NEW PRODUCT DIFFUSION IN CLOSED-LOOP SUPPLY CHAINS

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

NEW PRODUCT DIFFUSION IN CLOSED-LOOP SUPPLY CHAINS

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In this thesis, we develop a dynamic model for sales planning of a manufacturer who sells new and remanufactured versions of a product. Demand arrives over a finite life cycle according to the slightly modified Bass diffusion process. End-of-use product returns required for remanufacturing are constrained by the earlier sales. In this setting the manufacturer may simultaneously improve its economic and environmental performance by *partially* satisfying the initial demand. This can indeed occur when innovators contribute more heavily than imitators to the diffusion process, remanufacturing has a significantly large profit margin, or an unmet demand is very likely to be *backlogged* to be satisfied with a remanufactured product. But a very large backlogging rate may inflate the future demand if the initial sales volume is low, making it difficult to ensure a sufficient returns volume for remanufacturing. The manufacturer thus sells more under a very large backlogging rate, and a poorer environmental performance results. The optimal sales plans also differ across product types: The manufacturer of a *search* good has the advantage of keeping the future demand intact regardless of the initial sales, compared to the manufacturer of an *experience* good. Partially satisfying the demand can thus be desirable for search goods under a greater number of imitators, a lower margin from remanufacturing, or a lower backlogging rate. However, if partially satisfying the demand is desirable for both product types, the manufacturer of a search good sells more to enable a sufficient returns volume for the larger future demand.

Keywords: marketing-operations interface, new product diffusion, sales planning, closed-loop supply chains, remanufacturing.

ÖZET

KAPALI DEVRE TEDARİK ZİNCİRLERİNDE YENİ ÜRÜN YAYILIMI

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Endüstri Mühendisliği, Yüksek Lisans
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Bu tezde, bir ürünün yeni ve yeniden imal edilen halini satan bir üretici için dinamik bir satış modeli geliştirdik. Ürünün talebi sonlu bir ürün hayat döngüsü boyunca belli oranda değiştirilmiş Bass yayılım sürecine göre gerçekleşmektedir. Yeniden imalat için gerekli olan kullanımı tamamlanmış ürün miktarı erken dönemdeki satış ile kısıtlanmıştır. Bu modelde, üretici başlangıçtaki talebi *kısmen* karşılayarak ekonomik ve çevresel performansını aynı anda geliştirebilir. Bu durum yenilikçi müşteriler yayılım sürecine taklitçi müşterilerden daha fazla katkı yaptığında, yeniden imalatın kar payı çok yüksek olduğunda, veya karşılanmayan talep daha sonra yeniden imal edilen ürünle yüksek ihtimalle karşılanabildiğinde gerçekleşebilir. Ancak, çok yüksek birikmiş talep katsayısı eğer başlangıçtaki satış hacmi düşükse gelecekteki talebi şişirebilir ve yeterli miktarda kullanımı tamamlanmış ürünün geri dönüşünü zorlaştırabilir. Bu yüzden üretici yüksek birikmiş talep katsayısı altında daha fazla satış yapar ve bunun sonucunda çevresel performansını düşürür. En iyi satış planı ürünün tipine göre değişiklik göstermektedir: *araştırılan* ürünleri satan bir üretici, *deneyim* ürünlerini satan bir üreticiden farklı olarak, başlangıç satış miktarından bağımsız bir şekilde gelecek talebini koruyabilmektedir. Bu yüzden, araştırılan ürünlerin talebini kısmen karşılamak taklitçi müşteri sayısı yüksek olduğunda, yeniden imalatın kar payı düşük olduğunda ya da birikmiş talep katsayısı düşük olduğunda mümkündür. Ancak, eğer talebi kısmen karşılamak iki ürün tipi için de istenen durumsa, araştırılan ürünün üreticisi daha fazla satış yapar ve böylece gelecekteki talebi yeniden imalatla karşılamak için kullanımı tamamlanmış ürün miktarını yeterli seviyede tutar.

Anahtar sözcükler: pazarlama-işlemleri arayüzü, yeni ürün yayılımı, satış planlaması, kapalı devre tedarik zincirleri, yeniden imalat.

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To my beloved ones . . .

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Understanding consumer behaviour has always been a challenge for researchers in the area of management science. In a major advance, Bass (1969) has developed a behavioural rationale for the timing of initial purchase of new products. In the Bass diffusion model, initial purchases of the product are made by both "innovators" and "imitators." Innovators are not influenced in the timing of their initial purchase by the number of previous buyers. Imitators on the other hand are influenced by the number of previous buyers; imitators "learn" from those who have already bought the product. Innovators (or imitators) are thus likely to significantly contribute to the earlier (or later) stages of the adoption process. The likelihood of an initial purchase at any time, given that no purchase has yet been made, is a linear function of the number of previous buyers. Based on this assumption, Bass (1969) has formulated his famous diffusion dynamics over the product life cycle. See Bass (2004) for details.

Despite the popularity of the Bass diffusion model, very little is known about its application to sales planning in closed-loop supply chains. Unlike traditional supply chains, closed-loop supply chains collect, and recover value from, the end-of-use products. Remanufacturing and recycling are the two major disposition decisions for used products (Souza 2012). Remanufacturing replaces or reprocesses components of the used product to bring it to like-new condition (Atasu et al. 2008). Closed-loop supply chains have received much attention in the literature over the last two decades; we refer the reader to van der Laan et al. (2004), Guide and Wassenhove (2009), Ferguson and Souza (2010), Akçalı and Çetinkaya (2011), Hassini et al. (2012), Souza (2013), and Govindan et al. (2015) for comprehensive discussions. In this paper, we study the sales planning problem of a manufacturer who sells a product over a finite life cycle and is able to remanufacture any end-of-use product. Demand arrives over the product life cycle according to the slightly modified Bass diffusion process. In this setting, we investigate the effects of product, diffusion, consumer, and operational characteristics on optimal sales, manufacturing, and remanufacturing volumes.

Specifically, the manufacturer offers the product over two periods: Demand in period 1 can only be fulfilled with newly-manufactured (i.e., new) products. Demand in period 1 may also be rejected. A certain fraction of the consumers whose demands have been rejected in period 1 is willing to wait for adoption of the product in period 2 (the patient segment). A certain fraction of the consumers whose demands have been satisfied in period 1 is willing to return their used products to the manufacturer in period 2 (the green-minded segment). But no consumer in the market purchases more than once. The manufacturer is able to remanufacture a used product in period 2. A certain fraction of the consumers in period 2 switches from buying the new product to buying the remanufactured product if available (the functionality-oriented segment). Because remanufacturing often reduces the need for new materials and energy consumption for manufacturing (see Atasu et al., 2010, and Guide and Li, 2010), we assume that the manufacturer earns more by satisfying a demand with the remanufactured item rather than the new item. The manufacturer aims to maximize its total profit in periods 1 and 2.

Demand in period 2 depends on the sales volume in period 1 as in the Bass diffusion model modified to allow for rejection of some demand in period 1. We consider two modified versions of the Bass diffusion model for two product types: The first model assumes the product is a search good (see Nelson 1970); its value can be evaluated before the purchase. Customers who demand the product but are unable to purchase it can still generate the word-of-mouth effect, i.e., demand-based diffusion. The second model assumes the product is an experience good (see Nelson 1970); its value can only be evaluated after consumption. Customers who demand the product but are unable to purchase it cannot generate the word-of-mouth effect, i.e., sales-based diffusion. (See Ho et al., 2002, Kumar and Swaminathan, 2003, and Shen et al., 2011, for the latter extension of the Bass diffusion model. When all demand is satisfied in period 1, both of our extensions reduce to the two-period Bass diffusion model.) Our two-period setting effectively captures the word-of-mouth effect for product adoptions, enabling us to offer new insights into the optimal sales plans in different environments.

In both of our diffusion models, the manufacturer may want to reject some demand in period 1 in order to satisfy a portion of the unmet demand with remanufactured products in period 2. This can indeed occur under certain circumstances:

- The manufacturer rejects some demand at optimality if innovators contribute more heavily than imitators to the diffusion process: Even when some demand is rejected in period 1, the sales volume in period 1 can still be *high* (due to the large number of innovators), enabling a sufficiently large returns volume for fulfillment of the *low* demand in period 2 (due to the small number of imitators) with remanufactured products.
- The manufacturer also rejects some demand at optimality when most of the consumers who were unable to purchase the product in period 1 due to the firm's sales planning still want to buy it in period 2 (i.e., the patient segment is large), and most of the consumers who could purchase the product in period 1 return their used products in period 2 (i.e., the green-minded segment is large). When these two conditions hold, an unmet demand in period 1 is likely to be *backlogged* to be satisfied in period 2 through remanufacturing for which *ample* supply exists, making it profitable to reject some demand.
- Nevertheless, a *small* patient segment (or a *small* green-minded segment)

can still allow for optimality of partial demand fulfilment under a *large* (or *small*) functionality-oriented segment.

- Although the patient segment should exist in the market for optimality of partial demand fulfilment, the existence of a very large patient segment may lead to a *smaller* (non-zero) amount of unmet demand in period 1. This is because if the sales volume in period 1 is low, a *very large* patient segment inflates the demand in period 2, making it difficult to ensure a sufficient returns volume (bounded by the sales volume in period 1) for fulfilment of all demand in period 2 through remanufacturing.

The total energy consumption (over the product life cycles) is a common metric used for environmental assessment of remanufacturing in both academia and practice. See, for instance, Quariguasi-Frota-Neto and Bloemhof (2012), Gutowski et al. (2011), and Ovchinnikov et al. (2014). We also use the total energy consumption as our measure of environmental impact. Because remanufacturing often significantly reduces energy consumption in production, and our new and remanufactured items build upon the same product generation so that the initial use and secondary use stages have similar demands for energy (see Atasu et al., 2010, Quariguasi-Frota-Neto and Bloemhof, 2012, and Ovchinnikov et al., 2014), we assume that a remanufactured item is environmentally better than a new item. Therefore, as the manufacturer rejects more demand at optimality, it improves its environmental performance: The manufacturer rejects a demand in period 1 (potentially reducing the future demand due to diffusion) only if it is able to satisfy an extra possible backlogged demand with a remanufactured item in period 2. And a (possible) remanufactured item in period 2 is environmentally better than a (certain) new item in period 1.

If some demand is rejected at optimality in both diffusion models, demand-based diffusion leads to a worse environmental performance than sales-based diffusion. This is because the diffusion demand in period 2 is larger under demand-based diffusion, and thus the manufacturer under demand-based diffusion should reject less in period 1 in order to have a sufficient returns volume for the larger demand in period 2. But there are also cases in which some demand is rejected *only* under demand-based diffusion, leading to a better environmental performance under demand-based diffusion. Such cases occur because the firm under demand-based diffusion has the advantage of keeping the future diffusion demand intact regardless of the initial sales volume, making it less costly to reject some demand. One such case is when a large number of imitators exist in the market, i.e., any unmet demand greatly reduces the future diffusion rate under sales-based diffusion.

For both diffusion models, we have also examined the optimal sales plans when there is uncertainty in the number of consumers who create the word-of-mouth effect on the customers who have not yet demanded the product (i.e., probabilistic diffusion characteristics), and when there is uncertainty in the numbers of patient and green-minded consumers (i.e., probabilistic consumer behaviour). We have found that our insights continue to hold in both of these extensions. We have also found that the manufacturer tends to reject *less* demand in the latter extension than in our base model, demonstrating a *poorer* environmental performance.

Because the total diffusion demand is larger under demand-based diffusion than under sales-based diffusion when some demand is rejected in period 1, demand-based diffusion always leads to a better economic performance than sales-based diffusion. In both diffusion models, some demand is rejected in period 1, and the remanufacturing volume is increased, if the profit margin from remanufacturing is above a critical level that varies depending on the consumer and diffusion characteristics of the market: Subsidies for remanufacturing would be more attractive for firms with very cost-efficient remanufacturing technologies and/or high-value used products. From an environmental perspective, however, subsidies would only be effective if they allow the profit margin from remanufacturing to exceed the critical level. The profits tend to grow more with the number of innovators in the market than with the number of imitators, because innovators inflate the demand in *both* periods. The profits also grow with the magnitude of each of the patient, green-minded, and functionality-oriented segments.

Our work is related with the literature on the new product diffusion models applied to sales planning. In this literature, Ho et al. (2002), Kumar and

Swaminathan (2003), and Shen et al. (2011) study the sales planning problem of a manufacturer who aims to maximize the total profit over a product life cycle under supply constraints. Satisfying all current demand upon introduction of a new product may lead the future demand to grow rapidly (thanks to the positive word-of-mouth effect). These papers evaluate the use of two strategies as a remedy to the fast-growing demand that exceeds the available capacity: The firm can delay the launch time of the product in order to build up inventory until the launch time. Alternately, it can launch the product immediately and then reject some of the arriving demands, mitigating the word-of-mouth effect. In the latter strategy, a certain fraction of the unmet demand is backlogged and the remaining fraction is lost. Ho et al. (2002) show the latter strategy cannot be optimal in their setting, whereas Kumar and Swaminathan (2003) show it can be. Shen et al. (2011) compare the results in Ho et al. (2002) and Kumar and Swaminathan (2003), providing a counterexample to Ho et al. (2002), which supports the findings of Kumar and Swaminathan (2003). We focus on the latter strategy in this paper. Unlike the above papers, an ample supply exists for manufacturing in both periods and remanufacturing is possible in period 2: It may be desirable to reject some demand in period 1 so as to exploit the benefit of remanufacturing in fulfilment of the backlogged demand in period 2.

Our work is also related with the literature on closed-loop supply chain management with remanufacturing option. Inspired by the Bass diffusion model, Geyer et al. (2007) model the market demand over the product life cycle as following an isosceles trapezoid. In their setting, all demand is satisfied over the life cycle, and a certain fraction of the sold items returns to the manufacturer after a fixed market sojourn time. A remanufactured product is a perfect substitute for the new product. They investigate the profitability of remanufacturing when the end-of-use returns are remanufactured as long as there is a market demand. Georgiadis et al. (2006) numerically analyze the effects of the product life cycle pattern and average product usage time on capacity planning for collection and remanufacturing. Georgiadis and Athanasiou (2010) extend the model in Georgiadis et al. (2006) by allowing for two sequential product types. They study the impact of two-product joint life cycle on capacity planning in two cases: (i) the sequential products are identical and (ii) the market shows preference between the products. Unlike these papers, we explicitly incorporate the Bass diffusion process into our demand formulation, investigating the effects of product, diffusion, consumer, and operational characteristics on sales planning. Wang et al. (2017) consider a setting in which the demand arrives according to the Bass diffusion process and a certain fraction of the sold items returns to the manufacturer after a fixed market sojourn time. They characterize the optimal component reuse volume and acquisition costs. Unlike Wang et al. (2017), we study the sales planning problem in a two-period setting with possible backlogging.

Debo et al. (2006) examine the joint pricing of new and remanufactured products in an infinite-horizon setting with variable market sojourn time, imperfect substitution between new and remanufactured products, and supply constraints. They extend the generalized (price-dependent) Bass diffusion model (see Bass et al. 1994) by allowing for *repeat* purchases and modelling the coefficient of imitation as a function of the *installed base of new products*. They characterize the diffusion paths of new and remanufactured products, analyzing the impacts of remanufacturability level, capacity structure, and reverse channel speed on profitability. Akan et al. (2013) consider a manufacturer with ample manufacturing capacity who sells the new and remanufactured versions of a product over a finite life cycle. A remanufactured product is an imperfect substitute for the new product; demands arrive as a price-dependent diffusion process. They characterize the optimal pricing, production, and inventory policies of the manufacturer, establishing that partially satisfying demand for the remanufactured item is never optimal. Robotis et al. (2012) consider a manufacturer with a constrained production and service capacity who offers a leasing contract to consumers and remanufactures products. A remanufactured product is a perfect substitute for the new product. Demand arrives as a diffusion process that is controlled by the manufacturer through the leasing price and duration. They characterize the optimal pricing strategy of the manufacturer, investigating the effects of the remanufacturing option on the leasing price and duration. We depart from these papers by focusing on the traditional Bass diffusion model (1969) in a two-period setting with possible backlogging. We also postulate two different diffusion processes for two different product types, comparing the optimal sales plans in these two processes.

Although the new product diffusion models and closed-loop supply chains have received much attention in the literature, the impacts of product types (search vs. experience goods), diffusion characteristics (innovation and imitation coefficients), consumer characteristics (magnitudes of patient, green-minded, and functionality-oriented segments), and operational characteristics (profit margins from manufacturing and remanufacturing) on sales planning have not been dealt with in depth. Our research aims to fill this gap in the literature. The rest of this study is organized as follows: *Chapter 2* describes our diffusion models. *Chapter 3* establishes our analytical results. *Chapter 4* presents our numerical results. *Chapter 5* offers a summary and concludes. All proofs are contained in the appendix.

Chapter 2

The Diffusion Models

We consider a single manufacturer that offers a new product with a finite (two-period) life cycle. Demand evolves over time according to the slightly modified Bass diffusion model. In the original Bass diffusion model a population of consumers of size m gradually purchases the product. The rate at which consumers buy the product is determined by the fraction of innovators existing in the population and the word-of-mouth (or diffusion) effect that is a function of the number of previous purchases. *Innovators* buy the product independently of the other consumers' actions. *Imitators* are influenced in their timing of purchase by the other consumers' actions. In a discrete-time framework, given that the demand is fully satisfied by the manufacturer in every period, the sales volume in period T is

$$S_T = pm + (q-p)\sum_{t=1}^{T-1} S_t - \frac{q}{m}\left(\sum_{t=1}^{T-1} S_t\right)^2$$

where p is the fraction of innovators (coefficient of innovation) and q is a measure of the diffusion effect (coefficient of imitation). See Bass (1969) and Bass (2004) for details.

In this study, without loss of generality, we assume that the market consists of a continuum of consumers with total mass normalized to one, i.e., $m = 1$. Each customer purchases at most one unit of the product during the two-period selling

Table 2.1: Summary of notation.

