## Paraxial Space-Domain Formulation for Surface Fields on Large Dielectric Coated Circular Cylinders [1]

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**Introduction:** There are few asymptotic solutions in the literature that can efficiently evaluate surface fields excited on electrically large dielectric coated circular cylinders [1]-[2]. Furthermore, to the best of our knowledge, none of these asymptotic solutions yield accurate results along the paraxial (nearly axial) region. This is a well-known problem that has been observed for perfect electric conducting (PEC) and impedance cylinders in the past [3]-[4]. In this paper, a novel space-domain representation for the surface fields excited by an elementary current source is presented. These new expressions are very accurate along the paraxial region and in some cases can be made valid away from the paraxial region with some minor modifications

**Formulation:** Figure 1 shows the geometry of an electrically large dielectric coated circular cylinder. For an elementary surface electric current source in the $u$-direction ($u = \phi$ or $z$), the leading term of the circumferentially propagating ($\phi$ propagation) representation of the surface field component in the $l$-direction ($l = \phi$ or $z$) is given by

$$E_l(z,\phi) \approx \frac{1}{4\pi^2 d}\int_{-\infty}^{\infty} dk_z\, e^{-jk_z(z-z')}\int_{-\infty-j\epsilon}^{\infty-j\epsilon} G_{lu}(\nu,k_z) P_e^u\, e^{-j\nu(\phi-\phi')}d\nu\,,\ \epsilon > 0\,. \quad (1)$$

$G_{lu}(\nu, k_z)$ is the appropriate dyadic Green's function component which is explicitly given in [1] for both source and observation points located on the surface ($\rho = \rho' = d$). Performing a change of coordinates along with the geometrical relations given in [5], (1) is expressed in polar coordinates in the form of

$$E_l(s,\delta) \approx \frac{1}{2\pi}\int_0^{\infty}\int_0^{2\pi} \frac{G_{lu}(\zeta,\psi)}{2\pi}\, P_e^u\, e^{j\zeta s\cos(\psi-\delta)} d\psi\, \zeta\, d\zeta. \quad (2)$$

The Green's function components $G_{lu}(\zeta, \psi)$ are periodic with respect to $\psi$ with a period $\pi$ (i.e. $G_{lu}(\zeta,\psi) = G_{lu}(\zeta, \psi + \pi)$) and hence, can be approximated by a Fourier series (FS) where the FS coefficients are calculated using numerical integration. Based on numerical experimentation, only the two leading terms of the expansion are necessary in most cases. The net result of the above procedure is that $G_{lu}(\zeta, \psi)$ can be written as the product of two single variable ($\zeta$

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and $\psi$) functions. This leads to a closed-form analytical integration in $\psi$ and results in a considerable simplification in the evaluation of (2). Furthermore, if the abscissas ($\psi_p$) are chosen to be at $\psi_p = 0,\ \pi/2,\ \pi$ in the numerical evaluation of the FS coefficients, the function $G_{lu}(\zeta, \psi_p)$ is significantly simplified allowing a simpler numerical integration with respect to $\zeta$ in (2).

The $G_{zz}(\zeta, \psi)$ component is even with respect to $\psi$ and including the leading two terms of the FS expansion yields enough accuracy along the paraxial region. A 3-point trapezoidal rule is used in the interval $[0, \pi]$ for the calculation of the FS coefficients which are substituted into the series expansion to obtain the paraxial Green's function representation for this component [5]. The $G_{\phi z}(\zeta, \psi) = G_{z\phi}(\zeta, \psi)$ component, which is an odd function with respect to $\psi$, is rewritten by explicitly extracting the function $\frac{\zeta^2 \sin 2\psi}{2}$. The remaining expression is an even function with respect to $\psi$. However, unlike the $G_{zz}(\zeta, \psi)$ component, including only the leading term of the expansion for this component yields enough accuracy along the paraxial region. The leading FS coefficient is calculated performing a numerical integration in the $[0, \pi]$ interval using a 2-point trapezoidal rule. On the other hand, the $G_{\phi\phi}(\zeta, \psi)$ component requires a somewhat different approach. It is first written as the sum of *planar + curvature correction* terms. The planar term corresponds to a cylinder with an infinitely large radius of curvature. This planar term is already in the form of a two term FS expansion in $\psi$ and can be integrated in closed form along the $\psi$ integral. However, the curvature correction term, $G^{cc}_{\phi\phi}(\zeta, \psi)$, has a behavior similar to $G_{zz}(\zeta, \psi)$. Thus, it is approximated by a FS expansion where the FS coefficients are obtained numerically. As in the case for $G_{zz}(\zeta, \psi)$, numerical tests show that the two leading terms in the FS expansion yield accurate results. Details of this procedure can be found in [5].

