H_∞ BASED FILTERING FOR SYSTEMS WITH TIME DELAYS AND APPLICATION TO VEHICLE TRACKING

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By

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

H_∞ BASED FILTERING FOR SYSTEMS WITH TIME DELAYS AND APPLICATION TO VEHICLE TRACKING

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M.S. in Electrical and Electronics Engineering

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In this thesis, the filtering problem for linear systems with time delay is studied. The standard mixed sensitivity problem is investigated and the duality between H_∞ control problem and H_∞ filtering problem is established. By using this duality an alternative H_∞ filtering method is proposed.

An optimum H_∞ filter is designed for single output system. However the proposed technique does not only work for single delay in the measurement but also works for multiple delays in both state and measurement if the linear system has more than one outputs with delay. For this case, different suboptimal filters are designed.

This work also deals with an important aspect of tracking problems appearing in many different applications, namely state estimation under delayed and noisy measurements. A typical vehicle model is chosen to estimate the delayed state and the performances of the designed filters are examined under several scenarios based on different parameters such as amount of delays and disturbances.

ÖZET

ZAMAN GECİKMELİ SİSTEMLER İÇİN H_∞ TABANLI KESTİRİM VE ARAÇ TAKİP PROBLEMİNE UYGULANMASI

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Bu tez kapsamında zaman gecikmeli doğrusal sistemler için kestirim problemi üzerinde çalışılmıştır. Standart karışık hassasiyet problemi incelenmiş, burada anlatılan H_∞ kontrol problemi ile üzerinde çalışılan kestirim problemi arasında benzerlik kurulmuştur. Bu benzerlik kullanılarak alternatif bir kestirim metodu önerilmektedir.

Tek bir zaman gecikmesine sahip bir sistem için optimum süzgeç tasarlanmıştır. Bununla beraber önerilen yöntem birden çok gecikmeli çıktısı olan sistemlerde sadece ölçümde yer alan tek bir gecikme için değil, hem konumda hem de ölçümde yer alan birden fazla gecikme için çalışabilmektedir. Böyle bir durumda farklı alt optimum süzgeçler tasarlanmıştır.

Bu çalışmada ayrıca birçok uygulamada da görülebilecek olan izleme problemi yani gecikmeli ve gürültülü ölçümlerle konum kestirimi üzerine çalışılmıştır. Örnek bir araç modeli seçilmiş ve gecikmedeki farklılık ve benzeri değişik senaryolarda, tasarlanan süzgecin performansı test edilmiştir.

Anahtar Kelimeler: Zaman gecikmeli sistemler, Konum kestirimi, H_∞ süzgeç, Araç takibi

ACKNOWLEDGMENTS

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Chapter 1

INTRODUCTION

In this thesis we consider H_∞ filtering for systems with time delay. We use duality between control and estimation problems to propose a new structure for H_∞ based filters and apply to vehicle tracking under delayed and noisy measurements. In this chapter we give an overview of existing literature on estimation and filtering as well as on systems with delay and H_∞ techniques.

1.1   Estimation & Filtering

Estimation is the calculated approximation of a result which is usable even if input data may be incomplete, uncertain, or noisy. Estimation is defined as the process of inferring the value of a quantity of interest from indirect, inaccurate and uncertain observations, [21].

Some examples of estimation are:

- Statistical inference [40]

- Determination of the position and velocity of an aircraft (tracking) [35]

- Application of control to a plant in the presence of uncertainty (parameter identification, state estimation and stochastic control) [41]

- Determination of model parameters for predicting the state of a physical system (system identification) [39]

- Determination of characteristics of a transmitted message from noise-corrupted observation of the received signal (communication theory) [38]

- Determination of some parameters or characteristics of a signal or image (signal processing) [37]

In general, estimation can be classified into two categories related to the variable to be estimated.

- Parameter estimation for linear time invariant (LTI) systems

- State Estimation (Parameter estimation for time varying systems can be cast into this framework)

While parameter estimators for LTI systems deal with a time invariant quantity, state estimators are used in dynamic systems evolving in time and are more general than parameter estimators. In this thesis we will deal with state estimaton which is illustrated by Figure 1.1. The only part of the system that the estimator can access is the measurement which is affected by the measurement noise where the other parts are fixed transfer matrices. Estimation can be classified into three groups according to its nature and purpose [21]:

Filtering is the estimation of the current state of a dynamic system and provides an estimate of the output based on all data up to the current time.

Prediction is the estimation of the state ahead of the observed data. Prediction provides an estimate of the output at time k based on the data up to an earlier time $j < k$.
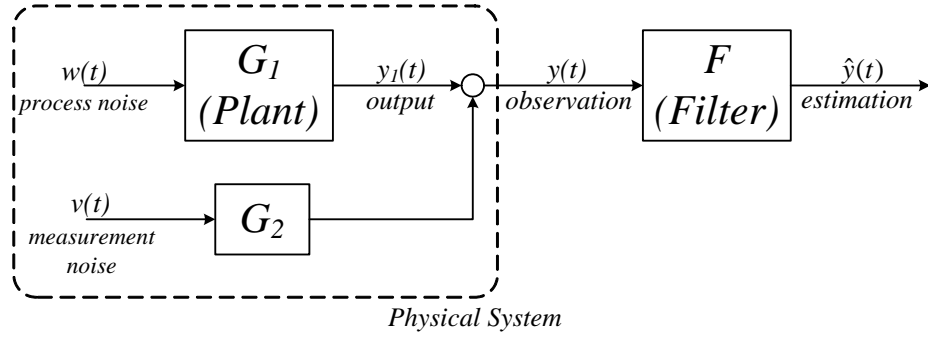
Figure 1.1: Estimation Problem in General

Smoothing is the estimation of the state behind of the observed data that are ahead in time of the estimate. Smoothing provides an estimate of the output at time k based on the data up to $j > k$.

Since we deal with real time estimation in this work we will use the term filtering. Actually the filter is a procedure of looking at a collection of data taken from a system or process. The term filtering mainly means that the process of removing undesired signals from the system, namely filtering the noise out.

The main concerns for someone, who does estimation, are the following [21]:

- A basic model of the system has to be available with unknown states to be estimated.

- Physical data are required to reduce the effect of measurement errors.

- In order to make the most accurate estimation, the residuals (difference between observation and estimation) should be as small as possible but one does not have control over this physical constraint.

1.2   Optimal Filtering

Optimal filtering is an innovative technique for detecting a signal against a background of noise or natural variability. While using optimal estimation techniques

someone has a model for some form of data that he or she observes or measures and also has an error criterion to minimize. Optimal estimation makes the best utilization of the data and the knowledge of the system and the disturbances under the optimization of some criterion. On the other hand, optimal estimation techniques can be sensitive to modeling errors and also computationally expensive. So it is very important to have a clear understanding of the assumptions under which an algorithm is optimal and also it is very important how these assumptions relate to real world.

1.3 H_∞ Optimal Filtering

In H_∞ filtering, H_∞ norm of the system which reflects the worst case gain of system is minimized. One does not need the exact knowledge of the statistics of the exogenous signals while designing an H_∞ optimal estimator. This estimation procedure ensures that the L_2 induced gain from the noise signals to the estimation error will be less than a prescribed γ level, where the noise signals are arbitrary energy-bounded signals. In the H_∞ setting, the exogenous input signal is assumed to be energy bounded rather than Gaussian white noise. Figure 1.1 is the general scheme of estimation. H_∞ filtering for such a system is made performing the following expressions:

$$y_1 = G_1 w$$

$$y = y_1 + G_2 v$$

Exogenous signals w and v prevent handling the measurement correctly. So we need to filter the measurement, y, with a filter denoted as F.

$$\hat{y} = Fy$$

Then the error between the output and estimated measurement is called estimation error of which L_2 norm over L_2 norm of noise signals will be less than

prescribed γ.

$$e = Fy - y_1 = (F - I)G_1 w + FG_2 v$$

$$\gamma = \sup \frac{\|e\|_2}{\left\| \begin{bmatrix} w \\ v \end{bmatrix} \right\|_2} \quad \text{such that} \quad \begin{bmatrix} w \\ v \end{bmatrix} \neq 0.$$

1.4 Time Delay Systems

Time-delay systems appear naturally in many engineering applications and, in fact, in any situation in which transmission delays cannot be ignored. Therefore, their presence must be taken into account in a realistic filter design. Moreover, stability and noise attenuation level guaranteed by filter design without considering time delays may collapse in the presence of non-negligible time delays. Time delays arise in several signal processing related problems, such as, echo cancellation, local loop equalization, multipath propagation in mobile communications, and array signal processing.

Time delays often also appears in many control systems (such as aircraft, chemical or process control systems) either in the state, the control input, or the measurements. Unlike ordinary differential equations, delay systems are infinite dimensional in nature and time-delay is, in many cases, a source of instability. The stability issue and the performance of control systems with delay are, therefore, both of theoretical and practical importance [36].

For a brief history of time delay systems, see [36] where it is mentioned that the delay equations were first considered in the literature in the XVIII century. Also it is mentioned in the same paper that systematic method study has began and Lyapunov's method was developed for stability of delay systems until 50's in XX century. Since 1950s, the subject of delay systems or functional differential

equations has received a great deal of attention in Mathematics, Biology and Control Engineering.

Over the past decade, much effort has been invested in the analysis and synthesis of uncertain systems with time-delay. Based on the Lyapunov theory of stability, various results have been obtained that provide, for example, finite-dimensional sufficient conditions for stability and stabilization. Departing from the classical linear finite-dimensional techniques which apply Smith predictor type designs, the new methods simultaneously allow for delays in the state equations and for uncertainties in both the system parameters and the time delays. During the early stages, delay-independent results were obtained which guarantee stability and prescribed performance levels of the resulting solutions. Recently, delay-dependent results have been derived that considerably reduce the overdesign entailed in the delay-independent solutions. In the present issue, new results are obtained for various control and identification problems for delay systems. These results are based either on Lyapunov methods or on frequency domain considerations.

In the next section we will focus on H_∞ filtering for time delay systems.

1.5 State Estimation for Time Delay Systems

1.5.1 H_∞ Techniques for State Estimation for Time Delay Systems

Many different H_∞ filtering techniques have appeared in the literature. All of them try to develop methodologies which ensure a prescribed bound on L_2 induced gain from noise signals to filtering error [4]-[15]. In this scope following Luenberger observer type filter is developed depending on a derived version of

bounded real lemma [5] and depending on a descriptor model transformation, Park's inequality, for the bounding of cross terms [6]. Some of them includes polytopic uncertainties in all matrices [6], [10], [11]. Consider a dynamic system as below:

$$\dot{x}(t) = Ax(t) + \sum_{i}^{m} A_i x(t - h_i) + Bw(t)$$

$$y(t) = Cx(t) + \sum_{i}^{m} C_i x(t - h_i) + Dw(t)$$

where $x(t)$ is the state vector, $y(t)$ is the measurement output vector, h is the fixed, known delay and $w(t)$ is the L_2 disturbance vector ($w \in L_2[0, \infty]$). Luenberger type observer for the above system is given by the following dynamical system:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \sum_{i}^{m} A_i \hat{x}(t - h_i) - L(\hat{y}(t) - y(t))$$

$$\hat{y}(t) = C\hat{x}(t) + \sum_{i}^{m} C_i \hat{x}(t - h_i)$$

where $\hat{x}(t)$ is the estimated state of $x(t)$, $\hat{y}(t)$ is the estimated output of $y(t)$ and L is the observer gain matrix. The estimated error, defined as $e(t) = x(t) - \hat{x}(t)$, obeys the following dynamical system:

$$\dot{e}(t) = (A - LC)e(t) + \sum_{i}^{m} (A_i - LC_i)(t - h_i) + (B - LD)w(t)$$

Then the γ observer for this system is designed under the conditions:

1. $\lim_{t \to \infty} e(t) \to 0$ for $w(t) \equiv 0$

2. $\|T_{ew}\|_{\infty} \leq \gamma$

where T_{ew} is the transfer function from disturbance w to the estimation error e and $\gamma > 0$. Now the problem is to reach L which is found by solving some set of linear matrix inequalities (LMI) and algebraic riccati equations in [5]-[9] as below:

$$L = \frac{1}{\varepsilon} P^{-1} C^T$$

where P is the solution of linear matrix inequalities or algebraic riccati equations.

Both instantaneous and multiple-time delayed measurements using the technique that re-organizes innovation analysis approach in Krein space is studied in [12]. It also gives the results in terms of the solutions of Algebraic Riccati and Matrix Differential Equations.

Full order and reduced order filters are designed for discrete time linear systems with delay [13], [14]. In order to design these type of filters LMI's are used similar to the ones in continuous time systems.

We should indicate that most of the above mentioned techniques involving LMIs are suboptimal in the sense that the filter can be obtained under the condition that the LMIs are solvable. In most situations the optimal performance level cannot be achieved. Besides the frequency domain method proposed in this paper, there are some time domain state-space based techniques leading to optimal H_∞ filters, see e.g. [17], [16]. In [16] a lifting technique is used to solve the associated Nehari problem (see Chapter 2 below). In [17], Mirkin solves the problem by parameterizing all solutions of the non-delayed problem and finding the ones which solve the delayed problem. This approach involves solving Riccati equations and checking a spectral radius condition. Among all available methods for the solution of the H_∞ suboptimal filtering problem under delayed and noisy measurements, Mirkin's approach [17] is the simplest. Moreover, his "central" filter's performance can get arbitrarily close to the optimum.

1.5.2 Other Techniques for State Estimation for Time Delay Systems

Although H_∞ filtering technique is widely used method for time delay systems, some other methods are used up to now. Darouach developed linear functional

observers for systems with time delays in state variables both in continuous time [22] and in discrete time [23]. Also similar approach is developed in [24] not for single delay as in [22] and [23] but multiple delays in state variables. Motivated by the previous works, similar but extended types of observers are developed [32]. Functional, reduced order and full order observers with and without internal delays are considered. However only the delay in the state equation is considered while designing observers. The observer converges, with any prescribed stability margin, to any number of linear functionals when some conditions are met in [33].

A robust observer is designed in [25] using LMI's. In order to provide the stability of observer it can be designed in two steps of which first step is to design a linear feedback control [29]. Then two Luenberger type observers are designed in the second step. Sufficient conditions are given in the form of Linear Matrix Inequalities.

State observers for systems with nonlinear disturbances where the non-linearities satisfy the Lipschitz conditions is considered in [34]. It is shown that the observation error is globally exponentially stable.

H_2 filtering for linear systems with time delays is issued in [26]-[28]. Since the filtering problem for time delay systems problem is infinite dimensional in nature, an attempt is made to develop finite dimensional methods that will guarantee a preassigned estimation accuracy. They give the sufficient conditions in the form of Linear Matrix Inequalities.

Kalman filter approach is extended to the case in which the linear system is subject to norm-bounded uncertainties and constant state delay then a robust estimator is designed for a class of linear uncertain discrete-time systems [30]. Estimation error covariance is guaranteed to lie within a certain bound for all admissible uncertainties solving two Riccati Equations. In this framework [20]

discusses trade off between optimality and computational burden of the filter and develops a method based on "extrapolating" the measurement to present time using past and present estimates of the Kalman filter and calculating an optimal gain for this extrapolated measurement.

