Nonlinear identification and optimal feedforward friction compensation for a motion platform

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\textit{A B S T R A C T}

In this study, we present a method of nonlinear identification and optimal feedforward friction compensation for an industrial single degree of freedom motion platform. The platform has precise reference tracking requirements while suffering from nonlinear dynamic effects, such as friction and backlash in the driveline. To eliminate nonlinear dynamic effects and achieve precise reference tracking, we first identified the nonlinear dynamics of the platform using Higher Order Sinusoidal Input Describing Function (HOSIDF) based system identification. Next, we present optimal feedforward compensation design to improve reference tracking performance. We modeled the friction using the Stribeck model and identified its parameters through a procedure including a special reference signal and the Nelder–Mead algorithm. Our results show that the RMS trajectory tracking error decreased from 0.0431 deg/s to 0.0117 deg/s when the proposed nonlinear identification and friction compensation method is utilized.

1. Introduction

Compensating the nonlinearities in system dynamics play a key role in achieving precise trajectory tracking. For some systems especially in industrial automation and defense industry, small errors due to nonlinear effects cause an intolerable loss in precision. These systems are often identified using the frequency response function (FRF), however most of the crucial nonlinear effects are ignored through linearization around limited range in this process. Hence, the identification and compensation of the nonlinear dynamic effects became essential for improving trajectory tracking performance of such systems.

Earliest system identification methods started with linear approximations of the dynamic systems. Using the assumption of linear time invariant (LTI) systems, identification techniques are comprehensively studied and a considerable amount of techniques became traditional approaches for control community such as Prediction Error methods [1], nonparametric and parametric models [2,3], and black-box identification tools [4]. In this context, frequency domain based system identification techniques like the FRF examined thoroughly for LTI systems previously [5]. Moreover, system identification techniques are expanded in order to capture the significant nonlinear dynamics, which are previously neglected through linear techniques [6].

To define the nonlinear behavior of the systems, initial studies in literature use Volterra series approximations [7,8]. Volterra series are generalizing the polynomial approximation of the dynamic systems within limited operating range [9–11]. Volterra series approximations investigate input–output dynamics of a nonlinear system with harmonic and inter-modulation frequency components.

Some other works proposed the generalized frequency response functions (GFRF) where FRF technique is generalized for nonlinear systems [12–15]. Additionally, some works utilized the best linear approximation (BLA) of nonlinear systems along with the idea of nonlinear distortions on FRFs using specialized multi-sine input signals and averaging techniques [16–19]. Moreover, the idea of nonlinear frequency response functions (NFRF) is investigated in [20]. In this approach, the output spectra depend also on the excitation amplitude along with the excitation frequency. The concept of NFRF generates a nonlinear Bode plot as a moderate approximation of the system gain based on its excitation amplitude and frequency.

As an alternative approach, describing function method is utilized to define the dynamic system response to a single sinusoidal input [21]. Based on the superposition principle of the harmonic responses, describing function method provides approximations of the steady-state solutions to relevant harmonic excitation. Depending on the nonlinearities of the system, different specialized types of the describing functions are defined such as Generalized Describing Functions (GDF) [22] and Higher Order Sinusoidal Input Describing Function (HOSIDF) [23].
HOSIDF extends the describing function theory to higher harmonics of the response of periodic input by introducing the notion of virtual harmonics generator. For a single periodic input with a fundamental frequency, HOSIDF analyzes all harmonics of the fundamental frequency. Further studies are presented for identification and compensation purposes in [24–27]. A comprehensive overview for frequency domain methods for nonlinear systems can be found in [28].

Our work contributes to the understanding of the nonlinear effects in an industrial motion platform with a single rotational degree of freedom (DoF). We present a standardized method for nonlinear identification and optimal feedforward compensation of low frequency disturbances resulting from friction. On the other hand, the procedure suggested here is not limited to a specific type of nonlinearity, and can be generalized to include nonlinearity based disturbances at other frequency regions. First, a study of friction compensation in a mechatronic system is discussed using the continuous Striebeck friction model [29]. Additionally, a method for friction identification is suggested using a special reference input and the Nelder–Mead Algorithm [30]. Then, a straightforward procedure for determining the optimal feedforward compensation to reduce the nonlinear effects is presented by applying the HOSIDF based frequency domain identification. The optimal case is achieved by decreasing the effect of higher order \( K > 1 \) harmonic spectral components \( K \omega_0 (K \in \mathbb{N}) \) to a sinusoidal input with frequency \( \omega_0 \). Experimental validation of the nonlinear identification and the feedforward friction compensation method is presented on a motion platform.

