JOINT TEST FOR STRUCTURAL MODEL SPECIFICATION

A Master's Thesis

by

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics

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ABSTRACT

JOINT TEST FOR STRUCTURAL MODEL SPECIFICATION

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Aim of this thesis is to propose a test statistic that can test for true structural model in time series. Main concern of the thesis is to suggest a test statistic, which has joint null of unit root and no structural break (difference stationary model). When joint null hypothesis is rejected, source of deviation from the null model may be structural break or (and) stationarity. Sources of the deviation correspond to different structural models: Pure stationary model, trend-break stationary model and trend-break with unit root model. The thesis suggests a test statistic that can discriminate null model from alternative models and more importantly, one alternative model from another. The test statistic that is proposed in the thesis is able to detect specific source of deviation from the null model. By doing so, we can determine the true structure model in time series. The thesis compares power properties of the test statistic that is proposed with the most favorable test in the literature. Simulation results indicate the power dominance over the test statistics in the literature. Moreover, we are able to specify true alternative model.

Key Words: Unit root, Structural Break, Joint Hypothesis Testing, Monte Carlo Simulations

ÖZET

YAPISAL MODEL BELİRLENMESİ İÇİN BİRLEŞİK TEST

Yüksel, Serkan

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Bu tezin amacı, zaman serilerinde yapısal modelin belirlenmesini sağlayabilecek bir test istatistiği önermektir. Bu amaç doğultusunda, boş hipotez olarak birim kök ve yapısal kırılmanın olmadığı (ilk fark durağan modeli) model belirlenmiştir. Bu boş hipotezin reddedilmesi durumunda, boş hipotezden sapmaya neden olan alternatifler durağan yapı veya (ve) yapısal kırılmadır. Sapmaya neden olan yapılar ise: Durağan model, yapısal kırılmalı durağan model ve yapısal kırılmalı birim kök modelleridir. Bu tezde boş hipotez altındaki modeli alternatif hipotezlerden ayırabilecek ve daha da önemlisi alternatif modelleri birbirinden ayırabilecek bir test istatistiği geliştirilmiştir. Böylece, zaman serilerinde doğru modelin sınanmasını sağlacak bir test istatistiği oluşturulmuştur. Ayrıca, geliştirilen test istatistiği ile literatürdeki en başarılı test istatistiği Monte Karlo simülasyonlarıyla karşılaştırılmış ve bu tezde geliştirilen test istatistiğinin daha başarılı olduğu

gözlenmiştir. Bu durum, geliştirilen test istatistiğinin kullanımsal geçerliliğine işaret etmektedir.

Anahtar Sözcükler: Birim Kök, Yapısal Kırılma, Birleşik Hipotez Testi, Mote Karlo Simülasyonları

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CHAPTER 1

INTRODUCTION

Time series analysis has been dealt with the properties of the many macroeconomic and financial time series. Main concern of the researches in the time series is the question that: how macroeconomic and financial time series move over time? A major ongoing debate started after Nelson and Plosser (1982) try to characterize the dynamic properties of macroeconomic and financial time series. Nelson and Plosser (1982) have claimed that, shocks hitting the economy have a permanent effect rather than temporary effect and the long run movement in the time series is altered by these permanent shocks. Using some statistical techniques that are proposed by Dickey and Fuller (1981), Nelson and Plosser (1982) have found that time series contains unit autoregressive root. Nelson and Plosser (1982) claim that time series follow a difference stationary model. Difference stationary model characterization of the macroeconomic variables indicates that, long run movement of the time series do not fluctuate around a steady state long run value but rather the movement is totally altered by the shocks that are hitting the economy. Then implications of the difference stationary model interrogate the steady state assumptions in the economics. Since justification of the difference stationary model deters underlying principles of economics, the research that is proposed by Nelson and Plosser (1982) has stimulated much interest.

Some researchers have challenged the characterization of the time series as a difference stationary framework which is suggested by Nelson and Plosser (1982). In particular, Rappaport and Reichlin (1989) and Perron (1989) argue that, log output is stationary around broken time trend whereas the date of break is the years of Great Depression. In brief, Perron (1989) shows that, Nelson and Plosser have failed to account for trend break in the GNP and they have accounted this one time innovation shift as long lasting rather than it was in fact one time innovation. If years of the Great Depression are specified as the time that structural change has occurred, then the unit root hypothesis is rejected in favor of the trend-break alternative. Perron (1989) claims that, the reason for failure to reject the unit root hypothesis is a consequence of misspecification in the trend function, especially a one time structural break in trend function. Perron (1989) has proven that, when in fact the trend break model is the true structure of the time series, unspecified structural break raises spurious evidence for unit root hypothesis. If trend-break alternative is not specified in the test procedure, unit root hypothesis cannot be rejected. After Perron (1989) made a critic of Nelson and Plosser, literature has been developed with the attempts to understand the true nature of time series: difference stationary model versus trend break stationary models.

These attempts and empirical findings are important for many reasons. First of all, if the trend stationary model is correct, then some studies as Cochrane (1988) and Cogley (1990) have put too much importance to the innovations in GNP. Non rejection of the unit root hypothesis is counter evidence against business-cycle hypothesis. When structural break is not accounted, then false empirical inferences may arise from the spurious conclusion of the unit root behavior. Cointegration analysis is based on the presumption that the time series follow a unit root pattern. In fact, if time series follow trend-break stationary pattern rather than difference stationary, then empirical relevance of the literature in econometrics on unit root and cointegration is brought into question.

Trend-break alternative model that is presented by Perron (1989) has been criticized for two reasons. First, Perron (1989) determines break date by presumption that, date of break coincides with years of Great Depression. Break date is specified prior to any knowledge up on data. The assumption that break date is known a priori was criticized by many authors. Christiano (1992) shows that the pretest examination of data can make important difference on Perron's conclusion. Christiano (1992) stated that, break date selection affects critical values of the test statistic which makes non rejection of the trend break stationary model dependent on the selection of the break date. Christiano states that, reliable test should consider break date as unknown a priori. The method that is suggested by Christiano (1992) relies on the standard sampling theory. The date of break is chosen independent of prior information about data. Also Banerjee, Lumsdaine, Stock (1992), Zivot and Andrews (1992) Perron and Vogelsang (1992) argue that the choice of break should be treated as unknown. Extensions of the trend break alternative model have been proposed by many authors. Specifically, Zivot and

Andrews (1992), Banerjee, Lumsdaine, Stock (1992), Perron and Vogelsang (1992), Perron (1997), Vogelsang and Perron (1998) adopted Perron's (1989) methodology for each possible break-date in the sample, which yields a sequence of the statistics that is in interest. Some algorithm that maximizes evidence against null hypothesis can be constructed from the sequence of the statistics, to determine break date.

Second criticism is based on the selection of the alternative form of trend break model. If the date of break is treated to be unknown, then the form of the break is also unknown. Then the determination of the form of break in alternative hypothesis becomes important. Sen (2003) notes that, if the alternative form of structural break does not coincide with the true form of break that time series follow, then test statistic will fail to reject difference stationary model because of wrong specification of the alternative hypothesis. Test for difference stationary model versus trend break stationary model should take into account all possible form of breaks in order to avoid specification errors that Perron has highlighted. Sen (2003) suggests that alternative form of break should be most general in order to avoid misspecifications. Alternative break forms that Perron has considered are: break in the mean of trend function (Crash Model), break in the slope of trend function (Changing Growth Model) and break in both mean and slope of trend function (Mixed Model). Sen (2003) has proposed a joint null hypothesis of unit root and no break in both mean and slope of trend function. Sen (2003) used the maximal F statistics that is proposed by Murray (1998) and Murray and Zivot (1998). Test is sequentially computed over range of possible break dates so maximum F test is also independent of break date specification. Then, joint null hypothesis corresponds to the difference stationary model. Non rejection of the null hypothesis indicates the appropriateness of the difference stationary model for time series. Joint null hypothesis incorporates with possible trend breaks of both types. Non rejection is not sourced by the unaccounted trend breaks because the null hypothesis includes restriction of no structural break with any possible form. Joint null hypothesis is not only unit root test but also test for no structural break pattern in the time series.

However, when joint null hypothesis is rejected, alternative hypothesis is too general to specify a structural form for time series. In other words, rejection of the test statistics does not provide us enough inference on the alternative hypothesis to conclude specific structural form. Since the null hypothesis is the joint mixture of unit root and no structural break, null hypothesis is too restrictive, when test is rejected, alternative may involve three model specifications according to source of deviation from the null model. According to source of deviation from the null model, there exist three alternative models: 1) Pure stationary model: Stationary and no break alternative. 2) Trend break stationary model: Stationary with some form of break. 3) Trend break model with unit root pattern: unit root behavior with structural break.

In our study, we aim to propose a test statistic that can exactly determine the specific structural model that time series pertain. We suggest a test statistic that can discriminate null model from these three alternatives. Moreover, our test statistic is able to discriminate one alternative from another. Additional to the difference stationary and trend break stationary models, we specify pure stationary model and trend break with unit root pattern model. Additional models are alternatives to the difference stationary null model. Additional alternative models are not covered in the literature. According to source of deviation from alternative, we specify two additional alternative models. When time series follow additional structure models, existence of their structure increases evidence against difference stationary model; they can be specified as alternatives to the difference stationary model. Secondly, we aim to propose a test statistic which has joint null of unit root and no structural break where break date is not determined a priori and break date is not affected by the unit root property of the time series.

Our test is motivated from the methodology that is suggested by Andrews (1992) and Andrews and Ploberger (1994). We use combination of the F maximum and the J test that has been proposed by Park (1989) and Park and Choi (1991). From the methodology of Vogelsang (1998), we utilize the property that J statistic converges to zero for stationary behavior of the time series and J statistic converges to a constant for unit root pattern. We adjust F max statistics with J value in order to determine one specific alternative hypothesis when joint null is rejected. By doing so, we can suggest a test statistic that can differentiate three different alternative structural models. Using joint null hypothesis of unit root and no structural break allow us to present a test statistics that can both test for unit root and structural break. We use similar methodology to Vogelsang (1998, 2003). But, rather than using only joint null of no break of both types; we propose joint null of unit root and no structural break. We are able to specify difference stationary model in the null hypothesis. The test statistic that we propose does not only test for unit root and (or) structural break, null and alternative hypotheses correspond to different structural models of time series. Moreover inclusion of unit root pattern in to the test statistic reduces the possible source of size and power distortions. Appendix section shows that our test statistic has better power properties than the test statistics in the literature.

Rest of the study is organized as follows. Chapter 2 consists of the literature survey and various comments on unit root hypothesis and structural break. Various model specifications and various test statistics that are presented in the literature are included in this section. Chapter 2 consists of various attempts to specify unit root or (and) structural break forms. Chapter ends up with the assessment of the literature and the shortcomings of the tests in the literature. Reason for inability to specify a structural model for time series has been discussed. Chapter 3 gives a detailed methodology of the test statistic that is presented in this study. Chapter 4 includes power and size Monte Carlo Simulations with comparison to the previous most powerful test. Chapter 5 concludes and includes the arguments for further research.

CHAPTER 2

LITERATURE SURVEY

In this section, the literature survey of the research is presented. First the literature on unit root hypothesis is discussed. Joint hypothesis test includes testing for unit root. Preliminary discussion of unit root hypothesis will clarify the developments in the joint test. Literature on unit root hypothesis has extended with testing for unit root with a linear time trend in the model. Secondly, trend break literature is included into agenda. Evolution of the trend break test into the joint hypothesis test holds particular importance for model specification of the time series. Literature of this evolutionary process has been presented in this research. Other part of the joint test is the test for structural break. Structural break literature is also summarized in this research. Literature has been discussed by the virtue of extensions to the hypothesis testing on the question of the true behavior of time series. This discussion has been concluded by the open questions that we aim to answer in this research.

2.1 LITERATURE ON UNIT ROOT HYPOTHESIS TESTING

Unit root hypothesis is particularly important in terms of the well established properties of the long-run equilibrium in economics. Therefore testing for unit root hypothesis has been one of the important areas of macro-econometrics. Unit root representation of the time series has been firstly presented by Dickey and Fuller (1979). In their seminal article, brief introduction to unit root autoregressive time series has been presented.

To follow their article, let T observations $Y_1, \ldots, Y_T$ be generated by the model $Y_t = \alpha Y_{t-1} + \varepsilon_t$, where ε_t is a sequence of independent normal random variables with zero mean and variance σ_ε^2 and t is time script. Properties of the regression estimator of α are obtained under the assumption that $|\alpha| \leq 1$. Because when $\alpha > 1$, time series is not stationary and the variance of the time series grows exponentially as t increases. Hence asymptotic distribution derivation may not be feasible. The time series with $\alpha = 1$ is sometimes called as random walk. The null hypothesis of unit root ($\alpha = 1$) holds interest in economic applications. The class of models presented in Dickey and Fuller (1979) are:

$$Y_t = \alpha Y_{t-1} + \varepsilon_t \tag{2.1a}$$

$$Y_t = \mu_0 + \alpha Y_{t-1} + \varepsilon_t \tag{2.1b}$$

$$Y_t = \mu_0 + \mu_2 t + \alpha Y_{t-1} + \varepsilon_t \tag{2.1c}$$

T-statistics for models are:

$$\hat{\tau} = (\hat{\alpha} - 1)(se_1^2 \ c_1)^{-1/2} \qquad (2.2a)$$

$$\hat{\tau}_\mu = (\hat{\alpha} - 1)(se_2^2 \ c_2)^{-1/2} \qquad (2.2b)$$

$$\hat{\tau}_t = (\hat{\alpha} - 1)(se_3^2 \ c_3)^{-1/2} \qquad (2.2c)$$

For $k = 1,2,3$ se_k^2 is the corresponding regression residual mean square from the models. Also c_k is lower-right element of u_k where $u_1 = (Y_{t-1})$, $u_2 = (1, Y_{t-1})$, $u_3 = (1, t, Y_{t-1})$. Limit distributions and their representations are shown in the Dickey and Fuller (1979).

After Box and Jenkins (1970) and Box and Pierce (1962) used the test of autocorrelation function of the deviations from fitted model, unit root testing proposed by Dickey and Fuller (1970) has been the test of appropriateness of the time series model. Non-rejection of the unit root null hypothesis is taken as an evidence for unit root behavior of the time series. Unit root pattern implies that, time series possess difference stationary model. First difference of the time series follow a stationary pattern. Then, time series have steady state values so that analysis of time series is feasible.

Autoregressive time series with unit root has taken much interest after Dickey (1976), Evans and Savin (1981, 1984) made forefront research. Random walk characterization such as $\Delta Y_t = \varepsilon_t$ where $\varepsilon_t \sim iid(0, \sigma^2)$, is a strong assumption. Hall (1978) showed the convenience and importance of the random walk hypothesis. Philips (1987) allows for more general weakly dependent and heterogeneous distribution theory for the random walk and allow for more general ARMA (1, 1) errors with single unit root. Philips (1987) notes that, the representation in (2.1a) can be a stochastic process generated as $Y_t = S_t + Y_0$ in terms of partial sums, $S_t = \sum_{t=1}^{T} \varepsilon_t$ for the innovation sequence $\{\varepsilon_t\}$ and initial condition Y_0. Initial condition Y_0 can have specific distribution so that $\alpha \geq 1$ alternative has been covered. Limiting distribution of the standardized sums is presented in second chapter of Philips's paper. From Dickey and Fuller (1970), OLS estimator of α in (2.1a) and t-statistic is:

$$T(\hat{\alpha}-1) = \{T^{-1}\sum_{1}^{T} y_{t-1}(y_t - y_{t-1})\} / \{T^{-2}\sum_{1}^{T} y_{t-1}^2\} \tag{2.3a}$$

$$t_\alpha = (\sum_{1}^{T} y_{t-1}^2)^{1/2}(\hat{\alpha}-1)/s \quad s^2 = T^{-1}\sum_{1}^{T}(y_t - \hat{\alpha} y_{t-1})^2 \tag{2.3b}$$

Philips derives new unit root test by defining new transformation estimator Z_α for $T(\hat{\alpha}-1)$ and transformation regression test statistic Z_t instead of regression t-statistic such as:

$$Z_\alpha = T(\hat{\alpha}-1) - (1/2)(s_T^2 - s_\varepsilon^2)/(T^{-2}\sum_{1}^{T} y_{t-1}^2) \tag{2.4a}$$

$$Z_t = (\sum_{1}^{T} y_{t-1}^2)^{1/2}(\hat{\alpha}-1)/s_T - (1/2)(s_T^2 - s_\varepsilon^2)[s_T(T^{-2}\sum_{1}^{T} y_{t-1}^2)^{1/2}]^{-1} \tag{2.4b}$$

s_T^2 and s_ε^2 are estimates of variance of errors (σ_ε^2) and variance of α (σ^2). These parameters should be estimated consistently. Philips showed that;

$$s_\varepsilon^2 = T^{-1}\sum_{1}^{T}(y_t - y_{t-1})^2 \tag{2.5a}$$

$$s_T^2 = T^{-1}\sum_{1}^{T}\varepsilon_\tau^2 - 2T^{-1}\sum_{\tau=1}^{L}\sum_{t=\tau+1}^{T}\varepsilon_t\varepsilon_{t-L} \tag{2.5b}$$

are consistent estimates of σ_ε^2 and σ^2. Philips (1987) has proposed an alternative unit root test to Dickey and Fuller's t-test. Theory of the research relies on the weak convergence. Characterization of the limiting distributions (2.3a) and (2.3b) are rather simple in terms of functionals of Brownian Motions.

Philip's unit root test transformation is related to the model (2.1a) corresponding to the class of models defined by Dickey and Fuller. But the models with drift and trend have not been covered in Philips (1987). This gap in the literature of unit root has been filled by Philips and Perron (1988). Two more models that correspond to the class of Dickey and Fuller are introduced by Phillips and Perron (1988). The models are:

$$Y_t = \mu_0 + \alpha Y_{t-1} + \varepsilon_t \tag{2.6}$$

$$Y_t = \mu_0 + \mu_2 (t-(1/2)T) + \alpha Y_{t-1} + \varepsilon_t \tag{2.7}$$

Then, regression t-statistics are:

$$t_{\hat{\alpha}} = (\hat{\alpha} - \alpha)\{\sum_{1}^{T}(y_{t-1} - y_t)^2\} / \hat{s} \tag{2.8}$$

$$t_{\tilde{\alpha}} = (\tilde{\alpha} - \alpha)/(\tilde{s}\ c_3)^{1/2} \tag{2.9}$$

Here, $\hat{s}$ and $\tilde{s}$ are the standard errors of regressions of (2.6) and (2.7) as before. c_3 is lower right element of $u_3 = (1, t-(\frac{1}{2}T), Y_{t-1})$.

Philips and Perron (1989) cite the importance of the innovations in the limiting distributions. When innovations are non-orthogonal and $\sigma^2 \neq \sigma_\varepsilon{}^2$, the Dickey and Fuller t test does not have the asymptotic size. Limiting distributions depend the nuisance parameters σ^2 and $\sigma_\varepsilon{}^2$. As denoted in Philips (1987), elimination of nuisance parameter is a result of having $s_T^{\ 2}$ rather than $\varepsilon_t = \Delta y_t$.

Extended models (2.6) and (2.7) accommodate with a fitted drift and a time trend so that they may be used to discriminate between unit root (difference stationarity) and stationarity around a deterministic trend. These extended models have better power compared to the previous no drift and no trend model. When many time series are simulated to be stationary about deterministic trend, percentage of the simulations that are rejected increase with extended model specification. The conclusion that Philips and Perron (1987) have reached made an influence on research. Many researchers suspect that unit root test is affected by inclusion of trend function into the model. When trend function is not included into the model, misspecification of trend parameter increases the evidence for non-stationary behavior and unit root hypothesis is not rejected erroneously. From the Monte Carlo simulations of the Perron and Philips (1987), one can claim that, maintenance of the trend parameter may affect the results of unit root tests. Theoretical literate have been developed with modifications of the unit root tests with trend function specified alternatives whereas empirical application of unit root hypothesis has attracted more attention. There is good summary of the research on this topic by Campbell and Perron (1991).

