Portfolio Selection Methods: An Application To İstanbul Securities Exchange Market

A THESIS

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PORTFOLIO SELECTION METHODS:

AN APPLICATION TO ISTANBUL SECURITIES EXCHANGE MARKET

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by
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August 1996

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ABSTRACT

PORTFOLIO SELECTION METHODS:

AN APPLICATION TO ISTANBUL SECURITIES EXCHANGE MARKET

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Master of Business Administration in Management

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In this study, Modern Portfolio Theory tools are used for constructing efficient portfolios. The Markowitz mean-variance model is presented and calculated for the construction of efficient portfolios from the Istanbul Securities Exchange Market stocks for the 1993-1994 period. The portfolios constructed are compared on the risk and return scales.

Keywords: Portfolio, Efficient Frontier, Diversification, Return, Risk, Capital Markets

ÖZET

Portföy Seçim Yöntemleri:

İstanbul Menkul Kıymetler Borsası İçin Bir Uygulama

Mert Çetin

İşletme Yönetimi Yüksek Lisans

Tez Yöneticisi: Dr. Ayşe Yüce

Ağustos 1996

Bu çalışmada Modern Portföy Teorisi Araçları, Etkinlik Sınırı oluşturulması için kullanılmıştır. Markowitz ortalama varyans modeli açıklanmış ve 1993-1994 donemi için, İstanbul Menkul Kıymetler Borsası hisse senetleri için hesaplanmıştır. Elde edilen etkin portföyler risk ve getiri boyutlarında karşılaştırılmştır.

<u>Anahtar Kelimeler</u>: Portföy, Etkinlik Sınırı, Çeşitlendirme, Getiri, Risk, Semaye Piyasaları.

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TABLE OF CONTENTS

NOMENCLATURE

$D_{i,t}$:	dividend and other payments for the i^{th} asset on the t^{th} period.
Ei :	Expected return on the i^{th} asset
Ep:	Expected rate of return on the portfolio.
Ii:	The market index
xi:	Proportion to invest from the i^{th} asset
Pi,n:	Price of the i^{th} asset on the n^{th} period
Pi:	Price of the old quoataion on the i^{th} period
Pssr:	Price of the stock split right owning quotation
$\bar{R}_i$:	Average rate of return on the i^{th} asset
$r_{i,t}$:	Rate of return on the i^{th} asset on the t^{th} period
Rf:	Riskfree rate of return
$\bar{R}_p$:	Average portfolio return
Rp:	Portfolio rate of return
R_i:	Rate of return on the i^{th} asset
m:	Number of stocks to be received as the result of the stock split
n:	Number of periods
λ:	Risk preferance coefficent
x_i:	Proportion to invest on the i^{th} asset
σ_p^2:	Portfolio variance
$\sigma_{i,j}$:	Covariance between the i^{th} and j^{th} asset returns
σ_p:	Portfolio standard deviation.

1. INTRODUCTION

Security markets have always captured many people's curiosity and imagination all over the world. The stories of sudden wealth are attractive that many people are moved to invest in the stock market. While investing is a complex subject, the potential rewards or risks can be large or small. The problem that an investor faces about which portfolio to hold, given that an infinite number of possibilities exist, is the subject of this study. In making his or her choice, the investor should evaluate alternative portfolios on the basis of their potential rewards and risks.

In 1952, Harry M. Markowitz published a now classic paper which marked the beginning of a new type of investment research and analysis. Markowitz provided a theoretical framework for the composition of optimum portfolios. The report was practically unintelligible to many investment professors and practitioners when it was published because it used mathematics and statistics. Although these changes at first affected the college classroom more than they affected the way people invest, their effect on the financial community has grown rapidly in the last decade.

In Turkey, today most of the investment institutions have their own portfolio management department, research department and investment consultants. But it is a fact that, most of them do not use modern portfolio selection methods in their analysis.

The first reason of their not using the portfolio selection methods may be stated as the problem with the investors. In Turkey there are investors that make their investment selections according to the speculative moves. That means their investment criteria is the news that they hear from outside. It is also very frustrating there is a possibility that the investor type described above may earn large amounts of profit in Istanbul Stock Exchange Market, so as a result it can be said that the investors that seek for a good portfolio management are very few.

The second reason is that many intermediaries do not find it necessary to develop their portfolio management methods as they do not face a big demand from the investors and prefer giving investment advises according to speculative moves.

The aim of this study is to find optimal portfolio in Istanbul Stock Exchange Market by using Markowitz efficient frontier. By using Markowitz Model (Mean Variance Model), I will try to maintain a portfolio that contains minimum expected deviation of return around the mean by using the risk diversification.

2. LITERATURE REVIEW

2.1. Risk and Common Stocks:

Common stocks are the securities that represent an ownership interest in a corporation. Compared to non marketable and fixed income investments, common stocks are risky assets because they produce rates of return quite difficult to forecast. This characterization is a generalization, however, for in certain respects common stocks may actually subject an investor to less risk than long term bonds and long term certificates of deposit. Also the risk of individual common stocks spans a wide range. Some common stocks are very risky in nearly every respect.

The degree of refuge that common stocks can provide, to protect investment returns from inflation is uncertain. A company's productive assets are often able to produce additional profits during periods of general price increases cash distributions and market values. The transformation of general inflation into increased profitability depends on a variety of considerations such as the ability of a firm to pass price increases along to customers and the kinds of inputs used by the firm in producing goods and services.

