decoder to provide better performance in the case of both A and Γ taking small values.

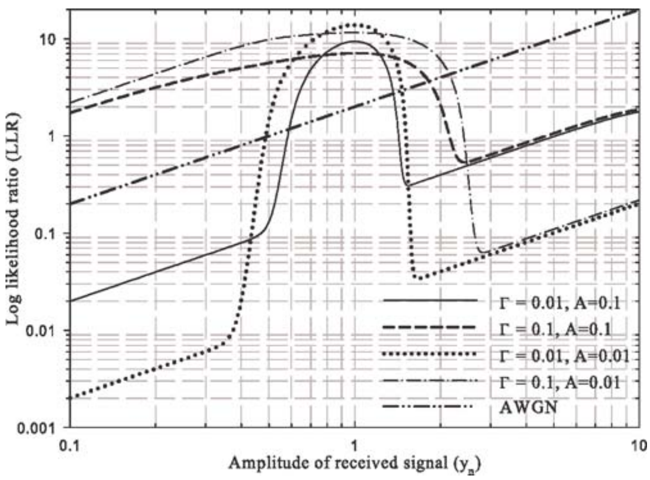

Fig. 2. LLR for AWGN and MAWCAIN channels with both and a equal to unity for various values of Γ and

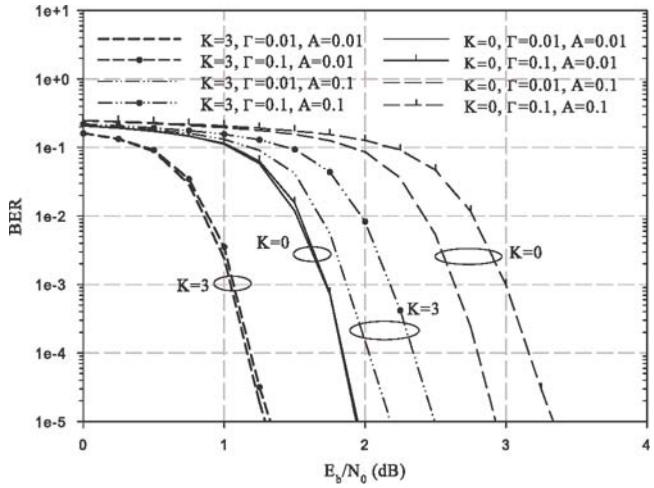

Fig. 3. Simulated BER performance of (1 13 15 13 15) turbo code over Rician fading channels under ICSI with MAWCAIN for a message block length of = 5000 bits after 5 decoding iterations.

For the practical use of turbo decoding over MAWCAIN channels, one need to estimate the impulsive index, Gaussian to impulsive noise power ratio Γ and the noise power σ^2. These parameters can be obtained through the second, fourth and sixth moments of the received envelopes [21]. Similarly, the fading statistics needs to be obtained using estimation techniques according to the decoder type which either uses ICSI (requires the knowledge of the fading coefficients) or NCSI (requires the knowledge of the mean value of the fading coefficients).

IV. Results And Discussion

Fig. 3 illustrate the simulation results of turbo codes over Rician fading channels under ICSI with MAWCAIN by using an input message block length of $N = 5000$ bits and five decoding iterations. For the analysis, two values of Rician fading parameter $K = 0$ and $K = 3$ are chosen to see the performance under severe and mild fading conditions. It can be seen from Fig. 3 that when $A = 0.01$ (i.e. more impulsive channel) there does not exist any noticeable performance difference for $\Gamma = 0.1$ and $\Gamma = 0.01$ when the channel is either mild ($K = 3$) or sever ($K = 0$). Contrary to this, when $A = 0.1$ (i.e. less impulsive channel than $A = 0.01$) smaller value of $\Gamma = 0.01$ provides better performance than the higher value of $\Gamma = 0.1$. This is due to the reason that the proposed turbo decoder suppresses more impulsive noise power. Similarly, Fig. 4 provides the simulation results of turbo codes over Rician fading channels under NCSI with MAWCAIN by using an input message block length of $N = 5000$ bits and five decoding iterations. Again, the two different values of Rician fading parameter $K = 0$ and $K = 3$ are chosen to see the performance under severe and mild fading conditions. From Fig. 4 one can easily notice that when $A = 0.01$ (i.e. more impulsive channel) there does not exist any noticeable performance difference for $\Gamma = 0.1$ and $\Gamma = 0.01$ when the channel is either mild ($K = 3$) or severe ($K = 0$). Contrary to this, when $A = 0.1$ (i.e. less impulsive channel than $A = 0.01$) smaller value of $\Gamma = 0.01$ provides better performance than the higher value of $\Gamma = 0.1$.

It is obvious from both Fig. 3 and Fig. 4 that turbo decoders for ICSI perform better than NCSI which is expected due to the availability of channel fading coefficients. It is also worth noticing that for mild fading scenario ($K = 3$) the ICSI performs approximately 0.4 dB better than NCSI at a BER of 10^{-5} whereas, the difference between the two for severe fading case ($K = 0$) is around 1 dB (for $\Gamma = 0.01$ and $A = 0.01$) again at a BER of 10^{-5}. The performance difference between ICSI and NCSI is less when the fading conditions are mild due to the fact that as the fading becomes mild, the multiplicative effect tends towards unity and the performance of ICSI and NCSI decoders become identical.

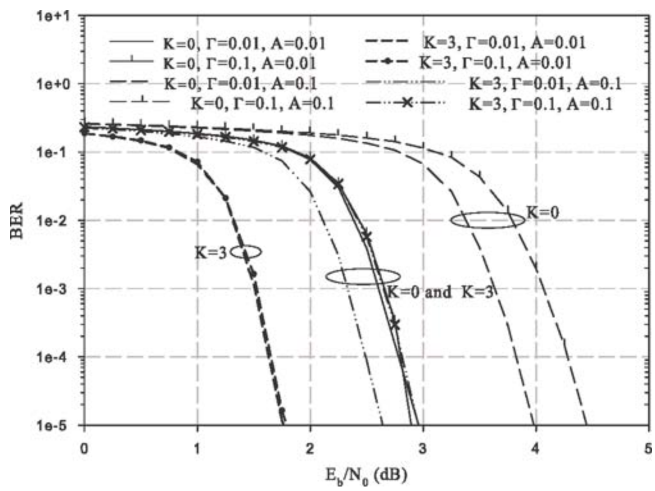

Fig. 4. Simulated BER performance of (1 13 15 13 15) turbo code over Rician fading channels under NCSI with MAWCAIN for a message block length of = 5000 bits after 5 decoding iterations.

V. Conclusion and Remarks

This article discusses the performance analysis of turbo codes over fully interleaved Rician fading channels under Middleton's additive white Class-A impulsive noise. The work provides the updated channel reliability expression along with its interpretation for the impulsive channel. A detailed performance analysis is provided for both the ICSI and the NCSI cases. The presented work is novel since the authors believe that it is a first attempt to provide such results and analysis over Rician fading channels under MAWCAIN.

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