2.2 STRUCTURAL MODEL SPECIFICATION – DIFFERENCE STATIONARY MODELS VERSUS TREND STATIONARY MODELS

Wide application of the unit root test put less importance to the structural model specification of the time series. After Nelson and Plosser's stimulating paper, one of the areas of interest in economics has been the application of unit root testing. The view that most economic time series are characterized by unit root behavior has become prevalent. Until Perron (1989) highlighted the importance of the structural model specification, literature has been developed on the empirical area of the unit root testing. After Philips and Perron's research, maintenance of trend functions in the models have not been considered seriously. However, Perron (1989) indicated that when true data generation has a one time change in trend function, unit root tests fail to reject the trend stationary model. Perron (1989) characterize unit root test with the trend break model alternative. Different characterizations of the trend break alternatives are presented in Perron's research. Perron has not only considered trend extended model, but also specified alternative trend-break stationary models. Then testing for unit root is enlarged to structural model specification with trend-break alternative. When unit root hypothesis is rejected in favor of the trend-break alternative, there is evidence for trend-break stationary model. Therefore unit root test has played the role for determining difference stationary versus trend-break stationary model specification. According to Perron, standard Dickey and Fuller (1979) unit root test cannot reject the unit root hypothesis, when in fact true data generation mechanism is that of trend break stationary. Spurious conclusion of the unit root testing may lead to incorrect empirical inference. Perron (1989) showed that even asymptotically stationary fluctuations of trend break model cannot be rejected.

Perron extends the analysis of Philip's to a more general case which allows for one time change in the trend function. When Philips has considered trend extended model, null hypothesis was unit root and alternative hypothesis was trend stationary model. Perron has allowed a break in the trend function which has three alternative forms. Null hypothesis of unit root has been considered for different three forms:

Model (A) $\quad y_t = \mu_0 + dD(T_B)_t + y_{t-1} + \varepsilon_t \qquad$ (2.10a)

Model (B) $\quad y_t = \mu_0 + y_{t-1} + (\mu_2 - \mu_0)DU_t + \varepsilon_t \qquad$ (2.10b)

Model (C) $\quad y_t = \mu_0 + y_{-1} + dD(T_B)_t + (\mu_2 - \mu_0)DU_t + \varepsilon_t \qquad$ (2.10c)

Here in these representations, $D(T_B)_t = 1$ if $t = T_B + 1$ and 0 otherwise; $DU_t = 1$ if $t > T_B$ and 0 otherwise. $A(L)\varepsilon_t = B(L)v_t$, $v_t \sim iid(0, \sigma^2)$. $A(L)$ and $B(L)$ are $p'th$ and $q'th$ order polynomials in the lag operator L. Corresponding alternative hypotheses are:

Model (A) $\quad y_t = \mu_0 + \beta t + (\mu_2 - \mu_0)DU_t + \varepsilon_t \qquad$ (2.11a)

Model (B) $\quad y_t = \mu_0 + \beta_1 t + (\beta_2 - \beta_0)DT_t + \varepsilon_t \qquad$ (2.11b)

Model (C) $\quad y_t = \mu_0 + \beta_1 t + (\mu_2 - \mu_0)DU_t + (\beta_2 - \beta_1)DT_t + \varepsilon_t \qquad$ (2.11c)

Here in these representations, $DT = t - T_B$ if $t > T_B$ and 0 otherwise. T_B is the break date. Perron (1989) has considered three alternative forms of breaks. First, it is crash model (Model (A)) which allows for one time change in the intercept of the trend. Second is changing growth model (Model (B)) which allows for one time change in slope of trend function. Third is mixed model (Model (C)) which allows a change in both intercept and slope of the trend function. Perron states, when structural break alternative included into specification of the time series, the unit root hypothesis can be rejected in favor of the trend-break alternative. Basic

Dickey Fuller unit root test gives false results due to omitted variable of trend function or misspecification of the trend with breaks. When breaks have not been taken into account, residuals increase so that unit root null cannot be rejected. However, if data is separated into sub-periods at the time of break, critical values of t-statistic decrease significantly so that unit root hypothesis is rejected.

Perron's analysis is important because of the statement: unaccounted structural breaks lead to spurious results of non-rejection of the unit root test. Estimation of nuisance parameter is highly affected by the trend behavior. Moreover, Perron has presented alternative representation of the time series such as trend-break stationary model. Perron suggests specification of the unit root test from these models:

$$y_t = \mu^A{}_0 + \theta^A DU_t + \beta^A t + dD(T_B)_t + \alpha^A y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \varepsilon_t \qquad (2.12a)$$

$$y_t = \mu^B{}_0 + \theta^B DU_t + \beta^B t + \gamma^B DT + \alpha^B y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \varepsilon_t \qquad (2.12b)$$

$$y_t = \mu^C{}_0 + \theta^C DU_t + \beta^C t + \gamma^C DT + d^C D(T_B)_t + \alpha^C y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \varepsilon_t \quad (2.12c)$$

Δy_{t-i} is included to reduce the autoregressive effect on nuisance parameter. Lag length k is selected by information criteria. The null hypothesis of unit root imposes restrictions $\alpha = 1,\ \beta = 0,\ \gamma = 0$ for representations in (2.14a-c). For the alternative models, asymptotic distributions of the t-statistics $t_{\bar{\alpha}^A}$, $t_{\bar{\alpha}^B}$, $t_{\bar{\alpha}^C}$ have been derived. (For i=A, B, C)

By specifying date of break as the years of Great Depression and Oil Crisis years, Perron uses same data set of Nelson and Plosser applying to his methodology. Perron finds evidence against unit root hypothesis. When alternative model is set up as a trend break model with specified break dates, unit root hypothesis is rejected.

Rappaport and Reichlin (1989) confirmed the Perron's conclusion. Rappaport and Reichlin noted that when alternative model is specified as segmented trend model rather than just trend stationary model, test statistic support segmented trend model. Both conclusions of Perron and Rappaport and Reichlin indicate that, difference stationary model cannot be approved by unit root test without putting trend-break into the test procedure. Empirical findings of Nelson and Plosser, Stulz and Wasserfallen (1985), Campbell and Mankiw (1987, 1988), Cochrane (1988), Hall (1987), Gould and Nelson (1974), Blanchard and Summers (1986) are brought into question.

After Perron has proven the spurious nature of the unit root test when alternative model is not specified as trend-break model, various questions have arisen in the literature. First of all, in Perron's analysis, choice of the break date depends on visual inspection of the data. Determining break date without any diagnostic tests has been attacked by many authors. Christiano (1992) argues that, the date of break should not be chosen independent of the test procedure. When break date is exogenous to the testing procedure, then critical values are higher according to true size. Hence, rejection of the null hypothesis in favor of the alternative may be also spurious.

$$\Delta y_t = \mu_0 + \theta dt^i + \beta t + \alpha y_{-1} + \sum_{j=0}^{k} c_j \Delta y_{t-j} + \varepsilon_t \qquad (2.13)$$

In equation (2.13), Perron's modified augmented Dickey and Fuller test which has null hypothesis of $\alpha = \beta = \theta = \gamma = 0$. Perron suggests comparing the t-statistic on α, t_α with critical values tabulated in his article where break date has been determined without pre-test of the data. Christiano suggests a methodology which allows for the selection of the break date as a function of the data being tested. He extends Perron's analysis testing the same null hypothesis with F statistics for $\forall t \in T$ with corresponding null hypothesis. Maximum value of F test will indicate the break date from the sequence of F statistics for $\forall t \in T$.

Secondly, Christiano suggests that, the lag length parameter k should also be selected with a methodology. Level of the lag truncation parameter k should be parallel to the significance of the parameter c_k. From maximum possible value of k, the latest insignificant value of k according to diagnostic tests should be excluded. Choosing appropriate level of k^* depends on the information criteria. Reason to include additional k^* extra parameters c_{k^*} is to get rid of the possible autocorrelation as explained before. According to Christiano, Perron's conclusion is a consequence of choosing break date a priori. When lag length parameter and break date are selected by data dependent methods, Christiano found contradictory evidence to trend stationarity model in GNP.

Perron and Vogelsang (1992) suggest another unit root test that is robust to the date of break specification. They have tested unit root hypothesis in the presence of mean break with application to the purchasing power parity hypothesis. According to Perron and Vogelsang (1992), assuming that date of break is known a priori is inappropriate. Their trend break alternative is crash model (Model A), corresponding to the Perron (1989). For one time change in the level at time T_B such that $1 < T_B < T$. They have parameterized the null hypothesis of unit root such that: $y_t = D(T_B)_t + \alpha y_{t-1} + \varepsilon_t$. $D(T_B)_t = 1$ if $t = T_B + 1$ and 0 otherwise. Assumption on ε_t is consistent with the previous literature. Under the mean break alternative:

$$y_t = \mu_0 + D(T_B)_t + \varepsilon_t \tag{2.14}$$

They use t statistic for $\alpha = 1$ in the following regression, which nests the null and alternative hypotheses:

$$y_t = \sum_{1}^{k} w_i D(TB)_{t-i} + \alpha y_{t-1} + \sum_{0}^{k} \Delta y_{t-i} + \varepsilon_t \tag{2.15}$$

They have suggested another method to choose date of break. T_B is chosen such that $t_\alpha(i, T_B, k)$ is minimized for i=A, B, C models and $k \in [0, k\max]$. $k\max$ is the upper bound of lag length. The selection of k was made with objective of getting the autocorrelation variance properties of the fitted residuals to resemble the assumptions made in the bootstrap simulations in the paper. As a result, $k = 4$ is the lag length that is chosen by Perron and Vogelsang (1992). When break occurs at time T_B, it increases the absolute value of t-statistics for null of $\alpha = 1$. Hence, the mean break date is determined when evidence against the null hypothesis is maximized. So for t-test T_B is determined from algorithm: $\inf_{T_B \in (1,T)} t_\alpha(i, T_B, k)$.

The method that is presented by Vogelsang and Perron is similar to the previous methods in which the date of break is specified according to date when evidence against null hypothesis is maximized. They apply their methodology to test for purchasing power parity hypothesis to find evidence for trend stationary model when mean break is utilized.

Another paper on this topic is proposed by Zivot and Andrews (1992). In their research they have cited the method proposed by Christiano (1992). Zivot and Andrews (1992) made critics of Christiano because of determining break dates solely by bootstrap methods. Zivot and Andrews (1992) used t-min statistic to determine date of break. Similar to Christiano (1992), Zivot and Andrews (1992) concluded that Perron's unit root tests are biased towards rejecting the unit root. But the break dates found by Zivot and Andrews are different from the break dates found by Perron. Unit root hypothesis is not rejected for some series of Nelson and Plosser (1982). Only nominal GNP, industrial production series are found to be trend stationary. However inability to reject the null hypothesis of unit root should not be taken as an evidence for accepting the null. Rejection indicates the inappropriateness of the null hypothesis against the alternative hypothesis. Specification of the alternative hypothesis is also important. Main reason for contradicting evidence in various papers is to specify different alternative forms of break. Perron and Vogelsang have considered crash model alternative whereas Christiano and Zivot and Andrews have specified mixed model alternative. Question of the break date specification should be incorporated with specification of the alternative form of break. In more general sense, when difference stationary versus trend-break stationary model is tested, alternative form of the trend-break plays important role.

More general attempt to determine unit root hypothesis and trend-break alternative has been given by Banerjee, Lumsdaine and Stock (1992). Different from previous literature, Banerjee, Lumsdaine and Stock (1992) tried to make unit root test and structural break tests together with general form. When date of break is assumed to be unknown, then existence of the trend-break should be tested. They perform a test that incorporates both unit root hypothesis and trend-break hypothesis in the null hypothesis. By doing so, they are able to specify the possible form of trend-break in the null hypothesis. Rejection of the null does not only indicates inappropriateness of the unit root null but also inappropriateness of the no trend-break restriction.

In order to test for unit root and structural break, sequential test is suggested. Sequential test has advantage of testing unit root with trend where alternative is trend-break for specific type of break. They consider both Crash Model alternative and Changing Growth Model alternative. Another advantage of the test is to have a joint null hypothesis. Also more than one algorithm is presented to choose break date. As an extension, recursive and rolling tests are suggested for unit root testing. But those tests do not incorporate with structural break test. Model is:

$$y_t = \mu_0 + \mu_1 \tau_1(T_B) + \mu_2 t + \alpha y_{t-1} + \beta(L)\Delta y_{t-1-L} + \varepsilon_t \qquad (2.16)$$

$\tau_1(T_B)$ captures the possibility of a trend-break. For Crash Model (Case (A)), $\tau_1(T_B) = 1(t > T_B)$. For Changing Growth Model (Case (B)), $\tau_1(T_B) = (t - T_B)(t > T_B)$. Testing for $\mu_1 = 0$ corresponds to structural break test for both case A and B. However under the null hypothesis joint null is $\mu_1 = 0,\ \alpha = 1$. A transformation regression Z_t for set of variables and transformed parameter vector θ are defined. The estimator of the test statistic computed over T observations for $k = k_0, k_0 + 1, ..., T - k_0$ where $k_0 = \lambda_0 T$. λ_0 is the trimming value of initial fraction of T. The stochastic process constructed from the sequential estimators and Wald test statistic is:

$$\theta(\lambda) = (\sum_1^T Z_{t-1}(\lambda T) Z_{t-1}(\lambda T)')^{-1} (\sum_1^T Z_{t-1}(\lambda T) y_t) \quad (2.17)$$

$$F_T(\lambda) = \{[R(\theta) - r][R(\sum_1^T Z_{t-1}(\lambda T) Z_{t-1}(\lambda T)^{-1}) R']^{-1} [R(\theta) - r]\} / q\sigma^2(\lambda) \quad (2.18)$$

where $\sigma^2(\lambda) = (T - q - m - 4)^{-1} \sum_1^T (y_t - \theta(\lambda) Z_{t-1})^2$.

$F_T(\lambda)$ is computed for every $\lambda \in (\lambda_0, 1 - \lambda_0)$. If any break of specified form exists, then it increases the $F_T(\lambda)$ statistics. Maximum value of the $F_T(\lambda)$ is the best candidate from the sequence of the $F_T(\lambda)$ statistics. $T_B = \lambda T$ is the break date such that $F_{T_B}(\lambda) = \max F_T(\lambda)$. Test statistic is denoted as $F_T^{MAX}(\lambda)$. $F_T^{MAX}(\lambda)$ is a Wald type test which also tests for existence of break. Banerjee, Lumsdaine and Stock (1992) have used Wald type $F_T^{MAX}(\lambda)$ test by utilizing the nature of the Wald test which allows testing for more than one parameter. Unit root and specific break type are tested together. By doing so, date of break is also determined by data dependent methods. Also, Banerjee, Lumsdaine and Stock (1992) compare various tests that take place in the literature. They propose Monte Carlo simulations to compare power properties of these various statistics. Monte Carlo simulations of Banerjee, Lumsdaine and Stock (1992) indicated that, sequential Wald type test is more accurate to detect break dates. Moreover $F_T^{MAX}(\lambda)$ statistic has power superiority over other test statistics. Their simulation results confirms

Perron's conclusion. Basic Dickey and Fuller t-test fails to reject unit root when in fact data generation process contains trend-break stationarity. According to Banerjee, Lumsdaine and Stock (1992), Christiano failed to reject unit root hypothesis because of using bootstrapped critical values. When they apply their methodology to the growth rates, they have concluded that some countries' GNP follows unit root.

DeLong etc. all. (1992) tested for I(1) versus trend-break stationary model with joint test. They have inverted unit root hypothesis to the joint hypothesis with null hypothesis $\alpha = 1,\ \mu_2 = 0$ with consistent notation with Banerjee, Lumsdaine and Stock (1992). DeLong etc.all. (1992) develop a similar test which has size adjusted power for nuisance parameter, they have analyzed the power results of the joint test concerning dependent variables α and $(y_0 - \mu_0)/\sigma$. They have reached the conclusion that unit root tests have low power against plausible trend stationary alternatives. Moreover, there are some cases unit root hypothesis is rejected in favor of the trend-break alternative and trend-break alternative is rejected in favor of the unit root alternative. This results does suggest that inferences of the test of integration is fragile and also using more restricted null of unit root with no breaks works better with this nature of fragility. Nature of the test affects results. When no break restriction is put in null hypothesis, form of the break that is specified in the null hypothesis is also important. But those papers have put limited effort to research on the null of unit root with general form of no structural break. DeLong etc. all. (1992) only concluded that it is premature to accept difference stationary model with basic unit root tests.

2.3 PARAMETER SHIFT LITERATURE

Banerjee, Lumsdaine and Stock (1992) shifted the unit root testing against trend-break alternative to more general joint test. When joint null is $\alpha = 1,\ \mu_2 = 0$, testing this null hypothesis is a kind of test for structural break. Trend break test is a distinct form of structural break. In order to understand model specification literature, it is of partial interest to understand structural break literature.

Before Banerjee, Lumsdaine and Stock (1992), structural break test is accounted for only change in parameters and it's linkage to the macroeconomic time series was not that much appealing in empirical literature. Structural break literature begins with Chow (1960) test for breaks in the regression. Main concern of the Chow (1960) was to asses the test of equality between set of coefficients in two linear regressions. By residual based test, regime stability or regime change has been concerned. Let $y_1 = X_1\beta_1 + \varepsilon_1$ and let m additional observations specified by the regression $y_2 = X_2\beta_2 + \varepsilon_2$. Test of equality between parameters has null hypothesis $H_0 : \beta_1 = \beta_2 = \beta$ such that $y = X\beta + \varepsilon$. Test depends:

$$d = y_2 - X_2\hat{\beta}_1 = X_2\beta_2 - X_2\beta_1 + \varepsilon_2 - X_2(X_1{}'X_1)^{-1}X_1{}'\varepsilon_1 \qquad (2.19)$$

$d^2 / \mathrm{var}(d)$ will be distributed by $F(1, n-p)$ for n observations and p regressors. This test is based on the prediction interval for one new observation. The sum of squares of the residuals under null hypothesis will be equal to the sum of squares of the residuals under alternative plus sum square deviations between two sets of estimates of y. Idea of the Chow Test is to obtain sum of squares of residuals under the assumption of equality between parameters and the sum squares without assumption of equality. The ratio of the difference between these two sums to the latter sum, adjusted for the corresponding degrees of freedom will be distributed as the $F(1, n-p)$ ratio under the null hypothesis. This test is the basic version of the Wald test. As a matter of fact, when null hypothesis is $\alpha = 1$, $\mu_2 = 0$, $\mu_2 = 0$ part of the null hypothesis tests for equality of parameters before and after the possible break date.

Another structural break test is proposed by Quandt (1959). Idea is to test for break in time t^* from T observations. Possible structural break date t^* has been chosen to minimize $\lambda = (\hat{\sigma}_1^t \hat{\sigma}_2^{T-t}) / \hat{\sigma}^T$. $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are estimated variance from first and second regressions which are given below and $\hat{\sigma}^T$ is the aggregated estimated variance. Model is presented in Quandt (1959) as:

$$y_i = a_1 + b_1 x_i + u_i \text{ for } i = 1, \ldots, t$$

$$y_i = a_2 + b_2 x_i + u_i \text{ for } i = t+1, \ldots, T$$

More popular test for structural change is introduced by Brown, Durbin and Evans (1975) abbreviated as CUSUM test. CUSUM test is named for cumulative sum of residuals. CUSUM test is often under critics for its low power so that CUSUM squares test has been developed. Whole literature of CUSUM test is beyond the scope of this research and reader is referenced to Kramer etc. all (1988) for a detailed discussion. CUSUM test cannot specify form of the break. Only information test brings is the location of the break.

Bai and Perron (1998) have considered more general form of structural break allowing for multiple breaks, occurring at unknown dates. Also, they have constructed another test statistics null of L breaks versus alternative $L+1$ breaks. Multiple structural breaks are treated to be unknown variables to be estimated. In Bai and Perron (1998), there is an example with three regimes. In this specific example they reach important results. When one of the breaks dominates (parameter shift is highly significant), sum of squared residuals is reduced. The reduction is highest when dominating break is correctly identified. They have further examined the relative importance of the dominating break. They have generated a data which has several breaks but one of the breaks has highest parameter value shift relative to the other breaks. When dominating break date correctly specified, test statistic finds evidence for only one dominating break rather than multiple breaks. This result is also found by Chong (1994).