In most respects common stocks are relatively risky assets to own, as investors may lose or earn great amounts of money during low or high inflation rates. Shares fluctuate in market value, dividend payments frequently change, and companies sometimes go out of business. A variety of things can occur to cause grief and financial loss to someone who has the residual claim of a common stock stockholder. Financial risk (the variability imparted to the earnings power of the operating entity because of the method of financing asset acquisition) and market risk (the risk that is caused by macroeconomic variables that influence all risky assets) are formidable uncertainties with which most common stockholders must cope.

The riskiness of a stockholder's portfolio of common stocks can be adjusted by careful selection of securities.. Old line firms with moderate amounts of debt and fairly large dividend pay outs subject their owners to less uncertainty than do new stocks issues and the stocks of firms in very unstable and highly competitive industries. Again however the severity of the risks that the investors encounter in common stocks investing varies, sometimes significant, from one stock to the next. Even the stocks of companies operating in the same industry can have diverse risk characteristics as every company shows a different level of performance during a specific period.

2.2. The Risk Diversification:

Risk is thought of as uncertainty regarding the expected rate of return from an investment. The risk of a stock portfolio depends on the proportions invested in individual stocks, their variances, their covariance's and their rates of returns. A change in any of these variables will change the risk of the portfolio. Early empirical studies, reached their conclusion by simulating the relation ship between risk and number of stocks. Elton, Gruber and Jensen is found that when stocks are randomly selected and combined in equal proportions into a portfolio, the risk of a portfolio declines as the number of different stocks in it increases. It is observed that the risk reduction effect diminishes rapidly as the number of stocks increases. Evans and Archer investigated the relationship between risk and number of stocks in a portfolio further and provided an analytical solution for the relationship between the two. They implied that 51 percent of a portfolio standard deviation is eliminated as diversification increases from 1 to 10 securities. Adding 10 more securities eliminates an additional 5 percent of the standard deviation. Increasing the number of securities to 30 eliminates only an additional 2 percent of the standard deviation. (Evans and Archer, 'diversification and the Reduction of Dispersion: an Emprical Analysis' , Journal of Finance ,Dec. 1968.

Diversification works because prices of different stocks do not move exactly together. The total risk in holding securities can be divided into two; the unique risk (unsystematic risk) and the market risk (systematic risk). Systematic risk is the proportion of an individual asset's total variance that is attributable to the variability of the portfolio. The variance that is due to unique features is called unsystematic risk. The only risk that remains for a portfolio is an overall aggregate market risk caused by macroeconomics variables that influence all risky assets. As the number of stocks in the portfolio increases, variance reduces until it reaches undiversifiable part. In a completely diversified portfolio, the unique risk is eliminated, so all that remains is systematic risk.

This risk is measured by the standard deviation of returns of the market portfolio. All rational, profit maximizing investors want to hold a completely diversified portfolio of risky assets, called the market portfolio. This market variability can change over time as the macroeconomics variables that affect the valuation of risky assets change.

2.3. Markowitz Portfolio Theory:

2.3.1. Harry M. Markowitz:

In every field of study it is possible to look back and identify a person or event that caused a major change in the direction or development of the field. In investments it is clear that the seminal work by Harry Markowitz on portfolio theory changed the field more than any other single event. His Ph.D. dissertation written at the University of Chicago dealt with portfolio selection; in it he developed the basic portfolio model. A brief presentation of the model was published in the Journal of Finance in 1952. Subsequently, a complete presentation of the theory and its implementation was published in Portfolio Selection Efficient Diversification of Investments by John Wiley and Sons in 1959 and by Yale University Press in 1962. Because of the this work, Markowitz is referred as the father of modern portfolio theory, and much subsequent research has been based on this development.

2.3.2.. Assumptions of Markowitz Model:

One basic assumption of the portfolio theory is that, investors want to maximize the returns from their portfolios of investment for a given level of risk (uncertainty).

It is also assumed that investors are basically risk averse, which simply mean that, given a choice between two assets with equal rate of returns, an investor will select the asset with the lower level of risk. It is also clear that the investor in a situation where two stocks have equal risk will chose the stock with the higher return. Investors generally require a higher rate of return to accept higher risk.

Other assumptions are that ; Markets are perfect, transaction costs and taxes do not exist, investors are price takers, and securities are infinitely divisible so that partial shares can be purchased if necessary and the investor has a single period horizon.

Although there is a difference in the specific definitions of risk and uncertainty, in this report, the two terms are used interchangeably. In fact, one way to define risk is uncertainty of future outcomes.

In the 1950' s and early 1960's a large segment of investment community talked about risk but there was no measurable specification for the term. One aspect of the portfolio model is that it requires investors to quantify their risk variable. The basic portfolio model, developed by Harry Markowitz, applied existing statistical theory to investments and thereby made it possible to compute the expected rate of return for a portfolio of assets and a risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of risk under a reasonable set of assumptions and derived the formulas for computing the variance of the portfolio. This portfolio variance formulation indicated the importance of diversification for reducing the risk measured by the variance of return and showed how to diversify properly. The Markowitz model is based on several assumptions regarding investor behavior.

1. Investors consider each investment alternative as being represented by a probability distribution of returns over some holding period.

2. Investors maximize one period expected utility and posses utility curves that demonstrate diminishing marginal utility wealth.

3. Investors estimate the risk on the basis of the variability of expected returns.

4. Investors base decisions solely on expected return and risk; that is; their utility curves are a function of expected return and the variance (standard deviation) of returns only.

