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A minimally altruistic refinement of Nash equilibrium

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HIGHLIGHTS

- A rather minimalist notion of altruism is introduced.
- It is based on a lexicographic preference structure.
- Minimal altruism is used to refine the set of Nash equilibria in normal form games.
- Three independent existence proofs are provided for this new refinement concept.
- An in-depth sensitivity analysis is conducted and expository examples are given.

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ABSTRACT

We introduce a minimal notion of altruism and use it to refine Nash equilibria in normal form games. We provide three independent existence proofs, relate minimally altruistic Nash equilibrium to other equilibrium concepts, conduct an in-depth sensitivity analysis, and provide examples where minimally altruistic Nash equilibrium leads to improved predictions.

1. Introduction

In the last thirty years, we witnessed a surge in experimental studies in economics reporting altruistic or other-regarding behavior (see Güth et al. (1982), Roth et al. (1991), Forsythe et al. (1994), Güth and Van Damme (1998), Fehr and Gächter (2002) and Charness and Rabin (2002) among others). This observation is in stark contrast with the *selfish man*, an assumption to which most theoretical models in economics, if not all, were implicitly or explicitly subscribed. Nevertheless, the experimental evidence in favor of other-regarding behavior is so overwhelming that we also see an increasing number of theoretical models explaining/incorporating altruism (see Rabin (1993), Levine (1998), Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Gintis et al. (2003), Fehr and Fischbacher (2003), Falk and Fischbacher (2006) and Cox et al. (2007, 2008) among others).

With a few exceptions (see Cox et al. (2008)), in most theoretical papers modeling other-regarding behavior, altruism is incorporated into their models with an additively separable utility function: an agent directly cares about others through altruism or indirectly cares about others through his inequality aversion. In this paper, we introduce a different notion of altruism and use it to refine Nash equilibria in normal form games. In modeling other-regarding behavior, we use agents' preferences as a work-horse rather than their utility functions. In particular, we assume that each agent *may* care about the well-being of a set of other agents in addition to his own well-being, in a *lexicographic* fashion: a set of agents (including the agent himself) each agent cares about and an order on these agents are defined, where an agent's own well-being is at the top of this *priority* order. Agents may best respond to others' strategies with lexicographic preferences on outcomes. Therefore, an agent in a strategic game first maximizes his own well-being and then among the set of outcomes that maximize his own well-being (i.e., his best-response set), he prefers the ones that maximize the well-being of the agent ranked second in his priority order and so on.[1] Clearly, this notion of altruism is much less demanding than the standard notion of altruism where all agents' utilities/payoffs enter into a utility function at the same level but possibly with varying weights. This is why we label it as *minimal altruism*.[2] If an agent's priority set is a singleton, then this means

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[1] A step in a similar direction is taken by Dutta and Sen (2012) in the social choice context. These authors introduce *partially honest* individuals, who strictly prefer telling the truth if doing so does not lead to an outcome worse than lying does. The presence of such individuals turns out to be crucial for obtaining Nash implementability. Similar minimal or costless honesty notions are also used by Laslier and Weibull (2013) and Dutta and Laslier (2010) in jury and voting contexts, respectively. Finally, Doğan (2013) introduce *responsible* agents who first care about their own utility and then social efficiency (in a lexicographic manner) in an allocation problem.

[2] Our *minimal altruism* notion is different and even less demanding than the notion Fishkin (1982) introduced: Fishkin's *principle of minimal altruism*, as a moral principle, stipulates that if an agent, by incurring minor personal costs, can bring about great benefits (or prevent great harm), then he/she is morally obligated to do so. It is also compatible with the *limits of altruism* Hardin (1977) put forward: "Never ask a person to act against his own self-interest". Finally and more closely, it is identical to the interdependence condition, *minimal altruism*, formulated by Knoblauch (2001).

he only cares about himself and thus he is a *selfish* agent. If his set is not a singleton, then he is a *minimally altruistic* agent.[3] Note that an agent's priority set does not have to include all other agents in this case: he may only care about the well-being of a subset of others in which case he is still considered to be a minimally altruistic agent. This case would also resemble *nepotism*.

The minimal altruism we introduce is more relevant (or useful) in situations where (i) agents cannot influence their own payoffs to a great extent but can still influence others' payoffs, (ii) agents are indifferent between multiple actions (i.e., there are multiple actions/strategies that maximize their own well-being), or (iii) some agents in a strategic situation have just a single action. In many such circumstances, it is plausible to assume that most people would also care about others' well-being as well. In fact, experimental findings in Engelmann and Strobel (2004), Güth et al. (2010, 2012) and Cappelen et al. (2013) and in many other experiments using *impartial spectators*, provide – at least a partial – support for our notion: in these experiments, subjects whose earnings are (at least locally) fixed and not affected by their decisions make distributive decisions that are in line with efficiency, equality and equity concerns.

We provide three independent existence proofs for our equilibrium concept: using Kakutani's fixed point theorem as in the existence proof of Nash equilibrium, using Zhou's fixed point theorem as in the existence proof of Nash equilibrium in games with strategic complementarities, and using the existence of Berge–Nash equilibrium. The first existence result is the most natural and standard one since Nash equilibrium is a special case in our setup. With adaptations of some of the sufficient conditions for the existence of Nash equilibrium and an additional condition (level-k empathy), we guarantee the existence of minimally altruistic Nash equilibrium. On the other hand, the reason why we resort to games with strategic complementarities is that our refinement has much to offer when the game has more coordination aspect than pure competition aspect and when players' potential to influence others' payoffs is substantial. Games with strategic complementarities satisfy these requirements to a great extent. Finally, we use Berge–Nash equilibrium for yet another existence result. The Berge equilibrium concept assumes a different – and a rather extreme – sort of altruistic behavior. Thus, it is reasonable to investigate the relationship between minimally altruistic Nash equilibrium and Berge–Nash equilibrium.

Our results show that minimally altruistic Nash equilibrium leads to better and sharper predictions than Nash equilibrium in many instances, if one believes that our notion of altruism is realistic. As a result of the richer structure of game definition that incorporates priority sets and priority orders, even players who have single strategies or players who are indifferent between all of their strategies can influence the set of minimally altruistic Nash equilibria, which is a feature of many real-life circumstances. Note that such players cannot influence the set of Nash equilibria. On the other hand, it is also important to emphasize that the minimally altruistic refinement operates on the set of individual best responses and not on the set of Nash equilibria.[4] This has important and interesting implications: even though players care about others' well-being only after maximizing their own well-being, this does not necessarily imply that they will be equally well off in cases where they are selfish and altruistic. For instance, a player who starts to care about others (or becomes selfish) may face a less or more favorable set of payoffs by doing that. Similarly, a player who nobody cared about before may face a less favorable set of payoffs after some (or even all) players start caring about him. Concerning the comparison between minimally altruistic Nash equilibrium and Berge–Nash equilibrium, we show that in many instances where Berge–Nash equilibrium does not exist, minimally altruistic Nash equilibrium exists. Finally, by conducting an in-depth sensitivity analysis we show that the set of equilibria is highly sensitive to the set of players each player cares about and the priority order each player has.

