An Underwater Acoustic Communications Scheme with Inherent Scale Diversity for Multiple Users

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Abstract—Wideband underwater acoustic communication channels can cause undesirable multipath and Doppler scaling distortions to propagating acoustic signals. In this paper, we propose to exploit a time-scale canonical representation for wideband time-varying channels to achieve joint multipathscale diversity. We design a signaling scheme with hyperbolic time-frequency signatures that is matched to the underwater acoustic environment to achieve scale diversity. The signaling scheme, combined with code-division multiple-access, is extended to multiple user transmission to improve multiuser detection performance, as demonstrated with simulations.

I. INTRODUCTION

The highly time-varying nature of the wideband underwater environment can cause undesirable distortions to propagating acoustic signals [1]–[3]. In particular, underwater acoustic (UWA) communication signals, with 0.3–20 kHz spectral components and thus bandwidths comparable to their central frequencies, have wideband properties due to the fast movement of scatterers in the UWA channel. The resulting signal distortions are multipath and Doppler-scale changes. A discrete time-scale canonical model was proposed to represent the wideband channel received signal as the linear superposition of discrete time shifts and Doppler scalings of the transmitted signal, weighted by the wideband spreading function [4].

Existing UWA communications schemes do not fully exploit the potential of matching signaling schemes to discrete timescale models of the wideband UWA environment. Orthogonal frequency-division multiplexing (OFDM) has been extensively used in UWA communication schemes, but coupled with appropriate post-processing as it is not robust to Doppler scaling effects (see for example [5], [6]). In [5], the distorted OFDM due to Doppler scaling is compensated at the receiver using resampling and then any remaining Doppler is removed using intercarrier interference reduction techniques. In [6], a multiple resampling OFDM receiver front-end is designed to have each resampling branch deal with each path with a different Doppler scaling factor. Code-division multiple access (CDMA) techniques have been widely-used for multiple user UWA communications (see, for example, [7], [8]). Adaptive multiuser detection techniques are applied in [7] instead of assuming an UWA channel model with multiple Doppler scaling paths at the receiver. In [9], a spread spectrum hyperbolic

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frequency modulation scheme is used that potentially matches the UWA communications channel. However, the assumed model only has a single Doppler scale path and the multiple time delay paths can distort any possible scale diversity as no conditions are provided on the signal.

In this paper, we apply the discrete time-scale canonical representation of wideband linear time-varying systems to the UWA communications channel to achieve an inherent joint multipath-scale diversity. We design a communications signaling scheme that is matched to the underwater acoustic environment using hyperbolic frequency-modulated (HFM) chirp signals. This is achieved using scale diversity by designing the HFM chirp rate parameters assigned for transmitting different symbols. HFM chirp signals are inherently Doppler scale invariant and have been used in other applications such as sonar [10]. We combine the signaling scheme with CDMA for multiple user UWA communications to improve multiuser detection performance.

This paper is organized as follows. In Section II, we provide the discrete time-scale canonical model for wideband linear time-varying systems, and we discuss the model's inherent time-scale diversity. In III, we consider the HFM chirp signal as a potential transmission waveform for UWA communications, and we design the HFM parameters in order to achieve the desirable scale diversity. We extend the signaling design with CDMA to multiple users in Section IV, and we provide simulations to demonstrate in Section V.

II. DISCRETE TIME-SCALE UWA CHANNEL MODEL

The output of a wideband linear time-varying system can be characterized by a superposition of time shifts and scale changes, weighted by the wideband spreading function (WSF), of the system. In particular, the wideband system can be considered as a collection of fast moving scatterers that are continuously distributed in range and velocity, with the WSF representing the reflection strength of the scatterers [11]. A discrete time-scale system representation was proposed to allow for efficient wideband system processing [4]. We apply it next to a wideband UWA communications channel with multipath delay spread T_d and Doppler scale spread A_d . Assuming a transmitted acoustic signal s(t) with duration Tand bandwidth W, the noiseless received signal y(t) of the wideband UWA communications channel is represented by [3], $y(t) = \sum_{m=M_0}^{M_1} \sum_{n=0}^{N(m)} \chi_{n,m} \ a_d^{m/2} \ s\left(a_d^m \ t - \frac{n}{W}\right) \,. \tag{1}$

Here, $a_d = \exp(1/\beta_d)$, β_d is the Mellin support of s(t), $A_d = A_1 - A_0$, $M_0 = \lfloor \ln A_0 / \ln a_d \rfloor$, $M_1 = \lceil \ln A_1 / \ln a_d \rceil$, $N(m) = \lceil a_d^m W T_d \rceil$ for integer m, and $\chi_{n,m}$ are the WSF coefficients. We assume that $T \gg T_d$ to ensure that there is no intersymbol interference. Assuming independence between the scatterers contributing to the *n*th time delay and the *m*th scale path, then Equation (1) can be considered to decompose the overall channel into a total of

$$M = \sum_{m=M_0}^{M_1} (N(m) + 1)$$

independent, flat-fading channels. This results in a potential joint multipath-scale diversity of order M [4].