5. For a given risk level, investors prefer higher returns to lower returns. Similarly for a given level of expected return investors prefer less risk to more risk.

Under these assumptions, a single asset or a portfolio of assets is considered to be ' efficient' if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

The Markowitz model data requires the following information for a set of n securities:

1. The expected rate of return of each security :n returns,
2. The variance of each security :n variance terms,
3. The covariance between any two securities : $\frac{(n^2 - n)}{2}$ covariance terms.

2.3.3. Alternative Risk Measures:

One of the best known measures of risk is the variance of expected returns. It is a statistical measure of the dispersion of returns around its expected value; that is, a larger value for the variance or standard deviation indicates greater dispersion and greater risk , all other factors being equal. The idea is that the more dispersed the returns, the greater the uncertainty of those returns in any future period. Another measure of risk is the range of returns based on the assumption that a large range returns, means greater uncertainty regarding future expected returns.

In contrast to using measures that analyze any deviation from expectations, some feel that the investor should only be concerned with returns below expectations, deviations below the mean value. A measure that only considers such adverse deviations is the semivariance. An alternative specification of this measure would be deviations below zero, or negative returns. Both measures (semivariance below the mean or semi variance below zero) implicitly assumes that investors want to minimize their regret from below average returns. It is implicit that investors would welcome positive returns or returns above expectations, so these are not considered when risk is measured.

Although there are numerous potential measures of risk in this report the variance or the standard deviation will be used in terms of risk.

2.3.4. Daily Returns:

Returns of course can be measured in a variety of ways, depending on the type of the asset being considered. In general return for a given type of asset can be calculated according to the formula:

$$R_i = \frac{EV - BV + CF}{BV}$$

where:

EV is the ending value,

BV is the beginning value,

CF is the cash flow during the period.

Based on the above formula the daily returns for common stocks are calculated according to the following formula:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1} + Div_{i,t}}{P_{i,t-1}}$$

where:

$P_{i,t}$ is the closing price of stock i values are day t,

$Div_{i,t}$ values are the dividend payments for a common stock during day t.

2.3.5. Average Returns:

The expected returns are the arithmetic mean of the daily returns.

$$\bar{R} = \sum_{t=1}^{N} R_{i,t} / N$$

N is the number of stocks

2.3.6. Variance of Return:

It was stated early in this report that the variance of the returns, is a measure of risk. Therefore, at this point I will demonstrate the computation of the variance of returns for an individual asset.

The variance and standard deviation (that is the square root of variance) are the measures of the variation of possible rates of return (R_i) from the expected rate of return [$E(R_i)$] and can be calculated as follows:

$$VARIANCE_i\left(\sigma_i^2\right) = \sum_{j=1}^{n}\left[R_{ij} - E(R_i)\right]^2 P_{ij}$$

where P_{ij} is the probability of the possible rate of return $\left(R_{ij}\right)$ on asset i in the state j

$$STANDARD\ DEVIATION_i\left(\sigma_i\right) = \sqrt{\sum_{j=1}^{n}\left[R_{ij} - E(R_i)\right]^2 P_{ij}}$$

2.3.7. Covariance of Returns:

Covariance is a measure of the degree to which two variables move together over time. In portfolio analysis, we usually are concerned with the covariance of rates of return rather than that of prices or some other variable. A covariance between the rates of return for two assets that is positive indicates that the rates of return tend to move in the same direction at the same time, if the covariance is negative the rates of return tend to move in the opposite directions. The magnitude of the covariance depends upon the variances of the individual rate of return series, as well as on the relationship between the series.

For two assets i and j, the covariance of the daily rate of return is defined as :

$$
\begin{aligned}
Cov_{i,j} &= E\{[R_i - E(R_i)][R_j - E(R_j)]\} \\
&= \frac{1}{n}\sum_{1}^{n}[R_i - E(R_i)][R_j - E(R_j)]
\end{aligned}
$$

where n is the number of days, that the daily rate of returns of the stocks are available for the specific period. In this report, n will be taken as 253.

2.3.8. Correlation Between the Securities:

Covariance is affected by the variability of the two return series but the size of the covariance depends on the unit of measurement. So in order to standardize this covariance the correlation among the two series can be used.

The correlation factor can be calculated as:

$$r_{i,j} = \frac{Cov_{i,j}}{\sigma_i \sigma_j}$$

A correlation of +1.00 would indicate perfect positive correlation, a value of -1.00 would mean that the rates of return moved in a completely opposite direction, and a value of zero would mean that there is no linear relationship between the returns. That is they are uncorrelated from a statistical standpoint: this does not mean that they are independent.

2.3.9. Portfolio Return:

The expected rate of return for a portfolio of assets is simply the weighted average of the expected rates of return of the individual assets in the portfolio. The weights are the proportion of total value invested in the asset. The computation of the expected return of a portfolio can be done by using the formula:

$$E\left(R_{port}\right) = \sum_{i=1}^{n} R_i x_i$$

where,

R_i = Rate of return on the i^{th} asset,

x_i = Proportion of the i^{th} asset in the portfolio,

n = number of assets in the portfolio,

R_{port} = Rate of return of the portfolio.

