Production, Manufacturing, Transportation and Logistics

# Planning sustainable routes: Economic, environmental and welfare concerns 

Okan Dukkanci ${ }^{\text {a,* }}$, Özlem Karsu ${ }^{\text {b }}$, Bahar Y. Kara ${ }^{\text {b }}$<br>${ }^{\text {a }}$ Faculty of Business Administration and Economics, European University Viadrina, 15230 Frankfurt (Oder), Germany<br>${ }^{\mathrm{b}}$ Department of Industrial Engineering, Bilkent University, Ankara 06800, Turkey

## A R TICLE I N F O

## Article history:

Received 2 December 2020
Accepted 25 September 2021
Available online xxx

## Keywords:

Routing
Sustainability
Multi-objective optimization
Fuel consumption
Customer and driver welfare


#### Abstract

We introduce a problem called the Sustainable Vehicle Routing Problem (SVRP) in which the sustainability notion is considered in terms of economic, environmental and social impacts. Inspired by real-world problems that large cargo companies face for their delivery decisions, we introduce a new facet to the classical vehicle routing problem by considering the welfare of all three stakeholders of the problem: an environmentally conscious company, the drivers, and the indistinguishable customers, as our setting assumes that all customers belong to the same delivery class. Thus, the proposed problem consists of three objective functions. The first one is to minimize the total fuel consumption and emission to represent the companies' main economic and environmental concerns. The second one is to maximize total welfare of the drivers through a function that encourages equitable payment across drivers while encouraging low total driver cost and the third one is to maximize total welfare of the customers through a function that encourages fairness in terms of delivery times. The last two objectives are measured using slots for tour lengths and delivery times. We implement an efficient solution approach based on the $\epsilon$ constraint scalarization to find the nondominated solutions of our triobjective optimization problem and present computational analysis that provide insights on the trade-off between the objectives. Our experiments demonstrate the potential of the suggested framework under the customer anonymity assumption to help decision makers make effective plans that all parties involved would give consent to.


© 2021 Elsevier B.V. All rights reserved.

## 1. Introduction and background

Vehicle routing problems (VRPs) have been around for decades and many variations have been defined. However, most of these studies only focus on the economic consequences of transporting goods from one point to another. Although in theory, logistics companies seem to have the only lead role in this problem, in practice, two significant supporting roles; customers and drivers also would like to protect their own interest. Therefore, companies have started to consider a more holistic point of view on their delivery operations by paying attention to other consequences such as environmental and social at the expense of additional economic burden (DHL, 2015), which extends the classical single objective setting to a multiobjective one.

Our research proposes a new sustainable routing system that focuses on economic, environmental and social impacts of freight transportation activities that can be adapted by real world cargo delivery companies that serve indistinguishable customers. In our

[^0]setting, the customers are considered as entities that are entitled to equitable service from the perspective of the company. They are delivery points, mostly individuals that the items have to delivered. We assume anonymity for customers in such an application, which can be also adapted by cargo companies that have delivery classes such as "Regular", "Prime", "Express" and "Over-Night" delivery. We assume that customers that we consider belong to one of the aforementioned delivery classes. Then, the service times to these customers are categorized in time slots and the company aims to finalize the deliveries at the earliest possible slot. Once the planning is finalized the company notifies the customers about their delivery slot. Here, avoiding extreme inequality in delivery times is a significant concern due to its effects on customer satisfaction. Hence, the overall aim is being efficient and fair in delivery time planning, which is ensured by serving as less customers as possible in the latest slots. Time slots/windows concepts have been previously used in VRP (Perugia, Moccia, Cordeau, \& Laporte, 2011) including the dial-a-ride problem (Matl, Hartl, \& Vidal, 2018) to ensure that equity between customers is considered. Several logistics companies around the world pay their drivers based on their mileage (Rodriguez, Rocha, Khattak, \& Belzer, 2003) and perform deliveries to their customers with a motivation to serve them as
soon as possible. The measure utilizing drivers mileage is considered via the mileage slots that the drivers are serving to. Since the payments are based on these slots, the company wants to ensure fairness among the drivers and determine an equitable allocation of mileage while keeping the total payment low. Thus, the proposed system is actually planning the routing decisions while considering three different performance measures: being economically sound and environmentally friendly; being fair to the drivers in terms of payments; and being fair to the customers in terms of delivery times (that are declared and notified after planning).

For the main objective, we use a fuel consumption function, which implicitly encourages (fuel) cost minimization for the company, hence both economic and environmental concerns are incorporate. In the last 10 years, due to raising concerns about the climate change and the global warming, environmental impacts of the transportation related activities have been considered in several VRPs. In these studies, environmental impacts are measured in terms of the amount of fuel consumed, which is directly proportional to $\mathrm{CO}_{2}$ emissions (Demir, Bektaş, \& Laporte, 2014b). The resulting functions do not only consider travelled distance but also include other factors such as vehicle payload and vehicle speed that affect the fuel consumption (Sbihi \& Eglese, 2010). Demir et al. (2014b) categorized these factors as vehicle, environment, traffic, driver and operations related factors and also presented different types of fuel consumption models in the literature. While estimating the amount of fuel consumed, including these factors increases the accuracy of the estimation, but it also increases the complexity of the fuel consumption model. Each actor mentioned above has an incentive to pay attention to economic and environmental impacts as it is crucial for everyone to have a sustainable environment in the future.

The first study that considers fuel consumption minimization in a routing problem is Kara, Kara, \& Yetis (2007), which introduced the "Green routing" concept by minimizing a cost function depending on both the distance traveled and also the load of the vehicle. Bektaş \& Laporte (2011) introduced the Pollution Routing Problem (PRP) with a more accurate fuel consumption model that considers speed and load as decisions. Variants of the "Green routing" problems or PRPs have since been studied, with efforts to address various factors observed in real-life such as time-dependency (Jabali, Woensel, \& de Kok, 2012), heterogeneous fleets (Koç, Bektaş, Jabali, \& Laporte, 2014), more than one objectives (Demir, Bektaş, \& Laporte, 2014a), pickup and delivery (Zachariadis, Tarantilis, \& Kiranoudis, 2015), inventory consideration (Mirzapour Al-e hashem \& Rekik, 2014), and location decisions (Dukkanci, Kara, \& Bektaş, 2019b). For comprehensive surveys on green routing and green network design problems, we refer the reader to Demir et al. (2014b) and Dukkanci, Bektaş, \& Kara (2019a).

Fairness concerns have been considered in many OR applications recently since they naturally arise in real life problems in different domains (Karsu \& Morton, 2015; Matl et al., 2018). Fairness concerns can be incorporated into mathematical models in a number of ways: One can use an inequality index and ensure that the index is optimized in an objective function, or bounded in a constraint to avoid extreme inequality. The classical minmax objectives or minmax type constraints is a typical example of this approach. Another method would be formulating the problem as a multiobjective optimization problem (MOP) in which the amount allocated to each entity is minimized (or maximized if a good is distributed). This approach is relatively unpopular since the corresponding MOP may be too difficult to handle in reasonable time. If there are $n$ entities and a single commodity, then a $n$-objective model will be formulated.

The third method would be defining a social welfare function (an equitable aggregation function) that encourages fair allocations over entities. In this work, we take this approach to ensure that
the customer delivery times and driver payments are equitably distributed. Note that for a function to be an equitable aggregation function, it should be in line with an equitable preference model and hence satisfy some well-defined properties (Argyris, Özlem Karsu, \& Yavuz, 2021; Karsu, Morton, \& Argyris, 2018; Kostreva, Ogryczak, \& Wierzbicki, 2004). First of all, it should be symmetric (this is a direct result of the anonymity assumption over the entities.). Recall that customers in the same delivery class is considered in our problem. Second, it should be in line with PigouDalton principle of transfers, which dictates that a transfer from a worse-off entity to a better-off one, which does not change the relative positions of these entities, should be preferred. Moreover, since such a function will encompass both efficiency and fairness concerns, it should be a nonincreasing function of the allocated amounts.

In line with the trend of acknowledging fairness in OR settings, more studies have started to consider social impacts of transportation activities in VRPs, fairness among drivers being one of them. As is the case with our proposed problem, to avoid any injustice between drivers who are usually paid by the distance that they need to travel, balancing the routes has been observed as an important challenge. Matl et al. (2018) presented a comprehensive survey and analysis for workload equity in VRPs. This study provided not only an extensive review on the related literature but also theoretical and numerical analysis on vehicle routing problems with equity objectives. The authors classified the literature based on the types of equity function, equity metric, optimization model and method. In terms of the equity function, the literature is divided into three categories; range, min-max and other.

The studies where equity is measured based on the range (the difference between the shortest and the longest tour) of the tours generally consider multi-objective optimization models by including equity as a new objective into the problem. Jozefowiez, Semet, \& Talbi (2002) introduce the route balancing concept into the VRP and evaluate the performance of several heuristic algorithms on the resulting bi-objective problem. To solve this problem, several heuristic algorithms are proposed (Jozefowiez, Semet, \& Talbi, 2007; 2009; Lacomme, Prins, Prodhon, \& Ren, 2015; Oyola \& Løkketangen, 2014). The studies using the min-max criterion as an equity function usually take the length of the longest (cost of the most expensive) tour as the primary objective and formulate a single objective optimization problem. Golden, Laporte, \& Taillard (1997) is the first study that considers the min-max objective in the VRP and the authors developed a tabu search based adaptive memory heuristic algorithm. Bertazzi, Golden, \& Wang (2015) compared different variants of the classical VRP and the min-max VRP and provided a worst-case analysis.

Huang, Smilowitz, \& Balcik (2012) studied a relief routing problem for a humanitarian setting. The authors analyse the impact of efficiency, efficacy and equity objectives on vehicle routes. For the latter objective, they define three different equity functions including a piecewise disutility function for unsatisfied deliveries. Halvorsen-Weare \& Savelsbergh (2016) is the first study that analyses the impacts of different equity functions on a bi-objective problem. The authors presented a bi-objective mixed capacitated general routing problem that minimizes the total cost and four different equity objectives. Lehuédé, Péton, \& Tricoire (2020) investigated a lexicographic minimax approach to solve a bi-objective vehicle routing problem with route balancing. Mancini, Gansterer, \& Hartl (2021) analysed multi-period collaborations between carriers in a vehicle routing problem where time and service consistency, and workload balance are considered. The workload balance among carriers is achieved by ensuring that the number of customer assigned to a carrier cannot be less than a minimum value set by the carrier. Campbell, Vandenbussche, \& Hermann (2008) studied two variants of the VRP for a post-disaster appli-

Table 1
Studies related to the SVRP.

|  | General |  | Fairness |  |  |  |  | Fuel Consumption |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Equity Function |  |  |  |  |  |  |  |  |
| References | \# of Obj. | Sol. A. | Cust. | Dri. | Range | Min-max | Other | Emission | Model |
| Kara et al. (2007) | One | E |  |  |  |  |  | $\checkmark$ | Factor |
| Bektaş \& Laporte (2011) | One | E |  |  |  |  |  | $\checkmark$ | Micro |
| Demir et al. (2014a) | Two | E\&H |  |  |  |  |  | $\checkmark$ | Micro |
| Jozefowiez et al. (2002) | Two | H |  | $\checkmark$ | $\checkmark$ |  |  |  |  |
| Golden et al. (1997) | One | H |  | $\checkmark$ |  | $\checkmark$ |  |  |  |
| Bertazzi et al. (2015) | One | - |  | $\checkmark$ |  | $\checkmark$ |  |  |  |
| Huang et al. (2012) | One | E | $\checkmark$ |  |  |  | $\checkmark$ |  |  |
| Halvorsen-Weare \& Savelsbergh (2016) | Two | E |  | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |
| Lehuédé et al. (2020) | Two | H |  | $\checkmark$ |  |  | $\checkmark$ |  |  |
| Mancini et al. (2021) | One | E\&H |  |  |  |  | $\checkmark$ |  |  |
| Campbell et al. (2008) | One | E\&H | $\checkmark$ |  |  | $\checkmark$ | $\checkmark$ |  |  |
| Eisenhandler \& Tzur (2019) | One | E\&H | $\checkmark$ |  |  |  | $\checkmark$ |  |  |
| Abdullahi et al. (2021) | Three | H |  |  |  |  |  | $\checkmark$ | Factor |
| Our study | Three | E | $\checkmark$ | $\checkmark$ |  |  | $\checkmark$ | $\checkmark$ | Micro |

cation where the first one minimizes the maximum arrival time of critical supplies and the second one minimizes the average arrival time. Eisenhandler \& Tzur (2019) studied a routing and allocation problem for a humanitarian application that includes collecting food donations and delivering them to food relief agencies. The problem aims to maximize the total amount food distributed and also be fair in the allocation of food. The authors aggregate these two concerns by using an objective function, in which the total amount is multiplied by a measure of equity (namely $1 \mathrm{mi}-$ nus the Gini index of the allocation) (Blackorby \& Donaldson, 1978; Ebert, 1987). In addition to several humanitarian applications, the studies that consider real-life problems with a fairness objective are waste collection (Kim, Kim, \& Sahoo, 2006), school bus routing (Li \& Fu, 2002), and home-to-work bus service (Perugia et al., 2011).