It is worth mentioning that minimally altruistic refinement is a complement rather than a substitute for other refinement concepts (e.g., essential equilibrium etc.). Our refinement is different than these in that it is *not* based on players making mistakes and hence does not use any perturbations. Moreover, minimally altruistic refinement is also different from coalition-proof and strong Nash refinements in that it is not based on a coalitional structure. Nevertheless, we do not see any element in the minimally altruistic refinement, which would pose a problem for applying it together with one of these other refinements.

The paper is structured as follows: in Section 2, we present some preliminaries, i.e., definitions and results we employ throughout the paper. In Section 3, we introduce minimal altruism, provide the formal definition for minimally altruistic Nash equilibrium and existence results. Section 4 provides further results. In Section 5, we present some examples where we refine the set of Nash equilibria using minimal altruism. Finally, Section 6 concludes.

2. Preliminaries

In what follows, we provide some definitions and theorems that we utilize throughout the paper. First, a set of definitions:

Definition 1 (*Quasiconcavity*)**.** A function $f : X \to \mathbb{R}$ on a convex set X is *quasiconcave* if for every $a \in \mathbb{R}$, $\{x \in X | f(x) \geq a\}$ is convex.

Definition 2 (*Closed Graph*)**.** Let X and Y be any topological spaces. A correspondence $F : X \to Y$ has a *closed graph* if $x \in F(y)$ for any two sequences $(x^n) \to x$ and $(y^n) \to y$ with for every n: $x^n \in F(y^n)$.

Definition 3 (*Upper Semi-Continuity*)**.** A function $f : X \to \mathbb{R}$ is *upper semi-continuous* if for every $x \in X$ and every sequence (x^n) with $(x^n) \to x$, $\limsup f(x^n) \leq f(x)$.

Definition 4 (*Lattice and Complete Lattice*)**.** A partially ordered set is a *lattice* if it contains the supremum and the infimum of all pairs of its elements. A lattice is *complete* if each nonempty subset has a supremum and an infimum.

Definition 5 (*Subcomplete Sublattice*)**.** Let X be a lattice and $Y \subset X$ be a sublattice. Y is a *subcomplete sublattice* of X if, for each nonempty subset Y' of Y, $\bigvee_X Y'$ and $\bigwedge_X Y'$ exist and are contained in Y.

Definition 6 (*Supermodularity*)**.** Let X be a lattice. A function $f : X \to \mathbb{R}$ is *supermodular* if for all $x, x' \in X$, $f(x)+f(x') \leq f(x \wedge x') + f(x \vee x')$.

[3] Mathematically speaking, modeling altruism in this fashion (i.e., with lexicographic preferences) is equivalent to taking the limit of CES family of utility functions (over agents' utilities), where the weight an agent attaches to an agent ranked at kth place in his priority order becomes infinitely greater than the weight he attaches to an agent ranked at $(k+1)$th place. Since all agents rank themselves at the top of their priority order, they care about themselves infinitely more than any other agent. Hence, the adjective, *minimal*, also makes sense from a mathematical point of view.

[4] First of all, refining the set of Nash equilibria by directly eliminating some of them would not be a significant innovation from an intellectual perspective. Moreover, what is modeled in that case would not be pure altruism since other-regarding behavior would not influence players' behavior in the game.

Definition 7 (*Increasing Differences Property*). Let X be a lattice and T be a partially ordered set. A function $f : X \times T \to \mathbb{R}$ has *increasing differences* in (x, t) if $f(x, t') - f(x, t)$ is increasing in x for every $t < t'$.

Definition 8 (*Veinott Increasingness*). Let X be a lattice. A correspondence $F : X \to X$ is *Veinott-increasing* if for each $x, y \in X$ with $x < y$, $a \in F(x)$ and $b \in F(y)$ implies $a \wedge b \in F(x)$ and $a \vee b \in F(y)$.

Definition 9 (*Fixed Point*). A *fixed point* of a function $f : X \to X$ is $x \in X$ such that $f(x) = x$. The *set of fixed points* of a correspondence $F : X \to X$ is defined as $\{x \in X | x \in F(x)\}$.

Finally, we refer to the following theorems throughout the paper:

Theorem 1 (Kakutani (1941)). *Let X be a nonempty, compact, and convex subset of Euclidean space. If $F : X \to X$ is a nonempty-valued and convex-valued correspondence with a closed graph, then F has a fixed point.*

Theorem 2 (Zhou (1994)). *Let X be a nonempty, complete lattice and $F : X \to X$ be a nonempty-valued correspondence. If F is Veinott-increasing and $F(x)$ is a subcomplete sublattice of X for every $x \in X$, then the fixed point set of F is a nonempty complete lattice.*

Theorem 3 (Topkis (1998)). *Let X be a nonempty, complete lattice, T be a partially ordered set, and $Y : X \times T \to X \times T$ be a correspondence. If Y is increasing, and $Y(x, t)$ is a nonempty subcomplete sublattice of $X \times T$ for each $(x, t) \in X \times T$, then*

(i) *for all $t \in T$, there exists a greatest (least) fixed point of $Y(x, t)$,*
(i) *the greatest (least) fixed point of $Y(x, t)$ is increasing in t on T.*

3. Minimally altruistic Nash equilibrium

In this section, we first define minimal altruism and minimally altruistic Nash equilibrium concepts. Then, we show that minimally altruistic Nash equilibrium is indeed a refinement of Nash equilibrium. Finally, we provide existence results for this refinement concept.

3.1. Minimal altruism and refinement

Let $\Gamma = (N, (X_i)_{i\in N}, (u_i)_{i\in N})$ be a normal form game where N is the finite set of players, X_i is the set of strategies for player i, and u_i is player i's payoff function. Let S_i be a subset of N that includes i and $(S_i)_{i\in N}$ be a collection of such subsets for all $i \in N$. Then, let $\succ_i$ be a strict order defined on S_i such that $i \succ_i j$, for all $j \in S_i \setminus \{i\}$. Finally, for any $i \in N$ and $j \in S_i$, let $\varphi(i, j)$ denote agent j's rank in agent i's priority order, $\succ_i$.[5] We define $\Gamma_{MA} = (N, (X_i)_{i\in N}, (S_i)_{i\in N}, (\succ_i)_{i\in N}, (u_i)_{i\in N})$ as a minimally altruistic version of a normal form game where each player is associated with one priority set and a priority order on it. To define the minimally altruistic Nash equilibrium, we first define minimal altruism.