First, a brief overview of the experimental setup utilized is presented in Section 2. In Section 3, the nonlinear behavior of the system is studied using Bode plots and HOSIDF analysis. Frequency domain identification based optimal feedforward compensation for the system is explained in Section 4. Finally, results and performance evaluations of our proposed method are presented in Section 5 and Section 6, respectively.

2. System description

The modeling and the experimental work are conducted for the single DoF industrial rotational motion platform shown in Fig. 1. As in the schematics illustrated in Fig. 1, system driveline starts with a servo motor. Motion is transferred from servo motor to the load through a gearbox and an additional pinion-ring gear couple. Angular position, speed and current data are acquired from the encoder, gyroscope and current transducer, respectively.

As our baseline, a speed feedback loop is implemented in order to meet the requirements of command tracking of the system, namely having a minimum bandwidth of 6 Hz, a gain margin of 3–6 dB, and a phase margin of 30–60 deg. An additional feedforward compensator is implemented in order to use the speed reference \( r(t) \) to generate the compensator output torque for this specific study. The outputs of the feedback and the feedforward compensators are combined to generate the torque signal, \( u(t) \), which is the input to our plant. The procedure introduced in this paper targets developing the optimal feedforward compensator to eliminate the effects of the strong nonlinearities in the experimental setup, most dominantly the friction in the driveline and the backlash of the transmission units. The block diagram of the control framework of the described system is given in Fig. 2. The feedback controller \( G_c \) is a classical PI controller and it is designed using loop shaping techniques so that the system meets the requirements mentioned previously. This feedback controller without the feedforward term provides the benchmark results we evaluate against the feedforward added controller. The structure of the feedforward controller is examined in the next sections.

3. Nonlinear plant dynamics

The system behavior is considered as nonlinear, causal and time invariant with an output that includes harmonic spectral components \( K \omega_0 (K \in \mathbb{N}) \) to a sinusoidal input with frequency \( \omega_0 \). The idea of the optimal compensator is simply based on the minimization of the harmonic components except the fundamental frequency, \( i.e., K > 1 \). This minimization would make the system as close to a linear system as possible. Consequently, using a PI-type linear controller would be appropriate.

In order to highlight the nonlinear dynamics of the system, a nonlinear bode plot \( \Phi(u, \gamma) \) is obtained through NFRF methods excluding phase data. System is considered as uniformly convergent and a bounded continuous input signal \( u(t) = \gamma \sin(\omega t) \) with the frequency \( \omega \) and the amplitude \( \gamma \) is used. To define the excitation level and the frequency for the HOSIDF measurements, nonlinear Bode plot is utilized. The nonlinear Bode plot of the plant is visualized in Fig. 3.

To clearly observe the friction-based nonlinearities, the system should be excited with an input at the friction dominated stick–slip amplitude region. As indicated in the nonlinear Bode plot, the lowest amplitude levels are distorted due to the friction characteristics. As the amplitude increases, amplitude dependency of the magnitude plot decreases. It can be concluded that, the lowest amplitudes suffer from

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**Fig. 1.** The industrial motion platform with one DoF, which is used as the experimental test setup, and the schematics of the driveline.

**Fig. 2.** The block diagram of the control strategy.
The corresponding nonlinear harmonic response can be defined as
\[ u(t) = \gamma \cos(\omega t + \phi_0) \]
along with the \( k \)th higher order sinusoidal input describing function
\[ H_k(\omega_0, \gamma) \] introduced in Eq. (4):
\[ H_k(\omega_0, \gamma) = \frac{Y(k\omega_0)}{U^k(\omega_0)}, \]
where \( Y(k\omega_0) \) is the corresponding higher order response and \( U^k(\omega_0) \) is defined as Eq. (5):
\[ U^k(\omega_0) = \prod_{l=1}^{k} U(\omega_0) \]
where \( U(\omega_0) \) is the sinusoidal input with fundamental frequency.

Using the data from Fig. 4, the HOSIDF measurements are obtained using Fast Fourier Transform (FFT) and presented in Fig. 5. Each higher order sinusoidal input describing function is calculated using Eqs. (3)–(5). From the magnitude plot of the first order sinusoidal input describing function, the relation of the gain with the amplitude can be examined. Although from the input amplitudes of 0.07 Nm–0.2 Nm, the dependency of the system gain to the excitation amplitude is observable, after 0.2 Nm, the system gain is not essentially dependent on the excitation amplitude. However, even after the excitation level of 0.2 Nm, nonlinear behavior can be observed from the magnitude plots of higher order sinusoidal input describing functions. In higher orders starting from \( H_3 \), the system gain decreases until a certain level of minimum (approx. –30 dB) due to the uncertainties based on low signal-to-noise ratio.