These results indicate that when one of the breaks is consistent break point which is dominant, correct specification of this break date allows greatest reduction in the sum squared residuals. So specification of this dominant break holds greater importance than finding all break points. Hence, literature is concentrated on the single break rather than multiple breaks. Presenting a test statistic that has high power to detect possible univariate break holds greater importance than multiple break concept. One exceptional research has been conducted by Bai, Lumsdaine and Stock (1998). Bai, Lumsdaine and Stock (1998) extended multivariate techniques for determining break dates precisely. If one interested in mean shift at multivariate models, Bai, Lumsdaine and Stock (1998) proposed pseudo-F test which is an extensions of Andrews' (1994) $\exp\_F$

statistics. Also some relationship between structural break and cointegration analysis has been presented in the paper. Following their methodology, there is evidence for break in the mean growth of consumption at the years of late 1960's and 1980's. For further discussion, reader is referenced to their research.

2.4 UNIT ROOT HYPOTHESIS AND STRUCTURAL BREAK

After Banerjee, Lumsdaine and Stock (1992) put unit root and trend break hypotheses together, researchers have tried to characterize joint hypothesis with various considerations. Attempts can be summarized in two sections. One family of the research consists of the investigation of the suitable test statistic which has persistent power properties and other family of the research consists of alternative break form specifications for joint test.

Sen (2001) has considered the F-test under the trend-break stationary alternative. He considers both alternatives of trend break (corresponding to changing growth model in Perron) and trend and mean break in trend function (corresponding to mixed model in Perron). Sen (2001) analyzed Perron's argument by considering the behavior of F-statistic. Sen (2001) reconsiders trend-break alternative by proposing a test that has joint null of unit root and no structural break. Sen utilized the joint test when true data generation process is trend-break stationary Sen (2001) replied the Perron's conclusion that τ_T test in (2.2c) has tendency to accept unit root hypothesis when true data generation process in fact trend-break stationary. Sen (2001) reconsiders Dickey and Fuller F-statistic for joint hypotheses $H_{02}: \mu_0 = \beta = 0,\ \alpha = 1$ and $H_{03}: \beta = 0,\ \alpha = 1$ in (2.16). Corresponding null hypotheses are random walk and random walk with drift. F-statistics is calculated for H_{02} and H_{03} denoted as ϕ_2 and ϕ_3.

Sen concludes that, under changing growth model alternative, when true data generation process is trend-break stationary, regardless of the parameter value or magnitude of break, ϕ_2 statistic can reject unit root hypothesis while ϕ_3 statistic may fail to reject unit root hypothesis when magnitude of break is small.

Analysis of Sen (2001) holds particular importance for alternative break form specification. When changing growth model is suitable alternative, both ϕ_2, ϕ_3 statistics reject unit root hypothesis. However if mixed model is true form of the data generation process, behavior of ϕ_2 and ϕ_3 depends on the parameter value of the mean break and trend break value. ϕ_3 statistic may fail to reject unit root hypothesis in some cases. Sen's analysis indicates that alternative model definition may lead to false inferences. Perron notes that, when trend-break is not accounted, unit root tests fail to reject unit root null hypothesis. However, Sen (2001) extends this statement. When alternative form of the break is misspecified, then unit root test still fails to reject unit root hypothesis.

Perron highlights the importance of the selection of the truncation lag on the outcome of the test. Test is performed using the t-statistic for the null hypothesis

$H_0 : \alpha = 1$ in the model (C).

Second method to choose lag truncation parameter is taken from Said and Dickey (1984). From maximum value of k, $k\max$ is specified and autoregression with $k\max$ and $k\max-1$ lags were estimated. F-test of $k\max$ versus $k\max-1$ iterated until F-test is insignificant for $k\max-c$ versus $k\max-c-1$. k^* is determined as $k^* = k\max-c$ which is the lag truncation value determined by these two procedures. Lag truncation selection holds particular importance according to Perron. Because selection of k^* may lead to size and power distortions. In application, Perron uses the suggested methodologies to select k^*. By doing so, Perron has reached same conclusion in his previous work. Contradicting results of Christiano (1992) are explained to be consequence of lacking an appropriate procedure to select k^*. Perron suggested that fixing k to some arbitrary value can lead to serious size distortions and power losses due to fact that, actual correlation structure of the data is not only unknown but also it is likely to be different for various time series. In application, Perron found that k^* level is different across countries. Using a fixed k^* in time series has important effect on the results of the test statistic.

Assessment of the researches that are conducted by Perron (1997), Christiano (1992), Banerjee, Lumsdaine and Stock (1992) prompt the literature into attempts to present a test statistic which directly assumes that break date and the lag truncation level are unknown. From the beginning of the trend-break stationary literature, testing for trend-break condemned with the proper mechanism to establish break date specification. Source of power distortions in the test statistics are thought to be caused by inappropriate break date specification. After literature has overcome the break date specification problem, area of research turned back to the question of appropriate trend-break modification.

Whenever dispute over power properties of the test statistics evaded from break date specification, question of the appropriate trend function hypothesis has come into agenda again. Turning back to the Banerjee, Lumsdaine and Stock (1992), joint test has been conducted for specific trend-break model from the three models presented by Perron. Regardless of specifying trend-break in the null hypothesis or alternative hypothesis, there is no serious treatment to determine trend function and power properties due to trend function. Vogelsang (1998) tried to fill this gap in the literature. Vogelsang examined the test for trend function hypothesis where errors follows I(1) or I(0) pattern. Alternatively, Vogelsang considered structural break test rather than joint test. Alternative treatment of trend function hypothesis is an attempt to make a test of structural break in the presence of I(1) or I(0) errors. The test proposed is robust to unit root behavior of the time series. Also statistics are asymptotically invariant to nuisance parameter. For difference stationary model, transformation of taking difference has been suspected by Perron. So test of structural break should incorporate with unit root or stationary behavior of the time series. Correct specification of trend function is required for reliable test statistic.

In order to represent in more general form, let $y_t = f(t)'\beta + u_t$ where $u_t = \alpha u_{t-1} + \eta_t + \theta\eta_{t-1}$. This assumption is general enough to permit polynomial trends possibly with a finite number of structural breaks. By forming partial sums of $\{y_t\}$, model can be transformed to $Z_t = g(t)'\beta + S_t$ where $g(t) = \sum_{j=1}^{t} f(j)$ and $Z_t = \sum_{j=1}^{t} y_j$. When u_t is I(1), Z_t has I(0) innovations. In canonical form $Y = X_1\beta + u$, $Z = X_2\beta + S$ where $Y = \{y_t\}$, $Z = \{Z_t\}$, $u = \{u_t\}$. $S = \{S_t\}$, $X_1 = \{f(t)'\}$, $X_2 = \{g(t)'\}$ are $(T \times k)$ matrices. Let $\hat{\beta}$ OLS estimate of β from the first form and β^* OLS estimate from second form.

$$y_t = f(t)'\beta + \sum_{i=j}^{m} \gamma_i t^i + u_t \text{ and } z_t = g(t)'\beta + \sum_{i=j+1}^{m} \gamma_i t^i + S_t \tag{2.20}$$

Let $J_T^1(m)$ denote standard OLS Wald statistics normalized by T^{-1} for testing the joint null hypothesis $\gamma_j = \gamma_{j+1} = ... = \gamma_m = 0$ and let $J_T^2(m)$ corresponding Wald type statistic test for the joint null $\gamma_{j+1} = \gamma_{j+2} = ... = \gamma_m = 0$. $J_T^1(m)$ is the unit root statistics proposed by Park and Choi (1988) as explained before. When errors are I(0), $J_T^1(m)$ converges to zero. When errors are I(1), $J_T^1(m)$ has a non-degenerate limiting distribution. m is maximum polynomial length which is determined solely on heuristic evidence. Consider testing the null hypothesis for β:

$$H_0 : R\beta = r \text{ versus } H_A : R\beta \neq r$$

Where R is a $(q \times k)$ matrix of constants; r is a $(q \times 1)$ matrix of q restrictions. Vogelsang proposes several test statistics for this null hypothesis such as:

$$T^{-1}W_T = T^{-1}(R\hat{\beta} - r)[R(X_1'X_1)^{-1}R']^{-1}(R\hat{\beta} - r)/s_y^2 \tag{2.21a}$$

$$PS_T = (R\beta^* - r)'[R(X_2'X_2)^{-1}R']^{-1}(R\beta^* - r)/(s_z^2 \exp(bJ_T(m))) \tag{2.21b}$$

$$PSW_T = (R\hat{\beta} - r)'[R(X_1'X_1)^{-1}R']^{-1}(R\hat{\beta} - r)/(T^{-1}100 s_z^2 \exp(bJ_T(m))) \tag{2.21c}$$

Here b is a constant. Statistics are normalized T^{-1} Wald type tests. 100 included in the last test for numerical fashion.

PS_T and PSW_T statistics are designed to have power when the errors are stationary. $J_T^1(m)$ statistic is included to make tests statistics robust to I(1) errors. Equality between the critical values of I(0) and I(1) errors is sustained by utilizing a parametric constant b. Inclusion of b does not affect the size or power of the statistics. Suppose that b=0, then $J_T^1(m)$ test has no effect so PS_T and PSW_T statistics have non-degenerate limiting distributions for both I(1) and I(0) errors. When b is some positive number, the $J_T^1(m)$ statistic smooth out the discontinuities of PS_T and PSW_T statistics by taking large values for I(1) errors and small values for I(0) errors. Therefore, the b's can be chosen to vanish the differences between I(0) and I(1) errors. Then, distributions of PS_T and PSW_T statistics come close to each other.

Choice of m depends up on Monte Carlo simulations and Vogelsang (1998) suggested that power is maximized when $m = 9$. Vogelsang (1998) applies the test methodology to GNP growth rates. When $q = 1$ statistics become Wald type t-statistics and these statistics have indicated considerable evidence for a shift in the slope function of many series. Limiting distributions and consistency conditions are established in Vogelsang (1998).

Vogelsang and Perron (1998) have considered the test for unit root allowing a break in the trend function at unknown time. This paper was extension to Vogelsang and Peron (1992). Previously in the literature, various researches considered innovation outlier and additive outlier models for the type of break occurring. Innovation model (IO) assumes that break occurs with gradual whereas additive outlier (AO) assumes that break occurs suddenly. This paper also uses AO framework. Break date endogenous choice has been made by concerning t-min statistic whose procedure and methodology have been explained before. Following Perron and Vogelsang's (1992) notation, the unit root statistics are asymptotically invariant to a mean shift under the null hypothesis, but this invariance property does not hold in finite samples. Effects of mean and slope changes on the limiting distributions are analyzed. Vogelsang and Perron (1998) brought many extensions to their previous work. Also one another extension is to model time series which has trend break under the null hypothesis of unit root. This extension adds literature the case that time series follow unit root pattern and trend-break behavior together. Asymptotic invariance of the statistic on to a mean shift under unit root has been explored. But this invariance does not hold for slope change.

Innovation outlier model presented in Vogelsang and Perron (1992) is a two step procedure. First series are detrended, and then the detrended regressions are estimated by OLS. Details are represented before in the literature. Extensions of outlier model consist of the null hypotheses:

$$y_t = y_{t-1} + \beta + \varphi^*(L)[\gamma DU_t + \varepsilon_t] \tag{2.22a}$$

$$y_t = y_{t-1} + \beta + \varphi^*(L)[\theta DT_t + \varepsilon_t] \tag{2.22b}$$

$$y_t = y_{t-1} + \beta + \varphi^*(L)[\theta DT_t + \gamma DU_t + \varepsilon_t] \tag{2.22c}$$

IO model is applicable to the cases where it is more reasonable to view the break as occurring more slowly over time. In principle, the dynamic path of adjustment of the shift could take any form so IO model is capable of defining shocks to the innovation process. But immediate and long run impact of the structural break is different in IO framework. Immediate impact is a shift in the slope is γ, while the long run impact is given by $\varphi^{*}(L)\gamma$. Under the corresponding alternative hypotheses y_t is given by:

$$y_t = \mu + \beta t + \varphi(L)[\theta DU_t + \varepsilon_t] \qquad (2.23a)$$

$$y_t = \mu + \beta t + \varphi(L)[\gamma DT_t + \varepsilon_t] \qquad (2.23b)$$

$$y_t = \mu + \beta t + \varphi(L)[\theta DU_t + \gamma DT_t + \varepsilon_t] \qquad (2.23c)$$

In general, unit root hypothesis $\alpha = 1$, $\theta = \gamma = 0$ in these three regressions. T_B and k selected as the same as in previous work of Perron and Vogelsang. Also power simulations of t-min test in the Vogelsang and Perron (1992) has been replied in Vogelsang and Perron (1998).

In Vogelsang and Perron (1998), it is argued that dependence upon θ and γ can be made precise. It can be shown (as shown in Perron and Vogelsang (1992)) that all of the statistics are asymptotically invariant to θ. But when $\gamma \neq 0$, the tests have asymptotic size less than the asymptotic level. A way to avoid this potential power loss is to first perform a pretest for shift in slope that is valid for both unit root and stationary errors. But as indicated in Vogelsang and Perron (1992), any time a pretest is conducted, the size of the ultimate test is likely to be distorted and detailed investigation is leaved for future research. So, before the future research is conducted on unit root and slope break hypothesis, one should keep in mind that; AO framework provide reliable tests with size that is variant to large shifts in intercept and slope.

Role of the error variance in power results has been researched by Vogelsang (1999) in which source of non-monotonic power is tried to be covered. Following from the early beginning of structural break literature, Vogelsang (1997) noted the tests for structural change in the trend function of a dynamic time series can have non-monotonic power functions. As distance between the null hypothesis and alternative hypothesis increases, power is decreasing in some ranges. Undetected shifts in trend bias the estimates and tests of dynamic parameters that may be of interest. From the literature of break in trend function, Bai (1994) noted that $\hat{\lambda}$ (date of break estimated) converges to the true break point at rate T, and this result holds for serially correlated errors that are stationary linear process. But this is not the case when errors are I(1) as shown in Vogelsang and Perron (1998). Unit root pattern and structural break tests mainly suffer from substantial power losses. It is shown that a wide variety of tests can have non-monotonic power functions, indicating that non-monotonic power is a serious problem in practice. Second, sources of non-monotonic power are uncovered in the literature. Here, the statistics are analyzed in a unified framework that allows direct comparisons. The statistics are expressed as a function of weighted Wald statistics. Since Wald statistics are scaled by estimates of variance parameters, their power functions are very sensitive to the behavior of the variance estimates. By examining the behavior of the Wald statistics and the weights, as the null and the alternative grow apart, two sources of non-monotonic power are pinpointed. If estimates of variance are not invariant to the shift parameter, non-monotonic power can result. The second source is the inclusion (or misspecification) of the lagged dependent variable.

Vogelsang considers testing for mean shift in regression,

$$y_t = \mu + \theta DU_t + u_t . \tag{2.24}$$

In similar notation we have: $u_t = \alpha u_{t-1} + \varepsilon_t$ and $DU_t = 1(t > T_B)$ with $\varepsilon_t \sim iid(0, \sigma^2)$. In order to test for structural break in the mean, it is necessary to estimate nuisance parameter associated with $\{u_t\}$. Variance estimate of θ, γ are functions of $\omega^2 = \sigma_\varepsilon^2 / (1-\alpha)^2$ where ω^2 is proportional to the spectral density of $\{u_t\}$ evaluated at frequency zero. ω^2 is estimated parametrically using estimates of α and σ_ε^2. In Vogelsang non-monotonic estimation is:

$$\hat{\omega}_{np}^2 = \sum_{j=-T+1}^{T-1} [K(j/L)T^{-1} \sum_{t=1}^{T-j} \hat{u}_t \hat{u}_{t+j}] \tag{2.25}$$

$\{\hat{u}_j\}$ is OLS residuals from regression imposing $\theta = 0$ and $k(.)$ is kernel function, L is lag truncation parameter. Formally, alternative representation of equation (2.24) has been given by Perron (1989):

$$y_t = \mu^* + \theta^* DU_t + \alpha y_{t-1} + \varepsilon_t \tag{2.26}$$

$\mu^* = \mu(1-\alpha)$, $\theta^* = \theta(1-\alpha)$, $\gamma^* = \gamma\alpha$. However the presence of y_{t-1} in regression has important consequences for power of some statistics. Following Wald type statistics that is proposed by Andrews and Ploberger (1994) for testing $\gamma = 0$, OLS estimate variance of $\{u_t\}$ is replaced by ω_{np}^2:

$$WS(T_B) = (\sum_{t=T_B+1}^{T} \hat{y}_t)^2 / (\hat{\omega}_{np}^2 \sum_{t=T+1}^{T} \hat{D}U_t^{\,2}) \tag{2.27}$$

$\hat{y}_t$ and $\hat{D}U_t$ are residuals from the regression of y_t and DU_t on constant and

$\hat{\omega}_{np}^2$. Define Wald statistic in the case where the parameter ω is assumed to be known. This exact Wald statistic is defined as:

$$WS^e(T_B) = (\sum_{t=T_B+1}^{T} \hat{y}_t)^2 / (\hat{\omega}^2 \sum_{t=T+1}^{T} \hat{D}U_t^{\,2}) \tag{2.28}$$

$WS(T_B) = (\omega^2 / \hat{\omega}_{np}^2) WS^e(T_B)$. Then the statistic is expressed as functions of weighted $WS^e(T_B)$. Vogelsang (1999) expresses the Wald type statistics that is proposed by Andrews and Ploberger (1994) as a test which is functions of weighted Wald statistics.

Other statistics considered in the Vogelsang (1999) are the statistics that are proposed by Vogelsang (1998). To summarize them again; let $z_t = \sum_{j=1}^{T} \hat{y}_t$ and $S_t = \sum_{j=1}^{t} u_j$ .Then (2.24) becomes:

$$Z_t = \mu t + \gamma DT + S_t \tag{2.29}$$

Define $J^* = \inf_{T_B \in \Lambda} J(T_B)$, $J(T_B)$ is the standard Wald statistic divided by T for testing $\beta_1 = ... = \beta_9 = 0$ in the regression $y_t = \mu + \theta DU_t + \sum_{i=1}^{9} \beta_i t^i + u_t$. The J^* is related to the class of unit root statistics proposed by Park and Choi (1988) as before. Represent ed as in compact notation $c_i(T_B) = 100 \exp(b_i J^*) T^{-2} \sum_{j=1}^{T} S_i(T_B)^2$. $i = 1,2$ for different values of b. The weighted Wald test statistics of interest are defined as:

$$MPSW = T^{-1} \sum_{T_B \in \Lambda} (\sum_{t=T_B+1}^{T} \hat{y}_t)^2 / [(\sum_{t=T_B+1}^{T} \hat{D}U_t^{\,2}) c_1(T_B)] \tag{2.30a}$$

$$SPSW = \sup_{T_B \in \Lambda} (\sum_{t=T_B+1}^{T} \hat{y}_t)^2 / [(\sum_{t=T_B+1}^{T} \hat{D}U_t^{\ 2}) c_2(T_B)] \qquad (2.30b)$$

In the first statistics, power decrease for an increase in θ. Source of non-monotonic power is explained to be the estimate of variance parameters. (Requirement an estimate of ω^2) As θ increases, $WS^e(T_B)$ also increases on average, regardless of T_B. The increase is larger the closer T_B to the true break date. But statistics (2.30a) and (2.30b) have the property that ω^2 need not be estimated to carry out the test. (See Vogelsang (1998) for details) This suggest that as long as the weights are not decreasing in θ, then the statistics will be, on average, increasing in θ and power will be monotonic. Therefore power can be non-monotonic because weights are decreasing on average as θ increases. This result simply reiterates the fact that power of the statistic is sensitive to the model in which variance estimated. Trend break models are suspected to have wrong specification of the break type. Then, specification of break type in the alternative becomes crucial for power results.

In order to compare power properties, Vogelsang uses Monte Carlo simulations for the tests (2.30a), (2.30b) and various eight statistics developed in the literature. Only $MPSW$ and $SPSW$ statistics seem to have monotonic power property. Also those two statistics have considerable power advantage. Because when errors are I(1), other statistics diverge and their power decreases significantly.

Role of y_{t-1} is important. From the result Perron (1989) and Vogelsang (1999), one can claim that, ignoring a mean shift in the autoregression biases the estimate of α towards 1, and estimated model appears to be I(1) without mean shift. Wrong date of shift may lead to some other bias as well. Those biases can cause non-monotonic power of statistics that are functions of estimate obtained from regression using wrong shift dates. ω^2 should be estimated under the alternative. But if the alternative form of break is not identified correctly, source of non-monotonicity still exists. This highlights the fact that alternative form determination is important issue in structural break literature.