Abdullahi, Reyes-Rubiano, Ouelhadj, Faulin, \& Juan (2021) studied a sustainable VRP where they also consider the economic, environmental and social impacts of freight transportation. In particular, the economic impacts include the fixed, variable and fuel cost of vehicles; the environmental impacts account for the fuel consumption based on a factor emission model and social impacts cover the risk related to traffic accidents instead of a fairness objective as in our problem. As a solution approach, the authors implemented a Biased-Randomised Iterated Greedy with Local Search algorithm.

For comprehensive surveys on vehicle routing problems and multi-objective routing problems, we refer the reader to Vidal, Laporte, \& Matl (2020) and Zajac \& Huber (2021), respectively.

Table 1 presents a summary of the related literature and categorizes the studies based on the following factors: (i) the number of objective functions (\# of Obj.), (ii) the proposed solution approaches (Sol. A.); exact (E) and/or heuristics (H), (iii, iv) having an equity function for customers (Cust.) and drivers (Dri.), respectively, ( v , vi, vii) the type of equity function (Range, Min-max or Other), (viii) including environmental impact (Emission) and (ix) the type of fuel consumption model used (Model). The table also demonstrates how our study fits into the current literature and shows that the proposed problem aims to fill a gap in the VRP literature by considering three objectives that cover economic, environmental and social impacts of transportation activities.

In this study, we introduce the Sustainable Vehicle Routing Problem (SVRP), an extension of the classical VRP, in which economic (fuel and driver cost), environmental ( $\mathrm{CO}_{2}$ emission) and social impacts (fairness to drivers and customers) are considered. The proposed SVRP consists of three objective functions; (i) to minimize the total amount of fuel consumption ( $\mathrm{CO}_{2}$ emissions), (ii) to
maximize welfare of the drivers and (iii) to maximize welfare of the customers.

As we elaborate in the upcoming sections, economic impacts are inherently addressed in the first and second objective functions as fuel and driver cost, respectively. We quantify the environmental impacts as $\mathrm{CO}_{2}$ emission by using a fuel consumption model. The fairness concern for the drivers is motivated by the observation that they are generally paid by the distance they travel. Any imbalance between tour lengths leads to unequal payments to the drivers. In order to ensure a fair and economically efficient payment system among drivers, we propose a profit function that awards shorter and balanced tours. Welfare of customers is measured using a function that favors quick and fair deliveries among customers in the same delivery class. The proposed welfare function is chosen as an equitable aggregation function of delivery time allocations to customers. Motivated by the observation that unhappy customers are more likely to remember this experience and take action, we lexicographically minimize the number of customers who receive their delivery in the late delivery slots of the day.

The contributions of this paper are as follows: (i) we define novel welfare functions for both customers and drivers that encourage both fairness and efficiency, (ii) we present the first multiobjective approach that considers economic, environmental and social impacts simultaneously from perspectives of all parties including companies, drivers and customers for a routing problem, (iii) we implement an efficient exact algorithm to find the nondominated solutions of this problem and conduct computational experiments on real road networks to analyse the trade-offs between three objectives.

The remainder of this paper is constructed as follows: Section 2 presents the problem definition including the description of emission model, customer and driver welfare functions. Section 3 provides a multi-objective mathematical model of the SVRP. Section 4 presents an exact solution approach to solve the SVRP and an illustration of this solution approach on an example. Computational results are discussed in Section 5, and conclusions and future research directions are given in Section 6.

## 2. Formal problem definition

The SVRP is defined on a complete directed graph $G=(N, A)$, where $N=\{0,1, \ldots, n\}$ denotes the set of customers (nodes), including the depot ( 0 ) and $A=\{(i, j): i, j \in N, i \neq j\}$ is the set of arcs. A fleet of $m$ identical vehicles, each with capacity $C$ serves the customers. The distance on $\operatorname{arc}(i, j) \in A$ is denoted by $d_{i j}$. Each
customer $i \in N$ has a nonnegative demand $q_{i}$. The vehicle speed is denoted by $v$.

The set of time slots for customers is denoted by $L^{C}$ and we let $l_{C}$ be the number of time slots considered for customers (i.e., the cardinality of set $L^{C}$ ). Similarly, $L^{D}$ and $l_{D}$ denote the set and number of route length slots for drivers. For instance, let us assume that the time slots for customers are as follows: 0 to 3,3 to 6 and 6 to 9 hours. Then, the set $L^{C}$ can be defined as $L^{C}=\{1,2,3\}$ and $l_{C}$ equals to 3 .

We maximize an equity-encouraging welfare function for the customers, where the welfare contribution received when delivery is made on time slot $l$ is denoted by $p_{l}^{c}$ for a customer. This could be seen as the amount of utility received by a customer when her delivery is made on time slot $l$. Similarly, in our equityencouraging welfare function for the drivers, the welfare contribution when route length of a driver is in a route length slot $l$ is $p_{l}^{D}$. This welfare contribution can be seen as the amount of utility that the central planner has when the length of the route of a driver is in slot $l$.

In the next sections, we explain the ways we define economic, environmental, and social impacts in terms of fuel consumption, $\mathrm{CO}_{2}$ emissions and fairness, respectively.

### 2.1. Fuel consumption and $\mathrm{CO}_{2}$ emissions

The model that we use to estimate the amount of fuel consumption and $\mathrm{CO}_{2}$ emissions is called Comprehensive Modal Emission Model (CMEM) proposed by Scora \& Barth (2006), Barth, Younglove, \& Scora (2005), Barth \& Boriboonsomsin (2008).

Based on the CMEM model, the fuel consumption rate $F_{r}$ in liters/second (L/s) can be calculated as,
$F_{r}=\xi(K \Upsilon V+P / \eta) / \kappa$,
where $\xi$ is the fuel-to-air mass ratio, $K$ is the engine friction factor, $\Upsilon$ is the engine speed, $V$ is the engine displacement (in L ), $\eta$ is the efficiency parameter for diesel engines and $\kappa$ is the heating value of a typical diesel fuel. Finally, $P$ is the second-by-second engine power output (in kW ) and it can be calculated as follows.
$P=P_{\text {tract }} / n_{t f}+P_{\text {acc }}$,
where $n_{t f}$ is the vehicle drive train efficiency and $P_{a c c}$ is the engine power demand associated with running losses of the engine and the operation of vehicle accessories such as air conditioning usage. $P_{\text {tract }}$ is the total tractive power requirement (in kW ) and it can be calculated as follows.
$P_{\text {tract }}=\left(M a+M g \sin \theta+0.5 C_{d} \rho S v^{2}+M g C_{r} \cos \theta\right) v / 1000$,
where $M$ is the total weight of the vehicle (in kg ) including the empty vehicle weight $w$ and weight of the goods carried, $a$ is the instantaneous acceleration (in $\mathrm{m} / \mathrm{s}^{2}$ ), $g$ is the gravitational constant (in $\mathrm{m} / \mathrm{s}^{2}$ ), $\theta$ is the road angle, $C_{d}$ is the coefficient of aerodynamic drag, $\rho$ is the air density (in $\mathrm{kg} / \mathrm{m}^{3}$ ), $S$ is the frontal surface area (in $\mathrm{m}^{2}$ ), $v$ is the vehicle speed (in $\mathrm{m} / \mathrm{s}$ ) and $C_{r}$ is the coefficient of rolling resistance.

We introduce some new parameters in order to simplify the above formulation: $\lambda=\xi / \kappa \psi$ where $\psi$ is the conversion factor of fuel, $\gamma=1 / 1000 n_{t f} \eta, \alpha=a+g \sin \theta+g C_{r} \cos \theta$ is a vehicle-arc specific constant and $\beta=0.5 C_{d} \rho S$ is a vehicle-specific constant. By using the new parameters, the total fuel consumption $F$ (in L ) for a vehicle traveling on a road segment of $d$ units (in m ) at a constant speed $v$ (in $\mathrm{m} / \mathrm{s}$ ) can be given as follows:
$F=\lambda K \Upsilon V d / v+\lambda \gamma d M \alpha+\lambda \gamma d \beta v^{2}$.
The emission model related parameter values are given in Appendix A .


Fig. 1. A concave down decreasing utility function for customers.

### 2.2. Fairness

This section introduces welfare functions that we use in order to incorporate the welfare concerns for the customers and drivers. Here, we assume that there is anonymity among customers, hence all customers will be treated the same without prioritization of some over the others. This is indeed the case where all customers considered belong to the same delivery class e.g., over-night delivery. With a similar approach, we assume that drivers, which are assigned to the same delivery class, have the same capabilities, so there is anonymity between drivers, as well.

### 2.2.1. Incorporating customer welfare

We use a welfare function for the customers, which is of the following form: $W F^{C}=\sum_{i=1}^{n} u_{i}^{C}\left(T_{i}\right)$, where $T_{i}$ is the delivery time of customer $i$. We let $u_{i}^{C}=u^{C}$ for all $i$, since this utility function is determined by a central decision maker. In that sense we are taking a central planning point of view (Karsu, 2016; Kaynar \& Karsu, 2018). We also assume that $u^{C}($.$) is concave down decreasing as$ seen in Fig. 1.

Since $u^{C}($.$) is monotonic in the sense that increasing the deliv-$ ery time of a customer (everything else being the same) decreases her utility, the welfare function $W F^{C}=\sum_{i=1}^{n} u^{C}\left(T_{i}\right)$ is monotonic. Such a function would also satisfy the Pigou-Dalton principle of transfers since the utility is decreasing in an increasing manner as the delivery time increases. To see why, consider $T_{1}, T_{2}$ as the delivery times of two customers 1 and 2 . Any convex combination of the delivery times would have larger utility. For example, when the delivery times are both $\left(\left(T_{1}+T_{2}\right) / 2\right)$, the overall welfare will increase as the decrease in $u_{1}$ (the utility of the better-off customer 1) will be smaller than the increase in $u_{2}$ (the utility of the worseoff customer 2).

