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Counting Composites

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ABSTRACT
I defend the thesis that Composition Entails Identity (CEI): that is, a whole is identical to all of its parts, taken together. CEI seems to be inconsistent, since it seems to require that the parts of a whole possess incompatible number properties (for instance, being one thing and being many things). I show that these number properties are, in fact, compatible.

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1. Introduction

A whole is distinct from each of its parts individually (for example, my desk isn't identical to its left half, nor is it identical to its right half).[1] Nonetheless, I maintain, a whole is identical to all of its parts collectively (my desk is identical to its left and right halves, taken together). Call this thesis 'Composition Entails Identity' (CEI).[2]

In this paper, I defend CEI against what I call 'The Counting Argument'. It seems that if the parts are identical to the whole that they compose, then the parts must instantiate distinct number properties: the parts are both n things and m things, where $n \neq m$. But distinct number properties are apparently incompatible: if $n \neq m$, then no thing(s) can be both n things and m things. So, it seems, CEI must be false.

While CEI requires that the parts of a whole are both n things and m things, the appearance of incompatibility is based on a misunderstanding of the logical form of number ascriptions. Properly understood, 'The parts are n things' and 'The parts are m things' can both be true.

2. A Plural Language

In what follows, I'll be using the resources of a plural first-order language, as follows.[3]

(i) *Singular Terms* ('*a*', '*b*', *etc.*) *and Plural Terms* ('*aa*', '*bb*', *etc.*). A singular term denotes only one thing (an 'individual'), while a plural term can denote many things (a 'plurality'). I treat the latter as 'inclusively plural' terms, capable of denoting

[1] Compare Baxter [1988a, 1988b].
[2] CEI is weaker than the more familiar thesis, 'Composition as Identity' (CAI). I say that $xx = y$ if xx compose y; CAI adds that $xx = y$ *only* if xx compose y. Only the weaker CEI is relevant to the problem discussed here.
[3] For a detailed survey, see Oliver and Smiley [2016].

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either an individual *or* a plurality. Throughout, I use '*aa* are *F*' as equivalent to '*aa* is, or are, *F*.'

Note that a predicate that applies to a plural term can be read either *distributively* or *collectively*. Roughly, '*F*' is read distributively just in case '*Faa*' implies '*Fa*' for each *a* among *aa* (as 'Alice and Beth are people' implies both 'Alice is a person' and 'Beth is a person'), and collectively otherwise (for instance, 'Alice and Beth lifted a piano' implies neither 'Alice lifted a piano' nor 'Beth lifted a piano').

(ii) *A term-forming operator* ('@'). '@' takes two old terms to a new term that denotes jointly what the old terms denote individually: for instance, '*a@b*' denotes *a* and *b*.[4]

(iii) *Singular variables* ('*x*', '*y*', *etc.*) *and plural variables* ('*xx*', '*yy*', *etc.*). Singular variables can take only singular terms as substitutions, and so '$\exists xFx$' is read as 'Something is *F*.' Plural variables can take both singular and plural terms as substitutions ('*Fa*' entails both '$\exists xFx$' and '$\exists xxFxx$'), and so '$\exists xxFxx$' is read as 'Some thing(s) is, or are, *F*.'

(iv) *A two-place 'inclusion' predicate* ('$\preccurlyeq$'). '$aa \preccurlyeq bb$' says that *aa* is/are included in *bb*. '$\preccurlyeq$' is read distributively at its first argument-place and collectively at its second: '$aa \preccurlyeq bb$' implies '$a \preccurlyeq bb$', for any *a* among *aa*; but it doesn't imply '$aa \preccurlyeq b$' for any *b* among *bb*. Inclusion is assumed to be both reflexive and transitive.

(v) *A two-place identity predicate* ('='). Identity is defined as mutual inclusion: '$aa = bb$' is equivalent to '$aa \preccurlyeq bb \ \& \ bb \preccurlyeq aa$'. Since '$\preccurlyeq$' is read collectively at its second argument-place, '=' is read collectively at each argument-place. This is just what we need, in order to formulate CEI:

Composition Entails Identity (CEI).
$\forall xx \forall y(Compose(xx, y) \supset xx = y)$

If *xx* compose *y*, then *y* is identical to *xx collectively*, not to each individual among *xx*.

3. The Counting Argument

Informally, the Counting Argument is simple: if CEI is true, then the very same thing(s) will instantiate distinct, and incompatible, number properties. Since no thing(s) can instantiate incompatible properties, CEI is false.

The argument comes in two variants, not always treated together (although see Wallace [2011: 818–22]). What distinguishes them is the motivation for thinking that, if CEI is true, the same thing(s) will instantiate different number properties.

First, there's what I'll call the 'Many-One Argument'. Composition is a relation between many things and one: if *xx* compose *y*, then, while *y* is one thing, *xx* are many things (that is, they're at least two). But if CEI is true, the parts just *are* the whole, and so the very same things are both one thing and many. So, CEI requires the parts of a whole to instantiate different, seemingly incompatible, number properties.[5]

Second, there's what I'll call the 'Many-More Argument'. In general, the number of things among the parts and the whole taken together is one greater than the number of

[4] On one view, that '*a@b*' denotes *a* and *b* implies both that '*a@b*' denotes *a* and that it denotes *b*. On a competing view, '*a@b*' denotes *a* and *b* together, and doesn't denote either of them individually. I remain neutral on this dispute; see Oliver and Smiley [2016: ch. 6] for discussion.

[5] See Baxter [1988a: 193], Lewis [1991: 87], Cotnoir [2013: 301], Varzi [2014: 49], and especially Yi [2014: 179–80].

things among the parts alone: if the parts are n things, then the parts and the whole taken together are n+1 things. (For example, if two mereological atoms, a and b, compose something, c, then, while a and b are two things, a, b, and c taken together are three things.) But if CEI is true, and the whole just *is* its parts, then the parts just *are* the parts and the whole taken together,[6] and so the very same things are both n things and n+1. Again, CEI requires the parts of a whole to instantiate different, seemingly incompatible, number properties.[7]

You might think that I could simply insist that the same thing(s) *can* instantiate different number properties. Isn't it simply part of my view that the parts of a whole can be both many things and one, or both n things and n+1?

This simple reply is unsatisfying. The Counting Argument deals in 'number ascriptions', claims of the form '*aa* are *n F*s.' It's standard practice among linguists and philosophers to treat number ascriptions as *cardinality ascriptions*: '*aa* are *n F*s' is true just in case there are exactly *n F*s among *aa*.[8] And there's a familiar method for regimenting cardinality ascriptions—easily adapted to plural languages—that seems to show that, *as a matter of logic*, no thing(s) can possess more than one cardinality, and so no thing(s) can instantiate different number properties.