Definition 10. Let R_i be a preference relation on $\mathbb{R}^{|N|}$. Let P_i denote the strict preference and I_i denote the indifference induced by R_i. For a given $(S_i, \succ_i)$, an agent i is a minimally altruistic agent if his preference among any two payoff vectors $e = (e_1, \ldots, e_i, \ldots, e_{|N|})$ and $e' = (e'_1, \ldots, e'_i, \ldots, e'_{|N|})$ is written as

$$\begin{aligned}
&eP_ie' \quad \text{if } e_i > e'_i\\
&eP_ie' \quad \text{if } e_i = e'_i \quad \text{and} \quad e_j > e'_j \quad \text{where } \varphi(i,j) = 2\\
&\cdots \quad \cdots\\
&\cdots \quad \cdots\\
&eP_ie' \quad \text{if } \forall k \in S_i \setminus \{m\}, e_k = e'_k \quad \text{and} \quad e_m > e'_m\\
&\quad \text{where } \varphi(i,m) = |S_i|\\
&eI_ie' \quad \text{if } \forall k \in S_i, \qquad e_k = e'_k.
\end{aligned}$$

Now, we can define minimally altruistic refinement of Nash equilibrium. For an agent $i \in N$ and for every $y \in X$, let

$$X_{i,1}(y) = \arg\max_{x\in X} u_i(x_i, y_{-i}).$$

Then, for every $y \in X$ and for every $k = 1, \ldots, |S_i| - 1$, let

$$X_{i,k+1}(y) = \arg\max_{x\in X_{i,k}(y)} u_{j_{k+1}}(x_i, y_{-i})$$

such that $j_1 = i$ and $\varphi(i, j_{k+1}) = k + 1$.

Definition 11 (*MANE*). A strategy profile $x^* \in X$ is a minimally altruistic Nash equilibrium (MANE) if for all $i \in N$: $x^* \in X_{i,|S_i|}(x^*)$.

Notice that S_i and $\succ_i$ are included in the definition of the game (Γ_{MA}) whereas R_i is included in the definition of the equilibrium concept. Thus, how the information provided by S_i and $\succ_i$ is processed is given in the equilibrium concept. It can be argued that we follow a *normative* approach here, by including altruism in the equilibrium concept, *MANE*. Alternatively, one can follow a *positive* approach by investigating the Nash equilibrium outcomes in games where players have altruistic preferences.[6] Finally, it is worthwhile emphasizing that minimally altruistic Nash equilibrium uses, whereas the Nash equilibrium concept ignores, the information provided by S_i and $\succ_i$.

Notice that for any player i, $X_{i,k}(\cdot)$ in this definition is a subset of the best response correspondence of i according to Nash, $BR_i(\cdot)$, which is equal to $X_{i,1}(\cdot)$. This directly implies the following result.

Proposition 1. *Minimally altruistic Nash equilibrium is a refinement of Nash equilibrium.*

Proof. Take any *MANE*, $x^* \in X$. Then for all $i \in N$, $x^* \in X_{i,|S_i|}(x^*)$. By definition, for all $i \in N$, $x^* \in X_{i,1}(x^*) = BR_i(x^*)$. Thus, x^* is a *NE*.

The following example demonstrates that a *NE* is *not* necessarily a *MANE*.

	x_2
x_1	2, 1
y_1	2, 0

Here, both strategy profiles are *NE*, but (y_1, x_2) is not a *MANE* given that $S_1 = \{1, 2\}$. ■

3.2. Existence

In the following parts, we provide three independent existence results: utilizing (i) Kakutani's fixed point theorem, (ii) Zhou's fixed point theorem, and (iii) the Berge–Nash equilibrium existence result.[7]

5 For the sake of completeness, for every $i, j \in N$ with $j \notin S_i$, we set $\varphi(i, j) = n+1$.

6 We thank Tarık Kara for bringing up this issue during our discussions.

7 A mixed strategy version of a normal form game is, by definition, a normal form game. Hence, these existence results are valid for equilibrium in mixed strategies as well.

3.2.1. Existence through Kakutani fixed point theorem

The following existence result builds on the fact that Nash equilibrium is a special case of minimally altruistic Nash equilibrium (i.e., selfish best-responses). Then, one intuitively expects that the existence of minimally altruistic Nash equilibrium can be guaranteed by taking sufficient conditions for Nash equilibrium (see Nash (1950)) as baseline and adding more conditions or modifying the existing ones. In fact, this is what we do in the following proposition.

We first define $B^i_j : X \to X$ such that for every $y \in X$:

$$B^i_j(y) = \{x \in X \mid \forall x'_i \in X_i : u_j(x_i, y_{-i}) \geq u_j(x'_i, y_{-i})\}.$$

That is, given a strategy profile of $-i$, agent j chooses a strategy for agent i from X_i in order to maximize his own payoff. This correspondence is used in the definition of *level-k empathy,* which relates the best responses of agents.

Axiom 1 (*Level-k Empathy*)**.** For $k \geq 2$, agent j is level-k empathetic towards agent i if for every $x \in X$: $X_{i,k-1}(x) \cap B^i_j(x)$ is nonempty.

If agent j is empathetic towards agent i, then agent i would not complain if agent j selfishly selects a strategy for agent i.

Proposition 2. *In game Γ_{MA}, if* (i) *each X_i is a nonempty, compact, and convex subset of a Euclidean space,* (ii) *each u_i is quasiconcave in x_j for every $j \in N$ with $i \in S_j$ and is continuous in x, and* (iii) *for every $i \in N$, each $j \in S_i$ is level-$\varphi(i, j)$ empathetic towards i, then MANE exists.*

Proof. Take any $i \in N$. It follows from the existence result of Nash equilibrium that $BR_i = X_{i,1}$ is nonempty-valued, convex-valued, and has a closed graph. This implies for all $x \in X$ that $X_{i,1}(x)$ is nonempty, compact, and convex.

Now, take any $x \in X$ and $i \in N$. Then, take some $j \in S_i$ such that $\varphi(i, j) = 2$. Since agent i maximizes u_j on a nonempty, compact, and convex set, $X_{i,1}(x)$, and u_j is quasiconcave in x_i and continuous in x by assumption, it follows that $X_{i,2}(x)$ is nonempty, compact, and convex. Recursively, for every $x \in X$, $X_{i,|S_i|}(x)$ is nonempty, compact, and convex as well.

We have that $X_{i,|S_i|}$ is nonempty-valued, convex-valued, and compact-valued. To utilize Kakutani's fixed point theorem we need to show that $X_{i,|S_i|}$ has a closed graph. For that, take any two sequences $(x^m) \to x$ and $(y^m) \to y$ such that for every m, $x^m \in X_{i,|S_i|}(y^m)$. Then, for every m, we have $x^m \in X_{i,k}(y^m)$ for every $k \in \{1, \ldots, |S_i| - 1\}$. Since for all m and for all $x'_i \in X_i$: $u_i(x^m_i, y^m_{-i}) \geq u_i(x'_i, y^m_{-i})$ (by optimality) and u_i is continuous, we have $u_i(x_i, y_{-i}) \geq u_i(x'_i, y_{-i})$ for every $x'_i \in X_i$. Then, $x \in X_{i,1}(y)$. Now, consider agent j with $\varphi(i, j) = 2$. Since j is level-2 empathetic towards i by assumption, we have for all m and for all $x' \in X_{i,1}(y)$: $u_j(x^m_i, y^m_{-i}) \geq u_j(x'_i, y^m_{-i})$ by optimality. Then, by continuity of u_j in x, we also have $u_j(x_i, y_{-i}) \geq u_i(x'_i, y_{-i})$ for every $x' \in X_{i,1}(y)$, which implies $x \in X_{i,2}(y)$. Since level-k empathy is assumed for every $j \in S_i$, it recursively follows that $x \in X_{i,k}(y)$ for every $k \in \{1, \ldots, |S_i|\}$. Hence, $x \in X_{i,|S_i|}(y)$, that is $X_{i,|S_i|}$ has a closed graph.