Parameters	
p	Coefficient of innovation, $p \in [0,1]$.
q	Coefficient of imitation, $q \in [0,1]$.
α	Fraction of unmet demands in period 1 that are satisfied in period 2, $\alpha \in [0,1]$.
β	Fraction of items sold in period 1 that are returned in period 2, $\beta \in [0,1]$.
γ	Fraction of consumers in period 2 who buy remanufactured items if available, and new items otherwise, $\gamma \in [0,1]$.
c_n	Unit manufacturing cost.
c_r	Unit remanufacturing cost.
r_n	Unit selling price of the new item, $r_n > c_n > 0$.
r_r	Unit selling price of the remanufactured item, $r_r > c_r > 0$.
Δ	Average profit margin ratio of manufacturing to remanufacturing, i.e., $\Delta = \frac{r_n - c_n}{(r_r - c_r)\gamma + (r_n - c_n)(1-\gamma)}$.
Decision Variables	
S_d	The sales volume in period 1 under demand-based diffusion.
S_s	The sales volume in period 1 under sales-based diffusion.
R_d	The remanufacturing volume in period 2 under demand-based diffusion.
R_s	The remanufacturing volume in period 2 under sales-based diffusion.
Q_d	The total manufacturing volume in periods 1–2 under demand-based diffusion.
Q_s	The total manufacturing volume in periods 1–2 under sales-based diffusion.

season. Demand in period 1 of the product life cycle, D_1, equals the estimated demand in period 1 according to the Bass diffusion model, i.e., $D_1 = p$. The manufacturer may reject a fraction of the demand in period 1. We denote by S the sales volume in period 1. A fraction α of the unmet demand is backlogged to be satisfied in period 2. The remaining fraction of the unmet demand is lost. This assumption has also been made in the sales planning literature (see, for instance, Ho et al., 2002, Kumar and Swaminathan, 2003, and Shen et al., 2011). Demand in period 2 of the product life cycle consists of two parts: the backlogged demand and the diffusion demand.

Our calculation of the diffusion demand in period 2 differs depending on the product type: If the product is a *search* good (see Nelson 1970), the diffusion demand in period 2 is based on the observed demand in period 1 (demand-based diffusion, Chapter 3.1). The diffusion demand in this case is $(p+qp)(1-p)$. The overall demand in period 2 thus becomes $D_{2,d}=(p+qp)(1-p)+\alpha(p-S)$. If the product is an *experience* good (again, see Nelson 1970), the diffusion demand in period 2 is based on the actual sales in period 1 (sales-based diffusion, Chapter 3.2). The diffusion demand in this case is $(p+qS)(1-p)$ where $S \leq D_1$. The overall demand in period 2 thus becomes $D_{2,s}=(p+qS)(1-p)+\alpha(p-S)$. (Since $m=1$, the maximum possible total diffusion demand cannot exceed one for both product types.) Because $S=D_1$ in the Bass diffusion model and $S \leq D_1$ in our model, we have formulated $D_{2,d}$ and $D_{2,s}$ by slightly modifying the Bass diffusion model. The latter formulation has also been proposed in the literature (again see, for instance, Ho et al., 2002, Kumar and Swaminathan, 2003, and Shen et al., 2011). When $S=D_1=p$, both $D_{2,d}$ and $D_{2,s}$ are identical to the estimated demand in period 2 according to the Bass diffusion model.

The demand in period 1 can only be fulfilled with new products. We define c_n as the unit manufacturing cost and r_n as the unit selling price of the new product. However, the demand in period 2 can be fulfilled with not only new products but also remanufactured products (to a limited extent). A fraction β of the used products that have been sold in period 1 is returned by the consumers to the manufacturer, and becomes available for remanufacturing and resale in period 2. This assumption has also been made in the closed-loop supply chain literature (see, for instance, Ovchinnikov et al., 2014, Abbey et al., 2015a, and Abbey et al., 2017). We define c_r as the unit remanufacturing cost and r_r as the unit selling price of the remanufactured product. A fraction γ of the consumers in period 2 buys remanufactured products if available, and new products otherwise. And the remaining fraction buys new products even if remanufactured products are available. Based on the breakdown of the market into these two segments, we calculate the *average* profit margin ratio of manufacturing to remanufacturing: $\Delta = \frac{r_n-c_n}{(r_r-c_r)\gamma+(r_n-c_n)(1-\gamma)}$. This notation facilitates our analysis in Chapters 3–4. Table 2.1 exhibits the notation we use throughout the paper.

To eliminate the trivial case in which it would be optimal to satisfy all demand in period 1, we assume $r_r-c_r > r_n-c_n$: The manufacturer is better off satisfying a demand with a remanufactured product (if available). (If $r_r - c_r \leq r_n - c_n$, there is no positive economic return from offering remanufactured items, and thus there is no incentive to delay fulfillment of some demand in period 1.) Experimental studies show that a positive fraction of the market only buys new products (the newness-conscious segment), while another positive fraction of the market displays indifference between new and remanufactured products (the functionality-oriented segment). The functionally-oriented segment is highly price sensitive and prefers remanufactured products at a discounted price. See Atasu et al. (2010), Guide and Li (2010), Ovchinnikov (2011), Ovchinnikov et al. (2014), Abbey et al. (2015a), and Abbey et al. (2015b). When $r_r < r_n$ in our model, the consumers might be grouped into these two distinct segments: The fraction γ of the consumers forms the functionality-oriented segment while the remaining fraction forms the newness-conscious segment. (Experimental studies also show that sellers with poor reputation should provide significant discounts to increase the attractiveness of their remanufactured products; see Subramanian and Subramanyam, 2012. And high brand equity may discourage purchases of the remanufactured products; see Abbey et al., 2015c). When $r_r = r_n$ and $\gamma = 1$ in our model, the remanufactured product might be considered a perfect substitute for the new product. Single use cameras and refillable cylinders are examples of perfect substitution (Atasu at al. 2010).

Chapter 3

Analytical Results

Chapters 3.1–3.2 establish our analytical results for our demand-based and sales-based diffusion models, respectively.

3.1 Demand-Based Diffusion

Let S_d denote the sales volume in period 1 in our demand-based diffusion model. Recall that $D_1 = p$ and $D_{2,d} = p + pq - p^2 - p^2q + \alpha(p - S_d)$. Our normalization of the total diffusion demand implies that $2p + pq - p^2 - p^2q \leq 1$. The returns volume that becomes available for remanufacturing and resale at the beginning of period 2 is βS_d. The demand that is satisfied with remanufactured items is $\min\{\beta S_d, \gamma D_{2,d}\}$. Thus the manufacturer's profit in period 2 as a function of $S_d \in [0, p]$ is

$$\pi_{2,d}(S_d) = \begin{cases} (r_r - c_r)\gamma D_{2,d} + (r_n - c_n)(1-\gamma)D_{2,d} & \text{if } \beta S_d \geq \gamma D_{2,d}, \\ (r_r - c_r)\beta S_d + (r_n - c_n)(D_{2,d} - \beta S_d) & \text{otherwise.} \end{cases}$$

The total profit in periods 1–2 as a function of $S_d \in [0, p]$ is $\pi_{1,d}(S_d) = (r_n - c_n)S_d + \pi_{2,d}(S_d)$. Proposition 3.1 characterizes the optimal sales volume (and the corresponding remanufacturing and manufacturing volumes) and the optimal total profit in our demand-based diffusion model.

Proposition 3.1. *Under demand-based diffusion:*

(a) The optimal sales volume in period 1 is

$$S_d^* = \begin{cases} \frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}} & \text{if } \alpha \geq \Delta \text{ and } \frac{\beta}{\gamma} \geq 1+q-p-pq, \\ p & \text{otherwise.} \end{cases}$$

(b) The optimal remanufacturing volume in period 2 is

$$R_d^* = \begin{cases} \beta\left(\frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}}\right) & \text{if } \alpha \geq \Delta \text{ and } \frac{\beta}{\gamma} \geq 1+q-p-pq, \\ \gamma(p+pq-p^2-p^2q) & \text{if } \alpha < \Delta \text{ and } \frac{\beta}{\gamma} \geq 1+q-p-pq, \text{ and} \\ \beta p & \text{otherwise, i.e., if } \frac{\beta}{\gamma} < 1+q-p-pq. \end{cases}$$

(c) The optimal total manufacturing volume in periods 1–2 is

$$Q_d^* = \begin{cases} \left(\frac{\beta+\gamma-\beta\gamma}{\gamma}\right)\left(\frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}}\right) & \text{if } \alpha \geq \Delta \text{ and } \frac{\beta}{\gamma} \geq 1+q-p-pq, \\ p+(1-\gamma)(p+pq-p^2-p^2q) & \text{if } \alpha < \Delta \text{ and } \frac{\beta}{\gamma} \geq 1+q-p-pq, \text{ and} \\ 2p+pq-p^2-p^2q-\beta p & \text{otherwise, i.e., if } \frac{\beta}{\gamma} < 1+q-p-pq. \end{cases}$$

(d) The optimal total profit is given by $\pi_{1,d}^* = (r_n - c_n)Q_d^* + (r_r - c_r)R_d^*$.

Proof. See Appendix A.

Proposition 3.1 establishes the minimum levels of α and β required for optimality of *partially* satisfying the demand in period 1: When $\alpha \geq \Delta$ and $\beta \geq \gamma(1+q-p-pq)$, $S_d^* < D_1 = p$. When $\alpha \geq \Delta$, only a small fraction of the unmet demand in period 1 is lost. Also, when $\alpha \geq \Delta$, remanufacturing is much more profitable than manufacturing. When $\beta \geq \gamma(1+q-p-pq)$, the satisfied demand in period 1 enables a significantly large returns volume so that some backlogged demand (in addition to some diffusion demand) can be fulfilled with remanufactured items in period 2. Hence, the manufacturer rejects some demand in period 1 in order to exploit the benefit of remanufacturing in fulfillment of some backlogged demand. Note that the minimum level of α *decreases* as

γ grows: When γ is high, the backlogged demand is more likely to prefer remanufactured items, and thus it is more desirable to reject some demand in period 1. But the minimum level of β *increases* as γ grows: When γ is high, because the demand for remanufactured items is larger, more used items should be returned in period 2 for partial demand fulfillment in period 1 to remain optimal. Based on Proposition 3.1, Corollary 3.2 identifies three different special cases of our demand-based diffusion model that lead to optimality of *fully* satisfying the demand in period 1.

Corollary 3.2. *Under demand-based diffusion, it is optimal to satisfy all demand in period 1 (a) when all unmet demand is lost (i.e., $\alpha = 0$), (b) when no used items are returned (i.e., $\beta = 0$), or (c) when no demand exists for remanufactured items (i.e., $\gamma = 0$).*

Proof. See Appendix A.

Corollary 3.3 characterizes the optimal solution in a special case of our demand-based diffusion model when the complete market penetration is achieved (i.e., the total diffusion demand is one) and all consumers in period 2 prefer the available remanufactured items to new items (i.e., $\gamma = 1$).

Corollary 3.3. *Under demand-based diffusion suppose that $2p+pq-p^2-p^2q = 1$ and $\gamma = 1$. Then:*

(a) The optimal sales volume in period 1 is

$$S_d^* = \begin{cases} \frac{1-p+\alpha p}{\alpha+\beta} & \textit{if } \alpha \geq \Delta \textit{ and } \beta \geq \frac{1-p}{p}, \\ p & \textit{otherwise.} \end{cases}$$

(b) The optimal remanufacturing volume in period 2 is

$$R_d^* = \begin{cases} \beta\left(\frac{1-p+\alpha p}{\alpha+\beta}\right) & \textit{if } \alpha \geq \Delta \textit{ and } \beta \geq \frac{1-p}{p}, \\ 1-p & \textit{if } \alpha < \Delta \textit{ and } \beta \geq \frac{1-p}{p}, \textit{ and} \\ \beta p & \textit{otherwise, i.e., if } \beta < \frac{1-p}{p}. \end{cases}$$

(c) The optimal total manufacturing volume in periods 1–2 is

$$Q_d^* = \begin{cases} \frac{1-p+\alpha p}{\alpha+\beta} & \text{if } \alpha \geq \Delta \text{ and } \beta \geq \frac{1-p}{p}, \\ p & \text{if } \alpha < \Delta \text{ and } \beta \geq \frac{1-p}{p}, \text{ and} \\ 1-\beta p & \text{otherwise, i.e., if } \beta < \frac{1-p}{p}. \end{cases}$$

(d) The optimal total profit is given by $\pi_{1,d}^* = (r_n - c_n)Q_d^* + (r_r - c_r)R_d^*$.

Proof. See Appendix A.

For the special case in Corollary 3.3, Corollary 3.4 shows how the optimal sales volume changes with respect to our model parameters. While most of the comparative statics results are in line with economic intuition, there are several interesting non-monotonicities driven by complex interactions of model parameters (even in this special case): The optimal sales volume may decrease with p when p is sufficiently high. And it may increase with α when α is sufficiently high. (See Chapter 4 for further comparative statics results in our general model and their interpretations.)

Corollary 3.4. *Under demand-based diffusion suppose that* $2p+pq-p^2-p^2q=1$ *and* $\gamma=1$. *Then:*

(a) Suppose that $\alpha \geq \Delta$: $S_d^* = p$ *as long as* $p \leq \frac{1}{1+\beta}$. S_d^* *drops from* $\frac{1}{1+\beta}$ *to* $\frac{\alpha}{\alpha+\beta}$ *as* p *increases from* $\frac{1}{1+\beta}$ *to one. Now suppose that* $\alpha < \Delta$: $S_d^* = p$.

(b) Suppose that $\alpha \geq \Delta$: $S_d^* = p$ *as long as* $\beta \leq \frac{1-p}{p}$. S_d^* *drops from* p *to* $\frac{1-p+\alpha p}{1+\alpha}$ *as* β *increases from* $\frac{1-p}{p}$ *to one. Now suppose that* $\alpha < \Delta$: S_d^* *is unaffected by* β.

(c) Suppose that $\beta \geq \frac{1-p}{p}$: S_d^* *is unaffected by* α *as long as* $\alpha < \Delta$*; it decreases if* α *increases to* Δ*; and it increases as* α *increases from* Δ. *Now suppose that* $\beta < \frac{1-p}{p}$: S_d^* *is unaffected by* α.

(d) Suppose that $\beta \geq \frac{1-p}{p}$: S_d^* *is unaffected by* Δ *as long as* $\Delta > \alpha$*; it decreases if* Δ *drops to* α*; and it is unaffected by* Δ *as long as* $\Delta < \alpha$. *Now suppose that* $\beta < \frac{1-p}{p}$: S_d^* *is unaffected by* Δ.

Proof. See Appendix A.

3.2 Sales-Based Diffusion

Let S_s denote the sales volume in period 1 in our sales-based diffusion model. Recall that $D_1 = p$ and $D_{2,s} = p + qS_s - p^2 - pqS_s + \alpha(p - S_s)$. Our normalization of the total diffusion demand implies that $\max_{S_s \le p}(2p + qS_s - p^2 - pqS_s) \le 1$. The returns volume that becomes available for remanufacturing and resale at the beginning of period 2 is βS_s. The demand that is satisfied with remanufactured items is $\min\{\beta S_s, \gamma D_{2,s}\}$. Thus the manufacturer's profit in period 2 as a function of $S_s \in [0, p]$ is

$$\pi_{2,s}(S_s) = \begin{cases} (r_r - c_r)\gamma D_{2,s} + (r_n - c_n)(1-\gamma)D_{2,s} & \text{if } \beta S_s \ge \gamma D_{2,s}, \\ (r_r - c_r)\beta S_s + (r_n - c_n)(D_{2,s} - \beta S_s) & \text{otherwise.} \end{cases}$$

The total profit in periods 1–2 as a function of $S_s \in [0, p]$ is $\pi_{1,s}(S_s) = (r_n - c_n)S_s + \pi_{2,s}(S_s)$. Proposition 3.5 characterizes the optimal sales volume (and the corresponding remanufacturing and manufacturing volumes) and the optimal total profit in our sales-based diffusion model.

Proposition 3.5. *Under sales-based diffusion:*

(a) The optimal sales volume in period 1 is

$$S_s^* = \begin{cases} \frac{p - p^2 + \alpha p}{\alpha + \frac{\beta}{\gamma} + pq - q} & \text{if } \alpha \ge \Delta + q - pq \text{ and } \frac{\beta}{\gamma} \ge 1 + q - p - pq, \\ p & \text{otherwise.} \end{cases}$$

(b) The optimal remanufacturing volume in period 2 is

$$R_s^* = \begin{cases} \beta\left(\frac{p - p^2 + \alpha p}{\alpha + \frac{\beta}{\gamma} + pq - q}\right) & \text{if } \alpha \ge \Delta + q - pq \text{ and } \frac{\beta}{\gamma} \ge 1 + q - p - pq, \\ \gamma(p + pq - p^2 - p^2 q) & \text{if } \alpha < \Delta + q - pq \text{ and } \frac{\beta}{\gamma} \ge 1 + q - p - pq, \text{ and} \\ \beta p & \text{otherwise, i.e., if } \frac{\beta}{\gamma} < 1 + q - p - pq. \end{cases}$$

(c) The optimal total manufacturing volume in periods 1–2 is

$$Q_s^* = \begin{cases} \left(\frac{\beta+\gamma-\beta\gamma}{\gamma}\right)\left(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right) & \textit{if } \alpha \geq \Delta + q - pq \textit{ and} \\ & \quad \frac{\beta}{\gamma} \geq 1+q-p-pq, \\ p + (1-\gamma)(p+pq-p^2-p^2q) & \textit{if } \alpha < \Delta + q - pq \textit{ and} \\ & \quad \frac{\beta}{\gamma} \geq 1+q-p-pq, \textit{ and} \\ 2p+pq-p^2-p^2q-\beta p & \textit{otherwise, i.e., if } \frac{\beta}{\gamma} < 1+q-p-pq. \end{cases}$$

(d) The optimal total profit is given by $\pi_{1,s}^* = (r_n - c_n)Q_s^* + (r_r - c_r)R_s^*$.

Proof. See Appendix A.

Proposition 3.5 establishes the minimum levels of α and β required for optimality of *partially* satisfying the demand in period 1: When $\alpha \geq \Delta + q - pq$ and $\beta \geq \gamma(1+q-p-pq)$, $S_s^* < D_1 = p$. The minimum level of α is higher under sales-based diffusion than under demand-based diffusion (see Proposition 3.1), while the minimum level of β is the same in both cases. Under sales-based diffusion, because the diffusion demand in period 2 increases with the sales volume in period 1, there is more incentive to satisfy all demand in period 1. Partial demand fulfillment in period 1 under sales-based diffusion can thus be justified if a greater fraction of the unmet demand in period 1 is to be backlogged. Based on Proposition 3.5, Corollary 3.6 identifies three different special cases of our sales-based diffusion model that lead to optimality of *fully* satisfying the demand in period 1.