First, we define a numerically specific plural quantifier phrase, '$\exists_n xxFxx$', read as 'There are exactly *n F*(s).' In English, 'There are exactly *n F*s just in case there are *n F*s such any *F* is identical to one of them.' More formally:

Definition of '$\exists_n xxFxx$'
Where '*F*' is read collectively:
(i) $\exists_1 xxFxx \equiv_{\text{df}} \exists xx(Fxx \,\&\, \forall yy(Fyy \supset yy = xx))$
(ii) For any $n \geq 2$,

$$\exists_n xxFxx \equiv_{\text{df}} \exists xx_1 \ldots \exists xx_n(Fxx_1 \ldots \& \ldots \& \ldots Fxx_n\ xx_1 \neq xx_2\ \& \ldots\ \&\ xx_{n-1} \neq xx_n\ \&$$
$$\forall yy(Fyy \supset (yy = xx_1 \vee \ldots \vee yy = xx_n)))$$

(To represent the claim that there are *at least n F*s, but possibly more, we drop the final, universally quantified conjunct.)[9]

Second, we introduce a cardinality function, '$\#_{\preccurlyeq}$', which maps a property, *F*, and some thing(s), *aa*, to that number, *n*, such that there are exactly *n F*(s) among *aa*:

Definition of '$\#_{\preccurlyeq}$'
$\#_{\preccurlyeq}(F, aa) =$ the n such that $\exists_n xx(Fxx \,\&\, xx \preccurlyeq aa)$

We then use this function to analyse number ascriptions:

Analysis of Number Ascriptions
aa are n Fs iff $\#_{\preccurlyeq}(F, aa) = n$

For example, '*aa* is one desk' reports that there's exactly one desk among *aa*. That is, for some *xx* among *aa*, *xx* are a desk, and any desk among *aa* is identical to *xx*. Or, $\exists xx(Dxx \,\&\, xx \preccurlyeq aa \,\&\, \forall yy((Dyy \,\&\, yy \preccurlyeq aa) \supset yy = xx))$.

The cardinality function takes a property as one of its arguments, but this doesn't commit us to the Geachian claim that we can never count objects *simpliciter* and can

[6] For example, if $a@b = c$, then $a@b@c = a@b@a@b$, which are just $a@b$ again.
[7] See Baxter [1988b: 578], van Inwagen [1994: 213–14], and Koslicki [2008: 42].
[8] See, e.g., Rothstein [2017: 28–35] and sources cited therein.
[9] Compare Barker-Plummer et al. [2011: 375–88].

only count objects of more specific kinds. To count how many objects *simpliciter* are among *aa*, we simply count instances of the property *being an individual*. We introduce a predicate, '*I*', read as '… are an individual' and defined as '$\exists x(x = aa)$'.[10] The number of individuals among *aa*, or $\#_{\preccurlyeq}(I, aa)$, is the *n* such that $\exists_n xx(Ixx \,\&\, xx \preccurlyeq aa)$. Equivalently, the number of individuals among *aa* is the *n* such that $\exists_n x(x \preccurlyeq aa)$, where '$\exists_n xFx$' is defined analogously to '$\exists_n xxFxx$', with singular variables in place of plural ones.

But now the force of the Counting Argument becomes clear. The crucial premise is that, if CEI is true, the parts of a whole instantiate different number properties. In particular, they're both *n* and *m* individuals, for some distinct *n* and *m*. Given the analysis of number ascriptions as cardinality ascriptions, this means that the function '$\#_{\preccurlyeq}$' maps the parts of a whole to some distinct *n* and *m*:

$$\text{(CA)}\quad \text{CEI} \supset \forall xx \forall y(Compose(xx, y) \supset \exists n \exists m(\#_{\preccurlyeq}(I, xx) = n \,\&\, \#_{\preccurlyeq}(I, xx) = m \,\&\, n \neq m))$$

(CA) is motivated, recall, by two seemingly incontrovertible claims—that composition is a relation between many things and one, and that if *xx* compose *y* and *xx* are *n* things then *xx* and *y* taken together are *n*+1 things. Drawing again on the analysis of number ascriptions:

$$\text{(MO)}\ \forall xx \forall y(Compose(xx, y) \supset (\#_{\preccurlyeq}(I, xx) \geq 2 \,\&\, \#_{\preccurlyeq}(I, y) = 1))$$

$$\text{(MM)}\ \forall xx \forall y(Compose(xx, y) \supset (\#_{\preccurlyeq}(I, xx) = n \supset \#_{\preccurlyeq}(I, xx@y) = n + 1))$$

Both (MO) and (MM) straightforwardly imply (CA): if CEI is true, then the parts of a whole will have more than one cardinality. But this is impossible. As a matter of logic, if $n \neq m$, '$\#_{\preccurlyeq}(I, aa) = n$' and '$\#_{\preccurlyeq}(I, aa) = m$' are jointly inconsistent: no thing(s) can have more than one cardinality.

Consider the simplest case, in which we attempt to say that some *aa* are both one individual and two individuals:

(1) *aa* is one individual.

$$(1^*)\ \#_{\preccurlyeq}(I, aa) = 1 \equiv_{\text{df}} \exists x(x \preccurlyeq aa \,\&\, \forall y(y \preccurlyeq aa \supset y = x))$$

(2) *aa* are two individuals.

$$(2^*)\ \#_{\preccurlyeq}(I, aa) = 2 \equiv_{\text{df}} \exists x_1 \exists x_2(x_1 \preccurlyeq aa \,\&\, x_2 \preccurlyeq aa \,\&\, x_1 \neq x_2 \,\&\, \forall y(y \preccurlyeq aa \supset (y = x_1 \vee y = x_2)))$$

(1) requires that there's a single thing to which anything among *aa* is identical, while (2) requires that there are two distinct things among *aa*. These requirements are jointly inconsistent. Likewise, *mutatis mutandis*, for any *n* and *m* such that $n \neq m$. As a matter of logic, no thing(s) can possess more than one cardinality [Yi 2014: 179–80].[11]

4. Ipseity Ascriptions

What's a defender of CEI to do? We've seen that, assuming the standard analysis of number ascriptions as cardinality ascriptions, distinct number properties are

[10] Note that '*Iaa*', or '$\exists x(x = aa)$', isn't trivially true. Granting $aa = aa$, what follows is '$\exists xx(xx = aa)$'. This is consistent with, but doesn't entail, '$\exists x(x = aa)$'.

