Journal of the Operational Research Society (2008) 59, 1460–1470 0160-5682/08

www.palgrave-journals.com/jors

Wide area telecommunication network design: application to the Alberta SuperNet

EA Cabral[1], E Erkut[2], G Laporte[3]* and RA Patterson[1]

[1]University of Alberta, Edmonton, Canada; [2]Bilkent University, Ankara, Turkey; and [3]HEC Montréal, Montréal, Canada

This article proposes a solution methodology for the design of a wide area telecommunication network. This study is motivated by the Alberta SuperNet project, which provides broadband Internet access to 422 communities across Alberta. There are two components to this problem: the network design itself, consisting of selecting which links will be part of the solution and which nodes should house shelters; and the loading problem which consists of determining which signal transport technology should be installed on the selected edges of the network. Mathematical models are described for these two subproblems. A tabu search algorithm heuristic is developed and tested on randomly generated instances and on Alberta SuperNet data.

Journal of the Operational Research Society (2008) **59,** 1460–1470. doi:10.1057/palgrave.jors.2602479
Published online 12 September 2007

Keywords: heuristic; network design; telecommunications

1. Introduction

Network design problems (NDPs) are central to planning telecommunication systems (see, eg Balakrishnan *et al*, 1997; Raghavan and Magnanti, 1997). Most network design research focuses on extracting from a network an optimal subnetwork that will satisfy various requirements. Here we use a broader network design definition that goes beyond the topological component and encompasses the loading aspect, that is the choice of equipment to be installed on the subnetwork.

Telecommunication networks are generally classified according to their geographical span. They include local area networks (LANs) connecting small areas, usually a single building or a set of buildings, metropolitan area networks (MANs) covering a city or a metropolitan area, and wide area networks (WANs) spanning large territories made up of several cities, states, or countries. Another important classification in network design is the subnetwork topology. The most common topologies are trees, rings, meshes and unstructured networks. Telecommunication networks are often composed of a *backbone network* linking primary nodes and of an *access network*, but this distinction does not apply to our study.

This article considers the design of a WAN tree network with a technological choice component. Our study is motivated by the Alberta SuperNet project, a partnership between the Alberta provincial government and a private consortium led by Bell West Inc. Their goal is to provide broadband Internet access to 422 communities across Alberta. Optical fibres in the SuperNet will be installed along existing roads, and therefore, the design problem uses the road network as an input. According to our GIS database, the Alberta road network comprises approximately 80 000 nodes and 280 000 edges. In practice, we solve the problem on a simplified network containing 21 714 nodes and 22 871 edges. The edges correspond to the shortest tree spanning the 422 communities; the nodes include these communities and intermediary locations on the spanning tree.

The Alberta SuperNet project requires no alternative paths or redundancy for communication flow, in the event of hardware or fibre failure. Thus the most cost-effective topology is a tree structure in which the digging and fibre installation costs are minimized, as suggested by Chamberland *et al* (2000). The project also allows for the coexistence of technologies along the same cable in different strands of optical fibres. With such freedom, signals can travel in parallel, as long as a sufficient number of fibres are available in the link for all signals. Because of multiple technologies, switches must sometimes be installed at the nodes to allow signals to pass between fibres of different transmission capacities. However, the presence of switches induces transmission delays. Also, it is sometimes necessary to locate multiplexers at the nodes to regenerate the signals.

The use of multiple technologies renders the telecommunication NDP complex. Our goal is to design a least-cost subnetwork that spans all communities and satisfies a number of technological constraints. Although the Alberta SuperNet is assumed to be tree-shaped, our formulations and algorithms do not assume any particular topology. They can therefore be applied to general contexts (Figure 1).

*Correspondence: G Laporte, HEC Montréal, 3000 chemin de la Côte Sainte Catherine, Montréal, Quebec H3T 2A7, Canada.
E-mail: gilbert@crt.umontreal.ca

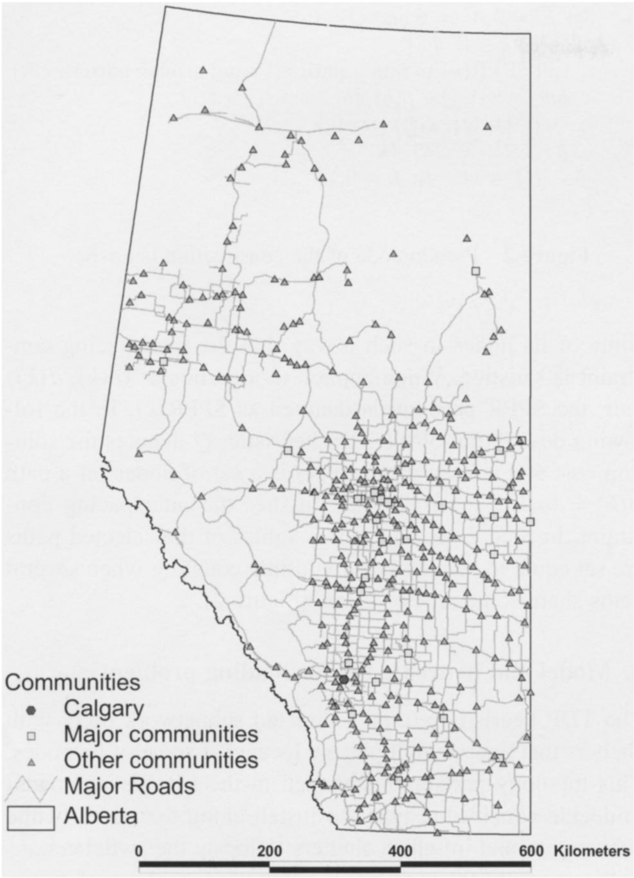

Figure 1 The simplified Alberta SuperNet.

An abundant literature exists on NDPs, particularly in the telecommunications area. A large body of the research addresses pure topological problems like the Steiner tree problem (STP) (eg Koch and Martin, 1998; Lucena and Beasley, 1998; Patterson *et al*, 1999; Polzin and Vahdati Daneshmand, 2001a,b, 2003; Costa *et al*, 2006) or problems defined on particular topologies like trees (Randazzo and Luna, 2001; Gzara and Goffin, 2005), rings (eg Armony *et al*, 2000; Chamberland and Sansò, 2000), or meshes (Costa, 2005; Kerivin and Mahjoub, 2005; Magnanti and Raghavan, 2005). Several papers address hierarchical problems that associate a particular technology with each level (eg Balakrishnan *et al*, 1998; Chamberland *et al*, 2000; Chamberland and Sansò, 2001; Chopra and Tsai, 2002; Labbé *et al*, 2004). Research on hierarchical network design is relevant to our case, but no existing paper addresses the problem we study. General articles and books on NDPs in telecommunications include Doverspike and Saniee (2000), Gavish (1992), and Sansò and Soriano (1999).