All of these four properties are preserved under finite intersections. Then the joint best response correspondence according to *MANE*, defined by $\bigcap_{i \in N} X_{i,|S_i|}$, satisfies the conditions of Kakutani's fixed point theorem. Therefore, the set of fixed points is nonempty, i.e. *MANE* exists. ■

Note that we just strengthen the quasiconcavity requirement compared to the sufficient conditions for the (standard) existence theorem for Nash equilibrium, and additionally assume level-k empathy. The former is a relatively minor and intuitive modification, whereas the latter is a very restrictive assumption.

3.2.2. Existence through games with strategic complementarities

Characterized by increasing joint best reply, games with strategic complementarities (GSC) rely on the extension of Tarski's fixed point theorem for correspondences (see Veinott (1992) and Zhou (1994)) and a lattice-based approach to monotone comparative statics (see Topkis (1978), Vives (1990) and Milgrom and Roberts (1990)). For our minimally altruistic refinement, GSC can be formalized in the following way:

Definition 12. A game Γ_{MA} has strategic complementarities *à la* minimally altruistic Nash (hence, is a GSC *à la* minimally altruistic Nash) if (i) each X_i is a complete lattice, (ii) each best response correspondence according to minimally altruistic Nash equilibrium, $X_{i,|S_i|}$, is nonempty-valued and Veinott-increasing, and (iii) for all $x \in X$, $X_{i,|S_i|}(x)$ is a subcomplete sublattice of X.

The lattice-based approach to monotone comparative statics under the notion of Nash equilibrium establishes that each player's payoff function needs to be supermodular in his own strategies and satisfy increasing differences in order to have Veinott-increasing joint best reply. Stemming from these sufficient conditions on payoffs, such classes of games with monotone best responses are referred to as supermodular games (see Topkis (1998)). However, under the notion of minimally altruistic Nash equilibrium, one needs further requirements to have Veinott-increasing joint best reply: the payoff to each player i needs to be supermodular and satisfy increasing differences with respect to the strategies of every player who cares about i.[8] In this respect, a minimally altruistic supermodular game can be defined as follows.

Definition 13. A game Γ_{MA} is minimally altruistic supermodular if (i) each X_i is a nonempty, compact, and complete lattice, (ii) each u_i is supermodular in x_j for every $j \in N$ with $i \in S_j$, (iii) each u_i has increasing differences in (x_j, x_{-j}) for every $j \in N$ with $i \in S_j$, and (iv) each u_i is upper semi-continuous in x_j for every $j \in N$ with $i \in S_j$.

Note that a minimally altruistic supermodular game is, by definition, a supermodular game. However, a GSC *à la* minimally altruistic Nash need not be a GSC.

Proposition 3. *The following statements on minimally altruistic refinement of Nash equilibrium are valid.*

(i) *A minimally altruistic supermodular game is a GSC à la minimally altruistic Nash.*
(ii) *In a GSC à la minimally altruistic Nash (hence, in a minimally altruistic supermodular game), the set of MANE is a nonempty complete lattice.*

Proof. For (i), take any $i \in N$ and $j \in S_i$ such that $\varphi(i, j) = 2$.

First, since u_i is upper semi-continuous in x_i, $X_{i,1}(x) = BR_i(x)$ is nonempty for all $x \in X$. Then, since u_j is upper semi-continuous in x_i, $X_{i,2}(x)$ is also nonempty for all $x \in X$. Similarly, we conclude that for every $k = 1, \ldots, |S_i|$, $X_{i,k}$ is nonempty-valued.

Now, take any $x, y \in X$ with $x < y$. Take $a \in X_{i,|S_i|}(x)$ and $b \in X_{i,|S_i|}(y)$ as well. This implies that $a \in X_{i,k}(x)$ and $b \in X_{i,k}(y)$ for every $k = 1, \ldots, |S_i|$. For Veinott-increasingness, we need to show that $a \wedge b \in X_{i,|S_i|}(x)$ and $a \vee b \in X_{i,|S_i|}(y)$. If $a < b$, the result is trivial. If not,

$$\begin{aligned} 0 &\leq u_i(a_i, x_{-i}) - u_i(a_i \wedge b_i, x_{-i}) \\ &\leq u_i(a_i \vee b_i, x_{-i}) - u_i(b_i, x_{-i}) \\ &\leq u_i(a_i \vee b_i, y_{-i}) - u_i(b_i, y_{-i}) \leq 0. \end{aligned}$$

8 In its essence, this modification is similar to the one we make in Proposition 2.

Here, the first and the last inequalities follow from optimality since $a \in X_{i,1}(x)$ and $b \in X_{i,1}(y)$. Supermodularity implies the second inequality and increasing differences property implies the third inequality. Then, $a \wedge b \in X_{i,1}(x)$ and $a \vee b \in X_{i,1}(y)$. Recalling that $a \in X_{i,2}(x)$ and $b \in X_{i,2}(y)$, we have

$$\begin{aligned} 0 &\leq u_j(a_i, x_{-i}) - u_j(a_i \wedge b_i, x_{-i}) \\ &\leq u_j(a_i \vee b_i, x_{-i}) - u_j(b_i, x_{-i}) \\ &\leq u_j(a_i \vee b_i, y_{-i}) - u_j(b_i, y_{-i}) \leq 0. \end{aligned}$$

Here, the first and the last inequalities are valid for every element of $X_{i,1}(x)$ and $X_{i,1}(y)$, including $a \wedge b \in X_{i,1}(x)$ and $a \vee b \in X_{i,1}(y)$. The second and the third inequalities follow from assumptions that u_j is supermodular in x_i and u_j has increasing differences. Then, $a \wedge b \in X_{i,2}(x)$ and $a \vee b \in X_{i,2}(y)$. Similar arguments will follow for every $j' \in S_i$. Then $a \wedge b \in X_{i,|S_i|}(x)$ and $a \vee b \in X_{i,|S_i|}(y)$, so that $X_{i,|S_i|}$ is Veinott-increasing.

Finally, consider any $x \in X$ and take any $a, b \in X_{i,|S_i|}(x)$. Note that $a, b \in X_{i,k}(x)$ for every $k = 1, \ldots, |S_i|$. First we have,

$$u_i(a_i, x_{-i}) = u_i(b_i, x_{-i}) \geq u_i(c_i, x_{-i}), \quad \forall c_i \in X_i$$

since $a, b \in X_{i,1}(x)$ and

$$u_i(a_i, x_{-i}) + u_i(b_i, x_{-i}) \leq u_i(a_i \wedge b_i, x_{-i}) + u_i(a_i \vee b_i, x_{-i})$$

by supermodularity of u_i in x_i. Then, it directly follows that $u_i(a_i, x_{-i}) = u_i(a_i \wedge b_i, x_{-i}) = u_i(a_i \vee b_i, x_{-i})$, i.e. $a \wedge b$, $a \vee b \in X_{i,1}(x)$. With this result and similar arguments, we have the following:

$$u_j(a_i, x_{-i}) = u_j(b_i, x_{-i}) \geq u_j(c_i, x_{-i}), \quad \forall c_i \in X_{i,1}(x)$$

since $a, b \in X_{i,2}(x)$ and

$$u_j(a_i, x_{-i}) + u_j(b_i, x_{-i}) \leq u_j(a_i \wedge b_i, x_{-i}) + u_j(a_i \vee b_i, x_{-i})$$

by supermodularity of u_j in x_i. Similar arguments will follow for every $j' \in S_i$. Then $a \wedge b \in X_{i,|S_i|}(x)$ and $a \vee b \in X_{i,|S_i|}(x)$, so that $X_{i,|S_i|}(x)$ is a subcomplete sublattice of X for all $x \in X$.[9] Then, the game is a GSC *à la* minimally altruistic Nash.