III. HFM SIGNALING SCHEME FOR UWA COMMUNICATIONS

A. HFM Design for Scale Diversity

We consider the hyperbolic frequency-modulated (HFM) chirp signal [12]

$$s_i(t) = e^{j2\pi c_i \ln\left((t+t_c)/t_r\right)}, \quad t \in [0, T]$$
 (2)

as an UWA communication signal. The chirp rate c_i of the HFM signal is chosen to represent the *i*th symbol, i=0, 1. Other parameters of the HFM signal in (2) include the normalization time constant t_r , the signal duration T, and the reference frequency $(1/t_c)$ that can be used to modulate the signal.

We want to use the HFM chirp signal to design a signaling scheme to match the underwater acoustic environment in a manner similar to sinusoids matching the wireless environment. Specifically, when a sinusoid $x(t) = e^{j2\pi f_0 t}$ is time shifted, $x(t - \tau) = e^{j2\pi f_0(t-\tau)} = e^{-j2\pi f_0\tau} x(t)$, the exponent is separable and the time shift does not change the magnitude of the signal. Considering the HFM signal as a dual signal and the scaling transformation, when the HFM signal $s(t) = e^{j2\pi c \ln (t/t_r)}$ is scaled by a factor a, $s(at) = e^{j2\pi c \ln (at/t_r)} = e^{j2\pi c \ln (a)} s(t)$, the exponent is also separable. Note, however, that for the joint UWA channel multipath and scaling transformations on the HFM signal, $e^{j2\pi c \ln ((at-\tau)/t_r)}$, the exponent is not separable. The separability is necessary for the signal to be invariant to the aforementioned distortions.

Although perfect separability is not possible, we want to derive constraints on the HFM chirp rates to minimize the correlation between any pair of UWA channel propagated HFM chirp signals. Specifically, consider the correlation function between two time-shifted and scaled HFM signals with different chirp rates

$$\Phi_{i,l,n,m} = \int_{T} \mathbf{s}_{i,n,m}(t) \, \mathbf{s}_{l,n,m}^{*}(t) \, dt \,, \quad i,l = 0, 1 \,, \qquad (3)$$

where

$$\mathbf{s}_{i,n,m}(t) = e^{j2\pi c_i \ln(a_{m,i} t - \tau_{n,i} + t_c)}$$

is the time-shifted and scaled transmitted signal in (2) with $t_r = 1$ s, $a_{m,i} = a_d^m$ and $\tau_{n,i} = n/W$ in (1) for the *i*th symbol. The correlation in (3) can be calculated as:

$$\mathbf{s}_{i,n,m}(t)\,\mathbf{s}_{l,n,m}^*(t) = A\,B(t)\,C(t),\tag{4}$$

where

$$A = \exp\left(j2\pi \ln \frac{(a_{m,i})^{c_i}}{(a_{m,l})^{c_l}}\right),\tag{5}$$

$$B(t) = \exp\left(j2\pi \ln\left(\frac{t - \tau_{n,i}/a_{m,i} + t_c/a_{m,i}}{t - \tau_{n,l}/a_{m,l} + t_c/a_{m,l}}\right)^{c_i}\right), \quad (6)$$

$$C(t) = \exp\left(j2\pi \ln\left(t - \tau_{n,l}/a_{m,l} + t_c/a_{m,l}\right)^{c_i - c_l}\right).$$
 (7)

Note that (5) is not a function of time. Equation (6) can be re-written as

$$B(t) = \exp\left(j2\pi c_i \ln\left(1 + \frac{t_c(1/a_{m,i} - 1/a_{m,l})}{t - \tau_{n,l}/a_{m,l} + t_c/a_{m,l}}\right)\right).$$
 (8)