Corollary 3.6. *Under sales-based diffusion, it is optimal to satisfy all demand in period 1 (a) when all unmet demand is lost (i.e., $\alpha = 0$), (b) when no used items are returned (i.e., $\beta = 0$), or (c) when no demand exists for remanufactured items (i.e., $\gamma = 0$).*

Proof. See Appendix A.

The maximum possible total demand (due to diffusion) occurs when $S_s = p$ in our sales-based diffusion model, i.e., $\arg\max_{S_s \leq p}(2p + qS_s - p^2 - pqS_s) = p$.

Corollary 3.7 characterizes the optimal solution in a special case of our sales-based diffusion model when the complete market penetration can be achieved by satisfying all demand in period 1 and all consumers in period 2 prefer the available remanufactured items to new items.

Corollary 3.7. *Under sales-based diffusion suppose that* $2p + pq - p^2 - p^2q = 1$ *and* $\gamma = 1$*. Then:*

(a) The optimal sales volume in period 1 is

$$S_s^* = \begin{cases} \frac{(1+\alpha)p^2 - p^3}{(2+\alpha+\beta)p - p^2 - 1} & \text{if } \alpha \geq \Delta + \frac{(1-p)^2}{p} \text{ and } \beta \geq \frac{1-p}{p}, \\ p & \text{otherwise.} \end{cases}$$

(b) The optimal remanufacturing volume in period 2 is

$$R_s^* = \begin{cases} \beta\left(\frac{(1+\alpha)p^2 - p^3}{(2+\alpha+\beta)p - p^2 - 1}\right) & \text{if } \alpha \geq \Delta + \frac{(1-p)^2}{p} \text{ and } \beta \geq \frac{1-p}{p}, \\ 1 - p & \text{if } \alpha < \Delta + \frac{(1-p)^2}{p} \text{ and } \beta \geq \frac{1-p}{p}, \text{ and} \\ \beta p & \text{otherwise, i.e., if } \beta < \frac{1-p}{p}. \end{cases}$$

(c) The optimal total manufacturing volume in periods 1–2 is

$$Q_s^* = \begin{cases} \frac{(1+\alpha)p^2 - p^3}{(2+\alpha+\beta)p - p^2 - 1} & \text{if } \alpha \geq \Delta + \frac{(1-p)^2}{p} \text{ and } \beta \geq \frac{1-p}{p}, \\ p & \text{if } \alpha < \Delta + \frac{(1-p)^2}{p} \text{ and } \beta \geq \frac{1-p}{p}, \text{ and} \\ 1 - \beta p & \text{otherwise, i.e., if } \beta < \frac{1-p}{p}. \end{cases}$$

(d) The optimal total profit is given by $\pi_{1,s}^* = (r_n - c_n)Q_s^* + (r_r - c_r)R_s^*$.

Proof. See Appendix A.

For the special case in Corollary 3.7, Corollary 3.8 shows how the optimal sales volume changes with respect to various model parameters. Similar to Corollary 3.4, Corollary 3.8 reveals that the optimal sales volume may increase with α when α is sufficiently high. (Again, see Chapter 4 for further comparative statics results in our general model and their interpretations.)

Corollary 3.8. *Under sales-based diffusion suppose that* $2p + pq - p^2 - p^2q = 1$ *and* $\gamma = 1$*. Then:*

(a) Suppose that $\alpha \geq \Delta + \frac{(1-p)^2}{p}$*:* $S_s^* = p$ *as long as* $\beta \leq \frac{1-p}{p}$*.* S_s^* *drops from* p *to* $\frac{(1+\alpha)p^2 - p^3}{(3+\alpha)p - p^2 - 1}$ *as* β *increases from* $\frac{1-p}{p}$ *to one. Now suppose that* $\alpha < \Delta + \frac{(1-p)^2}{p}$*:* S_s^* *is unaffected by* β*.*

(b) Suppose that $\beta \geq \frac{1-p}{p}$*:* S_s^* *is unaffected by* α *as long as* $\alpha < \Delta + \frac{(1-p)^2}{p}$*; it decreases if* α *increases to* $\Delta + \frac{(1-p)^2}{p}$*; and it increases as* α *increases from* $\Delta + \frac{(1-p)^2}{p}$*. Now suppose that* $\beta < \frac{1-p}{p}$*:* S_s^* *is unaffected by* α*.*

(c) Suppose that $\beta \geq \frac{1-p}{p}$*:* S_s^* *is unaffected by* Δ *as long as* $\Delta > \alpha - \frac{(1-p)^2}{p}$*; it decreases if* Δ *drops to* $\alpha - \frac{(1-p)^2}{p}$*; and it is unaffected by* Δ *as long as* $\Delta < \alpha - \frac{(1-p)^2}{p}$*. Now suppose that* $\beta < \frac{1-p}{p}$*:* S_s^* *is unaffected by* Δ*.*

Proof. See Appendix A.

Chapter 4

Numerical Experiments

In this chapter we conduct numerical experiments to illustrate our analytical results in Chapter 3, providing insights into both economic and environmental performances of the manufacturer under various conditions. Specifically, we investigate the impacts of operational characteristics ($r_n - c_n$ and $r_r - c_r$), diffusion characteristics (p and q), and customer characteristics (α, β, and γ) of our closed-loop supply chain on the optimal sales, manufacturing, and remanufacturing volumes (S_d^*, S_s^*, Q_d^*, Q_s^*, R_d^*, and R_s^*), and the optimal total profits ($\pi_{1,d}^*$ and $\pi_{1,s}^*$). We also compare our numerical results for the demand-based and sales-based diffusion models. In our study the manufacturer can demonstrate a superior environmental performance by remanufacturing more and manufacturing less. We assume that $\alpha = \beta = 0.6$, $p = 0.7$, $q = 0.3$, $r_n - c_n = 3$, $r_r - c_r = 7$, and $\gamma = 1$ in our base example. In our experiments we vary $r_n - c_n$ and $r_r - c_r$ between 0 and 10 such that $r_n - c_n \leq r_r - c_r$ (Chapter 4.1), p and q between 0 and 1 such that the maximum possible total demand (due to diffusion) is no greater than 1 (Chapter 4.2), and α and β between 0 and 1 (Chapter 4.3). We also perform the same experiments when $\gamma = 0.5$. We then extend our numerical analysis to probabilistic diffusion characteristics (i.e., when q is a word-of-mouth *probability* for an individual consumer, Chapter 4.4) and probabilistic consumer behavior (i.e., when α and β are backlogging and return *probabilities* for an individual consumer, respectively, Chapter 4.5). All the figures appear at the end of the chapter.

4.1 The Impacts of Operational Characteristics

Figure 4.1 shows how the profit margins $r_n - c_n$ and $r_r - c_r$ affect the pairs (R_d^*, R_s^*), (Q_d^*, Q_s^*), (S_d^*, S_s^*), and $(\pi_{1,d}^*, \pi_{1,s}^*)$, as well as their differences. In Figure 4.1, the following hold for the demand-based diffusion model: $\Delta > \alpha$ in region $A_{1,d}$ and $\Delta \le \alpha$ in region $A_{2,d}$. And the following hold for the sales-based diffusion model: $\Delta + q - pq > \alpha$ in region $A_{1,s}$ and $\Delta + q - pq \le \alpha$ in region $A_{2,s}$. (Recall that Δ is the average profit margin ratio of manufacturing to remanufacturing.) Figure 4.1 indicates that R_d^* and R_s^* are lower in regions $A_{1,d}$ and $A_{1,s}$ than in regions $A_{2,d}$ and $A_{2,s}$, respectively. However, Q_d^* and Q_s^* (and S_d^* and S_s^*) are greater in regions $A_{1,d}$ and $A_{1,s}$ than in regions $A_{2,d}$ and $A_{2,s}$, respectively.

When Δ is sufficiently low and the return rate is high enough ($\beta = 0.6$), the manufacturer rejects some demand in period 1 in both diffusion models. Because remanufacturing is significantly more profitable than manufacturing, and an ample supply will be available for remanufacturing in period 2 even under a low sales volume in period 1 (thanks to the high return rate), the manufacturer wants to reject some demand in order to fulfill a *fraction* of the unmet demand with remanufactured products in period 2 (rather than manufactured products in period 1). As the sales volume in period 1 drops, the diffusion demand in period 2 decreases under sales-based diffusion, but it stays the same under demand-based diffusion. In both diffusion models, because the diffusion demand in period 2 does *not* increase as the sales volume in period 1 drops, the unmet demand reduces the total manufacturing volume, increasing the remanufacturing volume by less than the reduction in the total manufacturing volume: The manufacturer has a better environmental performance when Δ is low. Conversely, when Δ is sufficiently high (even under the high return rate), the manufacturer satisfies all demand in both models, and a worse environmental performance results.

Remark 4.1. *R_d^* and R_s^* are greater, and Q_d^*, Q_s^*, S_d^*, and S_s^* are lower, when Δ is below a critical level. Both diffusion models lead to a better environmental* *performance when Δ drops below the critical level.*

Figure 4.1 also indicates that as $(r_n - c_n)$ or $(r_r - c_r)$ increases, both $\pi^*_{1,d}$ and $\pi^*_{1,s}$ increase. The effects of $(r_n - c_n)$ and $(r_r - c_r)$ vary with respect to regions $A_{1,d}$, $A_{2,d}$, $A_{1,s}$, and $A_{2,s}$: A unit increment in $(r_n - c_n)$ increases $\pi^*_{1,d}$ by 0.070 and 0.057 in regions $A_{1,d}$ and $A_{2,d}$, respectively. A unit increment in $(r_n - c_n)$ increases $\pi^*_{1,s}$ by 0.070 and 0.056 in regions $A_{1,s}$ and $A_{2,s}$, respectively. A unit increment in $(r_r - c_r)$ increases $\pi^*_{1,d}$ by 0.027 and 0.035 in regions $A_{1,d}$ and $A_{2,d}$, respectively. A unit increment in $(r_r - c_r)$ increases $\pi^*_{1,s}$ by 0.027 and 0.034 in regions $A_{1,s}$ and $A_{2,s}$, respectively. Recall that $(r_n - c_n)$ and $(r_r - c_r)$ are the multipliers of the manufacturing and remanufacturing volumes in the total profit function, respectively. Because the returns volume is constrained by the manufacturing volume in period 1, the remanufacturing volume is less than the manufacturing volume in all instances. Thus the total profits are affected by $(r_n - c_n)$ more than $(r_r - c_r)$. In addition, we know from Remark 4.1 that when Δ is below a critical level (and the return rate is high), the manufacturer remanufactures more and manufactures less. Thus $(r_n - c_n)$ affects the profits less, and $(r_r - c_r)$ affects the profits more, when Δ drops below the critical level.

Remark 4.2. *$r_n - c_n$ has a greater impact than $r_r - c_r$ on both $\pi^*_{1,d}$ and $\pi^*_{1,s}$. Also, $r_n - c_n$ has a lower impact, and $r_r - c_r$ has a greater impact, when Δ is below a critical level.*

Figure 4.1 shows that $(R^*_d - R^*_s)$, $(Q^*_d - Q^*_s)$, $(S^*_d - S^*_s)$, and $(\pi^*_{1,d} - \pi^*_{1,s})$ can be in one of the three different regions: Region $A_{1,d}$, region $A_{2,s}$, and the region between $A_{1,d}$ and $A_{2,s}$. Note that $\Delta > \alpha$ in region $A_{1,d}$, $\alpha \geq \Delta > \alpha - q + pq$ in the region between $A_{1,d}$ and $A_{2,s}$, and $\Delta \leq \alpha - q + pq$ in region $A_{2,s}$. We observe that when $\Delta > \alpha$, $S^*_d = S^*_s = p$, implying that $R^*_d = R^*_s$, $Q^*_d = Q^*_s$, and $\pi^*_{1,d} = \pi^*_{1,s}$. When $\Delta > \alpha$, the benefit of remanufacturing over manufacturing is so low that the manufacturer satisfies all demand in period 1 in both diffusion models (see Propositions 3.1 and 3.5). And thus our diffusion models become identical.

When $\alpha \geq \Delta > \alpha - q + pq$, R^*_d is greater than R^*_s, Q^*_d is smaller than Q^*_s, and S^*_d is smaller than S^*_s. In this case, $\pi^*_{1,d}$ is greater than $\pi^*_{1,s}$, and the difference

$(\pi^*_{1,d} - \pi^*_{1,s})$ increases as $(r_n - c_n)$ decreases or $(r_r - c_r)$ increases. When $\alpha \geq \Delta > \alpha - q + pq$, the manufacturer rejects some demand in period 1 under demand-based diffusion, whereas it satisfies all demand in period 1 under sales-based diffusion (see Propositions 3.1 and 3.5). This is because rejecting a demand in period 1 does not affect the future diffusion demand under demand-based diffusion, making it more desirable to reject some demand in period 1 and satisfy those backlogged demands (of the unmet demand) with remanufactured products in period 2. Thus the manufacturer can demonstrate a superior environmental performance under demand-based diffusion, without sacrificing its economic performance.

Finally, when $\Delta \leq \alpha - q + pq$, R^*_d, Q^*_d, and S^*_d are greater than R^*_s, Q^*_s, and S^*_s, respectively, implying that $\pi^*_{1,d}$ is greater than $\pi^*_{1,s}$. In this case the difference $(\pi^*_{1,d} - \pi^*_{1,s})$ decreases as $r_n - c_n$ or $r_r - c_r$ decreases. When $\Delta \leq \alpha - q + pq$, the benefit of remanufacturing over manufacturing is so high that the manufacturer rejects some demand in period 1 in *both* diffusion models (see Propositions 3.1 and 3.5). But the manufacturer rejects *less* under demand-based diffusion, in order to ensure the existence of a sufficient returns volume for fulfillment of the *larger* demand in period 2 (thanks to demand-based diffusion) through remanufacturing. In both diffusion models, the manufacturer reduces its sales in period 1 as long as a sufficient returns volume will be available for fulfillment of *all* the demand in period 2 (recall $\gamma = 1$): When the manufacturer rejects some demand, the sales volumes in periods 1 and 2 determine the manufacturing and remanufacturing volumes, respectively (again, see Propositions 3.1 and 3.5). Because the manufacturer rejects less under demand-based diffusion, the sales volumes are larger in both periods under demand-based diffusion, leading to a worse environmental performance. Because the manufacturer has the advantage of keeping the total diffusion demand intact under demand-based diffusion, it still has a better economic performance.

Remark 4.3. *When $\Delta > \alpha$, the optimal solutions are identical in both diffusion models. When $\alpha \geq \Delta > \alpha - q + pq$, the manufacturer has a better performance both economically and environmentally under demand-based diffusion than under sales-based diffusion. When $\Delta \leq \alpha - q + pq$, the manufacturer has a better economic performance under demand-based diffusion, and a better environmental* *performance under sales-based diffusion.*

4.2 The Impacts of Diffusion Characteristics

Figure 4.2 shows how the diffusion parameters p and q affect the pairs (S_d^*, S_s^*), (Q_d^*, Q_s^*), (R_d^*, R_s^*), and $(\pi_{1,d}^*, \pi_{1,s}^*)$, as well as their differences. As p increases, the demands in both periods increase. Thus the total sales volume increases with p. The increased sales not only improve the economic performance of the manufacturer but also lead to greater manufacturing and remanufacturing volumes. As q increases, the demand in period 1 stays the same, but the demand in period 2 increases. Thus the total sales volume does not increase with q as much as it does with p. The manufacturing and remanufacturing volumes, and the total profit, are therefore less affected by q than p. We also note that when p is low, the supply for remanufacturing is limited in period 2 due to the low sales volume in period 1. The remanufacturing volume is therefore affected less than the manufacturing volume by q.

Remark 4.4. *S_d^*, S_s^*, Q_d^*, Q_s^*, R_d^*, R_s^*, $\pi_{1,d}^*$, and $\pi_{1,s}^*$ tend to increase significantly with p, and only slightly with q. When p is low, R_d^* and R_s^* are affected by q less than Q_d^* and Q_s^*.*

When p is small, the manufacturer satisfies all demand in period 1 in both diffusion models (see Propositions 3.1 and 3.5). Because the demand in period 1 is low when p is small, the manufacturer should sell as much as possible in period 1 so that a sufficient number of returns will be available for remanufacturing in period 2. Thus when p is small, $S_d^* = S_s^* = D_1 = p$, implying that $Q_d^* = Q_s^*$, $R_d^* = R_s^*$, and $\pi_{1,d}^* = \pi_{1,s}^*$. However, when p is large and q is between 0.2 and 0.4, the manufacturer rejects some demand in period 1 in both diffusion models, in order to exploit the benefit of remanufacturing in period 2 from the backlogged demands. Unlike sales-based diffusion, the unmet demand does not reduce the future demand under demand-based diffusion. The manufacturer rejects *less* in period 1 under demand-based diffusion than under sales-based diffusion, in order to ensure the existence of a sufficient returns volume for fulfillment of the *larger* demand in period 2 through remanufacturing. Because S_d^* is greater than S_s^*, the sales volumes are larger in both periods under demand-based diffusion than under sales-based diffusion. This implies that Q_d^* and R_d^* are greater than Q_s^* and R_s^*, respectively. Finally, when p is large and q is 0.5, the manufacturer satisfies all demand in period 1 under sales-based diffusion. This is because increased sales in period 1 greatly improves the future diffusion demand when q is large under sales-based diffusion. But the manufacturer continues to reject some demand under demand-based diffusion, without sacrificing the future diffusion demand: $S_d^* < S_s^* = p$, implying that $Q_d^* < Q_s^*$, and $R_d^* > R_s^*$.

Remark 4.5. *When p is low, the optimal solutions are identical in both diffusion models. When p is high and q is low, the manufacturer has a better economic performance under demand-based diffusion, and a better environmental performance under sales-based diffusion. When both p and q are high, the manufacturer has a better performance both economically and environmentally under demand-based diffusion than under sales-based diffusion.*

4.3 The Impacts of Consumer Characteristics

In both diffusion models the parameter α can be viewed as a measure for the consumers' willingness to wait for adoption of the product when their demands are rejected in period 1. And the parameter β can be viewed as a measure for the consumers' willingness to return their used products when their demands are satisfied in period 1. Figure 4.3 shows how these consumer characteristics (α and β) affect the pairs (S_d^*, S_s^*), (Q_d^*, Q_s^*), (R_d^*, R_s^*), and $(\pi_{1,d}^*, \pi_{1,s}^*)$, as well as their differences. Each of S_d^*, S_s^*, Q_d^*, Q_s^*, R_d^*, R_s^*, $\pi_{1,d}^*$, and $\pi_{1,s}^*$ can be in one of the three different regions (see Propositions 3.1 and 3.5): The region when β is low ($\beta < 1 + q - p - pq$), the region when β is high ($\beta \geq 1 + q - p - pq$) and α is low ($\alpha < \Delta$ under demand-based diffusion and $\alpha < \Delta + q - pq$ under sales-based diffusion), and the region when β is high ($\beta \geq 1 + q - p - pq$) and α is high ($\alpha \geq \Delta$ under demand-based diffusion and $\alpha \geq \Delta + q - pq$ under sales-based diffusion).