One can incorporate such concave functions into the model via piecewise linear approximation. In our setting, since the planning is made considering time slots, we use a step function as follows: $u\left(T_{i}\right)=p_{j}^{C}, j: T_{j-1}^{C} \leq T_{i}<T_{j}^{C}$, where $p_{j}^{C}>p_{j+1}^{C}$ and $p_{j}^{C}-$ $p_{j+1}^{c}<p_{j+1}^{c}-p_{j+2}^{c} \quad \forall j=1, \ldots, l_{C}-2$. That is, we divide the range of possible delivery times into $l_{C}$ slots using thresholds and assign utility scores such that the customers in the same slot receive the same utility (Fig. 2). Since the utility scores decrease as slot increases, $u($.$) (and hence the total welfare W F^{C}$ ) is a nondecreasing function of delivery times (i.e., satisfies weak monotonicity).

Any delivery time distribution vector over customers is associated with a vector showing the number of customers served at each slot of the day. Let $n_{j}$ be the number of customers served at slot $j$ (Customer $i$ is served at slot $j$ if $T_{j-1}^{C} \leq T_{i}<T_{j}^{C}$ ).

Recall that $l_{C}$ is the number of time slots. Given a solution with an allocation vector ( $n_{1}, \ldots, n_{l_{C}}$ ) transferring one customer from a worse slot to a better slot can be considered as an efficiency encouraging transfer.


Fig. 2. Utility function of a customer for three slot case.

Definition 1. A welfare function WF is efficiency encouraging if for $n \in \mathbb{R}^{l_{C}}, W F(n)<W F\left(n+e_{i}-e_{j}\right)$, where $i, j: i<j$ and $e_{i}$ and $e_{j}$ are the ith and $j$ th unit vectors ${ }^{1}$ (Note that $n$ is the parameter showing the total number of customers. Here, with a slight abuse of notation we also denote the allocation vector by $n$. Which one is meant will be clear from the context).

We also ensure that the welfare function encourages equity increasing transfers. Consider for example two allocation vectors for a setting with 10 customers: $(3,2,5)$ and $(2,4,4)$. The second solution is obtained by transferring one customer from the best and worst slots to the middle one. Which solution is fairer is context specific, in this work, we take the view that decreasing the number of customers in the worst slots should be prioritized, hence consider the second one as more equitable. This is in line with a Rawlsian approach to fairness as it focuses on the worst-off members of a society (Rawls, 1971).

Definition 2. A customer welfare function WF is equity encouraging if for $n \in \mathbb{R}^{l}, W F(n)<W F\left(n-e_{i}+2 e_{j}-e_{k}\right)$, where $i, j$ and $k$ : $i<j<k$.

The definition implies that the utility gain from a unit transfer from $k$ to $j$ is larger than the utility loss from the transfer to $i$ to $j$. To encourage fair allocations of customers to time slots, we lexicographically minimize the number of customers assigned to time slots, starting from the worst one. That is, we solve the lexmin $\left(n_{l_{C}}, \ldots, n_{2}, n_{1}\right)$ problem. We ensure this by letting $p_{j}^{C}=\sum_{h=j-1}^{l_{c}-2} n^{h}+1$. The motivation behind this lexicographic minimization approach is the observation that customers can take more action in providing negative feedback when the delivery times are unacceptably high compared to other customers in the same delivery class as opposed to the case where they are lower since all customers are entitled to be treated equally. Our approach can be interpreted as avoiding the risk of losing these customers by preventing them from facing high delivery times as much as possible.

Proposition 1. Let $W F^{C}=\sum_{i=1}^{n} u^{C}\left(T_{i}\right)$ where $u^{C}\left(T_{i}\right)=p_{j}^{C}, j: T_{j-1}^{C} \leq$ $T_{i}<T_{j}^{C}$, and $p_{j}^{C}=\sum_{h=j-1}^{l_{c}-2} n^{h}+1$. The following holds: i) $W F^{C}$ is efficiency and equity encouraging. ii) A solution maximizing $W F^{C}$ is an optimal solution to lexmin $\left(n_{l_{C}}, \ldots, n_{2}, n_{1}\right)$ problem.

Proof. See Appendix B.
Corollary 1. In a three slot case, a solution maximizing $W F^{C}$ such that $p_{1}^{C}=n+2 ; p_{2}^{C}=n+1 ; p_{3}^{C}=1$, is an optimal solution to the $\operatorname{lexmin}\left(n_{3}, n_{2}, n_{1}\right)$ problem.

### 2.2.2. Incorporating driver welfare

Similarly, we define a welfare function for drivers $W F^{D}=$ $\sum_{i=1}^{m} u^{D}\left(r l_{i}\right)$, where $r l_{i}$ is the route length of driver $i . u^{D}$ is a step

[^1]function whose form is similar to $u^{C}$. Note that the function has a nonincreasing form as in the customer case since the utility of the central planner decreases as the route length of a driver increases. This is a direct result of the drivers being paid by the mileage they cover. The central planner would have economic concerns and hence prefer shorter routes, which is incorporated into the model through this nonincreasing welfare function for drivers. The structure of the welfare function $W F^{D}$ and the $u^{D}$ is analogous to the case discussed for customers. Dividing the possible route lengths into $l_{D}$ intervals once can define: $u^{D}\left(r l_{i}\right)=p_{j}^{D}, j: T_{j-1}^{D}<r l_{i}<T_{j}^{D}$, $p_{j}^{D}>p_{j+1}^{D} \quad \forall j=1, \ldots, l_{D}-1$.

Each route length distribution over the $m$ drivers is associated with a vector showing the number of drivers with route lengths in specific slots. Let $m_{j}$ be the number of drivers with route lengths in slot $j$. One can use two structures for $u^{D}$ depending on whether $\operatorname{lexmin}\left(m_{l_{D}}, \ldots, m_{2}, m_{1}\right)$ or $\operatorname{lexmax}\left(m_{1}, m_{2}, \ldots, m_{l_{D}}\right)$ is desired as follows:

Case 1: Lexicographically minimizing the number of drivers assigned to slots, starting from the maximum length slot $\operatorname{lexmin}\left(m_{l_{D}}, \ldots, m_{2}, m_{1}\right)$. This is analogous to the case described for customers hence the coefficients can be set as follows: $p_{j}^{D}=$ $\sum_{h=j-1}^{l_{D}-2} m^{h}+1$.

Proposition 2. Let $W F^{D}=\sum_{i=1}^{m} u^{D}\left(r l_{i}\right)$ where $u^{D}\left(r l_{i}\right)=p_{j}^{D}, j$ : $r l_{j-1}^{D} \leq r l_{i}<r l_{j}^{D}$, and $p_{j}^{D}=\sum_{h=j-1}^{l_{D}-2} m^{h}+1$. The following holds: i) $W F^{D}$ is efficiency and equity encouraging. ii) A solution maximizing $W F^{D}$ is an optimal solution to lexmin $\left(m_{l_{D}}, \ldots, m_{2}, m_{1}\right)$ problem.

The proof is omitted as it has the same structure as the proof of Proposition 1.

Case 2: Lexicographically maximizing the number of drivers assigned to slots, starting from the best slot lexmax $\left(m_{1}, m_{2}, \ldots, m_{l_{D}}\right)$. In this case, the coefficients can be set as follows: $p_{j}^{D}=$ $\sum_{h=0}^{L_{D}-1-j} m^{h}+1$ (See Proposition 1 in Appendix C).

Note that in the lexmax case (Case 2), the welfare function does not satisfy the equity encouraging transfers principle in Definition 2. This is because the priority changes from minimizing the number in the worst slots to maximizing the number in the best slots. Here, by worse, we mean longer length tour slots as we solve this problem for a central decision maker. Given two allocations $(3,2,5)$ and $(2,4,4)$ for an instance with 10 drivers, the second one will not be considered better since the number of drivers in the best slot decreases. This time, the welfare loss from decreasing the number of drivers in the best slot is larger than the gain from decreasing the number of drivers in worst slot. We investigate the implications of this alternative approach, i.e. lexmax approach in Section 5.2.1, and how to set the coefficients such that lexmax function is ensured in Appendix C. A similar lexmax approach can also be used regarding customers, i.e. maximizing the number of customers in the better slots can be prioritized over minimizing the ones in worse slots. We provide our main discussion using the lexmin approach since it is more preferred to achieve equitable distributions in various settings (Nace \& Orlin, 2007). We, however, also provide an example that shows how the solutions change in such an efficiency-oriented approach by implementing lexmax for customers in Section 5.2.2.

For the drivers' welfare function whether to use lexmax or lexmin is problem specific. If the payment made to the drivers is increasing with an increasing rate, then lexmin is a better approach. If the payment increases with a decreasing rate then lexmax should be preferred. To see why, one can check the difference in payment in the two allocations given in Definition 2.

Note that regardless of which approach is used, even if all tours are in the same mileage zone, that does not necessarily mean that
lengths of tours will be equal to each other. In a setting, where drivers are categorized as "better" and "worse" based on their capabilities, the decision maker can decide which driver will be assigned to which tour. That is, she for example can reward better drivers by assigning them longer tours with more payment. However, driver categorization does not deem our approach unimplementable, the decision maker can easily make such a postassignment.

## 3. Problem formulation

In this section, we present a multi-objective mixed integer programming formulation for the SVRP. The decision variables are defined as follows: A binary variable $x_{i j}$ equals 1 if a vehicle travels on $\operatorname{arc}(i, j) \in A$, and 0 otherwise. A continuous nonnegative variable $f_{i j}$ represents the flow on $\operatorname{arc}(i, j) \in A$. Another continuous nonnegative variable $t_{i j}$ represents the total time covered by a vehicle up to node $j \in N$ when the vehicle travels from node $i \in N$ to $j \in N$. Finally, a binary variable $w_{i l}$ equals 1 if the total time of a tour (driver time) where customer $i \in C$ is the last customer visited is on time slot $l \in L^{D} ; 0$ otherwise and another binary variable $y_{i l}$ equals 1 if the delivery for customer $i \in C$ is made on time slot $l \in L^{C} ; 0$ otherwise.

A multi-objective mathematical model of the SVRP is as follows:
Minimize $z_{1}(x)=\sum_{(i, j) \in A}\left[\left(\alpha \gamma \lambda d_{i j} \omega x_{i j}\right)+\left(\alpha \gamma \lambda d_{i j} f_{i j}\right)\right.$

$$
\begin{equation*}
\left.+\left(\beta \gamma \lambda d_{i j} v^{2} x_{i j}\right)+\left(K \Upsilon V \lambda d_{i j} \frac{x_{i j}}{v}\right)\right] \tag{1}
\end{equation*}
$$

