Chapter 6
Newton's Example of the Two Globes

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6.1 Introduction

At the end of the Scholium Newton includes a long paragraph about two globes revolving around their center of gravity and held together by a tensed cord. It has been interpreted as a thought experiment (Sect. 6.2) meant to show how the properties of true circular motion defined as absolute motion can be determined in a three-dimensional empty universe. I start by showing that this reading of Newton's example as a bona fide thought experiment is riddled with interpretation problems and that it is less straightforward than so far assumed (Sect. 6.3).

My alternative relies on understanding the contrast between considering the globes to be a fictional scenario and using it as a quantitative model of a dynamical interaction. I argue here against the former and for the latter: the scenario is an idealized model of a quasi-isolated system of two interacting bodies. I introduce and briefly motivate the reading in Sect. 6.4. Section 6.5 complements existing translations of the passage with a carefully analyzed manuscript source. Then I flesh out the picture: Newton's scenario shows us that it is possible to build a model of a two-body system using his definitions, with no reference to another body. Specifically, he provides a model for the analysis of true motion in terms of *quantities* (which I take to be the quantities invoked in the definitions) and their changes. We analyze changes in these quantities (and not their absolute values) by means of the three laws of motion, and in the process no other bodies are used as reference points. Section 6.6 briefly explains how my interpretation faces up to the problems which the standard reading faced in Sect. 6.3. Finally, in Sect. 6.7, I give other examples from Newton's works illustrating similar models. I conclude by

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pointing out that this example, far from being an obscure passage of Newton's, paves the way to understanding the theory of the solar system presented in the *Principia*.

6.2 The Reception of the Globes Scenario in the Literature

Newton begins the Scholium by saying that "although time, space, place, and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception." Unlike the entrenched use of quantities up until that moment, the *Principia* of 1687 puts forward new kinds of quantities,[1] and now it becomes "useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common." (Newton 1999, 408).

Then he introduces distinctions pertaining to time, space, place, and motion, and definitions of absolute 'time,'[2] 'space,'[3] 'place,'[4] and 'motion'.[5] The true and absolute motion of a body is distinct from its apparent and relative motions. Newton says that, although the parts of absolute space are not seen and make no impression on the senses, we are able to determine the true motion of bodies by means of the properties, causes, and effects of this motion. To show that the determination of true motion of bodies is not utterly hopeless, he discusses at the end of the Scholium the example of two globes revolving around the common center of gravity, while being held together by a tensed cord.

In the first stage of the description, the endeavor to recede from the center is known from the tension in the string. Then, using impressed forces on the faces of the globes, we can determine the direction of revolution (whether clockwise or counterclockwise from the perspective of an observer at rest with a bird's eye view on the globes). Finally, in the second part of the same paragraph, if we assume that there are some fixed bodies which maintain the same positions among themselves,

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[1] Some of the novel quantities, such as the quantity of matter and the various quantities of centripetal force, are described in the set of definitions at the beginning of the *Principia*. The space-time Scholium, as it is now called, is a commentary pertaining to the set of definitions. The definitions are of: quantity of matter, quantity of motion, inherent force of matter (*vis insita*), impressed force, centripetal force and three measures of it (absolute quantity of centripetal force, accelerative quantity and motive quantity). (See Newton 1999, 403–408)

[2] "Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration." (Newton 1999, 408)

[3] "Absolute space, of its own nature and without reference to anything external, always remains homogenous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space." (Newton 1999, 408–9)

[4] "Place is the part of space that a body occupies, and it is, depending on the space, either absolute or relative."(Newton 1999, 409)

[5] "Absolute motion is the change of position from one absolute place to another; relative motion is change of position from one relative place to another." (Newton 1999, 409)

we could compare the relative motions of the globes among these bodies with the tension in the cord and determine whether the motion belongs to the globes or not.

This is the example, in a nutshell. On the one hand, it faced - what I will call - the classical interpretation: on this reading, the example of the globes is lumped together with the example of a rotating water bucket. Their joint role is to show the *existence* of absolute motion (and by inference to the best explanation, the existence of absolute space).[6] The tension in the cord shows the endeavor to recede from the center. The existence of the endeavor to recede from the center signifies in turn the existence of real motion. Such motion is not motion with respect to any body, since it is implicitly assumed that there are no other bodies in the universe and there is no change in relative distance between themselves. Therefore, *this is* absolute motion. Some authors would go further and clarify the implicit inference: because absolute motion exists and since absolute motion is motion with respect to absolute space, then absolute space also exists.[7]

Naturally, the classical reading very often refers directly to the globes as an instance of a thought experiment:[8] the globes are moving in absolute space, which this interpretation takes to be an imagined empty universe. For instance:

> And if we accept the thought experiment with the globes in an otherwise empty space, the relevant motion cannot be motion with respect to any material body. Newton concludes that the motion must be motion with respect to absolute space: the spinning bodies successively occupy different locations in space itself. In this way, absolute motions are connected to forces and hence to observable effects. (Maudlin 2012, 23)

Recently however, against this classical reading, Laymon (1978) and Rynasiewicz (1995a, b, 2014, 2019) point out that, since the example comes at the end of the inquiry in the Scholium, it does not follow the pattern of argumentation of previous examples, such as the rotating bucket. According to this recent reading, we *assume* that true motion and absolute motion coincide; we no longer seek to prove either the existence of absolute motion, or that a body's true motion should be defined as motion with respect to absolute space.[9] The role of the globes example now becomes a matter of epistemology: how is one to distinguish absolute motion

[6] See Maudlin (2012), Nagel (1961), Van Fraassen (1970).

[7] See, for instance Maudlin (2012, 15): "Newton produces powerful empirical evidence for the existence of absolute motion (and hence absolute space and time) using considerations of the causes of motion."