When β is low, the returns volume in period 2 is limited in both diffusion models so that the manufacturer cannot satisfy any backlogged demand from period 1 with a remanufactured product. The manufacturer is thus better off satisfying all demand in period 1, implying that $S_d^* = S_s^* = p$. As β drops (further from $1 + q - p - pq$), the returns volume decreases in period 2. This leads to a greater manufacturing volume in period 2 (without affecting the manufacturing volume in period 1) and a smaller remanufacturing volume, and thus a lower total profit. Because all demand is satisfied in period 1 when β is low, the manufacturing and remanufacturing volumes, and the total profit, are not affected by α.

When β is high, the returns volume in period 2 is sufficient to satisfy a backlogged demand from period 1 with a remanufactured product. However, if α is low, it is very likely that the customer whose demand is rejected in period 1 does not want to purchase the product any longer. In this case the manufacturer trades off the loss of remanufacturing opportunity for protection of the total sales volume in period 1: $S_d^* = S_s^* = p$. Because all demand in period 1 is satisfied and all demand in period 2 is satisfied with remanufactured products when β is high and α is low, the manufacturing and remanufacturing volumes, and the total profit, are not affected by β (as long as $\beta \geq 1 + q - p - pq$) and α (as long as $\alpha < \Delta$ under demand-based diffusion and $\alpha < \Delta + q - pq$ under sales-based diffusion).

Remark 4.6. *When β or α is low, $S_d^* = S_s^* = p$. When β is high and α is low, Q_d^*, Q_s^*, R_d^*, R_s^*, $\pi_{1,d}^*$, and $\pi_{1,s}^*$ are unaffected by α and β. When β is low, as β further drops, Q_d^* and Q_s^* increase, R_d^*, R_s^*, $\pi_{1,d}^*$, and $\pi_{1,s}^*$ decrease. Q_d^*, Q_s^*, R_d^*, R_s^*, $\pi_{1,d}^*$, and $\pi_{1,s}^*$ are unaffected by α if β is low.*

When both β and α are high, the manufacturer rejects some demand in period 1 in order to exploit the benefit of remanufacturing. As β increases (further from $1 + q - p - pq$), the manufacturer rejects more demand in period 1, improving its economic and environmental performances. As α increases (further from Δ under demand-based diffusion and $\Delta + q - pq$ under sales-based diffusion), the manufacturer rejects *slightly* less demand in period 1. Because the total demand in period 2 is too high when α is very large, the manufacturer sells more in period 1 to increase the future returns volume so that it can fulfill all demand in period 2 with remanufactured products. This leads to a worse environmental performance, but a higher total profit. An important insight we gain here is that the profits tend to grow with the fraction of consumers who are patient *and* green-minded in the market, but too much patience may hurt the environmental performance.

Remark 4.7. *When both α and β are high, $S_d^* = S_s^* < p$. In this case the manufacturer improves its total profit as α or β increases, and its environmental performance as α drops or β increases.*

When $\beta < 1 + q - p - pq$ or $\alpha < \Delta$, $S_d^* = S_s^* = p$, implying that $Q_d^* = Q_s^*$, $R_d^* = R_s^*$, and $\pi_{1,d}^* = \pi_{1,s}^*$ (see Propositions 3.1 and 3.5). When $\beta \geq 1 + q - p - pq$ and $\Delta + q - pq > \alpha \geq \Delta$ (the I-shaped regions in the graphs of $S_d^* - S_s^*$, $Q_d^* - Q_s^*$, $R_d^* - R_s^*$ in Figure 4.3), $S_d^* < S_s^* = p$, implying that $Q_d^* < Q_s^*$, $R_d^* > R_s^*$, and $\pi_{1,d}^* > \pi_{1,s}^*$: The manufacturer under demand-based diffusion is able to improve its economic and environmental performances by manufacturing less and remanufacturing more. Our diffusion models show similar results if β is slightly above $1 + q - p - pq$ (and more demand is satisfied in period 1). When $\beta \geq 1 + q - p - pq$ and $\alpha \geq \Delta + q - pq$, $S_s^* < S_d^* < p$, implying that $Q_d^* > Q_s^*$, $R_d^* > R_s^*$, and $\pi_{1,d}^* > \pi_{1,s}^*$: The manufacturer under demand-based diffusion sells more, in order to increase the returns volume and satisfy the large demand in period 2 through remanufacturing. This leads to a better economic performance and a worse environmental performance. Our diffusion models show similar results if β is slightly above $1 + q - p - pq$ and α is too high.

Remark 4.8. *When α and β are high, the manufacturer performs economically better under demand-based diffusion, and the economic benefit of demand-based diffusion grows with β. When α and β are high, it performs environmentally better under sales-based diffusion if and only if α is above a threshold, and the environmental benefit of sales-based diffusion is highest if α equals the threshold. Diffusion models lead to markedly different environmental performances when β is too high.*

We have also examined the numerical results when $\gamma = 0.5$ on the same test bed: Most of our insights continue to hold in this case. See Figures 4.4–4.6 for our numerical results when $\gamma = 0.5$.

4.4 Probabilistic Diffusion Characteristics

Our results in Chapters 4.1-4.3 continue to hold if the innovation coefficient p is variable. This is because such uncertainty is resolved in the initial stage, and in our models the manufacturer has ample capacity to immediately fulfill any demand. However, if the word-of-mouth effect is variable, it is unclear how the optimal sales plan is affected. We now conduct additional numerical experiments to address this issue. We reformulate the problem in Chapter 2 for a market of m consumers by reinterpreting the parameter q as a probability rather than a deterministic coefficient. Namely, we redefine q as the *probability* that an individual consumer successfully creates the word-of-mouth effect on the customers who did not demand the product in period 1. Let Z denote the number of consumers who create the word-of-mouth effect. Recall that $D_{1,d} = D_{1,s} = \lfloor mp \rceil$ in a market of m consumers. We below formulate the profit functions for the two diffusion models.

- Recall that, in the case of demand-based diffusion, a consumer who demands the product in period 1 can create the word-of-mouth effect. Thus Z is a binomial random variable with parameters $\lfloor mp \rceil$ and q. When $Z = z$, $D_{2,d} = \lfloor (p + z/m)(m - mp) + \alpha(mp - S_d) \rceil$ and the manufacturer's profit in period 2 as a function of $S_d \in \{0, 1, .., D_{1,d}\}$ is given by

$$\pi_{2,d}(S_d, z) = \begin{cases} (r_r - c_r) \lfloor \gamma D_{2,d} \rceil \\ \qquad +(r_n - c_n) \lfloor (1-\gamma) D_{2,d} \rceil & \text{if } \lfloor \beta S_d \rfloor \geq \lfloor \gamma D_{2,d} \rceil, \\ (r_r - c_r) \lfloor \beta S_d \rfloor \\ \qquad +(r_n - c_n) \left(D_{2,d} - \lfloor \beta S_d \rfloor\right) & \text{otherwise.} \end{cases}$$

The *expected* total profit in periods 1–2 as a function of S_d is given by

$$\pi_{1,d}(S_d) = (r_n - c_n) S_d + \sum_{z=0}^{\lfloor mp \rceil} P(Z = z) \pi_{2,d}(S_d, z)$$

where $P(Z = z) = q^z(1-q)^{\lfloor mp \rceil - z}\binom{\lfloor mp \rceil}{z}$.

- Recall that, in the case of sales-based diffusion, a consumer who purchases the product in period 1 can create the word-of-mouth effect. Thus Z is a binomial random variable with parameters S_s and q. When $Z = z$, $D_{2,s} = \lfloor (p + z/m)(m - mp) + \alpha(mp - S_s) \rceil$ and the manufacturer's profit in period 2 as a function of $S_s \in \{0, 1, .., D_{1,s}\}$ is given by

$$\pi_{2,s}(S_s, z) = \begin{cases} (r_r - c_r)\lfloor \gamma D_{2,s} \rceil & \\ \qquad +(r_n - c_n)\lfloor (1-\gamma) D_{2,s} \rceil & \text{if } \lfloor \beta S_s \rfloor \geq \lfloor \gamma D_{2,s} \rceil, \\ (r_r - c_r)\lfloor \beta S_s \rfloor & \\ \qquad +(r_n - c_n)(D_{2,s} - \lfloor \beta S_s \rfloor) & \text{otherwise.} \end{cases}$$

The *expected* total profit in periods 1–2 as a function of S_s is given by

$$\pi_{1,s}(S_s) = (r_n - c_n)S_s + \sum_{z=0}^{S_s} P(Z = z)\pi_{2,s}(S_s, z)$$

where $P(Z = z) = q^z(1-q)^{S_s - z}\binom{S_s}{z}$.

We solve the above two problems on the same test bed when $m = 100$. We calculate the optimal sales volume in period 1, the optimal *expected* manufacturing and remanufacturing volumes, and the optimal *expected* total profit. See Figures 4.7–4.9 for our numerical results. We observe that the basic insights gained in Chapters 4.1–4.3 continue to hold in this case.

4.5 Probabilistic Consumer Characteristics

We now reformulate the problem introduced in Chapter 2 for a market of m consumers by reinterpreting the parameters α and β as probabilities rather than deterministic fractions. Namely, we redefine α as the *probability* that an individual consumer purchases the product in period 2 given that her demand is rejected in period 1, and β as the *probability* that an individual consumer returns the used product in period 2 given that her demand is satisfied in period 1. Let X

denote the number of consumers whose demands are backlogged in period 1, and Y denote the number of consumers who return their used products in period 2. Recall that $D_{1,d} = D_{1,s} = \lfloor mp \rceil$ in a market of m consumers. We below formulate the profit functions for the two diffusion models.

- In the case of demand-based diffusion, X is a binomial random variable with parameters $(D_{1,d} - S_d)$ and α, and Y is a binomial random variable with parameters S_d and β. When $X = x$ and $Y = y$, $D_{2,d} = \lfloor (p + pq)(m - mp) \rceil + x$ and the manufacturer's profit in period 2 is given by

$$\pi_{2,d}(x, y) = \begin{cases} (r_r - c_r)\lfloor \gamma D_{2,d} \rceil & \\ \quad +(r_n - c_n)\lfloor (1-\gamma) D_{2,d} \rceil & \text{if } y \geq \lfloor \gamma D_{2,d} \rceil, \\ (r_r - c_r)y + (r_n - c_n)(D_{2,d} - y) & \text{otherwise.} \end{cases}$$

  The *expected* total profit in periods 1–2 as a function of $S_d \in \{0, 1, .., D_{1,d}\}$ is given by

$$\pi_{1,d}(S_d) = (r_n - c_n)S_d + \sum_{x=0}^{D_{1,d}-S_d} \sum_{y=0}^{S_d} P(X = x)P(Y = y)\pi_{2,d}(x, y)$$

  where $P(X = x) = \alpha^x (1-\alpha)^{D_{1,d}-S_d-x} \binom{D_{1,d}-S_d}{x}$ and $P(Y = y) = \beta^y (1-\beta)^{S_d - y} \binom{S_d}{y}$.

- In the case of sales-based diffusion, X is a binomial random variable with parameters $(D_{1,s} - S_s)$ and α, and Y is a binomial random variable with parameters S_s and β. When $X = x$ and $Y = y$, $D_{2,s} = \lfloor (p + qS_s/m)(m - mp) \rceil + x$ and the manufacturer's profit in period 2 is given by

$$\pi_{2,s}(S_s, x, y) = \begin{cases} (r_r - c_r)\lfloor \gamma D_{2,s} \rceil & \\ \quad +(r_n - c_n)\lfloor (1-\gamma) D_{2,s} \rceil & \text{if } y \geq \lfloor \gamma D_{2,s} \rceil, \\ (r_r - c_r)y + (r_n - c_n)(D_{2,s} - y) & \text{otherwise.} \end{cases}$$

  The *expected* total profit in periods 1–2 as a function of $S_s \in \{0, 1, .., D_{1,s}\}$ is given by

$$\pi_{1,s}(S_s) = (r_n - c_n)S_s + \sum_{x=0}^{D_{1,s}-S_s} \sum_{y=0}^{S_s} P(X = x)P(Y = y)\pi_{2,s}(S_s, x, y)$$

where $P(X = x) = \alpha^x(1-\alpha)^{D_{1,s}-S_s-x}\binom{D_{1,s}-S_s}{x}$ and $P(Y = y) = \beta^y(1-\beta)^{S_s-y}\binom{S_s}{y}$.

We again solve the above two problems on the same test bed when $m = 100$. See Figures 4.10–4.12 for our numerical results. We observe that the basic insights gained in Chapters 4.1–4.3 again continue to hold in this case. We also note that the manufacturer tends to sell more when consumer behavior is probabilistic than when it is deterministic: The manufacturer rejects a demand if the unmet demand in period 1 is likely to be satisfied with a remanufactured item in period 2. But this is possible only when an unmet demand is backlogged *and* a consumer who purchases the product in period 1 returns it in period 2. In most of our instances, the probability that these two events occur together is significantly less than the probability that at most one of the two events occurs. Thus the manufacturer sells more when consumer behavior is probabilistic.

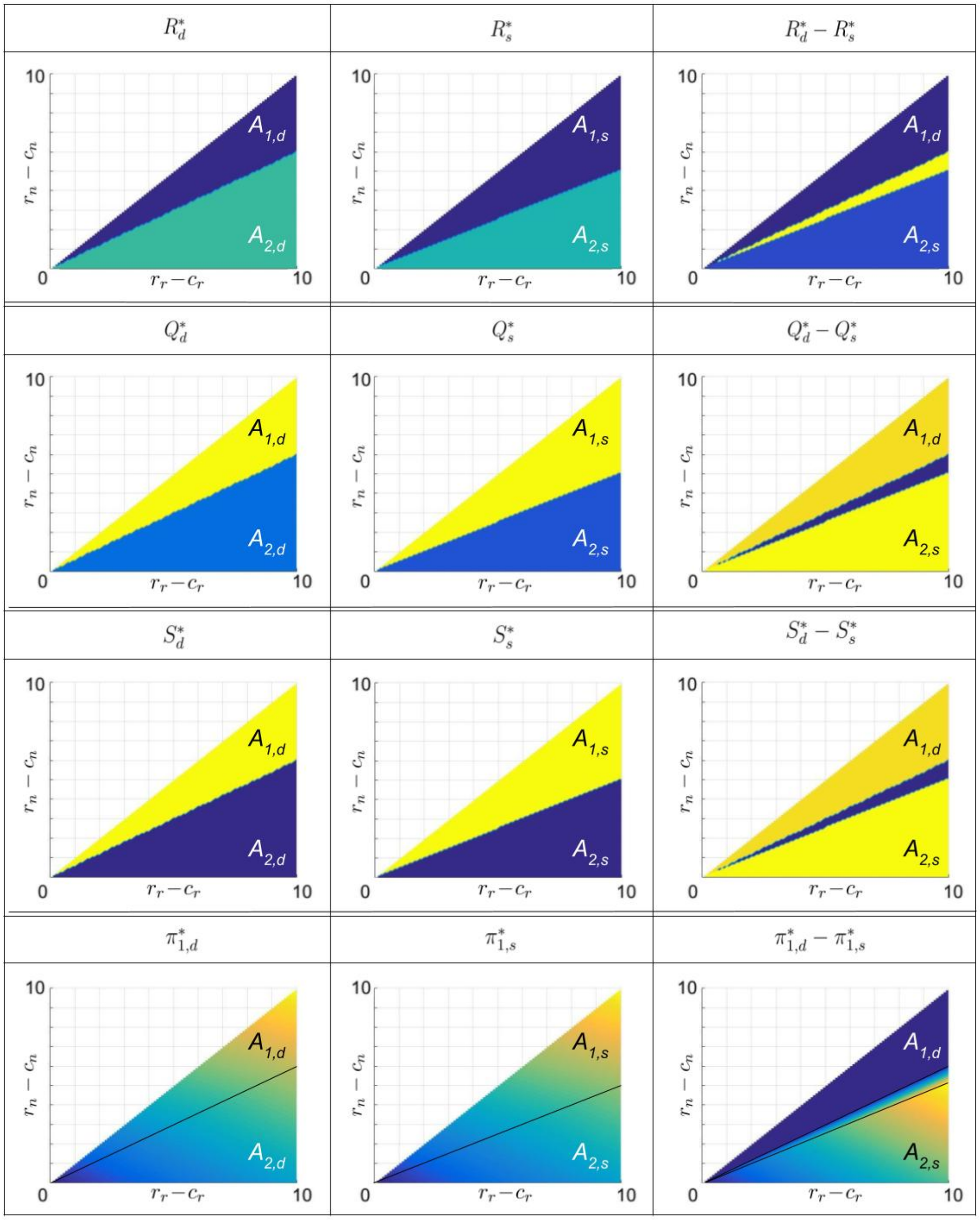

Figure 4.1: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. $(r_n - c_n)$ and $(r_r - c_r)$. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 1$, $p = 0.7$, $q = 0.3$. Darker color indicates a lower value.