Substituting the final expressions for the Green's function components into (2) and calculating the $\psi$ integrals in closed-form [5], the following expressions are obtained for the surface fields:

$$E_{zz}(\delta, s) \approx \frac{-Z_0}{2\pi k_0} \left\{ k_0^2 P(s) + \frac{\partial^2}{\partial z^2} [P(s) - Q(s)] \right\} \tag{3}$$

$$E_{\phi z}(\delta, s) \approx \frac{-Z_0}{2\pi k_0} \frac{\partial^2}{\partial z \partial l} \{M(s) - R(s)\} \tag{4}$$

$$E_{\phi\phi}(\delta, s) \approx \frac{-Z_0}{2\pi k_0} \left\{ k_0^2 U(s) + \frac{\partial^2}{\partial r_l^2} \left[ U(s) - \frac{\epsilon_r - 1}{\epsilon_r} W(s) \right] \right\} + \frac{jZ_0}{2\pi k_0} \left\{ S(s) + \frac{\partial^2}{\partial r_l^2} T(s) \right\} \tag{5}$$

where $P(s)$, $Q(s)$, $M(s)$, $R(s)$, $U(s)$, $W(s)$, $S(s)$ and $T(s)$ are special functions given explicitly in [5].

The integrals in $\zeta$ are evaluated numerically along the real axis using a Gaussian quadrature algorithm. An envelope extraction technique is used to overcome the difficulties in the numerical integration arising from the oscillatory as well as slowly decaying behavior of the integrands when necessary. The singularities of these integrands, which are located on the real axis (for lossless case) along the path of integration, are handled by regularizing the integrands. To implement this step, the singularities are found by means of a Newton-Raphson method.

**Results and Conclusion:** Figures 2 and 3 show the real and imaginary parts of the mutual impedance between two $z$-directed and two $\phi$-directed current modes, respectively, as a function of separation $s$ for $\alpha = 90°$ which corresponds to the axial direction. Note that

$$Z_{ij} = \int_{S_j} \mathbf{E}_i \cdot \mathbf{J}_j ds \qquad (6)$$

where $\mathbf{E}_i$ is the field due to source $\mathbf{J}_i$ and $S_j$ is the area occupied by source $\mathbf{J}_j$. In these figures, the results are compared with a standard eigenfunction solution for a large cylinder with $a = 3\lambda_0$, $t_h = 0.06\lambda_0$, $\epsilon_r = 3.25$ ($\lambda_0$ = free-space wavelength) and very good agreement is obtained. Additional numerical results concerning the source and observation points along the paraxial region as well as away from it will be presented.

To conclude, the scheme developed here yields field expressions that remain valid along the paraxial region for arbitrarily small and large separations between observation and source points. This work, in conjunction with the results of [1], can be used in a Method of Moments solution to design/analyze large arrays of conformal microstrip antennas.

**References:**

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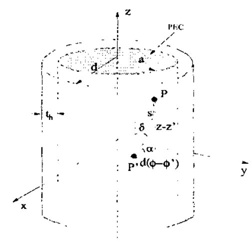

Figure 1: Dielectric coated PEC circular cylinder with inner radius $a$ and dielectric coating thickness $t_h = d - a$

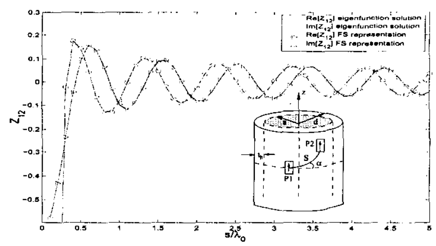

Figure 2: Real and imaginary parts of mutual impedance ($Z_{12}$) versus separation between two identical $z$−directed current sources for a coated cylinder with $a = 3\lambda_0$, $t_h = 0.06\lambda_0$, $\epsilon_r = 3.25$, $\alpha = 90°$

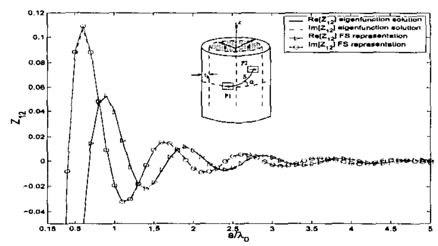

Figure 3: Real and imaginary parts of mutual impedance ($Z_{12}$) versus separation between two identical $\phi$−directed current sources for the same cylinder presented in Fig. 2, $\alpha = 90°$