Since both $\tau_{n,l}$ and t_c are very small, we can approximate

$$\ln\left(1 + \frac{(\tau_{n,l} + t_c)\left(\frac{1}{a_{m,i}} - \frac{1}{a_{m,l}}\right)}{t - \frac{\tau_{n,l}}{a_{m,l}} + \frac{t_c}{a_{m,l}}}\right)$$
$$\approx \frac{(\tau_{n,l} + t_c)\left(\frac{1}{a_{m,i}} - \frac{1}{a_{m,l}}\right)}{t - \frac{\tau_{n,l}}{a_{m,l}} + \frac{t_c}{a_{m,l}}}$$

and thus re-write (8) as

$$B(t) \approx \exp\left(j2\pi c_i \frac{(\tau_{n,l} + t_c)\left(\frac{1}{a_{m,i}} - \frac{1}{a_{m,l}}\right)}{t - \frac{\tau_{n,l}}{a_{m,l}} + \frac{t_c}{a_{m,l}}}\right) \approx 1.$$
(9)

The resulting correlation in Equation (3), after substituting Equations (5), (9) and (7) into (4), becomes

$$\Phi_{i,l,n,m} \approx \int_{\tau_{n,l}/a_{m,l}}^{T+\tau_{n,l}/a_{m,l}} \exp\left(j2\pi \ln\frac{(a_{m,i})^{c_i}}{(a_{m,l})^{c_l}}\right)$$
$$\cdot \exp\left(j2\pi \ln\left(t - \frac{\tau_{n,l}}{a_{m,l}} + \frac{t_c}{a_{m,l}}\right)^{c_i - c_l}\right) dt.$$

If we let $\zeta = \ln \left(t - \frac{\tau_{n,l}}{a_{m,l}} + \frac{t_c}{a_{m,l}} \right)$, then (3) can be approximated to

$$\Phi_{i,l,n,m} \approx e^{j2\pi \ln\left(a_{m,i}^{c_i}/a_{m,l}^{c_l}\right)}$$

$$\cdot \int_{\ln(t_c/a_{m,l})}^{\ln(t_c/a_{m,l}+T)} e^{j2\pi(c_i-c_l)\zeta} e^{\zeta} d\zeta$$

[4]



Fig. 1. Scale correlation in Equation (10) for increasing values of the chirp rate difference $\Delta c_{il} = c_i - c_l$. Four different cases are shown as two possible Doppler scaling paths are considered.

Thus, it can be shown that the correlation between any pair of UWA channel propagated hyperbolic chirp signals is mainly a function of scale transformed signals and depends on the difference between the two HFM chirp rates. The resulting correlation in (3) between the HFM with chirp rate c_i and the HFM with chirp rate c_l becomes

$$\Phi_{i,l,n,m} = \exp(j2\pi \ln \left(a_i^{c_i}/a_l^{c_l}\right))/(1+j2\pi\Delta c_{il})$$
$$\cdot \left(\left(\frac{t_c}{a_l}+T\right) \exp\left(j2\pi\Delta c_{il}\ln\left(\frac{t_c}{a_l}+T\right)\right)\right)$$
$$-\frac{t_c}{a_l} \exp(j2\pi\Delta c_{il}\ln\left(\frac{t_c}{a_l}\right) \right)$$
(10)

where $\Delta c_{il} = c_i - c_l$, $i \neq l$, and Plotting the scale correlation in (10) in Figure 1, it can be seen that as the chirp difference Δc_{il} increases, the scale correlation decreases significantly.

B. UWA Communications HFM Signaling Scheme

The HFM signaling scheme can be used for single user UWA communications provided we choose HFM signals with large difference in their chirp rates. The received signal r(t)of a wideband UWA communications channel can be written as

$$r(t) = y_i(t) + w(t)$$

= $\sum_{m=M_0}^{M_1} \sum_{n=0}^{N(m)} \chi_{n,m} a_d^{m/2} s_i \left(a_d^m t - \frac{n}{W} \right) + w(t)$

for symbol i=0, 1, where w(t) is additive white Gaussian noise. The correlated signal is obtain as

$$\Lambda(y_i) = \int_{-\infty}^{\infty} r(t) \, y_i^*(t) \, dt \,. \tag{11}$$

Since each symbol is transmitted with equal probability, the information bit of the kth user can be estimated as

$$\hat{i} = \underset{i=0,1}{\operatorname{arg\,max}} \Lambda(y_i^k).$$

Note that we assume that the UWA channel WSF coefficients are estimated *a priori*; methods for estimating the WSF coefficients can be found in [2], [3].