For (ii), since the properties satisfied by $X_{i,|S_i|}$ are preserved under finite intersections, $\bigcap_{i \in N} X_{i,|S_i|}$ satisfies the conditions of Zhou's fixed point theorem. Therefore, the set of *MANE* is a nonempty complete lattice. ∎

The utilization of strategic complementarities not only allows us to establish the existence of *MANE* but also provides a sharp characterization of the set of *MANE*. Moreover, referring to the constructive proof of Zhou's extension of Tarski's fixed-point theorem to set valued maps (see Echenique (2005)), it provides a simple iterative procedure to compute the extremal equilibria. Note also that we can provide a monotone comparative statics result on the set of *MANE*, utilizing Topkis' theorem (Topkis, 1998). In particular, letting T be a partially ordered set and $(\Gamma^t_{MA})_{t \in T}$ be a collection of GSC *à la* minimally altruistic Nash, the least *MANE* and the greatest *MANE* are increasing in t on T.

3.2.3. Existence through Berge–Nash equilibrium

We first provide a definition of Berge equilibrium (*BE*). The definition we provide below is commonly referred to as Berge equilibrium in the sense of Zhukovskii (1994) in the literature.[10] As it can be seen in the definition below, Berge equilibrium has a rather extreme version of altruism embedded in: for any i and given player i's strategy, all other players choose their strategies so as to maximize agent i's well-being.

Definition 14 (*Berge Equilibrium*)**.** In game Γ, x^* is a Berge equilibrium if for every $i \in N$, we have $u_i(x^*) \geq u_i(x^*_i, x_{-i})$ for every $x_{-i} \in X_{-i}$.

Berge–Nash equilibrium (*BNE*) directly follows as the intersection of Berge equilibrium with Nash equilibrium.

Definition 15 (*Berge–Nash Equilibrium*)**.** In game Γ, x^* is a Berge–Nash equilibrium if it is both a Berge equilibrium and a Nash equilibrium.

The following definition introduces the *reduced game* notion we utilize in the existence result that follows.

Definition 16. For given Γ_{MA}, $x \in X$ and $i, j \in N$ with $i \neq j$, we define the reduced game $\Gamma_{\{i,j\}}(x) = (\{i, j\}, X_i \times X_j, (v_i, v_j))$ such that $v_k : X_i \times X_j \to \mathbb{R}$ is given by $v_k(a_i, a_j) = u_k(a_i, a_j, x_{-\{i,j\}})$ for every $k \in \{i, j\}$ and for every $(a_i, a_j) \in X_i \times X_j$.

Lemma 1. *The strategy profile $x^* \in MANE(\Gamma_{MA})$ if for every $i, j \in N$ with $j \in S_i \setminus \{i\}$: $(x^*_i, x^*_j) \in B_i(x^*_i, x^*_j)$, where $B_i(\cdot)$ is a best response of i according to Berge–Nash equilibrium for the reduced game $\Gamma_{\{i,j\}}(x^*)$.*

Proof. Take any x^* such that for every $i, j \in N$ with $j \in S_i \setminus \{i\}$: $(x^*_i, x^*_j) \in B_i(x^*_i, x^*_j)$. Take any $i \in N$. We have $u_i(x^*) \geq u_i(x_i, x^*_{-i})$, $\forall x_i \in X_i$. Also, for every $j \in S_i \setminus \{i\}$: $u_j(x^*) \geq u_j(x_i, x^*_{-i})$, $\forall x_i \in X_i$. We know that $x^* \in X_{i,1}(x^*)$. Then, recursively, we can say that for every $k \in 2, \ldots, |S_i|$, $x^* \in X_{i,k}(x^*)$. Since i is arbitrarily chosen, the result follows. ∎

The following proposition uncovers an interesting relationship between Berge–Nash equilibrium and minimally altruistic Nash equilibrium.

Proposition 4. *If a normal form game Γ has a Berge–Nash equilibrium, then Γ_{MA} has a minimally altruistic Nash equilibrium. In fact, $BNE(\Gamma) = BNE(\Gamma_{MA}) \subset MANE(\Gamma_{MA})$.*

Proof. First of all, *BNE* of a game equals *BNE* of the minimally altruistic version of the game, since *BNE* does not depend on any S_i or $\succ_i$. For the latter, take $x^* \in BNE(\Gamma_{MA})$. Consider any relevant reduced game of Γ defined as above. Then, obviously, x^* satisfies the condition in Lemma 1 for every $i \in N$ and $j \in S_i \setminus \{i\}$. Thus, $x^* \in MANE(\Gamma_{MA})$. ∎

Larbani and Nessah (2008) provide sufficient conditions for the existence of *BNE*. Under the same set of conditions, the proposition above shows that *MANE* exists.

4. Further results

In this section, we provide further results on minimally altruistic Nash equilibrium.

The following remark relates *MANE* to *NE* and *BNE*. It focuses on the trivial case where players care *only* about themselves. In that case, *MANE* and *NE* are, not surprisingly, equivalent. On the other hand, the same remark also states that even if each player cares about all the other players, *MANE* may not be reduced to *BNE*.

Remark 1. In a game Γ_{MA}, if for every $i \in N$, $S_i = \{i\}$, then $MANE(\Gamma_{MA}) = NE(\Gamma_{MA})$. On the other hand, even if for all $i \in N$, $S_i = N$, it may be the case that $MANE(\Gamma_{MA}) \neq BNE(\Gamma_{MA})$.

[9] The best response correspondences are closed-valued in the interval topology (see Zhou (1994)). And a closed interval in a complete lattice X is a subcomplete sublattice of X.

[10] We refer to Zhukovskii's (1994) definition for Berge equilibrium since Berge (1957) himself offered only an intuitive/informal definition.

Proof. In the former case, for all $i \in N$, $X_{i,|S_i|} = X_{i,1} = BR_i$. The result follows. For the latter case, we again provide an example. Consider the following normal form game with three players in which Player 1 has two strategies whereas Player 2 and Player 3 have only one strategy.

	x_3
	x_2
x_1	2, 1, 0
y_1	2, 0, 1

Here, the set of *BNE* is empty but for every $(S_i)_{i\in N}$ and $(\succ_i)_{i\in N}$, *MANE* exists. ■

The following remark states that the set of *MANE* will be smaller as for a given player i, $|S_i|$ gets larger with a specific modification on the strict order, $\succ_i$. In particular, the remark indicates that if a player i, in addition to the current set of players he cares about, starts to care about some other players who he did not care about initially, the set of *MANE* of this new game will not be larger than the *MANE* of the initial game.

