Analysis of Calculus Textbook Problems via Bloom's Taxonomy

Feryal Alayont, Gizem Karaali & Lerna Pehlivan

To cite this article: Feryal Alayont, Gizem Karaali & Lerna Pehlivan (2023) Analysis of Calculus Textbook Problems via Bloom's Taxonomy, PRIMUS, 33:3, 203-218, DOI: 10.1080/10511970.2022.2048931

To link to this article: https://doi.org/10.1080/10511970.2022.2048931

Published online: 15 Apr 2022.
Analysis of Calculus Textbook Problems via Bloom’s Taxonomy

Feryal Alayont, Gizem Karaali and Lerna Pehlivan

ABSTRACT
In calculus courses, instructors often use the end-of-section problems in a textbook in homework assignments or other course assessments. As a result, these problems influence the teaching and learning of calculus. In this study, we examine the levels of cognitive demand of these problems in a mainstream calculus textbook and classify them within the framework of Bloom’s Taxonomy. We provide examples of the types of problems assigned to each of the six categories in this taxonomy and share some of the deliberations that led us to these assignments. Finally, we discuss the implications of our results for teaching calculus courses. We believe that our analysis will help calculus instructors be more cognizant of the cognitive demand of problems when assigning them for homework and, as a result, help them to appropriately support, assess, and enhance their students’ understanding of the topics.

KEYWORDS
Bloom’s taxonomy; calculus; textbook analysis

1. INTRODUCTION
Improving critical thinking skills of students is one of the essential goals of teaching mathematics. Instruction methods and tasks students are assigned while learning mathematics influence the types of mathematical reasoning students use, and hence affect the development of cognitive skills in students (cf. [13,18]). As a result, it becomes crucial that we, as instructors, pay attention to what tasks students are completing in our classes in order to help them develop their critical thinking skills.

In this paper, we investigate the levels of cognitive demand of end-of-section problems in a calculus textbook, which are often used as resources for the homework assignments or for practice. Our goal is to help calculus instructors be more aware of the types of problems available in a textbook. This will then help them to choose the type of questions that will better serve their learning goals and guide their students to develop higher-order critical thinking skills.

Similar analyses on mathematics textbook or exam problems have been conducted by many authors; see for example [2,5,12,17]. These articles analyze problems from the perspective of algorithmic/imitative vs. creative problem-solving,
types of reasoning used in solving problems and how students’ use of problem-solving strategies changed when problems in a test were similar to problems solved earlier. In [12], for example, authors found that 79% of tasks in common textbooks could be solved by imitating procedures, 13% by making minor modifications to procedures, and only 9% required new solution methods. See also [32] for more examples of studies and frameworks used in analyzing cognitive demand of tasks students encounter. Our analysis differs from these papers since we focus on categorizing the problems based on Bloom’s Taxonomy levels [4] and not on whether problems could be solved via imitating known procedures. In most cases though, there was some correlation between having a known procedure for solving a problem and the problem being categorized at a lower level in Bloom’s Taxonomy. However, this was not an exact correspondence at all times.

Analyses of textbook problems in other disciplines based on Bloom’s Taxonomy have been published (see for example [8,20]). There are also a handful of works that utilize the Bloom et al. framework for analyzing a selection of mathematics learning tasks (see for example [15,26]). However, we are not aware of such an analysis based on Bloom’s Taxonomy completed for calculus textbook end-of-section problems. That is the task of this work.

1.1. Why Calculus?

We have decided to focus our investigation on calculus for a variety of reasons. Our first reason was the ubiquity of calculus. Calculus is taught in a range of educational/institutional settings (high school, community college, and undergraduate institution). Thus teaching calculus is a common experience for most mathematics instructors. Furthermore, the teaching of calculus affects a large number of students [3,31].

College mathematics instructors of upper-division courses may assign different flavors of work to their students, such as homework assignments with problems assigned from the textbook, case studies, group projects, and creative writing assignments. In calculus, however, standard problem sets are almost universal. More often than not, these sets are composed of problems selected or adapted from the end-of-chapter problem lists of standard calculus textbooks. Thus a second reason for our decision to zero in on end-of-chapter calculus problems was that the typical calculus assignment often depended on these.