IV. HFM WITH CDMA MULTIUSER SIGNALING SCHEME

When HFM signaling is combined with CDMA it can be successfully used for multiuser detection in UWA communication channels. Assuming a K-user communication system, the *i*th transmitted symbol $b_i^k(t)$, of the *k*th user, k = 1, ..., K, i = 0, 1 can be written as

$$b_i^k(t) = e^{j2\pi c_i \ln((t+t_c)/t_r)}, \quad t \in [0, T]$$

The transmitted signal for the kth user is given by

$$s_i^k(t) = b_i^k(t) \operatorname{PN}_k(t) \,,$$

where $PN_k(t)$ is the pseudorandom noise (PN) code of the kth user.

After the wideband UWA channel, the observed signal at the receiver can be written as

$$r(t) = \sum_{k=1}^{K} y_i^k(t) + w(t),$$

where

$$y_i^k(t) = \sum_{m=M_0}^{M_1} \sum_{n=0}^{N(m)} \chi_{n,m} \ a_d^{m/2} \ s_i^k(a_d^m t - n/W),$$

and w(t) is assumed to be a white Gaussian noise. Following (11), the correlation between the received signal and the expected signal for the k user, assuming symbol i, is given by

$$\begin{split} \Lambda(y_i^k) &= \int_{-\infty}^{\infty} r(t) \, y_i^{k*}(t) \, dt \\ &= \sum_{m=M_0}^{M_1} \sum_{n=0}^{N(m)} \chi_{n,m} \, a_d^{m/2} \int_{-\infty}^{\infty} r(t) \\ &\quad \cdot \, s_i^k \Big(a_d^m t - \frac{n}{W} \Big) \, \operatorname{PN}_k \Big(a_d^m t - \frac{n}{W} \Big) \, dt \end{split}$$

The information bit of the kth user can be estimated as

$$\hat{i} = \underset{i=0,1}{\operatorname{arg\,max}} \Lambda(y_i^k).$$

V. SIMULATION RESULTS

A. HFM Signaling Scheme

In order to demonstrate the scale diversity using the HFM signaling scheme in UWA communications, we considered the following simulation example. We considered an HFM signal with duration T=0.1 s and 1.5 kHz maximum bandwidth. We assumed an UWA channel with $T_d=10$ ms multipath spread, $\beta_d=90$ Mellin spread resulting in $a_d=1.011$, and



Fig. 2. BER as a function of SNR per bit for varying chirp rate differences for single user in an UWA communications channel.



Fig. 3. CDMA with HFM signaling in an UWA communications channel using K=1 and K=4 users.

 $M_0 = 0$, $M_1 = 1$. Note that we assumed that the channel WSF coefficients were estimated at every iteration [3].

The user is assumed to transmit symbol 0 or 1 with equal probability. The HFM signals representing the two symbols differ by the value of the chirp rates. We considered chirp rates whose difference in values are given by $\Delta c_{01} = c_1 - c_0 = 0.1, 0.15, 0.5$. The simulation results of the bit error rate (BER) for different signal-to-noise ratio (SNR) values (in dB) per bit are shown in Figure 2. As expected from our derivations, scale diversity is achieved, and thus BER is reduced, as the chosen Δc_{01} value is increased from 0.1 to 0.5.

B. CDMA with HFM Signaling

We demonstrated the use of HFM signaling for multiuser detection in the following simulation example, where PN sequences are used to distinguish between users. The HFM signals we used in this example have T = 0.1 s duration and 3 kHz maximum bandwidth. We generated PN sequences of length 7 using the M-sequence generator [13]. We assumed that the UWA communications channel characteristics are the same as in the previous simulation in Section V-A. For each user, we considered the difference in chirp rate to vary as $\Delta c_{01} = c_1 - c_0 = 0.1, 1, 3$. The simulated BERs (in dB) per bit for different SNR values are shown in Figure 3. For comparison, we demonstrate the results for the case of K = 1user and K = 4 users. The scale diversity is better achieved when the difference in chirp rate increases; this holds both for the single and multiuser cases.

VI. CONCLUSION

In this paper, we proposed a matching signaling scheme to exploit the inherent diversity of the UWA communication channel. We first considered the time-scale canonical representation of wideband linear time-varying channels and their inherent joint multipath-scale diversity. We designed a signaling scheme, using hyperbolic frequency-modulated (HFM) signals, that is matched to this canonical representation. In particular, we derived conditions on the HFM chirp rates for representing different information symbols. Combined with CDMA, the signaling scheme was extended to multiple users and demonstrated high BER performance when the HFM chirp rate conditions were satisfied.

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