2.3.10. Standard deviation of a Portfolio:

As stated above the expected return of the portfolio was simply the weights multiplied by the average of the expected returns for the individual assets in the portfolio. Based upon this formula for expected return, one might assume that it is possible to derive the standard deviation of the portfolio in the same manner, that is, computing the standard deviation of the portfolio by computing the weighted average of the standard deviations of the individual assets. The fact is, this is not correct. When Markowitz applied the general formula for the standard deviation of a portfolio it was as follows:

$$\sigma_{port} = \sqrt{\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j Cov_{ij}}$$

where

σ_{port} = standard deviation of the portfolio

W_i = weights of the individual assets in the portfolio, where these weights are determined by the proportion of the value in the portfolio

σ_i^2 = variance of returns for asset i

Cov_{ij} = Covariance between the returns for assets i and j.

N = number of stocks or assets in the portfolio

In words, this formula indicates that the standard deviation for the portfolio is a function of the weights times average of the individual variances(where the weights are squared), plus the weights multiplied by the covariance's among all the assets in the portfolio. The point is that standard deviation for the portfolio encompasses not only the individual variances but also the covariance between pairs of individual securities. Further, it can be showed that, in a portfolio with a large number of securities this formula can be stated as the summation of weights multiplied by covariance's. This means that the important factor to consider when adding an asset to a portfolio with a number of other assets is not the individual's variance, but its average covariance with all other assets in the portfolio.

2.3.11. Minimum Variance Portfolio:

Minimum variance portfolio is the portfolio that has the lowest possible variance according to the given set of constraints.

2.3.12. Feasible Portfolio:

The portfolio that has the weights of the stocks $x_1, x_2, \ldots, x_n$ is said to be feasible if $\sum_{i=1}^{n} x = 1,\ x_i \geq 0$

2.3.13. Inefficiency of an Risk Return Combination:

A portfolio is inefficient if another portfolio has either higher return and no higher variance, or less variance and no less mean.

2.3.14. Infeasible:

If no portfolio can meet the constraints of the model is said to be infeasible.

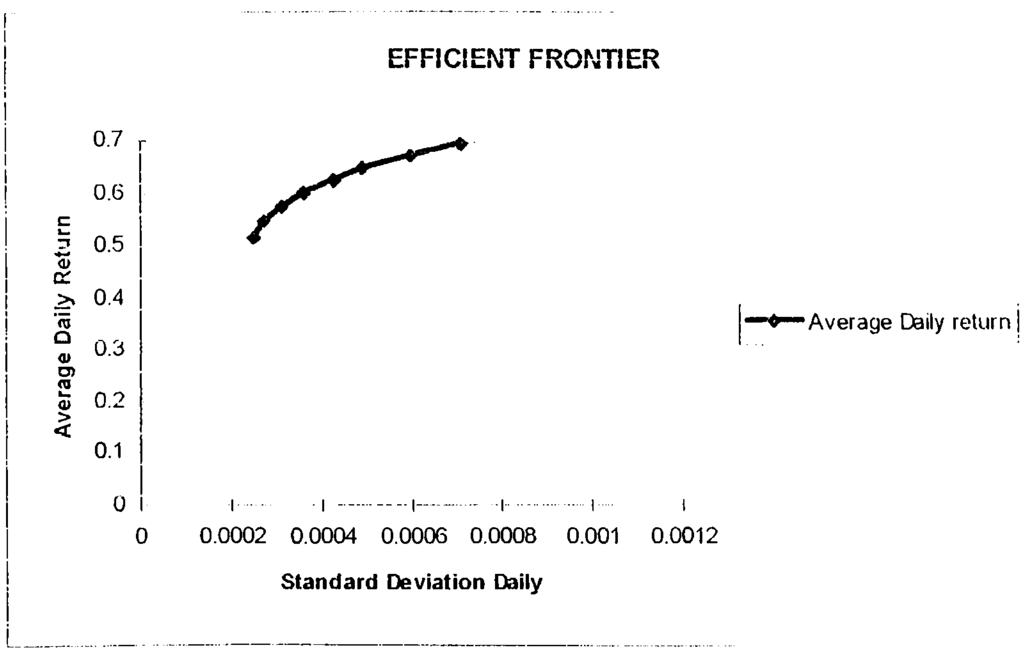
2.4. The Efficient Frontier and Investor Utility:

The efficient frontier indicates the set of portfolios that offer the highest attainable expected returns for each attainable risk level (or the lowest attainable risk for each attainable expected return level). Once the efficient frontier has been determined for portfolios formed from the securities under consideration, the investor has a choice to make. However, the shape of the efficient frontier for risky assets is generally such that one has to tolerate more and more risk to achieve higher returns. The slope of the efficient frontier decreases steadily as to move up the curve. This tendency implies that equal increments of added risk, as you move up the efficient frontier, will add progressively less of an increment in expected return. See figure 2.5.1.

The utility curves for an individual specify trade off that a person is willing to make between expected return and risk. An investor's utility curves are used in conjunction with the efficient frontier to determine which particular efficient portfolio is the best, given these risk return preferences. Two investors will not choose the same portfolio from the efficient set unless their utility curves are identical.

The optimal portfolio is the portfolio on the efficient frontier with the highest utility. The investor will choose the portfolio that will be found at the point of tangency between the efficient frontier and the curve with the highest possible utility for a given investor.

Figure 2.5.1.