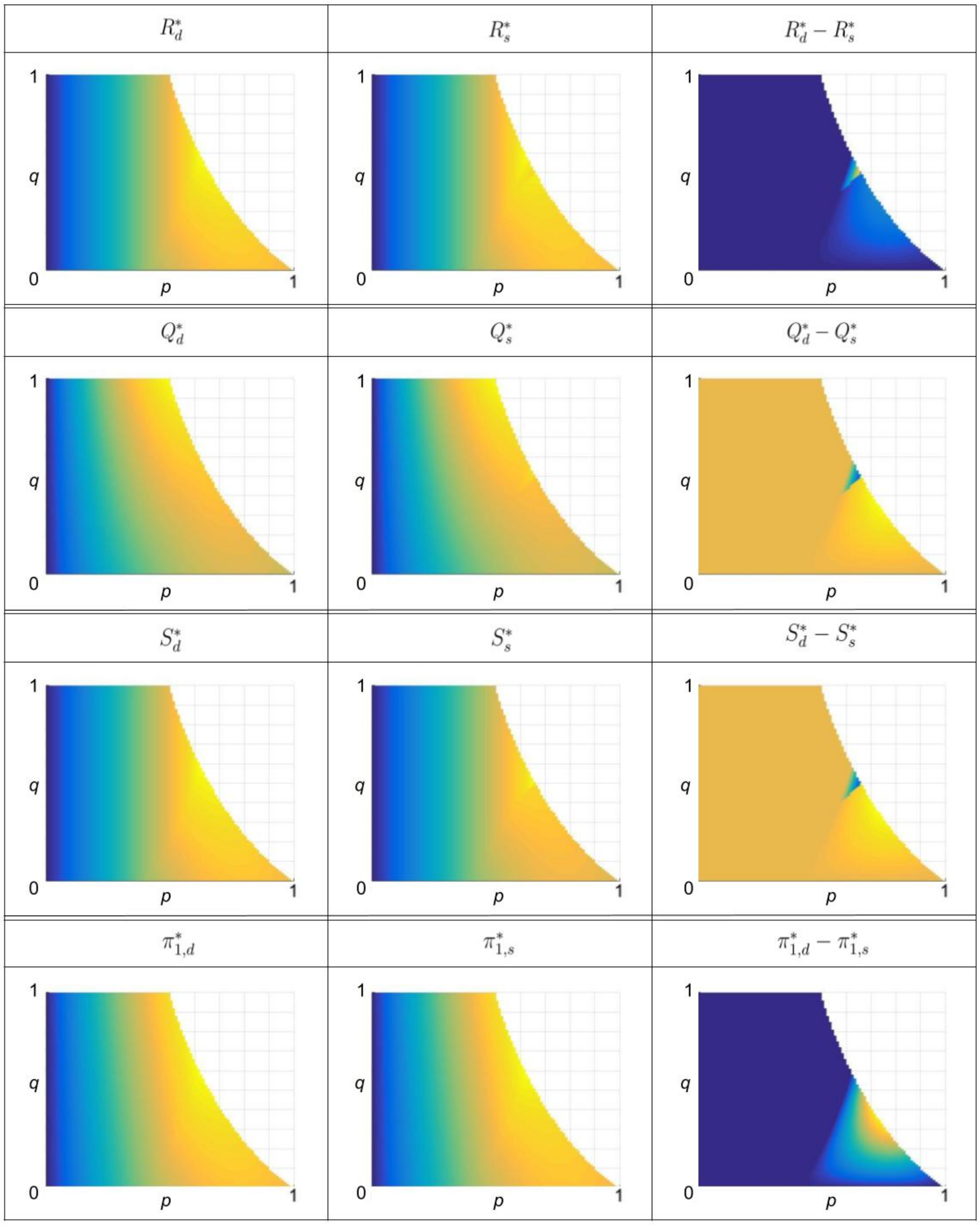

Figure 4.2: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. p and q. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 1$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value. The maximum possible total demand (due to diffusion) is no larger than one in the shaded region.

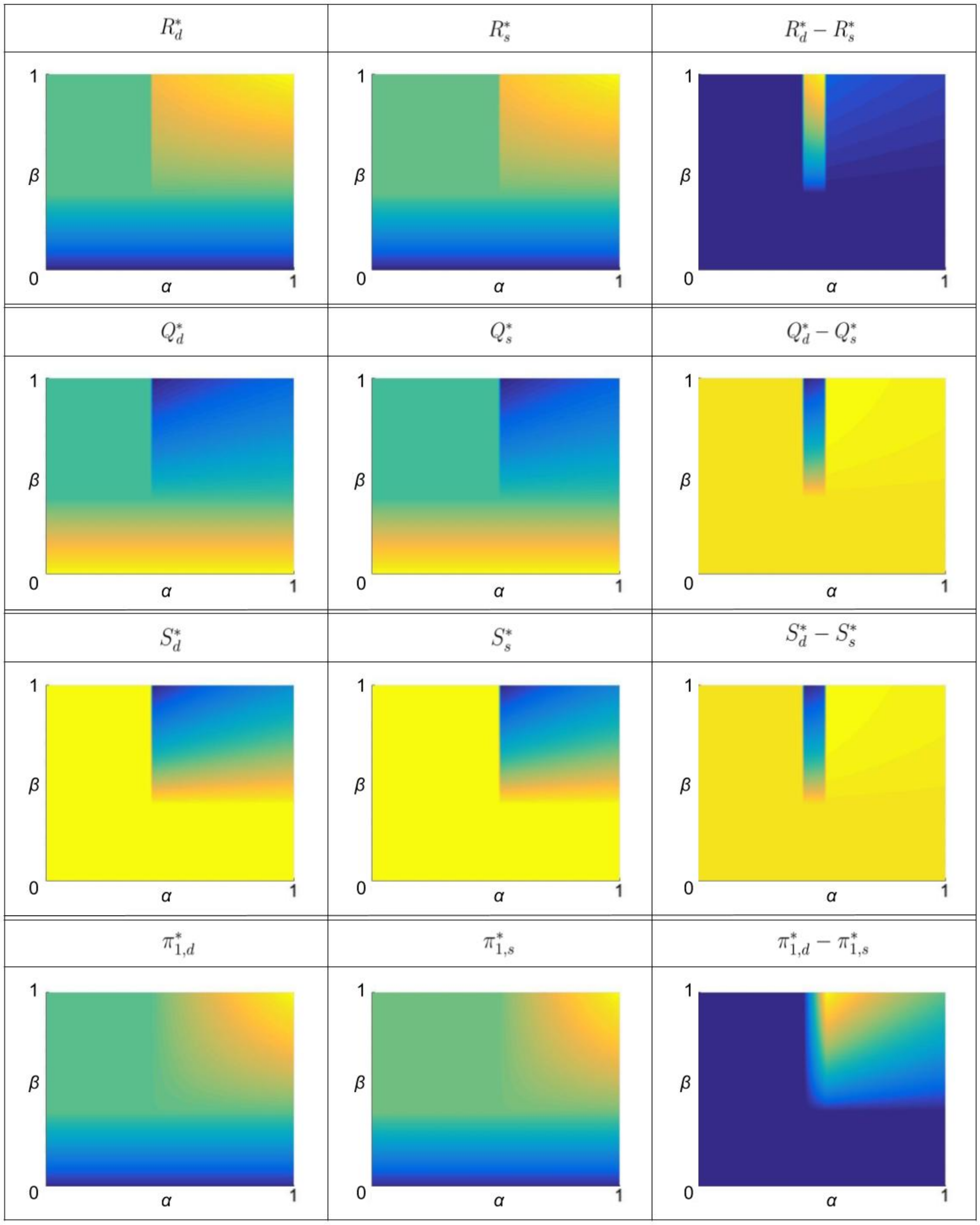

Figure 4.3: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. α and β. $\gamma = 1$, $p = 0.7$, $q = 0.3$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value.

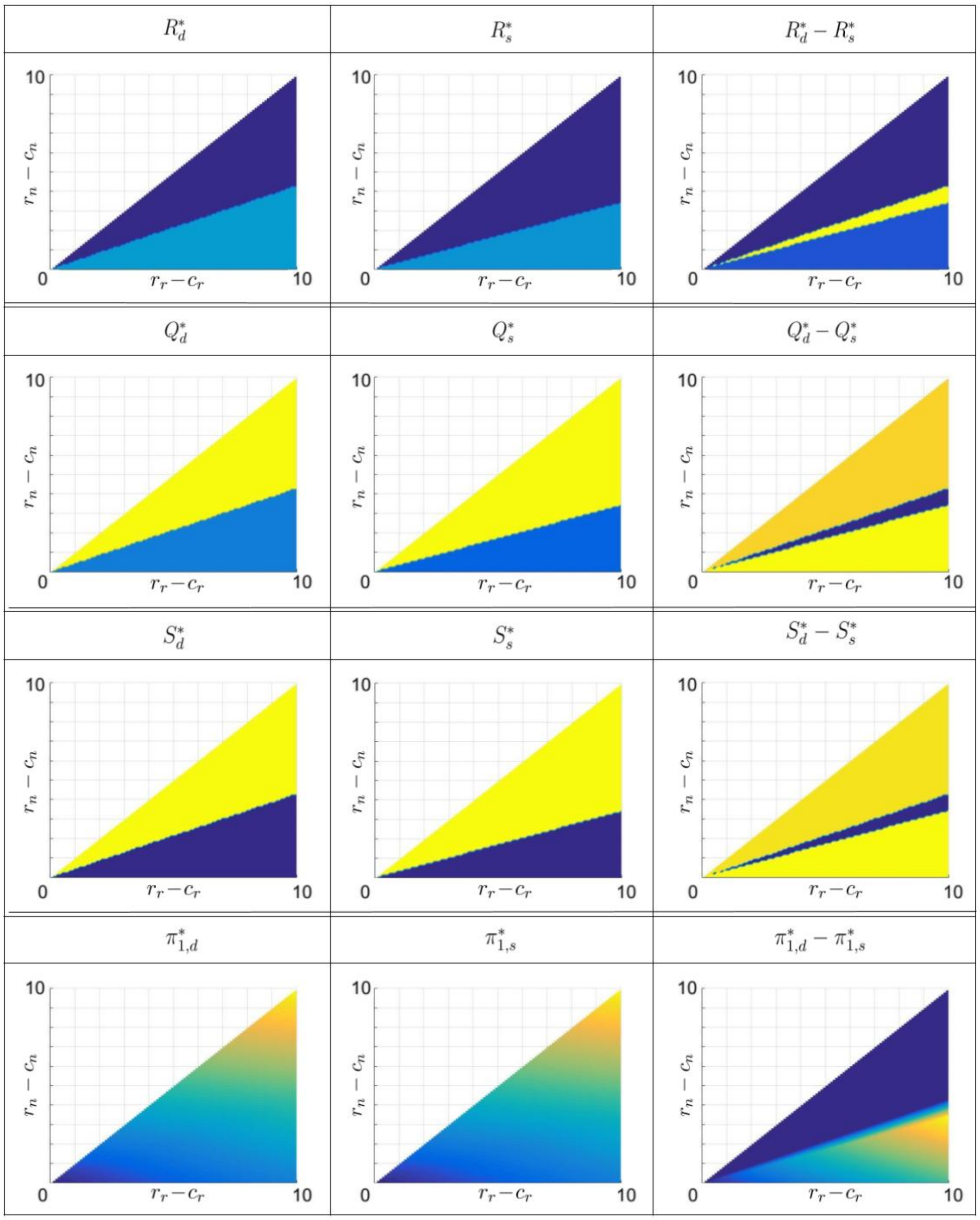

Figure 4.4: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. $(r_n - c_n)$ and $(r_r - c_r)$. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 0.5$, $p = 0.7$, $q = 0.3$. Darker color indicates a lower value.

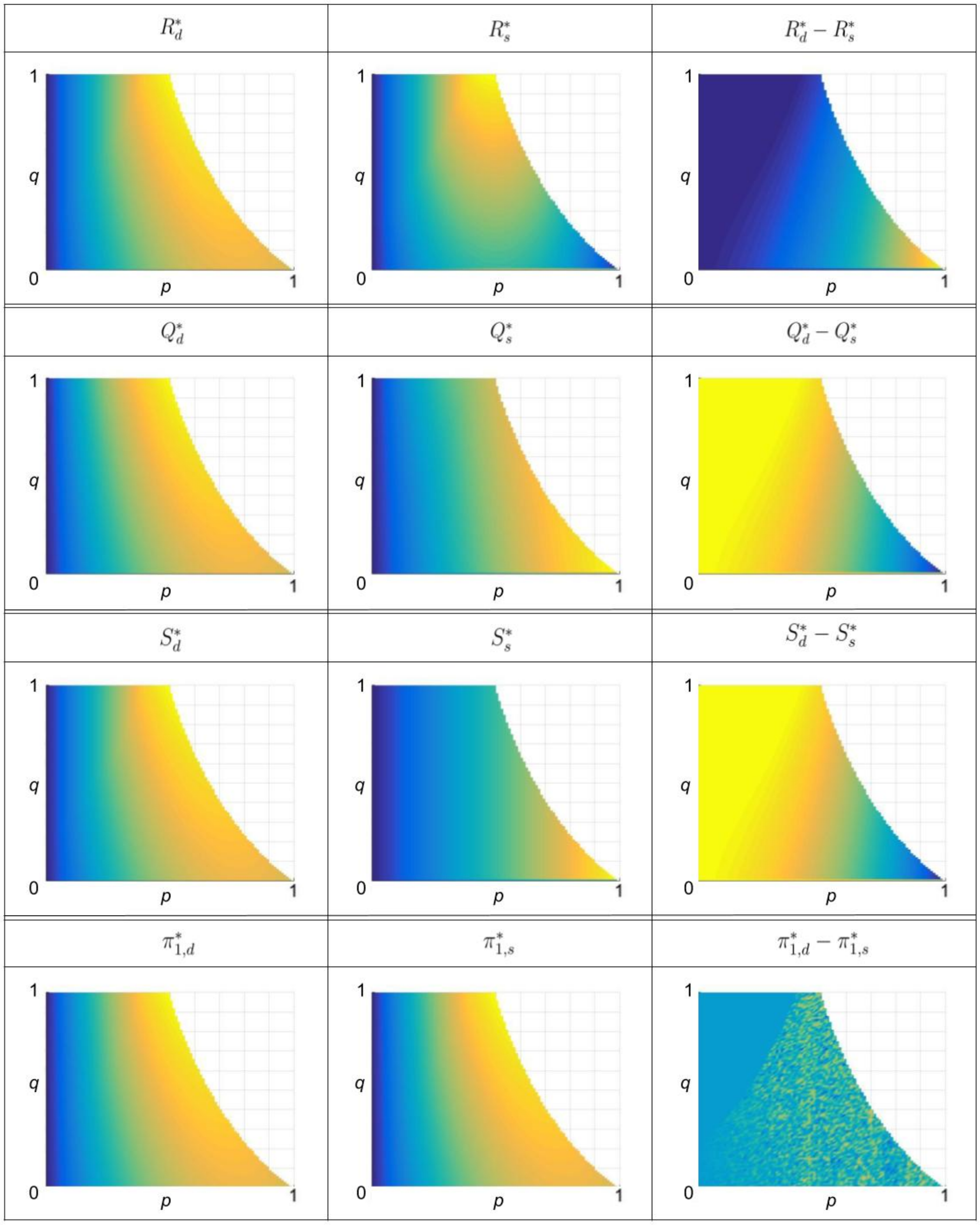

Figure 4.5: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. p and q. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 0.5$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value. The maximum possible total demand (due to diffusion) is no larger than one in the shaded region.

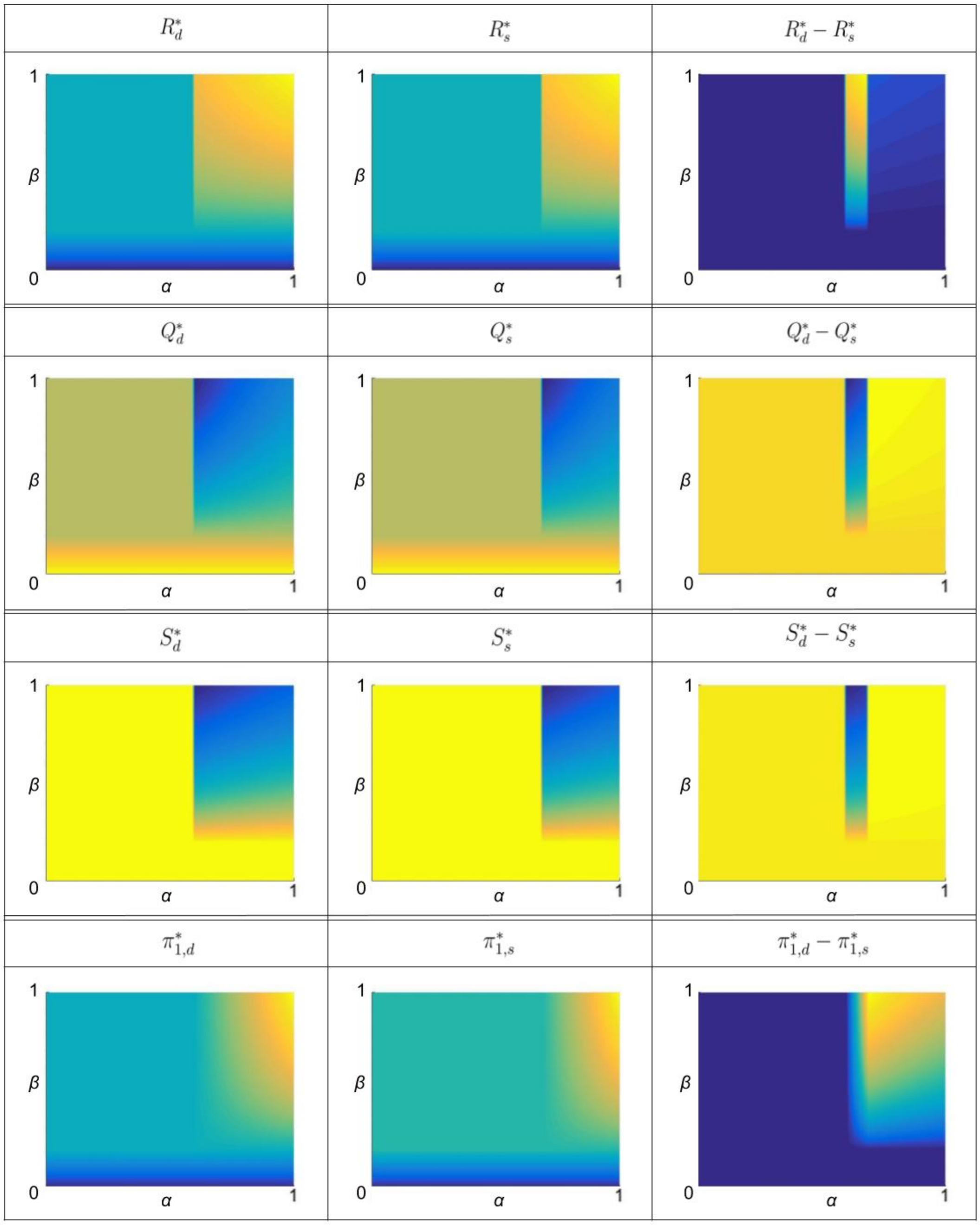

Figure 4.6: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. α and β. $\gamma = 0.5$, $p = 0.7$, $q = 0.3$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value.