1.6 Thesis Contribution and Organization

In the remaining parts of the thesis, using the frequency domain representations, we provide an alternative method to compute the H_∞ optimum filter directly. First, by using the duality between filtering and control, the problem at hand is transformed to a robust controller design for systems with time delays. The skew Toeplitz method developed earlier for the robust control of infinite dimensional systems, [1], [2], [3], is used to solve the H_∞ optimal filtering problem.

In Chapter 2, H_∞ estimation problem is described in detail and duality between estimation problem and standard mixed sensitivity H_∞ control problem is given. The solution process that is used in this work is mentioned and some number of approaches are developed for the solution. Chapter 3 includes vehicle tracking problem definition with the reason why we have chosen this problem as well as the illustrative examples and comparative results with the previous methods. Chapter 4 concludes the work and proposes some future work.

Chapter 2

FILTERING VIA DUALITY BETWEEN CONTROL AND ESTIMATION PROBLEMS

In this chapter a solution for H_∞ filtering problem is proposed establishing the duality between control and estimation problems. First the system for which the estimator will be designed is clarified and then the control problem is shortly mentioned and the solution is given at last.

2.1   System Architecture

Consider the linear system in Figure 2.1 with multiple time delays in state and output variables as described in (2.1), (2.2) and (2.3).

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{k} A_i x(t - h_i) + B w(t) \tag{2.1}$$

$$y_j(t) = C_j x(t - h_j) + D_j v_j(t) \qquad j = 1, \ldots, l \tag{2.2}$$
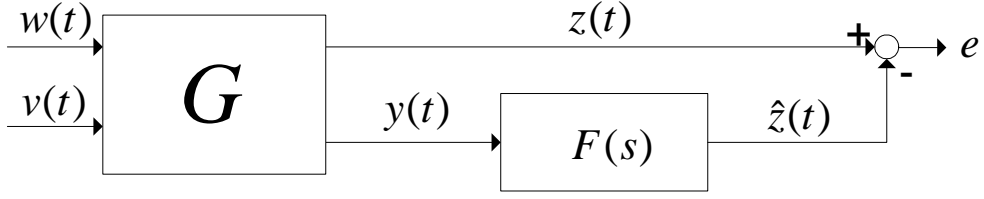
Figure 2.1: Dynamic System Model for Estimation

$$z(t) = Lx(t) \tag{2.3}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y_j(t) \in \mathbb{R}^p$ is j^{th} output vector, $z(t)$ is the state variable to be estimated, $w(t)$ and $v(t)$ are process noise and measurement noise respectively. Time delays h_i and h_j are assumed to be known. The matrices A_0, A_i, B, C_j, D_j and L are also known. Each sensor output available for filtering is described in (2.2).

Assumption: All the sensors that give information about the state are identical. They have all same characteristics other than the delays that differ due to the other factors such as distance and communication link properties. Therefore we assume that $h_j > 0$ for all j and C_j's are same with each other and also with L, namely:

$$C_1 = C_2 = C_3 = ... = C_l = L = C \tag{2.4}$$

Then (2.2) turns into;

$$y_j(t) = Cx(t - h_j) + D_j v_j(t) \qquad j = 1, \dots, l \tag{2.5}$$

2.2 H_∞ Filtering Problem

The term estimator is used for the function which gives the estimated value that we look for. For the system above, we will design an estimator in order to estimate $z(t)$ and this estimator must minimize the estimaton error which is the difference between estimate and the objective function, $z(t)$. Transfer functions

in (2.1) and (2.2) are found as following: Taking the Laplace Transform of both sides of (2.1) gives

$$sX(s) = A_0 X(s) + \sum_{i=1}^{k} e^{-h_i s} A_i X(s) + BW(s) \tag{2.6}$$

where the Laplace Transform of $x(t)$ is $X(s)$ and of $x(t-h)$ is $e^{-hs} X(s)$. Then the transfer funcion from w to state x is

$$X(s) = R(s)BW(s) \tag{2.7}$$

where

$$R(s) = (sI - A_0 - \sum_{i=1}^{k} A_i e^{-h_i s})^{-1} \tag{2.8}$$

The transfer function to the output is found as below:

$$Y_j(s) = Ce^{-h_j s} X(s) + D_j V_j(s) \tag{2.9}$$

Using $X(s)$ obtained from (2.7) turns (2.9) into

$$Y_j(s) = Ce^{-h_j s} R(s)BW(s) + D_j V_j(s) \tag{2.10}$$

In these type of systems, time delays in state and output prevent handling the output directly. Due to the this fact we have to minimize the estimation error corresponding to the estimate $\hat{z}(t)$, which is mainly composed of a filter affecting on the output $y(t)$ and expressed in frequency domain as

$$\hat{Z}(s) = \sum_{j=1}^{l} F_j(s)Y_j(s) \tag{2.11}$$

Thus the estimation error is written and found by the following equations:

$$e(t) = z(t) - \hat{z}(t) \tag{2.12}$$

$$
\begin{aligned}
E(s) &= Z(s) - \hat{Z}(s) \\
&= LX(s) - \sum_{j=1}^{l} F_j(s) Y_j(s) \\
&= LR(s)BW(s) - \sum_{j=1}^{l} F_j(s)(Ce^{-h_j s} R(s)BW(s) + D_j V_j(s))
\end{aligned}
\tag{2.13}
$$

Let us now assume that the measurement noise v is generated by a known coloring filter W_v, i.e. $V(s) = W_v(s)\hat{V}(s)$, where $\hat{v}(t)$ is an unknown finite energy signal. Similarly, let w be an unknown finite energy signal. Then the estimation error is

$$
\begin{aligned}
E(s) &= (I - \sum_{j=1}^{l} F_j(s) e^{-h_j s}) P_0(s) W(s) - \sum_{j=1}^{l} F_j(s) D_j W_v \hat{V}_j(s) \\
&= \begin{bmatrix} (I - \sum_{j=1}^{l} F_j(s) e^{-h_j s}) P_0(s) & -F_1(s)D_1(s)W_v & \dots F_l(s)D_l(s)W_v \end{bmatrix}
\begin{bmatrix} W(s) \\ \hat{V}_1(s) \\ \vdots \\ \hat{V}_l(s) \end{bmatrix} \\
&= \begin{bmatrix} (I - \mathbf{F}(s)\mathbf{H}(s))P_0(s) & -\mathbf{F}(s)\mathbf{D}(s) \end{bmatrix}
\begin{bmatrix} W(s) \\ \hat{\mathbf{V}}(s) \end{bmatrix}
\end{aligned}
\tag{2.14}
$$

where

$$
P_0 = CR(s)B, \qquad L = C
$$

and

$$
\mathbf{F}(s) = \begin{bmatrix} F_1(s) & F_2(s) & \dots & F_l(s) \end{bmatrix}
$$

$$
\mathbf{D}(s) = \begin{bmatrix} D_1(s)W_v & 0 & 0 & 0 \\ 0 & D_2(s)W_v & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & D_l(s)W_v \end{bmatrix}
\qquad
\mathbf{H}(s) = \begin{bmatrix} e^{-h_1 s} \\ e^{-h_2 s} \\ \vdots \\ e^{-h_l s} \end{bmatrix}
\tag{2.15}
$$

$$
\hat{\mathbf{V}}(s) = \begin{bmatrix} \hat{V}_1(s) & \hat{V}_2(s) & \dots & \hat{V}_l(s) \end{bmatrix}^T \qquad \mathbf{D}(s), \mathbf{D}^{-1}(s) \text{ are stable.}
$$

$F_j(s)$'s are the filters that will be designed to eliminate the effect of h_j's for each sensor output. D_j's are the are the weights associated with the noise v_j.

Then the L_2 induced norm from external signals w and $\hat{v}$ to the error e is

$$\gamma \;=\; \inf_{F \in H_\infty} \left\| \left[\; (I - \mathbf{F}(s)\mathbf{H}(s))P_0(s) \quad -\mathbf{F}(s)\mathbf{D}(s) \; \right] \right\|_\infty \qquad (2.16)$$

Multi-input multi-output (MIMO) solution of filtering problem will be reached via (2.16). However let us reduce it to a simpler solution which is Single-input single-output (SISO) case

$$\gamma_j \;=\; \inf_{F \in H_\infty} \left\| \; (\tfrac{1}{l} - F_j(s)e^{-h_j s})P_0(s) \quad -F_j(s)D_j W_v \; \right\|_\infty$$

$$= \;\; \sup_{\hat{v};w \neq 0} \frac{\|e_j\|_2}{\left\| \begin{bmatrix} w \\ \hat{v}_j \end{bmatrix} \right\|_2} \qquad (2.17)$$

However multi output case brings a suboptimal solution in other words sum of the optimal solutions for each output do not go to an optimal filter in general solution because;

$$\gamma_{opt} \leq \sum_{j=1}^{l} \gamma_j.$$

Clearly the following two conditions must be satisfied in order to have a finite γ_j:

$$F_j(s) \text{ is stable, and}$$
$$(\tfrac{1}{l} - F_j(s)e^{-h_j s})P_0(s) \text{ is stable} \qquad (2.18)$$

2.3   Mixed Sensitivity Control Problem

The standard mixed sensitivity H_∞ control problem associated with a stable plant $\widetilde{P}$ shown in Figure 2.2 can be defined as follows.

Transfer functions from the disturbance $\widetilde{w}$ to $\widetilde{y}$ and $\widetilde{u}$ are:

$$T_{\widetilde{w} \to \widetilde{y}} = W_1(1 + \widetilde{P}\widetilde{C})^{-1} \qquad (2.19)$$
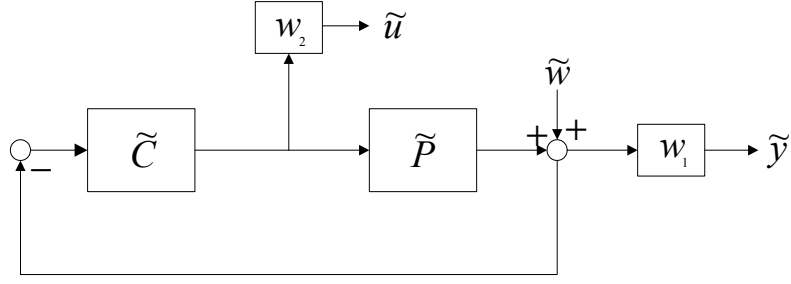
Figure 2.2: Feedback Control System

$$T_{\widetilde{w} \to \widetilde{u}} = -W_2 \widetilde{C}(1 + \widetilde{P}\widetilde{C})^{-1} \tag{2.20}$$

The optimal H_∞ controller design problem is:

$$\begin{aligned} \text{minimize} \quad & \gamma \\ \text{subject to} \quad & (\widetilde{P}, \widetilde{C}) \text{ is stable, and} \end{aligned} \tag{2.21}$$

$$\gamma = ||T_{\widetilde{w} \to \begin{bmatrix} \widetilde{y} \\ \widetilde{u} \end{bmatrix}}||_\infty \tag{2.22}$$

In order to find the smallest γ following is solved as below in case all the variables are constant (if not, transpose of that has to be solved):

$$\inf_{\widetilde{Q} \in H_\infty} \| \begin{bmatrix} W_1(1 - \widetilde{P}\widetilde{Q}) & -W_2\widetilde{Q} \end{bmatrix} \|_\infty \tag{2.23}$$

The free parameter $\widetilde{Q}$ is obtained from the controller

$$\widetilde{C} = \frac{\widetilde{Q}}{1 - \widetilde{P}\widetilde{Q}} \qquad \widetilde{Q} = \frac{\widetilde{C}}{1 + \widetilde{P}\widetilde{C}}.$$

The important point throughout this work is that the SISO result of the estimation problem (2.17) is the same problem with (2.23) provided that the following dualities are established:

$$\begin{aligned} W_1(s) &= P_0(s) \\ W_2(s) &= -I \\ \widetilde{P}(s) &= e^{-h_j s} D_j^{-1} W_v^{-1}(s) \\ \widetilde{Q}(s) &= F_j(s) D_j W_v(s) \end{aligned} \tag{2.24}$$

2.4 Solution

The problem of Section 2.3 can be solved using the technique in [1] where an optimal controller is designed for the system in Figure 2.2 in the form of:

$$\widetilde{C}_{opt}(s) = E_{\gamma_0}(s) m_d(s) \frac{N_0(s)^{-1} F_{\gamma_0}(s) L(s)}{1 + m_n(s) F_{\gamma_0}(s) L(s)} \tag{2.25}$$

The plant $P(s)$ admits a coprime inner/outer factorization of the form $P(s) = m_n(s) N_o(s)/m_d(s)$ where m_d and m_n are inner and N_o is outer. Set $E_{\gamma o}$ as

$$E_{\gamma o} := \left(\frac{W_1(-s) W_1(s)}{\gamma^2} - 1 \right)$$

Then define

$$F_{\gamma o} = G_{\gamma o}(s) \prod_{k=1}^{n_l} \frac{s - \eta_k}{s + \eta_k}$$

where $\eta_1, \ldots, \eta_k$ are the poles of $\widetilde{W}_1(-s)$ and $G_{\gamma o}$ is minimum phase and determined from the spectral factorization

$$G_{\gamma o}(s) G_{\gamma o}(-s) := \left(1 - \left(\frac{W_2(s) W_2(-s)}{\gamma^2} - 1 \right) E_{\gamma o}(s) \right)^{-1}$$

$L(s)$ is composed of the form $L(s) = L_2(s)/L_1(s)$ where $L_2(s)$ and $L_1(s)$ are the polynomials which satisfy the interpolation conditions given in [1].