[8] Arthur (2018), Barbour (1989, 629–40), Berkeley (1721), DiSalle (2006, 33–4), Earman (1989, Ch 4), Laymon (1978), Mach (1919, 229) Maudlin (2012, 22–5), Westfall (1971, 443–5)

[9] Rynasiewicz (2019) understands the distinction between the true and absolute motion of a body on the one hand, and the apparent and relative motions, on the other hand, as one of a metaphysical kind. The former has an elevated ontological status, more reality or existence perhaps, than the latter. See also Huggett (2012) and DiSalle (2002, 2006) on the connection between true and absolute motion. The clearest presentation I found in Brading, *Philosophy and the Physics Within*, Ch 3 (ms). My own view departs from all of these, but this is not the place to develop it. I take it from the recent literature that, at least in the case of the globes, Newton builds the description such that there is a single quantity of true motion pertaining to each globe, and that the challenge is to capture the factors which change this quantity, and only those.

from apparent motions, since we lack direct access through our senses to parts of space? Rynasiewicz (2019) argues that the globes scenario is a thought experiment supposed to illustrate "how to recognize the true motion of individual bodies and in actuality to separate [*discriminare*] from the apparent." (p. 18) The idea seems to be that, although in reality we do not have direct access to absolute motion and its properties, we can coherently conceive in imagination of true motion being different from apparent motions.

All available interpretations share a basic assumption: absolute motion can be conceived and changed in absolute space, where no bodies exist, whereas relative and apparent motions can be said to exist only by referring to changes in position among other bodies. And they all take the example to be one which is imagined unfolding in absolute space simpliciter.

We start by imagining an empty universe in which only the globes and the cord are present. Should the bodies not move, then there would be no tension in the cord (from Newton's own previous comments). But since there is tension in the cord, that means we know they *really* are in motion. (They have true motion.) This motion can only be conceived as motion with respect to absolute space, since there are no other bodies in the universe. In other words, they have a circular motion relative to absolute space. But a circular motion has a direction, so Newton shows how we can establish the direction by impressed forces on the faces of the bodies and see how the tension in the rope changes.

Now, for the second part of the scenario, imagine a different setup: the same globes and the cord, except now there is the "sphere" of fixed stars in the background. We notice a change in the relative position of the globes among the stars. Do we know if the globes really are moving only by attending to the relative changes of position? We do not; unless we attend to the effects of true motion (the endeavor to recede from the center). The tension in the string tells us that the motion belongs to the globes and not to the stars.[10] Since the stars are at rest in absolute space, we could use them as a backdrop reference frame: we now can infer the direction of motion from the relative changes of position of the globes among the stars. The first case was an instance of true motion conceived with respect to absolute space, while the second one illustrates how apparent motion is insufficient for the determination of true motion. They are both stages in a single thought experiment . . . or are they?

6.3 Is the Globes Scenario a Thought Experiment?

To my knowledge, there hasn't been an explicit justification of why we should interpret Newton's example in the manner introduced above, as a thought experiment. While some might find this reading natural and intuitive, as an interpretation of

[10] In Sect. 6.3 we shall see that this inference does not hold, given Newton's own qualifications about relativity of motions. The most direct criticism of it I found in Barbour (1989, 643–4).

Newton's text it deserves some scrutiny. Let me mention explicitly some possible reasons one might invoke for this interpretative category and point out some problems for it along the way. In the next section I introduce an alternative.

The reasoning goes presumably like this: the scenario is naturally a fictional thought experiment because it assumes some things which we cannot possibly observe in actuality. For instance, in the first part of the passage we assumed there were no other bodies in the universe. We start with absolute space imagined to be the space of the universe emptied of all material bodies. Then we "add" in imagination two bodies shaped like globes and a tensed cord between them.[11] Obviously, two bodies connected by a cord moving in an empty universe can only happen in a thought experiment.

This interpretation which relies strongly on imagination from the get-go is faced with several difficulties. To simplify, I will select two main challenges derived from the first part of the example (when the globes and the cord are supposed to be alone in absolute space), and two additional problems for the second part of the example (when we consider the fixed stars). The first problem for the empty-universe-with-globes reading derives in fact from the strongest textual support for it. The strongest supporting reason seems to be in a sentence Newton includes halfway through the paragraph. There, by the use of the tension in the cord and impressed forces we can conclude:

> In this way both the quantity and the direction of this circular motion could be found in any *immense vacuum*, where nothing external and sensible existed with which the balls could be compared. (Newton 1999, 414; my emphasis)

The idea is to consider the 'immense vacuum' above to be another name for the all-encompassing absolute space. Whether we want to equate a vacuum and absolute space is a different point, which I believe requires more argumentation, if only because Newton uses different terms for them and because nowhere does he define one in terms of the other. They could be assimilated to each other in imagination only if we *imagine* absolute space as a vacuum.

In any case, not every use of 'vacuum' should be taken to refer to absolute space. Specifically, this particular instance of vacuum makes perfect sense as an instance of a pocket of vacuum. Newton himself seems to have struggled with how and where to qualify the introduction of vacuum. He deleted an initial mention of vacuum ("two globes revolving in a vacuum") and qualified the single mention of the term ("*any* immense vacuum"; my emphasis). Consequently the emphasis shifted: from where we imagine the globes to be to what is involved in the determination of the direction of their motion. That is, the important point is that we do not use other bodies for the determination of motion. Clearly, then, the vacuum mentioned here is not the *single* all-encompassing universe emptied of matter in our imagination. It could be *any* pocket of vacuum (which could be quite immense) in which the two revolving

[11] A side note: we immediately face the question of how to understand the gravity of those two globes in such an empty universe.

globes find themselves because in *any vacuum the method introduced by Newton for determining the properties of their motions is the same* and it does not use reference to other bodies.[12]

The second problem involves the mention of impressed forces in the first part of the experiment (before the fixed stars are added to the scenario). The presence of impressed forces is in tension with the assumption that we supposed there are no other bodies in existence. The concept of 'impressed force' is a notion introduced by Newton in the *Principia* and it covers physical forces (such as pressure, impulses, or several kinds of centripetal force)—forces which have their seat or exist because of other material bodies. Hence there should be an action of other external bodies on the two-body system. On the assumption of empty space, however, this is not possible. If there are impressed forces in the system, then the space cannot be empty. How could it be both an empty space and non-empty at the same time?