Even though we believe that calculus was a worthwhile content area to pursue this type of analysis, other mathematical content areas such as college algebra, linear algebra and differential equations might also benefit from such investigations. In this way, our calculus textbook study can be an entry point to a broader conversation about assignments and tasks that facilitate the development of higher-order thinking skills of students of collegiate mathematics.
2. RESEARCH QUESTIONS AND METHODOLOGY

2.1. Theoretical Framework

Bloom’s Taxonomy is a classification of the types of educational tasks and objectives; in particular, it is used by educators to evaluate the learning tasks they design to achieve certain educational goals. The original taxonomy dates back to the 1950s when a committee of educators convened to develop language and models that could help educators design and improve curricula and assessment tasks. The final classification that became known as Bloom’s Taxonomy comes from the taxonomy of educational objectives in the cognitive domain developed by the committee and may be found in Bloom et al. [4].

In 2001, one of the original contributors to the conversation (D. R. Krathwohl), together with Lorin W. Anderson, completed a systematic revision of the by-now-well-known Bloom’s Taxonomy, see [1]. Nonetheless in our work, we decided to stick to the original taxonomy from 1956. Our main reason was that of simplicity. The new framework is perhaps more comprehensive but it is also much more complex and sophisticated. As busy mathematicians instructors hoping to reach other busy practitioners, we decided that keeping it simple would be a good idea.

Bloom’s Taxonomy provides instructors a straightforward scheme, and information on it is ubiquitously available. The underlying framework is translatable across the disciplines, and as such, it provides a common (albeit imperfect) language with scholars from other disciplines.

In the original Bloom et al. Taxonomy, the cognitive domain of educational objectives is divided into six hierarchical categories: Knowledge, Comprehension, Application, Analysis, Synthesis, Evaluation. Below is a brief description of each category from Bloom et al. [4]:

- **Knowledge:** “The recall of specifics and universals, … methods and processes, or … a pattern, structure, or setting.” (p. 62)
- **Comprehension:** “A type of understanding or apprehension such that the individual knows what is being communicated and can make use of the material or idea being communicated without necessarily relating it to other material or seeing its fullest implications.” (pp. 89–90)
- **Application:** “The use of abstractions in particular and concrete situations. The abstractions may be in the form of general ideas, rules of procedures, or generalized methods. The abstractions may also be technical principles, ideas, and theories which must be remembered and applied.” (pp. 120–123)
- **Analysis:** “The breakdown of a communication into its constituent elements or parts such that the relative hierarchy of ideas is made clear and/or the relations between the ideas expressed are made explicit.” (pp. 144–145)

---

1 The cognitive domain is one of three domains the committee decided to investigate. The other two are affective and psychomotor domains.
Synthesis: “The putting together of elements and parts so as to form a whole.” (pp. 162–165)

Evaluation: “Judgments about the value of material and methods for given purposes. Quantitative and qualitative judgments about the extent to which material and methods satisfy criteria.” (pp. 185–187)

2.2. Our Specific Research Questions

Our main guiding question as we began this project was: How can we offer our students diverse opportunities to learn? We were looking for ways that engaged students in cognitive activities that would require higher-level thinking, as captured in the three higher levels of Bloom’s Taxonomy. There are many activity-based tasks through which instructors can create learning opportunities. For example, Karaali [15] describes a class-wide debate where students are encouraged to use evaluative reasoning. Spindler [26] explores modeling projects in differential equations and engineering courses. Jaafar [11] focuses on writing-to-learn activities to facilitate deeper learning. However, the standard source for learning tasks that instructors use is the textbook. Besides, not all instructors have the chance to insert boutique activities into their curriculum as they wish. Thus our question evolved into: What kinds of calculus tasks are available in a standard calculus textbook?