After deriving the controller expression it is easy to find dual filter equation that we desire:

$$\widetilde{Q}_{opt} = \widetilde{C}_{opt}(1 + \widetilde{P}\widetilde{C}_{opt})^{-1}$$

2.4.1 Suboptimal Approach-I

Reconsider the estimation error under the special case restriction that is,

$$\hat{v}_j = \hat{v}_i = v$$

$$E(s) = \left[\ (I - \sum_{j=1}^{l} F_j(s)e^{-h_j s})P_0(s) \quad -\mathbf{F}(s)\mathbf{D}(s)\ \right] \begin{bmatrix} W(s) \\ \hat{\mathbf{V}}(\mathbf{s}) \end{bmatrix}$$

where $\mathbf{F}(s)$, $\mathbf{D}(s)$ and $\hat{\mathbf{V}}(\mathbf{s})$ are defined by (2.15). Let $\hat{F}_j(s) = F_j(s)D_j(s)W_v$ and $\widetilde{F}(s) = \sum_{j=1}^{l} \hat{F}_j(s)$ Then first part of the above expression becomes

$$\left[\ (I - \sum_{j=1}^{l} \hat{F}_j(s)D_j^{-1}(s)W_v^{-1}e^{-h_j s})P_0(s) \quad -\widetilde{F}(s)\ \right]$$

$$= \left[\ (I - D_1^{-1}(s)W_v^{-1}e^{-h_1 s}\mathcal{F}(s))P_0(s) \quad -\widetilde{F}(s)\ \right]$$

where

$$\mathcal{F}(s) = \hat{F}_1(s) + \widetilde{F}(s) - \widetilde{F}(s) + \sum_{j=2}^{l} F_j(s)D_1(s)D_j^{-1}(s)e^{-(h_j - h_1)s}$$

Assume that $h_1 \leq h_2 \leq \ldots \leq h_l$ (If not, following operations can be modified easily)

$$E(s) = \left[\ (I - D_1^{-1}(s)W_v^{-1}e^{-h_1 s}\widetilde{F}(s))P_0(s) \quad -\widetilde{F}(s)\ \right] \begin{bmatrix} w \\ v \end{bmatrix}$$

$$+ \left[\ D_1^{-1}(s)W_v^{-1}e^{-h_1 s}(I - D_1(s)D_2^{-1}(s)e^{-(h_2 - h_1)s})\hat{F}_2(s)P_0(s) \quad 0\ \right] \begin{bmatrix} w \\ v \end{bmatrix}$$

$$\vdots$$

$$+ \left[\ D_1^{-1}(s)W_v^{-1}e^{-h_1 s}(I - D_1(s)D_l^{-1}(s)e^{-(h_l - h_1)s})\hat{F}_l(s)P_0(s) \quad 0\ \right] \begin{bmatrix} w \\ v \end{bmatrix}$$

Now the problem will be solved in steps:

1. minimize $\|\ (I - D_1^{-1}(s)W_v^{-1}e^{-h_1 s}\widetilde{F}(s))P_0(s) \quad -\widetilde{F}(s)\ \|_\infty$ and find optimal $\widetilde{F}(s)$.

2. minimize $\|W_0 + D_1^{-1}(s)W_v^{-1}e^{-h_1 s}(I - D_1(s)D_2^{-1}(s)e^{-(h_2-h_1)s})\hat{F}_2(s)P_0(s)\|_\infty$

   where $W_0 = (I - D_1^{-1}(s)W_v^{-1}e^{-h_1 s}\widetilde{F}(s))P_0(s)$ and find optimal $\hat{F}_2(s)$.

3. minimize $\|W_1 + D_1^{-1}(s)W_v^{-1}e^{-h_1 s}(I - D_1(s)D_3^{-1}(s)e^{-(h_3-h_1)s})\hat{F}_3(s)P_0(s)\|_\infty$

   where $W_1 = W_0 + D_1^{-1}(s)W_v^{-1}e^{-h_1 s}(I - D_1(s)D_2^{-1}(s)e^{-(h_2-h_1)s})\hat{F}_2(s)P_0(s)$

   and find optimal $\hat{F}_3(s)$.

4. ...

Step 1 is the dual problem of the work solved before so $\widetilde{F}(s)$ is found easily. Step 2 and subsequent ones are more complicated than first step. They will also be solved by same technique, however some sort of extra works have to be done in order to obtain a solvable minimization problem.

2.4.2 Suboptimal Approach-II

Reconsider the estimation error equation (2.14):

$$E(s) = \begin{bmatrix} (I - \mathbf{F}(s)\mathbf{H}(s))P_0(s) & -\mathbf{F}(s)\mathbf{D}(s) \end{bmatrix} \begin{bmatrix} W(s) \\ \hat{\mathbf{V}}(s) \end{bmatrix}.$$

Above expression can be written as sum of l expressions:

$$E(s) = \sum_{j=1}^{l} \begin{bmatrix} (\alpha_j - F_j(s)e^{-h_j s})P_0(s) & -F_j(s)D_j W_v \end{bmatrix} \begin{bmatrix} W(s) \\ \hat{V}_j(s) \end{bmatrix} \qquad (2.26)$$

where $\sum_{j=1}^{l} \alpha_j = 1$

Each error expression namely the estimation error of jth output is solved in order to find $F_j(s)$ writing the L_2 induced norm from exogeneous signals to error:

$$\begin{aligned} \gamma_j &= \inf_{F \in H_\infty} \| \begin{bmatrix} (1 - \alpha_j^{-1}F_j(s)e^{-h_j s})\alpha_j P_0(s) & -F_j(s)D_j W_v \end{bmatrix} \|_\infty \\ &= \sup_{\hat{v},w \neq 0} \frac{\|e_j\|_2}{\left\| \begin{bmatrix} w \\ \hat{v}_j \end{bmatrix} \right\|_2} \end{aligned} \qquad (2.27)$$

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Optimal γ_j can be found easily using the method developed by dual control problem. However solving l-decoupled SISO H_∞ optimization problem gives a suboptimal solution since

$$\sup_{\hat{v},w \neq 0} \frac{\|e_j\|_2}{\left\| \begin{bmatrix} w \\ \hat{v}_j \end{bmatrix} \right\|_2} \leq \sum_{j=1}^{l} \| \ (\alpha_j - F_j(s)e^{-h_j s})P_0(s) \quad -F_j(s)D_j W_v \ \|_\infty$$

which means the sum of optimal γ_j's for each output is larger than the optimal γ of the complete problem.

$$\gamma_{opt} \leq \sum_{j=1}^{l} \gamma_j$$

Another problem about this type of sub-optimal solution is to determine α_j's. Actually α_j shows how the state affects the jth output. In other words it is related in what proportion of the state the jth filter will estimate. At first sight it seems that the output with smaller delay desires larger α ie. $h_i < h_k \Rightarrow \alpha_k < \alpha_i$. This condition is clearly satisfied unless we discard D_j. Therefore we can say that there should be a trade-off between α_j's that minimizes the sum of all. To summarize the problem, it is required to solve the minimization problem which is easier to solve than Approach-I:

$$\begin{aligned} \text{minimize} \quad & \sum_{j=1}^{l} \gamma_j \\ \text{subject to} \quad & \gamma_j = \| \ (\alpha_j - F_j(s)e^{-h_j s})P_0(s) \quad -F_j(s)D_j W_v \ \|_\infty \\ & \sum_{j=1}^{l} \alpha_j = 1 \end{aligned}$$

In this chapter, we derived the formulae of the filter to estimate the state of a linear system with time delay. Next section includes the illustration of these derivations with many different situations on a vehicle tracking problem.

Chapter 3

APPLICATION TO VEHICLE TRACKING PROBLEM

In this chapter, filter design method developed in Chapter 2 is applied to a vehicle tracking problem with delayed and noisy measurements. First the problem with the dynamic model of the vehicle is described. Then various versions of the problem are studied by varying some parameters such as amount of delay, number of delays, uncertainties. Finally some methods from literature are used to solve the same problem with comparative results.

3.1 Tracking Problem

The term *"tracking"* is the estimation of the state of a moving object based on remote measurements. This is done using one or more sensors located on different locations [21]. Vehicle tracking is widely used for safety purposes or military objectives such as determination of position and velocity of an aircraft.

3.1.1 Moving Vehicle Model

The vehicle model to be given here will be used in all simulations performed in this chapter. In order to obtain quick and clear results a simple vehicle model is chosen. The linear dynamic equations determining the behaviour of the vehicle are given as

$$\dot{x}(t) = Ax(t) + Bw(t)$$

$$y(t) = Cx(t) + Dv(t)$$

$$z(t) = Lx(t)$$

where x is the state of vehicle and composed of *position, velocity* and *acceleration*, y is the output vector and z is the state element to be estimated which is the *position* in our design. The signals w and v are process and measurement noise respectively. The matrices A, B, C, D and L are:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\varepsilon \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$C = L = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \qquad D = 1$$

$$x = \begin{pmatrix} p \\ v \\ a \end{pmatrix} \quad \begin{matrix} p : position \\ v : velocity \\ a : acceleration \end{matrix} \quad \Rightarrow \quad \dot{x} = \begin{pmatrix} \dot{p} \\ \dot{v} \\ \dot{a} \end{pmatrix} \quad \begin{matrix} \dot{p} = v : velocity \\ \dot{v} = a : acceleration \end{matrix}$$

Accordingly, rewrite the dynamic equations;

$$\begin{pmatrix} \dot{p} \\ \dot{v} \\ \dot{a} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\varepsilon \end{pmatrix} \begin{pmatrix} p \\ v \\ a \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} w(t)$$

This seems like a constant velocity model, but it has a disturbance in the acceleration derivative. The parameter ε determines the effect of initial value of the state, i.e.

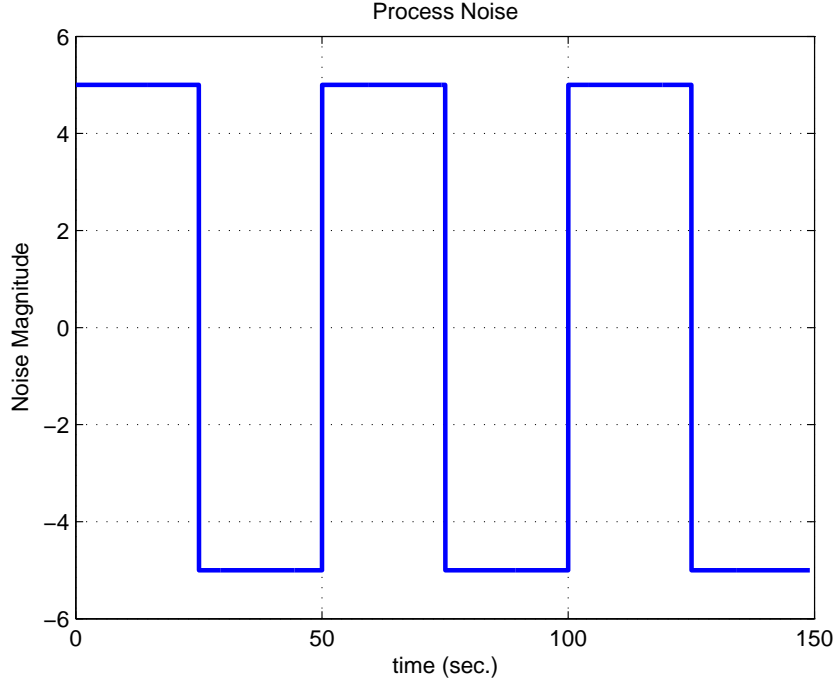$$\dot{a} + \varepsilon a = w(t) \qquad \text{Typically } a(t) \longrightarrow a$$

Figure 3.1: Process Noise $w(t)$

Output is the position with an unknown signal and it is the signal to be estimated.

$$y(t) = p(t) + v(t)$$

$$z(t) = p(t)$$

The disturbance $w(t)$, initial value of state and ε are the leading effects on the movement of the vehicle. In order to see the maneuvers in the movement $w(t)$ is chosen as finite energy signal as in Figure 3.1 which leads the maneuvers in Figure 3.3 which shows the vehicle behaviour in terms of its position in one dimension. Initial value of state and ε parameter are arbitrarily chosen as $x(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $\varepsilon = 2$ respectively. Additionally, different values of ε will be illustrated. Also in all simulations band-limited white noise $\hat{v}(t)$ is used as the measurement noise which is shown in Figure 3.2.

Our main objective throughout this work is to make an estimation of the state of a system with time delays. However it is aimed in this section to describe the
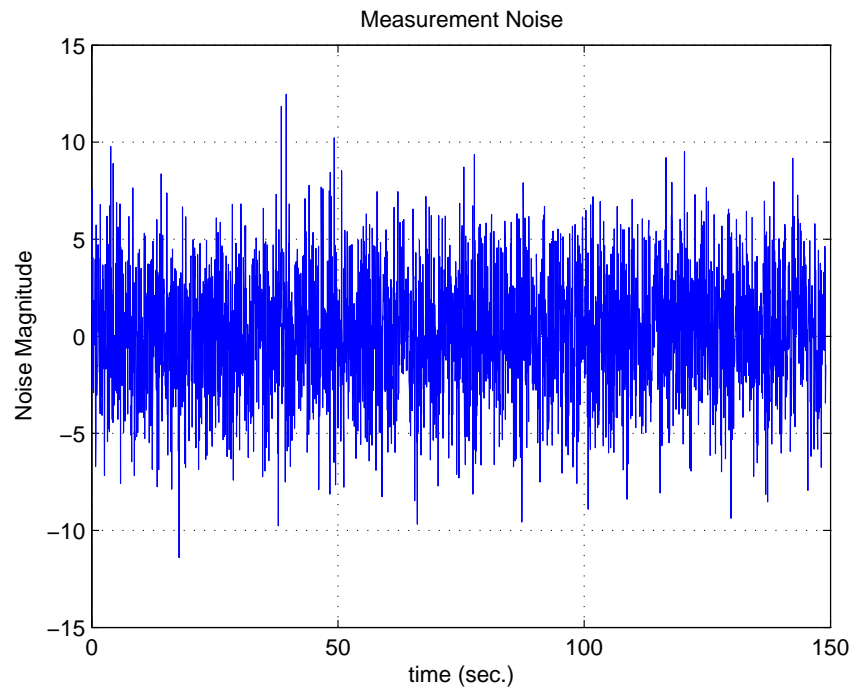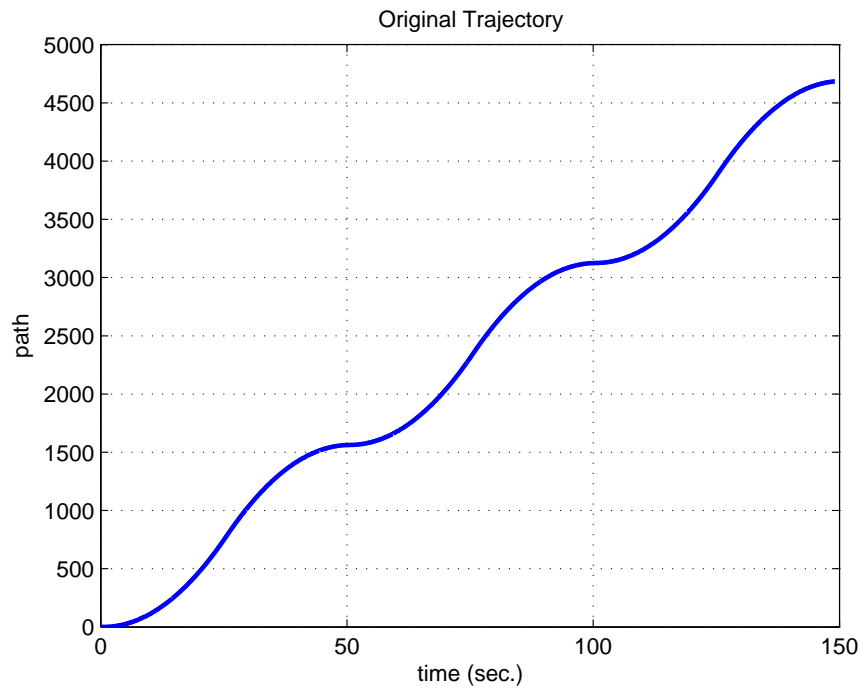
Figure 3.2: Measurement Noise $v(t)$

Figure 3.3: Original Trajectory to be Estimated

vehicle characteristics that will be used in simulations. That's why the delay issue has never been mentioned. Following sections include modified versions of the dynamic system which is issued here. Main difference will be the delay that will appear somewhere in the system.