The NDP considered in this article is NP-hard because it subsumes several NP-hard problems like the STP. Although it may be possible to integrate all aspects of the problem into a single formulation and to design a heuristic to generate solutions, such an approach would be ineffective in our case, due to the complexity of the resulting formulation. Instead, we opted for a decomposition approach in which we first solve the topological design problem (TDP) and then solve the loading problem (LP) on the TDP solution. We present models and algorithms for these problems in the next two sections, followed by computational results.

2. Model and heuristic for the TDP

The TDP is defined on an undirected network $G=(V, E, K)$, where V is a node set, and $E=\{(i, j) : i, j \in V, i<j\}$ is an edge set. The set $K=\{(o(k), d(k))\}$ is a set of *communication pairs* in which $o(k)$ and $d(k)$ are the respective origin and destination of the kth communication request. With each edge (i, j) is associated a cost c_{ij} and a length d_{ij}. Node j is associated with a fixed cost f_j of locating a *shelter* to house a multiplexer, a switcher, or both. Every $o(k)$ and $d(k)$ node requires a shelter. The TDP consists of determining a minimum cost subnetwork of G and of locating a shelter at some of its nodes in such a way that: (1) for every $(o(k), d(k))$ pair, the length of a path between $o(k)$ and the first shelter, between the last shelter and $d(k)$, or between two consecutive shelters does not exceed a preset bound λ; and (2) the total cost of the subnetwork, made up of edge costs and shelter fixed costs, is minimized. In the Alberta SuperNet project, the value of λ is 70 km. Note that this problem formulation disregards multiplexers. In other words, only shelters chosen by the TDP can house multiplexers in the solution of the LP.

The TDP can be formulated as an integer linear program in which the main variables correspond to directed paths associated with $(o(k), d(k))$ pairs. In order to handle directions, the number of communication pairs is first doubled, that is, we define $K'=\{(o'(k), d'(k)), (o''(k), d''(k))\}$, where $(o'(k), d'(k))=(o(k), d(k))$, and $(o''(k), d''(k))=(d(k), o(k))$, with $(o(k), d(k)) \in K$. Each edge $(i, j) \in E$ is replaced with two opposite arcs (i, j) and (j, i), with respective costs $c'_{ij}=c'_{ji}=c_{ij}/2$ and respective lengths $d'_{ij}=d'_{ji}=d_{ij}$. Denote the set of arcs by A. The problem definition is otherwise unchanged.

For each communication pair $k \in K'$, let $P(k)$ be the set of feasible paths from $o(k)$ to $d(k)$; given a path $p \in P(k)$, let $R(p)$ denote the set of feasible relay patterns of path p, that is an ordered subset of vertices on p separated by at most λ distance units, and let $r \in R(p)$ be a feasible relay pattern for path p. Define the binary variables

$$x_{ij}=\begin{cases}1 & \text{if arc } (i, j) \text{ belongs to the solution}\\ 0 & \text{otherwise}\end{cases}$$

$$y_i=\begin{cases}1 & \text{if a shelter is located at node } i\\ 0 & \text{otherwise}\end{cases}$$

$$z_k^{pr}=\begin{cases}1 & \text{if path } p \text{ with relay pattern } r \text{ is used by}\\ & \text{communication pair } (o(k), d(k))\\ 0 & \text{otherwise}\end{cases}$$

and the binary coefficients

$$a_{ij}^p = \begin{cases} 1 & \text{if arc } (i, j) \text{ belongs to path } p \\ 0 & \text{otherwise} \end{cases}$$

$$b_i^r = \begin{cases} 1 & \text{if a shelter is located at node } i \text{ in relay pattern } r \\ 0 & \text{otherwise} \end{cases}$$

The formulation of the TDP is then:
(TDP)

$$\text{Minimize} \quad \sum_{(i,j)\in A} c'_{ij} x_{ij} + \sum_{i\in V} f_i y_i \tag{1}$$

subject to

$$\sum_{\substack{p\in P(k)\\ r\in R(p)}} z_k^{pr} = 1 \qquad (k \in K') \tag{2}$$

$$\sum_{\substack{p\in P(k)\\ r\in R(p)}} a_{ij}^p z_k^{pr} \leqslant x_{ij} \qquad ((i, j)\in A, k\in K') \tag{3}$$

$$\sum_{\substack{p\in P(k)\\ r\in R(p)}} b_i^r z_k^{pr} \leqslant y_i \qquad (i \in V, k \in K') \tag{4}$$

$$x_{ij} = 0 \text{ or } 1 \qquad ((i, j) \in A) \tag{5}$$

$$y_i = 0 \text{ or } 1 \qquad (i \in V) \tag{6}$$

$$z_k^{pr} = 0 \text{ or } 1 \qquad (k \in K'). \tag{7}$$

In this formulation, the objective minimizes the total edge and shelter costs. Constraints (2) ensure that each $(o(k), d(k))$ pair is connected by a path. Constraints (3) imply that x_{ij} takes the value 1 whenever arc (i, j) belongs to path p, and constraints (4) guarantee that y_i is equal to 1 if a shelter is located at node i in path p. These two constraints lead to the correct cost calculation. We provide the above formulation for precision in problem definition. We do not use this formulation in solving the problem; instead we implement a heuristic.

2.1. Greedy heuristic

The heuristic we employ to solve the TDP was developed by Cabral *et al* (2007). It works on the directed graph in which each edge has been replaced by two opposite arcs. The heuristic is based on a procedure put forward by Takahashi and Matsuyama (1980) for the STP, which constructs a subnetwork in a greedy fashion, one $(o(k), d(k))$ pair at a time for every $k \in K'$. Because of the constraint imposed on the interspacing of shelters, the $(o(k), \ldots, d(k))$ paths are constructed by using the auxiliary pseudo-polynomial procedure suggested by Cabral *et al* (2005) for the shortest path problem with relays (SPPR). The input of the SPPR is a graph with arc (or edge) costs and weights, an interspacing limit of λ, and an origin–destination pair (o, d). The SPPR determines the shortest origin–destination path and the relay locations on

1. Set $\overline{E} := \emptyset$, $\overline{V} := \emptyset$ and $Q = 0$.
2. *for each* $k \in K$ *do* {
   call SPPR(k) to find a path $p(k)$ and a relay pattern $r(k)$
   for each $(i,j) \in p(k)$ *do*
     $\{Q := Q + c_{ij};\ c_{ij} = 0;\}$
   for each $i \in r(k)$ *do*
     $\{Q := Q + f_i;\ f_i = 0;\}$
   }

Figure 2 Pseudo-code of the construction heuristic.

some of its nodes in such a way that the interspacing constraint is satisfied. When applied to a particular $(o(k), d(k))$ pair, the SPPR problem is denoted as SPPR(k). In the following description of the TDP heuristic, Q denotes the solution cost and a relay pattern $r(k)$ is a set of nodes on a path $p(k) = (o(k), \ldots, d(k))$ that satisfies the interspacing constraint. In Step 2, the c_{ij} and f_i values of the selected paths are set equal to zero to avoid multiple counting when several paths share some arcs or nodes (Figure 2).