2.3. Method

To gain some insight into our research question, we considered various texts and in the end decided to use Stewart [28]. This specific textbook was chosen because Stewart’s Calculus textbooks are among the most commonly used calculus textbooks in the U.S. [22]. Among the different versions available, the Early Transcendentals version was the best-selling version at the time of the writing (as could be seen for example on a popular online bookstore). As such, we used the 7th edition of this version (ISBN: 978-0538497909 for combined single and multivariable calculus version, and 978-0538498678 for the single variable version).

We chose a variety of sections from a year-long single-variable calculus course. We focused on the mainstream year-long sequence because these calculus courses have more students than the other types of calculus courses. As a result, we decided to analyze tasks corresponding to both differential and integral calculus of functions of a single variable. It would also be interesting to complete a detailed analysis of each course separately.

We first included sections where the “big ideas” of calculus were introduced: limit (2.2: The Limit of a Function), differentiation (2.7: Derivatives and Rates of Change), integration (5.2: The Definite Integral) and series (11.2: Series). We also included two application sections, one for derivatives and the other for integration (4.3: How Derivatives Affect the Shape of a Graph; 8.1: Arc Length) and a section which we thought would by its nature have more algorithmic problems (7.1: Integration by Parts). We hypothesized that this breadth of sections, when
combined, would yield a more comprehensive result of the types of problems. Of course, other researchers might have chosen other sections, but we felt confident that the seven sections we chose would be representative of sections used by most calculus instructors, as they all involved standard topics.

In order to prepare for rating the problems based on Bloom’s Taxonomy, we first agreed on which documents to use for detailed descriptions of the levels. One resource was the adaptation of the original Bloom’s Taxonomy, published by Allyn and Bacon, Boston, MA, available online at https://www.uvic.ca/learningandteaching/assets/docs/instructors/for-review/Teaching%20Support/BloomsTaxonomy.pdf. A second resource [25] provides specific examples in applying Bloom’s Taxonomy to categorize single-variable calculus questions.

Following prior analogous work such as [8], we have aimed to ensure that the three authors would rate problems in a consistent and reliable manner by first reviewing all the problems in Section 2.2 (“The Limit of a Function”) individually and then having extensive discussions to come to a consensus on the ratings of the problems. After we reached a consensus on how to rate the items in this training section, for the remaining six sections, each of us rated all problems in two sections and each section was rated by two different people. Each researcher provided one main rating and one secondary rating for each problem they worked on, when appropriate. For problems with multiple parts, we assigned the highest rating available among all parts. For each section, one of the two people who rated that section combined the ratings into a file, assigning a rating when either the two main ratings or one of the main ratings and a secondary rating agreed. The third person, who had not initially rated the problems in that section, then reviewed the combined ratings and made final decisions on the remaining problems with controversial ratings. For a small number of questions that the third person was also not able to assign a final rating for, we discussed as a whole group how to assign the ratings.

When rating a problem, we assigned the rating from the perspective of how the student would solve it, assuming they had successfully completed the prerequisite courses and understood the topics that appeared before the section whose problems were being rated. When the problem allowed for a familiar, less conceptual solution method, we assumed the hypothetical student would likely use this familiar, less conceptual method. We also took into account if there was a very similar example in the section whose solution can be, in effect, followed procedurally. In the cases where some of us could foresee different methods of solving a problem, including those with technology, or when we knew there was a similar problem solved in the main text of the section, we made use of the “two possible ratings” option to allow for flexibility. For example, a problem could be considered at the analysis level without technology help while technology could turn it into an application problem. The flexibility in the ratings was narrowed, as described in the previous paragraph. However, we did not take into account differences between levels of preparation among the students that went beyond competence in standard prerequisites and acknowledge that student preparation also has an effect on the level of a problem.
Table 1. Number and percentage of problems in each Bloom level per section.