2.5. Other Methods for Portfolio Selection:

2.5.1. CAPM:

The capital asset pricing model is a major paradigm in the field of finance. Building on Markowitz's two parameter portfolio analysis model, researchers developed this model relatively simultaneously; necessary assumptions included homogenous expectations (that is all investors estimate identical probability distributions for future rates of return), perfect markets (buying or selling assets involves no taxes or transaction costs), and the existence of a risk -free borrowing and lending rate. After making these assumptions, the capital market line can be derived and the separation theorem (the division of the investment and the financing decision between risky and risk- free assets) can be proven. The result is that each investor will choose his or her optimal portfolio from combining two portfolios; one consisting of the risk free asset and the other consisting of the market portfolio (the portfolio that includes all risky assets). Evaluation of individual securities in this setting leads to the revelation that a security's expected return is positive linear function of its covariance with the market portfolio, a relationship that is referred to as the capital asset pricing model.

Capital Market theory builds upon the Markowitz portfolio model, and it has the following assumptions in relation to the Markowitz model which are expanded as follows:

1. All investors are Markowitz efficient investors who want to be somewhere on the efficient frontier. The exact location on the efficient frontier depends upon the utility function of the investor and differs among investors.

2. It is possible for investors to borrow or lend any amount of money at the risk free rate of return. Clearly, it is always possible to borrow money at the nominal risk free rate by buying risk free securities such as government T bills.

3. All investors have homogenous expectations; that is all investors estimate identical probability distributions for future rates of return. Again this expectation can be relaxed and as long as expectations are not vastly different the effect is minor.

4. All investors have the same one period time horizon, one month, one year and so on. The model will be developed for one hypothetical period, acknowledging that the results could be affected by a different assumption and the investors would have to derive risk measures consistent with their own horizons.

5. All investments are infinitely divisible; it is possible to buy and sell fractional shares of any asset or portfolio. This assumption simply allows us to discuss the various investment alternatives as continuos curves. Changing this assumption would have little impact on theory.

6. Buying or selling assets involves no taxes or transaction costs. This is reasonable assumption in number of instances. Specifically many investors do not have to pay taxes and the transaction costs for most financial institutions are less than 1 percent on most financial instruments. Again the relaxation of this assumption modifies the results but does not change the basic thrust.

7. There is no inflation or change in the interest rate, or inflation is fully anticipated. This is a reasonable initial assumption and can be modified.

8. Capital markets are in equilibrium. This means that we begin from a state in which all assets are properly priced in terms of the risk involved.

2.5.2. Arbitrage Pricing Theory:

The arbitrage pricing theory (APT) was developed by Ross in the early 1970's and initial published in 1976. The APT has three major assumptions:

1. Capital markets are perfectly competitive.

2. Investors always prefer more wealth to less wealth with certainty.

3. The stochastic process generating asset returns can be represented by a K factor model.

There are several major assumptions that are not required. This theory does not require that investors have a quadratic utility function, that security returns be normally distributed or that there be a market portfolio that contains all risky assets and is mean variance efficient. Obviously if such a theory were able to explain differential security prices, it would be considered a superior theory because it is simpler and it requires fewer assumptions.

The theory assumes that the process generating rates of return on assets can be represented as K factor model of the form:

$$R_i = E_i + b_{i1}\delta_1 + b_{i2}\delta_2 + \dots\dots\dots + b_{ik}\delta_k + \varepsilon_i \text{ for i = 1 to N}$$

where

R_i = return on asset i during a specified time period

E_i = expected return for asset i or risk free rate of return if all the other risk factors equal to zero.

b_{ik} = reaction in asset i returns to movements in the factor δ_k

δ_k = a common factor with a zero mean that influences the returns on all assets

ε_i = a unique effect on asset i return which, by assumption, is completely diversifable in large portfolios and has a mean of zero

N = number of assets

Two terms require elaboration δ and b. As indicated the δ terms are the multiple factors that are expected to have an impact on the returns of all assets. Examples may include inflation, growth in GNP, major political upheavals, or changes in interest rates. The point is that, the APT contends that there are a number of such factors that influence returns. This is in contrast to the CAPM, where the only variable of importance is the covariance of the asset with the market portfolio.

Given these common factors, the b's determine how each individual asset reacts to this common factor. This can be explained like: while all the assets may be affected by growth in GNP, the impact will differ between assets.

Similar to CAPM model, it is assumed that the unique effects ε's are independent and, therefore, that they will be diversified away in large portfolio.

2.5.3. Single Index Model :

Sharpe in 1963 suggested that the return on any security may be related to the performance of some index of business activity. He says that the major characteristic of the model is that the stock prices are dependent on the indexes that measure the volatility in security markets. The main aim of the study is to forecast the price changes for a given stock as a function of the overall market fluctuations, and these fluctuations are measured with an index that is an average of the stock returns.

The major characteristic of the model is the assumption that the returns of various securities are related only through common relationships with some basic underlying factor. The return from any security is determined solely by random factors and this single outside element. The return from any security is determined solely by random factors and this single outside element, more explicitly:

$$r_{i,t} = \alpha_i + \beta_i r_{I,t} + e_{i,t}$$

where α and β are constants for firm i, $r_{m,t}$ is the return on some underlying factor in period t, $r_{i,t}$ is the return on the stock of firm i, in period t. This is the single index model and is based on the assumption that the joint probability distribution between $r_{i,t}$ and $r_{m,t}$ is stationary and normal. As a result of this assumption, the error term has the following properties:

1. The e's average value is zero- that is E(e)=0
2. The variance of e is constant.
3. The error terms are uncorrelated with $r_{m,t}$
4. Securities should only be related through a common response to market. Error terms should not be correlated.