Alternative form specification leads to power loses. To analyze this result, first attempt has been made by Yang (2001) who suspects the misspecification in regression rather than the alternative hypothesis in interest. Yang (2001) has considered the asymptotic distortion of regression misspecification on the Sup_Wald test which is proposed by Andrews (1994). He adopts the idea of drifting data generation process which is introduced by Davidson and MacKinnon (1985, 1987). Then only source for dependence is possible breaks. The size of the Sup_Wald statistic is distorted by regression misspecification.

Vogelsang ‘s (2001) working paper deals with the test statistics that are proposed to test for shifts in the trend function of univariate time series. The test is valid in the presence of general forms of serial correlation in the errors. The tests are valid for both I(0) and I(1) errors. The tests are designed to detect a single break at an unknown time. A priori knowledge about innovations if they are I(0) or I(1) is not required. Partial sums of innovations play central role in testing so subjective choices like lag length, information criteria, kernel or truncation lag can be completely avoided. Approaches of Andrews (1993) and Andrews and

Ploberger (1994) are applied. The importance of uncovering instability of parameters in time series is well known. Failure to account for structural change in parameters can lead to inconsistent parameter estimates and biased forecasts. Models are extensions of the previous work of Vogelsang (1998). Test for trend break extended to all cases that are presented by Perron (1989). Power properties of the tests are presented in the working paper.

Sen (2003) has proposed his paper claiming that form of the break specification is the main reason for power distortions. Previously in the literature, it is argued that selection of the break date must not correlate with data. This argument extended the statement: choice of the alternative form must be general. If one assumes that location of the break is unknown, it is most likely that form of the break will be unknown. So turning back to the Perron's argument, unit root hypothesis should be tested for difference stationary model where alternative form is correctly specified. According to Sen (2003), one must proceed with the break specification according to the most general mixed model in order to prevent misspecification of the true alternative form. (Both slope and mean break denoted as mixed model). In addition, one may expect power distortions if the form of the break is wrongly specified. (For example, if one imposes the crash model where in fact changing growth model or mixed model is appropriate. On the other hand, if the crash model is the correct form then its use will yield superior power compared to mixed model.) Test statistic and the model specification is in similar fashion to Perron's notation. Sen slightly adjusts the equations (2.11a-c) for models A, B and C. Alternative hypotheses are consistent with Perron's IO framework.

Null hypothesis is;

$$y_t = \mu_1 + y_{t-1} + \varphi(L)v_t \quad (2.31)$$

To test for unit root in the IO framework the following methodology has been suggested; specify an interval $\Lambda = [\lambda_0, 1-\lambda_0] \subseteq (0,1)$ that is believed to contain the true break fraction. For each possible $\lambda \in \Lambda$, Sen (2003) estimate the regression that nests the null and the appropriate alternative following the same methodology of PAlgorithm to determine date of break is followed by Vogelsang (1998) and Zivot and Andrews (1992). Also $t_{DF}(i) = t_{DF}(\hat{\lambda}_i)$ is used to compare to other statistics, where $\hat{\lambda}_i (i = A,\ B,\ C)$ is the break date maximizes the Wald statistic $F_T[\lambda T]$ for null hypothesis as suggested by Banerjee, Lumsdaine and Stock (1992).

According to Sen (2003), correct specification of the alternative holds as much importance as the break date specification. Most general robust alternative of mixed model can increase likelihood, since; it is the most general form. Also mixed alternative model has admissible power advantage compared to other mistaken forms.

Sen (2003) puts together unit root hypothesis and structural break test. Sen considers Sup_Wald statistic that is proposed by Murray (1998) and Murray and Zivot (1998). Joint null hypothesis is unit root with no breaks. (Neither slope nor mean break) $F(T_B)$ statistic for joint null hypothesis,

$$H_0^j : \alpha = 1,\ \mu_1 = \mu_2 = 0 \quad (2.32)$$

is calculated as:

$$F_T(T_B) = (R\hat{\mu}(T_B) - r)[R(\sum_1^T x_t(T_B)x_t(T_B)')^{-1}R']^{-1}(R\hat{\mu}(T_B) - r)/q\sigma^2(T_B) \quad (2.33)$$

$\hat{\mu}(T_B)$ is the ordinary least squares estimator of $\mu = (\mu_0, \mu_1, \mu_2, \mu_3, \alpha, c_1, ..., c_k)$.

$x_t(T_B) = (1, DU_t, t, DT_t, y_{t-1}, \Delta y_{t-1}, ..., \Delta y_{t-k})$, $\quad r = (0,0,1)$.

$$\hat{\sigma}(T_B) = (T-5-k)^{-1}\sum_1^T [y_t - x_t(T_B)\hat{\mu}(T_B)]^2 \tag{2.34}$$

$(R\mu - r)$ corresponds to the restrictions imposed on the parameter vector μ by the joint null H_0.

$$R = \begin{pmatrix} 0\ 1\ 0\ 0\ 0\ 0_\mathrm{k} \\ 0\ 0\ 0\ 1\ 0\ 0_\mathrm{k} \\ 0\ 0\ 0\ 0\ 1\ 0_\mathrm{k} \end{pmatrix} \tag{2.35}$$

Where 0_k' is the zero row matrix with rank k. The sequence of the F statistics $\{F_T(T_B)\}_{T_B=\lambda_0 T}^{T_B=T-\lambda_0 T}$ is used to calculate the maximum F statistic for H_0^J as $F_T^{\max} = Max_{T_B \in \lambda_0 T ... T-\lambda_0 T} F_T(T_B)$. Power of three t-min and $t_{DF}(i) = t_{DF}(\hat{\lambda}_i)$ and $F_T^{\max}$ statistics are compared. Sen (2003) concluded that $F_T^{\max}$ statistic has the best power compared to the other statistics. Sen's research is a kind of extension to Vogelsang (1998). Rather than trying to present a test statistic robust to I(1) and I(0) errors, unit root behavior is endogenously tested in the joint test. Moreover, joint test is put as the most general joint null. According to Sen (2003), joint test is the most reliable test with significant power advantage.

2.7 ASSESMENT OF LITERATURE AND OPEN QUESTIONS

Literature of unit root hypothesis and structural break intersects at the research for model specification in time series. True nature of the time series behavior can be understand by hypothesis testing of the joint null hypothesis, $H_0^j : \alpha = 1,\ \mu_1 = \mu_2 = 0$. (In Sen's notation) Because this hypothesis nests difference stationary model testing and trend stationary model testing together. Since Perron suggested that misspecification for trend function leads to spurious results, literature has been developing by the research on alternative form specification. Sen (2003) claims that, if one specifies wrong alternative, joint test will also give wrong inferences. In order to avoid model misspecification, one should use general alternative. But important unanswered question arise from the fact that, when joint null rejected in favor of the alternative, we cannot reach specific conclusion of the time series in interest. Joint null puts three restrictions, so that there are three sources, which can lead to the rejection of the null. It is unambiguous without further investigation.

Second point is that, when purpose is to search for validity of trend stationary model, it cannot be set apart from unit root tests. Nuisance parameter (estimate of variance) is affected by innovations. When innovations follow I(1) behavior, test results will be different as shown by Vogelsang. Vogelsang attempted to bring this fact into research. Vogelsang developed a structural break test when innovations are I(1) or I(0).A priori we cannot know the true nature of the innovations. When

$u_t = u_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim iid(0,\sigma^2)$ then $y_t = \beta x_t + u_t$ has unit root so y_t is I(1). Then first we need to test for unit root. But turning back to Perron we know that unit root tests can give false results because of possible trend breaks. Hence, structural break tests should incorporate with unit root test.

Moreover, we found particular evidence for the fact that, when unit root behavior is not taken into account for structural break tests, then tests have tendency to reject no structural break null because of the increase in critical values due to unit root behavior. Perron's conclusion is two sided. When trend break is not taken into account, tests fail to reject unit root null hypothesis; but also when unit root is not taken into account, tests fail to reject the no break null hypothesis. Later research of Vogelsang includes a test that is robust to I(1) or I(0) innovations. But test is lack of conclusion for model specification. True break date- if exists- can be specified; but from the test results, we cannot conclude if the times series posses unit root behavior or not. These avoid us to determine the true structure of time series even we can test for structural break.

Alternative form is also important to find true break dates. Literature consists of contradicting evidence for break dates. Highly potential source for these inconveniences is wrong alternative specification. There are two main alternatives for break for trend function: $TD = \beta_0 + \beta_1 t$. Both or one of them may have shift. However omitted shift in the parameters will increase the innovation so that result of the test statistic may find wrong break date.

More generally, literature has developed on the question of the true structural nature of the time series. Does time series follow difference stationary model or trend break stationary? Even if we have joint null hypothesis $\alpha = 1,\ \mu_1 = \mu_3 = 0$ so that null model is difference stationary, it is not sure that alternative is trend break stationary. It is possible that $\alpha < 1$ and $\mu_1 = \mu_3 = 0$ so that time series is just stationary. When difference stationary null is rejected it does not indicate that trend break model is true nature. Moreover there is another possibility that $\alpha = 1,\ \mu_1 \neq 0$ or $\mu_3 \neq 0$. This is the case that time series follow unit root with trend break. Though there are four different possible structural models, they are compressed to two cases. A test statistic that can determine one of the four possible models will not suffer from misspecifications so we expect this statistic to be able to detect true date of break with true form of breaks (or break) if break exists.

CHAPTER 3

TEST STATISITIC AND METHODOLOGY

3.1 MOTIVATION

In this section, we briefly discuss the motivation of the test statistic suggested in this work. Assessments of the literature address one important question: Is it possible to present a test statistic that can conclude true structure of the time series? Our aim is to present a test statistic that allows us to perceive one specific pattern of the time series in question rather than just concluding possible combinations of many patterns. Null and alternative hypotheses should correspond to different patterns of the time series. Also, for the null hypothesis of difference stationary model, test statistic should also involve no structural break in the null hypothesis as Perron (1989) highlighted the biases arising from using a test statistic with unaccounted trend breaks. Moreover, all possible forms of breaks should be included in the null hypothesis in order to get rid of the biases arising from misspecification of the alternative break form. When the form of the break is assumed to be unknown, all possible forms of break should be included in the null hypothesis. On the other hand, if a structural break test is conducted with no break null hypothesis, test statistic is under effect of unit root property of the time series. Once unit root property is unknown, then structural break test can be biased by the source of unit root property. Hence, the test conducted here composes both unit root test and structural break test together. By presenting such a test, estimated break date (if exist) will not suffer from biases.

Motivation is utilized by presenting a test statistic for joint null hypothesis in (2.32). The null hypothesis includes both possible form of trend breaks, since non-rejection is not biased by any form of break, it indicates unit root pattern without break in the trend function. But when the joint null is rejected, it can not be taken as an evidence for trend break stationary model. Deviation from the null hypothesis has two main sources: Stationarity and structural break (mean break and (or) slope break in trend function-generalized as structural break). When both sources are specified in the null hypothesis, deviation from the null hypothesis in the test statistic can be caused by the combination of these sources. Rejection of the null hypothesis cannot specify one correct combination of these sources. In other words, only one alternative hypothesis is too general to determine true nature of the time series when joint null hypothesis is rejected. Then three possible alternatives are: Pure stationary time series without any form of break, stationary time series with trend break of some form and unit root behavior of time series with trend break of some form. These three alternatives are condemned to trend-break stationary model in the literature. We want to propose a test statistic that can differentiate these three alternatives.

Moreover, if specified alternative hypothesis is not correct form of break, there are serious power and size distortions as shown in Vogelsang (1998) and Sen (2003). This situation casts a doubt on the estimated date of breaks when a test is conducted with one specific alternative. Researchers reach contradicting results on same data set by estimating different break dates with different alternative specifications.

The test presented here nests all three alternatives. Test will be able to decide true form of break with accomplished date of break estimation. Literature has been developing with attempts to suggest a test statistic for difference stationary versus trend-break stationary test which has desired power properties without size distortions; we also aim to propose a test that has desired power properties without size distortions.

3.2 MODEL AND ASSUMPTIONS

Time series in interest is assumed to be generated by the following model:

$$y_t = \mu_0 + \mu_1 DU(T_B) + \mu_2 t + \mu_3 DT(T_B) + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + u_t \qquad (3.2)$$

y_t is the time series that is conceived. y_t is regressed on drift, mean break dummy, time trend, slope break dummy, its first lag and k 'th period difference of the y_t . The error u_t is specified to be an ARMA (1, 1) process defined as; $A(L)u_t = B(L)\varepsilon_t$ where $\varepsilon_t \sim i.i.d.(0, \sigma^2)$ under assumption 1. $A(L)$ and $B(L)$ are lag polynomial operators of order 1 and 1 where $A(L)$ component of u_t represents autoregression structure and $B(L)$ represents moving average component. Throughout, T_B denotes the date of break which is assumed to be unknown.

$DU(T_B) = 1(t > T_B)$ and $DT(T_B) = 1(t > T_B)(t - T_B)$ with $1(.)$ is indicator function. Before the date of break, indicator functions $DU(T_B)$ and $DT(T_B)$ are equal to zero and after the date of break $DU(T_B)$ and $DT(T_B)$ are 1.

The model that we consider in this section is driven by the sequence of error term $\{\varepsilon_t\}$. Similar to Perron (1989) and Philips and Perron (1988), classic assumption for $\{\varepsilon_t\}$ is:

ASSUMPTION 1:

i) $E(\varepsilon_t) = 0$ for all t. (3.1a)

ii) $\sup_t E(\varepsilon_t)^{\beta+e} < \infty$ for some $\beta > 2$ and any $e > 0$. (3.1b)

iii) as $T \to \infty$, $\sigma_\varepsilon^2 = \lim E(T^{-1}S_T^2)$ exists and $\sigma_\varepsilon^2 > 0$, where $S_t = \varepsilon_1, ..., \varepsilon_T$ (3.1c)

Assumption 1 is classic assumption which implies that a functional limit theorem applies to the partial sums of $\{\varepsilon_t\}$ so that asymptotic distribution of the test statistic exists. Here t is time script such as $t = 1, ..., T$. $E(.)$ is expectation operator. In this univariate set up, we assume that there is possible one time trend-break. Trend break is allowed to take three different forms. Break in the mean or (and) break in the slope of trend function. Trend break alternatives correspond to three forms that are represented in Perron (1989). Both segments of the trend functions are joined at time T_B. Change is presumed to occur gradually which corresponds to the IO model that is suggested by Perron (1989). In IO framework, effect of the break is sudden so that trend function changes its regime at the date of break. Δy_{t-i} is included into regression in order to eliminate possible correlation in the regression. Our representation is consistent with general mixed model (Model C) in Peron (1989) and Sen (2003).

The parameters μ_1 and μ_3 are the magnitude of the change in mean and slope of trend function. Since break date is assumed to be unknown, selection of the date depends on the test methodology. T_B is selected from the sequence of the statistic for every possible break date, testing for the joint null hypothesis. Following Banerjee, Lumsdaine and Stock (1992), T_B is the date that maximizes evidence against the joint null hypothesis. Details of the break date selection procedure are presented in the literature survey section.

Vogelsang (1992) suggested a data dependent method to chose lag truncation parameter k. More specifically, let selected value of k be k^*, such that; the coefficient on the last lag in an autoregression of order k^* is significant and the last coefficient on the last lag in an autoregression of order greater than k^* is insignificant, from the possible maximum order $k\max$. The significance of the coefficient is assessed using %5 critical values based on a standard normal distribution. Here in this study, we follow the same methodology to select lag length. After k^* has been determined, the test statistic is conducted.

Model specification consists of both types of breaks covered in the literature. Sen (2003) suggested that, wrong specification of the structural break type may lead to serious power distortions of the statistics. For example; if data generation process is stationary with break in the slope of the trend function, test statistic cannot reject the wrong null hypothesis of unit root because of specifying alternative hypothesis as mean break in the trend function. Also inaccurate specification of the break date estimation is a consequence of wrong specification of the break type. By including all types of breaks that is covered in the literature, we avoid the possible omitted variable biases. Sen (2003) suggest that, if true data generation process contains only one of the break types there will be power distortions. But compared to omitted break case, this distortion will be limited. Time trend is included into model, because we want to cover all possible alternative hypotheses. When time trend is placed into model, we can cover all the trend-break stationary alternative hypotheses.

3.3 TEST STATISTIC

The primary focus of the test statistic is to test for joint null hypothesis, $\alpha = 1,\ \mu_1 = \mu_3 = 0$. Test statistic presented here is very similar to the test proposed by Vogelsang (1998, 2003). Actually, this test statistic is an extension to the PSW_T statistic in (2.21c). Vogelsang proposes the test in order to test for $\mu_1 = \mu_3 = 0$ and rather than including y_{t-1} in model, unit root pattern is included in the errors. Then, unit root hypothesis is not tested. Test statistic's critical values are parametrically determined for different error patterns. Moreover, null hypothesis corresponds to no structural break. Test statistic does not give further information about the structural model of time series. In this extension, we want to fill these gaps in the literature.

Before we describe the test statistic, some preliminary notation is introduced.

Now let t^{j-1} be the highest order polynomial of time in $f(t)$ in the regression:

$$y_t = f(t)'\beta + \sum_{i=j}^{m} \gamma_i t^i + \alpha y_{t-1} + \varepsilon_t \tag{3.3}$$

$J_T(m)$ denote standard OLS Wald statistic testing for the null hypothesis :

$$\gamma_1 = \gamma_2 = \gamma_3 = ... = \gamma_m = 0 \tag{3.4}$$

in (3.3). $J_T(m)$ corresponds to the family of unit root statistics that is presented by Park and Choi (1988) and Park (1990). This test statistic test for unit root with polynomial trend fit in the model. $J_T(m)$ is included into the test statistic to utilize the respond of the $J_T(m)$ to the unit root pattern. For a detailed discussion of the use of $J_T(m)$ statistic, reader is referenced to the Vogelsang (1998). Similar to the PSW_T statistic, for the joint null hypothesis, test statistic is calculated as:

$$adj\_F(T_B) = \frac{[R\hat{\mu}(T_B) - r]\{R(\sum_{t=1}^{T} x_t(T_B)x_t(T_B)')^{-1}R'\}^{-1}[R\hat{\mu}(T_B) - r]}{q\sigma^2(T_B)\exp[J_T(m)]}$$

(3.5)

$\hat{\mu}(T_B)$ is the OLS estimator of the $\mu = (\mu_0, \mu_1, \mu_2, \mu_3, \alpha, c_1, ..., c_k)$, $x(T_B) = (1, DU(T_B), t, DT(T_B), y_{t-1}, \Delta y_{t-1}, ..., \Delta y_{t-k})$, $r = (0,0,1)'$, $\hat{\sigma}(T_B) = (T - 5 - k)^{-1}[\sum_{t=1}^{T} y_t - x_t(T_B)\hat{\mu}(T_B)]^2$. $(R\hat{\mu} - r)$ corresponds to the restrictions imposed on the parameter vector μ under the joint null hypothesis. $R = \begin{pmatrix} 0\ 1\ 0\ 0\ 0\ 0_k \\ 0\ 0\ 0\ 1\ 0\ 0_k \\ 0\ 0\ 0\ 0\ 1\ 0_k \end{pmatrix}$. Where $0_k'$ is the zero row matrix with rank k.

$adj\_F(T_B)$ statistic is a combination of classic $\max F(T_B)$ statistic presented in Murray (1998), Murray and Zivot (1998), Sen (2003) and Choi and Park (1988), Park (1989). The sequence of the $adj\_F(T_B)$ statistic $\{adj\_F(T_B)\}_{T_{B=\lambda_0 T}}^{T_{B=T-\lambda_0 T}}$ is used to determine break date. Algorithm is suggested by Vogelsang (1992), Banerjee, Lumsdaine and Stock (1992), Andrews (1992), Andrews and Ploberger (1994). Test is conducted for all possible break dates $T_B \in [\lambda_0 T, T-\lambda_0 T]$ where trimming value is λ_0. Discussion of trimming value takes place in Banerjee, Lumsdaine and Stock (1992), Perron (1992, 1997). We use trimming value $\lambda_0 = 0.15$ that is suggested by Sen (2003). By using sub-sample of the all data, break date assumed not to be at the ends of the sample. As discussed in Banerjee, Lumsdaine and Stock (1992), the choice of λ_0 entails a trade-off between needing enough observations in the shortest regressions to support the Gaussian approximation and wanting to capture possible breaks early and late at the sample. The date $T_B{}^C$ at which the sequence of the statistic $\{adj\_F(T_B)\}_{T_{B=\lambda_0 T}}^{T_{B=T-\lambda_0 T}}$ is maximized is determined to be the break date from the all possible break dates $T_B \in [\lambda_0 T, T-\lambda_0 T]$. So test statistic is $(adj\_F(T_B{}^C)) = Max_{T_B \in [\lambda_0 T, \lambda_0 T+1 \ldots, T-\lambda_0 T]} adj\_F(T_B)$.