Additional difficulties arise when we continue reading this as a thought experiment. There are problems concerning how to understand the transition between the first part and the second part of the scenario. For instance, about how those fixed stars come into being (a move that can make sense only in imagination), or whether we need to devise another thought experiment in which we no longer start with an empty universe. I will leave these kinds of worries aside for the moment because they are not directly relevant to my argument.[13]

More importantly, the inferences presented in the second part, after the introduction of the fixed bodies in the background, become problematic. Consider that we have the two globes revolving as they did in the beginning, only now we also have a set of bodies with fixed positions among themselves in the background. If we attend to the tension in the cord when the globes are revolving *uniformly* around their center of gravity, we could *not* infer that motion belongs to the globes *or* to the stars (contrary to Newton's own claim). To see why, consider the case of the stars revolving uniformly, but doing so at a different constant rate of rotation than the globes. To put it differently, if the motion of the globes and the stars' rotation are both uniform, but with different rates of revolution, Newton's own conclusion concerning true motion versus apparent motion is not correct. We could *not* infer that rotation pertains *only* to the globes.

This could not have been a minor blunder. Newton could have spotted this 'mistake' while revising the *Principia* for the second and third editions. In fact, the opposite happens: in the second edition Newton adds the sentence "and that the bodies at rest," strengthening his conclusion about which set of bodies are moving and which are at rest—a rather bold move, if all the motions are presumed uniform (and not varying) Smith (in-press b).

[12] There is a great similarity between this strategy and current methodology of studying the properties of binary star systems. Most stars are in fact binary systems. (csiro.au)

[13] For instance, Mach (1919) faults Newton with the assumptions entering into this thought experiment because it looks like the universe assumed is very different in crucial aspects from the universe we know to observe and inhabit. As he puts it, "the universe is not given twice."

Finally, received views relegate the example to obscurity: it is unclear how the connection with the project of the *Principia* proceeds, if at all. Given that it is a thought experiment, it is unlikely that it will be helpful for other real situations. Newton, then, seems to just pay an insincere lip service when he concludes the paragraph by saying:

> But in what follows, a fuller explanation will be given of how to determine true motions from their causes, effects, and apparent differences, and, conversely, of how to determine from motions, whether true or apparent, their causes and effects. For this was the purpose for which I composed the following treatise. (Newton 1999, 415)

As far as I can tell, nobody took these last sentences seriously. The argument of the *Principia*, the thought presumably goes, is far too complicated to have this scant example give us an insight into it. This is a fair point *if* we read the example as primarily a thought experiment concerning two bodies in a situation which is unlike any real instance in our physical universe. The example was presumed to have no actual connection to the "fuller explanation" provided by the *Principia*, despite Newton devoting a very long paragraph on it and saying exactly the opposite at the end of it. In the next section, I put forward a new approach that is in line with Newton's claim and argue that the passage can indeed be a guide to the methodology of the *Principia*.

6.4 Taking the Example as an Idealized Model: The Very Idea

The minimal description of the scenario is seen as an artifact of imagining the globes to be the sole inhabitants of absolute space. But it isn't the only way to read the example. Admittedly, Newton's description *is* minimal: the bodies lack any qualitative description, and any element introduced in the argument, such as the impressed forces or the fixed stars, shows up at a particular time, followed by specific inferences. But minimalism is justified, I argue, not because we start with the 'barest' environment of all: absolute space. Instead, the minimal description is the natural feature of idealized models, the result of considering a set of bodies as an isolated system under consideration in which measurement is performed of some quantities by means of other quantities. This section introduces the core idea of this interpretation.

Alongside recent scholarship, I take the example to have an epistemic function as well: it illustrates how true and apparent motions are to be presented and distinguished. I disagree, however, with existing views on how this role is achieved. Instead of conceivability, I rely on methodology. Instead of taking the example to describe actions and scenarios impossible to realize in reality,[14] I consider that the

[14] On the contrary, Newton says that the example aims to "actually" (*actu*) distinguish apparent and true motions. (See Sect. 6.4)

scenario is an idealized model of a system of two interacting bodies. Instead of relying on an individual's ability to conceive such motions in imagination starting from nothing, I argue that acts of imagination are subordinate to the process of idealization, in which features not under investigation or considered irrelevant are abstracted away. The last paragraph may look like an imagined case taking place in an empty universe, but that is because it is the result of idealization of physical situations in which two bodies affect each other's motions in determinate ways. Whereas received views take the lack of descriptive or contextual details as a problem, I suggest we should take them as features of the scenario curated to fit the role of a specific model.[15]

Specifically, there are two methodological factors incorporated in the scenario which help the example fulfill its function of an idealization.

> **The quasi-isolation feature:** According to it, some bodies are included in the description, and some are left out. Included bodies are interacting in an idealized manner: their actions are expressed by means of forces defined by Newton, and not by specifying a causal mechanism of sorts. Their interactions are fully captured by the three laws of motions, and there are no requirements on specifying a mechanical cause for how the actions are done. The isolation of a system of bodies is a deliberate act of investigation. We do not isolate some random set of bodies. Bodies not specifically included in the description are *not* annihilated: their existence and influence are simply bracketed for the time being. Actions of these other bodies are to be accounted for in terms of impressed forces only.

> **The quantitative feature:** All the relevant actions of bodies in our system are tracked solely by the set of quantities or forces Newton introduced previously in the *Principia*. Effects of actions always quantified. The quantities involved in the description will depend on the satisfaction of the quasi-isolation criterion. If the quasi-isolation is done in an appropriate manner, then the relations among quantities of motions and properties of forces have the strong form of a double conditional: one such quantity changes iff another quantity changes accordingly.