<table>
<thead>
<tr>
<th>Level/Sections</th>
<th>2.2</th>
<th>2.7</th>
<th>4.3</th>
<th>5.2</th>
<th>7.1</th>
<th>8.1</th>
<th>11.2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>4.1%</td>
<td>3.7%</td>
<td>4.6%</td>
<td>12.3%</td>
<td>4.2%</td>
<td>0%</td>
<td>0%</td>
<td>4.3%</td>
</tr>
<tr>
<td>Comprehension</td>
<td>18</td>
<td>24</td>
<td>4</td>
<td>35</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>36.7%</td>
<td>44.4%</td>
<td>4.6%</td>
<td>47.9%</td>
<td>1.4%</td>
<td>4.8%</td>
<td>7.8%</td>
<td>19.5%</td>
</tr>
<tr>
<td>Application</td>
<td>24</td>
<td>11</td>
<td>29</td>
<td>23</td>
<td>49</td>
<td>35</td>
<td>44</td>
<td>215</td>
</tr>
<tr>
<td></td>
<td>49.0%</td>
<td>20.4%</td>
<td>33.3%</td>
<td>31.5%</td>
<td>68.0%</td>
<td>83.3%</td>
<td>48.9%</td>
<td>46.0%</td>
</tr>
<tr>
<td>Analysis</td>
<td>2</td>
<td>15</td>
<td>39</td>
<td>4</td>
<td>12</td>
<td>5</td>
<td>25</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>4.1%</td>
<td>27.8%</td>
<td>44.8%</td>
<td>5.5%</td>
<td>16.7%</td>
<td>11.9%</td>
<td>27.8%</td>
<td>21.8%</td>
</tr>
<tr>
<td>Synthesis</td>
<td>3</td>
<td>2</td>
<td>10</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>13</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>6.1%</td>
<td>3.7%</td>
<td>11.5%</td>
<td>2.7%</td>
<td>8.3%</td>
<td>0%</td>
<td>14.4%</td>
<td>7.7%</td>
</tr>
<tr>
<td>Evaluation</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0%</td>
<td>0%</td>
<td>1.1%</td>
<td>0%</td>
<td>1.4%</td>
<td>0%</td>
<td>1.1%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

3. RESULTS AND ANALYSIS

Out of the 467 problems across 7 sections that we reviewed, 20 (4.3%) were rated at the knowledge level of Bloom’s Taxonomy, 91 (19.5%) at comprehension, 215 (46.0%) at application, 102 (21.8%) at analysis and 36 (7.7%) at synthesis levels. Finally, only three questions (0.6%) were at the evaluation level. The numbers and percentages of questions in each Bloom category per section are presented in Table 1.

We observe in Table 1 that the most common type overall are the application-level questions, even though this is not necessarily true in each section. More precisely, application-level questions are the most frequent in four out of seven sections while comprehension level questions are the majority in two and analysis level in one section.

Despite the common student assumption that math is all about facts and memory recall, knowledge-level problems occur very infrequently among the questions we have rated (4.3%). However, overall lower level questions make up almost 70% of all questions. The higher levels of Bloom’s Taxonomy are significantly underrepresented among the questions, only 8.3% of all questions falling into the top two levels: Synthesis and evaluation. Especially, the evaluation questions are very rare, with less than 1% of all questions falling into this category. Indeed, each section has at most one problem in this category.

In general, the later questions in each section are rated in the higher categories, although in some sections, this does not hold. For example, in Section 5.2 (“The Definite Integral”), many questions numbered 41 and higher are placed in the knowledge level. While synthesis and evaluation level questions are often the later questions in a section, analysis questions are sometimes scattered among all questions. For example, in Sections 8.1 (“Arc Length”) and 11.2 (“Series”), some of the earlier questions are in the analysis category.

Now we provide a sample of questions from each of the Bloom et al. categories.

An example of a knowledge level task is Problem #4 from Section 4.3 (“How Derivatives Affect the Shape of a Graph”):
#4. (a) State the First Derivative Test.
(b) State the Second Derivative Test. Under what circumstances is it inconclusive? What do you do if it fails?

This problem is categorized at the knowledge level because it simply asks the student to state the two derivative tests, both of which are available in the section. The student will simply copy those definitions and note the case when one test is inconclusive.