3. Model and heuristic for the loading problem

The TDP heuristic returns a directed subnetwork of G with shelters that houses multiplexers located at some of its nodes. This topology remains unchanged in the LP. We now need to decide which fibre types to install along the edges of the subnetwork and in which shelters to locate the switchers.

We consider three different types of optical signal transport technologies: Gigabit Ethernet (GE), Synchronous Optical Network (SONET), and Dense Wavelength Division Multiplex (DWDM). Among these, only GE is Internet compatible, and therefore, in order to have an Internet network in place, users must receive and transmit signals in GE technology. SONET and DWDM are well-established technologies for telecommunication, and provide more capacity per fibre and add less delay to the signal than GE technology.

GE technology has a per-fibre transmission capacity of 2.5 Gbps (Gigabits per second), compared to 10 Gbps/fibre for SONET and 40 Gbps/fibre for DWDM. The most expensive technology is DWDM, followed by SONET, then GE. Our model assumes the use of simple-mode optical fibres that are suitable for all three technologies. Our industrial partner informed us that, compared to GE, SONET and DWDM add an insignificant delay to the signal, but our model is capable of distinguishing between different signal delays. We assumed that GE repeaters, GE/SONET and GE/DWDM switchers add a delay of 1 ms to the signals, whereas all the other equipment add no delay. If a signal leaves an origin in GE or arrives at a destination in GE, no switcher is necessary at these nodes. However, if another technology is used, a switcher is necessary.

We used the following equipment prices: a GE repeater costs \$10 000 (all montetary amounts are in Canadian dollars), a SONET repeater costs \$15 000, a DWDM repeater costs \$35 000, a GE/SONET switcher costs \$20 000, a GE/DWDM

Table 1 Costs per metre in \$ per strand

Cable type h	1	2	3	4	5	6	7
α_h: # of strands/ cable	12	24	36	48	72	96	144
β_h: cost/ meter	2.25	2.85	3.58	5.35	6.50	7.90	10.63

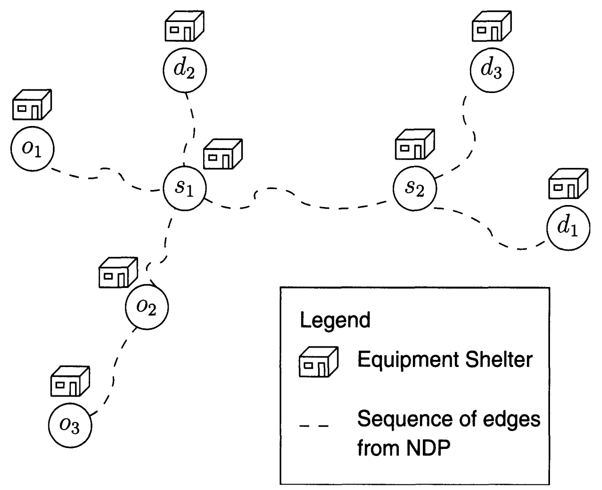

Figure 3 Subgraph from TDP.

switcher costs \$40 000, and a SONET/ DWDM switcher costs \$25 000. Cable prices are a stepwise function of the number of strands they contain. Table 1 provides costs per metre of each line of type $h \in H = \{1, 2, 3, 4, 5, 6, 7\}$ considered in our problem. If the transmission load requires a cable with a minimum of 20 strands on a 1-km road segment, one would use a type 2 cable, which would cost \$2850.

The solution procedure must be able to account for capacity, cost, and signal delays.

3.1. Network simplification

The subnetwork generated by the TDP heuristic (see Figure 3) can be simplified to remove intermediate nodes between any two successive shelter locations i and j on an $(o(k), d(k))$ path, to yield the simplified network in Figure 4. In other words, the subpath $(i, \ldots, j)$ is replaced with a single edge (i, j) of length $\bar{c}_{ij}$. This makes sense because it never is sub-optimal to use a single cable type on $(i, \ldots, j)$: if one cable type is best for a subpath of $(i, \ldots, j)$, then the same type is best for the entire path. Furthermore, the case of multiple cable types would require the location of multiplexers along the way. Thus a shelter exists at all nodes of the network on which the LP is solved.

3.2. Formulation

Denote by $G = (N, A)$ the subnetwork resulting from the simplification, when N is a set of nodes and A is a set of

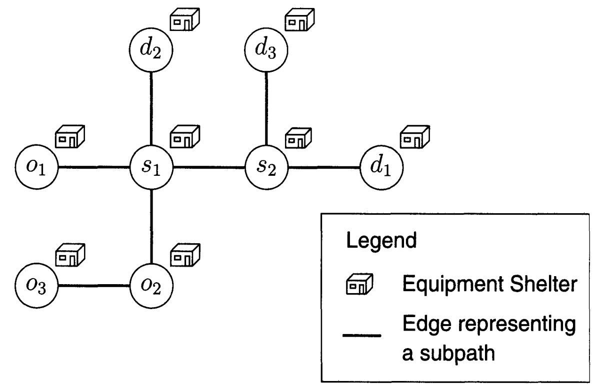

Figure 4 Graph for LP.

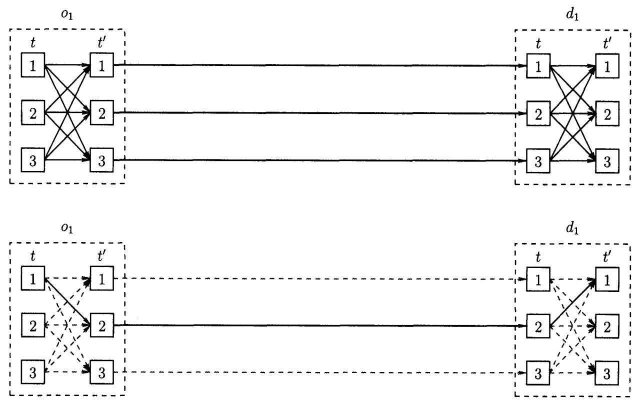

Figure 5 Technology pairs along a path $(o(k), \ldots, d(k))$.

arcs. For a given $k \in K'$, let $N'(k) = N \backslash \{o(k), d(k)\}$. Let T be the set of available technologies. In our application, $T = \{1 = \text{GE}, 2 = \text{SONET}, 3 = \text{DWDM}\}$. Denote by σ^t the fibre capacity of technology t. In our application, $\sigma^1 = 2.5$, $\sigma^2 = 10$, and $\sigma^3 = 40$. The set $T^2 = \{(t, t') : t, t' \in T\}$ represents all possible technology pairs associated with a shelter: t is the entering technology and t' is the exiting technology (Figure 5). If $t \neq t'$, then a switcher of cost $p^{tt'}$ must be located in the shelter. With each pair $(t, t') \in T^2$ is associated a delay $\delta^{tt'}$. In order to account for origins and destinations, it is useful to introduce a technology 0 at these nodes. Consequently, we define $T' = T \cup \{0\}$, and $T'^2 = \{(t, t') : t, t' \in T'\}$. If $t = 1$, then $\delta^{01} = \delta^{10} = 0$ because no switcher is necessary to send or receive a signal in GE. A communication flow demand ϕ_k (in Gbps) is given for each $(o(k), d(k))$ pair. The maximum allowed signal delay has the same value $\Delta_{\max}$ for each $(o(k), d(k))$ pair. Denote by β_h the cost per meter of cable of type h.