3.2   Illustrative Examples and Simulation Results

Many different variations of vehicle tracking problem are illustrated in this section to investigate the filter performance against different situations. Simulations were performed by the help of MATLAB-Simulink. First, we will see how the amount of delay affects the filter performance and try to find whether the filter has a limitation in terms of delay amount. Then the ε parameter will be changed in order to see how the different initial values of acceleration impact the filter performance. It will be shown that the existence of delay in state variable does not create difficulties for the solution.

In order to get comparable results of different situations all the simulations were performed with dynamic system mentioned in previous section changing the related parameter in each simulation. Moreover all the simulations are one-dimensional and can be easily extended to 2 or 3 dimensional cases.

3.2.1 Single Delay in Output

Delay in Output - Case 1

Consider the linear system defined in (2.1) and (2.2) with no delay in state and only one delay in output. In this case we have

$$\dot{x}(t) = Ax(t) + Bw(t) \tag{3.1}$$

$$y(t) = Cx(t - h) + Dv(t) \tag{3.2}$$

In order to reach the error expression in Chapter 2, it is necessary to take the Laplace transforms of (3.1) and (3.2) which are given in (3.3) and (3.5).

$$X(s) = (sI - A)^{-1}B + W(s) \tag{3.3}$$

$$\begin{aligned} Y(s) &= CX(s)e^{-hs} + DV(s) \\ &= C((sI - A)^{-1}B + W(s)) + DV(s) \end{aligned} \tag{3.4}$$

We know that $z(t) = Lx(t)$ will be estimated by a filter denoted by $F(s)$ and $L = C$ which is assumed previously. Then

$$\begin{aligned} E(s) &= Z(s) - \hat{Z}(s) = LX(s) - F(s)Y(s) \\ &= LX(s) - F(s)(CX(s)e^{-hs} + DV(s)) \\ &= CX(s) - F(s)e^{-hs}CR(s)W(s) - F(s)DV(s) \\ &= (1 - F(s)e^{-hs})P_0(s)W(s) - F(s)DW_v(s)\hat{V}(s) \end{aligned} \tag{3.5}$$

where $R(s) = (sI - A)^{-1}B$, $V(s) = W_v(s)\hat{V}(s)$ and $P_0(s) = CR(s)$. Then the L_2 induced norm is

$$\gamma = \|(1 - F(s)e^{-hs})P_0(s) \qquad - F(s)DW_v(s)\|_{H_\infty} \tag{3.6}$$

Note that A, B, C and D matrices were given in Section 3.1.1: Delay in the output is equal to 0.4 ($h = 0.4$) and ε is taken as 2.

$$P_0(s) = CR(s) = C(sI - A)^{-1}B$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 0 & s+2 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.7}$$

$$= \frac{1}{s^2(s+2)}$$

$$DW_v(s) = 1 \tag{3.8}$$

According to the optimal filter solution in Chapter 2, filter equation is:

$$F(s) = E_\gamma(s)m_d(s)\frac{N_0^{-1}F_\gamma(s)L(s)}{1 + e^{-0.4s}F_\gamma(s)L(s)(1+E_\gamma(s))} \tag{3.9}$$

$$E_\gamma(s) = (\frac{1}{\gamma^2}P_0(s)P_0(-s) - 1) = -\frac{1+\gamma^2 s^4(s^2-4)}{\gamma^2 s^4(s^2-4)}$$

with $\gamma_{opt} = 1.1486$. For the above numerical values $F_\gamma(s)$ and $L(s)$ are found as:

$$F_\gamma(s) = \frac{nF_\gamma(s)}{dF_\gamma(s)} = \frac{s^4 - 4s^2}{0.8706s^4 + 4.086s^3 + 6.108s^2 + 3.263s + 0.8566}$$

$$L(s) = \frac{nL(s)}{dL(s)} = -\frac{s^2 + 2.326s + 0.662}{s^2 - 2.326s + 0.662}$$

After some operations desired filter turns into

$$F(s) = (\frac{e^{-0.4s} - \gamma^2 s^2 dF_\gamma \frac{dL}{nL}}{1 + \gamma^2 s^4(s^2-4)})^{-1}$$

$$= \gamma(1 + \frac{\gamma e^{-0.4s} - 1 - \gamma^2 s^4(s^2-4) - \gamma^2 s^2 dF_\gamma \frac{dL}{nL}}{1 + \gamma^2 s^4(s^2-4)})^{-1} \tag{3.10}$$

If $s^2(s^2-4) + dF_{\gamma 0}$ and $-1 + L_0 = -1 + \frac{dL_0}{nL_0}$ are used instead of γdF_γ and L^{-1} respectively then the resulting filter turns into the form of

$$F(s) = \gamma\frac{R_1(s)}{1 + R_1(s)R_2(s)} \tag{3.11}$$

where $R_1(s)$ and $R_2(s)$ are Infinite Impulse Response (IIR) and Finite Impulse Response (FIR) filters respectively, i.e. impulse response of R_2 is zero outside

the time interval $[0, 1]$. For the above numerical values of the problem we have

$$R_1(s) = \frac{s^2 + 2.326s + 0.662}{s^2 + 2.368s + 0.7603}$$

$$R_2(s) = \frac{0.8706e^{-0.4s}}{s^6 - 4s^4 + 0.758}$$

$$+ \frac{10^{-2}(0.0074s^5 - 0.09476s^4 + 0.9289s^3 - 6.965s^2 + 34.83s - 87.06)}{s^6 - 4s^4 + 0.758}$$

The signal namely the position of the vehicle to be estimated $z(t)$ is given in Figure 3.3 of which maneuvered shape is provided by process noise $w(t)$ in Figure 3.1.

While Figure 3.4 shows the difference between output and original trajectory $(z(t) - y(t))$, Figure 3.5 is the difference between the estimation and original trajectory $(z(t) - \hat{z}(t))$. Figure 3.4 is the error in the output of the system. It is clearly shown the effect of time delay and this delayed information needs to be corrected. As a result, corrective effect of the filter is obviously seen in Figure 3.5.

Figure 3.5 shows that the filter successfuly eliminates the effect of time delay. Since the problem is not just a simple noise elimination problem, noise in Figure 3.2 still remains at the output of filter.

Delay in Output - Case 2

The only difference in this case according to Case 1 is the time delay which is used as $h = 2$ which leads γ to a greater value. $\gamma = 2.4333$ is found and Figure 3.7 is the estimation error of the filter which is designed to suppress the effect of time delay seen in output error in Figure 3.6. Resulting filter used in simulations is composed of following expressions which are placed into 3.11.

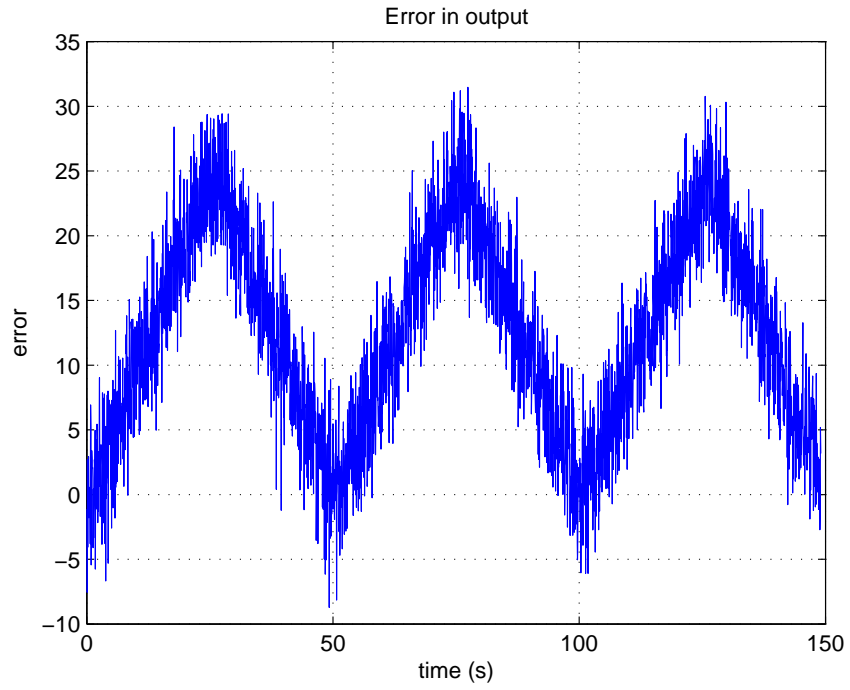$$R_1(s) = \frac{s^3 + 4.309s^2 + 5.246s + 1.255}{s^3 + 4.814s^2 + 7.234s + 3.213}$$
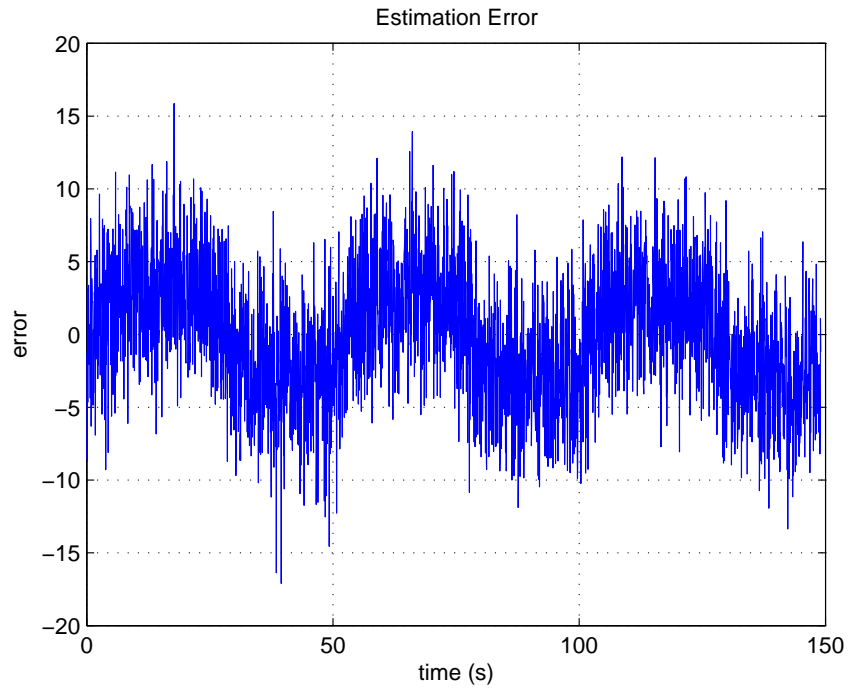
28

Figure 3.4: $z(t) - y(t)$ for Case 1

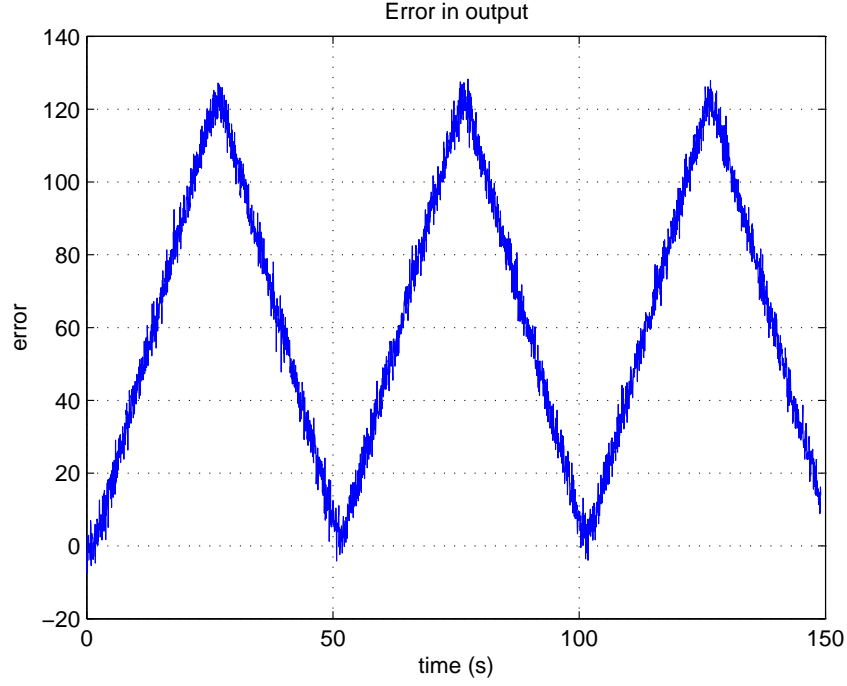Figure 3.5: Estimation Error $(z(t) - \hat{z}(t))$ for Case 1

Figure 3.6: $z(t) - y(t)$ for Case 2

$$R_2(s) = \frac{0.411e^{-2s}}{s^6 - 4s^4 + 0.1689}$$

$$- \frac{0.1908s^4 - 0.6158s^3 + 0.6785s^2 - 0.5263s + 0.2165}{s^5 - 1.997s^4 - 0.01061s^3 + 0.0212s^2 - 0.04233s + 0.08456}$$

We have obtained similar effect as in Case 1. Greater time delay causes larger magnitude of error. However it can be said that the filter works successfully against time delay.

Delay in Output - Case 3

Let $h = 6$ be the only difference for this case as in Case 2, then the resulting filter expression is same with Case 1 and 2. Time delay has the largest value of these three cases so does γ which we have $\gamma = 9.086$. Optimal filter in 3.11 has
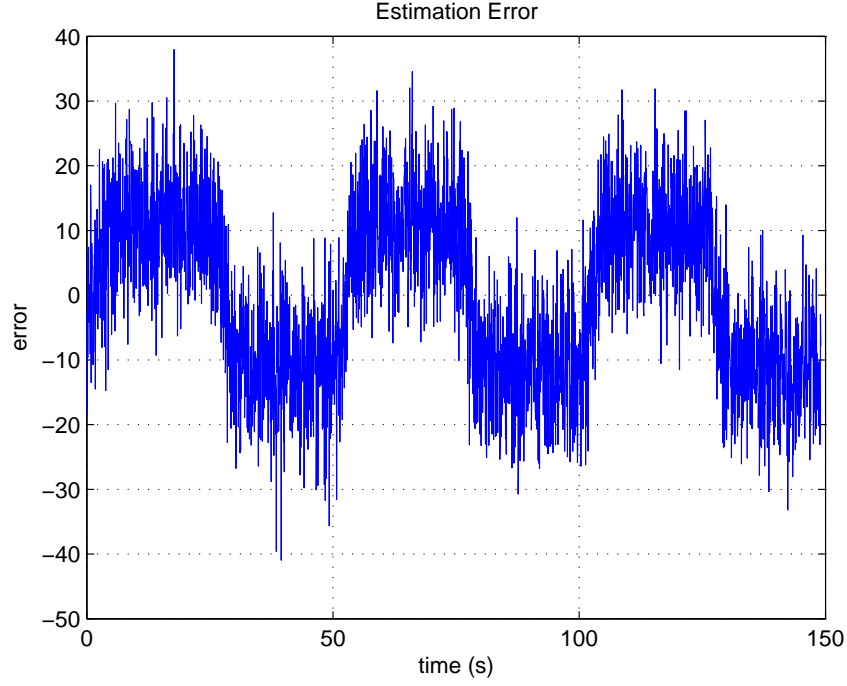
Figure 3.7: Estimation Error $(z(t) - \hat{z}(t))$ for Case 2

the following expressions:

$$R_1(s) = \frac{s^3 + 4.171s^2 + 4.685s + 0.6858}{s^3 + 5.846s^2 + 11.36s + 7.341}$$

$$R_2(s) = \frac{0.11e^{-6s}}{s^6 - 4s^4 + 0.0121}$$

$$- \frac{1.04s^4 - 2.589s^3 + 1.191s^2 - 0.375s + 0.06484}{s^5 - 2s^4 - 0.0007574s^3 + 0.001515s^2 - 0.003s + 0.006}$$

Figure 3.8 and Figure 3.9 are the output error and estimation error respectively, namely $z(t) - y(t)$ and $z(t) - \hat{z}(t)$.