Among comprehension-level questions, we picked Problem #33 from Section 5.2 (“The Definite Integral”), which is the section with the highest proportion of comprehension questions.

This problem is rated as a comprehension-level question because the student simply uses the interpretation of the definite integral as signed area. As long as the student understands this interpretation, there is no further step needed.

#33. The graph of \( f \) is shown. Evaluate each integral by interpreting in terms of areas.

(a) \( \int_0^2 f(x) \, dx \)
(b) \( \int_0^5 f(x) \, dx \)
(c) \( \int_0^7 f(x) \, dx \)
(d) \( \int_0^9 f(x) \, dx \)

We will now provide one more comprehension level question: Question #47 in Section 7.1 (“Integration by Parts”). This question is an example for the rating process where initially two raters chose different categories and reached an agreement after discussions among all three raters.
#47. (a) Use the reduction formula in Example 6 to show that

\[ \int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin(2x)}{4} + C \]

(b) Use part (a) and the reduction formula to evaluate \( \int \sin^4 x \, dx \)

Example 6 requires proof of the reduction formula

\[ \int \sin^n x \, dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \, dx, \]

where \( n \geq 2 \) is an integer. On the one hand, Example 6 includes a formula into which the student could simply plug \( n = 2 \) and hence one could rate Question #47 at the comprehension level. On the other hand, one could rate Question #47 at the application level since the student could imitate the method in Example 6 for part (a) to obtain the formula, which is an application-level question for integration by parts. Due to these two different approaches, the two raters chose different categories for this question. After a group discussion, all three raters agreed to consider Question #47 as a follow-up to Example 6 and, therefore, rated it at the comprehension level.

Section 8.1 (“Arc length”) has the highest proportion of application level questions (35 out of 42). We present Question #36 of this section as an example for this level:

#36. A steady wind blows a kite due west. The kite’s height above ground from horizontal position \( x = 0 \) to \( x = 80 \) ft is given by \( y = 150 - \frac{1}{40} (x - 50)^2 \). Find the distance traveled by the kite.

This question is rated at the application level since the student applies the arc length formula after identifying the appropriate function and integral limits from the given information, and evaluates the definite integral.

For the analysis level, we selected Question #43 from Section 7.1 (“Integration by Parts”):

#43. Evaluate the indefinite integral \( \int x e^{-2x} \, dx \). Illustrate, and check that your answer is reasonable, by graphing both the function and its antiderivative (take \( C = 0 \)).

This problem is rated as an analysis level question because the student identifies the method of solution as integration by parts, considering a substitution first.
Moreover, the student, by graphing the function and its antiderivative, comes to a conclusion that her integral evaluation is reasonable. The main analysis tasks here, then, involve the student seeing a pattern when trying substitution and analyzing the two graphs to be created to reach the desired conclusion.

Section 11.2 (“Series”), with 14.4%, has the highest proportion of synthesis level questions. We offer Question #76 in this section as a sample for this level:

#76. Graph the curves $y = x^n$, $0 \leq x \leq 1$, for $n = 0, 1, 2, 3, 4, \ldots$ on a common screen. By finding the areas between successive curves, give a geometric demonstration of the fact, shown in Example 7 that

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1.$$ 

This question is rated at the synthesis level since the student creates the connection within the $n$th term of the infinite sum and the areas in between two curves $x^{n-1}$ and $x^n$. Establishing this relation leads to the conclusion that the infinite sum of Example 7 can be formulated as the infinite sum of all the in-between areas which by the graph can be seen to be 1.