In order to formulate the LP, we introduce the following variables:

$$x_{ij}^{kt} = \begin{cases} 1 & \text{if technology } t \text{ is selected for communication} \\ & \text{pair } (o(k), d(k)) \text{ along arc } (i, j) \in A \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^{ktt'} = \begin{cases} 1 & \text{if for communication pair } (o(k), d(k)) \\ & \text{and node } i \in N, t \text{ is the entering technology} \\ & \text{and } t' \text{ is the exiting technology} \\ 0 & \text{otherwise} \end{cases}$$

$$z_i^{tt'} = \begin{cases} 1 & \text{if a switcher from technology } t \text{ to technology } t' \\ & \text{is installed at node } i \in N \\ 0 & \text{otherwise} \end{cases}$$

$v_{ij}^t =$ the number of fibre strands required for technology t on arc $(i, j) \in A$

$$w_{ij}^h = \begin{cases} 1 & \text{if a cable of type } h \text{ is installed on arc } (i, j) \in A \\ 0 & \text{otherwise.} \end{cases}$$

The formulation of the loading problem is then:
(LP)

$$\text{Minimize} \quad \sum_{h\in H}\sum_{(i,j)\in A} \bar{c}_{ij}\beta_h w_{ij}^h + \sum_{(t,t')\in T^2}\sum_{i\in N} \rho^{tt'} z_i^{tt'} \tag{8}$$

subject to

$$\sum_{t'\in T} x_{ij}^{kt'} = 1 \qquad (k \in K', (i, j) \in A) \tag{9}$$

$$y_{o(k)}^{k0t'} = x_{o(k)j}^{kt'} \qquad (k \in K', t' \in T, (o(k), j) \in A) \tag{10}$$

$$\sum_{t\in T} y_i^{ktt'} = x_{ij}^{kt'} \qquad (k \in K', t' \in T, i \in N'(k), (i, j) \in A) \tag{11}$$

$$x_{id(k)}^{kt} = y_{d(k)}^{kt0} \qquad (k \in K', t \in T, (i, d(k)) \in A) \tag{12}$$

$$x_{ij}^{kt} = \sum_{t'\in T} y_j^{ktt'} \qquad (k \in K', t \in T, j \in N'(k), (i, j) \in A) \tag{13}$$

$$z_i^{tt'} \geqslant y_i^{ktt'} \qquad (k \in K', (t, t') \in T^2 \text{ and } t \neq t', i \in N) \tag{14}$$

$$\sum_{i\in N'(k)}\sum_{(t,t')\in T^2} \delta^{tt'} y_i^{ktt'} + \sum_{t'\in T} \delta^{0t'} y_{o(k)}^{k0t'} + \sum_{t \int T} \delta^{t0} y_{d(k)}^{kt0} \leqslant \Delta_{\max} \qquad (k \in K') \tag{15}$$

$$\sigma^t v_{ij}^t \geqslant \sum_{k\in K} \phi_k x_{ij}^{kt} \qquad ((i, j)\in A, t\in T) \tag{16}$$

$$\sum_{t\in T} v_{ij}^t \leqslant \sum_{h\in H} \alpha_h w_{ij}^h \qquad ((i, j) \in A) \tag{17}$$

$$x_{ij}^{kt} = 0 \text{ or } 1 \qquad (k \in K', t \in T, (i, j) \in A) \tag{18}$$

$$y_i^{ktt'} = 0 \text{ or } 1 \qquad (k \in K', (t, t') \in T'^2, i \in N) \tag{19}$$

$$z_i^{tt'} = 0 \text{ or } 1 \qquad ((t, t') \in T^2, i \in N) \tag{20}$$

$$v_{ij}^t \geqslant 0 \text{ and integer} \qquad (t \in T, (i, j) \in A) \tag{21}$$

$$w_{ij}^h = 0 \text{ or } 1 \qquad (h \in H, (i, j) \in A) \tag{22}$$

In this formulation, the objective function computes the total fibre cost plus the cost of installing multiplexers. Constraints (9) state that exactly one technology will be selected for each communication pair and arc over the network. Constraints (10) ensure consistency between the technology change at the origin node of a communication pair and the technology used over the arcs leaving that node; similarly, constraints (11) ensure consistency between the technology leaving a node that is not a communication origin and the type of technology change provided at the given node. Constraints (12) and (13) are similar to (10) and (11) but apply to destination nodes. By constraints (14) switcher is installed at node i if a technology change takes place at that node. Constraints (15) impose the maximum delay requirement on any telecommunication pair. On the left hand side, the first term considers the technology change delays inside the path, the second term considers the technology change delay at the origin node, and the last term considers the technology change delay at the destination node. Constraints (16) ensure that sufficient fibre is installed on arc (i, j) to carry the flow passing on that arc; whereas constraints (17) guarantee that an appropriately sized cable is installed on (i, j) to accommodate the required number of fibre optical strands.

This integer program is of large scale even for small network examples, and is impractical for the Alberta SuperNet project. We have therefore opted to solve it by means of a tabu search (TS) heuristic.

3.3. TS algorithm

TS is a metaheuristic introduced by Glover (1986), which has become one of the most popular tools to a host of hard combinatorial optimization problems. It is based on the notion of neighbourhood. The neighbourhood $N(s)$ of solution s is the set of all solutions that can be reached from s by performing a certain type of move. Starting from an initial solution, TS moves at each iteration to the best solution in a subset $M(s)$ of $N(s)$. To prevent cycling, all solutions possessing a certain attribute of the new solution are declared *tabu* (or forbidden).