Crucial important extension in our statistic is to adjust $\max F(T_B)$ statistic with $J_T(m)$ statistic. Discussion of the $J_T(m)$ statistic takes place in Vogelsang (1994, 1998, 2003, and 2004). The argument is developed by Andrews (1992), Andrews and Zivot (1994). The test statistic is optimal in the sense that, test statistic is not under effect of nuisance parameter $\sigma^2 / \sigma_\varepsilon^2$ so that central limit theorem can be applied to create critical values after Monte Carlo simulations. Vogelsang has generalized a structural break test statistic by adjusting $\max F(T_B)$ with $J_T(m)$. However unit root behavior in Vogelsang (1998) is exogenous. Test is robust to I(1) and I(0) errors but the question of I(0) versus I(1) is not included in the test statistics. This situation cast a doubt on the test estimated break dates which may be distorted from the unit root behavior. Moreover, as discussed before, test can not conclude the true form of the time series; trend stationary or difference stationary models.

Vogelsang uses $J_T(T_B)$ statistic because $J_T(T_B)$ converges to zero when errors are I(0) and $J_T(T_B)$ has non-degenerate limiting distribution. When errors follow I(0) pattern, then test statistic is just Wald type $F\max$ statistic same as in Sen (2003). Asymptotic derivations are shown in Vogelsang (1998) for both I(1) and I(0) errors. Vogelsang uses some parametric adjustment to bring both distributions' critical values equal; parameter "b" in PSW_t statistic (2.21c). Though $J_T(T_B)$ is not added for a statistical test, it can be used as an statistic to make an adjustment in Wald type test. We aim to utilize nature of the $J_T(T_B)$ statistic in our test statistic. When $J_T(T_B) \neq 0$, null hypothesis presented in (3.4) corresponds to the unit root hypothesis with trend. Alternative hypothesis corresponds to the higher order polynomial fit of the trend.

$$J_T(m) = \frac{RSS_y - RSS_j}{RSS_j} \tag{3.6}$$

Where RSS_y -the residual sum is squares- under null hypothesis (3.4). RSS_j is the sum of squares for higher polynomial trend fit. Number m has been determined by practical appropriateness. Vogelsang (1998, 2003) has shown that $m=9$ maximizes power of the test statistics. We also found evidence that $m=9$ performs better. When $m>9$ has been determined some loss of power has been observed. Simulations are available up on request.

$J_T(m)$ test that is presented in Ouliaris, Park and Philips (1988), Park (1989, 1990), can be considered as another unit root test which has specific polynomial trend fit alternative. Purpose to adjust $\max F(T_B)$ with $J_T(m)$ statistic is to utilize the sharp difference of statistical value in the $J_T(m)$ for I(1) and I(0) behavior of the time series. As noted before, when y_t is stationary, then $J_T(m)$ converges to zero rapidly. However, when y_t follows unit root pattern, then $J_t(m)$ jumps to a positive number. This sharp difference help us to distinguish I(1) versus I(0) pattern in the time series.

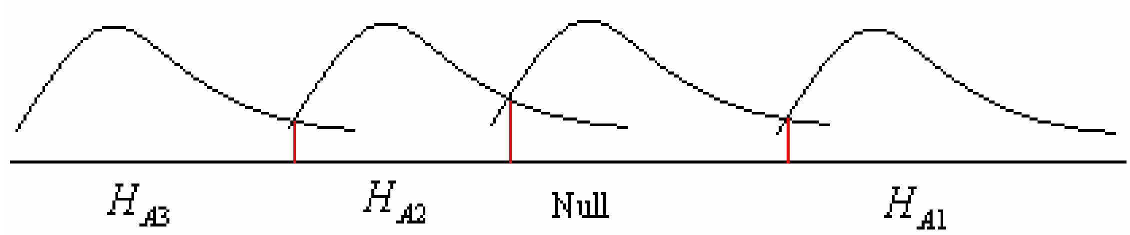
When $\alpha<1$, then $J_T(m)$ converges to zero. So that, test statistic (3.5) get closer to (2.33). Hence, if time series is stationary our test will be same as in Sen (2003). Sen (2003) presented that $\max F(T_B)$ statistic does not suffer from size distortions. So that under the null hypothesis our test does not suffer from size distortions. But if $\alpha=1$, $adj\_F(T_B{}^C)$ is different than $\max F(T_B)$ which is weighted by factor $\exp[J_T(m)]^{-1}$.

We have:

$$adj\_F(T_B{}^C)^* = \max F(T_B) D(F(T_B)_{CV}) \tag{3.7}$$

Where $D(F(T_B)_{CV})$ is an indicator function. When $\max F(T_B) \le F(T_B)_{CV}$, $D(F(T_B)_{CV})$ is equal to 1; else if $\max F(T_B) > F(T_B)_{CV}$, $D(F(T_B)_{CV})$ is equal to $\exp[J_T(m)]^{-1}$. Indicator function utilizes the weighting effect only if the joint null hypothesis of difference stationary model is rejected by $\max F(T_B)$ statistic. $F(T_B)_{CV}$ is %5 critical value of $\max F(T_B)$ which is derived by Sen (2003), Murray (1998) and Murray and Zivot (1998). Hence, when $\max F(T_B) \le F(T_B)_{CV}$, $adj\_F(T_B{}^C)^* = \max F(T_B)$. We have two step test statistic. When joint null of unit root and no break is rejected, difference stationary model is rejected in favor of the one of the alternative hypotheses. In order to differentiate alternative hypotheses, we need to be sure that first joint null is rejected. When joint null and three alternatives (stationary and no trend break model, stationary with some type of trend break model, unit root and trend break model) have been considered all together, serious power distortions arises from the inclusion of null hypothesis into consideration. By construction two step test methodology we can discriminate different alternatives without size distortions. Reason for such situation is that, $J_T(m)$ increases rapidly with unit root pattern and any type of trend break or any regime change in trend function. $J_T(m)$ value exponentially decreases the value of (3.5) so that any type of the break and unit root decreases our test statistic. When stationary without break is the true nature of the time series, $J_T(m)$ decreases according to parameter value of α. But $\max F(T_B)$ increases as α decreases as the data generation process deviates from the joint null hypothesis.

Figure 3.1- Various alternative Hypotheses



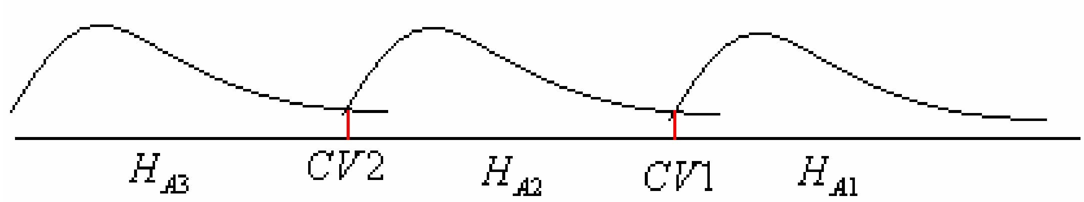
Let $H_{A3}: \alpha = 1,\ \mu_1 \neq 0$ or (and) $\mu_3 \neq 0$, $H_{A2}: \alpha < 1,\ \mu_1 \neq 0$ or (and) $\mu_3 \neq 0$ and $H_{A1}: \alpha < 1,\ \mu_1 = \mu_3 = 0$ as shown in figure 1. These alternative hypotheses correspond to different structural models. H_{A3} is trend-break unit root model, H_{A2} is trend-break stationary model and H_{A1} is pure stationary model. Under the null hypothesis, $adj\_F(T_B{}^C)^*$ statistic has its asymptotic distribution.

When alternative hypothesis H_{A1} is the true data generation process, with stationary and no break alternative, test statistic has higher value since $J_T(m) \to 0$. $J_T(m)$ statistic looses its domination. As $J_T(m) \to 0$, $\exp[J_T(m)] \to 1$. when y_t is stationary and $\max F(T_B)$ is higher than $F(T_B)_{CV}$ so then (3.5) has higher value than its critical value. Our test statistic discriminates the pure stationary alternative from other alternatives by the property of $J_T(m)$ statistic. Under H_{A1}, weighting function does not have any affect so only source for rejection is the deviation from the null hypothesis is stationarity ($\alpha < 1$). Under H_{A1}, $\max F(T_B)$ has higher value than its critical value. In this case, our test statistic has significantly higher values.

When true data generation process is H_{A2}, due to structural break, $J_T(m)$ deviates from its null value and jumps to a positive constant. Weighting factor $\exp[J_T(m)]^{-1}$ decreases the value of $adj\_F(T_B{}^C)^*$. Both structural break and stationarity are the sources of deviation from the null hypothesis. Though structural break and stationarity increases the value of $\max F(T_B)$ statistic, dominance of weighting factor decreases $adj\_F(T_B{}^C)^*$ value. $J_T(m)$ statistic jumps to a positive constant due to structural break. Weighting function has some positive constant.

When source of deviation is only structural break, weighting factor decreases test value more. Under the H_{A3}, test statistic has lowest value when both trend break and unit root pattern exists. Though $\max F(T_B)$ increases as a deviation from the null hypothesis, decrease in the $J_T(m)$ statistic dominates the $adj\_F(T_B{}^C)^*$ value. Both unit root pattern and structural break increases $J_T(m)$ value. Hence, weighting factor decreases test value sharply. If we reconsider figure 1 with indicator function:

Figure 3.2-Critical Values



When null hypothesis is rejected, $adj\_F(T_B{}^C)^* = \max F(T_B)\exp(J_T(m))$. $J_T(m)$ is left-scaled test in $adj\_F(T_B{}^C)$, and the test statistic is affected by break and unit root behavior by an increase RSS_y in (3.6). For stationary and no break alternative, $adj\_F(T_B{}^C)^*$ increases by the source of deviation from joint null as α decrease, $\max F(T_B)$ increases and $J_T(m)$ converges to zero rapidly. Under H_{A1}, test statistics is bounded above since $\alpha = 0$ is the case that deviation from the joint null is maximized. When data generation process is under H_{A2}, $adj\_F(T_B{}^C)^*$ decrease from the joint null value of $F(T_B)_{CV}$ by the source of deviation of stationarity and structural break. Under H_{A1} alternative test statistic has higher value than its null value; under H_{A2} alternative, test statistic has lower value than its null value. By utilizing the opposite directional value for the source of deviation from the null hypothesis, we can parametrically find a threshold critical value which discriminates H_{A1} and H_{A2}. Denoted as $CV1$ is the threshold critical value between H_{A1} and H_{A2}. Choice of $CV1$ was made on the basis of power since there does not appear an analytical method of maximizing power for infinite parametric space for breaks.

When data generation process shifts from the stationary and trend-break alternative (H_{A2}) to unit root and trend-break alternative (H_{A3}), the $adj\_F(T_B{}^C)^*$ statistic decreases further. Only source of deviation from the null hypothesis is the trend-break. Data generation process holds I(1) property. Hence, $J_T(m)$ value jumps to a higher positive constant due to both unit root property and structural break, under H_{A3} alternative test statistic has lowest values. $\max F(T_B)$ increases due to trend-break, but dominating increase in $J_T(m)$ due to both unit root and structural break decreases $adj\_F(T_B{}^C)^*$ further. Hence under H_{A3}, values of $adj\_F(T_B{}^C)^*$ is reduced more. Then, we can find another threshold value ($CV2$) to discriminate alternatives H_{A2} and H_{A3}. $CV2$ is determined on the same basis of $CV1$.

Limiting distributions of the statistic $adj\_F(T_B{}^C)^*$ under the null hypothesis is expressed as functional of standard Brownian motions by Vogelsang (1998). Reader is referenced for asymptotic results and the proofs of the functional convergences to the Vogelsang (1998). (Null hypothesis is expressed as $\mu_1 = \mu_3 = 0$ under $\alpha = 1$ in the second theorem of the paper by Vogelsang. So exogenously limiting distribution is derived under joint null so there is no change for asymptotical results. We left investigation for further research.)

Critical values of the $J_T(m)$ statistic are presented in Table 3.1 and critical values of $adj\_F(T_B{}^C)^*$ are presented in Table 3.2. Critical values are derived for 5000 replications of Monte Carlo simulations under the joint null hypothesis. $m=9$ is taken for maximum value. Lag length is taken zero for critical values. Critical values are derived for finite sample sizes are: $T=50,100,150,250$. $T=100$ critical value will be used for Monte Carlo simulations in chapter 4.

CHAPTER 4

MONTE CARLO SIMULATIONS

In this chapter, we present finite sample size and power simulations for $adj\_F(T_B^{\ C})^*$ statistics. In order to compare test statistics performance, we also include the test statistics that are presented in Sen (2003) and Vogelsang (1998). Simulations show that $adj\_F(T_B^{\ C})^*$ statistic has better power results. To stay consistent with literature, we follow the experimental data generation process that is suggested by Vogelsang and Perron (1998) and Sen (2003). Time series are assumed to be generated by the data:

$$[1-(\alpha+\rho)L+\rho L^2]y_t = (1+\psi L)[\mu_1 DU(T_B)+\mu_3 DT(T_B)+\varepsilon_t] \qquad (4.1)$$

Where $t = 1,...,T$. (4.1) is general enough to include errors following ARMA (1, 1) errors. In (4.1) ρ is AR (1) parameter and ψ is MA (1) component. As discussed before, the presentation of (4.1) is equivalent to IO model presented in Perron and Vogelsang (1992). For the size simulations joint null hypothesis

$\alpha=1,\ \mu_1=\mu_3=0$ is imposed into (4.1) then the data generation becomes $[\rho L+\rho L^2]y_t=(1+\psi L)\varepsilon_t$. This is the same as $A(L)\Delta y_t=B(L)\varepsilon_t$ in time series presented in (3.2). k^* is determined by the procedure explained above. However our simulations indicate that when k^*=0, results are identical. The sample sizes for simulations are set to be $T=50,100,150,250$ in order to analyze size distortions in detail. The following combinations of $\rho,\ \psi$ are used: $(\rho,\psi)=\{(0,0),(0.6,0),(-0.6,0),(0,0.5),(0,-0.5)\}$. Parameter values in the combinations are the same combinations that were in the literature. Under different trimming values λ_0 finite sample results are replicated. $\lambda_0\in\{0.15,0.10,0.5\}$. Only $\lambda_0=0.15$ is presented in tabulations. Trimming value determination does not affect general results. Other trimming value results are available up on request.

For the power simulations, the break date is specified to be in the middle of the data generation process. Initial value of the data generation process $y_0=0$. For three alternative hypotheses, we consider the combinations of the parameters $\alpha,\ \mu_1,\ \mu_3$ for the parameter space as: $\alpha=\{1,0.95,0.9,0.8,0.7\},\ \mu_1=\{0,1,2,4,6,8,10\}$, $\mu_3=\{0,0.1,0.2,0.3,04,0.5\}$. For the case that H_{A1} is the true data generation process, we simulate finite sample power for $\alpha=\{0.95,0.9,0.8,0.7\}$ and $\mu_1=\mu_3=0$. For the case that H_{A2} is true data generation process we simulate combinations of $\alpha=\{0.95,0.9,0.8,0.7\}$, $\mu_1=\{0,1,2,4,6,8,10\}$ and $\mu_3=\{0,0.1,0.2,0.3,04,0.5\}$ excluding the cases $\alpha=\{0.95,0.9,0.8,0.7\}$ with $\mu_1=\mu_3=0$. When H_{A3} is true data generation process, we consider combinations of the parameters $\alpha = 1$, $\mu_1 = \{0,1,2,4,6,8,10\}$ and $\mu_3 = \{0, 0.1, 0.2, 0.3, 04, 0.5\}$ excluding the case $\alpha = 1$, $\mu_1 = \mu_3 = 0$ which corresponds to the joint null hypothesis. Parameters in the simulations are selected from the early literature in order to confer the better comparison. For every case, 5.000 replications were generated with $N(0,1)$ random numbers which are generated by the same seed. Also first 300 generations are dropped from the sample.

4.1 SIMULATIONS IN GENERAL

Simulations are considered for null hypothesis and three alternatives. In the literature, Monte Carlo simulations are condemned to size and power simulations. Size simulation shows the rejection rate of the null hypothesis when true data generation process follow null model which corresponds to difference stationary model. Critical value for the size simulation is the %5 critical value under the null hypothesis. In the previous literature, power simulations only show the ability of the test statistic to reject null hypothesis under alternative data generation process. Data generation processes of H_{A1}, H_{A2} and H_{A3} are considered together. So, power results of the test statistics only indicate the power of the test statistic to reject wrong null hypothesis. There is no further information for the true structure of the time series.

In this research, we consider various alternative data generation processes in order to show the ability of our test statistic to reject wrong null hypothesis and ability of our test statistic to conclude true alternative data generation process which corresponds to different structural model. Then, our power simulations have three alternative hypotheses. We aim to visualize the rejection rate of the test statistic in favor of the three alternatives. Power of the test statistic is not only high rejection rate but also rejection of the null hypothesis in favor of the true alternative model in the data generation process. In the power simulation results tables, all three alternative hypotheses considered. Rejection rate of the test statistic in favor of the one specific alternative has been shown. For detailed parameter space, reader is referenced to the appendix section.

4.2 FINITE SAMPLE SIZE SIMULATIONS

Table 4.2 Finite Sample Size of $\max F(T_B)$ **and** $adj\_F(T_B{}^C)^*$

(ρ,ψ)	0,.0	.6,.0	-.6,.0	.0,.5	.0,-.5	.6,.5	.6,-.5	.6,.5
$\max F(T_B)$	0.05	0.069	0.051	0.086	0.412	*	*	*
$adj\_F(T_B)$	0.050	0.073	0.053	0.18	0.397	0.08	0.05	0.53

Table 4.2 shows that, when $(\rho,\psi)=(0,0)$, so that errors follow similar normal distribution, $adj\_F(T_B{}^C)^*$ does not suffer from size distortions. Sen (2003) has indicated that negative moving average component creates a size distortion. This size distortion even increased in the $adj\_F(T_B{}^C)^*$ statistic because $adj\_F(T_B{}^C)^*$ statistic has tendency to reject joint null hypothesis when true data

generation process deviates from the null. $adj\_F(T_B{}^C)^*$ statistic penalizes any change in trend function. When we consider more cases which have not been covered in the literature, we see that size distortion of negative component of moving average component depends on the autoregressive component.

Table 4.2.2 shows that, when sample size has been increased, $\max F(T_B)$ statistic over rejects the null hypothesis for ARMA (1, 1) errors. Tendency to reject null hypothesis is imported to the performance of the $adj\_F(T_B{}^C)^*$ statistics. The rejection mostly favors H_{A1}. Nature of the ARMA (1, 1) errors is beyond of the aim of this research so left the inspection for further research.

4.3 FINITE SAMPLE POWER SIMULATIONS WHEN H_{A1} IS TRUE DGP

Table 4.3.1 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$\mu_1 = 0, \mu_3 = 0$	$adj\_F(T_B{}^C)$			$\max F(T_B)$	$J_T(m)$
ρ ψ α	H_{A3}	H_{A2}	H_{A1}		
0 0 0.95	0,00	37,80	62,20	5,70	9,30
0 0 0.9	0,00	14,20	85,80	7,80	20,70
0 0 0.8	0,00	0,60	99,40	23,40	56,80
0 0 0.7	0,00	0,00	100,00	55,20	86,30
0 0 0.6	0,00	0,00	100,00	86,00	97,40

Power simulations are categorized in three alternatives. When data generation process is specified as H_{A1} which is stationary and no break alternative, power simulations are presented in Table 4.3.