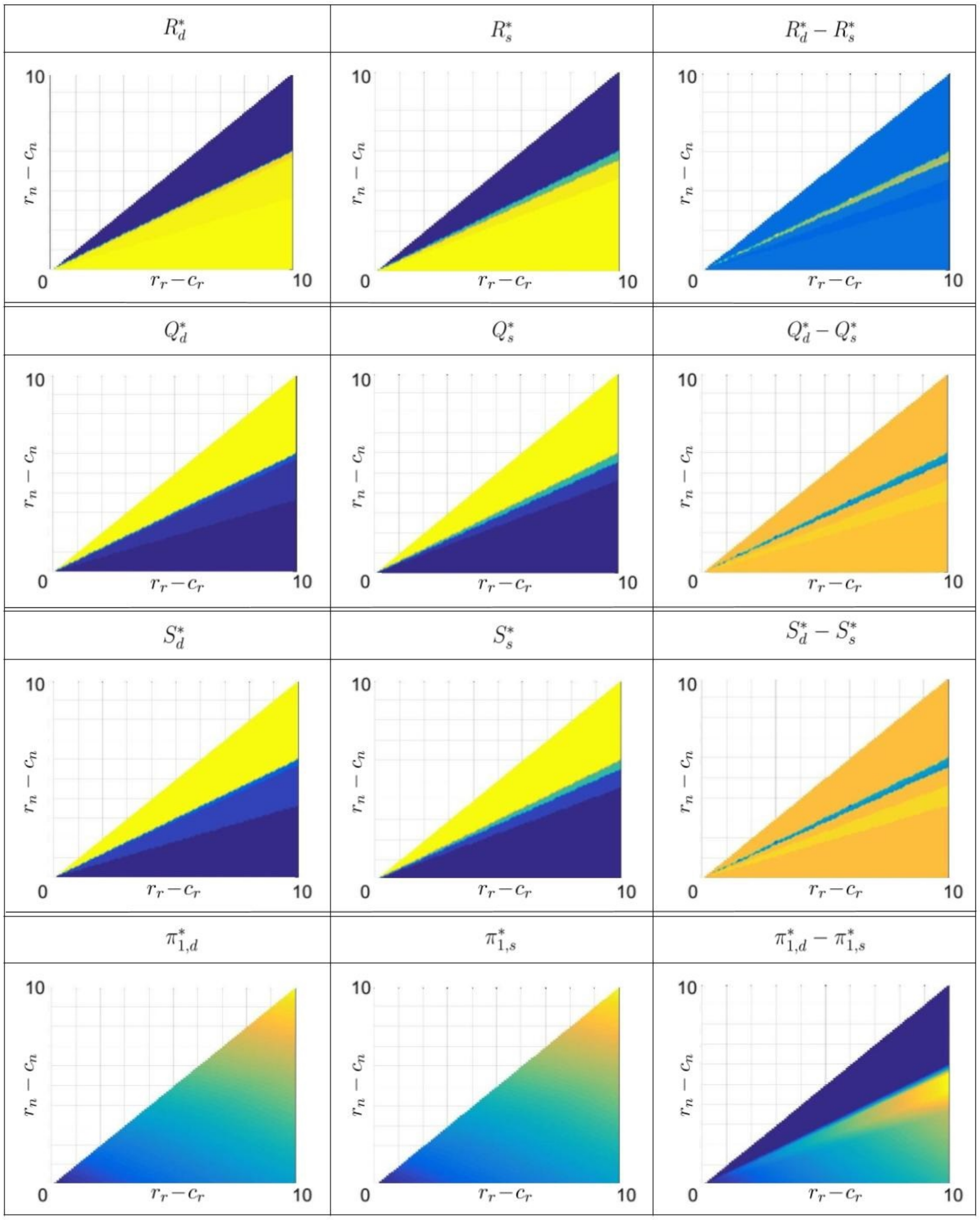

Figure 4.7: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. $(r_n - c_n)$ and $(r_r - c_r)$. R_d^* and R_s^* are the optimal expected remanufacturing volumes. Q_d^* and Q_s^* are the optimal expected total manufacturing volumes. $\pi_{1,d}^*$ and $\pi_{1,s}^*$ are the optimal expected total profits. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 1$, $p = 0.7$, $q = 0.3$. Darker color indicates a lower value.

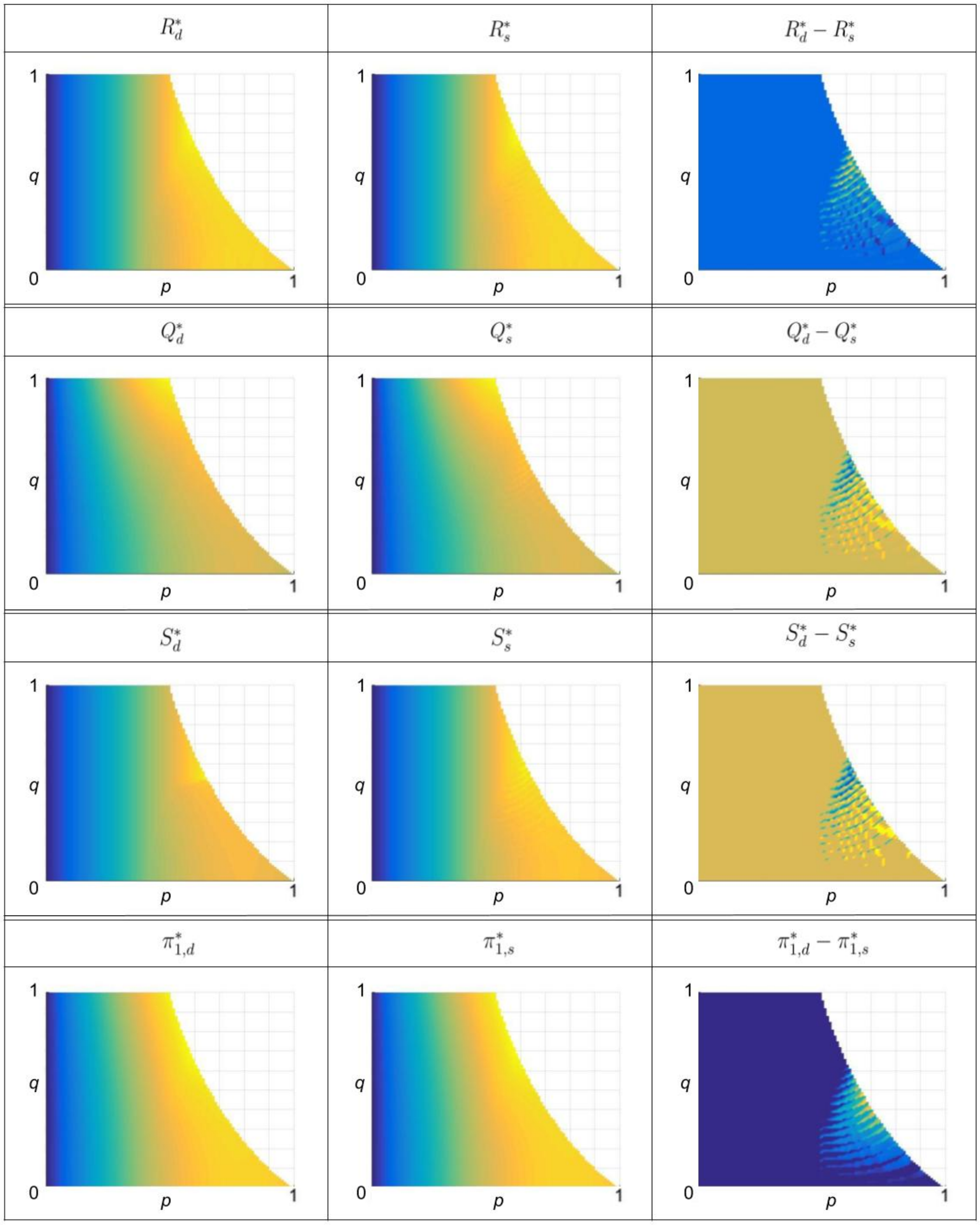

Figure 4.8: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. p and q. R_d^* and R_s^* are the optimal expected remanufacturing volumes. Q_d^* and Q_s^* are the optimal expected total manufacturing volumes. $\pi_{1,d}^*$ and $\pi_{1,s}^*$ are the optimal expected total profits. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 1$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value. The maximum possible total demand (due to diffusion) is no larger than 100 in the shaded region.

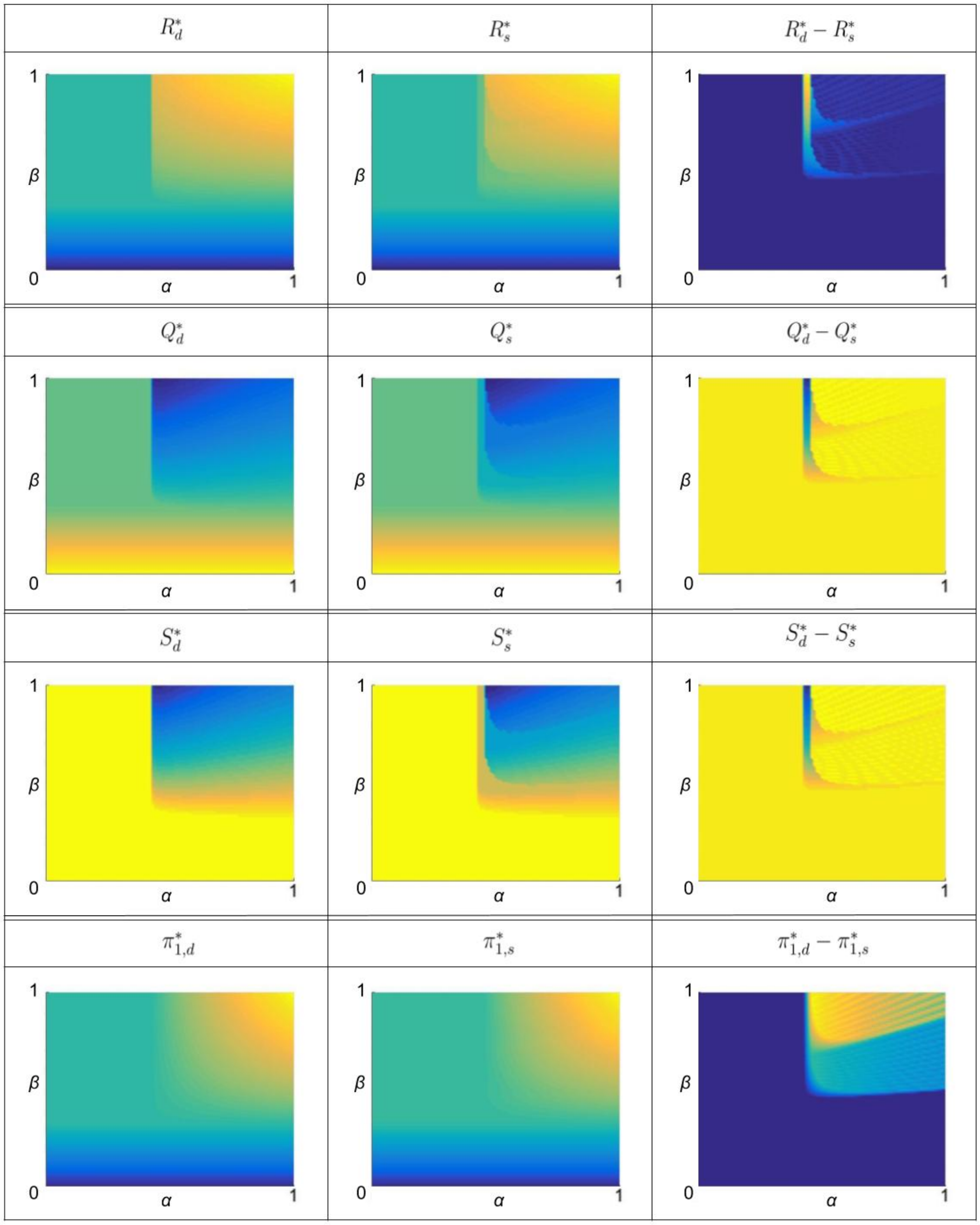

Figure 4.9: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. α and β. R_d^* and R_s^* are the optimal expected remanufacturing volumes. Q_d^* and Q_s^* are the optimal expected total manufacturing volumes. $\pi_{1,d}^*$ and $\pi_{1,s}^*$ are the optimal expected total profits. $\gamma = 1$, $p = 0.7$, $q = 0.3$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value.

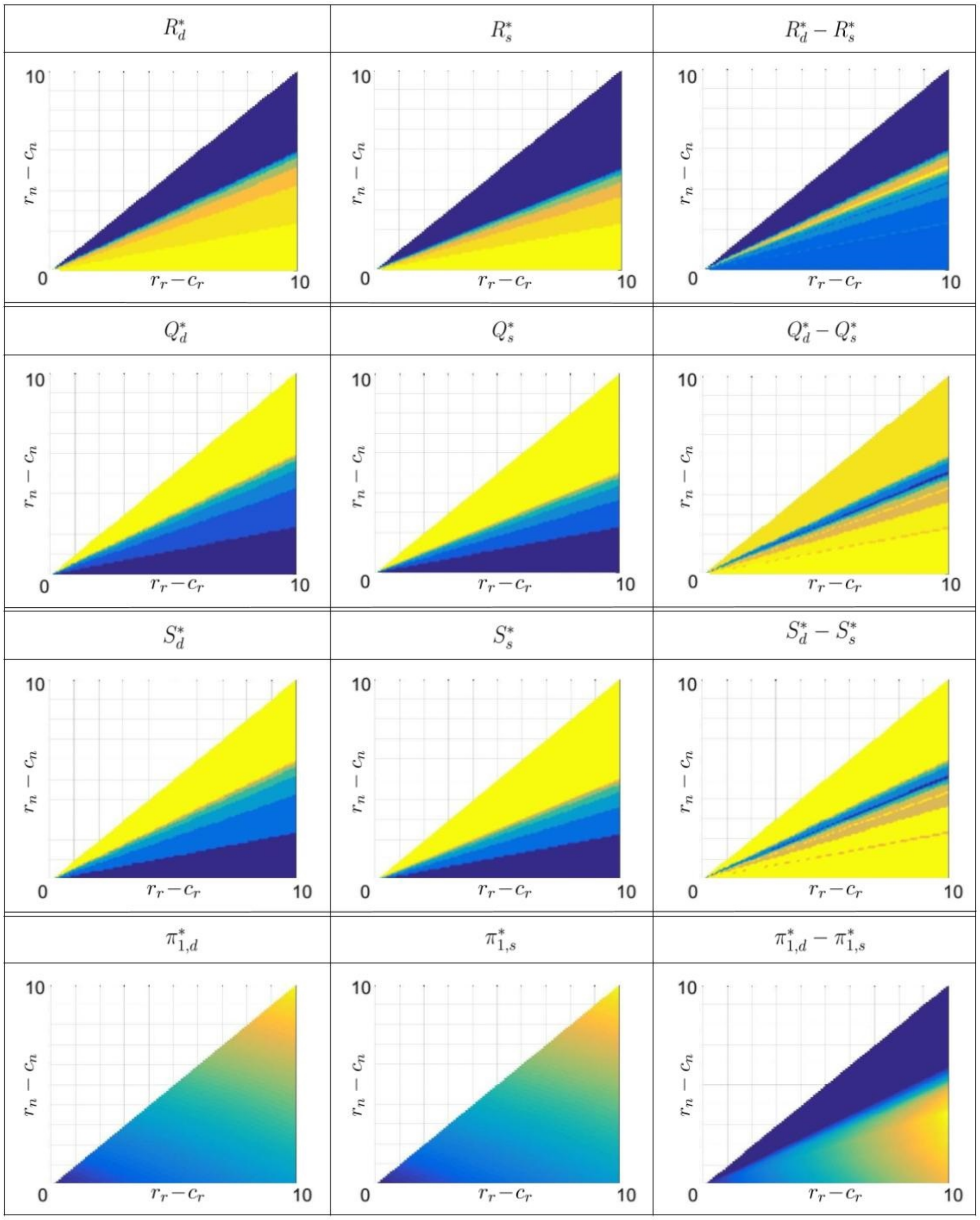

Figure 4.10: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. $(r_n - c_n)$ and $(r_r - c_r)$. R_d^* and R_s^* are the optimal expected remanufacturing volumes. Q_d^* and Q_s^* are the optimal expected total manufacturing volumes. $\pi_{1,d}^*$ and $\pi_{1,s}^*$ are the optimal expected total profits. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 1$, $p = 0.7$, $q = 0.3$. Darker color indicates a lower value.