Based on the amplitude dependency of the HOSIDF results given in Fig. 5, HOSIDF analysis can be considered as a descriptive tool for the examination of the amplitude dependency of our system. Hence, the HOSIDF technique can be utilized on our system in order to obtain the optimal feedforward compensation for friction compensation, similar to the studies presented in [32] and [33]. In this context, the optimality of the feedforward compensation is the minimization of the difference between the nonlinear system and its linearized counterpart (i.e. the system whose output do not include higher order harmonics).

4. Feedforward compensation

The friction has a great influence on the performance and the precision of our motion platform, especially at slow speeds. Different friction models [34] can be implemented as a model for feedforward compensation [35–37]. Even though the pure Coulomb friction based feedforward controller design improves the system performance, as implemented in [32], discontinuous characteristics of the model is not well-suited for optimization methods. More complex friction models
like LuGre model are utilized in some feedforward controller design methods [33] and adaptive techniques are studied as compensation tools [38,39]. However, complex friction models are highly dependent on the parametric changes on the system due to manufacturing processes or environmental conditions by its very nature. In this work, Stribeck friction model given in Eq. (6) is utilized in order to obtain a continuous and relatively simple feedforward signal compared to the Coulomb and LuGre friction models. The Stribeck friction model describes the friction torque in a rotational system as:

\[
T_f = \begin{cases} 
\sigma_2 \omega, & \text{if } \omega \neq 0 \\
T_a, & \text{if } \omega = 0 \text{ and } |T_a| < T_s \\
T_s \times \text{sgn}(T_a), & \text{otherwise}
\end{cases}
\]

(6)

where \( \omega \) is the angular velocity, \( T_f, T_a \) and \( T_s \) are the friction, applied input and stiction torques, respectively, and \( \sigma_2 \) is the viscous term. The Stribeck exponential curve \( s(\omega) \) in Eq. (6) is defined as:

\[
s(\omega) = (T_a + (T_s - T_a) e^{\frac{-\omega^2}{\omega_s^2}}) \times \text{sgn}(\omega),
\]

(7)

where \( T_s, \omega_s \) and \( \delta \) are the Coulomb torque, Stribeck velocity and Stribeck exponent, respectively. The parameters of the Stribeck model are identified using the Nelder–Mead optimization algorithm. The special excitation signal is generated from a band limited random signal up to 4 Hz, enveloped by the sum of first three non-zero terms of the Fourier series of a rectangular wave [30]. This speed reference enables the system to cross zero velocity line with different amplitudes and frequencies within a short time period. The excitation signal used as a speed reference is shown in Fig. 6 and the identified Stribeck model is illustrated in Fig. 7. The parameters of the model are given in Table 1. After the identification, instead of using the friction model output directly, we normalized the model’s output to the interval of \([-1,1]\) for low velocities by dividing the output by \( T_s \) and we utilized a gain \( K \) to acquire the final feedforward signal. The block diagram of the feedforward compensation \( G_{ff} \) is given in Fig. 8.

The experimental setup is initially subjected to a PI feedback controller in order to satisfy the requirements mentioned previously, i.e. a
minimum bandwidth of 6 Hz, a gain margin of 3–6 dB, and a phase margin of 30–60 deg. Next, the feedforward compensation design, shown in Fig. 8, is applied to the system with an increasing $K$ value in order to understand the effect of the feedforward gain to the harmonic response measurements. In order to find the optimal feedforward gain, the linear and the nonlinear systems are compared. The aim of the feedforward compensator is to force the nonlinear system to approximate the linear and the nonlinear systems are compared. The optimal feedforward gain can be stated as $K_{opt}$.

$K_{opt}$ can be obtained using the cost function: $\min \sum_{k \in N} \left| E[Y(kw_0)] \right|^2$, where $E[k \in N \geq 2|E[Y(kw_0)] \neq 0]$ with $\eta$–confidence level as introduced in [32]. This cost function, when minimized, guarantees that the feedforward constant $K$ found will maximize the relative magnitude of the fundamental frequency and force the system to behave as linear as possible.

Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Identified value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stribeck torque (Nm)</td>
<td>$T_s = 0.3160$</td>
</tr>
<tr>
<td>Coulomb torque (Nm)</td>
<td>$T_c = 0.4243$</td>
</tr>
<tr>
<td>Stribeck velocity (rad/s)</td>
<td>$w_s = 0.0247$</td>
</tr>
<tr>
<td>Stribeck exponent</td>
<td>$\delta = 1$</td>
</tr>
</tbody>
</table>

Fig. 8. The block Diagram of Feedforward Compensator ($G_{ff}$).