[11] You might think that this is already secured, since '$\#_{\preccurlyeq}$' is a *function*. But see Oliver and Smiley [2016: 4–7] on multi-valued functions.

incompatible. Faced with this analysis, I can't simply *insist* that the parts of a whole can instantiate distinct number properties.

Fortunately, I can do better. The standard analysis is false: some things might fail to be *n F*s even if there are exactly *n F*s among them. An alternative analysis is needed. And, as we'll see, there's a compelling analysis that renders distinct number properties compatible.

To see why the standard analysis fails, let *aa* be an ordinary deck of fifty-two playing cards. If number ascriptions are cardinality ascriptions, then (3) is analysed as (3*):

(3) *aa* are four aces.

$$
\begin{aligned}
(3^*)\ & \#_{\preccurlyeq}(A, aa) = 4 \\
& \equiv_{\mathrm{df}} \exists_4 xx(Axx \ \&\ xx \preccurlyeq aa) \\
& \equiv_{\mathrm{df}} \exists xx_1 \ldots \exists xx_4(Axx_1 \ \&\ \ldots\ \&\ Axx_4 \ \&\ xx_1 \preccurlyeq aa \ \&\ldots\&\ xx_4 \preccurlyeq aa\ \& \\
& \quad xx_1 \neq xx_2 \ \&\ \ldots\ xx_3 \neq xx_4 \ \&\ \forall yy(Ayy \supset (yy = xx_1 \vee \ldots \vee yy = xx_4)))
\end{aligned}
$$

In English: '*aa* are four aces' is true just in case there are exactly four distinct aces among *aa*. But this is wrong: there are exactly four aces among *aa*, but *aa* aren't *themselves* four aces.

Other examples can be given. For instance, if I have twenty books on my shelf, ten hardcover and ten softcover, then there are exactly ten hardcovers among the books. But those books aren't *themselves* ten hardcovers.

What these cases have in common is that, while there are exactly *n F*s among *aa*, those *n F*s don't *exhaust aa*; *aa* includes the *n F*s and more besides. (The fifty-two cards include the four aces *and* the other forty-eight cards.) A satisfying analysis of number ascriptions must yield the result that '*aa* are *n F*s' is false in such cases.

As a first attempt, we might say that '*aa* are *n F*s' is true just in case two conditions are met: (i) there are exactly *n F*s among *aa*, and (ii) any thing(s) among *aa* are an *F*. On this approach, (3) is analysed as (3**):

$$(3^{**})\ \exists_4 xx(Axx \ \&\ xx \preccurlyeq aa) \ \&\ \forall yy(yy \preccurlyeq aa \supset Fyy)$$

(3**) implies that the *only* things among the fifty-two cards are the four aces.[12] (3**) is false, since the forty-eight non-aces are *also* among the fifty-two cards. So, the approach gets the right result in this case (and, as the reader can check, in the case of the twenty books).

But the approach fails in other cases. We can count pluralities as well as individuals. For example, a pair of shoes is a plurality, and we can count pairs of shoes by saying things like 'There are two pairs of shoes in my closet.' But when *F* is a predicate that applies (collectively) to pluralities, we can't stipulate that *aa* are *n F*s just in case *aa* include nothing other than those *n F*s.

Consider this case: I have two matching pairs of shoes in my closet. Let '*aa*' denote the four shoes that make up these two pairs. On the approach being considered, (4) is analysed as (4*):

(4) *aa* are two (matching) pairs of shoes.

$$(4^*)\ \exists_2 xx(Pxx \ \&\ xx \preccurlyeq aa) \ \&\ \forall yy(yy \preccurlyeq aa \supset Pyy)$$

In English: '*aa* are two (matching) pairs of shoes' is true just in case (i) there are exactly two (matching) pairs of shoes among *aa* and (ii) the only thing(s) among *aa* are

[12] More formally: $\exists xx_1 \ldots \exists xx_4(Axx_1 \ \&\ldots\&\ Axx_4 \ \&\ xx_1 \neq xx_2 \ \&\ldots\&\ xx_3 \neq xx_4 \ \&\ \forall yy(yy \preccurlyeq aa \supset (yy = xx_1 \vee \ldots \vee yy = xx_4)))$.

(matching) pairs of shoes. But, while (4) is true, condition (ii) isn't satisfied: each pair of shoes just *is* the shoes that make it up;[13] so, if *aa* includes the pairs xx_1 and xx_2, it also includes the individual shoes that make up those pairs, none of which is itself a pair of shoes. The analysis fails.

The point generalizes. Where '*F*' is a predicate that applies collectively to many things, we can't treat '*aa* are *n F*s' as equivalent to 'there are exactly *n F*s among *aa* and anything(s) among *aa* are an *F*.' For an *F* might simply *be*, and hence *include*, some non-*F*s (as a pair of shoes just *is*, and hence *includes*, some non-pairs). But then it might be true that *aa* are *n F*s, but false that *aa* includes only those *n F*s.

We began with the thought that, when there are exactly *n F*s among *aa*, and yet *aa* aren't themselves *n F*s, this is because *aa* includes the *n F*s and more besides. We can't cash this out by saying that *aa* are *n F*s just in case any thing(s) among *aa* are one of those *F*s. We do better to cash it out in terms of collective identity: *aa* are *n F*s just in case there are *n F*s to which *aa* are collectively identical. Since identity is mutual inclusion, *aa* are *n F*s just in case (i) *aa* includes the *n F*s and (ii) the *n F*s include *aa*. *That's* the sense in which *aa* includes the *n F*s and nothing more besides: it's not that any thing(s) among *aa* are one of those *F*s; rather, it's that *aa* and the *F*s are the same thing(s), and so *aa* doesn't include any thing(s) that aren't also included in the *F*s.

More formally, we can introduce a three-place predicate, '$\#_{=}$', which takes a property, *F*, some thing(s), *aa*, and a number, *n*, to the proposition that there are *n F*s to which *aa* are (collectively) identical.[14]

Definition of '$\#_{=}(F, aa, n)$'
Where '*F*' is read collectively,

(i) $\#_{=}(F, aa, 1) \equiv_{\mathrm{df}} \exists xx(Fxx \;\&\; xx = aa)$

(ii) For any $n \geq 2$,
$\#_{=}(F, aa, n) \equiv_{\mathrm{df}} \exists xx_1 \ldots \exists xx_n(Fxx_1 \;\&\; \ldots \;\&\; Fxx_n \;\&\; xx_1 \neq xx_2 \;\&\; \ldots \;\&\; xx_{n\text{-}1} \neq xx_n \;\&\; xx_1@ \ldots @xx_n = aa)$

We can use '$\#_{=}$', rather than '$\#_{\leqslant}$', to analyse number ascriptions.