Maximize $z_{2}(x)=\sum_{i \in C} \sum_{l \in L^{D}} p_{l}^{D} w_{i l}$
Maximize $z_{3}(x)=\sum_{i \in C} \sum_{l \in L^{C}} p_{l}^{C} y_{i l}$
subject to
$\sum_{j \in C} x_{0 j}=m$
$\sum_{j \in N \backslash\{i\}} x_{i j}=1 \quad \forall i \in C$
$\sum_{i \in N \backslash\{j\}} x_{i j}=1 \quad \forall j \in C$
$\sum_{j \in N \backslash\{i\}} f_{i j}-\sum_{j \in N \backslash\{i\}} f_{j i}=q_{i} \quad \forall i \in C$
$q_{i} x_{i j} \leq f_{i j} \leq\left(C-q_{j}\right) x_{i j} \forall i, j \in N: j \neq i$
$\sum_{j \in N \backslash\{i\}} t_{i j}-\sum_{j \in N \backslash\{i\}} t_{j i}=\sum_{j \in N \backslash\{i\}}\left(\frac{d_{i j}}{v} x_{i j}\right) \quad \forall i \in C$
$t_{0 i}=\frac{d_{0 i}}{v} x_{0 i} \quad \forall i \in C$
$t_{i j} \leq M x_{i j} \quad \forall i, j \in N: j \neq i$
$T_{l-1}^{D} w_{i l} \leq t_{j 0} \leq T_{l}^{D} w_{i l}+M\left(1-w_{i l}\right) \quad \forall i \in C, l \in L^{D}$
$T_{l-1}^{C} y_{i l} \leq \sum_{j \in N \backslash\{i\}} t_{j i} \leq T_{l}^{C} y_{i l}+M\left(1-y_{i l}\right) \quad \forall i \in N, l \in L^{C}$
$\sum_{l \in L^{D}} w_{i l}=1 \quad \forall i \in C$
$\sum_{l \in L^{C}} y_{i l}=1 \quad \forall i \in N$
$x_{i j} \in\{0,1\} \quad \forall i, j \in N: j \neq i$
$f_{i j} \geq 0 \quad \forall i, j \in N: j \neq i$
$t_{i j} \geq 0 \quad \forall i, j \in N: j \neq i$
$w_{i l} \in\{0,1\} \quad \forall i \in C, l \in L^{D}$
$y_{i l} \in\{0,1\} \quad \forall i \in N, l \in L^{C}$.
The first objective function (1) minimizes the total amount of fuel consumption and emission calculated by the emission model. The second (2) and the third (3) objective functions maximize the total welfares of the drivers and customers, respectively. Constraint (4) ensures that a fleet of $m$ vehicles leaves the depot. With constraint (5) and (6), it is guaranteed that every customer will be visited exactly once by only one vehicle. Constraint (7) ensures flow conservation between customers. Constraint (8) provides a lower bound on flow variable and also it imposes vehicle capacity constraint. Constraints (9)-(11) calculate the arrival time at customers in a tour. Constraints (12)-(13) allocate the driver time and customer delivery time into the predetermined time slots, respectively. Constraints (14)-(15) ensure that the driver time and the customer delivery time are allocated to only one predetermined time slot, respectively. Here, we remark that in the problem description, we state that fairness objective for drivers will be evaluated based on the tour lengths. Since vehicle speed is constant in our problem and to avoid any additional auxiliary variables, we consider the total time that driver spent instead of the total distance that driver travelled in a tour in the formulation. Constraints (16)-(20) are the domain constraints.

## 4. Solution approach

Consider the following multiobjective integer programming problem
maximize $z(x)$
subject to $x \in \mathcal{X}$.
where $z: \mathbb{R}^{n} \rightarrow \mathbb{R}^{3}$ is a vector valued function and $\mathcal{X} \subset \mathbb{R}^{n}$ is the feasible region. In particular we assume that $\mathcal{Z}:=z(\mathcal{X})=\{z(x) \mid x \in$ $\mathcal{X}\}$ consists of vectors with at least two integer components. The aim is finding the set of nondominated points $\left(\mathcal{Z}_{N} \subset \mathcal{Z}\right)$ of problem P.

The solution algorithm we implement is provided in Algorithm 1. It works on the projected space where each point $z \in \mathcal{Z}$ is projected onto $\mathbb{R}^{2}$ with respect to its second and third components. In that sense, it is a variant of the solution algorithm discussed in Kirlik \& Sayın (2014), which we modified considering the specifics of the SVRP. The proposed algorithm searches predefined sub-regions in this projected space. The sub-regions yet

```
Algorithm 1 Algorithm to solve (P).
    Let \(z_{2}^{N}=m \times p_{l_{D}}^{D}\) and \(z_{3}^{N}=n \times p_{l_{C}}^{C}\), feasible \(=1, \mathcal{Z}_{N}=\emptyset, I S=\emptyset\),
    \(N S=\emptyset, \mathcal{R}=\emptyset, \epsilon=1\).
    Solve \(M\left(z_{2}^{N}, z_{3}^{N}\right)\). If the model is infeasible, feasible \(=0\).
    while feasible do
        Let an optimal solution be \(x . \mathcal{Z}_{N} \leftarrow \mathcal{Z}_{N} \cup\{z(x)\}\). Set \(l_{2}=\)
        \(z_{2}(x)+\epsilon\). Solve \(M\left(l_{2}, z_{3}^{N}\right)\).
        if The model is infeasible then
            feasible \(=0\).
        end if
    end while
    Sort \(\mathcal{Z}_{N}\) with respect to \(z_{2}\) (.) in decreasing order. Let \(Z:=\)
    \(\left\{\left(z_{2}, z_{3}\right):\left(z_{1}, z_{2}, z_{3}\right) \in \mathcal{Z}_{N}\right\}\).
    for \(i=1, \ldots,|Z|-1\) do
        \(\mathcal{R}=\mathcal{R} \cup\left(z_{2}^{i+1}+\epsilon, z_{3}^{i}+\epsilon\right)\)
    end for
    \(\mathcal{R}=\mathcal{R} \cup\left(z_{2}^{N}, z_{3}^{|Z|}+\epsilon\right)\)
    while \(|\mathcal{R}| \geq 1\) do
        Take the first element in \(\mathcal{R}\). Let it be \(\left(l_{2}, l_{3}\right) . \mathcal{R}=\mathcal{R} \backslash\left(l_{2}, l_{3}\right)\).
        flagsolve=1.
        for \(i=1, \ldots,|I S|\) do
            if \(I S(i, 1) \leq l_{2} \& I S(i, 2) \leq l_{3}\) then
                flagsolve=0. BREAK;
            end if
        end for
        for \(i=1, \ldots,|N S|\) do
            if \(N S(i, 1) \leq l_{2} \leq N S(i, 3) \& N S(i, 2) \leq l_{3} \leq N S(i, 4)\) then
                flagsolve \(=0 . \quad \mathcal{R}=\left\{\left(N S(i, 3)+\epsilon, l_{3}\right),\left(l_{2}, N S(i, 4)+\epsilon\right)\right\} \cup\)
                \(\mathcal{R}\), BREAK;
            end if
        end for
        for \(\left(\overline{\bar{L}}_{\underline{2}}, \bar{l}_{3}\right) \in \mathcal{R}\) do
            if \(\bar{l}_{2} \leq l_{2}\) and \(\overline{l_{3}} \leq l_{3}\) then
                flagsolve=0. BREAK;
            end if
        end for
        if flagsolve \(=1\) then
            Solve \(M\left(l_{2}, l_{3}\right)\).
            if The model is feasible then
                Let an optimal solution be \(x\). \(\mathcal{Z}_{N}=\mathcal{Z}_{N} \cup z(x)\).
                \(\mathcal{R}=\left\{\left(N S(i, 3)+\epsilon, l_{3}\right),\left(l_{2}, N S(i, 4)+\epsilon\right)\right\} \cup \mathcal{R}\).
                \(N S=N S \cup\left(l_{2}, l_{3}, z_{2}(x), z_{3}(x)\right) ;\)
            else
                \(I S=I S \cup\left(l_{2}, l_{3}\right)\)
            end if
        end if
    end while
```

to be explored is kept as a list $\mathcal{R}$, which is initialized as a region that is guaranteed to include the projections of all elements of $\mathcal{Z}_{N}$. At an arbitrary iteration, the algorithm explores a sub-region in $\mathcal{R}$ for new nondominated points. If new points are found, the set of sub-regions to be explored ( $\mathcal{R}$ ) is updated accordingly. A sub-region $R \in \mathcal{R}$ can be defined by two numbers, which are thresholds for the second and third objective function values as follows: $R\left(l_{2}, l_{3}\right):=\left\{z \in \mathbb{R}^{3}: z_{2} \geq l_{2}, z_{3} \geq l_{3}\right)$. We search for new points in $R\left(l_{2}, l_{3}\right)$ by solving the following scalarization:
maximize $z_{1}(x)+\alpha z_{2}(x)+\beta z_{3}(x)$
subject to $x \in \mathcal{X}$
$\left(M\left(l_{2}, l_{3}\right)\right)$

$$
\begin{aligned}
& z_{2}(x) \geq l_{2} \\
& z_{3}(x) \geq l_{3} .
\end{aligned}
$$



Fig. 3. Region that can be eliminated when $x^{*}$ is an optimal solution to $M\left(l_{2}, l_{3}\right)$.

This scalarization is the well-known (augmented) epsilon constraint scalarization. The optimal solution of this scalarization problem is an efficient solution of problem P and its image in the objective space is a nondominated point. If a new nondominated point is found in a given region $R\left(l_{2}, l_{3}\right)$, we exclude subregions that are guaranteed not to include a nondominated point (i.e. its projection) based on Proposition 3.

Proposition 3. Let $x^{*}$ be an optimal solution of $M\left(l_{2}, l_{3}\right)$. $\nexists z \in \mathcal{Z}_{N}$ : $l_{2} \leq z_{2} \leq z_{2}\left(x^{*}\right), l_{3} \leq z_{3} \leq z_{3}\left(x^{*}\right)$.

Proof. To the contrary assume that $\exists x \in \mathcal{X}: z(x) \in \mathcal{Z}_{N}, l_{2} \leq z_{2}(x) \leq$ $z_{2}\left(x^{*}\right), l_{3} \leq z_{3}(x) \leq z_{3}\left(x^{*}\right)$. For $z(x)$ to be nondominated, $z_{1}(x)>$ $z_{1}\left(x^{*}\right)$ should hold (Since $x$ is at most as good as $x^{*}$ with respect to $z_{2}($.$\left.) and z_{3}().\right)$. Since $x$ is feasible for $M\left(l_{2}, l_{3}\right)$, this contradicts the optimality of $x^{*}$. See Fig. 3.

We provide the pseudocode of the algorithm in Algorithm 1. The algorithm starts with finding lower bounds for the second and third objective function values by assigning their possible lowest values which are $m \times p_{l_{D}}^{D}$ and $n \times p_{l_{C}}^{C}$, respectively. Throughout the algorithm a number of sets are utilized as follows: Set of regions yet-to-be explored $\mathcal{R}$; set of regions explored before, for which a non-dominated point is found $N S$; set of regions explored before, for which no nondominated point is found (i.e. the scalarization model is infeasible) IS. The first epsilon-constraint scalarization model (at line 2) is solved using the overall lower bounds for $z_{2}(x)$ and $z_{3}(x)$, which guarantees that the nondominated point with the best $z_{1}($.$) value is returned. Starting from this solution,$ the algorithm first "sweeps" the search region by putting bounds only on $z_{2}($.$) in the scalarization model (lines 3-8). After line 8$, we obtain points that are nonincreasing with respect to $z_{1}$ (.) and increasing with respect to $z_{2}$ (.). The points do not have any order with respect to $z_{3}($.$) .$

After finding an initial set of solutions, the algorithm determines regions yet-to-be explored, making use of these solutions. These regions are defined such that their union is guaranteed to contain projections of all the remaining nondominated points. To sweep the image set $\mathcal{Z}$ this time in the opposite direction, the initial solution set is sorted in decreasing order with respect to $z_{2}$ (.). The regions are defined accordingly and in line 13, an additional region having the lowest possible value for $z_{2}($.$) is added. At each$ iteration of the main loop (starting at line 14), the first element in set $\mathcal{R}$ is chosen to be explored. We first check some filtering rules to make use of information obtained from the previous iterations relying on the following results:

Proposition 4. For two regions $\left(l_{2}, l_{3}\right)$ and $\left(\overline{l_{2}}, \overline{l_{3}}\right): \overline{l_{2}} \leq l_{2}$ and $\overline{l_{3}} \leq$ $l_{3}:$

- If there are no nondominated points in $\left(\overline{l_{2}}, \bar{l}_{3}\right)$, then there are no nondominated points in $\left(l_{2}, l_{3}\right)$.
- If the optimal solution of $M\left(\overline{l_{2}}, \overline{l_{3}}\right)$ is feasible for $M\left(l_{2}, l_{3}\right)$ then it is optimal for $M\left(l_{2}, l_{3}\right)$.
Both statements are a direct result of the feasible region of $M\left(l_{2}, l_{3}\right)$ being a subset of that of $M\left(\overline{l_{2}}, \bar{l}_{3}\right)$. Similar rules are discussed in other studies see e.g. Kirlik \& Sayın (2014) and Klamroth, Lacour, \& Vanderpooten (2015). If the first case is observed for a region $R\left(l_{2}, l_{3}\right)$, since no solution is found, no new regions are added to the list. In the second case, the region is divided into two smaller sub-regions and these regions are added to the yet-to-be explored list.