Fig. 7. The experimentally identified Stribeck model.

Fig. 8. The experimentally identified Stribeck model.

![Fig. 7. The experimentally identified Stribeck model.](image1)

![Fig. 8. The block Diagram of Feedforward Compensator ($G_{ff}$).](image2)

In this calculation, the noise in the experimental data should be considered and only the harmonics with a high signal to noise ratio should be taken into account [33]. We take only the harmonics with $\eta$–confidence level into account in the HOSIDF method. Harmonics without $\eta$–confidence level are neglected due to the fact that the noise level of those harmonics are higher than the harmonic signal. This can be considered as the quality of the measurements on harmonics. The expected value of the sample mean $\left| E[Y(w_0)] \right|$ is not equal to 0 with at least $\eta$–confidence level if the criteria given in Eq. (9) is greater than the corresponding cumulative distribution function.

$$\frac{\bar{Y}(kw_0)^2}{\sigma_Y^2(kw_0)} > F_{2.2N-1}$$.

where $F_{2.2N-1}$ defined as the cumulative $F_{2.2N-1}$ distribution $\text{cdf}(F_{2.2N-1}) = \eta$ [32]. The sample mean and the variance on sample mean are defined as Eqs. (10) and (11), respectively.

$$\bar{Y}(kw_0) = \frac{1}{N} \sum_{n=1}^{N} Y_n(kw_0)$$

$$\sigma_Y^2(kw_0) = \frac{1}{N(N-1)} \sum_{n=1}^{N} (Y_n(kw_0) - \bar{Y}(kw_0))^2$$

To determine the optimal value, $K$ values in the range of $[0,0.14]$ with 50 equal intervals are tested until overcompensating the setup, i.e., observing a system output leading the reference velocity during direction changes. In the experiments, a sinusoidal speed reference of $\gamma = 2$ and $w_0 = 0.5$ is used as the excitation signal.

5. Results and discussion

Using the experimental data, the frequency spectra of the input and the output are calculated using the FFT. Then, the output spectrum of the system is evaluated using Eqs. (8) and (9). Only the harmonics that satisfy the confidence level criteria given in Eq. (9) are used in the calculation of the optimal feedforward gain. The harmonic responses and the confidence level of the harmonics without the feedforward compensation are given along with the chosen harmonics in Fig. 9.

As it can be observed from Fig. 9, the response to the sinusoidal input with a fundamental frequency consist of higher harmonics for our system. Using only the chosen harmonics, the optimal feedforward gain is calculated using Eq. (8). The output of the cost function with respect to varying $K$ values is shown in Fig. 10. Once the feedforward gain is set to the optimal gain, the magnitude in chosen higher harmonics
decrease. As shown in Fig. 10, the feedforward gain of $K = 0.0808$ minimizes the contributions of the higher order harmonics at the output of the system, hence force our system to behave “more linear”.

The experiments shown in Fig. 9 are repeated after the feedforward gain is set to the optimal value of 0.0808. The harmonic responses and the confidence level of the harmonics are given in Fig. 11. The same harmonics selected for the case without the feedforward compensation are highlighted for the optimal feedforward compensation case. As it can be seen from the magnitude graphs, the magnitudes of all the higher order harmonics decreased in contrast of the fundamental harmonic frequency. This desired optimal case stands for the scenario where the effects of the higher harmonics are minimum compared to the fundamental harmonic frequency.

In order to observe the magnitude decrease in the relevant harmonics for the optimal feedforward compensation, $|\bar{Y}(k\omega_0)|$ values are illustrated in Fig. 12. By only applying the optimal feedforward compensation along with the PI feedback controller, significant decrease in $|\bar{Y}(k\omega_0)|$ for higher order harmonics can be achieved. For instance, the third and the seventh harmonics decreased by 22 and 14 dB, respectively. In contrast, the $|\bar{Y}(k\omega_0)|$ for the fundamental frequency is increased almost 10 dB in the optimal feedforward compensation case. The increase in the contribution of the fundamental frequency is a mathematical side effect of the feedforward compensation [33].

In the optimal case, the response of the system includes almost the same fundamental response along with decreased amplitudes of the higher harmonics, leading to an increase in the fundamental frequency contribution, which can be observed in Fig. 12.