Evaluation-level questions are very rare. There are solely three questions that we included in this level within all the sections. These are Question #76 in Section 4.3 (“How Derivatives Affect the Shape of a Graph”), Question #72 in Section 7.1 (“Integration by Parts”), and Question #79 in Section 11.2 (“Series”). We pick Question #79 of Section 11.2 as an example for this level:

#79. What is wrong with the following calculation?

$$0 = 0 + 0 + 0 + \ldots$$

$$= (1 - 1) + (1 - 1) + (1 - 1) + \ldots$$

$$= 1 - 1 + 1 - 1 + 1 - 1 + \ldots$$

$$= 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \ldots$$

$$= 1 + 0 + 0 + 0 + \ldots = 1$$

Question #79 is rated at the evaluation level because this question requires the students to judge the validity of an argument. (Recall that Bloom et al. describe evaluation tasks as those that involve “judgments about the value of material and methods for given purposes” [4, p. 185].)
4. DISCUSSION

4.1. Implications for the Classroom

Possibly the most striking feature of Table 1 is that application is the most common category of questions. This has both positive and negative implications.

Most mathematics instructors de-emphasize memorization and focus on whether students can apply the main ideas of a course in relevant contexts. Especially for courses such as calculus deemed by most people to be service courses, this expectation is natural. In other words, an emphasis on application problems will resonate with many instructors. We want our students to be able to use the math they are learning; asking them application-oriented questions may help us do just that.

However, we need to observe that application in Bloom’s Taxonomy is the third category among the lower-level cognitive tasks. And looking through the concrete examples we worked with, we can see that indeed these problems are too low-level, often remaining at or very near the level of plug-and-chug once a student finds a solved problem following a similar template. Using the “five strands of mathematical proficiency” terminology of [16], we can say that these problems focus on procedural fluency.

A closer look at the Bloom’s Taxonomy description of application reveals a more concerning issue. Application in Bloom’s Taxonomy is not really directly related to “applications to the real world”. In particular in the context of calculus teaching, it refers to an application of an abstract concept, theory, or construct that is being taught or learned within a specific (and still mathematical) context. As a result, the predominance of these types of questions in our learning and assessment tasks do not fulfill the expectations of our students and client departments of real-world applications.

Another issue we can see relates to the remark we made in Footnote 2. It should be noted here that we have had students who were able to plug and chug but unable to articulate the distinct steps of the process or display understanding of the reasons for what they were doing when asked. Thus in some mathematical contexts, we have seen students find it more challenging to perform knowledge or comprehension tasks than related application tasks. This points towards a distinct incompatibility of Bloom’s Taxonomy with the pedagogical purposes of mathematics instruction: Surely we want our students to be able to do both, but it is not clear that in the mathematical context, application is always “higher level” than knowledge or comprehension.

For all the reasons mentioned above, choosing assignment problems randomly from the end-of-section problems is not sufficient to help students develop full

---

2 One might think that plug-and-chug activity should be relegated to the knowledge category. However, Bloom et al. describe these two levels in specific language, and knowing a procedure or a process (knowledge) is not the same as knowing how to apply it in an abstract or concrete context (application). A learner displaying knowledge of a process might be to be able to list the steps that they have memorized, while the capacity to perform it as an application task might involve an actual context where the process needs to be used.
mathematical proficiency. Instructors might wish to be intentional in their choices to make sure to specifically include problems in the higher levels of analysis, synthesis, evaluation as they are rare yet essential for developing proficiency in conceptual understanding, strategic competence and adaptive reasoning of the five strands of mathematical proficiency [16]. Including even knowledge and comprehension problems in addition to application problems might be appropriate at times since some of the knowledge and comprehension problems might focus more on conceptual understanding than an application problem. (This also resonates with Schoenfeld’s “balanced diet” perspective on problem solving and teaching for robust understanding; see for example [24] for more.)

Although we have been advocating for instructors to be cognizant of the level of the learning tasks assigned, it is important to note that learner level also has an impact on choosing appropriate tasks. It might be argued that at times, assigning application problems similar to examples provided in a section might be more beneficial to the learners than assigning higher level tasks. This is due to the more beneficial effect of “worked examples” than the “generation effect” for novices for certain material. More specifically “complex material may need explicit guidance to assist learners to understand the material” [7]. However, we might want to eventually bring our students to a point where they can approach more challenging problems after they pass the novice stage.