Such is the view that I propose. Although separating these two factors is my own doing, I suggest here that they belong to Newton's methodology proper. Moreover, I would argue that they are constitutive of Newtonian idealizations in general.[16] The globes example may be the first of such idealizations, but certainly not the last in the *Principia*.

In many of his writings, Smith has emphasized in various ways the crucial role of Newtonian idealizations in testing the theory of Newtonian gravity throughout its long history. Here is one such example:

[15] The model I have in mind is akin the two-body problem in physics, and not, say, mechanical models for causal interaction of two bodies. Newton, of course, does not use the word "model." For recent work on this understanding of Newtonian models see Ducheyne (2005) and Ducheyne (2012, esp. chap. 2). Ducheyne focuses on planetary models. I share much with Ducheyne's arguments, especially the idea that the models in Book 1 are not restricted to mathematics. But I also think that the two features which I introduce here are to be more systematically applied and embedded into Newton's natural philosophy, going beyond models of planetary motions.

[16] This claim will be developed elsewhere. This paper restricts itself to arguing that the presentation of the globes scenario is best understood as an illustration of the result of applying these two features. Specifically, it demotes the understanding of the scenario as a thought experiment. (That is not say that thought experiments in natural science do not use models.) Briefly put, this scenario is closer to reasoning in physics proper than we have seen it so far represented in the literature.

> The purpose is to shift the focus of ongoing research onto systematic discrepancies between the idealizations and observation, asking in a sequence of successive approximations, what further forces or density variations are affecting the actual situation? The theory of gravity and deductions from it become not so much explanations or representations of known phenomena, but instruments in ongoing research, revealing new discrepancies between, for example, true and idealized orbital motions. [. . .] The theory of gravity gets tested in this process through its requiring that every deviation from any Newtonian idealization be *physically significant* – that is, every deviation has to result from some unaccounted for density variation or force, gravitational or otherwise. So the test question is not whether calculation agrees with observation, but whether robust physical sources can be found for the discrepancies between calculation and observation. (Smith 2014, 277)

My interpretation builds on Smith's insight that Newton's methodology for natural philosophy in the *Principia* aims to form the laws of forces of nature by modeling ever more complex systems of bodies interacting through those forces. In this process, idealizations of a specific kind take center stage.

Let's see how they could briefly play out in Newton's example. In principle, the two globes could also be affected by other bodies acting by means of impressed forces. The idealization consists in bracketing the existence of these bodies at different times and taking the two globes as a (possibly) isolated system of interacting bodies. In such a system, any changes in motion are described only quantitatively. The changes in the quantity of true motion could be compared with changes in two other different quantities. One is the tension in the rope, the other is the relative rate of rotation among the fixed stars. Newton's goal in the scenario, I argue, is to separate these two types of correlations in the first stage, and then to connect them, in the latter part. The former set of quantities (the quantity of true circular motion and the tension in the cord) could indicate the source of physically significant changes (caused by other bodies affecting the quasi-isolated system). The latter set (the quantity of true circular motion and relative rate of rotation), by itself, could not. They could be combined under certain conditions. But making this distinction clear is achieved not by a thought experiment, but by a carefully constructed and worded scenario, to which we now turn.

6.5 The Original Text and the Model of Interaction

Let us now have a closer look at Newton's own words. Instead of using Cohen and Whitman's authoritative translation, I include an (unpublished) translation by George Smith and Anne Whitman with notes sending to the original Latin or to deleted passages:[17]

> It is most difficult [*difficillimum est*] indeed to identify [*cognoscere*] the true motions of individual bodies, and ~~in practice~~[18] ?actually? to discriminate them from apparent ones,

[17] ?- shows insertions

[18] The adverb deleted and replaced by "*actu*" cannot quite be made out, and hence this translation is a guess.

because those parts of that immobile spaces in which bodies are truly moved do not strike against [*incurrunt*] the senses. ~~Xxxx xxxx xxxx which individual immobile xxxx xxxx true nevertheless are to be reckoned [verum tamen disputando], and that we can sometimes gather something partly from the forces that are the causes and effects of the true motions, partly~~ ?Nevertheless, the case [*causa*] is not absolutely hopeless [*prorsus desperata*]. For arguments are forthcoming [*suppetunt*] partly? from the apparent motions that are the differences of the true motions [and] ?partly from the forces that are the causes and effects of the true motions?. For example [*ut*], if two globes at a given distance from one another connected by an intervening cord were being revolved [*revolverentur*] ~~in a vacuum~~ around the common center of gravity, the endeavor of the globes to recede from the axis of motion ~~will~~ would become known [*innotesceret*] from the tension in the cord, and then the quantity of circular motion could be computed. Thereupon if ?no matter what? equal forces were to be impressed [*imprimerentur*] at the same time on alternate faces of the globes increasing or lessening the circular motion, the increase or decrease of the motion ~~could be still be learned~~ ?would become known? from the added or diminished tension in the cord, and therefrom ?finally? on which faces of the globes the forces would have to be [*deberent*] impressed for the motion to be increased maximally could be found, that is, the posterior faces, or those which follow [*sequuntur*] in the circular motion. Moreover, learning the face that follows and the opposite face that precedes, the direction of the motion would be identified [*cognosceretur*]. ~~This circular [motion] in an immense vacuum~~ In this manner both the quantity and the direction of this circular motion could be found in ?whatever [*quovis*]? immense vacuum where nothing external and sensible exists with which the globes could be compared. If now some distant bodies were set in that space maintaining given positions among themselves [*inter se*], as the fixed stars are in our regions, it could indeed not be known from the relative translation of the globes among the bodies [*inter corpora*] whether this motion was to be attributed to these [globes] or those [bodies]. But if the cord were examined and its tension were found to be that which the motion of the globes would require [*requireret*], it would be legitimate [*liceret*] to conclude the motion to be of the globes,[19] and finally from the translation of the globes among the bodies to gather the direction of this motion. Moreover, to gather the true motions from their causes, effects and apparent differences, and conversely their causes and effects from motions whether true or apparent, will be shown more extensively [*fusior*] in the following. For to this end I composed the Following Treatise.[20]