Remark 2. Let Γ be a normal form game. Take an arbitrary $i \in N$, and define $\succ'_i$ on N. Let Γ^k_{MA} with $(S_{i,k}, \succ_{i,k})$ be such that $S_{i,k}$ consists of the top k elements in $\succ'_i$ and $\succ_{i,k} = \succ'_i|_{S_{i,k}}$ for given $(S_j)_{j\in N\setminus\{i\}}$ and $(\succ_j)_{j\in N\setminus\{i\}}$. Then, $MANE(\Gamma^\beta_{MA}) \subset MANE(\Gamma^\alpha_{MA})$ if $\alpha < \beta$.

Proof. Take $x^* \in MANE(\Gamma^\beta_{MA})$ for some $\beta \in \{2, \dots, |N|\}$. Consider an arbitrary α with $\alpha < \beta$. By definition, for every $i \in N$ and $k \in \{1, \dots, \beta\}$, $x^* \in X_{i,k}(x^*)$. Since $\alpha < \beta$, it trivially follows that $x^* \in MANE(\Gamma^\alpha_{MA})$. ■

The next remark is, in fact, a corollary to the remark above. It implies that starting with completely selfish players and increasing the cardinality of their priority sets in the way described in Remark 2, *MANE* refines the set of *NE* in a monotonic fashion.

Remark 3. Let $(\Gamma^k_{MA}) = ((N, (X_i)_{i\in N}, (S^k_i)_{i\in N}, (\succ^k_i)_{i\in N}, (u_i)_{i\in N}))$ be a collection of minimally altruistic versions of the same normal form game Γ. Let $S^0_i = \{i\}$ for every $i \in N$ and define Γ^{k+1}_{MA} by a modification on Γ^k_{MA} for some $i \in N$ such that $\sum_{i\in N} |S^{k+1}_i \setminus \{i\}| = \sum_{i\in N} |S^k_i \setminus \{i\}| + 1 = k + 1$. Then, the cardinality of the set of MANE is nonincreasing in k.

Proof. The result follows from a recursive application of the exercise described in Remark 2. ■

The common message of the following remarks (Remarks 4–9) is that the set of *MANE* is, as expected, very sensitive to each player's priority set and the priority orders on these sets. Remarks here show that even very minor changes on priority sets and orders may lead to changes in the set of equilibria in ways that cannot be systematically predicted.[11]

Remark 4. Let Γ_{MA} and $\Gamma_{MA'}$ be minimally altruistic versions of the same normal form game Γ such that the only difference is that for some $i \in N$, $S'_i \subset S_i$ and $\succ'_i = \succ_i|_{S'_i}$. Then, it may be the case that $MANE(\Gamma_{MA}) \not\subset MANE(\Gamma_{MA'})$ and $MANE(\Gamma_{MA'}) \not\subset MANE(\Gamma_{MA})$.

[11] The sensitivity of the predictions of our refinement concept to priority sets and orders should not be seen as a weakness. This sensitivity is of the same sort Nash equilibrium has with respect to payoffs. It is a known fact that improving the ex-ante position of a player in a game by, for instance, increasing his payoffs in some strategy combinations does not necessarily imply a higher equilibrium payoff for the player.

Proof. Consider the following normal form game:

	x_3
	x_2
x_1	2, 1, 0
y_1	2, 0, 1

Let $S_1 = \{1, 2, 3\}$, $S'_1 = \{1, 3\}$, and $1 \succ_1 2 \succ_1 3$. Define Γ_{MA} and $\Gamma_{MA'}$ accordingly. Now, $MANE(\Gamma_{MA}) = \{(x_1, x_2, x_3)\}$ and $MANE(\Gamma_{MA'}) = \{(y_1, x_2, x_3)\}$. ■

Remark 5. Let Γ_{MA} and $\Gamma_{MA'}$ be minimally altruistic versions of the same normal form game Γ such that the only difference is that for some $i \in N$, $\succ_i \neq \succ'_i$. Then, it may be the case that $MANE(\Gamma_{MA}) \not\subset MANE(\Gamma_{MA'})$ and $MANE(\Gamma_{MA'}) \not\subset MANE(\Gamma_{MA})$.

Proof. Consider the following normal form game:

	x_3
	x_2
x_1	2, 1, 0
y_1	2, 0, 1

Let $S_1 = \{1, 2, 3\}$, $1 \succ_1 2 \succ_1 3$ and $1 \succ'_1 3 \succ'_1 2$. Define Γ_{MA} and $\Gamma_{MA'}$ accordingly. Now, $MANE(\Gamma_{MA}) = \{(x_1, x_2, x_3)\}$ and $MANE(\Gamma_{MA'}) = \{(y_1, x_2, x_3)\}$. ■

Remark 6. Let Γ_{MA} and $\Gamma_{MA'}$ be minimally altruistic versions of the same normal form game Γ such that the only difference is that for some $i \in N$, $S'_i \setminus S_i = \{j'\}$ and $S_i \setminus S'_i = \{j\}$ with $\varphi(i, j) = \varphi'(i, j')$. Then, it may be the case that $MANE(\Gamma_{MA}) \not\subset MANE(\Gamma_{MA'})$ and $MANE(\Gamma_{MA'}) \not\subset MANE(\Gamma_{MA})$.

Proof. Consider the following normal form game:

	x_3
	x_2
x_1	2, 1, 0
y_1	2, 0, 1

Let $S_1 = \{1, 2\}$ and $S'_1 = \{1, 3\}$. Define Γ_{MA} and $\Gamma_{MA'}$ accordingly. Now, $MANE(\Gamma_{MA}) = \{(x_1, x_2, x_3)\}$ and $MANE(\Gamma_{MA'}) = \{(y_1, x_2, x_3)\}$. ■

Remark 7. Let Γ_{MA} and $\Gamma_{MA'}$ be minimally altruistic versions of the same normal form game Γ such that the only difference is that for some $i, j \in N$ with $i \notin S_j \cup S'_j$ and $j \notin S_i \cup S'_i$: $\varphi(i, k) = \varphi'(j, k)$ and $\varphi(j, k) = \varphi'(i, k)$ for every $k \in N \setminus \{i, j\}$. Then, it may be the case that $MANE(\Gamma_{MA}) \not\subset MANE(\Gamma_{MA'})$ and $MANE(\Gamma_{MA'}) \not\subset MANE(\Gamma_{MA})$.