It is also assumed that, the firms error term in period t is uncorrelated with any other firm's error term in period t.

It can be said that Single Index Model is a simple model for the portfolio selection problem. The Single Index model defined by Sharpe can be described as:

$$R_i = A_i + B_i I + c$$

$$I = A_{n+1} + C_{n+1}$$

$$E_i = A_i + B_i(A_{n+1})$$

$$V_i = (B_i^2)(Q_{n+1} + Q_i)$$

$$C = (B_i)(B_j)(Q_{n+1})$$

where

R_i = rate of return on the ith *asset*

A_i = A constant return term which is independent of the market fluctuations.

B_i = The beta coefficent

I_i = The index

C_i = The error term

E_i = Expected return on the ith *asset*
c = the error term of the rate of return on the ith asset.

V_i = Variance of any security i.

Q_{n+1} = is the variance of I

Q_i = is the variance of the random error

A_{n+1} = is the expected value of the market index

C = is the covariance between any two securities

C_{n+1} = is the error term of the market index.

Portfolio Return:

$$E_p = \sum_{i=1}^{n} x_i E_i = \sum_{i=1}^{n} x_i \left(A_i + B_i I + C_i\right)$$

E_p: *Portfolio* rate of return

Portfolio Variance:

$$\alpha_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j B_i B_j \alpha_m^2 + \sum_{i=1}^{n} x_i^2 \alpha_{e,i}^2$$

or,

$$\alpha_p^2 = \sum_{i=1}^{n} x^2 B_i^2 \alpha_m^2 + \sum_{i=1}^{n}\ \sum_{J=1 i\neq j}^{n} x_i x_j B_i B_j \alpha_m^2 + \sum_{i=1}^{n} x_i^2 \alpha_{e,i}^2$$

where

α_p^2 = Portfolio Variance

α_m = Variance of the market index

$\alpha_{e,i}^2$ = Standard error of estimator

Portfolio Beta:

As individual securities have beta coefficients, constructed portfolios have beta coefficients as well. Portfolio beta will measure the percent change in the portfolio's return when there is one percent change in the market index.

$$B_p = \sum_{i=1}^{n} x_i B_i$$

where

B_p = Portfolio Beta

Portfolio Alpha:

Portfolio alpha measures the rate of portfolio returns which is independent of the market fluctuations that a portfolio can gain.

$$A_p = \sum_{i=1}^{n} x_i A_i$$

where

A_p = Portfolio Alpha

A_i = Alpha of the individual asset.

2.5.4. The Black Model:

The Black model is very similar to the Markowitz model except the removal of the non negativity constraints of the Markowitz model. This removal of the nonnegativity constraint changed the shape and composition of the efficient frontier.

When short selling securities, typically the investor must not only leave the short sale proceeds with the brokerage firm, but must also meet initial margin requirements by depositing a portion of his or her own funds with the broker. Removal of the nonnegativity constraint on the weights allows an investor to sell securities short without having to meet these margin requirements. Furthermore the investor would get to use the short sale proceeds, in addition to his or her own funds, to purchase long positions in other securities.

In the Black model aside from the shape of the minimum variance boundary changing from a parabola to hyperbola ,this condition hold regardless of whether variance or standard deviation is used as the risk measure.

2.5.5. The Tobin Model:

The Tobin model differs from both the Black model and the Markowitz model as it removes the assumption that all securities had positive variance. The Tobin model allows the existence of a security that has no risk. But the nonnegativity constraint of the Markowitz model is imposed on all securities except for this risk free asset. Short selling of the risk free rate is similar to borrowing funds at a cost equal to the risk free asset's rate of return.

3. THE MARKOWITZ MODEL

Markowitz portfolio analysis requires that many equations be solved simultaneously, by using a mathematical procedure that minimizes the portfolio's risk at each level of aggregate return. Markowitz diversification applied on a large scale to hundreds of assets is called Markowitz portfolio analysis and its results may be quite interesting. Since it considers both the risk and return of dozens, hundreds and thousands of different securities simultaneously (the number is limited only by the size of the computer used and the number of securities for which the portfolio analyst has risk and return statistics.), in some cases it may be a more powerful method of analyzing a portfolio than using some executive' s brain power or selecting investment with a committee.

A diversified portfolio derived by Markowitz portfolio analysis usually be diversified across industries. However, it will not usually contain a large number of different securities. Some of the securities will be added to the portfolio primarily to obtain risk reduction due to their low correlation with other assets in the portfolio. Thus, not all the assets in a dominant portfolio will have either high rates of return or low standard deviation's- some may low correlation instead.

For the application of the Markowitz theory, a quadratic programming is used. The quadratic program is developed based on the assumptions and structure of the Markowitz theory and has the following form:

$$Minimize\ \alpha^2_{port} = \sum_{i=1}^{n}\left[\sum_{j=1}^{n}\left(x_i x_j Cov_{i,j}\right)\right]$$

subject to

1. $$\sum_{i=1}^{n}\left[(x_i)E(R_i)\right] = E(R_{pi})$$

2. $$\sum_{i=1}^{n}(x_i) = 1$$

3. $$x_i, x_j \geq 0$$

where,

x_i, x_j = weights of securities,

$E(R_i)$ = Expected return of security i,

$E(R_{pi})$ = Expected return of portfolio i,

α_{port} = Standard deviation of the portfolio,

$Cov_{i,j}$ = Covariance between security i and j

The objective function of this quadratic program is to minimize the variance of the portfolio. As stated above the variance for the portfolio is the function of the weighted average of the individual variances, plus the weighted covariance's among all assets in the portfolio. But it can be shown that, as the number of assets in the portfolio increases, the portfolio variance decreases and approaches the average covariance. This means that as portfolios that have large number of assets are formed the covariance term becomes relatively more important.