The text is fairly clean, with very few changes. One modification involves qualifications of the use of 'vacuum' and the insertion of a stronger conclusion which attributes motion to the globes and rest to the fixed stars. We also witness the use of the subjunctive throughout the passage directing the reader to hypothetical situations. My approach is to consider this hypothetical situation as a description of an idealized case, in which given certain assumptions, some consequences follow. In other words, the hypothetical situation is not *fabled* one. It has a hypothetical form where "ut" has the power of introducing an example. And it describes a carefully abstracted instance of interacting bodies, not the result of freely running creative

[19] In the second edition of the *Principia* Newton inserted at this point the further clause, "and that the bodies were at rest."

[20] These passages are quoted from a longer manuscript by George Smith and Anne Whitman. Appendix 5 consists of variorum translations of selections from the version of *Liber Primus* Newton submitted to Cambridge Library under the auspices of Lucasian Lectures, Dd. 9.46 (pp. 36–215 of Whiteside 1989). See Whiteside (1989, 45–6), translated by George E. Smith; and the folio numbers (11 and 12) in Dd. 9.46. Smith (in press-b).

imaginative powers. We start from reality and abstract away some features, whereas in fabled imaginings we start from scratch and add features as we please.

It is worth noting, for instance, that there is nothing in the language to direct our thoughts to a scenario *created* by imagination. There is no language of creation when it comes to the fixed stars either: the bodies are set (not imagined, not created). By comparison, in the earlier manuscript *De Gravitatione*, the famous passage that invites the reader to imagine some entities similar to the bodies of our experience starts by saying, "Fingamus itque spatia vacua per mundum disseminari quorum [....]. Sed si fingamus [...]." (Hall and Hall, 1963, 106) In the case of *De Gravitatione* the wording elicits the act of imagination, hypothesis, and fabrication, whereas the Scholium confines its language to that of putting forward the givens of the problem and describing hypothetical results (but not imagined): the verbs are such as "revolverentur," "innotesceret," "cognosceretur," etc.- all expressing the hypothetical of a problem setting.

Let me flesh out more my interpretation on offer here, by emphasizing the two methodological factors mentioned previously. First, the quasi-isolation factor: we start with the world of experience as we encounter it, a universe with (many) bodies in motion. Somewhere, in a part of this universe, in a pocket of vacuum, two globes interact with each other by means of the tensed rope between them. They revolve around their common center of gravity as Newton suggests. Our imagination functions as abstraction: in the first part of the scenario (before considering the fixed stars), it is required to bracket the existence of other bodies as points of reference, but not to annihilate them altogether. This, I argue, is how the role of imagination is in service of using Newton's powerful concept of impressed force in order to provide a fairly complete mathematical description of the motions (to a first approximation) without invoking other bodies.

Second, there is the quantitative dynamics. According to the view on offer here, the quantity of **true** motion will be a quantity ascribed to bodies in a carefully specified system of interaction (and not with respect to absolute space). The changes in such quantities are going to be considered **absolute** because they are changed by impressed forces *acting explicitly from outside the system*. Basically, for my reading, all that matters are variations in quantities, and the causes and the effects of such changes. To put it differently, we ask ourselves: What are the things that make a difference to the system of bodies under investigation, and what difference do they make?[21]

[21] This question has often showed up during George's course and it is a recurrent pattern in the history of testing Newtonian gravity. See Smith 2014.

Some quantities can be thought of in a purely mathematical manner, even though they are measured by appealing to sensible quantities.[22] To my mind, this means that quantities involving absolute space, for instance, cannot be evaluated by reference to parts of absolute space, but they can still be used to model a particular system of interacting bodies. This kind of measurement is done by reference to sensible things, and the parts of absolute space "do not impinge on the senses."

I suggest that in this case we know or measure absolute quantities pertaining to some true motions in a *specialized sense*: by evaluating whether the changes in such quantities characterizing motions are caused by impressed forces in harmony with expected effects of such motions (such as the endeavor to recede from the center as a physical, real effect on bodies).[23] This means for me that we will be using mainly inferences concerning *relations among quantities*: the quantity of true circular motion of the globes is not to be known as an absolute number or value on its own, in isolation from any other kinds of quantities.

Thus, I take it that the epistemic challenge of the last paragraph of the Scholium is a challenge that we encounter when in the physical world we try to *measure* certain quantities (not when we try to conceive them). I suggest that the challenge of measuring true and absolute quantities of a particular, concrete instance of real bodies interacting is answered *not* by providing a number or a specific value. But by modeling the situation as a specific quasi-isolated system of two interacting bodies using only the definitions and his laws of motion. Provided we built such an idealization, then we answer the challenge when we fully identify all the other quantities which determine uniquely the quantity of true motion and which are explicitly stated as interdependent by means of mathematical relations, within a particular system of bodies.

This is how this methodology is illustrated by the two globes scenario. Each of the bodies is assumed to have a quantity of matter (Definition 1). Their revolution is around the center of gravity which suggests that gravity is also part of this scenario. The bodies are in circular motion which means that their motion is the result of two forces – the inertial force and the centripetal force keeping them on the trajectory (Definitions 3, 5, and the first two laws of motion). The centripetal force is clearly determined: we have its center. Physically, each body attracts the other one; mathematically, this interaction is described by considering the center of gravity as the center of attraction. The tension in the cord is a measure of the strength of the action of each body on the other (Law 3). What the scenario idealizes is the interaction of two bodies in which all the three laws of motion concur.