Table 2 Computation times (in minutes)

a	b	$\|V\|$	$\|E\|$	$\|K\| = 25$	$\|K\| = 50$	$\|K\| = 75$	$\|K\| = 100$
25	50	1250	2425	0.1	0.2	0.3	0.3
	100	2500	4875	0.2	0.5	0.8	1.1
	150	3750	7325	0.5	0.9	1.5	2.0
75	50	3750	7375	0.4	1.0	1.5	2.1
	100	7500	14825	1.4	3.1	4.9	6.4
	150	11250	22275	2.5	5.4	8.5	11.8
125	50	6250	12325	1.0	2.3	3.6	4.7
	100	12500	24775	3.1	6.8	10.5	14.1
	150	18750	37225	5.9	13.1	20.3	26.9

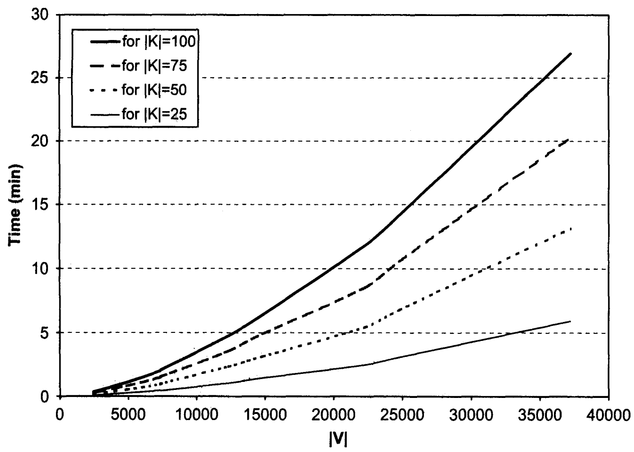

Figure 6 TDP heuristic computation time (in minutes).

The set $M(s)$ is the set of non-tabu solutions reachable from s. The process ends with the best solution encountered during the search whenever a given stopping criterion is met. The most common stopping criteria are a set number of iterations, a set number of consecutive iterations without improvement, or a time limit. In order to prevent the search process from stalling, tabu tenures are lifted after a number of iterations, at which time the risk of cycling has been virtually eliminated. The tabu status of a candidate solution can always be revoked without risk of cycling if this candidate solution is the best one encountered during the search. The success of any TS implementation depends largely on a careful exploitation of the structure and features of the problem at hand. We have applied this technique to the LP as described in the remainder of this section.

Given a TDP solution, a test is first performed to determine if it is feasible for the LP, ie if (12) can be satisfied for some technology $t \in T$. Otherwise the instance is infeasible. We have developed three heuristics to construct a feasible solution. The first, called delay bound heuristic (DBH), initially assigns the GE technology to all communications. Any $(o(k), \ldots, d(k))$ path violating (12) is then upgraded to SONET, and then to DWDM if necessary. This heuristic quickly produces a solution but does not take advantage of bundling signals to reduce the number of switchers along the communication paths. The second heuristic, called delay bound equipment saver heuristic (DBESH), corrects this deficiency by first identifying the highest value of k assigned to an arc of A and promoting all signals passing on that arc to technology k. The third heuristic, called *SONET Heuristic* (SH) initially assigns the GE technology to the entire network. It then upgrades each communication path $(o(k), \ldots, d(k))$ to SONET between the first and the last shelter, excluding the two arcs incident to $o(k)$ and $d(k)$. Although this heuristic does not guarantee feasibility in principle, it has always yielded feasible solutions in our test problems and has provided the best starting points to the TS algorithm.

We have used two types of move to define neighbour solutions. In a *single move*, the current technology associated with an arc (i, j) on a given path k is replaced by another; in a *trunk move*, the technologies on all paths sharing the same arc (i, j) are changed to the same technology on that arc. Single moves enable a fast recalculation of the objective function, because only a local update is required. Trunk moves are more time consuming because they require a full recalculation. After some experimentation we have opted to set the tabu tenure of a move equal to θ, where θ is randomly generated in [25, 50] according to a discrete uniform distribution. This means that once a move is performed, it cannot normally be undone for θ iterations.

Three versions of the TS algorithms were created. The first one, named *SingleTS*, used only the single moves; the second one, named *TrunkTS*, used solely trunk moves; finally, the third one combined both single and trunk moves, and was simply named *SingleTrunkTS*.

We have also tested two stopping criteria with several parameter values: the total number of iterations spent in the search and the total number of iterations without improvement in the value of the best known solution. We found that the second criterion with a value of 50 produced the best results.

Table 3 TDP heuristic percentage gap between the best and worst solutions

a	b	$\|V\|$	$\|E\|$	$\|K\| = 25$	$\|K\| = 50$	$\|K\| = 75$	$\|K\| = 100$
25	50	1250	2425	14.1	11.6	8.8	9.2
	100	2500	4875	9.9	10.4	8.8	7.6
	150	3750	7325	10.9	10.5	11.9	9.7
75	50	3750	7375	10.3	8.8	7.9	8.0
	100	7500	14 825	13.1	9.7	8.0	6.4
	150	11 250	22 275	9.5	8.0	8.4	8.4
125	50	6250	12 325	11.9	9.3	8.1	7.4
	100	12 500	24 775	10.9	8.0	7.6	7.1
	150	18 750	37 225	12.2	9.4	7.7	7.9

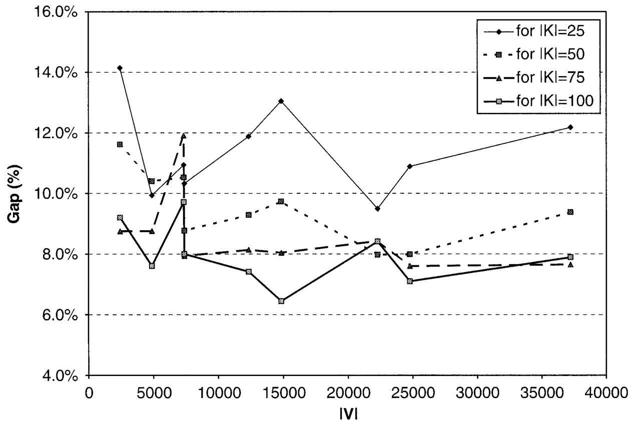

Figure 7 TDP heuristic percentage gap between the best and worst solutions.

4. Computational results

We carried out all computational tests on computers with AMD Opteron 250 2.4 GHz processors, 16 gigabytes of RAM and CentOS 4.2 operational system. We coded the algorithms in C++ and compiled them with GNU gcc compiler, version 3.4.4 20050721.

Tests were divided into two major groups, one using randomly generated test graphs, and another using the Alberta SuperNet project network.

4.1. Randomly generated tests

The test graphs follow a grid structure, with a rows and b columns and randomly (uniformly) generated integer values for costs and lengths. For these tests we used $|K|$ values of 25, 50, 75, and 100, a values of 25, 75, and 125, and b values of 50, 100, and 150. Parameters λ and $\Delta_{\max}$ were fixed to 70 km and 5 ms, respectively. Cost and edge length values were selected from [10, 30]. Communication pairs were randomly chosen. All communication pairs originated at a same point, in accordance with the Alberta SuperNet project, which has the centre of communication located in Calgary. The relay fixed costs were set at \$100 000, and the path costs were defined as \$10 per m. Communication flows were randomly generated, with a probability of 0.9 of being 1 Gbps and a probability of 0.1 of being 64 Gbps. With this communication flow probabilities, 10% of the communication pairs receive a communication load equivalent to those of internet highways, introducing a bias for DWDM technology usage in their final solution.