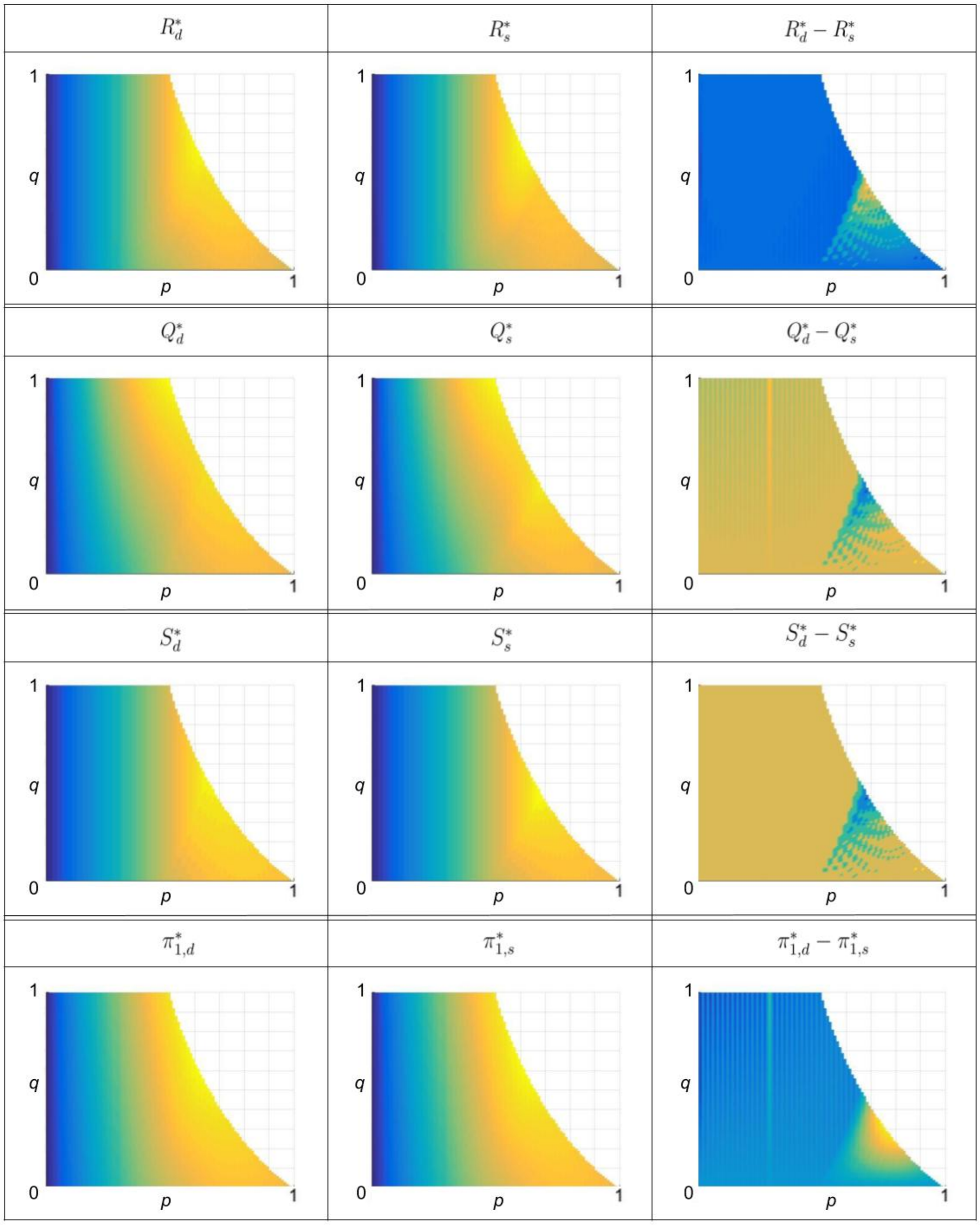

Figure 4.11: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. p and q. R_d^* and R_s^* are the optimal expected remanufacturing volumes. Q_d^* and Q_s^* are the optimal expected total manufacturing volumes. $\pi_{1,d}^*$ and $\pi_{1,s}^*$ are the optimal expected total profits. $\alpha = 0.6$, $\beta = 0.6$, $\gamma = 1$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value. The maximum possible total demand (due to diffusion) is no larger than 100 in the shaded region.

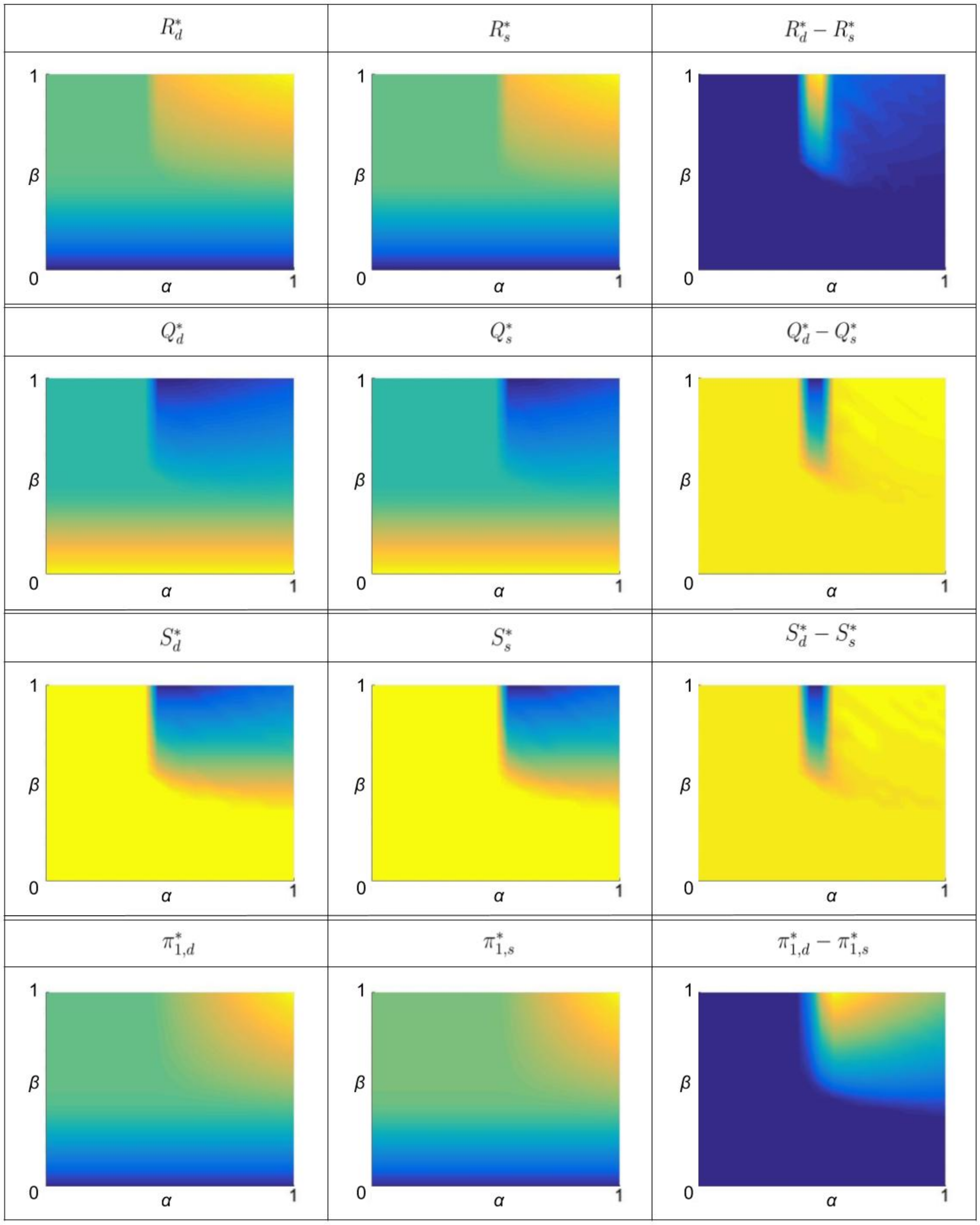

Figure 4.12: 2D projections of 3D graphs of R_d^*, R_s^*, $(R_d^* - R_s^*)$, Q_d^*, Q_s^*, $(Q_d^* - Q_s^*)$, S_d^*, S_s^*, $(S_d^* - S_s^*)$, $\pi_{1,d}^*$, $\pi_{1,s}^*$, $(\pi_{1,d}^* - \pi_{1,s}^*)$ vs. α and β. R_d^* and R_s^* are the optimal expected remanufacturing volumes. Q_d^* and Q_s^* are the optimal expected total manufacturing volumes. $\pi_{1,d}^*$ and $\pi_{1,s}^*$ are the optimal expected total profits. $\gamma = 1$, $p = 0.7$, $q = 0.3$, $r_n - c_n = 3$, $r_r - c_r = 7$. Darker color indicates a lower value.

Chapter 5

Conclusion

We have studied the sales planning problem of a manufacturer who offers both new and remanufactured products. Demand arrives according to the Bass diffusion model over the finite product life cycle. We consider two different diffusion processes for two product types: search vs. experience goods. In both diffusion models, the manufacturer may reject some demand in the initial stage to exploit the benefit of remanufacturing in the future, improving its environmental performance. This can indeed occur when the value that can be recovered from an end-of-use product is above a threshold, which varies depending on the diffusion and consumer characteristics of the market. But the optimal sales volume remains the same as long as the value recovered stays below (or above) this threshold: From an environmental perspective, the policymakers may want to induce the firms to undertake remanufacturing by facilitating the used product collection and/or providing tax benefits to remanufactured items. We have found that such strategies can only be useful if the value recovered from an end-of-use product will exceed the threshold.

The manufacturer also rejects some demand when innovators contribute more heavily than imitators to the diffusion process. Early adopters of the product are innovators, whose used products are required for remanufacturing. Late adopters of the product are, to a large extent, imitators, who are affected by the earlier sales or demand in their purchasing decisions. The manufacturer rejects a demand in the initial stage of the life cycle if it is able to satisfy the potentially backlogged demand with a remanufactured product (from the used products of innovators), but without a large decline in the future demand (greatly shaped by imitators). This is possible when the diffusion process relies heavily on innovators rather than imitators. Finally, the manufacturer rejects some demand when both the backlogging and return rates are large. However, a very large backlogging rate may increase the initial sales volume, leading to a poorer environmental performance. This is because if the initial sales volume is low under a very large backlogging rate, the backlogged demand inflates the future demand, making it difficult to ensure a sufficient returns volume (bounded by the initial sales volume) for fulfillment of all the future demand through remanufacturing.

The manufacturer performs economically better under demand-based diffusion than under sales-based diffusion. The sales volume can be optimally reduced under a lower margin from remanufacturing, a greater imitation coefficient, or a lower backlogging rate in the demand-based diffusion, compared to the sales-based diffusion. And if it is optimal to reduce the sales only in the demand-based diffusion, the sales-based diffusion leads to a worse environmental performance than the demand-based diffusion. If optimal in both models, the reverse is true.

Future extensions of this study could allow for variable used product condition in remanufacturing operations and incorporate product acquisition decisions into our modeling framework. See Guide and Wassenhove (2001) for a discussion of product acquisition management for remanufacturing. See Galbreth and Blackburn (2006), Galbreth and Blackburn (2010), and Mutha et al. (2016) for reactive, planned, and sequential acquisition strategies, respectively. Future research could also extend our models to successive product generations and/or competitive markets. See Norton and Bass (1987) for an extension of the Bass diffusion model to successive product generations. See Majumder and Groenevelt (2001), Debo et al. (2005), Ferguson and Toktay (2006), Atasu et al. (2008), and Ovchinnikov et al. (2014) for remanufacturing under competition.

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Appendix A

Proofs of the Analytical Results

Proof of Proposition 3.1. (a) Suppose that $\frac{\beta}{\gamma} \geq 1 + q - p - pq$. This implies that $\frac{\beta S_d}{\gamma} \geq p + pq - p^2 - p^2 q + \alpha(p - S_d)$ if $S_d = p$. Note that $\frac{\beta S_d}{\gamma} \geq p + pq - p^2 - p^2 q + \alpha(p - S_d)$ if and only if $S_d \geq \frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}}$. Let $\tilde{S}_d = \frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}} \leq p$. The optimal total profit can be written as

$$
\begin{aligned}
\pi^*_{1,d} = \max\Bigg\{ & \max_{p \geq S_d \geq \tilde{S}_d} \{(r_n - c_n)S_d + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2 q \\
& + \alpha(p - S_d))\}, \max_{\tilde{S}_d \geq S_d \geq 0} \{(r_n - c_n)S_d + (r_r - c_r)\beta S_d + (r_n - c_n)(p + pq - p^2 \\
& - p^2 q + \alpha(p - S_d) - \beta S_d)\} \Bigg\} \\
= \max\Bigg\{ & \max_{p \geq S_d \geq \tilde{S}_d} \{S_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2 q \\
& + \alpha p)\}, \max_{\tilde{S}_d \geq S_d \geq 0} \{S_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
& + (r_n - c_n)(p + pq - p^2 - p^2 q + \alpha p)\} \Bigg\}.
\end{aligned}
$$

- Suppose that $\alpha < \Delta$, i.e., $(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) > 0$. Thus:

$$\max_{p \geq S_d \geq \tilde{S}_d} \left\{ S_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2 q + \alpha p) \right\}$$
$$= p(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2 q + \alpha p).$$

Since $(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r) > 0$:

$$\begin{aligned}
&\max_{\tilde{S}_d \geq S_d \geq 0} \{ S_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&+ (r_n - c_n)(p + pq - p^2 - p^2 q + \alpha p) \} \\
&= \tilde{S}_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&+ (r_n - c_n)(p + pq - p^2 - p^2 q + \alpha p).
\end{aligned}$$

Since $p \geq \tilde{S}_d$:

$$\begin{aligned}
&p(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2 q + \alpha p) \\
&\geq \tilde{S}_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2 q + \alpha p) \\
&= \tilde{S}_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)\tilde{S}_d\left(\alpha + \frac{\beta}{\gamma}\right) \\
&= \tilde{S}_d\left[(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) - (r_n - c_n)\left(\alpha + \frac{\beta}{\gamma}\right)\left(1 - \frac{1}{\Delta}\right)\right] \\
&+ (r_n - c_n)\tilde{S}_d\left(\alpha + \frac{\beta}{\gamma}\right) \\
&= \tilde{S}_d\left[(1 - \alpha)(r_n - c_n) + \frac{\beta}{\gamma}\left(\frac{1}{\Delta} - 1\right)(r_n - c_n)\right] \\
&+ (r_n - c_n)\tilde{S}_d\left(\alpha + \frac{\beta}{\gamma}\right) \\
&= \tilde{S}_d\left[(1 - \alpha)(r_n - c_n) + \beta(r_r - c_r - r_n + c_n)\right] \\
&+ (r_n - c_n)\tilde{S}_d\left(\alpha + \frac{\beta}{\gamma}\right) \\
&= \tilde{S}_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&+ (r_n - c_n)(p + pq - p^2 - p^2 q + \alpha p).
\end{aligned}$$

Thus if $\alpha < \Delta$, then $S_d^* = p$.

- Now suppose that $\alpha \geq \Delta$, i.e., $(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) \leq 0$. Thus:

$$\max_{p \geq S_d \geq \tilde{S}_d} \left\{ S_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2q + \alpha p) \right\}$$
$$= \tilde{S}_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2q + \alpha p).$$

Since $(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r) > 0$:

$$\begin{aligned}
&\max_{\tilde{S}_d \geq S_d \geq 0} \{S_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&+(r_n - c_n)(p + pq - p^2 - p^2q + \alpha p)\} \\
&= \tilde{S}_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&+(r_n - c_n)(p + pq - p^2 - p^2q + \alpha p).
\end{aligned}$$

Recall that $\tilde{S}_d(r_n - c_n)\left(1 - \frac{\alpha}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p + pq - p^2 - p^2q + \alpha p) = \tilde{S}_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] + (r_n - c_n)(p + pq - p^2 - p^2q + \alpha p)$. Thus if $\alpha \geq \Delta$, then $S_d^* = \tilde{S}_d$.

Next suppose that $\frac{\beta}{\gamma} < 1 + q - p - pq$. This implies that $\tilde{S}_d = \frac{p + pq - p^2 - p^2q + \alpha p}{\alpha + \frac{\beta}{\gamma}} > p$. The optimal total profit can be written as

$$\begin{aligned}
\pi_{1,d}^* &= \max_{p \geq S_d \geq 0} \{(r_n - c_n)S_d + (r_r - c_r)\beta S_d \\
&\quad +(r_n - c_n)(p + pq - p^2 - p^2q + \alpha(p - S_d) - \beta S_d)\} \\
&= \max_{p \geq S_d \geq 0} \{S_d[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&\quad +(r_n - c_n)(p + pq - p^2 - p^2q + \alpha p)\} \\
&= p[(1 - \beta - \alpha)(r_n - c_n) + \beta(r_r - c_r)] \\
&\quad +(r_n - c_n)(p + pq - p^2 - p^2q + \alpha p).
\end{aligned}$$

Thus if $\frac{\beta}{\gamma} < 1 + q - p - pq$, then $S_d^* = p$.

(b) Suppose that $\alpha \geq \Delta$ and $\frac{\beta}{\gamma} \geq 1 + q - p - pq$: The optimal sales volume in period 1 is $S_d^* = \frac{p + pq - p^2 - p^2q + \alpha p}{\alpha + \frac{\beta}{\gamma}}$. The returns volume is $\frac{\beta(p + pq - p^2 - p^2q + \alpha p)}{\alpha + \frac{\beta}{\gamma}}$ and demand in period 2 is $p + pq - p^2 - p^2q + \alpha\left(p - \frac{p + pq - p^2 - p^2q + \alpha p}{\alpha + \frac{\beta}{\gamma}}\right)$. It can be easily shown that $\frac{\beta(p + pq - p^2 - p^2q + \alpha p)}{\alpha + \frac{\beta}{\gamma}} = \gamma\left[p + pq - p^2 - p^2q + \alpha\left(p - \frac{p + pq - p^2 - p^2q + \alpha p}{\alpha + \frac{\beta}{\gamma}}\right)\right]$.

Thus the optimal remanufacturing volume is $\beta\left(\frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}}\right)$. Now suppose that $\alpha < \Delta$ and $\frac{\beta}{\gamma} \geq 1+q-p-pq$: The optimal sales volume in period 1 is $S_d^* = p$. The returns volume is βp and demand in period 2 is $p + pq - p^2 - p^2q$. Since $\beta p \geq \gamma(p+pq-p^2-p^2q)$, the optimal remanufacturing volume is $\gamma(p+pq-p^2-p^2q)$. Lastly, suppose that $\frac{\beta}{\gamma} < 1 + q - p - pq$: The optimal sales volume in period 1 is $S_d^* = p$. The returns volume is βp and demand in period 2 is $p + pq - p^2 - p^2q$. Since $\beta p < \gamma(p + pq - p^2 - p^2q)$, the optimal remanufacturing volume is βp.

(c) Suppose that $\alpha \geq \Delta$ and $\frac{\beta}{\gamma} \geq 1 + q - p - pq$: Recall that the fraction γ of the demand in period 2 is satisfied with remanufactured products in this case. Manufactured products are used in period 1. Manufactured products are also used to satisfy the fraction $(1-\gamma)$ of the demand in period 2. Thus the optimal total manufacturing volume is $Q_d^* = \frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}} + (1-\gamma)\left[p + pq - p^2 - p^2q + \alpha\left(p - \frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}}\right)\right] = \frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}} + \left(\frac{1-\gamma}{\gamma}\right)\frac{\beta(p+pq-p^2-p^2q+\alpha p)}{\alpha+\frac{\beta}{\gamma}} = \left(\frac{\beta+\gamma-\beta\gamma}{\gamma}\right)\left(\frac{p+pq-p^2-p^2q+\alpha p}{\alpha+\frac{\beta}{\gamma}}\right)$. Now suppose that $\alpha < \Delta$ and $\frac{\beta}{\gamma} \geq 1+q-p-pq$: Recall that the fraction γ of the demand in period 2 is again satisfied with remanufactured products in this case. Manufactured products are used to satisfy all demand in period 1, but also the fraction $(1-\gamma)$ of the demand in period 2. Thus the optimal total manufacturing volume is $Q_d^* = p+(1-\gamma)(p+pq-p^2-p^2q)$. Lastly, suppose that $\frac{\beta}{\gamma} < 1+q-p-pq$: Recall that the returns volume is less than the demand for remanufactured products in period 2, i.e., $\beta p < \gamma(p + pq - p^2 - p^2q)$, in this case. Manufactured products are used to satisfy all demand in period 1, but also the demand in period 2 that is not satisfied with remanufactured products. Thus the optimal total manufacturing volume is $p + p + pq - p^2 - p^2q - \beta p$.