6. Friction compensation performance

Performance measurements are conducted for a reference sinusoidal speed input signal with $\gamma = 2$ deg/s and $\omega_0 = 0.5$ Hz, as in the identification phase. The results are obtained for the case without feedforward compensation and the case with the optimal feedforward compensation ($K = 0.0808$). The sinusoidal reference tracking performance of the system in the two cases are shown in Figs. 13 and 14.

The results show that, the optimal feedforward compensation enables the system to achieve better reference tracking for zero crossing references, which is the region where the friction and the backlash effects are most dominant. In order to quantify this performance enhancement using only the speed reference tracking error initially, RMS tracking error is used, which is given in (12):

$$E_{RMS} = \sqrt{\frac{1}{M} \sum_{m=1}^{M} (u_m(t) - y_m(t))^2}$$

(12)
For the experiment shown in Fig. 13, the average tracking errors without feedforward compensation cases and with the optimal feedforward compensation are 0.0431 deg/s and 0.0117 deg/s, respectively. This difference in errors is mainly originated from the nonlinearities of the system and the performance of the feedforward compensation. The reference tracking error is illustrated in Fig. 15.

The RMS error is a descriptive tool for the error signal analysis for the speed reference tracking error as presented in Fig. 15. On the other hand, positioning accuracy during the same experiment can be quantified using the standard deviation of error integral. To calculate the performance of the method for the positioning accuracy, the error data is integrated. However, polynomial trend is observed in the error integral due to effects like the sensor drift. In a sense, the position data collected needs to be filtered through a high pass filter before making observations related to the positioning performance of the proposed method. Therefore, the polynomial fit over the error integral is calculated using the Least Squares Method and subtracted from the error integral. Then, the standard deviation over the integral of the error is calculated to obtain positioning accuracy for the speed reference tracking scenario. By this way, undesired sensor effects are omitted. For the error integral in Fig. 16, the standard deviations without feedforward compensation and with the optimal feedforward compensation cases are calculated as 0.0382 deg and 0.0051 deg, respectively.

Fig. 17 illustrates the torque input generated by the controller. Only the PI feedback controller generates the torque in the case without the feedforward compensation. Significant difference in zero crossing can be observed from the torque data.

The optimal feedforward gain can be also verified using the odd harmonic responses due to odd nonlinear characteristics of the system. The decrease in the contribution of the normalized, odd, higher order harmonics can be observed from Fig. 18.

7. Conclusion

This study describes a standardized method for the nonlinear identification and the optimal feedforward friction compensation of a single DoF industrial motion platform. Frequency domain identification based
on the HOSIDF concept is utilized to obtain the optimal friction compensation. The optimal compensation is achieved by minimizing the contributions of higher order ($K > 1$) harmonic spectral components $K\omega_0$ to the output for a sinusoidal input with frequency $\omega_0$. To represent the friction and the backlash in the platform, the simple and continuous Stribeck friction model is identified using a special reference signal. The model is normalized and used for feedforward compensation together with an optimal gain. The identification of the plant dynamics and the design of the LTI PI controller can be considered as the limitations of the methodology described in this study. Using different identification techniques to obtain a higher fidelity system model and designing a higher performance feedback controller might improve the performance of the overall methodology. Additionally, the feedback and the feedforward controllers can be iteratively redesigned to improve the results further. However, the identification and the feedback controller design described in this study is sufficient to demonstrate the optimality of the feedforward compensator for our system.

The contribution of this work is to utilize the HOSIDF based frequency domain identification techniques from the literature in the optimization of feedforward friction compensation design with the Stribeck friction model. The study presents a comprehensive method for the optimal nonlinear friction compensation of dynamic systems with harmonic responses. The experimental results demonstrate the improved system performance and higher precision in reference tracking. Further research can be directed to the identification of relevant harmonics of HOSIDF method with real-time data. Although the proposed procedure of identification and the optimal feedforward gain determination is a straightforward method, online techniques will enable the optimal feedforward compensation to be resistant to changes in the system due to the manufacturing processes or the environmental conditions.

CRediT authorship contribution statement

Ahmet Furkan Guc: Conceptualization, Methodology, Software, Investigation, Writing - original draft. Zafer Yumrukcal: Conceptualization, Resources, Writing - review & editing. Onur Ozcan: Conceptualization, Supervision, Writing - review & editing, Project administration, Funding acquisition.

Declaration of competing interest

No author associated with this paper has disclosed any potential or pertinent conflicts which may be perceived to have impending conflict with this work. For full disclosure statements refer to https://doi.org/10.1016/j.mechatronics.2020.102408.

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References

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