Table 2 presents the computational effort for the greedy heuristic. Each row contains the average computation time in minutes for ten instances. In Figure 6, we can see that the computation time grows with the number of nodes $|V|$, and also with the number of communication pairs $|K|$.

Table 3 reports the gap between the best and the worst solution found by the TDP heuristic. As one can observe in Figure 7, the gap decreases as the number of nodes $|V|$ increases, and it decreases as the number of communication pairs $|K|$ increases.

Once the topological NDP is solved for each instance, the resulting network is simplified, yielding a graph with $|N|$ nodes. As each test set results in a different number of nodes, we present the average and standard deviation of $|N|$ for given combinations of $|K|$, a and b in Table 4. This table also shows the average computation time for each of the LP heuristics. Each row represents the average computation time of 10 instances in seconds for given values of $|K|$, a and b. Table 5 presents the gap between the best and the worse solutions obtained by those heuristics.

As one can observe from Table 4, the LP routines DBH, DBESH, SH, and SingleTS were very fast, usually performing calculations in seconds even for the largest test set problems. TrunkTS and SingleTrunkTS were comparatively slower, with an average of 8.5 min and of over an hour in the worst case scenario. By comparing the gaps of Table 5, one can see that neither the TrunkTS nor the SingleTrunkTS algorithms yielded solutions that were significantly better than the SingleTS algorithm. Figures 8 and 9 show that the SingleTS heuristic is the most promising.

Table 4 Heuristic computation time (in seconds)

$\|K\|$	a	b	ab	$\|N\|$	$\sigma^2_{\|N\|}$	*DBH*	*DBESH*	*SH*	*SingleTS*	*TrunkTS*	*SingleTrunkTS*
25	25	50	1250	55.9	3.5	0.1	0.0	0.0	0.1	2.5	2.6
		100	2500	76.9	2.5	0.3	0.0	0.0	0.1	10.7	10.7
		150	3750	95.7	5.2	0.4	0.0	0.0	0.1	18.3	18.4
	75	50	3750	92.1	4.8	0.4	0.0	0.0	0.1	11.9	12.0
		100	7500	120.8	7.0	1.6	0.0	0.0	0.2	24.5	24.6
		150	11 250	148.0	9.5	2.5	0.0	0.0	0.3	38.4	38.6
	125	50	6250	110.0	6.2	0.9	0.0	0.0	0.2	23.5	23.7
		100	12 500	153.3	9.5	3.0	0.0	0.0	0.3	41.8	42.0
		150	18 750	181.3	16.6	6.1	0.0	0.0	0.4	61.0	61.6
50	25	50	1250	90.4	3.3	0.2	0.0	0.0	0.1	13.0	13.1
		100	2500	120.3	4.4	0.5	0.0	0.0	0.2	54.7	54.8
		150	3750	141.6	5.1	0.8	0.0	0.0	0.3	84.9	85.0
	75	50	3750	140.4	3.4	0.9	0.0	0.0	0.3	69.2	69.3
		100	7500	184.4	7.2	3.1	0.0	0.0	0.3	108.5	108.9
		150	11 250	220.9	6.0	5.7	0.0	0.0	0.5	241.1	240.5
	125	50	6250	167.5	7.4	2.2	0.0	0.0	0.3	92.0	91.9
		100	12 500	228.1	10.3	6.8	0.0	0.0	0.5	239.1	240.6
		150	18 750	277.6	9.6	13.6	0.0	0.0	0.7	632.0	631.8
75	25	50	1250	121.9	4.0	0.3	0.0	0.0	0.2	32.9	33.0
		100	2500	153.9	2.4	0.9	0.0	0.0	0.4	142.3	143.1
		150	3750	178.9	6.2	1.4	0.0	0.0	0.5	296.3	297.1
	75	50	3750	179.1	6.2	1.5	0.0	0.0	0.4	155.9	156.1
		100	7500	234.2	8.5	4.9	0.0	0.0	0.6	443.9	443.6
		150	11 250	272.9	8.1	8.9	0.0	0.0	0.8	741.3	752.7
	125	50	6250	215.3	7.1	3.6	0.0	0.0	0.7	468.7	468.0
		100	12 500	286.4	10.9	10.4	0.0	0.0	0.7	767.9	786.1
		150	18 750	343.1	8.1	19.1	0.0	0.0	1.5	1665.8	1666.0
100	25	50	1250	147.4	2.7	0.4	0.0	0.0	0.3	65.9	66.1
		100	2500	185.8	3.5	1.1	0.0	0.0	0.5	344.9	353.2
		150	3750	216.2	4.7	1.9	0.0	0.0	0.8	815.4	818.9
	75	50	3750	216.4	2.5	2.1	0.0	0.0	0.6	368.3	368.4
		100	7500	280.1	8.5	6.4	0.0	0.0	0.8	911.2	912.2
		150	11 250	326.1	9.7	12.9	0.0	0.0	1.1	1924.2	1961.1
	125	50	6250	253.5	5.3	4.7	0.0	0.0	1.0	1186.5	1190.0
		100	12 500	343.7	6.1	14.2	0.0	0.0	1.2	2215.5	2241.5
		150	18 750	396.4	8.1	24.8	0.0	0.0	1.9	4122.6	4150.2

Table 5 LP heuristic percentage gap between the best and worst solutions

$\|K\|$	a	b	*DBH*	*DBESH*	*SH*	*SingleTS*	*TrunkTS*	*SingleTrunkTS*
25	25	50	0.58	0.56	0.48	0.03	0.08	0.02
		100	0.45	0.43	0.30	0.01	0.03	0.01
		150	0.27	0.22	0.22	0.01	0.04	0.01
	75	50	0.39	0.34	0.25	0.02	0.05	0.00
		100	0.23	0.21	0.19	0.02	0.06	0.00
		150	0.21	0.19	0.14	0.01	0.06	0.00
	125	50	0.26	0.25	0.24	0.02	0.07	0.00
		100	0.21	0.19	0.11	0.02	0.05	0.00
		150	0.08	0.07	0.07	0.00	0.03	0.00
50	25	50	0.45	0.37	0.30	0.03	0.04	0.01
		100	0.39	0.35	0.24	0.02	0.02	0.00
		150	0.29	0.27	0.18	0.02	0.03	0.00
	75	50	0.31	0.32	0.24	0.05	0.02	0.00
		100	0.22	0.19	0.13	0.02	0.03	0.00
		150	0.13	0.10	0.08	0.00	0.03	0.00