Our investigation may also point towards other challenges of using Bloom’s Taxonomy for mathematics content teaching. A particular challenge becomes clearer when one starts a broader conversation about why we teach calculus. Many of us do not teach calculus only for its content value. Some of us see calculus as an outstanding human achievement that significantly impacted human civilization and its evolution in the last few centuries and want to impress upon the students its role in making the modern world. Furthermore teaching our students responsibility, accountability, perseverance, self-awareness, respect, communication skills, learning skills, audience awareness, and the value of intrinsic motivation is always on our minds when we enter our classrooms, even when calculus is the declared context. In other words, our goals and motivations go beyond specific content and related cognitive skills, and as such, Bloom’s Taxonomy seems innately incapable of capturing a significant portion of our particular teaching aspirations.

4.2. Limitations of the Study

We have used Bloom’s Taxonomy as a standard measuring stick known in most education circles as a tool to categorize cognitive levels of learning tasks. However, its direct applicability to mathematics instruction or its optimal effectiveness might be questioned. Indeed mathematics education researchers have developed alternative frameworks for measuring the cognitive demand of mathematical learning and assessment tasks. For example, see
Stein, Grover, and Henningsen [27], whose framework on examining mathematical tasks was recommended for international comparison studies [9]. The Stein–Grover–Henningsen framework includes four levels of cognitive demand: Memorization (M), Procedures without connections (P), Procedures with connections (PC), and Doing mathematics (DM). Here M and P are considered lower-level demand and PC and DM are higher-level demand. These types of subject-specific frameworks might offer mathematics instructors more flexible tools.

Another aspect of this study that might limit the power of its conclusions is the amount of content we picked out of a full year-long calculus sequence. Numerically speaking, the number of sections of [28] we reviewed were seven sections total, out of 80 available sections. Therefore, our viewpoint is naturally limited. Additionally, as we chose the first or early sections on each main topic, this might have led to a lower number of analysis or synthesis questions. However, it is possible to ask questions requiring creative thinking even in a procedural topic as \( u \)-substitution, as discussed below. Therefore, we believe the effect of choosing early sections on the distributions of levels of the problems was minimal.

Finally, the way we decided on our ratings was by thinking how a student might solve a problem. This is highly varying, dependent on student background and competence, instructor’s teaching style, and more. We even found ourselves changing our ratings as we discussed certain problems and with the proliferation of available online tools. As a result, we do not think that we can claim the ratings we decided were set-in-stone, ‘applying to all situations and students’.

4.3. Future Directions

In this study, we considered questions from one popular print textbook for a whole-year calculus sequence. Similar studies can be undertaken for other courses, such as linear algebra, college algebra, differential equations; for online calculus books, for free, open-source books, for databases of calculus questions such as those in WebWork, etc. There are many other instances where a careful analysis of questions available in a medium with representative samples can allow instructors to become more adept at making the analysis themselves when assigning homework to their students.

5. CONCLUSION

This study analyzed the end-of-section problems in a calculus textbook according to Bloom’s Taxonomy. What are some possible implications of this analysis for our own teaching?

When we teach a concept, our objective is for our students to develop a deep level of understanding. To get there, they need to practice solving problems at the intended level on their own. Thus, we need to be aware of the cognitive levels of the problems we assign for practice, from the student’s perspective, and not assume
that “the problems closer to the end are at a higher level” for a group of end-of-section problems. Thinking carefully about the students’ background, including what is available to them as example problems within the section and what their calculators are capable of doing, we need to quickly imagine how a problem would be solved by a student and assess its cognitive level accordingly.

If we are writing our own problems, we can utilize creative problem formats. For example, instead of asking a student to perform a $u$-substitution, which is a really procedural calculus technique, a student can be asked to determine whether the substitution method is appropriate for a given integral and to justify the reasons for why or why not. Or a student can be asked to analyze a hypothetical student solution applying the substitution method and asked to justify whether the solution is correct, and if not correct, to give suggestions to the hypothetical student for how to correct their solution. Asking a student to explain their solutions in writing will often raise the cognitive level of the problems. Alternatively, a student can be asked to come up with two-three integrals which reduce to a given integral after applying the substitution method (essentially reversing the substitution method), or a student can be asked to find an integral from class notes that does not require substitution and to modify this integral minimally to make it an appropriate integral that can be solved using substitution. All of these problem formats remove the procedural familiarity of the substitution method and require the student to focus on understanding how, why and when the process really works. Such problems would help students practice the “creative mathematically founded reasoning” as advocated in [19] as well as deter cheating in assessments and help “balance conceptual understanding, procedural skills, and fluency with real-world application” in the assignments as discussed in [33].