Table 4.3.1 implicates the stationary and no break data generation processes. Null hypothesis of $J_T(m)$ statistics is unit root. $J_T(m)$ statitics is included in order to show the consistency of the Perron's (1989) argument for $J_T(m)$ statistic in power simulations. Table 4.3.1 shows that, $adj\_F(T_B{}^C)^*$ statistic concludes true alternative form with better power than the $\max F(T_B)$ statistic for every case. When true data generation process is stationary without break $adj\_F(T_B{}^C)^*$ statistic rejects the joint null hypothesis in favor of the true alternative H_{A1}. This situation holds even the case that; true data generation process is near unit root. (For $\alpha = 0.95$). For $(\rho,\psi) = (0.6,0)$, power of the test statistic is distorted. Both $adj\_F(T_B{}^C)^*$ and $\max F(T_B)$ statistics fait to reject null hypothesis though data generation process is stationary without break. When AR component of ARMA (1, 1) error is included into the data generation process, positive AR component increases spurious evidence for the null hypothesis. Hence, the test statistics fail to reject the joint null hypothesis. This situation is reversed for negative AR component. For $(\rho,\psi) = (-0.6,0)$, Table 4.2.3 shows that $adj\_F(T_B{}^C)^*$ statistic can reject false null in favor of the true alternative. Also $\max F(T_B)$ statistic does not suffer from power distortions. Effect of MA component is similar to the AR component. Tables present the similar results. In summary, when H_{A1} is true, $adj\_F(T_B{}^C)^*$ statistic has desired property such that it can conclude true alternative with high power.

4.4 FINITE SAMPLE POWER SIMULATIONS WHEN H_{A2} IS TRUE DGP

Table 4.4.11 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.8	0,00	99,90	0,10	14,70	100,00
0	0.2	0.8	0,00	100,00	0,00	23,10	100,00
0	0.3	0.8	0,00	100,00	0,00	44,20	100,00
0	0.4	0.8	7,20	92,80	0,00	76,40	100,00
1	0	0.8	0,00	6,30	93,70	15,60	100,00
1	0.1	0.8	0,00	100,00	0,00	20,70	100,00
1	0.2	0.8	0,00	100,00	0,00	36,10	100,00
1	0.3	0.8	0,10	99,90	0,00	60,50	100,00
1	0.4	0.8	4,20	95,80	0,00	85,80	100,00
2	0	0.8	0,00	37,20	62,8	36,50	100,00
2	0.1	0.8	0,00	99,90	0,10	43,00	100,00
2	0.2	0.8	0,00	100,00	0,00	66,90	100,00
2	0.3	0.8	0,00	100,00	0,00	85,80	100,00
2	0.4	0.8	1,30	98,70	0,00	92,30	100,00

Second family of the alternative is the H_{A2}. When data generation process designed to be stationary around broken trend function, it is observed that, the power of the $adj\_F(T_B{}^C)^*$ statistic has been affected by the level values of the shift parameters of μ_1 and μ_3- Especially when data generation process follows a near unit root pattern. This data generation process is not included in the literature. However we are suspicious to differentiate a unit root behavior from trend break. In order to do that, we set a small deviation from unit root behavior when break is presented in data generation process. Table 4.4.1 indicates that, when true data generation process follow near unit root pattern, $adj\_F(T_B{}^C)^*$ test statistic may conclude wrong alternative. Especially in the cases that, a high break in the slope of trend is presented. Slope break shifts the test statistic leftwards in figure 1, but near unit root pattern with highly persistent trend break increases the effect of this shift more than the indication of stationary break alternative. But rejection rates also show that even small mean break can be detected by $adj\_F(T_B{}^C)^*$ statistic where $\max F(T_B)$ statistic fail to reject null hypothesis.

However, when $\alpha = 0.9$ even for a high trend break $adj\_F(T_B{}^C)^*$ statistic concludes true alternative hypothesis which is presented in Table 4.4.2. But this situation is not general. For the case that slope break is high, mean break is low and α is close to 1, the test statistic favors H_{A3}. Power of the test statistic grows with the break values of μ_1 and μ_3. This is the desired property that a test statistic should hold. However, most of the test statistics in the literature is lack of this property. We consider low break values. If $adj\_F(T_B{}^C)^*$ performs good under these scenarios, $adj\_F(T_B{}^C)^*$ will have better power properties for higher values of break. For high α and small breaks, $\max F(T_B)$ statistic sometimes fails to reject false null hypothesis. However, in the same case, $adj\_F(T_B{}^C)^*$ has high rejection rate but only weakness that test statistic suffers is the conclusion of wrong alternative.

We investigate analysis in detailed context by searching various values of parameters. For $\alpha = 0.9$ high trend break dominates the test statistic and $adj\_F(T_B{}^C)^*$ favors unit root and trend break alternative. Table 4.4.2 shows that for $\alpha = 0.9$, $\mu_3 = 0.5$, $\mu_1 = \{0,1,2,3,4\}$ joint null is rejected in favor of the H_{A3}. Trend slope break plays the role of unit root behavior in the errors only if the mean break is dominated by slope break. This result is consistent with Sen (2003). It is observed that when α decreases to 0.8, regardless of the magnitude of the break $adj\_F(T_B{}^C)^*$ statistic has high power in true alternative. (Table 4.5.1) This shows that, only under high α value, $adj\_F(T_B{}^C)^*$ statistic has disadvantage of concluding wrong alternative.

Another implication arises from Table 4.4.2 is that, when small mean break is presented in the data generation process, $adj\_F(T_B{}^C)^*$ fails to reject no break pattern. In other words, small mean break is not detected by test statistic, so that, $adj\_F(T_B{}^C)^*$ rejects the null hypothesis in favor of the H_{A1}. Even in these small mean break cases, $\max F(T_B)$ fails to reject the joint null. Not only break is not detected but also stationary behavior of the time series is falsified. Also inability of the $adj\_F(T_B{}^C)^*$ statistic is limited. For the case that $\alpha = 0.9$, $\mu_1 = 1$, $\mu_3 = 0$

rejection rate is 53.9 percent. This rate decreases to 7.9 percent in $\max F(T_B)$ statistics. (Table 4.3.2) When value of mean break increases and $\alpha = 0.9, \mu_1 = 2, \mu_3 = 0$, rejection rate of $adj\_F(T_B{}^C)^*$ statistic increases to 92.6 percent whereas $\max F(T_B)$ has 47.3 percent rejection rate. $adj\_F(T_B{}^C)^*$ statistic loses power when slope break jumps to significant level such that $\alpha = 0.9, \mu_1 = 2, \mu_3 = 0.5$ and rejection rate decreases to 1 percent. But note that this case is specific to high α value. When α decreases to 0.8 so data generation process is $\alpha = 0.8, \mu_1 = 2, \mu_3 = 0.5$, rejection rate jumps to 86.2. (Table 4.5.1)

Limited power distortions in $adj\_F(T_B{}^C)^*$ statistic decreases with the presence of AR (1) errors. Table 4.4.3 includes simulations when AR component $\rho = 0.6$. If we compare Table 4.4.3 with previous Table 4.4.2, we see that wrong alternative conclusion rate has been decreased. But low mean break is not detected in the presence of AR component. The direction of the impact is reserved when AR component is negative. Next table (Table 4.4.4) reveals that, negative AR component has an impact on test statistic such that $adj\_F(T_B{}^C)^*$ rejects the null hypothesis in favor of H_{A3}. Spurious unit root pattern evidence is caused by negative AR component. Tables 4.4.5 and 4.4.6 show that MA component does not have significant effect on power properties of the test statistic.

Superiority of the $adj\_F(T_B{}^C)^*$ statistic is clearer in the presentations in Tables 4.4.7-11 where simulations have same parameters as in the work of Sen (2003). It is possible to compare power simulations with Sen (2003). Wrong alternative specifications follow the same pattern for previous case. However the distortions are decreased in number. Simulations are the same as in the case $(\rho,\psi)=(0,0)$, for positive AR component but negative AR component has effect on $adj\_F(T_B{}^C)$ statistic. For the data generation process: $\alpha=0.8,\ \mu_1=8,\ \mu_3=0$; when $\rho=0$, rejection rate is 98.6 for H_{A2} but 3.30 for same alternative. (Table 4.4.8) This is an evidence for the distortional effect of AR component. Simulations in Table 4.3.2 where $\alpha=0.9$ there is no power distortion. This result indicates that, source of the distortions in the statistic is dependent up on the parameter values of the data generation process. Then someone should be cautious to test for both structural break and unit root. Parameter space should be general enough to reach a reliable conclusion. Undetermined cases may involve contradicting results. This is another motivation for our research. We wanted to include all alternatives such that parameter effect up on the test statistic is clearer which is not wholly discovered in the literature. For example Sen (2003) found that $\max F(T_B)$ statistic has serious size distortions for negative MA component and erratic power results for positive MA component. However in his power simulations $\alpha=0.8$ is the only case. We reach the same erratic nature of negative MA component of errors when $\alpha=0.8$. But the erratic nature is removed when $\alpha=0.9$. So power results will highly depend up on parameter values. When alternative form of break is not general enough to capture all forms of break, reliability of the results will be brought to question.

4.5 FINITE SAMPLE POWER SIMULATIONS WHEN H_{A3} IS TRUE DGP

Table 4.5.1 Finite Sample Power of $adj\_F(T_B^{\ C})^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0)$			$adj\_F(T_B^{\ C})^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	1	92,5	7,50	0,0	88,70	5,00
0	0.2	1	100,	0,00	0,0	100,00	5,00
0	0.3	1	100,	0,00	0,0	100,00	5,00
0	0.4	1	100,	0,00	0,0	100,00	5,00
0	0.5	1	100,	0,00	0,0	100,00	5,00
1	0	1	0,00	100,00	0,0	16,60	5,00
1	0.1	1	99,6	0,40	0,0	90,40	5,00
1	0.2	1	100,	0,00	0,0	100,00	5,00
1	0.3	1	100,	0,00	0,0	100,00	5,00
1	0.4	1	100,	0,00	0,0	100,00	5,00
1	0.5	1	100,	0,00	0,0	100,00	5,00
2	0	1	6,90	93,10	0,0	71,10	5,00
2	0.1	1	99,9	0,10	0,0	100,00	5,00
2	0.2	1	100,	0,00	0,0	100,00	5,00
2	0.3	1	100,	0,00	0,0	100,00	5,00
2	0.4	1	100,	0,00	0,0	100,00	5,00
2	0.5	1	100,	0,00	0,0	100,00	5,00

Under data generation process that is unit root and structural break, power results are presented in Table 4.5.1-5. Results are very similar to the case where data generation process is stationary and break. (Tables 4.4.2-11) $adj\_F(T_B^{\ C})^*$ statistic can conclude true alternative except that data generation process is only mean break with unit root alternative. Note that this exception is disappear when value of mean break is higher than 6 or there is also slope trend break. In our unreported simulations, we consider $\max F(T_B)$ statistic. $\max F(T_B)$ has good power under this data generation process except there is low mean break. It is interesting to note that when $adj\_F(T_B^{\ C})^*$ concludes wrong alternative,

$\max F(T_B)$ has some power distortion. When magnitude of mean break is really low, $adj\_F(T_B{}^C)^*$ rejects the null hypothesis in favor of the stationary break alternative. Because mean break is not significant enough to deviate from the null hypothesis, $adj\_F(T_B{}^C)^*$ is partially lower than the null value so that it falls into the second rejection area in Figure 1. Those cases include $\alpha = 1,\ \mu_3 = 0,\ \mu_1 = \{0,1,2\}$. These few exceptions deviate from true alternative; even under different combinations of ARMA errors. However there is one case that $adj\_F(T_B{}^C)^*$ statistic fails to reject joint null hypothesis. When $\alpha = 1,\ \mu_1 = 2,\ \mu_3 = 0.1$, joint null is not rejected for any of the three alternatives. For all simulations, this is the only case that $adj\_F(T_B{}^C)^*$ statistic fails to reject.

CHAPTER 5

SUMMARY AND CONCLUSION

In this work, we try to build a test statistic that can conclude the true behavior of the time series. Testing for the true virtue of the time series has been the one important area in time series. Previous literature focus on the difference stationary versus trend break stationary models after Perron (1989) found that, spurious non-rejection of difference stationary model is a consequence of unaccounted trend break alternative. However trend stationary models that are covered in the previous literature specify different alternative forms. In order to diminish misspecification errors, general mean and slope of the trend break in trend function is specified which is suggested by Sen (2003). When alternative form does not include true form of break, the other forms are not able to detect the structural break so that test statistic may suffer from spurious results which makes the inferences arises from the test statistics unreliable. But then, alternative hypothesis is too general to conclude the true structure of the time series. Practical use of this specification is limited. We propose a test statistic which has joint null hypothesis so that null hypothesis is difference stationary model. However alternative form is determined according to the source of the deviation from the null hypothesis. Hence we were able to set up all possible alternative hypotheses which are not put together in the literature. So we have specific alternative hypotheses which correspond to different models for time series. This is the main motivation of the work that is presented. By imposing different alternatives to test statistic we can extend the alternative behavior to the difference stationary models. As an extension to the trend break stationary model, pure stationary model and trend break with unit root pattern behavior of the time series is also covered in our test statistic.

Misspecification of the alternative hypothesis brings the size and power distortions in test statistics. The test statistic proposed in this work has better power results compared to most general test statistic proposed in the literature. Also test statistic does not suffer from the misspecification of the alternative form of break since it is general enough to capture all forms of break in the alternative. Power results are quite evidence for the practical use of the test statistic. Our test has generally better power than the $\max F(T_B)$ statistic which is proposed by Sen (2003). Note that Sen showed that $\max F(T_B)$ statistic has better power properties from the previous test that are presented in the literature. Monte Carlo simulations highlight that only weakness of the $adj\_F(T_B{}^C)^*$ statistic is that, test statistic concludes wrong alternative when there is only small mean break with high lag dependency. In other cases dominance of the $adj\_F(T_B{}^C)^*$ statistic is presented in the Monte Carlo simulations. Even the low trend break is detected by $adj\_F(T_B{}^C)^*$ statistic.

Test statistic does not have only power dominance but also test has practical advantages. Alternatives can specify if there is only break or unit root with break. So that breaks date specification practically more reliable. However detailed analysis of the break date specification properties are beyond the scope of this research and left for further research. But since the source of deviation is known by using $adj\_F(T_B{}^C)^*$ statistic, one can reach more reliable break specification. Though we did not make an empirical application of the $adj\_F(T_B{}^C)^*$ statistic, it will be interesting to address an empirical problem. Inferences that arise from the test statistic may have interesting results. Any empirical application of $adj\_F(T_B{}^C)^*$ test is another idea for further research. $adj\_F(T_B{}^C)^*$ statistic is simulated by Monte Carlo replication. Asymptotic distribution is not derived since previous literature captures the asymptotic derivation of modification of this statistic which is not robust to I(1) property. Derivation under the joint null hypothesis will prove the invariance of the $adj\_F(T_B{}^C)^*$ statistic to nuisance parameter which is discussed in Vogelsang (1992).

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APPENDIX

TABLES

Table 3.1. $J_T(m)$ Statistic: Critical Values

Percentage	T=50	T=100	T=150	T=250	∞
1%	0.524	0.499	0.500	0.494	0.479
2.5%	0.732	0.701	0.682	0.668	0.684
5%	0.966	0.926	0.927	0.909	0.916
10%	1.306	1.269	1.250	1.229	1.275
20%	1.894	1.798	1.809	1.777	1.839
50%	3.836	3.556	3.577	3.499	3.585
80%	7.314	7.097	6.865	6.895	6.929
90%	10.064	9.712	9.745	9.282	9.525
95%	12.835	12.746	12.528	12.026	12.435
97.5%	15.938	15.535	15.674	15.294	15.497
99%	20.565	19.831	19.787	19.777	20.464

Table 3.2. $adj\_F(T_B{}^C)^*$ **Statistic: Critical Values**

Percentage	T=50	T=100	T=150	T=250	
1%	4.99E+00	4.73E+00	4.85E+00	4.32E+00	4.01E+00
2.5%	3.89E+00	3.58E+00	3.86E+00	3.59E+00	3.51E+00
5%	2.90E+00	2.84E+00	2.94E+00	2.84E+00	2.73E+00
10%	1.91E+00	1.94E+00	2.02E+00	2.00E+00	1.90E+00
20%	1.01E+00	1.05E+00	1.10E+00	1.14E+00	1.18E+00
50%	1.30E-01	1.53E-01	1.79E-01	1.89E-01	1.91E-01
80%	4.21E-03	4.69E-03	7.28E-03	6.82E-03	6.42E-03
90%	2.34E-04	3.01E-04	5.96E-04	5.16E-04	5.23E-04
95%	1.28E-05	1.95E-05	2.78E-05	3.51E-05	3.71E-05
97.5%	4.57E-07	1.28E-06	8.47E-07	1.50E-06	1.49E-06
99%	1.37E-09	2.36E-08	8.82E-09	2.89E-08	2.69E-08

4.2 Finite Sample Size Simulations

Table 4.2.2 Finite Sample Size of $\max F(T_B)$ when T=250

(ρ,ψ)	0.0,0.0	0.6,0.0	-0.6,0.0	(0.0,0.5)	.0,-.5	-0.6,0.5
$\max F(T_B)$	0.042	0.406	0.901	0.07	0.936	0.080

4.3 Finite Sample Power Simulations when H_{A1} is True DGP

Table 4.3.1 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$\mu_1 = 0, \mu_3 = 0$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
ρ	ψ	α	H_{A3}	H_{A2}	H_{A1}		
0	0	0.95	0,00	37,80	62,20	5,70	9,30
0	0	0.9	0,00	14,20	85,80	7,80	20,70
0	0	0.8	0,00	0,60	99,40	23,40	56,80
0	0	0.7	0,00	0,00	100,00	55,20	86,30
0	0	0.6	0,00	0,00	100,00	86,00	97,40

Table 4.3.2 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$\mu_1 = 0, \mu_3 = 0$	$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
ρ ψ α	H_{A3}	H_{A2}	H_{A1}		
0.6 0 0.95	0,00	39,10	60,90	15,10	100,00
0.6 0 0.9	4,90	0,00	0,00	4,90	100,00
0.6 0 0.8	0,90	0,00	0,00	0,90	100,00
0.6 0 0.7	0,40	0,00	0,00	0,40	100,00
0.6 0 0.6	1,20	0,00	0,00	1,20	100,00

Table 4.3.3 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$\mu_1 = 0, \mu_3 = 0$	$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
ρ ψ α	H_{A3}	H_{A2}	H_{A1}		
-0.6 0 0.95	0,00	10,90	89,10	91,40	100,00
-0.6 0 0.9	0,00	3,20	96,80	96,60	100,00
-0.6 0 0.8	0,00	0,00	100,00	99,70	100,00
-0.6 0 0.7	0,00	0,00	100,00	100,00	100,00
-0.6 0 0.6	0,00	0,00	100,00	100,00	100,00

Table 4.3.4 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$\mu_1 = 0, \mu_3 = 0$	$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
ρ ψ α	H_{A3}	H_{A2}	H_{A1}		
0 0.5 0.95	4,50	0,00	0,00	4,50	100,00
0 0.5 0.9	2,40	0,00	0,00	2,40	100,00
0 0.5 0.8	1,70	0,00	0,00	1,70	100,00
0 0.5 0.7	2,90	0,00	0,00	2,90	100,00
0 0.5 0.6	0,00	0,00	100,00	8,50	100,00

Table 4.3.5 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$\mu_1 = 0, \mu_3 = 0$	$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
ρ ψ α	H_{A3}	H_{A2}	H_{A1}		
0 -0.5 0.95	0,00	2,10	97,90	94,20	100,00
0 -0.5 0.9	0,00	0,10	99,90	98,30	100,00
0 -0.5 0.8	0,00	0,00	100,00	100,00	100,00
0 -0.5 0.7	0,00	0,00	100,00	100,00	100,00
0 -0.5 0.6	0,00	0,00	100,00	100,00	100,00