When we have a tensed rope between two bodies, we find ourselves in a lucky epistemic position because the tension is the expression of an equilibrium between

[22] The preceding paragraph stressed again the distinction between relative and "actual" quantities. The former are sensible measures of the latter. When we refer to quantities involving time, space, place, motion in the absolute sense, we use a "manner of expression which is out of the ordinary and purely mathematical."

[23] According to the reading on offer here, the bucket experiment follows the same method.

two endeavors to recede from the center. Those endeavors are quantities directly correlated with the amount of quantity of circular motion of each body, because they both are known to vary directly with the mass of the body, the period of rotation, and the radius of circular motion (and only with those quantities.) So, on my account, a quantity of circular motion can be assumed to be "computed" by a different quantity when variations in all the quantities which feature into the mathematical understanding of the former determine variations in the other quantity as well; and vice versa.

Notably, true motion is not known as a specific number or a specific value simpliciter. The true motion of the two bodies can be known only insofar as it changes. And it varies if and only if impressed forces act on those two bodies. I take it that this is also the insight provided by Hoek (2022) concerning motion according to Law 1. Rather than conceiving the motion invoked in Law 1 as the motion of force-free bodies, we conceive of it as the motion that is changed only by impressed forces. Similarly, the quantity of circular motion has one value only insofar as it does not change by means of impressed forces.[24]

Once the bodies and their relevant quantities are set to a first approximation (which involves both an attempt at quasi-isolation and quantitative modeling), we can now use a geometrical space to represent their motions and changes in those quantities. Note how in Newton's description, all reference points are fully determined by the quantities internal to the system of bodies, and not the other way around. The center of motion is not a point picked at random, but chosen by the properties of the system. In this case, the center of gravity depends only on the quantity of matter of the two bodies and their spatial separation.

But when represented in a geometrical space, the motion of the two globes gains two other features: the plane and the direction of rotation. Why are these two geometrical features important? Because they complete the geometrical description of the motion of the globes and they can be determined by the pairs of impressed forces (Definition 3, and the first corollary to the laws of motion). Again, the idea of properties being determined by other entities means that there is a one-to-one mathematical correlation. Only one pair of (equal but of opposite directions) impressed forces corresponds to the plane and the direction of rotation: those that maximize the change in the quantity of circular motion.

However, the fact that we use impressed forces tells us that the system is only *quasi*-isolated: impressed forces act as causes for changes in the quantity of motion within the system, while their physical source is abstracted away. The crucial bit is that we do *not* need other bodies when describing those changes: neither to describe their influence on the two globes, nor to use them as reference points for the motion. The description relying only on Newton's definitions and laws is self-contained and

[24] Focusing on changes in the motions of bodies as quantities also shows how specifying the bodies the motion to take place in a vacuum is a significant detail. It points out to the lack of resistance for the motions and, therefore, it provides a separate reason for considering the bodies a system unto itself. That is, the isolation of the system is well supported from a dynamical point of view as well.

complete, provided we start with a quasi-isolated system. The idea is that the true motion is to be fully determined by interactions internal to the system and changed by external actions on the system which are captured by means of impressed forces.

Let us now move to the second part of the scenario. Consider the *same system* and the *same changes in the tension* produced by impressed forces, but with a sphere of fixed stars surrounding the system of two globes. Or to put it differently, for the second part of the paragraph, we use imagination for a different abstraction: we bracket the existence of any other bodies except the globes, the rope, and the fixed stars. If we focus only on the changes in the relative positions of the globes among the stars, we will still notice changes in the rate of revolution. Based on these relative motions, this (apparent) rate of revolution would seem to increase or decrease. However, if we use only these changes in spatial positions, we could not say whether the quantity of true motion changes or not. That is, *we could not say whether impressed forces do act on our system or not*. Accounting for external physical influences on our system can only be determined when we consult the measure of true motion internal to the system (the tension in the cord).

6.6 Answering the Previous Challenges

So far, I have explained how the system of revolving globes becomes an idealization in virtue of it being quasi-isolated and because the inferences mentioned in the scenario rely on relations between quantities. But does my interpretation fare any better with respect to the previous interpretative challenges? I believe it does and now I turn to address them directly.

First, on my account, the two parts of the scenario (before and after introducing the fixed stars) are integrated. Consistent with Newton's words, the bodies are in motion from the get-go. We do not face the problem of creating new fixed stars in the scenario, messing up the dynamics recreated in our imagination: my reading does not assume we talk about two different universes, with different matter distributions, at different times in the argument. Additionally, it was assumed, from the beginning, by the isolation factor, that the fixed bodies which are considered in the second part of the scenario do not act causally on the globes.[25]

Second, the inference on which Newton relied in the last part of the scenario is no longer problematic. Previous readings assumed that in the second part of the scenario, the globes are taken to be in *uniform* circular motion when they are compared with the fixed stars. In that case, we could not say whether the motion pertains to them or to the stars. It could be that both the globes and the fixed stars revolve, but at different rates of rotation. Simply put, the objection is the following:

[25] These kinds of assumptions are crucial in idealizations of dynamically interacting systems. As George Smith's works show, when we clearly spell out assumptions, we make it easier for ourselves to subject these idealizations to systematic revision Smith (2007, 2012, 2014).

unless the tension in the cord *changes in a manner non-correlated with changes in the relative positions*, we would not be able to conclude that the bodies are moving and the stars are at rest.

However, the situation is different if we witness a correlation between the change in the tension in the cord and the variations in relative rotation when systematic pairs of impressed forces act on the globes. We might be tempted to think that we are looking to determine whether the stars *truly* are at rest: staying in the same place in absolute space from infinity to infinity. But this would go against Newton's previous qualification about true rest.[26] Newton does not conclude that the fixed bodies are at true rest. We are not interested in finding the true rest of the stars, but whether they can be *taken to be at rest* (i.e., provide a workable standard of rest) for the dynamical interactions involving the two globes. This allows us to conclude that, for all intents and purposes of studying the system of the globes, the fixed bodies are at rest in a restricted sense, not in a true sense.