Revised Analysis of Number Ascriptions.
aa are *n F*s iff $\#_{=}(F, aa, n)$

Number ascriptions are best understood, not as *cardinality ascriptions*, but as what I'll call '*ipseity ascriptions*':[15] to say that *aa* are *n F*s isn't to say how many *F*s are *among aa*, but simply to say that there are some *n F*s to which *aa* are (collectively) identical.[16]

[13] We needn't assume CEI here, since we needn't think that a pair of shoes is a composite object. Instead, we might think that 'pair of shoes' is grammatically singular but applies only to pluralities [Oliver and Smiley 2016: 305–7].

[14] This analysis gives the 'exact' reading of '*aa* are *n F*s'. To say that *aa* are *at least n F*s, but possibly more —say, that there are *n F*s such that those *F*s, together with some (possibly identical) *F*s, are collectively identical to *aa*. For example, *aa* are at least one *F* just in case $\exists xx\exists yy(Fxx \;\&\; Fyy \;\&\; xx@yy = aa)$, while *aa* are least two *F*s just in case $\exists xx_1\exists xx_2\exists yy(Fxx_1 \;\&\; Fxx_2 \;\&\; Fyy \;\&\; xx_1 \neq xx_2 \;\&\; xx_1@xx_2@yy = aa)$.

[15] 'Ipseity' is derived from the Latin 'ipse' (roughly equivalent to 'self') and means 'identity' or 'self'. The term is sometimes used, e.g., in the phenomenological tradition, to refer to a subject's sense of themselves as a unified self or consciousness. No such connotation is intended here.

[16] It's crucial that '*F*' be read collectively in the definition of '$\#_{=}(F, aa, n)$'. Otherwise, e.g., the fifty-two cards in a deck will count as one card (since they're identical to some *xx* such that 'card' is true of *xx* on the distributive reading). Thanks to an anonymous referee for discussion.

This analysis gets the right results in the original cases. (3) is now analysed as (3***):

$$(3^{***})\ \#_{=}(A, aa, 4)$$
$$\equiv_{\mathrm{df}} \exists xx_1 \exists xx_2 \exists xx_3 \exists xx_4 (Axx_1\ \&\ \ldots\ \&\ Axx_4\ \&\ xx_1 \neq xx_2\ \&\ \ldots\ \&\ xx_3 \neq xx_4\ \&\ xx_1@xx_2@xx_3@xx_4 = aa)$$

In English: '*aa* are four aces' is true just in case there are four distinct aces to which *aa* are (collectively) identical. So, '*aa* are four aces' comes out *false*, for the intuitively correct reason: the fifty-two cards include some things that aren't included in the four aces (namely, the forty-eight other cards), and so the fifty-two cards aren't *identical* to the four aces. (Likewise, *mutatis mutandis*, for the case of the twenty books.)

The analysis also gets the right results in cases where '*F*' is a predicate that applies collectively to many non-*F*s. (4) is now analysed as (4**):

$$(4^{**})\ \#_{=}(P, aa, 2)$$
$$\equiv_{\mathrm{df}} \exists xx_1 \exists xx_2 (Pxx_1\ \&\ Pxx_2\ \&\ xx_1 \neq xx_2\ \&\ xx_1@xx_2 = aa)$$

In English: '*aa* are two (matching) pairs of shoes' is true just in case there are two distinct (matching) pairs of shoes to which *aa* are collectively identical. So, '*aa* are two (matching) pairs of shoes' can come out as *true*, as it should, despite the fact *aa* include things other than pairs of shoes (namely, the individual shoes that make up the pairs).

So, number ascriptions are best analysed by using the predicate '$\#_{=}$' rather than the function '$\#_{\preccurlyeq}$'. Before we see how this affects the Counting Argument, three points about the relationship between cardinality and ipseity are in order.

(i) *Cardinality is always unique; ipseity needn't be*. As a matter of logic, '$\#_{=}(F, aa, n)$' and '$\#_{=}(F, aa, m)$' are jointly consistent, even if $n \neq m$.

This is good news for me, as a defender of CEI. Consider a case in which four squares, *a*–*d*, are arranged to compose a larger square, *e*. Given CEI, the four smaller squares are identical to the larger square ($a@b@c@d = e$), and so the four smaller squares are also identical to themselves and the larger square taken together ($a@b@c@d = a@b@c@d@e$). Thus, letting '*aa*' denote the squares, I say that *aa* are one square (*e*) and four squares ($a@b@c@d$) and five squares ($a@b@c@d@e$). If these claims were all cardinality ascriptions, then they would be inconsistent. However, if they're ipseity ascriptions, then they're consistent. While no thing(s) can possess two or more distinct cardinalities, they *can* possess two or more distinct ipseities.

(ii) *In general, cardinality and ipseity are independent*. That is, *in general,* neither '$\#_{\preccurlyeq}(F, aa) = n$' nor '$\#_{=}(F, aa, n)$' implies the other.

The cases with which I began this section show that '$\#_{\preccurlyeq}(F, aa) = n$' doesn't always imply '$\#_{=}(F, aa, n)$': there are exactly four aces among the fifty-two cards that make up an ordinary deck, but those cards aren't themselves four aces. The case of the four squares shows that '$\#_{=}(F, aa, n)$' doesn't always imply '$\#_{\preccurlyeq}(F, aa) = n$', either: '*aa* is one square' and '*aa* are four squares' are both true, but 'There's exactly one square among *aa*' and 'There are exactly four squares among *aa*' are both false, since there are exactly *five* squares among *aa*.

Corollary: in general, '$\#_{\preccurlyeq}(F, aa) = n$' and '$\#_{=}(F, aa, m)$' are jointly consistent, even where $n \neq m$. The ipseity ascription '*aa* are *n* *F*s' implies that there are *at least n F*s among *aa*;[17] it doesn't imply that there are *at most n F*s among *aa*.

[17] '$\#_{=}(F, aa, n)$' is true only if there are some *F*s, xx_1–xx_n, to which *aa* are collectively identical. Since identity is mutual inclusion, it follows that $xx_1@\ldots@xx_n \preccurlyeq aa$, from which it follows that $xx_1 \preccurlyeq aa\ \&\ \ldots\ \&\ xx_n \preccurlyeq aa$.