4.4 Finite Sample Power Simulations when H_{A2} is True DGP

Table 4.4.1 Finite Sample Power of $adj\_F(T_B^{\ C})^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0)$			$adj\_F(T_B^{\ C})^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.95	0,50	99,50	0,00	16,80	0,00
0	0.2	0.95	90,80	9,20	0,00	87,90	0,00
0	0.3	0.95	100,00	0,00	0,00	100,00	0,00
0	0.4	0.95	100,00	0,00	0,00	100,00	0,00
0	0.5	0.95	100,00	0,00	0,00	100,00	0,00
1	0	0.95	0,00	87,60	12,40	8,40	0,40
1	0.1	0.95	2,90	97,10	0,00	35,30	0,00
1	0.2	0.95	90,60	9,40	0,00	89,60	0,00
1	0.3	0.95	99,90	0,10	0,00	100,00	0,00
1	0.4	0.95	100,00	0,00	0,00	100,00	0,00
1	0.5	0.95	100,00	0,00	0,00	100,00	0,00
2	0	0.95	0,00	99,80	0,20	66,30	0,00
2	0.1	0.95	4,90	95,10	0,00	85,30	0,00
2	0.2	0.95	84,20	15,80	0,00	98,30	0,00
2	0.3	0.95	99,80	0,20	0,00	100,00	0,00
2	0.4	0.95	100,00	0,00	0,00	100,00	0,00
2	0.5	0.95	100,00	0,00	0,00	100,00	0,00
4	0.5	0.95	0,00	100,00	0,00	100,00	0,00
4	0.1	0.95	2,00	98,00	0,00	100,00	0,00
4	0.2	0.95	50,80	49,20	0,00	100,00	0,00
4	0.3	0.95	97,10	2,90	0,00	100,00	0,00
4	0.4	0.95	99,90	0,10	0,00	100,00	0,00
4	0.5	0.95	100,00	0,00	0,00	100,00	0,00
6	0	0.95	0,00	100,00	0,00	100,00	0,00
6	0.1	0.95	0,30	99,70	0,00	100,00	0,00
6	0.2	0.95	14,40	85,60	0,00	100,00	0,00
6	0.3	0.95	77,10	22,90	0,00	100,00	0,00
6	0.4	0.95	99,30	0,70	0,00	100,00	0,00
6	0.5	0.95	100,00	0,00	0,00	100,00	0,00
8	0	0.95	0,00	100,00	0,00	100,00	0,00
8	0.1	0.95	0,00	100,00	0,00	100,00	0,00
8	0.2	0.95	1,70	98,30	0,00	100,00	0,00
8	0.3	0.95	33,90	66,10	0,00	100,00	0,00
8	0.4	0.95	90,10	9,90	0,00	100,00	0,00
8	0.5	0.95	99,80	0,20	0,00	100,00	0,00

Table 4.4.2 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.9	0,00	100,00	0,00	8,80	0,00
0	0.2	0.9	5,20	94,80	0,00	39,30	0,00
0	0.3	0.9	76,50	23,50	0,00	84,70	0,00
0	0.4	0.9	98,40	1,60	0,00	94,90	0,00
0	0.5	0.9	99,90	0,10	0,00	100,00	0,00
1	0	0.9	0,00	53,90	46,10	7,90	3,40
1	0.1	0.9	0,00	100,00	0,00	19,70	0,00
1	0.2	0.9	5,40	94,60	0,00	58,60	0,00
1	0.3	0.9	69,00	31,00	0,00	83,10	0,00
1	0.4	0.9	96,30	3,70	0,00	93,90	0,00
1	0.5	0.9	99,80	0,20	0,00	100,00	0,00
2	0	0.9	0,00	92,60	7,40	47,30	0,00
2	0.1	0.9	0,00	100,00	0,00	65,10	0,00
2	0.2	0.9	3,30	96,70	0,00	91,10	0,00
2	0.3	0.9	48,40	51,60	0,00	99,10	0,00
2	0.4	0.9	89,00	11,00	0,00	100,00	0,00
2	0.5	0.9	99,00	1,00	0,00	100,00	0,00
4	0.5	0.9	0,00	99,90	0,10	100,00	0,00
4	0.1	0.9	0,00	100,00	0,00	100,00	0,00
4	0.2	0.9	0,20	99,80	0,00	100,00	0,00
4	0.3	0.9	7,60	92,40	0,00	100,00	0,00
4	0.4	0.9	43,10	56,90	0,00	100,00	0,00
4	0.5	0.9	83,30	16,70	0,00	100,00	0,00
6	0	0.9	0,00	100,00	0,00	100,00	0,00
6	0.1	0.9	0,00	100,00	0,00	100,00	0,00
6	0.2	0.9	0,00	100,00	0,00	100,00	0,00
6	0.3	0.9	0,30	99,70	0,00	100,00	0,00
6	0.4	0.9	4,10	95,90	0,00	100,00	0,00
6	0.5	0.9	27,80	72,20	0,00	100,00	0,00
8	0	0.9	0,00	100,00	0,00	100,00	0,00
8	0.1	0.9	0,00	100,00	0,00	100,00	0,00
8	0.2	0.9	0,00	100,00	0,00	100,00	0,00
8	0.3	0.9	0,00	100,00	0,00	100,00	0,00
8	0.4	0.9	0,10	99,90	0,00	100,00	0,00
8	0.5	0.9	1,60	98,40	0,00	100,00	0,00

Table 4.4.3 Finite Sample Power of $adj\_F(T_B^{\;C})^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0.6,0)$			$adj\_F(T_B^{\;C})^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.9	0,00	100,00	0,00	72,30	100,00
0	0.2	0.9	3,60	96,40	0,00	42,90	100,00
0	0.3	0.9	93,50	6,50	0,00	33,70	100,00
0	0.4	0.9	100,00	0,00	0,00	56,60	100,00
0	0.5	0.9	100,00	0,00	0,00	86,00	99,40
1	0	0.9	0,00	38,00	62,00	75,20	100,00
1	0.1	0.9	0,00	100,00	0,00	80,20	100,00
1	0.2	0.9	7,50	92,50	0,00	50,00	100,00
1	0.3	0.9	92,90	7,10	0,00	25,80	100,00
1	0.4	0.9	99,90	0,10	0,00	55,90	100,00
1	0.5	0.9	100,00	0,00	0,00	95,70	99,60
2	0	0.9	0,00	95,30	4,70	57,10	100,00
2	0.1	0.9	0,00	100,00	0,00	94,40	100,00
2	0.2	0.9	8,20	91,80	0,00	82,80	100,00
2	0.3	0.9	86,40	13,60	0,00	67,10	100,00
2	0.4	0.9	99,80	0,20	0,00	82,40	100,00
2	0.5	0.9	100,00	0,00	0,00	98,00	99,90
4	0.5	0.9	0,00	100,00	0,00	79,20	100,00
4	0.1	0.9	0,00	100,00	0,00	90,30	100,00
4	0.2	0.9	2,20	97,80	0,00	94,00	100,00
4	0.3	0.9	53,10	46,90	0,00	98,80	100,00
4	0.4	0.9	96,60	3,40	0,00	99,80	100,00
4	0.5	0.9	99,90	0,10	0,00	99,90	100,00
6	0	0.9	0,00	100,00	0,00	98,80	100,00
6	0.1	0.9	0,00	100,00	0,00	99,90	100,00
6	0.2	0.9	0,20	99,80	0,00	99,90	100,00
6	0.3	0.9	12,80	87,20	0,00	100,00	100,00
6	0.4	0.9	71,60	28,40	0,00	100,00	100,00
6	0.5	0.9	98,50	1,50	0,00	100,00	100,00
8	0	0.9	0,00	100,00	0,00	100,00	100,00
8	0.1	0.9	0,00	100,00	0,00	100,00	100,00
8	0.2	0.9	0,00	100,00	0,00	100,00	100,00
8	0.3	0.9	0,90	99,10	0,00	100,00	100,00
8	0.4	0.9	23,30	76,70	0,00	100,00	100,00
8	0.5	0.9	80,40	19,60	0,00	100,00	100,00

Table 4.4.4 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(-0.6,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.9	0,00	100,00	0,00	72,30	100,00
0	0.2	0.9	3,60	96,40	0,00	42,90	100,00
0	0.3	0.9	93,50	6,50	0,00	33,70	100,00
0	0.4	0.9	100,00	0,00	0,00	66,60	100,00
0	0.5	0.9	100,00	0,00	0,00	96,00	99,40
1	0	0.9	0,00	38,00	62,00	65,20	100,00
1	0.1	0.9	0,00	100,00	0,00	83,20	100,00
1	0.2	0.9	7,50	92,50	0,00	50,00	100,00
1	0.3	0.9	92,90	7,10	0,00	35,80	100,00
1	0.4	0.9	99,90	0,10	0,00	65,90	100,00
1	0.5	0.9	100,00	0,00	0,00	95,70	99,60
2	0	0.9	0,00	95,30	4,70	57,10	100,00
2	0.1	0.9	0,00	100,00	0,00	94,40	100,00
2	0.2	0.9	8,20	91,80	0,00	84,80	100,00
2	0.3	0.9	86,40	13,60	0,00	67,10	100,00
2	0.4	0.9	99,80	0,20	0,00	82,40	100,00
2	0.5	0.9	100,00	0,00	0,00	98,00	99,90
4	0.5	0.9	0,00	100,00	0,00	79,20	100,00
4	0.1	0.9	0,00	100,00	0,00	97,30	100,00
4	0.2	0.9	2,20	97,80	0,00	97,70	100,00
4	0.3	0.9	53,10	46,90	0,00	99,80	100,00
4	0.4	0.9	96,60	3,40	0,00	99,80	100,00
4	0.5	0.9	99,90	0,10	0,00	99,90	100,00
6	0	0.9	0,00	100,00	0,00	100,00	100,00
6	0.1	0.9	0,00	100,00	0,00	100,00	100,00
6	0.2	0.9	0,20	99,80	0,00	100,00	100,00
6	0.3	0.9	12,80	87,20	0,00	100,00	100,00
6	0.4	0.9	71,60	28,40	0,00	100,00	100,00
6	0.5	0.9	98,50	1,50	0,00	100,00	100,00
8	0	0.9	0,00	100,00	0,00	100,00	100,00
8	0.1	0.9	0,00	100,00	0,00	100,00	100,00
8	0.2	0.9	0,00	100,00	0,00	100,00	100,00
8	0.3	0.9	0,90	99,10	0,00	100,00	100,00
8	0.4	0.9	23,30	76,70	0,00	100,00	100,00
8	0.5	0.9	80,40	19,60	0,00	100,00	100,00

Table 4.4.5 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0.5)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.9	0,00	100,00	0,00	14,40	100,00
0	0.2	0.9	18,20	81,80	0,00	74,90	100,00
0	0.3	0.9	86,00	14,00	0,00	89,70	100,00
0	0.4	0.9	99,10	0,90	0,00	100,00	99,60
0	0.5	0.9	99,90	0,10	0,00	100,00	96,30
1	0	0.9	0,00	64,50	35,50	11,40	100,00
1	0.1	0.9	0,00	100,00	0,00	32,70	100,00
1	0.2	0.9	17,70	82,30	0,00	74,50	100,00
1	0.3	0.9	79,80	20,20	0,00	99,80	100,00
1	0.4	0.9	97,90	2,10	0,00	100,00	99,70
1	0.5	0.9	99,90	0,10	0,00	100,00	98,00
2	0	0.9	0,00	94,10	5,90	78,40	100,00
2	0.1	0.9	0,00	100,00	0,00	86,80	100,00
2	0.2	0.9	9,00	91,00	0,00	90,50	100,00
2	0.3	0.9	60,20	39,80	0,00	100,00	100,00
2	0.4	0.9	92,70	7,30	0,00	100,00	100,00
2	0.5	0.9	99,50	0,50	0,00	100,00	99,40
4	0.5	0.9	0,00	99,90	0,10	100,00	100,00
4	0.1	0.9	0,00	100,00	0,00	100,00	100,00
4	0.2	0.9	0,70	99,30	0,00	100,00	100,00
4	0.3	0.9	12,30	87,70	0,00	100,00	100,00
4	0.4	0.9	51,90	48,10	0,00	100,00	100,00
4	0.5	0.9	87,30	12,70	0,00	100,00	100,00
6	0	0.9	0,00	100,00	0,00	100,00	100,00
6	0.1	0.9	0,00	100,00	0,00	100,00	100,00
6	0.2	0.9	0,00	100,00	0,00	100,00	100,00
6	0.3	0.9	0,40	99,60	0,00	100,00	100,00
6	0.4	0.9	6,50	93,50	0,00	100,00	100,00
6	0.5	0.9	35,10	64,90	0,00	100,00	100,00
8	0	0.9	0,00	100,00	0,00	100,00	100,00
8	0.1	0.9	0,00	100,00	0,00	100,00	100,00
8	0.2	0.9	0,00	100,00	0,00	100,00	100,00
8	0.3	0.9	0,00	100,00	0,00	100,00	100,00
8	0.4	0.9	0,20	99,80	0,00	100,00	100,00
8	0.5	0.9	2,40	97,60	0,00	100,00	100,00

Table 4.4.6 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,-0.5)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.9	0,00	100,00	0,00	88,60	100,00
0	0.2	0.9	0,00	100,00	0,00	78,60	100,00
0	0.3	0.9	0,10	99,90	0,00	71,60	100,00
0	0.4	0.9	28,30	71,70	0,00	71,00	100,00
0	0.5	0.9	89,10	10,90	0,00	80,20	100,00
1	0	0.9	0,00	3,00	97,00	90,60	100,00
1	0.1	0.9	0,00	100,00	0,00	93,80	100,00
1	0.2	0.9	0,00	100,00	0,00	87,70	100,00
1	0.3	0.9	0,10	99,90	0,00	80,80	100,00
1	0.4	0.9	24,70	75,30	0,00	70,80	100,00
1	0.5	0.9	84,30	15,70	0,00	86,60	100,00
2	0	0.9	0,00	41,80	58,20	82,60	100,00
2	0.1	0.9	0,00	100,00	0,00	93,80	100,00
2	0.2	0.9	0,00	100,00	0,00	97,70	100,00
2	0.3	0.9	0,10	99,90	0,00	97,10	100,00
2	0.4	0.9	13,70	86,30	0,00	90,70	100,00
2	0.5	0.9	70,90	29,10	0,00	96,90	100,00
4	0.5	0.9	0,00	99,20	0,80	93,90	100,00
4	0.1	0.9	0,00	100,00	0,00	97,90	100,00
4	0.2	0.9	0,00	100,00	0,00	99,70	100,00
4	0.3	0.9	0,00	100,00	0,00	100,00	100,00
4	0.4	0.9	1,10	98,90	0,00	100,00	100,00
4	0.5	0.9	10,20	89,80	0,00	100,00	100,00
6	0	0.9	0,00	100,00	0,00	100,00	100,00
6	0.1	0.9	0,00	100,00	0,00	100,00	100,00
6	0.2	0.9	0,00	100,00	0,00	100,00	100,00
6	0.3	0.9	0,00	100,00	0,00	100,00	100,00
6	0.4	0.9	0,00	100,00	0,00	100,00	100,00
6	0.5	0.9	0,90	99,10	0,00	100,00	100,00
8	0	0.9	0,00	100,00	0,00	100,00	100,00
8	0.1	0.9	0,00	100,00	0,00	100,00	100,00
8	0.2	0.9	0,00	100,00	0,00	100,00	100,00
8	0.3	0.9	0,00	100,00	0,00	100,00	100,00
8	0.4	0.9	0,00	100,00	0,00	100,00	100,00
8	0.5	0.9	0,00	100,00	0,00	100,00	100,00

Table 4.4.7 Finite Sample Power of $adj\_F(T_B^{\ C})^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0)$			$adj\_F(T_B^{\ C})^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.8	0,00	99,90	0,10	14,70	100,00
0	0.2	0.8	0,00	100,00	0,00	23,10	100,00
0	0.3	0.8	0,00	100,00	0,00	44,20	100,00
0	0.4	0.8	7,20	92,80	0,00	76,40	100,00
0	0.5	0.8	41,70	58,30	0,00	94,60	100,00
1	0	0.8	0,00	6,30	93,70	15,60	100,00
1	0.1	0.8	0,00	100,00	0,00	20,70	100,00
1	0.2	0.8	0,00	100,00	0,00	36,10	100,00
1	0.3	0.8	0,10	99,90	0,00	60,50	100,00
1	0.4	0.8	4,20	95,80	0,00	85,80	100,00
1	0.5	0.8	30,20	69,80	0,00	98,90	100,00
2	0	0.8	0,00	37,20	62,80	36,50	100,00
2	0.1	0.8	0,00	99,90	0,10	43,00	100,00
2	0.2	0.8	0,00	100,00	0,00	66,90	100,00
2	0.3	0.8	0,00	100,00	0,00	85,80	100,00
2	0.4	0.8	1,30	98,70	0,00	92,30	100,00
2	0.5	0.8	13,80	86,20	0,00	94,80	100,00
4	0	0.8	0,00	84,40	15,60	96,50	100,00
4	0.1	0.8	0,00	99,90	0,10	98,60	100,00
4	0.2	0.8	0,00	100,00	0,00	99,80	100,00
4	0.3	0.8	0,00	100,00	0,00	99,90	100,00
4	0.4	0.8	0,00	100,00	0,00	100,00	100,00
4	0.5	0.8	0,50	99,50	0,00	100,00	100,00
6	0	0.8	0,00	96,20	3,80	100,00	100,00
6	0.1	0.8	0,00	99,90	0,10	100,00	100,00
6	0.2	0.8	0,00	100,00	0,00	100,00	100,00
6	0.3	0.8	0,00	100,00	0,00	100,00	100,00
6	0.4	0.8	0,00	100,00	0,00	100,00	100,00
6	0.5	0.8	0,00	100,00	0,00	100,00	100,00
8	0	0.8	0,00	98,60	1,40	100,00	100,00
8	0.1	0.8	0,00	100,00	0,00	100,00	100,00
8	0.2	0.8	0,00	100,00	0,00	100,00	100,00
8	0.3	0.8	0,00	100,00	0,00	100,00	100,00
8	0.4	0.8	0,00	100,00	0,00	100,00	100,00
8	0.5	0.8	0,00	100,00	0,00	100,00	100,00

Table 4.4.8 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0.6,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.8	0,80	0,00	0,00	0,80	100,00
0	0.2	0.8	2,50	0,00	0,00	2,50	100,00
0	0.3	0.8	0,00	100,00	0,00	14,10	100,00
0	0.4	0.8	0,00	100,00	0,00	46,90	100,00
0	0.5	0.8	0,20	99,80	0,00	83,20	100,00
1	0	0.8	0,90	0,00	0,00	0,90	100,00
1	0.1	0.8	2,10	0,00	0,00	2,10	100,00
1	0.2	0.8	0,00	100,00	0,00	7,50	100,00
1	0.3	0.8	0,00	100,00	0,00	26,80	100,00
1	0.4	0.8	0,00	100,00	0,00	62,90	100,00
1	0.5	0.8	0,10	99,90	0,00	89,90	100,00
2	0	0.8	0,00	2,10	97,90	11,70	100,00
2	0.1	0.8	0,00	61,30	38,70	19,50	100,00
2	0.2	0.8	0,00	100,00	0,00	36,10	100,00
2	0.3	0.8	0,00	100,00	0,00	62,40	100,00
2	0.4	0.8	0,00	100,00	0,00	85,40	100,00
2	0.5	0.8	0,00	100,00	0,00	96,80	100,00
4	0	0.8	0,00	7,20	92,80	94,70	100,00
4	0.1	0.8	0,00	41,30	58,70	94,20	100,00
4	0.2	0.8	0,00	98,20	1,80	97,00	100,00
4	0.3	0.8	0,00	100,00	0,00	98,90	100,00
4	0.4	0.8	0,00	100,00	0,00	99,80	100,00
4	0.5	0.8	0,00	100,00	0,00	99,90	100,00
6	0	0.8	0,00	6,10	93,90	100,00	100,00
6	0.1	0.8	0,00	23,80	76,20	100,00	100,00
6	0.2	0.8	0,00	85,40	14,60	100,00	100,00
6	0.3	0.8	0,00	100,00	0,00	100,00	100,00
6	0.4	0.8	0,00	100,00	0,00	100,00	100,00
6	0.5	0.8	0,00	100,00	0,00	100,00	100,00
8	0	0.8	0,00	3,30	96,70	100,00	100,00
8	0.1	0.8	0,00	10,40	89,60	100,00	100,00
8	0.2	0.8	0,00	58,90	41,10	100,00	100,00
8	0.3	0.8	0,00	99,00	1,00	100,00	100,00
8	0.4	0.8	0,00	100,00	0,00	100,00	100,00
8	0.5	0.8	0,00	100,00	0,00	100,00	100,00