Table 5 Continued

$\lvert K \rvert$	a	b	*DBH*	*DBESH*	*SH*	*SingleTS*	*TrunkTS*	*SingleTrunkTS*
	125	50	0.19	0.14	0.12	0.01	0.03	0.00
		100	0.14	0.12	0.09	0.02	0.02	0.00
		150	0.05	0.05	0.04	0.00	0.01	0.00
75	25	50	0.50	0.45	0.20	0.00	0.01	0.00
		100	0.32	0.30	0.18	0.01	0.00	0.00
		150	0.22	0.17	0.13	0.01	0.00	0.00
	75	50	0.29	0.27	0.18	0.00	0.02	0.00
		100	0.19	0.15	0.11	0.01	0.01	0.00
		150	0.13	0.10	0.08	0.01	0.01	0.00
	125	50	0.19	0.17	0.14	0.01	0.02	0.00
		100	0.13	0.10	0.08	0.01	0.01	0.00
		150	0.08	0.06	0.05	0.00	0.02	0.00
100	25	50	0.54	0.49	0.19	0.00	0.01	0.00
		100	0.28	0.26	0.16	0.01	0.01	0.00
		150	0.18	0.16	0.12	0.00	0.01	0.00
	75	50	0.29	0.27	0.17	0.00	0.01	0.00
		100	0.12	0.11	0.11	0.00	0.01	0.00
		150	0.11	0.09	0.08	0.01	0.01	0.00
	125	50	0.19	0.15	0.13	0.00	0.01	0.00
		100	0.09	0.09	0.08	0.00	0.01	0.00
		150	0.06	0.05	0.04	0.00	0.00	0.00

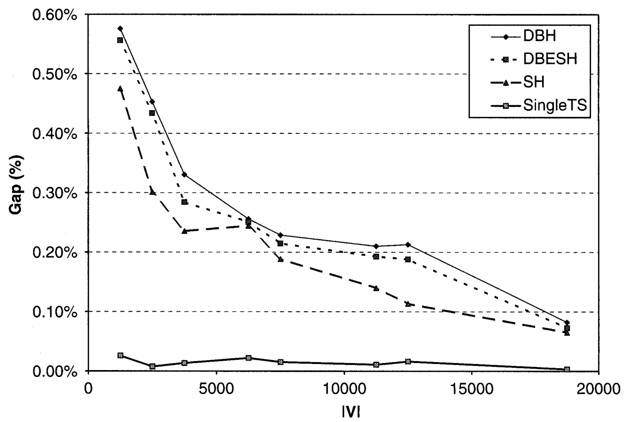

Figure 8 LP Heuristic Gap for $|K| = 25$.

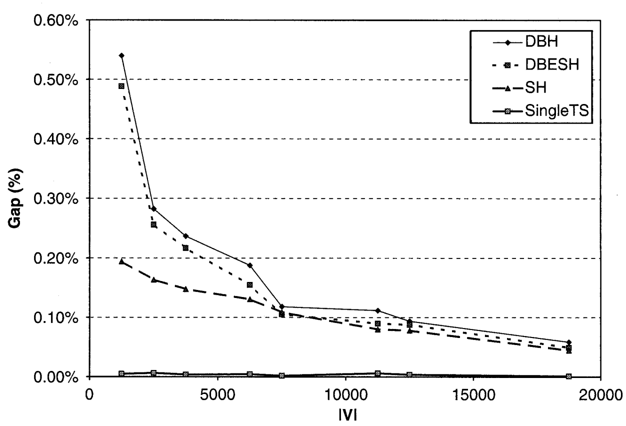

Figure 9 LP Heuristic Gap for $|K| = 100$.

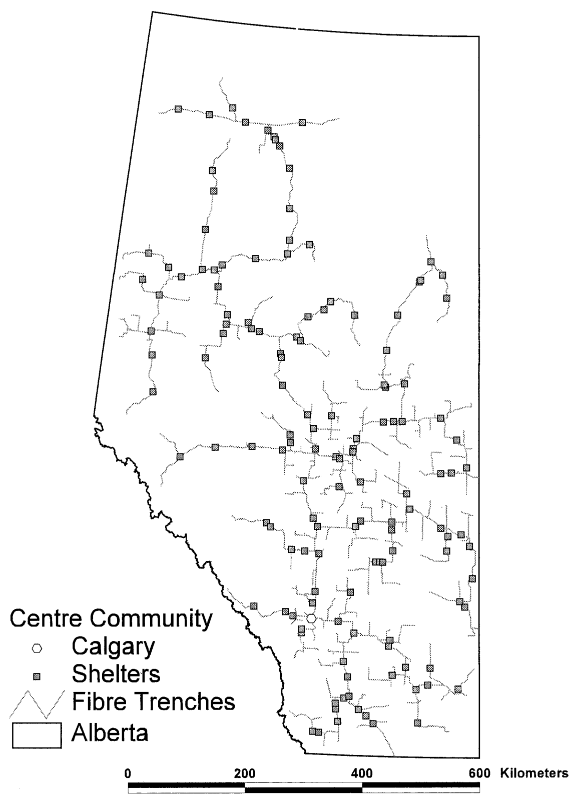

Figure 10 Placing fibre trenches and equipment shelters.

4.2. The Alberta SuperNet data

The Alberta SuperNet instance was constructed from GIS data for Alberta and information provided by Bell Canada. The network contains 21 714 nodes and 22 871 edges. The

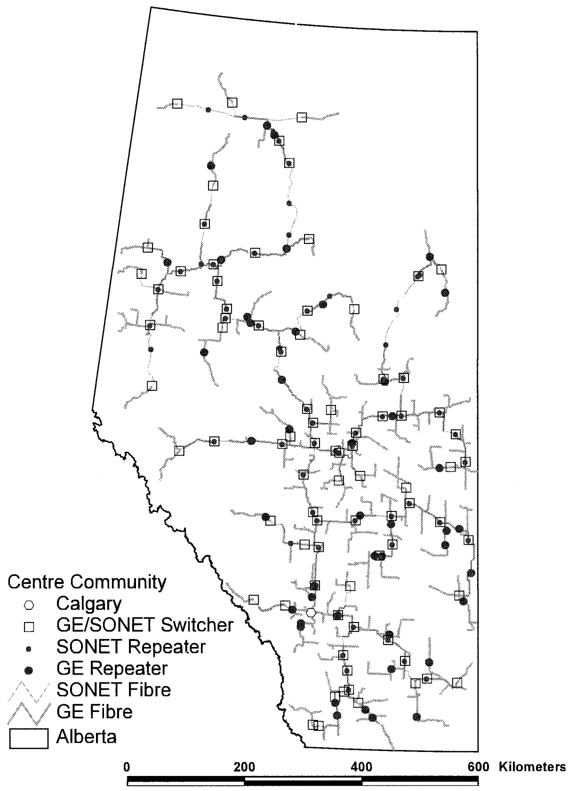

Figure 11 Defining technology on fibre and equipment.