Finally, we do not explore a region if a super set of it is yet-tobe explored (see lines 26-30).

If for a region $R\left(l_{2}, l_{3}\right)$ none of the above cases apply, then the scalarization problem $M\left(l_{2}, l_{3}\right)$ is solved so as to find nondominated points with projections in the region. If found, new regions are defined and added and set NS is updated; if not, set IS is updated.

Proposition 5. The algorithm finds all nondominated points in a finite number of iterations (i.e. solving a finite number of models). The number of models solved is bounded by $\left(\left|\mathcal{Z}_{N}\right|+1\right)^{2}$.

The proof is provided in Appendix D. Our computational experiments show that the actual values of the number of models solved is well below this bound.

We illustrate the algorithm on an example problem in Appendix E.

## 5. Computational study

This section presents the computational analysis on the SVRP. We first describe the data set used in the computational experiments and the settings of the parameters. We then analyse the solutions of problem instances of different sizes, which we obtain by varying number of customers $(|N|)$ and number of vehicles ( $m$ ), for the setting where we lexicographically minimize the number of drivers assigned to slots, starting from the worst slot. We perform further experiments to see the effect of time slot lengths on the results and also the settings, where a lexmax approach is used to assess total welfare of the drivers. Finally, we provide results on how the solutions change when the planner is assumed to be only efficiency-oriented and implement lexmax approach for the customers.

All computational experiments are implemented in Java platform and solved by Cplex 12.7 .1 on a Linux OS environment with Dual Intel Xeon E5-2690 v4 14 Core 2.6 GHz processors with 128 GB of RAM.

### 5.1. Data set and parameter setting

The computational experiments are conducted over the instances of PRP Library by Demir, Bektaş, \& Laporte (2012). The data set is based on real road networks of randomly selected cities of the United Kingdom where demands are randomly generated. We consider instances with $10,15,20,25$ customers $(|N|)$ and $2,3,4$, $5(\mathrm{~m})$ drivers. For each problem size, we solve five randomly generated instances.

The fixed vehicle speed ( $v$ ) is set as approximately $55 \mathrm{~km} / \mathrm{h}$, which is the optimal vehicle speed that minimizes the amount of fuel consumption and emission for the specific parameter values in Appendix A, when there are no time related constraints.

The number of time slots $\left(l_{C}\right)$ is set to three for customers and threshold values $\left(T_{j}^{C}\right)$ for time slots (customers) are three, six and nine hours. The welfare function coefficients for customers are set as in Corollary 1 , as $p_{1}^{C}=n+2 ; p_{2}^{C}=n+1 ; p_{3}^{C}=1$. Similarly, the number of route length slots $\left(l_{D}\right)$ is set to three for drivers. Recall that the vehicle speed is assumed to be constant; hence using route length slots is equivalent to using time slots for the drivers. We, therefore, use time slots for drivers with threshold values ( $T_{j}^{D}$ ) of three, six and nine. The welfare function coefficients for drivers are set as $p_{1}^{D}=m+2 ; p_{2}^{D}=m+1 ; p_{3}^{D}=1$ when the lexmin approach is utilized and they are set as $p_{1}^{D}=m+3 ; p_{2}^{D}=2 ; p_{3}^{D}=1$ when the lexmax approach is used as described in Corollary 1 in Appendix C.

The coefficients $\alpha$ and $\beta$ in $\mathrm{M}\left(l_{2}, l_{3}\right)$ are set as small numbers (to ensure that minimizing cost is prioritized) but not too small (in order not to lead to weakly nondominated solutions). In the experiments we set $\alpha$ and $\beta$ as $0.01 /((m+1) m)(0.01 /((m+2) m)$ for lexmax approach) and $0.01 /((n+1) n)$, respectively. Since the objective function values for the second and third objectives are integer-valued, in the proposed algorithm, the step size value ( $\epsilon$ ) is taken as one in order to ensure that all nondominated solutions are found.

### 5.2. Analysis

In this section, we first analyse the trade-off between the three objectives of minimizing fuel consumption and emission and maximizing customer and driver welfare. To do so, we observe how the best solution of an objective function performs with respect to the other objectives. Table 2 presents the absolute percent deviations of the best solution of an objective function from the best attainable levels of the other two objective functions. We report the average and maximum deviations over five different 15 -customer instances with two, three and four drivers. As it can be seen in Table 2, we report two different average and maximum values for the best solutions of the second and third objectives. The reason is that for the corresponding objective function, there are two solutions with the highest (for maximization) objective function value. For example, for $m=2$ instances, there are two solutions maximizing driver welfare ( $z_{2}$ ), one is better with respect to emission and the other is better with respect to customer welfare. To ensure a fair and meaningful comparison, we show the deviations of all such maximizers, separately in two rows. The extra column titled "Comp." indicates the solution for which the deviations are calculated. For example, for $m=2$ instances, $z_{1}$ indicates that deviations are reported for the ones that are better with respect to emission, among the solutions maximizing driver welfare $\left(z_{2}\right)$.

The results on Table 2 suggest that for smaller number of vehicles ( $m=2$ ), minimizing fuel consumption and emission leads to more reduction in the welfare of the customers compared to drivers. Overall, using more vehicles leads to an increase in the customer welfare, however the driver welfare gets relatively worse, as expected. The results also indicate that maximizing the welfare of the drivers has a larger impact on the emission amount compared to the customer welfare, especially for higher number of vehicles. A similar observation can be made for solutions maximizing customer welfare: when $m>2$, the trade-off between emission and customer welfare is more notable than the one between driver and customer welfares. This is due to the fact that maximizing driver and customer welfare encourages tour length (distance) minimization: the former directly and the latter through delivery time minimization, as the vehicle speed is constant. Minimizing fuel consumption, however, considers the load-based total distance rather than the total distance, due to the emission model used.

Table 2
Deviations between best solutions for each objective function.

| $\|N\|=15$ | Min $z_{1}$ |  |  |  | $\operatorname{Max} z_{2}$ |  |  |  |  | Max $z_{3}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{2}$ |  | $z_{3}$ |  | Comp. | $z_{1}$ |  | $z_{3}$ |  | Comp. | $z_{1}$ |  | $z_{2}$ |  |
| $m$ | Avg | Max | Avg | Max |  | Avg | Max | Avg | Max |  | Avg | Max | Avg | Max |
| 2 | 0.00 | 0.00 | 13.42 | 19.60 | $z_{1}$ | 0.00 | 0.00 | 13.42 | 19.60 |  | 10.05 | 16.84 | 37.50 | 50.00 |
|  |  |  |  |  | $z_{3}$ | 6.05 | 7.84 | 4.51 | 6.05 |  |  |  |  |  |
| 3 | 8.10 | 16.67 | 5.13 | 19.76 | $z_{1}$ | 3.14 | 11.88 | 1.42 | 1.97 | $z_{1}$ | 6.64 | 17.61 | 5.00 | 25.00 |
|  |  |  |  |  | $z_{3}$ | 8.65 | 17.61 | 0.00 | 0.00 | $z_{2}$ | 8.65 | 17.61 | 0.00 | 0.00 |
| 4 | 8.27 | 18.18 | 1.57 | 1.96 | $z_{1}$ | 4.36 | 11.39 | 1.65 | 1.96 | $z_{1}$ | 7.81 | 16.19 | 4.55 | 9.09 |
|  |  |  |  |  | $z_{3}$ | 8.79 | 19.91 | 0.55 | 1.57 | $z_{2}$ | 9.33 | 19.91 | 2.73 | 9.09 |

Table 3
Driver and customer distributions for the best solution of each objective function.

| $\|N\|=15$ |  | Min $z_{1}$ |  | $\operatorname{Max} z_{2}$ |  |  | Max $z_{3}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $m$ | I | D. Dist. | C. Dist. | Comp | D. Dist. | C. Dist. | Comp. | D. Dist. | C. Dist. |
| 2 | 1 | (0,1,1) | $(6,6,3)$ | $z_{1}$ | $(0,1,1)$ | $(6,6,3)$ |  | $(0,0,2)$ | (10,5,0) |
|  |  |  |  | $z_{3}$ | $(0,1,1)$ | $(10,4,1)$ |  |  |  |
|  | 2 | (0,2,0) | $(7,8,0)$ | $z_{1}$ | $(0,2,0)$ | $(7,8,0)$ |  | $(0,2,0)$ | $(13,2,0)$ |
|  |  |  |  | $z_{3}$ | $(0,2,0)$ | $(13,2,0)$ |  |  |  |
|  | 3 | (0,1,1) | $(6,6,3)$ | $z_{1}$ | $(0,1,1)$ | $(6,6,3)$ |  | $(0,0,2)$ | $(10,5,0)$ |
|  |  |  |  | $z_{3}$ | $(0,1,1)$ | $(10,4,1)$ |  |  |  |
|  | 4 | Infeasible |  |  |  |  |  |  |  |
|  | 5 | (0,1,1) | $(8,5,2)$ | $z_{1}$ | $(0,1,1)$ | $(8,5,2)$ |  | $(0,0,2)$ | $(8,7,0)$ |
|  |  |  |  | $z_{3}$ | $(0,1,1)$ | $(8,6,1)$ |  |  |  |
| 3 | 1 | (1,1,1) | $(8,4,3)$ | $z_{1}$ | $(0,3,0)$ | $(9,6,0)$ |  | $(0,3,0)$ | $(13,2,0)$ |
|  |  |  |  | $z_{3}$ | $(0,3,0)$ | $(13,2,0)$ |  |  |  |
|  | 2 | $(1,2,0)$ | $(9,6,0)$ | $z_{1}$ | $(2,1,0)$ | $(10,5,0)$ |  | $(2,1,0)$ | $(14,1,0)$ |
|  |  |  |  | $z_{3}$ | $(2,1,0)$ | $(14,1,0)$ |  |  |  |
|  | 3 | $(0,3,0)$ | $(11,4,0)$ | $z_{1}$ | $(0,3,0)$ | $(11,4,0)$ |  | $(0,3,0)$ | $(14,1,0)$ |
|  |  |  |  | $z_{3}$ | $(0,3,0)$ | $(14,1,0)$ |  |  |  |
|  | 4 | $(0,3,0)$ | $(9,6,0)$ | $z_{1}$ | $(0,3,0)$ | $(9,6,0)$ |  | $(0,3,0)$ | $(14,1,0)$ |
|  |  |  |  | $z_{3}$ | $(0,3,0)$ | $(14,1,0)$ |  |  |  |
|  | 5 | (1,1,1) | $(10,5,0)$ | $z_{1}$ | $(0,3,0)$ | $(10,5,0)$ | $z_{1}$ | $(0,2,1)$ | $(12,3,0)$ |
|  |  |  |  | $z_{3}$ | $(0,3,0)$ | $(12,3,0)$ | $z_{2}$ | $(0,3,0)$ | $(12,3,0)$ |
| 4 | 1 | (2,1,1) | $(11,4,0)$ | $z_{1}$ | $(1,3,0)$ | $(10,5,0)$ | $z_{1}$ | $(0,4,0)$ | $(15,0,0)$ |
|  |  |  |  | $z_{3}$ | $(1,3,0)$ | $(15,0,0)$ | $z_{2}$ | $(1,3,0)$ | $(15,0,0)$ |
|  | 2 | $(2,2,0)$ | $(10,5,0)$ | $z_{1}$ | $(3,1,0)$ | $(12,3,0)$ | $z_{1}$ | $(2,2,0)$ | $(15,0,0)$ |
|  |  |  |  | $z_{3}$ | $(3,1,0)$ | $(15,0,0)$ | $z_{2}$ | $(3,1,0)$ | $(15,0,0)$ |
|  | 3 | $(1,3,0)$ | $(11,4,0)$ | $z_{1}$ | $(1,3,0)$ | $(11,4,0)$ |  | $(1,3,0)$ | $(15,0,0)$ |
|  |  |  |  | $z_{3}$ | $(1,3,0)$ | $(15,0,0)$ |  |  |  |
|  | 45 | $(1,3,0)$ | $(13,2,0)$ | $z_{1}$ | $(2,2,0)$ | $(10,5,0)$ |  | $(1,3,0)$ | $(15,0,0)$ |
|  |  |  |  | $z_{3}$ | $(2,2,0)$ | $(12,3,0)$ |  |  |  |
|  |  | (2,1,1) | $(10,5,0)$ |  | $(2,2,0)$ | $(11,4,0)$ |  | $(0,4,0)$ | $(15,0,0)$ |