Therefore, in my reading, both the existence of an immense vacuum and the existence of the fixed stars are compatible in the example. The first stage of the scenario deliberately excluded the fixed stars as reference points. The later stage adds them back (as reference points) and explains the conditions under which they can be used as a standard of rest for the true motions of the system. Imagination, as I said, can still be used, but not for a thought experiment.

The scenario serves the role of an idealization or a model for other systems of interacting bodies. In the following section I show that this model shows up in two other examples in Newton's own writings and how they all share the focus on quantities of motion and forces of a quasi-isolated system.

Before proceeding further, however, let me address a possible problem for my interpretation. Newton mentions a very specific pair of impressed forces and how their direction can be changed to map the direction of the circular motion. One could say that this can only be done by design: that is, we do it as an intervention. Thought experiments thrive on deliberate, fictional interventions or on the participation of the agent in the imagined scenario, whereas the idealized model put forward here has less room for actions caused by intentions.

My answer is that, as far as I can tell, there is nothing in Newton's own language[27] to suggest that the reader is assumed to *do* something. He is positing a situation in which certain kinds of impressed forces have a particular kind of effect.

[26] See Newton's discussion of the distinction between absolute space and relative space. Newton (1999, pp. 409–10)

[27] "Thereupon if?no matter what? equal forces were to be impressed [*imprimerentur*] at the same time on alternate faces of the globes increasing or lessening the circular motion, the increase or decrease of the motion ~~could be still be learned~~?would become known? from the added or diminished tension in the cord, and therefrom?finally? on which faces of the globes the forces would have to be [*deberent*] impressed for the motion to be increased maximally could be found, that is, the posterior faces, or those which follow [*sequuntur*] in the circular motion." (See Sect. 6.4)

Granted, the reasoning presented is of a hypothetical kind,[28] but the focus is on the inferences connecting the direction of rotation to hypothetical actions of impressed forces. That is, the hypothetical language is an answer to the following question: "*what could we infer* if we start with these givens about the situation?" In contrast, it does *not* address a wholly different kind of question, pertaining to fictional actions, such as "*what would happen* if we do this (*imagines herself giving them a push with her hands in imagination*) to the globes?"

6.7 The Idealization in Action

If we take the example of the globes to be an idealized model for interactions of bodies in general, then we are no longer surprised to find surprisingly similar instances in two unlikely places. This section will describe two such instances, one from the Queries to the *Opticks*, the other one from a draft of *Liber Secundus*. For lack of space, I will not discuss these examples in detail. My focus is limited to the conceptual connections which the globes example illustrates. Since they are deep and significant for the methodology of natural philosophy, I hope to rescue Newton's scenario from obscurity and to do justice to Newton's final sentences of the Scholium.

First, a strikingly similar example shows up in the famous Query 31, added to the second edition of the *Opticks*, where Newton discusses active and passive principles. I call it 'striking' because the Queries (and especially Query 31) have always been the starting point for understanding Newton's more speculative thoughts on the principles of nature and procedures of experimental philosophy, and not usually associated with methodological considerations. Here is the passage:

> And thus Nature will be very conformable to herself and very simple, performing all the great Motions and the heavenly Bodies by the Attraction of Gravity which intercedes those Bodies, and almost all the small ones of their Particles by some other attractive and repelling Powers which intercede the Particles. The *Vis inertiae* is a passive Principle by which Bodies persist in their Motion or rest, receive Motion in proportion to the Force impressing it, and resist as much as they are resisted. By this Principle alone there never could have been any Motion in the World. Some other Principle was necessary for putting Bodies into Motion; and now they are in Motion, some other Principle is necessary for conserving the Motion. For from the various Composition of two Motions, 'tis very certain that there is not always the same quantity of Motion in the World. For if two Globes joined by a slender Rod, revolve about their common Center of Gravity with an uniform Motion, while the Center moves on uniformly in a right Line drawn in the Plane of their circular Motion; the Sum of the Motions of the two Globes, as often as the Globes are in the right Line described by their common Center of Gravity, will be bigger than the Sum of their Motions, when they are in a Line perpendicular to that right Line. By this Instance it appears that Motion may be got or lost. (Newton 1718, 397)

[28] The general hypothetical form is basically an inference: if such-and-such effects are present, then such-and-such claims are true. See Sect. 6.5.

Above, the example of two globes joined by a rod is introduced as a test model for composing kinematically the motions of each body. The simple composition of motions, without regard to their generation or physical causes, has the unpalatable consequence that, in the motion of the globes just described, we get different evaluations for the same quantity of motion. Thus, just as in the globes scenario, understanding circular motion in light of Newton's laws and definitions is again important.

We take that the significant component motion, (the vis inertiae), of the globes is on the tangent (that is where they move should they be released). The motion of the center of gravity will be perpendicular on those motions when the globes are in the right line described by the center of gravity. And the motion will be in the same line when they are in a perpendicular line. If we compare the effect of the uniform motion on the direction of the tangential motion of each of the globes, we get smaller and larger values for each globe. And thus, we might (rashly) conclude, that each globe equally loses and gains motion.

Following this passage, Newton explains that, in fact, bodies *lose* more motion than they have. So, kinematics by itself is not a guide for analyzing systems of bodies. We need to consider forces from the start, to focus on quantities of true motion, and *afterwards* evaluate the composition of motions.

The quoted passage shows that forces (as active and passive principles) are necessary in accounting for the true quantities of motion, while apparent variation in such quantities derive from misapplied compositions of motions. Similarly, the key component for changes in the system of the two globes were impressed forces. By contrast, the kind of mathematical composition of motion that disregards forces as causes and effects ends up in conclusions contrary to experience. We need to take into account what motion is communicated to the bodies within the system and what motion is taken from them by other bodies (no matter how fluid or viscous). This is in part what Newton's laws of motion help us model: how motion is communicated among bodies.