(iii) *However, when counting individuals, cardinality implies ipseity*. That is, '$\#_{\leqslant}(I, aa) = n$' implies '$\#_{=}(I, aa, n)$' (although not vice-versa).

The reason why '$\#_{\leqslant}(F, aa) = n$' doesn't *generally* imply '$\#_{=}(F, aa, n)$' is that the n Fs included in aa might not be identical to aa. But, where 'I' is substituted for 'F', this is impossible: any aa are collectively identical to all of the individuals included among them. Thus, '$\#_{\leqslant}(I, aa) = n$' implies '$\#_{=}(I, aa, n)$'.

By contrast, '$\#_{=}(I, aa, n)$' doesn't imply '$\#_{\leqslant}(I, aa) = n$'. When the atoms a and b compose c, both '$a@b$ are one thing' and '$a@b$ are two things' are true, since there are both one thing (c) and two things ($a@b$) to which they're identical. But both 'There's exactly one thing among $a@b$' and 'There are exactly two things among $a@b$' are false, since there are exactly *three* things among aa.

Corollary: '$\#_{\leqslant}(I, aa) = n$' and '$\#_{=}(I, aa, m)$' are jointly consistent, even where $n \neq m$. While the ipseity ascription 'aa are n individuals' implies that there are *at least* n individuals among aa, it doesn't imply that there are *at most* n individuals among aa.

5. Resisting the Counting Argument

Analysing number ascriptions as ipseity ascriptions allows me to resist both variants of the Counting Argument against CEI. Each variant is meant to show that, if I identify wholes with their parts, then I must assign two distinct cardinalities to the same things:

$$\text{(CA)} \quad \text{CEI} \supset \forall xx \forall y(Compose(xx, y) \supset \exists n \exists m(\#_{\leqslant}(I, xx) = n \;\&\; \#_{\leqslant}(I, xx) = m \;\&\; n \neq m))$$

However, while I'm committed to assigning distinct *numbers* to the same things, I needn't treat both numbers as *cardinalities*. I can instead treat at least one of them as an *ipseity*, rendering jointly consistent what seemed to be incompatible number ascriptions.

5.1. The Many-One Argument Revisited

The Many-One Argument relies on (MO):

$$\text{(MO)} \; \forall xx \forall y(Compose(xx, y) \supset (\#_{\leqslant}(I, xx) \geq 2 \;\&\; \#_{\leqslant}(I, y) = 1))$$

(MO) is meant to capture the fact that composition is a relation between many things and one. Now, I can grant that the parts of a whole have at least two things among them.[18] But if number ascriptions are ipseity ascriptions, then to say that the whole is one thing is to say, *not* that there's exactly one thing among it, but rather that there's a thing to which it's identical. That is, instead of (MO), we have (MO*):

$$\text{(MO*)} \; \forall xx \forall y(Compose(xx, y) \supset (\#_{\leqslant}(I, xx) \geq 2 \;\&\; \#_{=}(I, y, 1))$$

But then what follows from CEI is that the parts of a whole have many individuals among them, and that they're also identical to some individual. This doesn't imply (CA), and, as a matter of logic, it's perfectly consistent.

You might insist that (MO) is true, even if it goes beyond the literal meaning of 'the parts are many and the whole is one.' That is, you might insist that a whole *is* one thing, in the sense of having exactly one thing among it. But we have good reason to reject this claim.

[18] Recall that 'aa are n individuals' implies that there are at least n individuals among aa, and so to say that the parts of a whole *are* at least two individuals implies that there are at least two individuals among them.

Van Inwagen [1990] famously poses the Special Composition Question: under what conditions do some *xx* compose some *y*? The beginning of an answer is that wholes are distinguished from 'mere pluralities' by exhibiting a certain kind of 'unity' or 'oneness': a whole is 'unified' or 'one' in a way in which mere pluralities aren't [ibid.: 118].[19] But a whole is unified in this way only if its *parts* are unified in that way (the parts of my desk form a unified whole only because those parts themselves are unified), and so the parts of a whole must be one thing in a way in which mere pluralities aren't. They can't be one thing in the sense of there being exactly one thing among them, since that contradicts the fact that they're many. They can only be one thing in the sense of there being a thing to which they're identical.[20]

If number ascriptions are ipseity ascriptions, then the fact that composition is a relation between many things and one only motivates (MO*), not (MO). I can accept it without attributing distinct cardinalities to any thing(s).

5.2. The Many-More Argument Revisited

The Many-More Argument relies on (MM):

(MM) $\forall xx \forall y(Compose(xx, y) \supset (\#_{\preccurlyeq}(I, xx) = n \supset \#_{\preccurlyeq}(I, xx@y) = n + 1))$

(MM) is meant to capture the fact that, if *n xx* compose *y*, then *y* is an *n*+1-th thing (for instance, if *xx* are two things, then *y* is a third thing). But, with the distinction between cardinality and ipseity ascriptions in hand, I can reject (MM).

Suppose that two mereological atoms, *a* and *b*, compose something, *c*. In this case, it seems that *a@b* are two things while *a@b@c* are three things. If we regiment both of these claims as cardinality ascriptions, we get (5*) and (6*), which are jointly inconsistent if *a@b* = *a@b@c*:

(5) *a* and *b* are two things.
(5*) $\#_{\preccurlyeq}(I, a@b) = 2$
$\equiv_{\mathrm{df}} \exists x_1 \exists x_2(x_1 \preccurlyeq a@b \;\&\; x_2 \preccurlyeq a@b \;\&\; x_1 \neq x_2 \;\&\; \forall y(y \preccurlyeq a@b \supset (y = x_1 \vee y = x_2)))$

(6) *a*, *b*, and *c* are three things.
(6*) $\#_{\preccurlyeq}(I, a@b@c) = 3$
$\equiv_{\mathrm{df}} \exists x_1 \exists x_2 \exists x_3(x_1 \preccurlyeq a@b@c \;\&\; x_2 \preccurlyeq a@b@c \;\&\; x_3 \preccurlyeq a@b@c \;\&\; x_1 \neq x_2 \;\&\; x_1 \neq x_3 \;\&\; x_2 \neq x_3 \;\&$
$\forall y(y \preccurlyeq a@b@c \supset (y = x_1 \vee y = x_2 \vee y = x_3)))$

I accept (6*), since I distinguish *c* from each of *a* and *b* individually. But I reject (5*). Since I believe that *a@b* = *a@b@c*, I infer that there are exactly *three* things among *a@b* —namely, *a*, *b*, and *c*.