Above three cases show that noise characteristics in terms of the noise magnitude of the estimation error changes proportional to time delay and also to γ. Actually, the infinite frequency response of the filter creates this effect. Since $|F(j\infty)| = \gamma$ as derived in (3.11), magnitude of noisy components of the estimation error signal increases with the increase in γ.
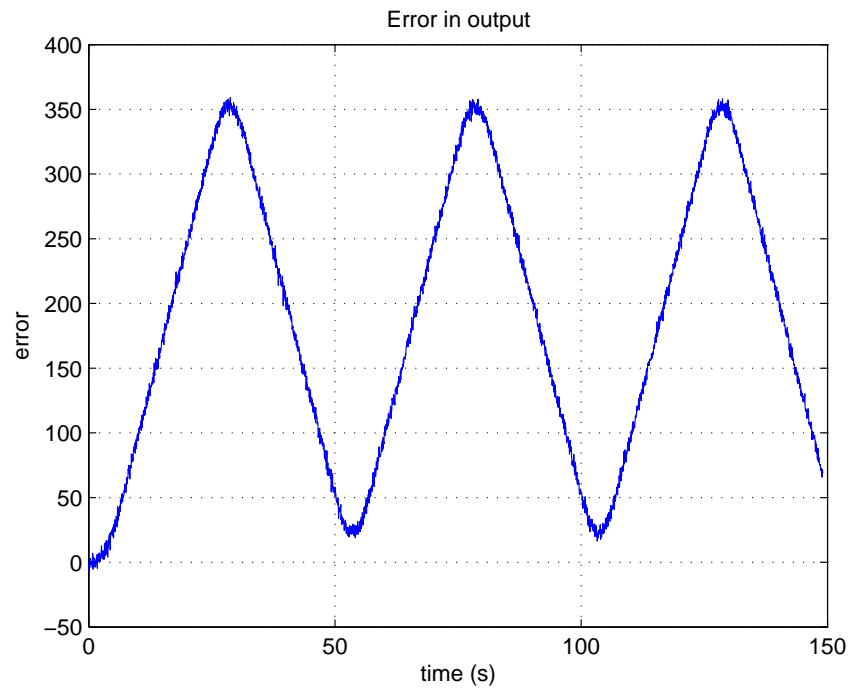
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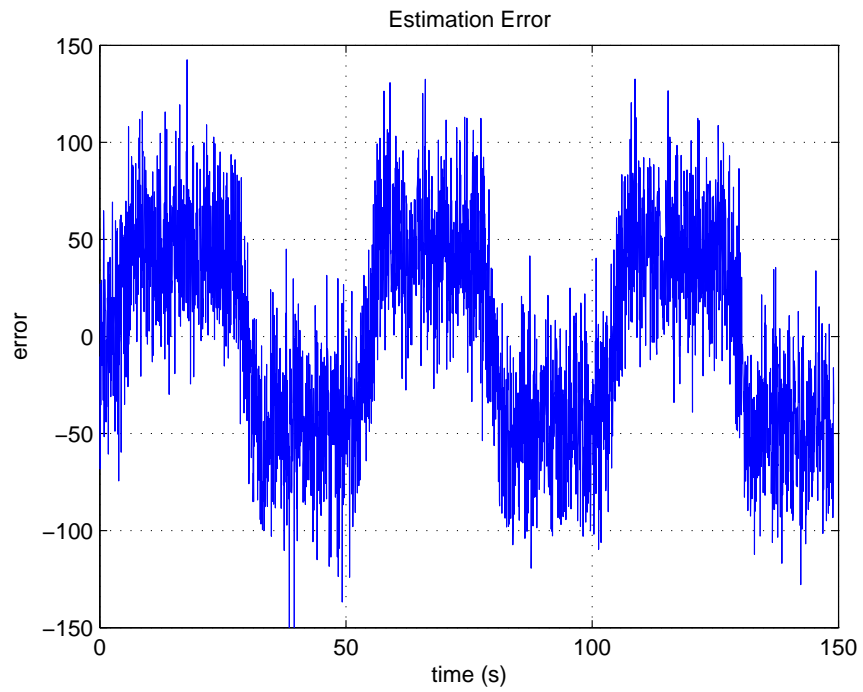Figure 3.8: $z(t) - y(t)$ for Case 3

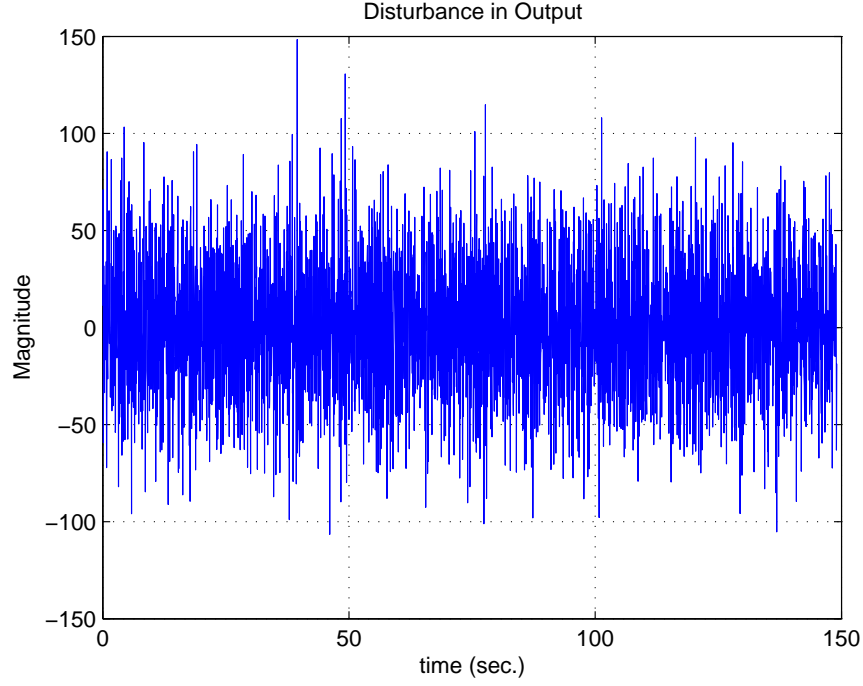Figure 3.9: Estimation Error $(z(t) - \hat{z}(t))$ for Case 3

Figure 3.10: Measurement Noise associated with a High Pass Filter

Delay in Output with High Pass Characteristics Disturbance - Case 4

In order to reduce the effect of the measurement error which were seen in estimation error in the above cases, we may consider using a weight $W_v(s)$ which generates $v(t)$. Let $h = 2$ and $DW_v(s) = \frac{10s+1}{s+10}$ which shapes the measurement noise in Figure 3.2 as in Figure 3.10. $\gamma = 2.8245$ is found for this case and derivation of the filter is same as the above filters.

Figure 3.11 and Figure 3.12 are the output error and estimation error respectively. In this case the noise seen at the output is amplified relative to earlier cases however the filter succeeds suppressing the noise in partially in addition to eliminating the effect of time delay.
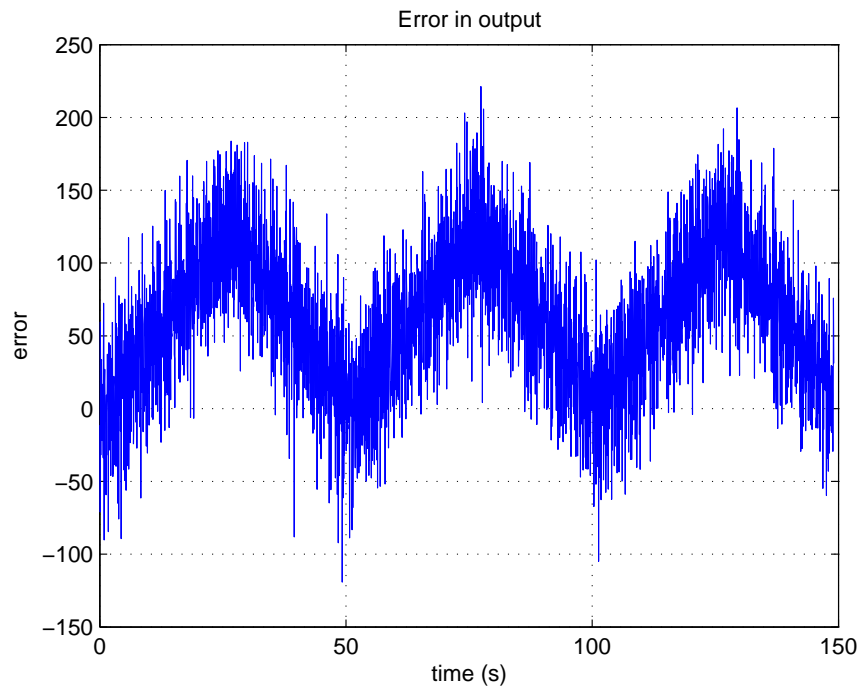
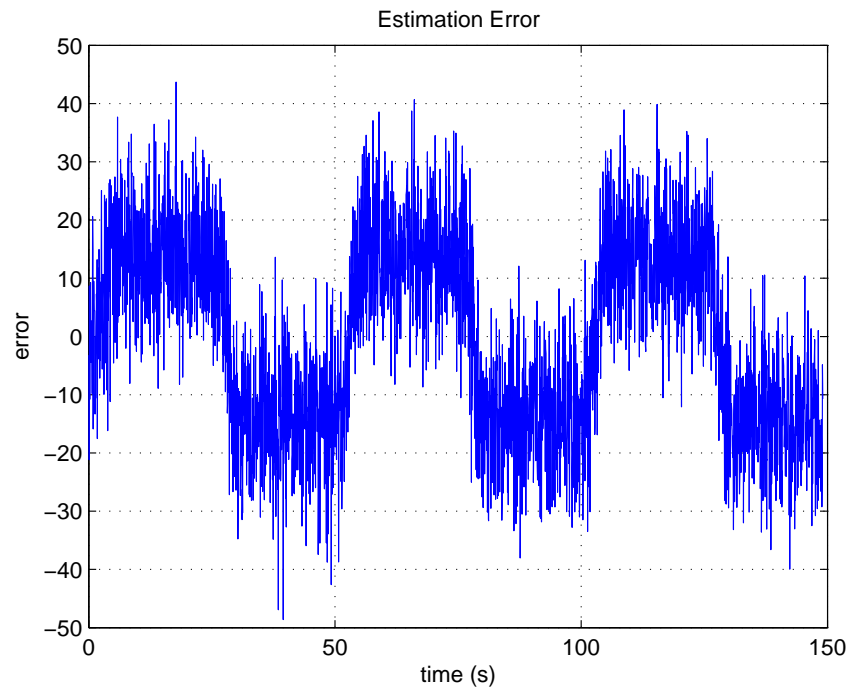Figure 3.11: $z(t) - y(t)$ for Case 4

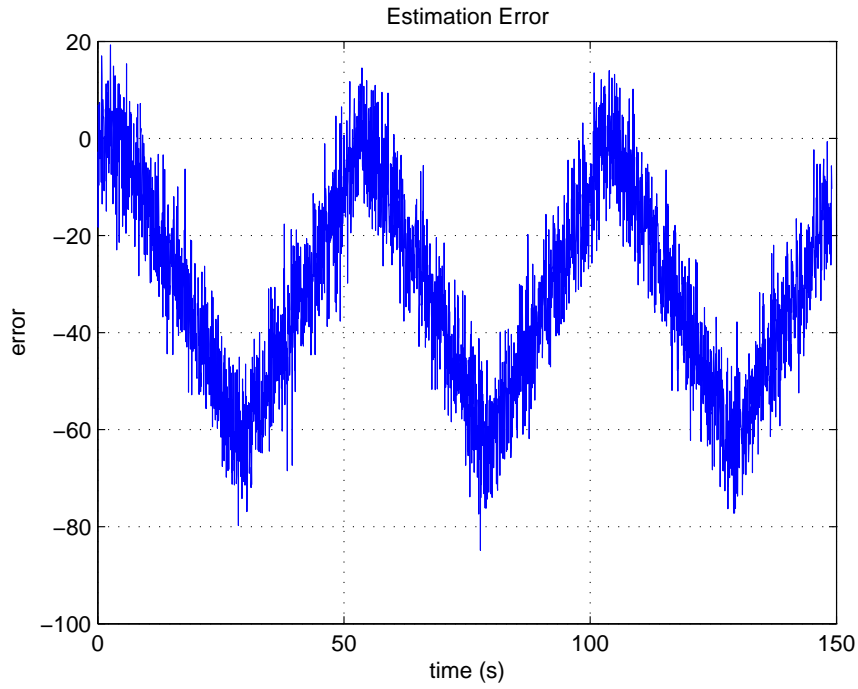Figure 3.12: Estimation Error $(z(t) - \hat{z}(t))$ for Case 4

Figure 3.13: Estimation Error $(z(t) - \hat{z}(t))$ for the System with Actual Delay is 1

What happens if the actual delay is different than the delay used in filter design?

All the above filters were designed for exact value of delay. One can wonder what will happen if the actual delay is not exactly same as the delay that the filter is designed for. To illustrate this situation following simulations were performed.

Consider the problem in Case 2 which has 2 seconds time delay. However the delay in the system is taken different than 2. Figure 3.13 and Figure 3.14 show the cases where the time delays are 1 second and 3 seconds respectively.