Although the results reported in Table 2 give us an idea on the level of impact of considering just one objective on the others, the exact figures for the second and third objectives may not show the real trade-off due to the fact that these percentage deviations mainly depend on the welfare function coefficients ( $p^{D}$ and $p^{C}$ ). To present a more accurate analysis, we also provide the driver and customer distributions for the best solutions in Table 3, for each of the five instances solved. The results in this table are in line with the observations made above. We see that an increase in the number of vehicles leads to more customers being served in the earlier slots, increasing performance with respect to customer welfare. On the other hand, as the number of vehicles increases, minimizing fuel consumption and emission results in reduction in fairness between drivers: the driver distributions deviate from the driver welfare maximizing solution, which tends to assign similar route lengths to drivers while avoiding very long routes. We also observe that maximizing customer welfare leads to a less impact on drivers' welfare compared to its impact on pollution; indeed, in most instances with $m>2$ the same driver distributions are seen in the best solutions of $z_{3}$ and $z_{2}$. This indicates that if only two objectives can be handled, the ones to be chosen should be fuel consumption minimization and one of the social-impact based ob-
jectives; driver welfare maximization or customer welfare maximization.

Further analysis on the first instance $(I=1)$ with 15 customers and three drivers show that allowing a $0.38 \%$ increase in the emission increases the customer welfare around $16 \%$ by completing services of two more customers within first time slot ( $10,4,1$ ) instead of the last one $(8,4,3)$. An additional $0.72 \%$ increase in the emission leads to an approximately $23 \%$ increase in the customer welfare having all 15 customers served within first two time slot $(9,6,0)$ and also a $20 \%$ increase in the driver welfare having all three drivers completing their deliveries in the second route length slot $(0,3,0)$ instead of having each of them in different slots $(1,1,1)$.

Table 4 summarizes the main results for the SVRP, where lexmin approach is used for the drivers. For each problem size ( $|N|$, $m$ combination) we report the average and the minimum values of the number of nondominated vectors found, and the average and maximum values of the number of (single objective) mathematical models solved. We also report the average and maximum solution times per nondominated solution over the five instances solved. We use a time limit of 7200 seconds and report the number of instances that could not be solved due to infeasibility in parentheses in column $m$.

Table 4
Summary of the results.

| $\|N\|$ | $m$ | $\left\|\mathcal{Z}_{N}\right\|$ |  | \# models |  | $\mathrm{CPU} /\left\|\mathcal{Z}_{N}\right\|$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Avg | Min | Avg | Max | Avg | Max |
| 10 | 2 | 4.20 | 3 | 7.80 | 10 | 1.92 | 3.59 |
| 10 | 3 | 3.80 | 2 | 7.40 | 11 | 0.86 | 1.55 |
| 10 | 4 | 3.40 | 2 | 6.40 | 8 | 0.72 | 1.97 |
| 15 | 2 (1) | 9.75 | 7 | 14.25 | 22 | 404.01 | 658.38 |
| 15 | 3 | 7.60 | 3 | 11.40 | 18 | 282.83 | 844.29 |
| 15 | 4 | 7.60 | 5 | 11.80 | 15 | 575.75 | 1643.65 |
| 20 | 3 | 7.40 | 4 | 11.20 | 14 | 1747.92 | 2859.54 |
| 20 | 4 | 11.40 | 6 | 16.20 | 29 | 1690.71 | 2341.45 |
| 20 | 5 | 12.00 | 8 | 16.80 | 26 | 1900.89 | 3276.88 |
| 25 | 3 (2) | 9.00 | 8 | 12.67 | 14 | 3653.11 | 5400.96 |
| 25 | 4 | 9.80 | 5 | 14.00 | 21 | 4694.22 | 6230.38 |
| 25 | 5 | 9.80 | 5 | 14.00 | 21 | 4815.70 | 6692.22 |



Fig. 4. Nondominated solutions found by different step size values $(|N|=25$, $m=3, I=4)$.

Note that even the cost minimizing variant of VRP is difficult to solve, making the triobjective variant even harder. As expected, increasing the number of customers results in a notable increase in the solution times. Increasing the number of drivers does not have a foreseeable effect on the solution time. For example, for $|N|=15$ instances, increasing the number of drivers from 2 to 3 , reduces the solution time, while a further increase of $m$ from 3 to 4 , increases the solution time. On the other hand, we see that the solution time increases as $m$ increases for $|N|=25$ instances. We should note here that the total solution time is determined by two factors: the number of nondominated points (and hence the number of models solved) and the time required to solve each model. It is seen that the single objective models become increasingly harder to solve as problem size increases. Nevertheless, the solution algorithm is able to return solutions for problems up to 25 customers and 5 drivers.

As can be seen from the results in Table 4, for larger-sized instances with higher number of customers, both solution times and the number of nondominated solutions found increase, which limits the computational experiments to relatively small-sized instances. For larger instances, our algorithm can be used to obtain a well-representative subset of the Pareto set with less computational effort compared to finding the whole set. This can be achieved by setting step size value $(\epsilon)$ to a larger number. In the original experiments, $\epsilon$ is taken as one in order to ensure that all nondominated solutions are found. Here, we analyse how the set of nondominated solutions change with larger step size values. We conduct experiments over one instance with 25 customers and three drivers, considering three different step size values; one, two and three. Fig. 4 shows the nondominated solutions found by using

Table 5
$\mathcal{Z}_{N}$ for the instance $(|N|=15, m=3)$.

| Index | $z_{1}(x)$ | $z_{2}(x)$ (D. Dist.) | $z_{3}(x)$ (C. Dist.) |
| :--- | :--- | :--- | :--- |
| 1 | 126.67 | $10(1,1,1)$ | $250(10,5,0)$ |
| 2 | 129.94 | $9(0,2,1)$ | $251(11,4,0)$ |
| 3 | 130.14 | $12(0,3,0)$ | $250(10,5,0)$ |
| 4 | 130.48 | $12(0,3,0)$ | $251(1,4,0)$ |
| 5 | 134.48 | $9(0,2,1)$ | $252(12,3,0)$ |
| 6 | 147.22 | $12(0,3,0)$ | $252(12,3,0)$ |

different step size values. When the step size is set as one (original case), the number of nondominated solutions found is ten. When the step size is increased to two and three, then the number of nondominated solutions decreases to five and three, respectively. Fig. 4 indicates that the subset of nondominated solutions obtained with larger stepsizes is well-dispersed. Having a representative subset may also be preferred to having the whole set by the decision maker, as it would reduce the cognitive burden when choosing the alternative to implement.

To demonstrate how a potential user can use the original approach where step size is one, we analyse one of the instances with 15 customers and three drivers in more detail. Solving this instance results in six nondominated solutions reported in Table 5 and their geographical representations are depicted in Fig. 5. Table 5 presents the values for each objective function ( $\left.z_{1}(x), z_{2}(x), z_{3}(x)\right)$ and also distributions of drivers (D. Dist.) and customers (C. Dist.) in the route length and time slots, respectively. On the maps shown in Fig. 5, the red house image shows the location of the depot and routes of the vehicles are differentiated from each other by the colours of images on the customer locations. We use letters in alphabetical order to show the visiting sequence of the vehicles, where depot is labeled as A , the first customer visited is B and so on.

In the first nondominated solution with the least amount of emission, drivers are equally distributed among route length slots and 10 out of 15 customers are served within the first time slot while remaining five are visited in the second one. As it can be seen from Fig. 5(a), the (orange) tour located in the lower-right of the map with only two customers is completed in the first route length slot and all of the customers visited in this tour are served within the first time slot. The (green) tour located in the upperright of the map with seven customers, two of which are visited within the second time slot, is finished in the second route length slot. The last (purple) tour located in the left part of the map with six customers, three of which are visited within the second time slot, is completed in the third route length slot. In the second nondominated solution (Fig. 5(b)) with the expense of consuming three more liters of fuel, one of the customers, who was


Fig. 5. Nondominated solutions of instance $(|N|=15, m=3)$.
visited in the second time slot in the first nondominated solution, is now served during the first time slot in the purple tour, which also causes one more driver to complete his route in the second route length slot (orange tour) and so worsens the second objective function. The third nondominated solution (Fig. 5(c)) increases the amount of fuel consumed by 0.2 L (compared to the second solution) and serves one more customer in the second time slot instead of in the first, but in this solution, all drivers complete their journeys within the second route length slot resulting a fairer distribution for drivers. This is achieved by removing a customer from purple tour and inserting it to orange tour. By looking at Figs. 5(c) and 5(d), one can say that they are the same, except the visiting sequence of the customers in the purple tour: the purple tour is re-
versed, resulting in 0.34 L of more fuel consumption, but one more customer served within the first time slot compared to the third nondominated solution. The fifth nondominated solution (Fig. 5(e)) is much more similar to the first nondominated solution compared to the fourth one in terms of vehicle routes. The only difference between first and fifth solutions is that two customers, which are visited at the end of the green tour in the first solution, are served at the end of the orange tour. This change on routes results in more fuel consumption (around 8 L ) and one more driver to finish his journey during the second time slot instead of the first. However, now in the fifth solution, two customers, which are now served in the orange tour, are visited in the first time slot instead of the second time slot as it is in first solution. In the last nondom-

Table 6
Lexmin vs. Lexmax analysis for driver utility.

| \| $N$ \| | m | lexmin |  |  |  |  |  | lexmax |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\left\|\mathcal{Z}_{N}\right\|$ |  | \# models |  | $\mathrm{CPU} /\left\|\mathcal{Z}_{N}\right\|$ |  | $\left\|\mathcal{Z}_{N}\right\|$ |  | \# models |  | $\mathrm{CPU} /\left\|\mathcal{Z}_{\mathrm{N}}\right\|$ |  |
|  |  | Avg | Min | Avg | Max | Avg | Max | Avg | Min | Avg | Max | Avg | Max |
| 15 | 2 (1) | 9.75 | 7.00 | 14.25 | 22.00 | 404.01 | 658.38 | 12.00 | 7.00 | 17.25 | 22.00 | 687.71 | 1010.44 |
| 15 | 3 | 7.60 | 3.00 | 11.40 | 18.00 | 282.83 | 844.29 | 8.60 | 5.00 | 13.00 | 21.00 | 796.45 | 2054.36 |
| 15 | 4 | 7.60 | 5.00 | 11.80 | 15.00 | 575.75 | 1643.65 | 8.40 | 6.00 | 13.20 | 17.00 | 1371.03 | 2913.06 |

Table 7
Lexmin vs. Lexmax for driver utility $(|N|=15, m=4)$.