Table 4.4.9 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(-0.6,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.8	0,80	0,00	0,00	0,80	100,00
0	0.2	0.8	2,50	0,00	0,00	2,50	100,00
0	0.3	0.8	0,00	100,00	0,00	14,10	100,00
0	0.4	0.8	0,00	100,00	0,00	46,90	100,00
0	0.5	0.8	0,20	99,80	0,00	83,20	100,00
1	0	0.8	0,90	0,00	0,00	0,90	100,00
1	0.1	0.8	2,10	0,00	0,00	2,10	100,00
1	0.2	0.8	0,00	100,00	0,00	7,50	100,00
1	0.3	0.8	0,00	100,00	0,00	26,80	100,00
1	0.4	0.8	0,00	100,00	0,00	62,90	100,00
1	0.5	0.8	0,10	99,90	0,00	89,90	100,00
2	0	0.8	0,00	2,10	97,90	11,70	100,00
2	0.1	0.8	0,00	61,30	38,70	19,50	100,00
2	0.2	0.8	0,00	100,00	0,00	36,10	100,00
2	0.3	0.8	0,00	100,00	0,00	62,40	100,00
2	0.4	0.8	0,00	100,00	0,00	85,40	100,00
2	0.5	0.8	0,00	100,00	0,00	96,80	100,00
4	0	0.8	0,00	7,20	92,80	91,70	100,00
4	0.1	0.8	0,00	41,30	58,70	94,20	100,00
4	0.2	0.8	0,00	98,20	1,80	97,00	100,00
4	0.3	0.8	0,00	100,00	0,00	98,90	100,00
4	0.4	0.8	0,00	100,00	0,00	99,80	100,00
4	0.5	0.8	0,00	100,00	0,00	99,90	100,00
6	0	0.8	0,00	6,10	93,90	100,00	100,00
6	0.1	0.8	0,00	23,80	76,20	100,00	100,00
6	0.2	0.8	0,00	85,40	14,60	100,00	100,00
6	0.3	0.8	0,00	100,00	0,00	100,00	100,00
6	0.4	0.8	0,00	100,00	0,00	100,00	100,00
6	0.5	0.8	0,00	100,00	0,00	100,00	100,00
8	0	0.8	0,00	3,30	96,70	100,00	100,00
8	0.1	0.8	0,00	10,40	89,60	100,00	100,00
8	0.2	0.8	0,00	58,90	41,10	100,00	100,00
8	0.3	0.8	0,00	99,00	1,00	100,00	100,00
8	0.4	0.8	0,00	100,00	0,00	100,00	100,00
8	0.5	0.8	0,00	100,00	0,00	100,00	100,00

Table 4.4.10 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0.5)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.8	2,50	0,00	0,00	2,50	100,00
0	0.2	0.8	0,00	100,00	0,00	18,30	100,00
0	0.3	0.8	1,60	98,40	0,00	65,80	100,00
0	0.4	0.8	24,90	75,10	0,00	96,60	100,00
0	0.5	0.8	62,30	37,70	0,00	99,90	100,00
1	0	0.8	4,10	0,00	0,00	4,10	100,00
1	0.1	0.8	0,00	100,00	0,00	7,80	100,00
1	0.2	0.8	0,00	100,00	0,00	34,60	100,00
1	0.3	0.8	1,00	99,00	0,00	79,30	100,00
1	0.4	0.8	16,50	83,50	0,00	98,50	100,00
1	0.5	0.8	49,00	51,00	0,00	99,90	100,00
2	0	0.8	0,00	51,60	48,40	43,70	100,00
2	0.1	0.8	0,00	100,00	0,00	53,80	100,00
2	0.2	0.8	0,00	100,00	0,00	78,20	100,00
2	0.3	0.8	0,30	99,70	0,00	96,20	100,00
2	0.4	0.8	6,70	93,30	0,00	99,80	100,00
2	0.5	0.8	26,90	73,10	0,00	100,00	100,00
4	0	0.8	0,00	86,10	13,90	99,90	100,00
4	0.1	0.8	0,00	99,90	0,10	100,00	100,00
4	0.2	0.8	0,00	100,00	0,00	100,00	100,00
4	0.3	0.8	0,00	100,00	0,00	100,00	100,00
4	0.4	0.8	0,10	99,90	0,00	100,00	100,00
4	0.5	0.8	1,90	98,10	0,00	100,00	100,00
6	0	0.8	0,00	95,80	4,20	100,00	100,00
6	0.1	0.8	0,00	99,90	0,10	100,00	100,00
6	0.2	0.8	0,00	100,00	0,00	100,00	100,00
6	0.3	0.8	0,00	100,00	0,00	100,00	100,00
6	0.4	0.8	0,00	100,00	0,00	100,00	100,00
6	0.5	0.8	0,00	100,00	0,00	100,00	100,00
8	0	0.8	0,00	98,40	1,60	100,00	100,00
8	0.1	0.8	0,00	100,00	0,00	100,00	100,00
8	0.2	0.8	0,00	100,00	0,00	100,00	100,00
8	0.3	0.8	0,00	100,00	0,00	100,00	100,00
8	0.4	0.8	0,00	100,00	0,00	100,00	100,00
8	0.5	0.8	0,00	100,00	0,00	100,00	100,00

Table 4.4.11 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,-0.5)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	0.8	0,00	38,50	61,50	99,60	100,00
0	0.2	0.8	0,00	100,00	0,00	99,40	100,00
0	0.3	0.8	0,00	100,00	0,00	99,40	100,00
0	0.4	0.8	0,00	100,00	0,00	99,40	100,00
0	0.5	0.8	0,00	100,00	0,00	99,60	100,00
1	0	0.8	0,00	0,00	100,00	99,80	100,00
1	0.1	0.8	0,00	50,60	49,40	99,70	100,00
1	0.2	0.8	0,00	100,00	0,00	99,70	100,00
1	0.3	0.8	0,00	100,00	0,00	99,70	100,00
1	0.4	0.8	0,00	100,00	0,00	99,70	100,00
1	0.5	0.8	0,00	100,00	0,00	99,70	100,00
2	0	0.8	0,00	0,10	99,90	99,70	100,00
2	0.1	0.8	0,00	68,20	31,80	99,80	100,00
2	0.2	0.8	0,00	100,00	0,00	99,90	100,00
2	0.3	0.8	0,00	100,00	0,00	99,90	100,00
2	0.4	0.8	0,00	100,00	0,00	99,90	100,00
2	0.5	0.8	0,00	100,00	0,00	99,90	100,00
4	0	0.8	0,00	16,20	83,80	99,90	100,00
4	0.1	0.8	0,00	90,70	9,30	99,90	100,00
4	0.2	0.8	0,00	100,00	0,00	100,00	100,00
4	0.3	0.8	0,00	100,00	0,00	100,00	100,00
4	0.4	0.8	0,00	100,00	0,00	100,00	100,00
4	0.5	0.8	0,00	100,00	0,00	100,00	100,00
6	0	0.8	0,00	59,40	40,60	100,00	100,00
6	0.1	0.8	0,00	97,70	2,30	100,00	100,00
6	0.2	0.8	0,00	100,00	0,00	100,00	100,00
6	0.3	0.8	0,00	100,00	0,00	100,00	100,00
6	0.4	0.8	0,00	100,00	0,00	100,00	100,00
6	0.5	0.8	0,00	100,00	0,00	100,00	100,00
8	0	0.8	0,00	83,60	16,40	100,00	100,00
8	0.1	0.8	0,00	99,00	1,00	100,00	100,00
8	0.2	0.8	0,00	100,00	0,00	100,00	100,00
8	0.3	0.8	0,00	100,00	0,00	100,00	100,00
8	0.4	0.8	0,00	100,00	0,00	100,00	100,00
8	0.5	0.8	0,00	100,00	0,00	100,00	100,00

4.5 Finite Sample Power Simulations when H_{A3} is True DGP

Table 4.5.1 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	1	92,50	7,50	0,00	88,70	5,00
0	0.2	1	100,00	0,00	0,00	100,00	5,00
0	0.3	1	100,00	0,00	0,00	100,00	5,00
0	0.4	1	100,00	0,00	0,00	100,00	5,00
0	0.5	1	100,00	0,00	0,00	100,00	5,00
1	0	1	0,00	100,00	0,00	16,60	5,00
1	0.1	1	99,60	0,40	0,00	90,40	5,00
1	0.2	1	100,00	0,00	0,00	100,00	5,00
1	0.3	1	100,00	0,00	0,00	100,00	5,00
1	0.4	1	100,00	0,00	0,00	100,00	5,00
1	0.5	1	100,00	0,00	0,00	100,00	5,00
2	0	1	6,90	93,10	0,00	71,10	5,00
2	0.1	1	99,90	0,10	0,00	100,00	5,00
2	0.2	1	100,00	0,00	0,00	100,00	5,00
2	0.3	1	100,00	0,00	0,00	100,00	5,00
2	0.4	1	100,00	0,00	0,00	100,00	5,00
2	0.5	1	100,00	0,00	0,00	100,00	5,00
4	0.5	1	55,90	44,10	0,00	100,00	5,00
4	0.1	1	100,00	0,00	0,00	100,00	5,00
4	0.2	1	100,00	0,00	0,00	100,00	5,00
4	0.3	1	100,00	0,00	0,00	100,00	5,00
4	0.4	1	100,00	0,00	0,00	100,00	5,00
4	0.5	1	100,00	0,00	0,00	100,00	5,00
6	0	1	81,80	18,20	0,00	100,00	5,00
6	0.1	1	100,00	0,00	0,00	100,00	5,00
6	0.2	1	100,00	0,00	0,00	100,00	5,00
6	0.3	1	100,00	0,00	0,00	100,00	5,00
6	0.4	1	100,00	0,00	0,00	100,00	5,00
6	0.5	1	100,00	0,00	0,00	100,00	5,00
8	0	1	93,60	6,40	0,00	100,00	5,00
8	0.1	1	100,00	0,00	0,00	100,00	5,00
8	0.2	1	100,00	0,00	0,00	100,00	5,00
8	0.3	1	100,00	0,00	0,00	100,00	5,00
8	0.4	1	100,00	0,00	0,00	100,00	5,00
8	0.5	1	100,00	0,00	0,00	100,00	5,00

Table 4.5.2 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0.6,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	1	99,20	0,80	0,00	100,00	5,00
0	0.2	1	100,00	0,00	0,00	100,00	5,00
0	0.3	1	100,00	0,00	0,00	100,00	5,00
0	0.4	1	100,00	0,00	0,00	100,00	5,00
0	0.5	1	100,00	0,00	0,00	100,00	5,00
1	0	1	4,70	95,30	0,00	69,70	5,00
1	0.1	1	100,00	0,00	0,00	100,00	5,00
1	0.2	1	100,00	0,00	0,00	100,00	5,00
1	0.3	1	100,00	0,00	0,00	100,00	5,00
1	0.4	1	100,00	0,00	0,00	100,00	5,00
1	0.5	1	100,00	0,00	0,00	100,00	5,00
2	0	1	47,10	52,90	0,00	90,70	5,00
2	0.1	1	100,00	0,00	0,00	100,00	5,00
2	0.2	1	100,00	0,00	0,00	100,00	5,00
2	0.3	1	100,00	0,00	0,00	100,00	5,00
2	0.4	1	100,00	0,00	0,00	100,00	5,00
2	0.5	1	100,00	0,00	0,00	100,00	5,00
4	0	1	88,60	11,40	0,00	100,00	5,00
4	0.1	1	100,00	0,00	0,00	100,00	5,00
4	0.2	1	100,00	0,00	0,00	100,00	5,00
4	0.3	1	100,00	0,00	0,00	100,00	5,00
4	0.4	1	100,00	0,00	0,00	100,00	5,00
4	0.5	1	100,00	0,00	0,00	100,00	5,00
6	0	1	98,30	1,70	0,00	100,00	5,00
6	0.1	1	100,00	0,00	0,00	100,00	5,00
6	0.2	1	100,00	0,00	0,00	100,00	5,00
6	0.3	1	100,00	0,00	0,00	100,00	5,00
6	0.4	1	100,00	0,00	0,00	100,00	5,00
6	0.5	1	100,00	0,00	0,00	100,00	5,00
8	0	1	99,80	0,20	0,00	100,00	5,00
8	0.1	1	100,00	0,00	0,00	100,00	5,00
8	0.2	1	100,00	0,00	0,00	100,00	5,00
8	0.3	1	100,00	0,00	0,00	100,00	5,00
8	0.4	1	100,00	0,00	0,00	100,00	5,00
8	0.5	1	100,00	0,00	0,00	100,00	5,00

Table 4.5.3 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(-0.6,0)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	1	41,90	58,10	0,00	7,00	5,00
0	0.2	1	100,00	0,00	0,00	92,20	5,00
0	0.3	1	100,00	0,00	0,00	100,00	5,00
0	0.4	1	100,00	0,00	0,00	100,00	5,00
0	0.5	1	100,00	0,00	0,00	100,00	5,00
1	0	1	0,00	99,70	0,30	70,30	5,00
1	0.1	1	4,00	0,00	0,00	4,00	5,00
1	0.2	1	100,00	0,00	0,00	93,40	5,00
1	0.3	1	100,00	0,00	0,00	100,00	5,00
1	0.4	1	100,00	0,00	0,00	100,00	5,00
1	0.5	1	100,00	0,00	0,00	100,00	5,00
2	0	1	0,00	100,00	0,00	80,70	5,00
2	0.1	1	99,30	0,70	0,00	12,50	5,00
2	0.2	1	100,00	0,00	0,00	96,80	5,00
2	0.3	1	100,00	0,00	0,00	100,00	5,00
2	0.4	1	100,00	0,00	0,00	100,00	5,00
2	0.5	1	100,00	0,00	0,00	100,00	5,00
4	0	1	25,80	74,20	0,00	99,70	5,00
4	0.1	1	99,90	0,10	0,00	90,20	5,00
4	0.2	1	100,00	0,00	0,00	100,00	5,00
4	0.3	1	100,00	0,00	0,00	100,00	5,00
4	0.4	1	100,00	0,00	0,00	100,00	5,00
4	0.5	1	100,00	0,00	0,00	100,00	5,00
6	0	1	64,60	35,40	0,00	100,00	5,00
6	0.1	1	100,00	0,00	0,00	100,00	5,00
6	0.2	1	100,00	0,00	0,00	100,00	5,00
6	0.3	1	100,00	0,00	0,00	100,00	5,00
6	0.4	1	100,00	0,00	0,00	100,00	5,00
6	0.5	1	100,00	0,00	0,00	100,00	5,00
8	0	1	85,90	14,10	0,00	100,00	5,00
8	0.1	1	100,00	0,00	0,00	100,00	5,00
8	0.2	1	100,00	0,00	0,00	100,00	5,00
8	0.3	1	100,00	0,00	0,00	100,00	5,00
8	0.4	1	100,00	0,00	0,00	100,00	5,00
8	0.5	1	100,00	0,00	0,00	100,00	5,00

Table 4.5.4 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,0.5)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	1	95,80	4,20	0,00	100,00	5,00
0	0.2	1	100,00	0,00	0,00	100,00	5,00
0	0.3	1	100,00	0,00	0,00	100,00	5,00
0	0.4	1	100,00	0,00	0,00	100,00	5,00
0	0.5	1	100,00	0,00	0,00	100,00	5,00
1	0	1	0,00	100,00	0,00	30,70	5,00
1	0.1	1	99,70	0,30	0,00	100,00	5,00
1	0.2	1	100,00	0,00	0,00	100,00	5,00
1	0.3	1	100,00	0,00	0,00	100,00	5,00
1	0.4	1	100,00	0,00	0,00	100,00	5,00
1	0.5	1	100,00	0,00	0,00	100,00	5,00
2	0	1	15,30	84,70	0,00	84,80	5,00
2	0.1	1	99,90	0,10	0,00	100,00	5,00
2	0.2	1	100,00	0,00	0,00	100,00	5,00
2	0.3	1	100,00	0,00	0,00	100,00	5,00
2	0.4	1	100,00	0,00	0,00	100,00	5,00
2	0.5	1	100,00	0,00	0,00	100,00	5,00
4	0	1	63,20	36,80	0,00	100,00	5,00
4	0.1	1	100,00	0,00	0,00	100,00	5,00
4	0.2	1	100,00	0,00	0,00	100,00	5,00
4	0.3	1	100,00	0,00	0,00	100,00	5,00
4	0.4	1	100,00	0,00	0,00	100,00	5,00
4	0.5	1	100,00	0,00	0,00	100,00	5,00
6	0	1	86,40	13,60	0,00	95,00	5,00
6	0.1	1	100,00	0,00	0,00	100,00	5,00
6	0.2	1	100,00	0,00	0,00	100,00	5,00
6	0.3	1	100,00	0,00	0,00	100,00	5,00
6	0.4	1	100,00	0,00	0,00	100,00	5,00
6	0.5	1	100,00	0,00	0,00	100,00	5,00
8	0	1	95,30	4,70	0,00	100,00	5,00
8	0.1	1	100,00	0,00	0,00	100,00	5,00
8	0.2	1	100,00	0,00	0,00	100,00	5,00
8	0.3	1	100,00	0,00	0,00	100,00	5,00
8	0.4	1	100,00	0,00	0,00	100,00	5,00
8	0.5	1	100,00	0,00	0,00	100,00	5,00

Table 4.5.5 Finite Sample Power of $adj\_F(T_B{}^C)^*$, $\max F(T_B)$, $J_T(m)$

$(\rho,\psi)=(0,-0.5)$			$adj\_F(T_B{}^C)^*$			$\max F(T_B)$	$J_T(m)$
μ_1	μ_3	α	H_{A3}	H_{A2}	H_{A1}		
0	0.1	1	3,20	0,00	0,00	3,20	5,00
0	0.2	1	100,00	0,00	0,00	96,30	5,00
0	0.3	1	100,00	0,00	0,00	100,00	5,00
0	0.4	1	100,00	0,00	0,00	100,00	5,00
0	0.5	1	100,00	0,00	0,00	100,00	5,00
1	0	1	0,00	99,30	0,70	69,00	5,00
1	0.1	1	0,70	0,00	0,00	0,70	5,00
1	0.2	1	100,00	0,00	0,00	90,70	5,00
1	0.3	1	100,00	0,00	0,00	100,00	5,00
1	0.4	1	100,00	0,00	0,00	100,00	5,00
1	0.5	1	100,00	0,00	0,00	100,00	5,00
2	0	1	0,00	100,00	0,00	80,10	5,00
2	0.1	1	3,10	0,00	0,00	3,10	5,00
2	0.2	1	100,00	0,00	0,00	94,80	5,00
2	0.3	1	100,00	0,00	0,00	100,00	5,00
2	0.4	1	100,00	0,00	0,00	100,00	5,00
2	0.5	1	100,00	0,00	0,00	100,00	5,00
4	0	1	4,70	95,30	0,00	93,60	5,00
4	0.1	1	99,50	0,50	0,00	70,40	5,00
4	0.2	1	100,00	0,00	0,00	100,00	5,00
4	0.3	1	100,00	0,00	0,00	100,00	5,00
4	0.4	1	100,00	0,00	0,00	100,00	5,00
4	0.5	1	100,00	0,00	0,00	100,00	5,00
6	0	1	30,50	69,50	0,00	100,00	5,00
6	0.1	1	99,90	0,10	0,00	100,00	5,00
6	0.2	1	100,00	0,00	0,00	100,00	5,00
6	0.3	1	100,00	0,00	0,00	100,00	5,00
6	0.4	1	100,00	0,00	0,00	100,00	5,00
6	0.5	1	100,00	0,00	0,00	100,00	5,00
8	0	1	54,90	45,10	0,00	100,00	5,00
8	0.1	1	100,00	0,00	0,00	100,00	5,00
8	0.2	1	100,00	0,00	0,00	100,00	5,00
8	0.3	1	100,00	0,00	0,00	100,00	5,00
8	0.4	1	100,00	0,00	0,00	100,00	5,00
8	0.5	1	100,00	0,00	0,00	100,00	5,00