Of course, I could have made this move even without distinguishing cardinality ascriptions from ipseity ascriptions. But then I would have needed to reject (5). The move would have amounted to biting the bullet and denying the obvious fact that *a@b* are two things. With the distinction in hand, I can accommodate that fact: '*a@b* are two things' is analysed as (5**).

(5**) $\#_{=}(I, a@b, 2)$
$\equiv_{\mathrm{df}} \exists x_1 \exists x_2(x_1 \neq x_2 \;\&\; x_1@x_2 = a@b).$

[19] See also Koslicki [2008: ix] and Spencer [2017: 866].
[20] I defend this view in more detail elsewhere [forthcoming a].

a and *b* are two things, in the sense that there are two things to which they're collectively identical (namely, *a* and *b* themselves).[21]

Now, just as I accommodated the intuitive thought behind (MO) by using (MO*), I need to accommodate the intuitive thought behind (MM). The problem with (MO) was that it casts both of the relevant number ascriptions, '*xx* are many things' and '*y* is one thing', as cardinality ascriptions, when the latter should be cast as an ipseity ascription. You might think that the problem with (MM) is similar: it casts both of the relevant number ascriptions, '*xx* are *n* things' and '*xx@y* are *n*+1 things', as cardinality ascriptions, when the former should be cast an ipseity ascription.

$$\text{(MM*)}\ \forall xx \forall y(Compose(xx, y) \supset (\#_{=}(I, xx, n) \supset \#_{\preccurlyeq}(I, xx@y) = n + 1))$$

(MM*) fits with my claim that *a@b* are two things but have exactly three things among them. But it fails in more complex cases.

Suppose that there are three mereological atoms, *a*, *b*, and *c*, that each pair of *a*–*c* composes something (*d*, *e*, and *f*), and that *a@b@c* together compose something (*g*). Given CEI, the atomic parts of *g*, *a@b@c*, are three things—or $\#_{=}(I, a@b@c, 3)$. But, *contra* (MM*), there aren't just four things among *a@b@c@g*. There are exactly *seven* things among *a@b@c@g*—namely, each of *a*–*g*.

You might think that I should simply reject the intuition behind (MM), instead of trying to accommodate it. Indeed, it seems that the intuition will be difficult to accommodate. It can be expressed as follows: if the parts 'considered apart from the whole' are *n* things, then the parts and the whole 'taken together' must be *n*+1 things. But, according to CEI, the parts *are* the parts and the whole taken together. So, it might seem that there's no distinction to be drawn between counting the parts 'considered apart from the whole' and counting them 'considered together with the whole'. What could it mean, to consider some things apart from themselves?

Here's what I propose. To say, in the case of the two atoms, that *a* and *b* are two things 'considered apart from *c*' is to say that they're two *things distinct from c*. That is, they're two instances, not of the property *being an individual*, but of the property *being an individual distinct from c*. Letting '$\lambda xx.[Ixx \ \&\ xx \neq c]$' denote that property, we have this:

(7) *a* and *b* are two things distinct from *c*.

$$\text{(7*)}\ \#_{\preccurlyeq}(\lambda xx.[Ixx \ \&\ xx \neq c], a@b) = 2$$
$$\equiv_{\text{df}} \exists x_1 \exists x_2(x_1 \preccurlyeq a@b \ \&\ x_2 \preccurlyeq a@b \ \&\ x_1 \neq c \ \&\ x_2 \neq c \ \&\ x_1 \neq x_2 \ \&$$
$$\forall y((y \preccurlyeq a@b \ \&\ y \neq c) \supset (y = x_1 \vee y = x_2)))$$

(7) is true, in this case: since *a* and *b* are atoms, the only thing included in *a* is *a*, and the only thing included in *b* is *b*; so, other than *c*, there are only two individuals included in *a@b*—namely, *a* and *b*.[22]

By contrast, to say that *a@b@c* are three things 'considered together' is to say, simply, that *a@b@c* are three instances of the property *being an individual*, as in (6).

[21] Since '$\#_{=}(I, aa, n)$' doesn't imply '$\#_{\preccurlyeq}(I, aa) = n$', (5*) doesn't imply (5).

[22] Since '$\#_{\preccurlyeq}(I, aa) = n$' entails '$\#_{=}(I, aa, n)$', (7) entails that there are two things distinct from *c* to which *a@b* are collectively identical:

$$\#_{=}(\lambda xx.[Ixx \ \&\ xx \neq c], a@b, 2)$$
$$\equiv_{\text{df}} \exists x_1 \exists x_2(x_1 \neq c \ \&\ x_2 \neq c \ \&\ x_1 \neq x_2 \ \&\ x_1@x_2 = a@b).$$

This straightforwardly implies (5*), and so (7) is consistent with the claim that *a@b* are two things in the sense captured by (5*).

This is how I distinguish counting the parts 'considered apart from the whole' from counting the parts and the whole 'taken together'. Both involve cardinality ascriptions, but the ascriptions are relativized to different properties.

Generalizing, the problem with (MM) isn't that it casts both of the relevant number ascriptions, '*xx* are *n* things' and '*xx@y* are *n*+1 things', as cardinality ascriptions, but rather that it casts the first of these as a cardinality ascription *of the wrong sort*. The intuitive thought behind (MM) is better cast as this:

$$(\mathrm{MM}^{**})\ \forall xx \forall y(Compose(xx, y) \supset (\#_{\leqslant}(\lambda zz.[Izz\ \&\ zz \neq y], xx) = n \supset \#_{\leqslant}(I, xx@y) = n + 1)),$$

where '$\lambda zz.[Izz\ \&\ zz \neq y]$' denotes the property of being an individual distinct from *y*. If *xx* compose *y*, and there are exactly *n* individuals distinct from *y* among *xx*, then there are exactly *n*+1 individuals *simpliciter* among *xx@y*.

We've seen that (MM**) gets the right result in the case of the two atoms. Unlike (MM*), it also gets the right result in the case of the three atoms: there are exactly six individuals distinct from *g* among *a*–*g*, (namely, *a*–*f*), and so (MM**) correctly predicts that there are exactly seven individuals *simpliciter* among *a*–*g*.

Thus, I can accept the claim that if the parts are *n* things then the parts and the whole together are *n*+1 things, without attributing distinct cardinalities to the parts.

Summing up this section, the distinction between cardinality ascriptions and ipseity ascriptions allows us to reject both (MO) and (MM), and thereby reject (CA). CEI does *not* require that the parts of a whole instantiate distinct, and incompatible, cardinalities. The Counting Argument fails.