| Index | lexmin, $\left(p_{1}^{D}, p_{2}^{D}, p_{3}^{D}\right)=(6,5,1)$ |  |  |  |  | lexmax, ( $p_{1}^{D}, p_{2}^{D}, p_{3}^{D}$ ) $=(7,2,1$ ) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. | $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. |
| 1 | 125.24 | 18 | 251 | $(2,1,1)$ | $(11,4,0)$ | 125.24 | 17 | 251 | $(2,1,1)$ | $(11,4,0)$ |
| 2 | 125.35 | 18 | 252 | $(2,1,1)$ | $(12,3,0)$ | 125.35 | 17 | 252 | $(2,1,1)$ | $(12,3,0)$ |
| 3 | 126.36 | 21 | 250 | $(1,3,0)$ | $(10,5,0)$ |  |  |  |  |  |
| 4 | 126.65 | 21 | 251 | $(1,3,0)$ | $(11,4,0)$ |  |  |  |  |  |
| 5 | 127.12 | 21 | 252 | $(1,3,0)$ | $(12,3,0)$ |  |  |  |  |  |
| 6 | 127.23 | 21 | 253 | $(1,3,0)$ | $(13,2,0)$ | 127.23 | 13 | 253 | $(1,3,0)$ | $(13,2,0)$ |
| 7 | 127.90 | 21 | 254 | $(1,3,0)$ | $(14,1,0)$ | 127.90 | 13 | 254 | $(1,3,0)$ | $(14,1,0)$ |
| 8 | 134.48 | 20 | 255 | $(0,4,0)$ | $(15,0,0)$ | 134.48 | 8 | 255 | $(0,4,0)$ | $(15,0,0)$ |
| 9 |  |  |  |  |  | 135.00 | 17 | 253 | $(2,1,1)$ | $(13,2,0)$ |
| 10 | 139.36 | 21 | 255 | $(1,3,0)$ | $(15,0,0)$ | 139.36 | 13 | 255 | $(1,3,0)$ | $(15,0,0)$ |

inated solution (Fig. 5(f)) compared the fifth one, one of the customers, which was visited in the purple tour, is now served in the orange tour and also both purple and orange tours are reversed resulting in almost 13 more liters of fuel consumption. The customer distribution to time slots remains the same as in the fifth solution, but now all drivers complete their routes at the same (second) time slot leading to more balanced tours for drivers. Through this analysis, the decision maker can, for example, see that, with $2.58 \%$ increase in fuel cost (emissions), it is possible to improve customer satisfaction and serve one more customer in the best slot; while an extra improvement requires a further $3.58 \%$ increase in emission that is around $6.2 \%$ increase from the minimum emission solution.

Even this small example illustrates the possibility of obtaining alternative solutions for the decision maker to choose from, considering the interests of all stakeholders involved. This multiobjective framework allows the decision makers analyse the tradeoffs between the three criteria that reflect these interests and come up with good compromise solutions to ensure a sustainable system that will make all parties satisfied.

### 5.2.1. Lexmin vs. lexmax comparison for drivers

Note that depending on the payment structure, the decision makers may also use a lexicographic maximization approach to assess driver welfare and hence lexicographically maximize the number of drivers with routes in the slots, starting with the best slot. Table 6 compares the average results of lexicographic minimization (lexmin) and lexicographic maximization (lexmax) approaches for driver utility functions. The figures given in Table 6 are the average results over five instances with 15 customers and two, three and four drivers resulting 15 instances in total, one of which is infeasible.

The results indicate that using lexmax approach instead of lexmin approach increases the number of nondominated solutions found and hence the number of models solved. The solution time per nondominated solution also increases by around $130 \%$ on average, suggesting that using lexmax approach makes the problem more difficult to solve.

We also analyse how the set of solutions change when the approach changes. For four out of 14 instances, the same nondominated solution sets are found by both lexmin and lexmax approaches. For one of the remaining instances with 15 customers
and four drivers, we present a detailed comparison between these two approaches (Table 7). As it can be seen in the table, six nondominated solutions are common to both approaches (1,2,6,7,8,10). Lexmin approach finds three additional solutions ( $3,4,5$ ), which cannot be found when lexmax approach is used. Solutions 3, 4 and 5 with driver distribution $(1,3,0)$ are not found by the lexmax approach because all of them are dominated by solution 2 in the lexmax sense. On the other hand, lexmax approach finds nondominated solution 9 , which cannot be found by using lexmin approach. This is again due to the change in the preference structure for driver distribution.

### 5.2.2. Lexmin vs. lexmax comparison for customers

We now consider the case where the decision maker is not fairness-oriented and implement a lexicographic maximization approach to maximize the number of customers, starting with the best slot. We conduct experiments over five instances with 15 customers and two, three and four drivers to compare the results of lexicographic minimization (lexmin) and lexicographic maximization (lexmax) approaches for customer welfare functions. Overall, the results suggest that for small number of drivers, the two approaches yield different nondominated solutions. When the number of drivers is increased, similarity between the Pareto frontiers of both approaches increases.

We further analyse how the nondominated solutions change when a different (lexmin or lexmax) approach is used to quantify customer welfare. We present a comparison between these two approaches on one instance with 15 customers and two drivers (Table 8). The figures given in the table indicate that there are seven nondominated solutions found by both approaches ( $1,2,3,4,5,15,17$ ). Lexmin approach finds eight additional nondominated solutions ( $6,7,8,9,11,12,13,16$ ), the first four of which are dominated by solutions 5 and the last four of them are dominated by solution 10, when lexmax approach is used. On the other hand, lexmax approach finds three additional nondominated solutions $(10,14,18)$, which cannot be found when lexmin approach is used since the first solution (10) is dominated by solution 9 and the last two solutions ( 14 and 18) are dominated by solution 13. As expected, we see more solutions with no (resp. most) customers in the latest (resp. earliest) slot in lexmin (resp. lexmax) approach.

Table 8
Lexmin vs. Lexmax for customer utility $(|N|=15, m=2)$.

| Index | lexmin, $\left(p_{1}^{C}, p_{2}^{C}, p_{3}^{C}\right)=(17,16,1)$ |  |  |  |  | lexmax, $\left(p_{1}^{C}, p_{2}^{C}, p_{3}^{C}\right)=(18,2,1)$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. | $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. |
| 1 | 124.69 | 4 | 201 | $(0,1,1)$ | $(6,6,3)$ | 124.69 | 4 | 123 | $(0,1,1)$ | $(6,6,3)$ |
| 2 | 125.00 | 2 | 216 | $(0,0,2)$ | $(6,7,2)$ | 125.00 | 2 | 124 | $(0,0,2)$ | $(6,7,2)$ |
| 3 | 125.11 | 4 | 202 | $(0,1,1)$ | $(7,5,3)$ | 125.11 | 4 | 139 | $(0,1,1)$ | $(7,5,3)$ |
| 4 | 125.29 | 4 | 218 | $(0,1,1)$ | $(8,5,2)$ | 125.29 | 4 | 156 | $(0,1,1)$ | $(8,5,2)$ |
| 5 | 125.72 | 4 | 219 | $(0,1,1)$ | $(9,4,2)$ | 125.72 | 4 | 172 | $(0,1,1)$ | $(9,4,2)$ |
| 6 | 126.26 | 2 | 230 | $(0,0,2)$ | $(5,9,1)$ |  |  |  |  |  |
| 7 | 127.72 | 4 | 232 | $(0,1,1)$ | (7,7,1) |  |  |  |  |  |
| 8 | 127.90 | 2 | 233 | $(0,0,2)$ | $(8,6,1)$ |  |  |  |  |  |
| 9 | 128.15 | 4 | 233 | $(0,1,1)$ | $(8,6,1)$ |  |  |  |  |  |
| 10 |  |  |  |  |  | 129.36 | 4 | 188 | (0,1,1) | (10,3,2) |
| 11 | 129.52 | 2 | 245 | $(0,0,2)$ | $(5,10,0)$ |  |  |  |  |  |
| 12 | 129.72 | 4 | 234 | $(0,1,1)$ | $(9,5,1)$ |  |  |  |  |  |
| 13 | 130.83 | 2 | 248 | $(0,0,2)$ | $(8,7,0)$ |  |  |  |  |  |
| 14 |  |  |  |  |  | 131.30 | 2 | 189 | $(0,0,2)$ | $(10,4,1)$ |
| 15 | 133.63 | 4 | 235 | $(0,1,1)$ | $(10,4,1)$ | 133.63 | 4 | 189 | (0,1,1) | (10,4,1) |
| 16 | 133.68 | 2 | 249 | $(0,0,2)$ | $(9,6,0)$ |  |  |  |  |  |
| 17 | 134.53 | 2 | 250 | $(0,0,2)$ | $(10,5,0)$ | 134.53 | 2 | 190 | $(0,0,2)$ | $(10,5,0)$ |
| 18 |  |  |  |  |  | 135.95 | 2 | 205 | $(0,0,2)$ | (11,3,1) |

Table 9
Customers from different delivery classes ( $|N|=15, m=2$ ).

| Original |  |  |  |  | 10 prime customers |  |  |  |  |  | 5 prime customers |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. | $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. | C. Dist. (Prime) | $z_{1}$ | $z_{2}$ | $z_{3}$ | D. Dist. | C. Dist. | C. Dist. (Prime) |
| 117.93 | 4 | 201 | $(0,1,1)$ | $(6,6,3)$ | 117.93 | 4 | 83 | $(0,0,1)$ | $(6,6,3)$ | $(3,4,3)$ | 117.93 | 4 | 28 | $(0,1,1)$ | $(6,6,3)$ | (3,1,1) |
| 118.11 | 2 | 232 | $(0,0,2)$ | $(7,7,1)$ | 118.11 | 2 | 114 | $(0,0,2)$ | $(7,7,1)$ | $(4,6,0)$ | 118.11 | 2 | 33 | $(0,0,2)$ | $(7,7,1)$ | $(3,2,0)$ |
| 118.35 | 2 | 245 | $(0,0,2)$ | $(5,10,0)$ | 118.38 | 4 | 84 | $(0,1,1)$ | $(8,4,3)$ | $(4,3,3)$ | 124.47 | 4 | 33 | $(0,1,1)$ | $(8,4,3)$ | $(3,2,0)$ |
| 118.38 | 4 | 203 | $(0,1,1)$ | $(8,4,3)$ | 119.91 | 2 | 117 | $(0,0,2)$ | $(10,4,1)$ | $(7,3,0)$ | 127.51 | 2 | 34 | $(0,0,2)$ | $(7,7,1)$ | $(4,1,0)$ |
| 120.15 | 4 | 204 | $(0,1,1)$ | $(9,3,3)$ | 120.15 | 4 | 106 | $(0,1,1)$ | $(9,3,3)$ | $(6,3,1)$ | 129.58 | 4 | 34 | $(0,1,1)$ | $(7,5,3)$ | $(4,1,0)$ |
| 120.16 | 2 | 248 | $(0,0,2)$ | $(8,7,0)$ | 121.61 | 4 | 107 | $(0,1,1)$ | $(10,4,1)$ | $(7,2,1)$ |  |  |  |  |  |  |
| 121.17 | 4 | 233 | $(0,1,1)$ | $(8,6,1)$ | 125.37 | 4 | 108 | $(0,1,1)$ | $(10,4,1)$ | $(8,1,1)$ |  |  |  |  |  |  |
| 121.61 | 4 | 235 | $(0,1,1)$ | $(10,4,1)$ | 125.48 | 4 | 116 | $(0,1,1)$ | $(8,6,1)$ | $(6,4,0)$ |  |  |  |  |  |  |
| 133.37 | 2 | 249 | $(0,0,2)$ | $(9,6,0)$ | 125.93 | 4 | 117 | $(0,1,1)$ | $(10,4,1)$ | $(7,3,0)$ |  |  |  |  |  |  |
| 137.79 | 2 | 250 | $(0,0,2)$ | $(10,5,0)$ | 129.68 | 4 | 118 | $(0,1,1)$ | $(10,4,1)$ | $(8,2,0)$ |  |  |  |  |  |  |

### 5.2.3. Customers from different delivery classes

This section analyses a case where we partially relax the assumption of customer anonymity and consider customers from different delivery classes such as "regular" and "prime" customers together, where the customers within their respective classes are still indistinguishable. Our model can be trivially modified so as to consider fairness among "prime" customers only. We conduct some experiments and Table 9 shows the results for an instance with two drivers and 15 customers. Here, we consider two different cases; with 10 and five "prime" customers out of 15 total customers and compare it with the original case, where fairness is considered for all 15 "prime" customers. Overall results indicate that in some cases, we find the same nondominated solutions that is found by the original version of the problem, while new nondominated solutions can also be added to the set when the assumption of full customer anonymity is relaxed.