4. DATA

For the calculations and for the construction of the efficient frontier daily closing price of all the stocks that are available during the period of 1/11/93 and 28/10/94 are used. The closing prices are taken from the Istanbul Securities Exchange publications. The stocks that are used in the study are the ones from Istanbul Stock Exchange market. For the available 254 days of the selected period stock split and capital increase data are obtained from the Capital Markets Board bulletins.

In the calculation of the rates of return, stock prices are adjusted for stock splits stock dividends or capital increases.

$$r_i = \frac{\left(P_{i+1} - \left(P_{ssr} / m\right)\right)}{\left(P_{ssr} / m\right)}$$

r_i = Rate of return on the ith period

P_i = Price of old quotation on the ith period

P_{ssr} = Price of the stock split right on quotation

m = number of shares to be received as the result of the stock split

During the period of 1/11/93 and 28/10/94 there are 151 available stocks in Istanbul Securities Exchange Market that can be used for the construction of the efficient frontier. But because of the structure of the objective function of the quadratic efficient frontier. But because of the structure of the objective function of the quadratic programming; that is each stock covariance is multiplied by all the others; the objective function gets an enormous size which makes it very time consuming to be solved. For that reason a computer algorithm that is shown below is used. The algorithm, in every application eliminates the stocks that is worse than other which means:' having higher risk but less return; or having equal risk but less return; or having equal return but higher risk'. Besides that algorithm the stocks that have negative correlation with the other stocks are also be included which are Akbank and Aksa in this study. (See appendix B)

After these procedures we have 37 stocks available in hand to apply to the quadratic program to find the weights of each stock in each portfolio for the construction of the efficient frontier.

Daily returns of each stock are found and used in order to find the covariance of stocks with each other. Covariance's are used as the inputs for the objective function of the quadratic programming.

Average daily return of each stock is found in order to use as the portfolio returns.

Daily standard deviations of each stock are calculated and used as input for the quadratic programming in the constraint part.

For the calculations of the covariance daily returns and average returns Microsoft Excel version 5.0 is used. For the solution of the quadratic programming problems Microsoft Solver is used.

For the construction of the efficient frontier 15 portfolios are constructed having daily returns in the range of %0.1852 to %0.694471 are formed. The slices of average historical rate of yearly return is selected as 30 %, meaning that the historical yearly rate of return will increase by 30 % for each portfolio starting from 60 % (portfolio #1) up to 480% of yearly return (portfolio # 15). The corresponding historical daily return of each portfolio is also calculated. The weight of each individual stock in each portfolio, the standard deviation of all the portfolios are found by the quadratic program and then the efficient frontier is formed.

5. FINDINGS

If the results that are presented below are examined it can be first seen that the efficient frontier is quadratic and convex. The standard deviation of the portfolios first decreases than increases, and this means that the portfolios from 1 to 8 have inferior performance as there are other feasible portfolios with the same standard deviations that have higher returns.(See Table 5.2, and Figure 5.1)

If the dominating stocks in the portfolios are examined, it is seen that Kent Gida, Banvit and TSKB have higher weights in all of the portfolios relative to the other stocks. For the dominating sector food industry must be considered as both Kent and Banvit belongs to that industry.(See Appendix A.)

Another point is that, as the average historical return increases among the portfolios, the number of the stocks that enters in the portfolios also increases in order to diversify risk. While the minimum number of stocks in a portfolio is two (portfolio #1) , the maximum number is fourteen (portfolio # 15).