The second example speaks directly to the role of idealization for the methodology of the *Principia*. In article 21 of *Liber Secundus*,[29] Newton struggles to describe the kind of interaction Jupiter and the Sun share in virtue of mutual gravitational attraction. He explains that the action of gravity between any two planets (say, Jupiter and the Sun) is a simple action, which can be treated twofold. But in order to exemplify this conceptualization, Newton uses the model of two globes hooked together by a rope revolving around their common center of gravity.

> The Sun attracts [*trahit*] Jupiter and the other Planets, Jupiter attracts [*trahit*] its Satellites and similarly the Satellites act on one another and on Jupiter, and all the Planets act on themselves mutually. And although, in a pair of Planets, the action of each on the other can be distinguished and can be considered as paired actions by which each attracts [*trahi*] the other, ~~they are not two but a simple operation between two termini. By the contraction of one rope insofar as between~~[30] yet inasmuch as these are actions between two bodies, they are not two but a simple operation between two termini. **Two bodies can be drawn [*trahi*]**

[29] Smith (in press-a), '*Liber secundus*.'

[30] The bolded emphasis is mine throughout.

> **to each other by the contraction of a single rope between them**. The cause of the action is two-fold, namely the disposition of each of the two bodies; the action is likewise two-fold, insofar as it is upon two bodies: but ~~the operation by which the Sun~~ insofar as it is between two bodies it is simple and single. There is not, for example, one operation by which the Sun for example attracts [*trahit*] Jupiter and another operation by which Jupiter attracts the Sun, but a single operation by which the Sun and Jupiter endeavor to approach each other. By the action by which the Sun attracts [*trahit*] Jupiter, Jupiter and the Sun endeavor to approach each other (by Law 3), and by the action by which Jupiter attracts the Sun, Jupiter and the Sun also endeavor to approach each other. Moreover, the Sun is not attracted [*attrahitur*] by a twofold action towards Jupiter, nor Jupiter by a twofold action towards the Sun, but there is one action between them by which both approach each other. [. . .][31]

I will cut through the complexity of the actual argument to point out that the passage relies on a model of the interaction between Sun and Jupiter which is almost identical to the system of the two globes connected by a rope. Here Newton tries to offer a model for conceptualizing the interaction of two such bodies through gravity.[32] The operations of each globe on the other are exerted at the same time through the means of the rope. Not without its problems, this model becomes a paradigm for thinking about gravitational attraction more broadly.[33] The tension in the string now acts as a placeholder for the equality of two actions: of how much each body attracts the other.[34] But this makes it also a model for the equality of action and reaction. Similarly, in the globes example, the tension in the cord was a measure both of the endeavor to recede from the center and of the quantity of true circular motion (which mathematically is determined by the inherent force and the centripetal force).

6.8 Conclusions

Let us take stock of this journey. I started by introducing the current interpretations of the last paragraph of the Scholium (Sect. 6.2), and the problems we encounter if we read the example of the two globes as a thought experiment (Sect. 6.3). My approach is to consider the scenario to be an idealization of two quasi-isolated interacting bodies illustrating the inferences which such an idealization allows based on Newton's definitions of laws of motion (Sect. 6.4). I think that the original

[31] This is yet another one of the great contributions for which I am grateful to George Smith. On the one hand, there is the dedication to the analysis of the text. On the other hand, there is the generosity in sharing these materials with generations of researchers.

[32] Recall that the two globes were taken to revolve around their center of gravity (and not some arbitrary point), a well-defined mathematical point which assumes some understanding of gravity.

[33] Newton proposes here a view of analyzing the motion of an isolated system of two bodies acting through a central potential. Our current physics sensibilities recognizes this as a two-body problem and it is one the paradigmatic examples taught in celestial physics. Yet until Newton there was nobody who formulated the motion of two bodies under gravity in this manner.

[34] Law 3 is inevitably included in the model.

language in the manuscript source supports my reading (Sect. 6.5) and does not describe a fictional thought experiment. I flesh out more fully my approach (Sect. 6.5) and then explain how it faces up to the original interpretation challenges (Sect. 6.6). Section 6.7 provides the proof that Newton himself took the system of the globes to be a model by introducing two other examples from his writings: one from the Queries to the *Opticks* and the other from a manuscript about the System of the World, predating the *Principia*.

We are now, I believe, in a position to draw a final connecting line between the two-globes scenario, other examples which Newton used, and Newton's methodology more generally. I think this connection could be described using George Smith (2020)'s charming (adopted) phrase of "putting questions to nature." When summing up his discussion of experiments in Newton's *Principia*, Smith writes that the experiments

> [. . .] posed questions about the relationship between forces of some kind, which are not directly accessible, and various other quantities more accessible to observation. They all, of course, involved intricate design and hence contrived situations. But what enabled them to have at least the promise of extracting answers from nature about forces was their presupposing, in every case, Newton's first two laws of motion. Their design involved, first, one or more theoretical relationships derived from these two laws between forces and quantities that would be accessible to observation in certain circumstances and, then, the physical realization of those circumstances. To say this in a more customary manner, all five of them involved one or more relationships derived from the first two laws that licensed a theory-mediated measurement of something pertaining to forces in specific circumstances. (Smith 2020, 18)

My aim for this paper was to argue that the scenario of the globes in the *Principia* is a model for how to "cross-examine Nature herself" as well (Bacon 1620, 232). In my view the example shows how to describe a system of bodies and forces relying on Newton definitions and laws and what kind of inferences are licensed by its idealization factors. I suggest that measuring the true motion of the globes by two different kinds of quantities is a case of successful cross-examination. The later procedure of putting this model to work in ever more complex, actual physical circumstances is a fuller development of this strategy, just as Newton himself concluded. George Smith has sometimes described this methodological strategy "theory-mediated measurement" and proved that it is a staple of continuous success of physical inquiry into the nature of gravitation.

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