## 6. Conclusion

In this study, we propose a multiobjective framework that helps the planners to consider the interests of the main stakeholders in a logistics setting. Each objective reflects the concerns related to one of the three parties: the company, the customers served and the drivers delivering the good. Our problem assumes that customers are indistinguishable in terms of the service they are entitled to receive and so they belong to the same delivery class. We reflect the company's economic and environmental concern via minimizing a fuel consumption (emission) function and incorporate the welfare of the customers, and drivers by suggesting novel welfare function forms. Our framework is motivated by a real-life problem that a
large logistics company is facing; hence we formulate the welfare functions accordingly. The company divides the planning period (typically one day) into time slots and informs the customers about which slot their delivery will be performed. In line with this, we measure the welfare of a customer based on the slot she is served and define the total welfare function so as to encourage quick and fair delivery across customers. Observing that the company pays the drivers based on mileage, we define a welfare function for the drivers that will ensure quick and balanced routes. We implement an efficient objective-space based algorithm to solve the resulting triobjective optimization problem and demonstrate the applicability of the framework.

Future research could explore use of heuristic algorithms so as to find quick solutions to the three objective programming problem that we introduce in this work. Moreover, alternative welfare function forms and their effects on the recommended solutions can be investigated.

## Acknowledgments

The authors thank anonymous reviewers and the editor for their valuable comments on an earlier version of this paper that resulted in improved content and exposition.

## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2021.09.036

## References

Abdullahi, H., Reyes-Rubiano, L., Ouelhadj, D., Faulin, J., \& Juan, A. A. (2021). Modelling and multi-criteria analysis of the sustainability dimensions for the green vehicle routing problem. European journal of operational research, 292(1), 143-154.
Argyris, N., Özlem Karsu, \& Yavuz, M. (2021). Fair resource allocation: Using welfare-based dominance constraints. European journal of operational research. https://doi.org/10.1016/j.ejor.2021.05.003. In Press https://www.sciencedirect. com/science/article/pii/S0377221721003970
Barth, M., \& Boriboonsomsin, K. (2008). Real-world carbon dioxide impacts of traffic congestion. Transportation Research Record: Journal of the Transportation Research Board, 2058(1), 163-171.
Barth, M., Younglove, T., \& Scora, G. (2005). Development of a heavy-duty diesel modal emissions and fuel consumption model. Technical Report. California Partners for Advanced Transit and Highways (PATH), Institute of Transportation Studies, University of California at Berkeley.
Bektaş, T., \& Laporte, G. (2011). The pollution-routing problem. Transportation Research Part B: Methodological, 45(8), 1232-1250.
Bertazzi, L., Golden, B., \& Wang, X. (2015). Min-max vs. min-sum vehicle routing: A worst-case analysis. European journal of operational research, 240(2), 372-381.
Blackorby, C., \& Donaldson, D. (1978). Measures of relative equality and their meaning in terms of social welfare. Journal of economic theory, 18(1), 59-80.
Campbell, A. M., Vandenbussche, D., \& Hermann, W. (2008). Routing for relief efforts. Transportation Science, 42(2), 127-145.
Demir, E., Bektaş, T., \& Laporte, G. (2012). An adaptive large neighborhood search heuristic for the pollution-routing problem. European journal of operational research, 223(2), 346-359.
Demir, E., Bektaş, T., \& Laporte, G. (2014a). The bi-objective pollution-routing problem. European journal of operational research, 232(3), 464-478.
Demir, E., Bektaş, T., \& Laporte, G. (2014b). A review of recent research on green road freight transportation. European journal of operational research, 237(3), 775-793.
DHL (2015). Fair and responsible logistics: A DHL perspective on how to create lasting competitive advantage. Available from: https://discover.dhl.com/content/ dam/dhl/downloads/interim/full/dhl-trend-report-fair-responsible.pdf [Accessed 22 September 2021].
Dukkanci, O., Bektaş, T., \& Kara, B. Y. (2019a). Green network design problems. In Sustainable transportation and smart logistics (pp. 169-206). Elsevier.
Dukkanci, O., Kara, B. Y., \& Bektaş, T. (2019b). The green location-routing problem. Computers \& Operations Research, 105, 187-202.
Ebert, U. (1987). Size and distribution of incomes as determinants of social welfare. Journal of economic theory, 41(1), 23-33.
Eisenhandler, O., \& Tzur, M. (2019). A segment-based formulation and a matheuristic for the humanitarian pickup and distribution problem. Transportation Science, 53(5), 1389-1408.
Golden, B. L., Laporte, G., \& Taillard, É. D. (1997). An adaptive memory heuristic for a class of vehicle routing problems with minmax objective. Computers \& Operations Research, 24(5), 445-452.
Halvorsen-Weare, E. E., \& Savelsbergh, M. W. (2016). The bi-objective mixed capacitated general routing problem with different route balance criteria. European journal of operational research, 251(2), 451-465.
Mirzapour Al-e hashem, S., \& Rekik, Y. (2014). Multi-product multi-period inventory routing problem with a transshipment option: A green approach. International Journal of Production Economics, 157, 80-88.
Huang, M., Smilowitz, K., \& Balcik, B. (2012). Models for relief routing: Equity, efficiency and efficacy. Transportation research part E: logistics and transportation review, 48(1), 2-18.
Jabali, O., Woensel, T., \& de Kok, A. (2012). Analysis of travel times and $\mathrm{CO}_{2}$ emissions in time-dependent vehicle routing. Production and Operations Management, 21(6), 1060-1074.
Jozefowiez, N., Semet, F., \& Talbi, E.-G. (2002). Parallel and hybrid models for multi--objective optimization: Application to the vehicle routing problem. In International conference on parallel problem solving from nature (pp. 271-280). Springer.
Jozefowiez, N., Semet, F., \& Talbi, E.-G. (2007). Target aiming pareto search and its application to the vehicle routing problem with route balancing. Journal of Heuristics, 13(5), 455-469.
Jozefowiez, N., Semet, F., \& Talbi, E.-G. (2009). An evolutionary algorithm for the vehicle routing problem with route balancing. European journal of operational research, 195(3), 761-769.

Kara, I., Kara, B. Y., \& Yetis, M. K. (2007). Energy minimizing vehicle routing problem. In A. Dress, X. Yinfeng, \& Z. Binhai (Eds.), Combinatorial optimization and applications, lecture notes in computer science, vol 4616 (pp. 62-71).
Karsu, Ö. (2016). Approaches for inequity-averse sorting. Computers \& Operations Research, 66, 67-80.
Karsu, Ö., \& Morton, A. (2015). Inequity averse optimization in operational research. European journal of operational research, 245(2), 343-359.
Karsu, Ö., Morton, A., \& Argyris, N. (2018). Capturing preferences for inequality aversion in decision support. European journal of operational research, 264(2), 686-706.
Kaynar, N., \& Karsu, Ö. (2018). Equitable decision making approaches over allocations of multiple benefits to multiple entities. Omega, 81, 85-98. https://doi. org/10.1016/j.omega.2017.10.001. https://www.sciencedirect.com/science/article/ pii/S0305048317301330.
Kim, B.-I., Kim, S., \& Sahoo, S. (2006). Waste collection vehicle routing problem with time windows. Computers \& Operations Research, 33(12), 3624-3642.
Kirlik, G., \& Sayın, S. (2014). A new algorithm for generating all nondominated solutions of multiobjective discrete optimization problems. European journal of operational research, 232(3), 479-488.
Klamroth, K., Lacour, R., \& Vanderpooten, D. (2015). On the representation of the search region in multi-objective optimization. European journal of operational research, 245(3), 767-778. https://doi.org/10.1016/j.ejor.2015.03.031. https://www. sciencedirect.com/science/article/pii/S0377221715002386.
Koç, Ç., Bektaş, T., Jabali, O., \& Laporte, G. (2014). The fleet size and mix pollution-routing problem. Transportation Research Part B: Methodological, 70, 239-254.
Kostreva, M. M., Ogryczak, W., \& Wierzbicki, A. (2004). Equitable aggregations and multiple criteria analysis. European journal of operational research, 158(2), 362-377.
Lacomme, P., Prins, C., Prodhon, C., \& Ren, L. (2015). A multi-start split based path relinking (msspr) approach for the vehicle routing problem with route balancing. Engineering applications of artificial intelligence, 38, 237-251.
Lehuédé, F., Péton, O., \& Tricoire, F. (2020). A lexicographic minimax approach to the vehicle routing problem with route balancing. European journal of operational research, 282(1), 129-147.
Li, L., \& Fu, Z. (2002). The school bus routing problem: A case study. Journal of the Operational Research Society, 53(5), 552-558.
Mancini, S., Gansterer, M., \& Hartl, R. F. (2021). The collaborative consistent vehicle routing problem with workload balance. European journal of operational research, 293(3), 955-965.
Matl, P., Hartl, R. F., \& Vidal, T. (2018). Workload equity in vehicle routing problems: A survey and analysis. Transportation Science, 52(2), 239-260.
Nace, D., \& Orlin, J. B. (2007). Lexicographically minimum and maximum load linear programming problems. Operations research, 55(1), 182-187.
Oyola, J., \& Løkketangen, A. (2014). Grasp-asp: An algorithm for the cvrp with route balancing. Journal of Heuristics, 20(4), 361-382.
Perugia, A., Moccia, L., Cordeau, J.-F., \& Laporte, G. (2011). Designing a home-to-work bus service in a metropolitan area. Transportation Research Part B: Methodological, 45(10), 1710-1726.
Rawls, J. (1971). A theory of justice. Cambridge Mass: Harvard University Press.
Rodriguez, D. A., Rocha, M., Khattak, A. J., \& Belzer, M. H. (2003). Effects of truck driver wages and working conditions on highway safety: Case study. Transportation research record, 1833(1), 95-102.
Sbihi, A., \& Eglese, R. W. (2010). Combinatorial optimization and green logistics. Annals of operations research, 175(1), 159-175.
Scora, G., \& Barth, M. (2006). Comprehensive modal emissions model, version 3.01 User's guide. Technical Report. Centre for Environmental Research and Technology. University of California, Riverside, United States of America.
Vidal, T., Laporte, G., \& Matl, P. (2020). A concise guide to existing and emerging vehicle routing problem variants. European journal of operational research, 286(2), 401-416.
Zachariadis, E. E., Tarantilis, C. D., \& Kiranoudis, C. T. (2015). The load-dependent vehicle routing problem and its pick-up and delivery extension. Transportation Research Part B: Methodological, 71, 158-181.
Zajac, S., \& Huber, S. (2021). Objectives and methods in multi-objective routing problems: A survey and classification scheme. European journal of operational research, 290, 1-25.


[^0]:    * Corresponding author.

    E-mail address: dukkanci@europa-uni.de (0. Dukkanci).

[^1]:    ${ }^{1}$ ith unit vector in $\mathbb{R}^{l_{c}}$ has a value of 1 in ith dimension and 0 everywhere else.