□

Proof of Corollary 3.2. By Proposition 3.1, $S_d^* = p$ when $\alpha = 0$ or $\beta = 0$. Note that $\Delta = 1$ if $\gamma = 0$. By Proposition 3.1, $S_d^* = p$ when $\gamma = 0$ and $\alpha < 1$. By Proposition 3.1, $S_d^* = 0$ and $\pi_{1,d}^* = (2p + pq - p^2 - p^2q)(r_n - c_n)$ when $\gamma = 0$ and $\alpha = 1$. But it can be shown that $\pi_{1,d}(p) = (2p+pq-p^2-p^2q)(r_n-c_n)$ also when

$\gamma = 0$ and $\alpha = 1$. Thus $S_d^* = p$ when $\gamma = 0$ and $\alpha = 1$.

$\square$

Proof of Corollary 3.3. Note that $2p + pq - p^2 - p^2q = 1$ implies $q = \frac{1-p}{p}$. The proof follows by plugging $q = \frac{1-p}{p}$ and $\gamma = 1$ into Proposition 3.1.

$\square$

Proof of Corollary 3.4. (a) Suppose that $\alpha \geq \Delta$. By Corollary 3.3, $S_d^* = p$ if $\beta < \frac{1-p}{p}$, i.e., $p < \frac{1}{1+\beta}$. And, again by Corollary 3.3, $S_d^* = \frac{1-p+\alpha p}{\alpha+\beta}$ if $\beta \geq \frac{1-p}{p}$, i.e., $p \geq \frac{1}{1+\beta}$. Note that $S_d^* = \frac{1-p+\alpha p}{\alpha+\beta} = p$ when $p = \frac{1}{1+\beta}$. Thus, and since $\partial\left(\frac{1-p+\alpha p}{\alpha+\beta}\right)/\partial p \leq 0$, S_d^* drops from $\frac{1}{1+\beta}$ to $\frac{\alpha}{\alpha+\beta}$ as p increases from $\frac{1}{1+\beta}$ to 1. Now suppose that $\alpha < \Delta$. By Corollary 3.3, $S_d^* = p$.

(b) Suppose that $\alpha \geq \Delta$. By Corollary 3.3, $S_d^* = p$ if $\beta < \frac{1-p}{p}$. And, again by Corollary 3.3, $S_d^* = \frac{1-p+\alpha p}{\alpha+\beta}$ if $\beta \geq \frac{1-p}{p}$: Note that $S_d^* = \frac{1-p+\alpha p}{\alpha+\beta} = p$ when $\beta = \frac{1-p}{p}$. Thus, and since $\partial\left(\frac{1-p+\alpha p}{\alpha+\beta}\right)/\partial\beta \leq 0$, S_d^* drops from p to $\frac{1-p+\alpha p}{1+\alpha}$ if β increases from $\frac{1-p}{p}$ to one. Now suppose that $\alpha < \Delta$. By Corollary 3.3, $S_d^* = p$: S_d^* is unaffected by β.

(c) Suppose that $\beta \geq \frac{1-p}{p}$. By Corollary 3.3, $S_d^* = p$ if $\alpha < \Delta$: S_d^* is unaffected by α as long as $\alpha < \Delta$. By Corollary 3.3, $S_d^* = \frac{1-p+\alpha p}{\alpha+\beta}$ if $\alpha \geq \Delta$: S_d^* drops from p to $\frac{1-p+\alpha p}{\alpha+\beta}$ if α increases to Δ. Since $\partial\left(\frac{1-p+\alpha p}{\alpha+\beta}\right)/\partial\alpha \geq 0$ when $\beta \geq \frac{1-p}{p}$, S_d^* increases as α increases from Δ. Now suppose that $\beta < \frac{1-p}{p}$. By Corollary 3.3, $S_d^* = p$: S_d^* is unaffected by α.

(d) Suppose that $\beta \geq \frac{1-p}{p}$. By Corollary 3.3, $S_d^* = p$ if $\Delta > \alpha$: S_d^* is unaffected by Δ as long as $\Delta > \alpha$. By Corollary 3.3, $S_d^* = \frac{1-p+\alpha p}{\alpha+\beta}$ if $\Delta \leq \alpha$: S_d^* drops from p to $\frac{1-p+\alpha p}{\alpha+\beta}$ if Δ drops to α. And it is unaffected by Δ as long as $\Delta \leq \alpha$. Now suppose that $\beta < \frac{1-p}{p}$. By Corollary 3.3, $S_d^* = p$: S_d^* is unaffected by Δ.

$\square$

Proof of Proposition 3.5. (a) Suppose that $\frac{\beta}{\gamma} \geq 1 + q - p - pq$. This implies that $\frac{\beta S_s}{\gamma} \geq p + qS_s - p^2 - pqS_s + \alpha(p - S_s)$ if $S_s = p$. Note that $\frac{\beta S_s}{\gamma} \geq p + qS_s - p^2 - pqS_s + \alpha(p - S_s)$ if and only if $S_s \geq \frac{p-p^2+\alpha p}{\frac{\beta}{\gamma}+\alpha+pq-q}$. Let $\tilde{S}_s = \frac{p-p^2+\alpha p}{\frac{\beta}{\gamma}+\alpha+pq-q} \leq p$. The optimal total profit can be written as

$$\pi^*_{1,s} = \max\left\{ \max_{p \geq S_s \geq \tilde{S}_s} \{(r_n - c_n)S_s + \left(\frac{r_n - c_n}{\Delta}\right)(p + qS_s - p^2 \right.$$
$$-pqS_s + \alpha(p - S_s))\}, \max_{\tilde{S}_s \geq S_s \geq 0} \{(r_n - c_n)S_s + (r_r - c_r)\beta S_s + (r_n - c_n)(p$$
$$\left. +qS_s - p^2 - pqS_s + \alpha(p - S_s) - \beta S_s)\} \right\}$$
$$= \max\left\{ \max_{p \geq S_s \geq \tilde{S}_s} \{S_s(r_n - c_n)\left(1 + \frac{q - \alpha - pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 \right.$$
$$+\alpha p)\}, \max_{\tilde{S}_s \geq S_s \geq 0} \{S_s[(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r)]$$
$$\left. +(r_n - c_n)(p - p^2 + \alpha p)\} \right\}.$$

- Suppose that $\Delta > \alpha + pq - q$, i.e., $(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) > 0$. Thus:

$$\max_{p \geq S_s \geq \tilde{S}_s} \left\{ S_s(r_n - c_n)\left(1 + \frac{q - \alpha - pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 + \alpha p) \right\}$$
$$= p(r_n - c_n)\left(1 + \frac{q - \alpha - pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 + \alpha p).$$

  Since $(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r) > 0$:

$$\max_{\tilde{S}_s \geq S_s \geq 0} \{S_s[(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r)]$$
$$+(r_n - c_n)(p - p^2 + \alpha p)\}$$
$$= \tilde{S}_s[(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r)]$$
$$+(r_n - c_n)(p - p^2 + \alpha p).$$

Since $p \geq \tilde{S}_s$:

$$\begin{aligned}
&p(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 + \alpha p) \\
&\geq \tilde{S}_s(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 + \alpha p) \\
&= \tilde{S}_s(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)\tilde{S}_s\left(\frac{\beta}{\gamma} + \alpha + pq - q\right) \\
&= \tilde{S}_s[(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) \\
&-(r_n - c_n)\left(\frac{\beta}{\gamma} + \alpha + pq - q\right)\left(1 - \frac{1}{\Delta}\right)] \\
&+(r_n - c_n)\tilde{S}_s\left(\frac{\beta}{\gamma} + \alpha + pq - q\right) \\
&= \tilde{S}_s\left[(1 + q - \alpha - pq)(r_n - c_n) + \frac{\beta}{\gamma}\left(\frac{1}{\Delta} - 1\right)(r_n - c_n)\right] \\
&+(r_n - c_n)\tilde{S}_s\left(\frac{\beta}{\gamma} + \alpha + pq - q\right) \\
&= \tilde{S}_s[(1 - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r - r_n + c_n)] \\
&+(r_n - c_n)(p - p^2 + \alpha p) \\
&= \tilde{S}_s[(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r)] \\
&+(r_n - c_n)(p - p^2 + \alpha p).
\end{aligned}$$

Thus if $\Delta > \alpha + pq - q$, then $S_s^* = p$.

- Now suppose that $\Delta \leq \alpha + pq - q$, i.e., $(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) \leq 0$. Thus:

$$\begin{aligned}
&\max_{p \geq S_s \geq \tilde{S}_s}\left\{S_s(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 + \alpha p)\right\} \\
&= \tilde{S}_s(r_n - c_n)\left(1 + \frac{q-\alpha-pq}{\Delta}\right) + \left(\frac{r_n - c_n}{\Delta}\right)(p - p^2 + \alpha p).
\end{aligned}$$

Since $(1 - \beta - \alpha + q - pq)(r - c_n) + \beta(r - c_r) > 0$:

$$\begin{aligned}
&\max_{\tilde{S}_s \geq S_s \geq 0}\{S_s[(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r)] \\
&+(r_n - c_n)(p - p^2 + \alpha p)\} \\
&= \tilde{S}_s[(1 - \beta - \alpha + q - pq)(r_n - c_n) + \beta(r_r - c_r)] \\
&+(r_n - c_n)(p - p^2 + \alpha p).
\end{aligned}$$

Recall that $\tilde{S}_s(r_n-c_n)\left(1+\frac{q-\alpha-pq}{\Delta}\right)+\left(\frac{r_n-c_n}{\Delta}\right)(p-p^2+\alpha p)=\tilde{S}_s[(1-\beta-\alpha+q-pq)(r_n-c_n)+\beta(r_r-c_r)]+(r_n-c_n)(p-p^2+\alpha p)$. Thus if $\Delta \leq \alpha+pq-q$, then $S_s^*=\tilde{S}_s$.

Next suppose that $\frac{\beta}{\gamma}<1+q-p-pq$. This implies that $S_s(\frac{\beta}{\gamma}+\alpha+pq-q)<p-p^2+\alpha p$ for all $S_s \leq p$. The optimal total profit can be written as

$$\begin{aligned}
\pi_{1,s}^* &= \max_{p\geq S_s\geq 0}\{(r_n-c_n)S_s+(r_r-c_r)\beta S_s \\
&\quad +(r_n-c_n)\left(p+qS_s-p^2-pqS_s+\alpha(p-S_s)-\beta S_s\right)\} \\
&= \max_{p\geq S_s\geq 0}\{S_s[(1-\beta-\alpha+q-pq)(r_n-c_n)+\beta(r_r-c_r)] \\
&\quad +(r_n-c_n)(p-p^2+\alpha p)\} \\
&= p[(1-\beta-\alpha+q-pq)(r_n-c_n)+\beta(r_r-c_r)]+(r_n-c_n)(p-p^2+\alpha p).
\end{aligned}$$

Thus if $\frac{\beta}{\gamma}<1+q-p-pq$, then $S_s^*=p$.

(b) Suppose that $\alpha \geq \Delta+q-pq$ and $\frac{\beta}{\gamma}\geq 1+q-p-pq$: The optimal sales volume in period 1 is $S_s^*=\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}$. The returns volume is $\frac{\beta(p-p^2+\alpha p)}{\alpha+\frac{\beta}{\gamma}+pq-q}$ and demand in period 2 is $p+q\left(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)-p^2-pq\left(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)+\alpha\left(p-\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)$. It can be easily shown that $\frac{\beta(p-p^2+\alpha p)}{\alpha+\frac{\beta}{\gamma}+pq-q}=\gamma\left(p+q\left(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)-p^2-pq\left(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)+\alpha\left(p-\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)\right)$. Thus the optimal remanufacturing volume is $\beta\left(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}\right)$. Now suppose that $\alpha<\Delta+q-pq$ and $\frac{\beta}{\gamma}\geq 1+q-p-pq$: The optimal sales volume in period 1 is $S_s^*=p$. The returns volume is βp and demand in period 2 is $p+pq-p^2-p^2q$. Since $\beta p\geq\gamma(p+pq-p^2-p^2q)$, the optimal remanufacturing volume is $\gamma(p+pq-p^2-p^2q)$. Lastly, suppose that $\frac{\beta}{\gamma}<1+q-p-pq$: The optimal sales volume in period 1 is $S_s^*=p$. The returns volume is βp and demand in period 2 is $p+pq-p^2-p^2q$. Since $\beta p<\gamma(p+pq-p^2-p^2q)$, the optimal remanufacturing volume is βp.

(c) Suppose that $\alpha\geq\Delta+q-pq$ and $\frac{\beta}{\gamma}\geq 1+q-p-pq$: Recall that the fraction γ of the demand in period 2 is satisfied with remanufactured products in this case. Manufactured products are used in period 1. Manufactured products are also used to satisfy the fraction $(1-\gamma)$ of the demand in period 2. Thus the optimal total manufacturing volume is $Q_s^* = \frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q} + (1-\gamma)(p+q(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}) - p^2 - pq(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q}) + \alpha(p - \frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q})) = (\frac{\beta+\gamma-\beta\gamma}{\gamma})(\frac{p-p^2+\alpha p}{\alpha+\frac{\beta}{\gamma}+pq-q})$. Now suppose that $\alpha < \Delta+q-pq$ and $\frac{\beta}{\gamma} \geq 1+q-p-pq$: Recall that the fraction γ of the demand in period 2 is again satisfied with remanufactured products in this case. Manufactured products are used to satisfy all demand in period 1, but also the fraction $(1-\gamma)$ of the demand in period 2. Thus the optimal total manufacturing volume is $Q_s^* = p+(1-\gamma)(p+pq-p^2-p^2q)$. Lastly, suppose that $\frac{\beta}{\gamma} < 1+q-p-pq$: Recall that the returns volume is less than the demand for remanufactured products in period 2, i.e., $\beta p < \gamma(p+pq-p^2-p^2q)$, in this case. Manufactured products are used to satisfy all demand in period 1, but also the demand in period 2 that is not satisfied with remanufactured products. Thus the optimal total manufacturing volume in periods 1-2 is $p+p+pq-p^2-p^2q-\beta p$.

□

Proof of Corollary 3.6. By Proposition 3.5, $S_s^* = p$ when $\alpha = 0$ or $\beta = 0$. Note that $\Delta = 1$ if $\gamma = 0$. By Proposition 3.5, $S_s^* = p$ when $\gamma = 0$ and $\alpha < 1+q-pq$. By Proposition 3.5, $S_s^* = 0$ and $\pi_{1,s}^* = (2p-p^2)(r_n - c_n)$ when $\gamma = 0$ and $\alpha \geq 1+q-pq$ (which is possible only when $q-pq = 0$ and $\alpha = 1$). But it can be shown that $\pi_{1,s}(p) = (2p-p^2)(r_n-c_n)$ also when $\gamma = 0$ and $\alpha = 1$. Thus $S_s^* = p$ when $\gamma = 0$ and $\alpha = 1$.

□

Proof of Corollary 3.7. Note that $2p+pq-p^2-p^2q = 1$ implies $q = \frac{1-p}{p}$. The proof follows by plugging $q = \frac{1-p}{p}$ and $\gamma = 1$ into Proposition 3.5.

□

Proof of Corollary 3.8. (a) Suppose that $\alpha \geq \Delta + \frac{(1-p)^2}{p}$. By Corollary 3.7, $S_s^* = p$ if $\beta < \frac{1-p}{p}$. And, again by Corollary 3.7, $S_s^* = \frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}$ if $\beta \geq \frac{1-p}{p}$: Note that $S_s^* = \frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1} = p$ when $\beta = \frac{1-p}{p}$. Thus, and since $\partial\left(\frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}\right)/\partial\beta <$

0, S_s^* drops from p to $\frac{(1+\alpha)p^2-p^3}{(3+\alpha)p-p^2-1}$ if β increases from $\frac{1-p}{p}$ to one. Now suppose that $\alpha < \Delta + \frac{(1-p)^2}{p}$. By Corollary 3.7, $S_s^* = p$: S_s^* is unaffected by β.

(b) Suppose that $\beta \geq \frac{1-p}{p}$. By Corollary 3.7, $S_s^* = p$ if $\alpha < \Delta + \frac{(1-p)^2}{p}$: S_s^* is unaffected by α as long as $\alpha < \Delta + \frac{(1-p)^2}{p}$. By Corollary 3.7, $S_s^* = \frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}$ if $\alpha \geq \Delta + \frac{(1-p)^2}{p}$: S_s^* drops from p to $\frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}$ if α increases to $\Delta + \frac{(1-p)^2}{p}$. Since $\partial\left(\frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}\right)/\partial\alpha \geq 0$ when $\beta \geq \frac{1-p}{p}$, S_s^* increases as α increases from $\Delta + \frac{(1-p)^2}{p}$. Now suppose that $\beta < \frac{1-p}{p}$. By Corollary 3.7, $S_s^* = p$: S_s^* is unaffected by α.

(c) Suppose that $\beta \geq \frac{1-p}{p}$. By Corollary 3.7, $S_s^* = p$ if $\Delta > \alpha - \frac{(1-p)^2}{p}$: S_s^* is unaffected by Δ as long as $\Delta > \alpha - \frac{(1-p)^2}{p}$. By Corollary 3.7, $S_s^* = \frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}$ if $\Delta \leq \alpha - \frac{(1-p)^2}{p}$: S_s^* drops from p to $\frac{(1+\alpha)p^2-p^3}{(2+\alpha+\beta)p-p^2-1}$ if Δ drops to $\alpha - \frac{(1-p)^2}{p}$. And it is unaffected by Δ as long as $\Delta \leq \alpha - \frac{(1-p)^2}{p}$. Now suppose that $\beta < \frac{1-p}{p}$. By Corollary 3.7, $S_s^* = p$: S_s^* is unaffected by Δ.

$\square$