Proof. Consider the following normal form game:

	x_3
	x_2
x_1	2, 1, 0
y_1	2, 0, 1

Let $S_1 = \{1, 2\}$, $S_3 = \{3\}$, $S'_1 = \{1\}$ and $S'_3 = \{3, 2\}$. Define Γ_{MA} and $\Gamma_{MA'}$ accordingly. Now, $MANE(\Gamma_{MA}) = \{(x_1, x_2, x_3)\}$ and $MANE(\Gamma_{MA'}) = \{(x_1, x_2, x_3), (y_1, x_2, x_3)\}$. Hence, $MANE(\Gamma_{MA}) \not\subset MANE(\Gamma_{MA'})$. Note that the converse is symmetric. ■

Remark 8. Let Γ_{MA} and $\Gamma_{MA'}$ be minimally altruistic versions of the same normal form game Γ such that the only difference is that for some $j \in N$ and for every $i \in N \setminus \{j\}$: $\varphi(i, j) > \varphi'(i, j)$. Then, it may be the case that maximum equilibrium payoff for player j in $MANE(\Gamma_{MA'})$ is smaller than minimum equilibrium payoff for player j in $MANE(\Gamma_{MA})$.

Proof. Consider the following normal form game:

x_3

	x_2	y_2
x_1	1, 3, 0	1, 4, 0
y_1	1, 2, 1	2, 0, 1
z_1	0, 0, 0	2, 1, 0

Let $S_1 = \{1, 2, 3\}$, $1 \succ_1 3 \succ_1 2$ and $1 \succ'_1 2 \succ'_1 3$. Let $S_3 = \{1, 2, 3\}$, $3 \succ_3 1 \succ_3 2$ and $3 \succ'_3 2 \succ'_3 1$. Define Γ_{MA} and $\Gamma_{MA'}$ accordingly. Now, for $\Gamma_{MA'}$, there is a unique *MANE*, (z_1, y_2, x_3), which yields 1 to Player 2. For Γ_{MA}, there is also a unique *MANE*, (y_1, x_2, x_3), which yields 2 to Player 2. ■

Remark 9. Let Γ_{MA} and $\Gamma_{MA'}$ be minimally altruistic versions of the same normal form game Γ such that the only difference is that for some $i \in N$, $S'_i \setminus S_i = \{j\}$ and $\succ_i = \succ'_i |_{S_i}$. Then, it may be the case that an equilibrium that yields player *i* the highest (the lowest) payoff is eliminated.

Proof. Consider the following normal form game:

	x_2	y_2
x_1	a, 2	1, 3
y_1	a, 1	2, 0
z_1	0, 0	2, 1

Let $S_1 = \{1\}$, $S'_1 = \{1, 2\}$ and $S_2 = S'_2 = \{2\}$. Define Γ_{MA} and $\Gamma_{MA'}$ accordingly. Now, for Γ_{MA}, the set of *MANE* is $\{(y_1, x_2), (z_1, y_2)\}$. For $\Gamma_{MA'}$, (z_1, y_2) is the unique *MANE* if $a > 0$. Therefore, if $a > 2$ ($a < 2$), then the equilibrium that yields Player 1 the highest (the lowest) payoff is eliminated. ■

The remark below states that if each and every player in a game has a unique best response to some *NE*, then that *NE* is also a *MANE*.

Remark 10. If x^* is a NE in which $BR_i(x^*) = \{(x^*_i, \cdot)\}$ for every $i \in N$, then x^* is also a MANE.

Proof. Take some *NE*, x^*, in which $BR_i(x^*) = \{(x^*_i, \cdot)\}$ for every $i \in N$. Take any $i \in N$. Regardless of S_i and $\succ_i$, $X_{i,k}(x^*) = \{(x^*_i, \cdot)\}$ for every $k \in \{1, \ldots, |S_i|\}$. Since *i* is arbitrary, the result follows. ■

Finally, the following remark shows that in some games *MANE* and *NE* are not different. In particular, if for each player in the game, any two different strategies always give different payoffs, then the *MANE* of this game will be equivalent to the *NE* of the game.

Remark 11. In a game Γ_{MA}, if $\forall i \in N$, $\forall x_{-i} \in X_{-i}$, and $\forall x_i, y_i \in X_i$: $u_i(x_i, x_{-i}) \neq u_i(y_i, x_{-i})$, then $MANE(\Gamma_{MA}) = NE(\Gamma_{MA})$.

Proof. Any *NE* of Γ_{MA} satisfies the condition in Remark 10, hence $NE(\Gamma_{MA}) \subset MANE(\Gamma_{MA})$. The converse is true since *MANE* is a refinement of *NE*. ■

5. Examples

In this section, we provide examples where minimally altruistic refinement leads to significantly different predictions than Nash equilibrium.

The following example shows an instance where a player with a unique strategy can influence the set of minimally altruistic Nash equilibria. We know that this cannot happen when Nash equilibrium is employed. Moreover, in this example, *NE* gives practically uninformative predictions whereas *MANE* makes a sharp prediction about the outcome of the game.

Example 1. Consider the normal form game where $N = \{1, 2, 3\}$, first and second players have two strategies and the third player has only one strategy. Obviously, the third player has no influence on the determination of *NE*. In contrast, we show that he (indirectly) influences the set of *MANE*.

x_3

	x_2	y_2
x_1	3, 0, 5	4, 0, 4
y_1	3, 4, 0	4, 4, 4

Here, any strategy profile is a pure strategy *NE*. Note that this game is a minimally altruistic supermodular game given that $x_i > y_i$ for every $i \in \{1, 2\}$, $S_1 = \{1, 2\}$, and $S_2 = \{2, 3\}$. We have the set of pure strategy *MANE* as $\{(y_1, y_2, x_3)\}$. Thus, the equilibrium set is refined to a singleton. ♢

The following example shows an instance where, again, any strategy profile is a *NE*. On the other extreme, neither *BE* nor *BNE* exists in this game. Nevertheless, *MANE* exists and moreover it is unique.

Example 2. Consider the normal form game where $N = \{1, 2, 3\}$ and set $X_i = \{x_i, y_i, z_i\}$ for every $i \in N$. Let $S_1 = \{1, 2\}$, $S_2 = \{2, 3\}$, and $S_3 = \{3, 1\}$. Let $u_1(\theta) = 2$ if $\theta_{-1} = (x_2, x_3)$, $u_1(\theta) = 1$ if $\theta_{-1} = (x_2, \cdot)$ or $\theta_{-1} = (\cdot, x_3)$ but not $\theta_{-1} = (x_2, x_3)$, and $u_1(\theta) = 0$ otherwise. Define u_2 and u_3 similarly using y_j and z_j, respectively. Note that player *i* cannot affect his own payoff in this game. Also note that this game is a GSC *à la* minimally altruistic Nash for any orders defined on X_i's. It is easy to see that in this game, any strategy profile is a pure strategy *NE*. Moreover, there exists no *BE*. Hence, there exists no *BNE* either. However, there is a unique pure strategy *MANE*, which is (y_1, z_2, x_3). ♢

In the *divide-the-dollar game* (a special case of the Nash demand game where the bargaining frontier is linear), using *NE* does not give sharp predictions. In fact, there are infinitely many Nash equilibria of this game: any point on the bargaining frontier is a *NE*. In the literature on such problems, researchers usually modify the rules of the game such that the equilibrium set is a singleton.[12] In the following example, we also modify this game, *but* in a way that the set of *NE* remains unchanged yet *MANE* refines *NE* significantly.