From the figures it is clarified that the different delays in the system and that for design can take the results further than the desired ones. Actually the greater time difference between assumed (for what the filter is designed) and actual delays cause greater error. So it can be said that the actual time delay
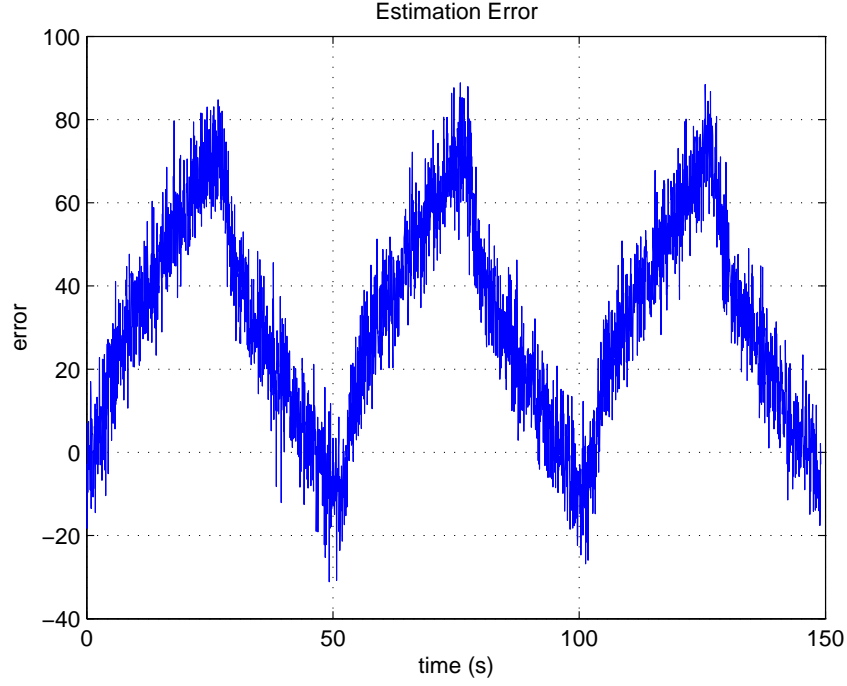
Figure 3.14: Estimation Error $(z(t) - \hat{z}(t))$ for the System with Actual Delay is 3

has to be determined with maximum accuracy in order to get best performance from the filter.

Change in ε-parameter

ε-parameter which determines how much the initial value of the state impacts system dynamics is used as 2 in the above simulations. The next two simulations will show the capability of filter against smaller and larger ε's. While Figure 3.15 and Figure 3.16 show the output and estimation errors when $\varepsilon = 0.2$, Figure 3.17 and Figure 3.18 are the output and estimation errors when $\varepsilon = 10$ respectively. In these simulations all the numeric values except than ε are same with the values in Case 4.

Small ε creates poles near to the imaginary axis in $P_0(s)$ than it seems that small ε leads the output somewhat different than the original state. However the filter is able to track the original trajectory in both cases.

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Figure 3.15: $z(t) - y(t)$ for the system with $\varepsilon = 0.2$

Figure 3.16: Estimation Error $(z(t) - \hat{z}(t))$ for the system with $\varepsilon = 0.2$
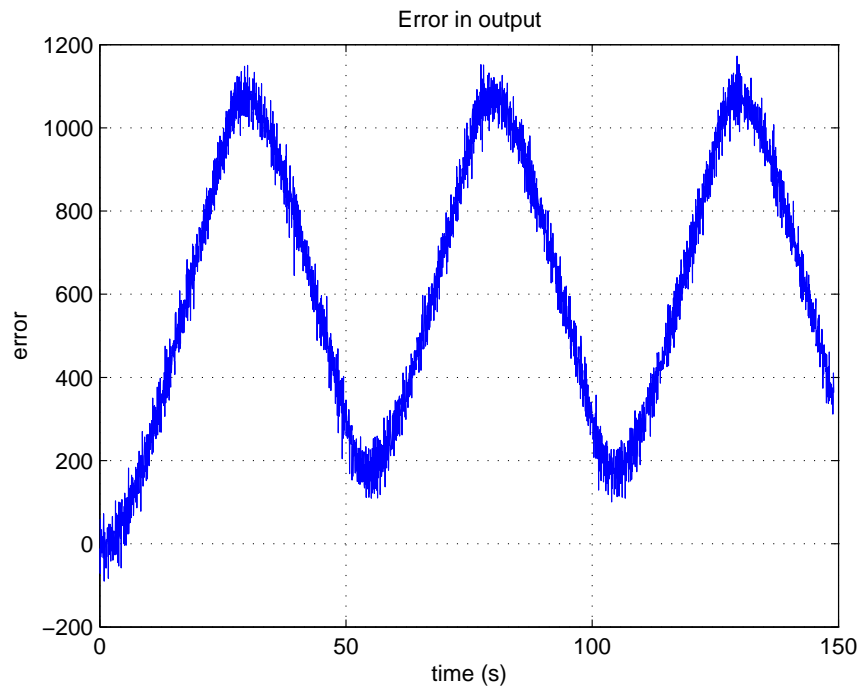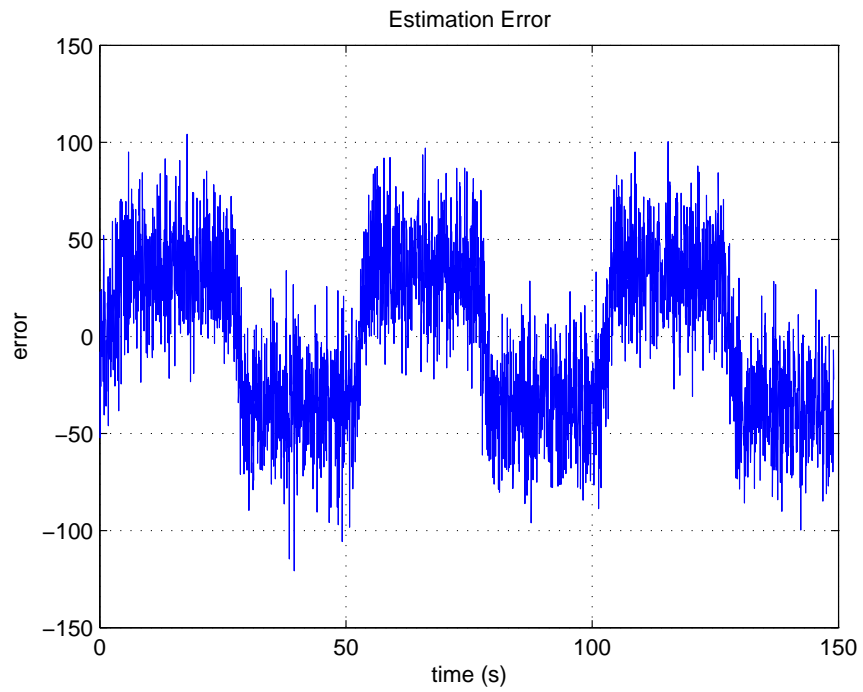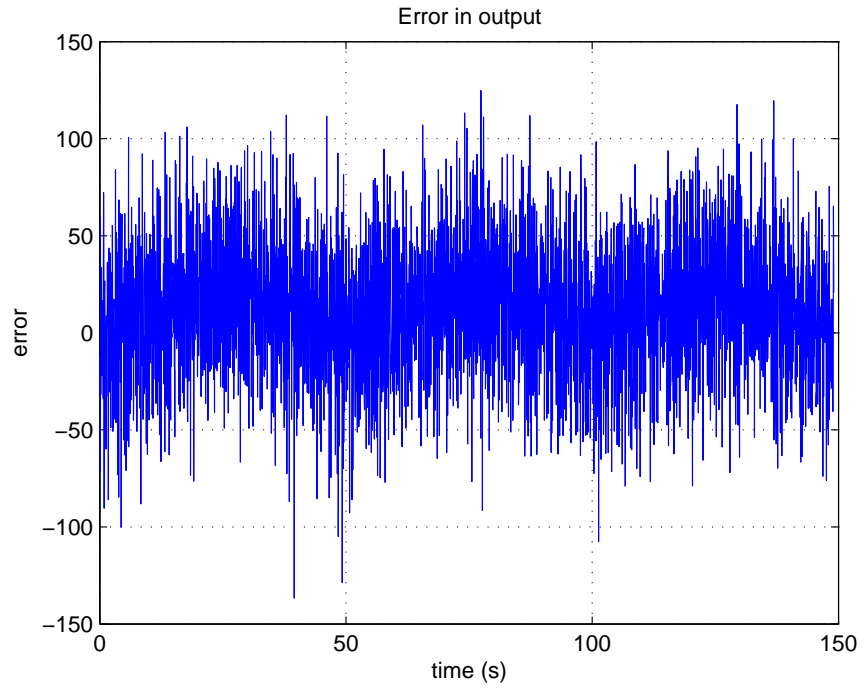
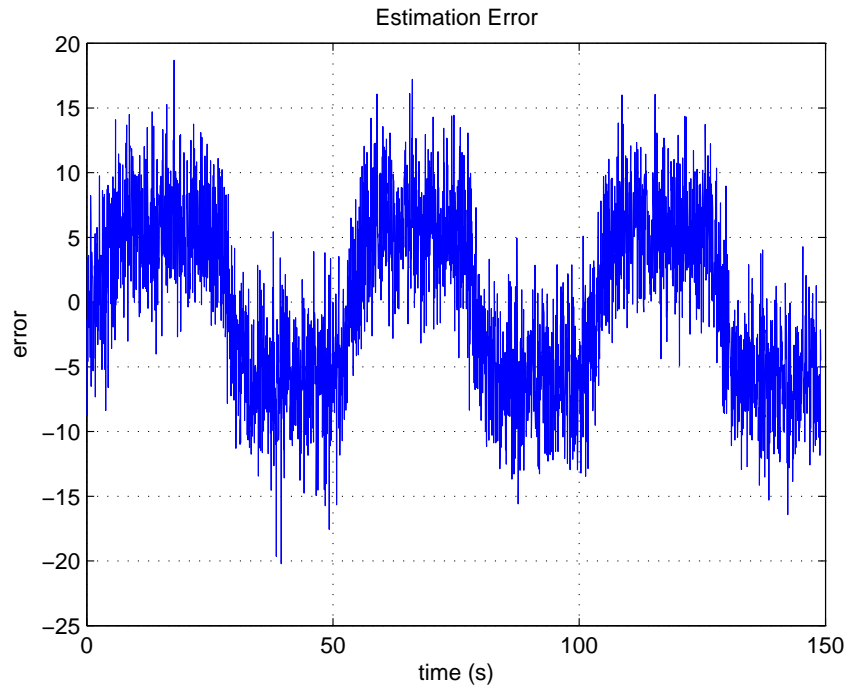Figure 3.17: $z(t) - y(t)$ for the system with $\varepsilon = 10$

Figure 3.18: Estimation Error $(z(t) - \hat{z}(t))$ for the system with $\varepsilon = 0.2$
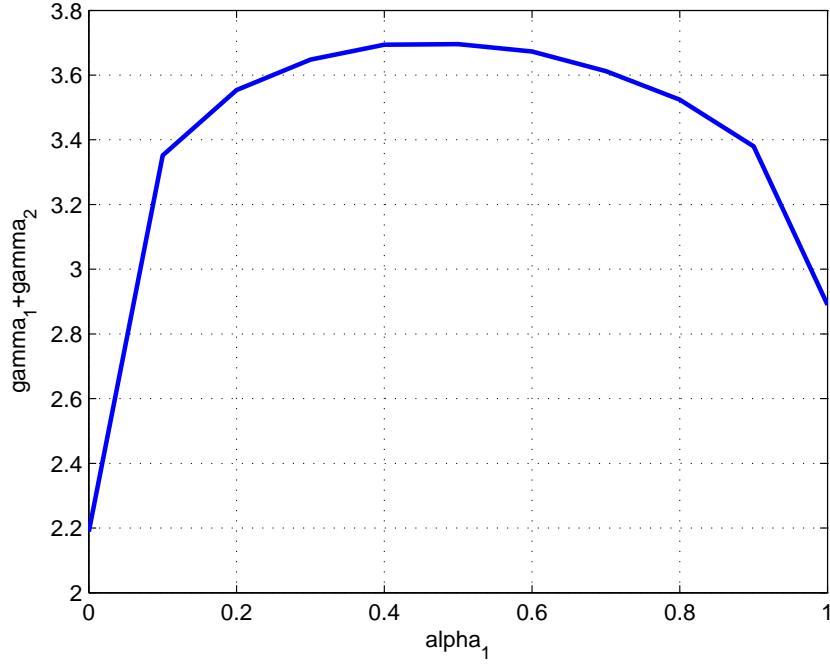
Figure 3.19: $\gamma_1 + \gamma_2$ for Suboptimal Filter Approach

3.2.2 Multiple Delay in Output

Consider the dynamic system with two outputs with delay $h_1 = 0.4$ and $h_2 = 2$. The solution method mentioned in Section 2.4.2 is applied onto this problem. Corresponding parameters α_1 and α_2 related with h_1 and h_2 are searched between $[0, 1]$ with

$$D_1 W_v = \frac{20s + 1}{s + 5} \qquad D_2 W_v = \frac{5s + 1}{s + 0.8}$$

in order to minimize $\gamma_1 + \gamma_2$.

Figure 3.19 is $\gamma_1 + \gamma_2$ versus α_1 which shows that γ values corresponding to $\alpha_1 = 0$ and $\alpha_2 = 1$ are the optimum values to form the suboptimal filter.

In fact, there may be some other examples for different values of h_1, h_2, D_1, D_2 such that the optimal value of $\gamma_1 + \gamma_2$ is attained in the interval $0 < \alpha < 1$. Figure 3.19 can be minimized anywhere which means γ_1 and γ_2 can take different values in the interval $[0, 1]$.

3.2.3 Delay in both State and Output

Simulations up to now are very similar in terms of delay characteristics other than the amount of delays. In this section we consider time delay in the state in addition to one in the output. We expect that state delay will clearly change the original trajectory. Therefore the change in state has a little effect on output but no more effort will be needed since it does not disturb the general structure mentioned and derived above. Case 4 of the previous section is repeated here by adding one delay to the state. So new structure of (2.1)and (2.2) are as in (3.12) and (3.13).