6. Alternatives

In this section, I compare my response to the Counting Argument to three recent alternatives.

6.1. Bohn

Bohn [2014: 145–6, forthcoming: 4–6] doesn't distinguish cardinality ascriptions from ipseity ascriptions. Instead of reinterpreting the number ascriptions that motivate the Many-One and Many-More arguments, he denies that they're true.

Recall that the Counting Argument concerns how many *individuals* or *things* are included in a given plurality. According to Bohn, claims to the effect that *aa* have exactly *n* *individuals* or *things* among them are never true, and might even be ill-formed: 'No ordinary thing has a particular cardinality independent of how it is conceptualized' [2014: 145]; 'for any cardinal number *n*, *being n* only holds of something relative to it falling under certain concepts' [forthcoming: 5].

We've seen that there's an innocuous sense in which cardinality ascriptions are relativized to properties: one of the arguments of '$\#_{\leqslant}$' is a property, and the function tells us how many instances of a given property are included among some thing(s). Bohn's point must be that the property of being an individual can't serve as an argument of this function: we can't say that there are *n* individuals among *xx*; we can say only that there are *n* instances of a more specific kind among them. Moreover, Bohn thinks, we shouldn't be tempted to say that the parts of a whole are both *n* and *m* instances of the same kind: in

the case of the two atoms, for example, *a@b* are two instances of one kind (*atom*) and one instance of another (*composite*). Thus, there's no contradiction: in general, '$\#_{\preccurlyeq}(F, aa) = n$' and '$\#_{\preccurlyeq}(G, aa) = m$' are jointly consistent, even where $n \neq m$.[23]

There are two problems with this response. First, it's not clear what motivates the ban on counting *individuals* or *things*. Bohn can hardly object to my introduction of the predicate '*I*', since it was defined by using well-understood notions of quantification and identity: '*Iaa*' is true just in case '$\exists x(x = aa)$' is. There must be some reason why that predicate, while perfectly well-defined, can't be used for counting (besides the *ad hoc* reason that this seems to land a defender of CEI in contradiction).[24]

Second, we can't always avoid contradiction by relativizing different cardinality ascriptions to different properties or concepts. Recall the case in which four smaller squares, *a@b@c@d*, compose a fifth square, *e*. A defender of CEI should say that these are one square (*e*) and four squares (*a@b@c@d*) and five squares (*a@b@c@d@e*). My approach renders these claims jointly consistent by treating them as ipseity ascriptions. But, by Bohn's lights, they're inconsistent, since no thing(s) can have different cardinalities relative to the same property or concept. Generalizing, Bohn's solution only works if it's impossible for many *F*s to compose another *F*. Since that's clearly possible, the proposed solution fails.[25]

6.2. Spencer

Spencer [2017] blocks the Counting Argument by providing an alternative analysis of number ascriptions (he doesn't distinguish cardinality and ipseity ascriptions). All number ascriptions are analysed by using predicates at the level of logical form that correspond directly to ordinary-language numerical predicates: '*aa* are one thing' is analysed as '*One*(*aa*)'; '*aa* are two things' is analysed as '*Two*(*aa*)'; etc. These predicates are given no logical analysis, and the truth-conditions of '*N*(*aa*)' aren't cashed out in terms of quantification and identity. The truth-condition of '*One*(*aa*)', for instance, is simply that *aa* are one thing.

Since Spencer's number predicates aren't subject to logical analysis, there's no way to show that '*Nxx* and '*Mxx*' are jointly inconsistent, when $N \neq M$. So, he says, he can simply grant that the parts of a whole are both many things and one, and both *n* things and *n*+1 [ibid.: 864–5].[26]

This response is no more satisfying that the simple one that I considered and rejected in section 3. In the absence of an analysis of number ascriptions of the sort that I provided in section 4—that is, one that shows *how* the same thing(s) can possess distinct number properties—an opponent of CEI can rightfully insist that 'No thing(s) can possess distinct number properties' is a plausible arithmetical principle, more deserving of our adherence than is a controversial metaphysical thesis about parts and wholes.

[23] See also Kleinschmidt [2012].

[24] One might insist that the identity predicate, '=', which I used to define '*I*', is illegitimate, by adopting Geach's [1967] doctrine of relative identity: nothing is ever identical to anything else *simpliciter*, but only relative to a concept, property, or kind. However, that view faces serious difficulties, and Bohn himself rejects it: 'Relative identity is worse than death' [2014: 146n10].

[25] Baxter [2005: 378] and Varzi [2014: 52] anticipate this point.

[26] Spencer doesn't consider the Counting Argument as I've presented it; he considers only the Many-One Argument. However, his response to the latter naturally extends to the former.

Moreover, the Counting Argument *as regimented in section 3* remains unanswered. Even if Spencer's approach to number ascriptions is correct, and that formal argument isn't a faithful regimentation of the informal Counting Argument, it might be a sound *independent* argument against CEI.[27]

Spencer might try to answer this second worry by giving responses to the Many-One and Many-More arguments that are similar to mine, thereby rejecting (CA). But this is bound to be unsatisfying.

Recall that, in my response to the Many-More argument, I claimed that the sense in which $a@b$ are two things is captured by using my predicate '$\#_{=}$'. Spencer would say, instead, that it's captured by using the unanalysed predicate, '*Two*'. But, since Spencer provides no informative account of the truth-conditions of '$Two(a@b)$', he can't explain why it's consistent with '$\#_{\leqslant}(a@b) = 3$' (or indeed why it's consistent with '$Three(a@b)$', to which he's apparently committed). So, while Spencer might insist that there's a sense in which $a@b$ are two things, even though there are three things among them, he can't explain what this sense is, or demonstrate that the claim that $a@b$ are two things in this sense raises no trouble for CEI.[28]

6.3. Wallace

Instead of insisting that we can't count individuals *sans phrase*, or reject any logical analysis of number ascriptions, Wallace [2011] insists that, while we can *sometimes* count the individuals in a plurality by using the cardinality function, this method is inappropriate if the plurality includes both a whole and its parts. In such cases, we can't count those objects directly; we must count them indirectly, by counting the variables that occur in the many-one identity sentences that we accept (assuming that we accept CEI).