Example 3 (*Pie Division Game*)**.** Consider the simple pie division game (usually called 'divide the dollar' game) in which every player $i \in N$ simultaneously claims $c_i \in [0, 1]$ of a pie of size 1. If $\sum_{i \in N} c_i > 1$, then each player receives 0 and if $\sum_{i \in N} c_i \leq 1$, then each player *i* receives his claim, c_i.[13] We modify this game as follows: (i) (efficiency) if $\sum_{i \in N} c_i < 1$ then $1 - \sum_{i \in N} c_i$ is equally divided between players, i.e. player *i* receives $c_i + k$ where $k = (1 - \sum_{i \in N} c_i)/n$, and (ii) (satiation) each player is indifferent between getting more than 3/4 and getting 3/4.[14] In this modified version, any strategy profile $(c_i)_{i \in N}$ such that $\sum_{i \in N} c_i = 1$ is still a *NE* (as it is in the standard version). However, *MANE* refines the set of *NE* to

$$\left\{ (c_i)_{i \in N} \mid \sum_{i \in N} c_i = 1 \text{ and } \forall i \in N \text{ with } S_i \setminus \{i\} \neq \varnothing : c_i \leq 3/4 \right\}.$$

[12] For this approach, the reader is referred to Brams and Taylor (1994), Anbarcı (2001), Ashlagi et al. (2012) and Cetemen and Karagözoğlu (2013).

[13] In this standard version of the game, the strategy profile with $\forall i \in N : c_i = 1$ is a *NE*, but it is not a *MANE*. In the modified version, the set of equilibria is refined further.

[14] This can be justified in a setting where nobody is able to consume more than 3/4 (e.g., the pie will go bad before a single person can eat the three quarters of it). Alternatively, one can also think about consumption capacities.

In fact, if everyone cares about someone else and every player is indifferent between receiving more than $1/n$ and receiving $1/n$, then there would be a unique pure strategy *MANE*. Note that the set of *NE* would still not change. $\diamondsuit$

The following example is a version of the well-known *threshold public good game*. It shows that if players in this game care about each other's well-being, *MANE* gives a different prediction than *NE*, and in a special case, *MANE* predicts that a socially optimal level of public good will be produced whereas some *NE* still predict a suboptimal level of public good production.

Example 4 (*Threshold Public Good Game*)**.** In this game, players contribute some amount, $c_i \in [0, C]$, to cooperatively produce two public goods, labeled k and m. If $\sum_{i\in N} c_i < K$, then no public good is produced. Thus, each player receives 0, but incurs the cost c_i, leading to a net payoff of $-c_i$. If $\sum_{i\in N} c_i \geq K$ but $\sum_{i\in N} c_i < M$, then k will be produced but not m. Thus, each player receives his value $v_{i,k} < K$ and incurs the cost c_i, leading to a net payoff of $v_{i,k} - c_i$. Besides, the residual amount, $\sum_{i\in N} c_i - K$, is realized as additional payoff by some players. In particular, assume that there are two types of players which differ in these additional utilities. Type t_m agents realize $\pi(t_m) = 0$ in case $\sum_{i\in N} c_i < M$ and they realize $\pi(t_m) = \sum_{i\in N} c_i - K$ in case $\sum_{i\in N} c_i \geq M$; whereas type t_k agents realize $\pi(t_k) = \sum_{i\in N} c_i - K$ as long as $\sum_{i\in N} c_i \geq K$. In this game, if $\sum_{i\in N} v_{i,k} < K$, then public good k cannot be produced in equilibrium since $c_i = 0$ for each i. If not, letting $C \geq M$, in some pure strategy *NE*, public good m cannot be produced. However, in all pure strategy *MANE* except the one with $\forall i \in N : c_i = 0$, both public goods are produced under the condition that at least one type t_k player cares about some type t_m player. The special case in which the optimal production, $c_i = C$ for every $i \in N$, will be realized is the case where every player cares about some type t_m agent. $\diamondsuit$

Finally, we study a simple version of the famous *congestion game* (Rosenthal, 1973). In the following example, six out of eight possible strategy profiles are *NE*, whereas with the given priority structure, only two of them are *MANE*.

Example 5 (*Congestion Game*)**.** In this game, the set of players is $N = \{1, 2, 3\}$. There are two possible routes from their originating point to a destination, labeled A and B. Players simultaneously choose a route. Hence, the strategy set is $X_i = \{A, B\}$ for every $i \in N$. If a route is used by more than one player, there will be a congestion. In that case, none of these players using the same route can reach the destination on time. If a player can reach the destination on time, he receives a payoff of 1. Otherwise, he receives a payoff of 0. Clearly, there are six pure strategy *NE* of this game: (A, B, B), (A, B, A), (B, A, A), (B, A, B), (B, B, A), and (A, A, B). Now, let us assume that Player 3 is in an emergency situation and hence both Player 1 and Player 2 care about Player 3 in a minimally altruistic sense. Moreover, Player 3 is still assumed to be selfish. It follows that the set of pure strategy *MANE* is $\{(A, A, B), (B, B, A)\}$. Hence, even if Player 1 and 2 cared about Player 3 in a minimal fashion, this leads to the elimination of *NE* that were favorable to them. Moreover, note that neither Berge equilibrium nor Berge–Nash equilibrium exists in this game. $\diamondsuit$

6. Conclusion

In this paper, we first introduce a minimal yet very reasonable notion of altruism. It is *minimal* since it stipulates that people care about others only after maximizing their own well-being.[15] Then, we use this notion to refine the set of Nash equilibria in normal form games. We prove the existence of equilibrium under this new refinement through three different channels, relate it to Nash equilibrium and Berge–Nash equilibrium, and provide examples showing the usefulness of this refinement concept. The definition of minimally altruistic Nash equilibrium contains Nash equilibrium as a special case (e.g., selfish best-responses). It gives sharper and more reasonable predictions than Nash equilibrium if players not only care about themselves but also care about other players' well-being in a lexicographic fashion when best-responding to others' strategies in a strategic game. Minimally altruistic Nash equilibrium shares a flavor similar to that of Berge–Nash equilibrium in that it is also based on altruistic behavior. However, from an empirical point of view we believe that it is much easier to support minimal altruism.[16] Moreover, from a theoretical perspective, there are games where Berge–Nash equilibrium (or Berge equilibrium) does not exist whereas minimally altruistic Nash equilibrium exists. Future theoretical work may study simultaneous application of the concept together with other refinement concepts, investigate general families of games where its use is fruitful, and characterize the family of games where the effects of changes in priority structure on equilibrium payoffs is more systematic. The investigation of the relationship (e.g., the domain of games where they coincide) between minimally altruistic Nash equilibrium and Kantian equilibrium (see Roemer (2010)) may also be of interest. Finally, future experimental work may shed light on the empirical validity of its predictions.

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[15] Despite the fact that we focus on altruism in this paper, the scope of the paper could be more general than altruism. In particular, it could concern games where each player's preferences over outcomes are lexicographic over a finite collection of utility functions. We would like to thank an anonymous reviewer for pointing this out.

[16] See Fehr and Fischbacher (2003) and the references therein for *partial* support from experimental work, which shows that as the cost of altruism decreases altruistic behavior is more frequently observed.

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