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - h_1) + B w(t) \tag{3.12}$$

$$y(t) = C x(t - h_2) + D v(t) \tag{3.13}$$

Choose

$$A_0 = A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}, \ h_1 = 1, \ h_2 = 2, \ DW_v(s) = \frac{10s + 1}{s + 10}$$

Then P_0 changes accordingly:

$$P_0(s) = \frac{(1 + e^{-2s})^2}{s^2(s + 2 + 2e^{-2s})} \quad \text{and} \quad \gamma = 4.4089 \ \text{ are found.}$$

We approximate $P_0(s)$ using high order Pade approximation so that this approach work. Remaining part of the solution is performed same with the previous ones. Resulting estimation error of the filter $z(t) - \hat{z}(t)$ is shown in Figure 3.21 where the error is in delayed state $(z(t) - y(t))$ is given in Figure 3.20.
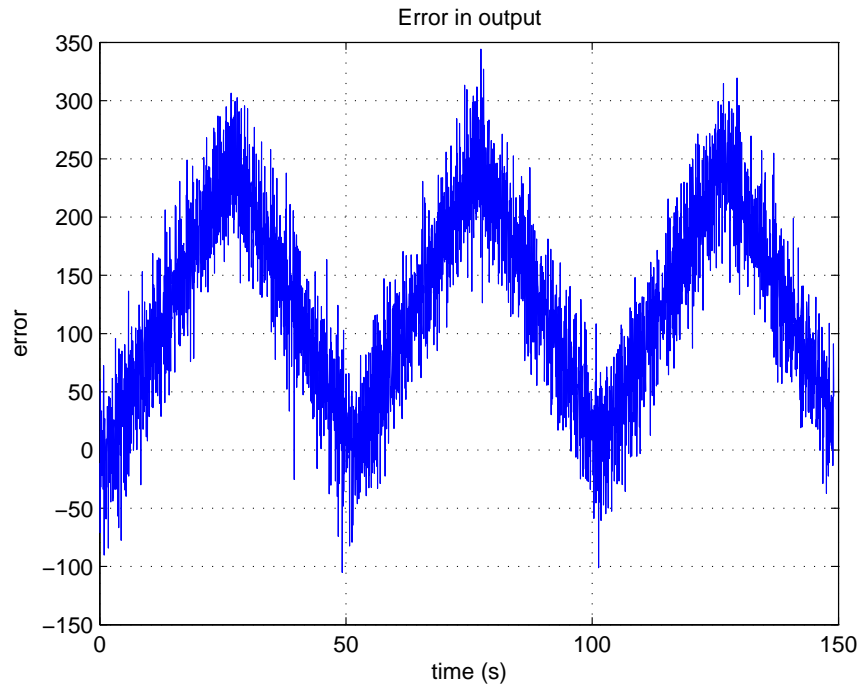
Figure 3.20: $z(t) - y(t)$ if the State has Time Delay in addition to that in Measurement
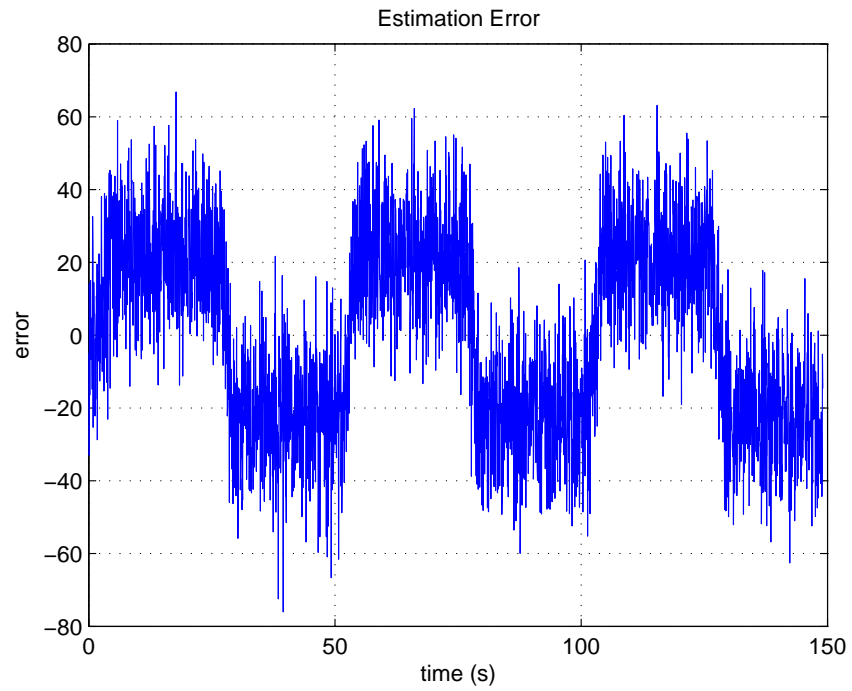

Figure 3.21: Estimation Error $(z(t) - \hat{z}(t))$ if the State has Time Delay in addition to that in Measurement

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3.3 Comparative Results

In this section the simulation results obtained with newly derived filter will be compared to ones from the literature. Since Kalman Filter is best known method for estimation purposes it will be illustrated first, then the method of Mirkin will be applied to the same vehicle tracking problem because we expect similar results to ours with his near optimal filter.

3.3.1 Kalman Filter Approach

Kalman Filter is the well known filtering method for Estimation applications. In order to see how it behaves in case of existence of delay in the system we will firstly discretize the continuous time linear system which is defined with (3.1) and (3.2).

$$\dot{x}(t) = Ax(t) + Bw(t)$$

$$y(t) = Cx(t - h) + Dv(t)$$

Kalman Filter works with white process and measurement noise with zero mean Gaussian distribution. Since the process noise shown with the Figure 3.1 is a finite energy signal, it is needed to make simple modifications on this noise to use it as the process noise for Kalman filter.

Write $w(t)$ as below:

$$w(t) = x_w(t) + n_1$$

$$\dot{x}_w(t) = n_2$$

where $x_w(t)$ is a finite energy signal with zero derivative and noises n_1 and n_2 are assumed to be white with identical zero mean gaussian distribution.

The dynamic equations defined by (3.1) and (3.2) turns into the below expressions:

$$\dot{\tilde{x}}(t) = \mathcal{A}\tilde{x}(t) + \mathcal{B}\tilde{w}(t)$$

$$y(t) = \mathcal{C}\tilde{x}(t - h) + Dv(t)$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x_w \end{bmatrix} \quad \tilde{w} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad \mathcal{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \quad \mathcal{C} = \begin{bmatrix} C & 0 \end{bmatrix}.$$

Single Delay in Output Case is handled firstly to investigate and compare the performance of two methods. Let sampling time be T_s and $h \gg T_s$ then $h = NT_s$. When the differential equation is solved $x(t)$ is found as

$$\tilde{x}(t) = e^{\mathcal{A}t}\tilde{x}(0) + \int_0^t e^{\mathcal{A}(t-\tau)}\mathcal{B}\tilde{w}(\tau)d\tau$$

Writing this expression for $x(t_{k+1})$ and $x(t_k)$ gives

$$\tilde{x}((k+1)T_s)) = e^{\mathcal{A}T_s}\tilde{x}(kT_s) + \int_{kT_s}^{(k+1)T_s} e^{A((k+1)T_s-\tau)}\mathcal{B}\tilde{w}(\tau)d\tau$$

use $\tilde{w}(\tau) \approx \tilde{w}(kT_s)$ on $\tau \in [kT_s \quad (k+1)T_s]$ then

$$\tilde{x}(k+1) = A_d\tilde{x}(k) + B_d\tilde{w}(k)$$

where $A_d = e^{\mathcal{A}T_s}$ and $B_d = \int_0^{T_s} e^{\mathcal{A}\tau}\mathcal{B}d\tau$

(3.1) and (3.2) can further be written in discrete time as

$$\tilde{x}(k+1) = A_d\tilde{x}(k) + B_d\tilde{w}(k)$$

$$y(k) = \mathcal{C}\tilde{x}(k - N) + Dv(k)$$

It is better to write in augmented state with below relations

$$\tilde{x}_1(k) = \tilde{x}(k)$$

$$\tilde{x}_2(k) = \tilde{x}(k-1)$$

$$\tilde{x}_3(k) = \tilde{x}(k-2)$$

$$\vdots$$

$$\tilde{x}_{N+1}(k) = \tilde{x}(k-N)$$

$$\bar{x}(k) = \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \\ \vdots \\ \tilde{x}_{N+1}(k) \end{bmatrix} \implies \bar{x}(k+1) = \begin{bmatrix} \tilde{x}_1(k+1) = \tilde{x}(k+1) \\ \tilde{x}_2(k+1) = \tilde{x}_1(k) \\ \vdots \\ \tilde{x}_{N+1}(k+1) = \tilde{x}_N(k) \end{bmatrix}$$

$$\bar{A}_d = \begin{bmatrix} A_d & 0 & \dots & \dots & 0 \\ I & 0 & \dots & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & I & 0 \end{bmatrix} \quad \bar{B}_d = \begin{bmatrix} B_d \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad \begin{array}{c} \bar{C}_d = \begin{bmatrix} 0 & \dots & 0 & \mathcal{C} \end{bmatrix} \\ \\ \bar{D}_d = D \end{array}$$

Augmented system equations are expressed as:

$$\bar{x}(k+1) = \bar{A}_d \bar{x}(k) + \bar{B}_d \bar{w}(k)$$

$$y(k) = \bar{C}_d \bar{x}(k) + \bar{D}_d \bar{v}(k)$$

$w(k)$ and $v(k)$ are process and measurement noises assumed to be zero mean Gaussian distribution with covariances Q_k and R_k.

$$\bar{w}(k) \sim N(0, Q_k)$$

$$\bar{v}(k) \sim N(0, R_k)$$

Since we do not have a restriction in terms of noise characteristics in H_∞ solution we do not use the exactly same noise used in H_∞ filtering solution. Then $Q_k = \epsilon_k \bar{B}_d \bar{B}_d^T$ and $R_k = \rho_k I$ are used as covariance.

Numerical Example

Case 1 of Section 3.2.1 is repeated in order to compare the performance of two filters. Firstly it is needed to find new system dynamics. Sampling time $T_s = 0.05$ seconds and time delay $h = 0.4$ which results $N = 8$ are used. After necessary calculations A_d and B_d are found as;

$$A_d = \begin{bmatrix} 1 & 0.05 & 0.0012 & 2 \cdot 10^{-5} \\ 0 & 1 & 0.0476 & 0.0012 \\ 0 & 0 & 0.9048 & 0.0476 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_d = \frac{1}{8} \begin{bmatrix} 2T_s^2 - 2T_s - e^{-2T_s} + 1 & \frac{2}{3}T_s^3 - T_s^2 + T_s + \frac{1}{2}e^{-2T_s} - \frac{1}{2} \\ 2(2T_s + e^{-2T_s} - 1) & 2T_s^2 - 2T_s - e^{-2T_s} + 1 \\ 4(1 - e^{-2T_s}) & 2(2T_s + e^{-2T_s} - 1) \\ 0 & 8T_s \end{bmatrix}$$

$$= 10^{-2} \begin{bmatrix} 2 \cdot 10^{-3} & 2 \cdot 10^{-5} \\ 0.12 & 2 \cdot 10^{-3} \\ 4.7 & 0.12 \\ 0 & 0.05 \end{bmatrix}$$

Different Kalman filter updates are applied to each 25 seconds period of original trajectory where the process noise $w(t)$ of H_∞ filter problem which is shown by Figure 3.1 is either 5 or -5. The first 25 seconds period is handled here and error of H_∞ filter and Kalman filter in this interval are shown by Figure 3.22 and by Figure 3.23 respectively. Kalman Filter updates begin with initial state $\begin{bmatrix} 0 & 0 & 0 & 5 \end{bmatrix}^T$.

Kalman Filter error according to Figure 3.22 seems better than H_∞ filter result. However Kalman filter needs to know exact time delay to form the state space representation in discrete time and also time delay must be an integer multiple of sampling time. Otherwise H_∞ filtering method does not regard such
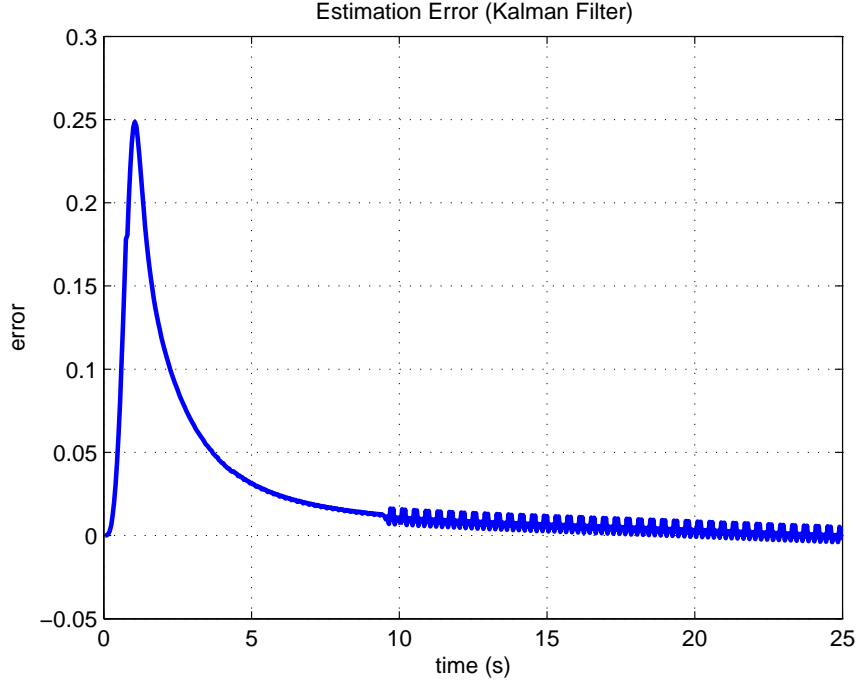
45

Figure 3.22: Estimation Error $(z(t) - \hat{z}(t))$ for the Kalman Filter with initial state with process noise information

restrictions on time delay. In addition to this, the main difference between two methods is the statics of process noise. While H_∞ filtering method works without any information about process noise, Kalman filter takes it into account as seen in above derivations and as the last element of initial state. Figure 3.24 is the Kalman filter error in case that the initial state is $\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$ in the interval $t \in [0, 25]$ which means there is no information about the signal, $w(t)$, of Figure 3.1. Figure 3.25 shows Kalman Filter error in complete time duration where process noise, $w(t)$, is 5 and -5 consecutively. Thus, when there is no initial condition information H_∞ works better.

3.3.2 Mirkin's H_∞ Suboptimal Filter Approach

Extraction of dead-time estimators from parametrization of all delay-free estimators is proposed in Mirkin's method [17]. This approach enables a straightforward reduction of various dead-time problems to a special one-block distance problem.
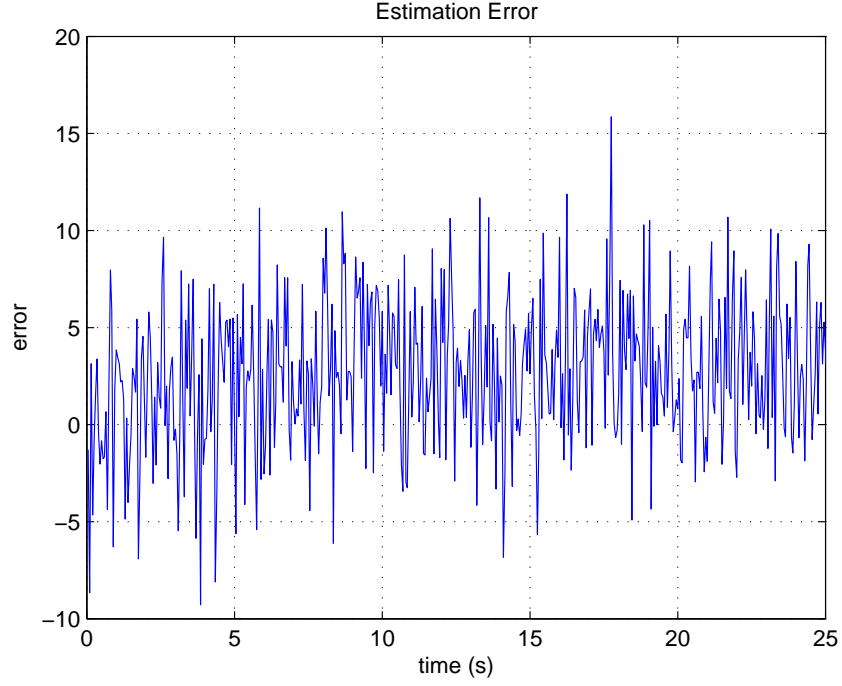
46

Figure 3.23: Estimation error $(z(t) - \hat{z}(t))$ for $t \in [0, 25]$ of Case 1 in Section 3.2.1

Also near optimal H_∞ estimator is a generalized Smith predictor. In Figure 3.26 it is shown that delay is augmented to the plant P resulting in the design of an unconstrained estimator for infinite dimensional system. Classical Nehari problem is solved during solution process of this problem. Solvability conditions are applied on the problem to find K_h which is in the form of the Figure 3.27.