More concretely, suppose that a and b compose c. According to CEI, $a@b = c$, and so (10) is true:[29]

(8) $\exists x_1 \exists x_2 \exists x_3 (x_1@x_2 = x_3 \ \& \ x_1@x_2@x_3 = a@b)$

The first conjunct, '$x_1@x_2 = x_3$', contains two variables embedded in a string ('x_1' and 'x_2'), and one variable not so-embedded ('x_3'). According to Wallace, we count the objects among $a@b$ by counting these variables. Thus, we can say that there are two objects among $a@b$ (the values of the two embedded variables—namely, a and b) and that there's one (the value of the unembedded variable—namely, c).

This approach gives Wallace a response to the Many-One argument that resembles mine. A whole is one thing, not in the sense that there's exactly one thing among it, but rather in the sense that it serves as the value of the single, unembedded variable in a many-one identity claim. Thus, $a@b$ are one thing, even though '$\#_{\leqslant}(I, a@b) = 1$' is false, and this is consistent with their also being two things.

Wallace's response to the Many-More argument is more complicated. When we confront a sentence of the form '$x_1@ \ldots @x_n = y$', we count the objects among $x_1@ \ldots @x_n@y$ by counting the variables in that sentence. But, crucially, we can't *add* the number of embedded variables to the number of unembedded ones, yielding a

[27] Thanks to David Liebesman for this point.
[28] Lipman [forthcoming: 4] anticipates this point.
[29] Wallace [2011: 820–1] only discusses quantificational number claims—i.e. claims of the form 'There are *n Fs*.' Here, I consider an extension of her view to number ascriptions.

count of n+1 objects. We can only *conjoin* them: there are n objects among x_1@ … @x_n@y (namely, x_1–x_n), and there's one (namely, y), and that's the end of the matter. Thus, Wallace allows that '$\#_{\leqslant}(I, a@b) = 3$' is true, but denies that there are three objects among $a@b$: there are two objects among $a@b$, and there's one, and that's the end of the matter [ibid.: 818].

Unfortunately, Wallace provides no independent linguistic evidence for her approach to number ascriptions. Her approach is motivated entirely by the fact that it affords a response to the Counting Argument, and so it threatens to be *ad hoc*.

Nor does the linguistic evidence to which I appealed in section 4 support Wallace's distinction between direct and indirect counting. I appealed to sentence-pairs like (9) and (10), in order to show the difference between cardinality and ipseity ascriptions (where '*aa*' denotes a collection of twenty books):

(9) There are exactly ten hardcover books among *aa*.
(10) *aa* are ten hardcover books.

My approach explains why (10) is false even though (9) is true. Wallace's doesn't. Recall that we're supposed to count objects indirectly when we're counting both wholes and their parts; otherwise, counting objects by using '$\#_{\leqslant}$' is unobjectionable. But this case doesn't seem to involve composition (a book collection doesn't seem to be a composite object) or to depend on the identification of wholes with their parts. By Wallace's lights, her method of indirect counting doesn't apply in this case, and so it can't be used to explain the intuitive truth-values of (9) and (10).

Finally, my view captures some intuitive claims that Wallace's doesn't. Wallace can say that $a@b$ are two things while c is one, and so she can accommodate the intuition behind (MO). But since she denies that we can *add* a whole to its parts, she denies that $a@b@c$ are three things. So, she can't accommodate the intuition behind (MM). My view accommodates both.

7. A Worry about Plural Languages

You might worry about my reliance on the resources of first-order plural languages throughout this paper, since Sider [2007, 2014] and others have argued that CEI is incompatible with standard approaches to the meaning and logic of plural expressions.

The point is most vivid for plural definite descriptions, phrases of the form 'the *F*s', where '*F*' is taken to distribute somehow over the plurality being denoted. Consider a brick wall. It seems that we can collectively refer to the bricks that compose it as 'the bricks'. Now, according to the standard analysis of plural definite descriptions, 'the *F*s' denotes that plurality *aa* that includes all and only the *F*s. But if CEI is true, there's no plurality that includes all and only the bricks. If CEI is true, then each brick is identical to the atoms that compose it. Therefore, any plurality that includes at least one brick also includes the atoms that compose it, and so can't include *only* bricks. It seems, then, that there are no such things as *the bricks*. Generalizing, if an *F* is composed of non-*F*s, then, if CEI is true, there are no such things as *the Fs*; there are far fewer pluralities than we ordinarily think there are [Sider 2007: 57–9, 63–6, 2014: 215–16].[30]

[30] See also Cotnoir [2013: 313–17], Calosi [2018: 282–7], and Loss [forthcoming: 4].

My response to this problem is similar to my response to the Counting Argument. I develop the details elsewhere [forthcoming b]. Here, I'll give only a sketch.

On the standard analysis of plural definite descriptions, 'the *F*s' denotes that plurality *aa* that includes all and only the *F*s. But the standard analysis fails. Just as we can count pluralities as well as individuals (recall section 4), we can group them together, and refer to them with definite descriptions. Consider, again, the matching pairs of shoes in my closet. Since a pair of shoes just *is* the shoes that make it up, the two matching pairs just *are* the four shoes. So, 'the matching pairs of shoes in my closet' denotes those four shoes. But it's *false* that this plurality includes all and only the pairs of shoes in my closet, since it also includes each of the individual shoes.

I defend an alternative analysis of plural definite descriptions. 'The *F*s' needn't denote a plurality that includes all and only the *F*s. It simply denotes the plurality of *F*s, or the *F*s taken together. That is, if there are some things, aa_1–aa_n, such that (i) each of aa_1–aa_n is an *F*,[31] and (ii) each *F* is identical to one of them,[32] then 'the *F*s' denotes the plurality $aa_1@\ldots@aa_n$. On this analysis, it's still true that *all* *F*s are included in the denotation of 'the *F*s'. But it needn't be true that *only* *F*s are included in it. Letting aa_1 and aa_2 be the pairs of shoes in my closet, 'the matching pairs of shoes in my closet' denotes $aa_1@aa_2$, even though this plurality includes some things that aren't pairs of shoes.

Apply this to the bricks that make up the wall. According to CEI, each of the bricks (call them 'bb_1'–'bb_n') is identical to the atoms that compose it, and so the plurality $bb_1@\ldots@bb_n$ includes some non-bricks. Nonetheless, on my analysis, that plurality can still serve as the denotation of 'the bricks'.

So, a defender of CEI has little to fear from the plural resources on which I've relied, in responding to the Counting Argument.[33]

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[31] That is, aa_1 are an *F*, and aa_2 are an *F*, and … and aa_n are an *F*.

[32] That is, if *xx* are an *F*, then either $xx = aa_1$ or $xx = aa_2$ or … or $xx = aa_n$.

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