We also need to be aware of the total cognitive load placed on students in a problem set. If the load is too high, students can give up prematurely. Varying the cognitive levels of assigned problems in a set is also beneficial as this will help students practice their skills at all levels.

In addition to the tips we provided above, we recommend two special issues of PRIMUS that focus on homework assignments for further strategies on homework creation and implementation [10].

An important goal in education is the successful transfer of material learned in the classroom to other contexts, including real-life contexts. This is especially true in service courses such as calculus. In some cases, this might involve what Rebello et al. [23] describe as “horizontal transfer,” a well-defined situation with variables clearly identified and a specific solution method, similar to the problems categorized as comprehension or application in Bloom’s Taxonomy. Often, though, students will be expected to perform “vertical transfer,” an ill-defined situation where the variables or the solution method or both are unclear and some novel thinking is involved. Rebello et al. [23] argue for training students for both types of transfer, although they also note that student preparation can make a significant difference in whether a problem is perceived as a horizontal or vertical transfer problem from that student’s perspective. In light of this, we also need to account for the variability
of contexts in the problems we assign for practice. In other words, not only do we need to vary the cognitive demand but also provide multiple, and sometimes novel, contexts for students to apply what they are learning. This will help them be more nimble in applying their knowledge in brand new situations once they are done with our courses.

One final note to make is that with so many online tools and sources providing end-of-section problem solutions freely or for a small fee (see e.g., [6,14,21,29]), we might ponder whether using these problems as our go-to resources for our assignments is serving our students well. With the recent sudden online switch due to COVID-19 social distancing restrictions, many mathematics instructors had to face the challenges of these services being accessible easily, even possibly during exams; see [30,33] for a sample of the many conversations that took place regarding these challenges.

Are single-answer computational problems really benefiting students? If students can find answers to our problems online, how do we ensure academic integrity? These are tough questions, but we prefer to be optimistic. In the end, we can use this online switch as an opportunity to re-imagine what a homework assignment can be instead of going back to old habits once the online challenge is over.

**Acknowledgments**

The authors would like to thank the editors and the referees for their useful comments and suggestions on an earlier version of the manuscript.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

**ORCID**

Gizem Karaali  
http://orcid.org/0000-0002-0502-8358

**REFERENCES**


**BIOGRAPHICAL SKETCHES**

Feryal Alayont received her undergraduate mathematics degree from Bilkent University, Turkey, and her Ph.D. in mathematics from the University of Minnesota. She is currently a professor of mathematics at Grand Valley State University, where she also serves as the Mathematics Advising and Engagement Coordinator.

Gizem Karaali completed her undergraduate studies at Boğaziçi University, Istanbul, Turkey. After receiving her Ph.D. in Mathematics from the University of California Berkeley, she taught at the University of California Santa Barbara for two years. She is currently a professor of mathematics at Pomona College where she enjoys teaching a wide variety of courses and working with many interesting people. She has most recently been involved with promoting the humanistic aspects of mathematics via her work through the *Journal of Humanistic Mathematics*. Gizem Karaali is a Sepia Dot (a 2006 MAA Project NExT Fellow).

Lerna Pehlivan received her undergraduate degree in Mathematics from Boğaziçi University, Istanbul, Turkey and her Ph.D degree in Mathematics from the University of Southern California, Los Angeles. She held a Fields Institute Postdoctoral Fellowship at Carleton University, Ottawa and a postdoctoral position at York University, Toronto. She is currently a lecturer at the Faculty of Electrical Engineering, Mathematics and Computer Science at University of Twente, Enschede, The Netherlands.