Numerical Examples

Case 1 in Section 3.2.1

Solution of this approach gets a central suboptimal filter. Basically the filter elements G_a and J_a are found as following:

$$J_a(s) = \frac{3.31s^2 + 7.843s + 2.483}{s^3 + 4.826s^2 + 6.85s + 2.483}$$

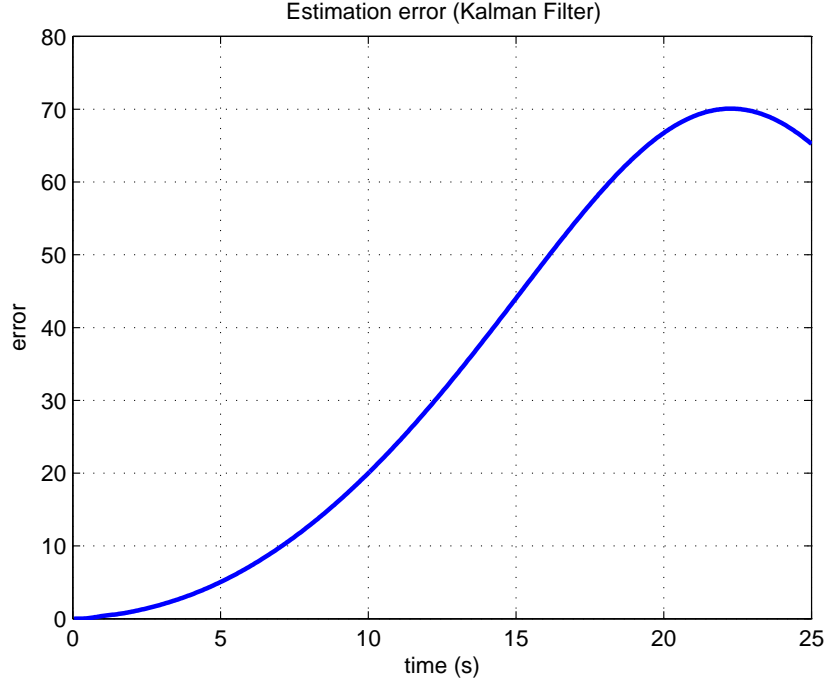$$G_a(s) \;=\; \frac{e^{-0.4s}}{s^6 - 4s^4 + 0.5102}$$

Figure 3.24: Estimation Error $(z(t) - \hat{z}(t))$ for the Kalman Filter without process noise information

$$-\frac{4.42 \ 10^{-5}s^5 - 5.56 \ 10^{-4}s^4 + 5.44 \ 10^{-3}s^3 - 4.08 \ 10^{-2}s^2 + 0.2041s - 0.5102}{s^6 - 4s^4 + 0.5102}$$

Then the resulting filter is

$$K_h = \frac{J_a(s)}{1 - J_a(s)G_a(s)}$$

Suboptimal γ which is satisfied by suboptimal filter is found as ~ 1.4 which is larger than optimal γ which is 1.1486.

Figure 3.28 is the estimation error of the optimal and central suboptimal filters together where the darker one with smaller amplitude is the result of Mirkin's method. It is seen that central suboptimal filter reduces the effect of noise by the help of low pass filter characteristics. This is due to the gain of the optimal filter at $s = +\infty$, i.e., in Case 1 to 2 we have $F(\infty) = \gamma$, which means that the high frequency component of the noise is amplified/attenuated by a factor of γ. Whereas the central suboptimal filter of [17] is always strictly
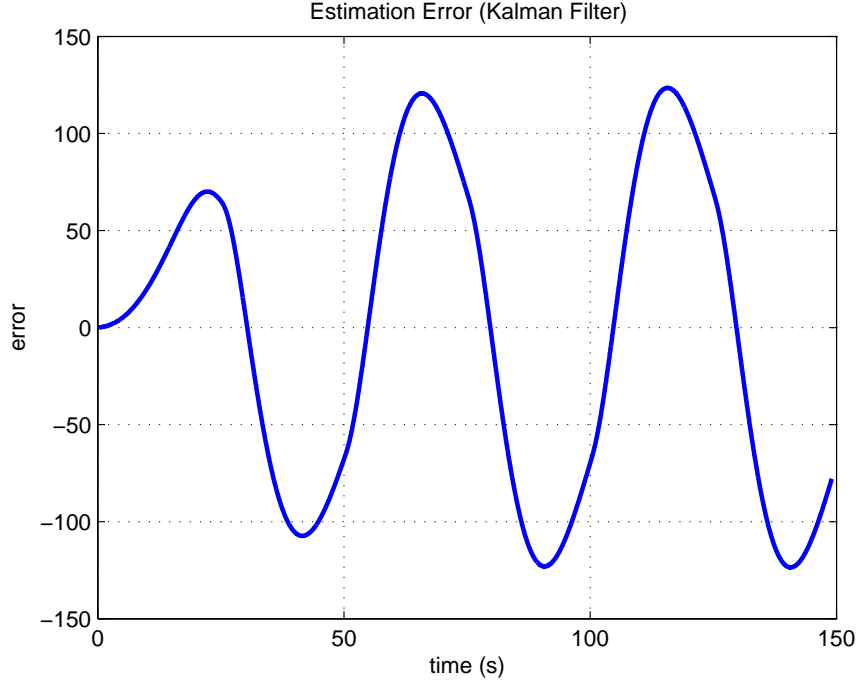
Figure 3.25: Estimation Error $(z(t) - \hat{z}(t))$ for the Kalman Filter without process noise information

proper, hence high frequency noise is always filtered. However avarage value of estimation error of suboptimal filter is larger than that of the optimal filter where average error values of optimal filter and suboptimal values are 2.611 and 4.076 respectively in the time period $[0, 25]$ and -2.176 and -3.77 respectively in the time period $[26, 50]$.

Case 4 in Section 3.2.1

Necessary calculations have been performed as in above example to compare with the another designed filter. Suboptimal $\gamma \approx 3$ which is greater than optimal γ which has been found as 2.8245. The expressions that form the estimator are found as below:

$$J_a(s) = -\frac{21.16s^3 + 50.68s^2 + 17.4s + 1.254}{s^4 + 10.32s^3 + 26.56s^2 + 24.77s + 12.55}$$

$$G_a(s) = \frac{0.051(s + 10)e^{-2s}}{s^7 + 0.1s^6 - 4s^5 - 0.4s^4 + 0.5102s + 0.05102}$$
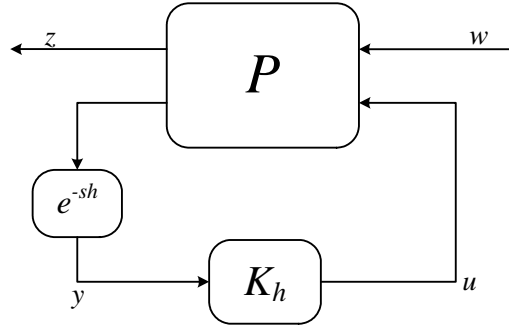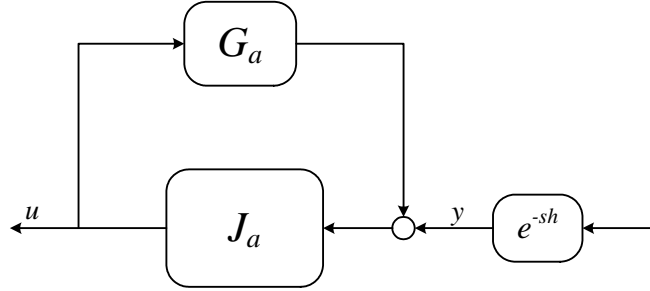
49

Figure 3.26: Dynamic System for the Mirkin's Approach

Figure 3.27: Estimator to be Designed by Mirkin's Approach

$$+ \quad \frac{0.042s^6 - 0.143s^5 + 0.268s^4 - 0.578s^3 + 0.92s^2 - 0.97s + 0.517}{s^7 + 0.1s^6 - 4s^5 - 0.4s^4 + 0.5102s + 0.05102}$$

Figure 3.29 shows the performance of the filters in comparison with previosly designed filter. Similar to the previous graph, darker one with smaller bandwidth is the result of Mirkin's method. Similar to previous one, avarage error value of Mirkin's suboptimal filter is larger than the error of optimal filter where average error values of optimal filter and suboptimal values are 12.623 and 13.522 respectively in the time period $[0, 25]$ and -11.05 and -11.44 respectively in the time period $[26, 50]$.

This chapter ends with these comparisons which make us confident with our technique. It seems that the proposed filter makes successful state estimation for time delay systems. One can find the main conclusions of this work as well as probable future works on this study in the next chapter.
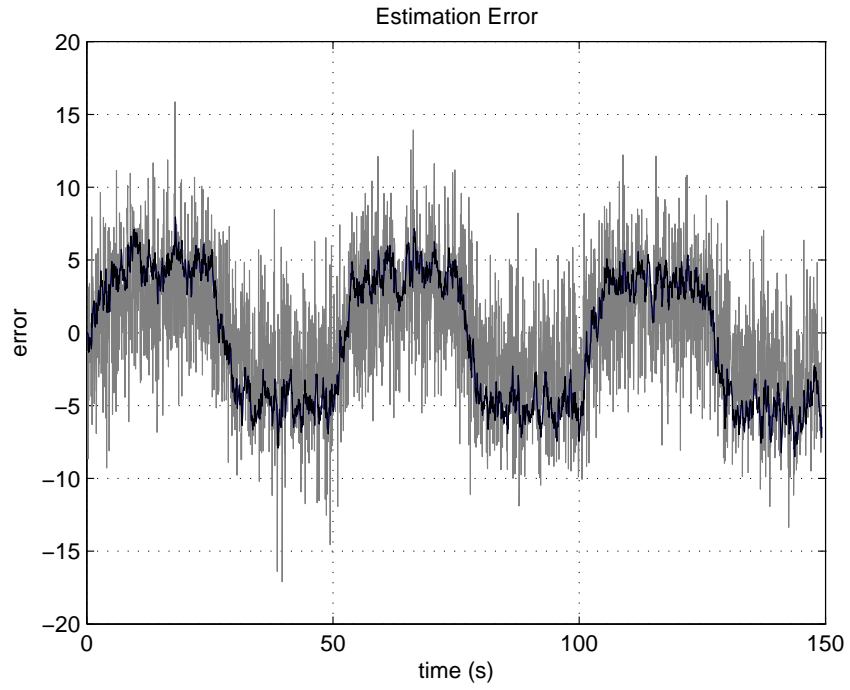
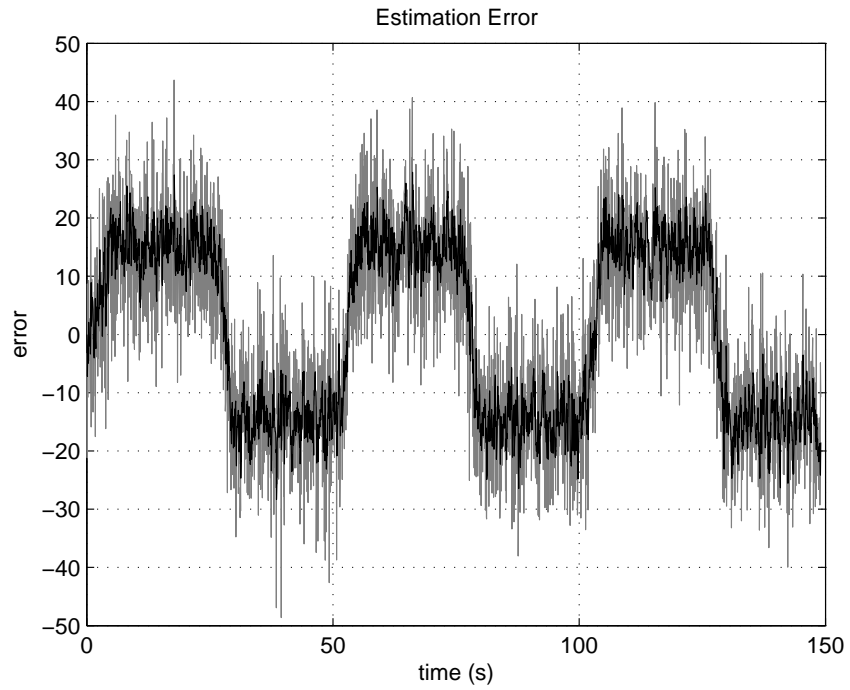Figure 3.28: Estimation Error $(z(t) - \hat{z}(t))$ for Case 1 with Mirkin's Approach

Figure 3.29: Estimation Error $(z(t) - \hat{z}(t))$ for Case 4 with Mirkin's Approach

Chapter 4

CONCLUSIONS

In this study, different aspects of filter design for time delay systems are discussed. The goal is to eliminate the effect of time delay and noise as much as possible. The advantages of H_∞ filtering against time delay and noisy data are discussed. The methods from the literature are considered. Also, the mixed sensitivity problem for H_∞ controller design is examined. The similarity between filtering and control problems are realized and a duality between two problems is established. In this framework, a novel approach was proposed to estimate the state of a linear system with time delay in state variables or measurements. In addition to single output case multi-sensor case with different delay and noise characteristics is discussed. Some optimal and sub-optimal solution methods are proposed. The developed method was applied on a vehicle tracking problem and the state of the vehicle is estimated under noisy and delayed measurements. Different amounts of delays are examined with successful results. Some advantages of the proposed method in this work are:

The designed filter has a simple structure (3.11); one needs to compute the performance level γ and two functions R_1 and R_2. In the examples, R_1 is a low order rational function whereas R_2 is an FIR filter whose coefficients can

be computed using the formulae in Chapter 2. This technique does not need to solve any type of Linear Matrix Inequality or Algebraic Riccati Equation which are proposed by all other techniques in previous works.

This method can also handle delays in the state equations. Here we have approximated the state delay to obtain a quick solution. However an exact solution is still possible by using the method proposed by [2]. This technique does not assume any noise statistics.

A number of improvements can be made for this proposed technique. In multi-delay case proposed method designs a suboptimal filter which is the combination of many optimal filters. So, as a continuation of this study, multi delay case solution can be upgraded to seek an optimal filter for the linear system. Also, this approach can be used in other applications where estimation problems play an important role, e.g. estimating the queue length of a network traffic in active queue management, where many delayed measurements can be available to make the estimation.

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