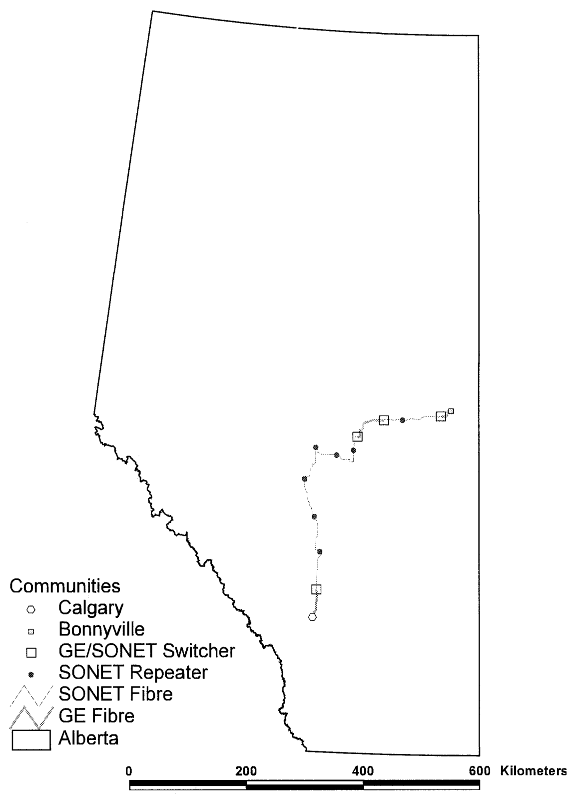

Figure 12 Path between Bonnyville and Calgary.

major road network of Alberta was used as an input graph. Each execution of the algorithm described in Figure 2 took, on average, 43 min, and the total gap between the best and the worst solution was 2.55%, corresponding to $3.25 million. It took 7.2 h to obtain this solution. Fibre trench costs totaled $113.61 million, whereas sheltering costs represented $13.80 million. This topology design is presented in Figure 10.

Once the TDP was solved, we constructed the simplified graph which contained 553 nodes and 3508 edges. Running DBH, DBESH, SH, and SingleTS took less than 2 s. The network loading solution is presented in Figure 11. Figure 12 depicts the configuration of a particular communication path in the network. This path connects the cities of Bonnyville and Calgary; the total delay to the signal is 4 ms.

5. Conclusions

We solved a telecommunication network design with a technological choice component, motivated by the Alberta SuperNet project. The problem naturally divides into a topological design subproblem and a loading subproblem. The first subproblem was formulated as an integer linear program and was solved by means of a greedy heuristic. The second subproblem was formulated as a non-linear mixed integer program and was solved by means of a TS heuristic. Computational tests conducted on randomly generated instances and on data received from the SuperNet project confirm the feasibility of the proposed methodology.

Acknowledgements—This work was partially supported by the Canadian Natural Sciences and Engineering Research Council under grants CRD 268431, OGP 25481 and 39682-05. This support is gratefully acknowledged. Thanks are due to Fatma Gzara, Osman Alp, Erla Anderson and two anonymous referees for their valuable comments.

References

Armony M, Klincewicz JG, Luss H and Rosenwein MB (2000). Design of stacked self-healing rings using a genetic algorithm. *J Heuristics* **6**: 85–105.

Balakrishnan A, Magnanti TL and Mirchandani PB (1997). Network design. In: Dell'Amico M, Maffioli MF and Martello S (eds). *Annotated Bibliographies in Combinatorial Optimization*. Wiley: Chichester.

Balakrishnan A, Magnanti TL and Mirchandani PB (1998). Designing hierarchical survivable networks. *Opns Res* **46**: 116–136.

Cabral EA, Erkut E, Laporte G and Tjandra SA (2005). The shortest path problem with relays. Unpublished manuscript.

Cabral EA, Erkut E, Laporte G and Patterson RA (2007). The network design problem with relays. *Eur J Opl Res*, **180**: 834–844.

Chamberland S and Sansò B (2000). Topological expansion of multiple-ring metropolitan area networks. *Networks* **36**: 210–224.

Chamberland S and Sansò B (2001). On the design problem of multitechnology networks. *INFORMS Journal on Computing* **13**: 245–256.

Chamberland S, Sansò B and Marcotte O (2000). Topological design of two-level telecommunication networks with modular switches. *Opns Res* **48**: 745–760.

Chopra S and Tsai C-Y (2002). A branch-and-cut approach for minimum cost multi-level network design. *Disc Math* **242**: 65–92.
Costa AM (2005). A survey on Benders decomposition applied to fixed-charge network design problems. *Comput Opl Res* **32**: 1429–1450.
Costa AM, Cordeau J-F and Laporte G (2006). Steiner tree problems with profits. *INFOR* **44**: 99–115.
Doverspike R and Saniee I (2000). *Heuristic Approaches for Telecommunications Network Management, Planning and Expansion*. Kluwer: Boston.
Gavish B (1992). Topological design of computer communication networks—The overall design problem. *Eur J Opl Res* **58**: 149–172.
Glover F (1986). Future paths for integer programming and links to artificial intelligence. *Comput Opl Res* **13**: 533–549.
Gzara F and Goffin J-L (2005). Exact solution of the centralized network design problem on directed graphs. *Networks* **45**: 181–192.
Kerivin H and Mahjoub AR (2005). Design of survivable networks: A survey. *Networks* **46**: 1–21.
Koch T and Martin A (1998). Solving Steiner tree problems in graphs to optimality. *Networks* **32**: 207–232.
Labbé M, Laporte G, Rodrigues Martín I and Salazar González JJ (2004). The ring star problem: Polyhedral analysis and exact algorithm. *Networks* **43**: 177–189.
Lucena A and Beasley JE (1998). A branch and cut algorithm for the Steiner problem in graphs. *Networks* **31**: 39–59.
Magnanti TL and Raghavan S (2005). Strong formulations for network design problems with connectivity requirements. *Networks* **45**: 61–79.
Patterson RA, Pirkul H and Rolland E (1999). A memory adaptive reasoning technique for solving the capacitated minimum spanning tree problem. *J Heuristics* **5**: 159–180.
Polzin T and Vahdati Daneshmand S (2001a). A comparison of Steiner tree relaxations. *Disc Appl Math* **112**: 241–261.
Polzin T and Vahdati Daneshmand S (2001b). Improved algorithms for the Steiner problem in networks. *Disc Appl Math* **112**: 263–300.
Polzin T and Vahdati Daneshmand S (2003). On Steiner trees and minimum spanning trees in hypergraphs. *Opns Res Lett* **31**: 12–20.
Raghavan S and Magnanti TL (1997). Network connectivity. In: Dell'Amico M, Maffioli F and Martello S (eds). *Annotated Bibliographies in Combinatorial Optimization*. Wiley: Chichester,
Randazzo CD and Luna HPL (2001). A comparison of optimal methods for local access uncapacitated network design. *Ann Opns Res* **106**: 263–286.
Sansò B and Soriano P (1999). *Telecommunications Network Planning*. Kluwer: Boston.
Takahashi H and Matsuyama A (1980). An approximate solution for the Steiner problem in graphs. *Math Japon* **24**: 573–577.

Received December 2006;
accepted June 2007 after one revision