Table 5.1

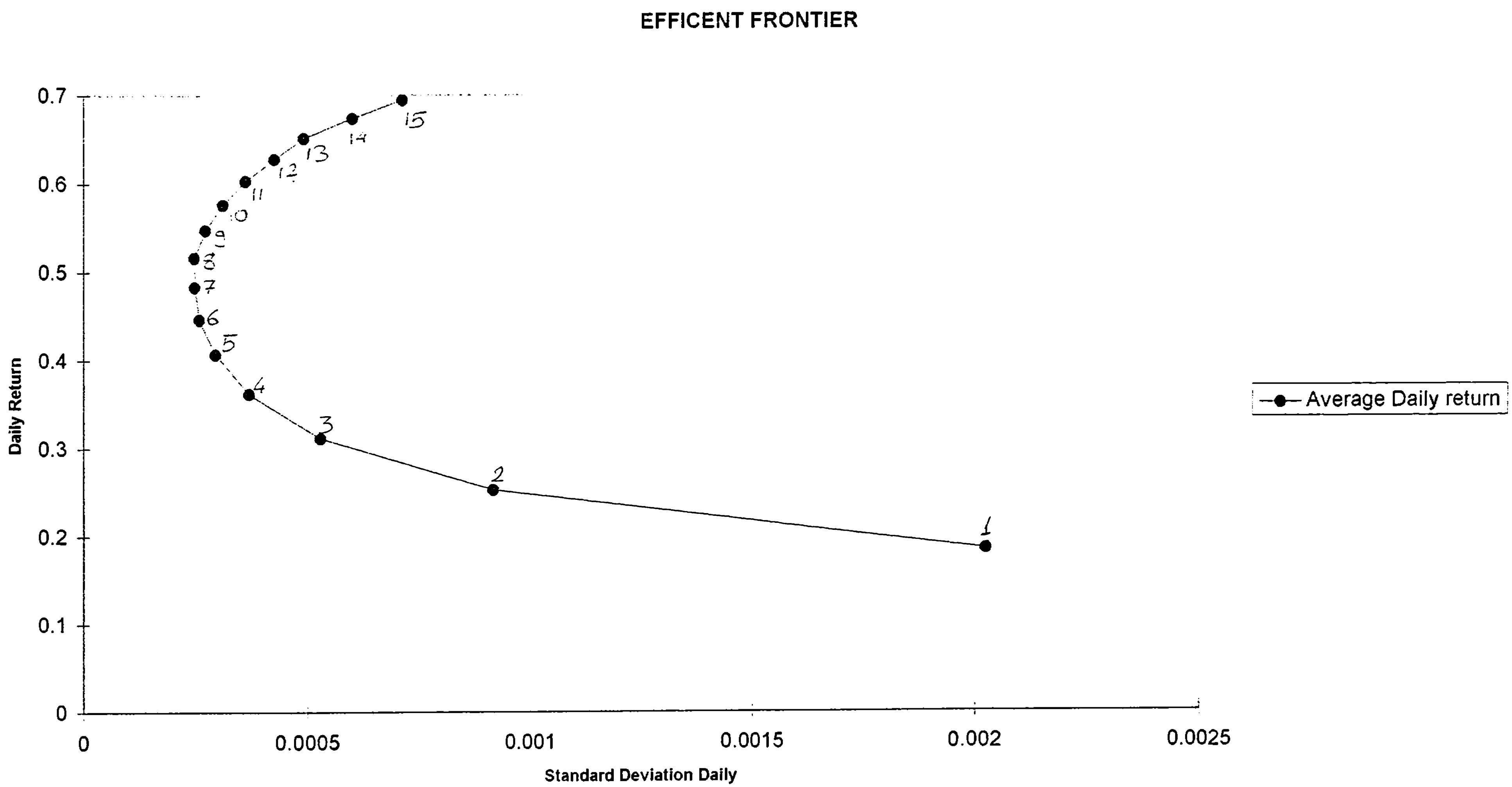
FINDINGS:
Yearly Return, Average daily return, And standard deviation of the individulal stocks in the portfolios:

	Yearly Return %	AV Daily Returns %	Standard deviation daily:
ADANA ÇIMENTO	398.491	0.634451	0.0582
KENT GIDA	377.9035	0.617742	0.029592
TAT KONSERV.	839.1003	0.885691	0.053348
UCAK	1932.725	1.192871	0.060002
BANVIT	220.6128	0.45974	0.038709
BOLU ÇIMENTO	516.8443	0.718889	0.054947
CIMSA	269.7722	0.516177	0.043112
CIMENTAŞ	348.6985	0.592766	0.04956
DENIZLI CAM	1561.638	1.112596	0.060804
EGE SERAMIK	519.3182	0.720476	0.055781
KARTONSAN	458.0606	0.679185	0.053479
LUKS KADIFE	1587.824	1.118821	0.062296
MIGROS	373.8373	0.614357	0.052563
PETKIM	1664.494	1.136508	0.071946
AKSA	273.7459	0.520407	0.049317
ANADOLU SIG.	1232.009	1.024612	0.127919
ERCIYES BIRA	360.3544	0.602923	0.054229
KORDSA	731.3882	0.837315	0.061171
SARKUSYAN	301.2714	0.548534	0.053458
SIEMENS	421.7552	0.652525	0.057801
TRAKYA CAM	194.8847	0.426661	0.041727
UNYE ÇIMENTO	267.8277	0.51409	0.049534
USAŞ	1085.966	0.978434	0.06179
AKBANK	184.7236	0.412798	0.051846
DURAN OFSET	1037.741	0.961932	0.07628
EGE BIRA	245.8197	0.489678	0.053567
GOOD YEAR	99.09086	0.271467	0.045394
GUNEY BIRA	325.9119	0.572127	0.058081
IZMIR DEMIR Ç.	326.953	0.573094	0.05979
KRUMA	187.0177	0.41597	0.05263
KUTAHYA PORS.	807.5909	0.872137	0.062535
PINAR UN	926.7496	0.921139	0.065719
RABAK	177.3996	0.402496	0.044635
THY	1031.733	0.959827	0.075326
TIRE KUTSAN	852.6104	0.891365	0.061907
TSKB	59.45436	0.183865	0.00456
YASAŞ	389.4706	0.627216	0.060016

Table 5.2.

FINDINGS:
Yearly Return, Average daily return, and Standard deviation of the portfolios:

Potrfolios	Average *Daily return %*	*Yearly return %*	Standard deviation daily:
1	0.1852	60	0.002022
2	0.253	90	0.000915
3	0.3108	120	0.000528
4	0.36139	150	0.000372
5	0.40618	180	0.000295
6	0.4462	210	0.000259
7	0.4829	240	0.000249
8	0.516	270	0.000248
9	0.547	300	0.000273
10	0.5759	330	0.000312
11	0.6026	360	0.000363
12	0.6276	390	0.000426
13	0.651	420	0.000491
14	0.67341	450	0.000599
15	0.694471	480	0.000709

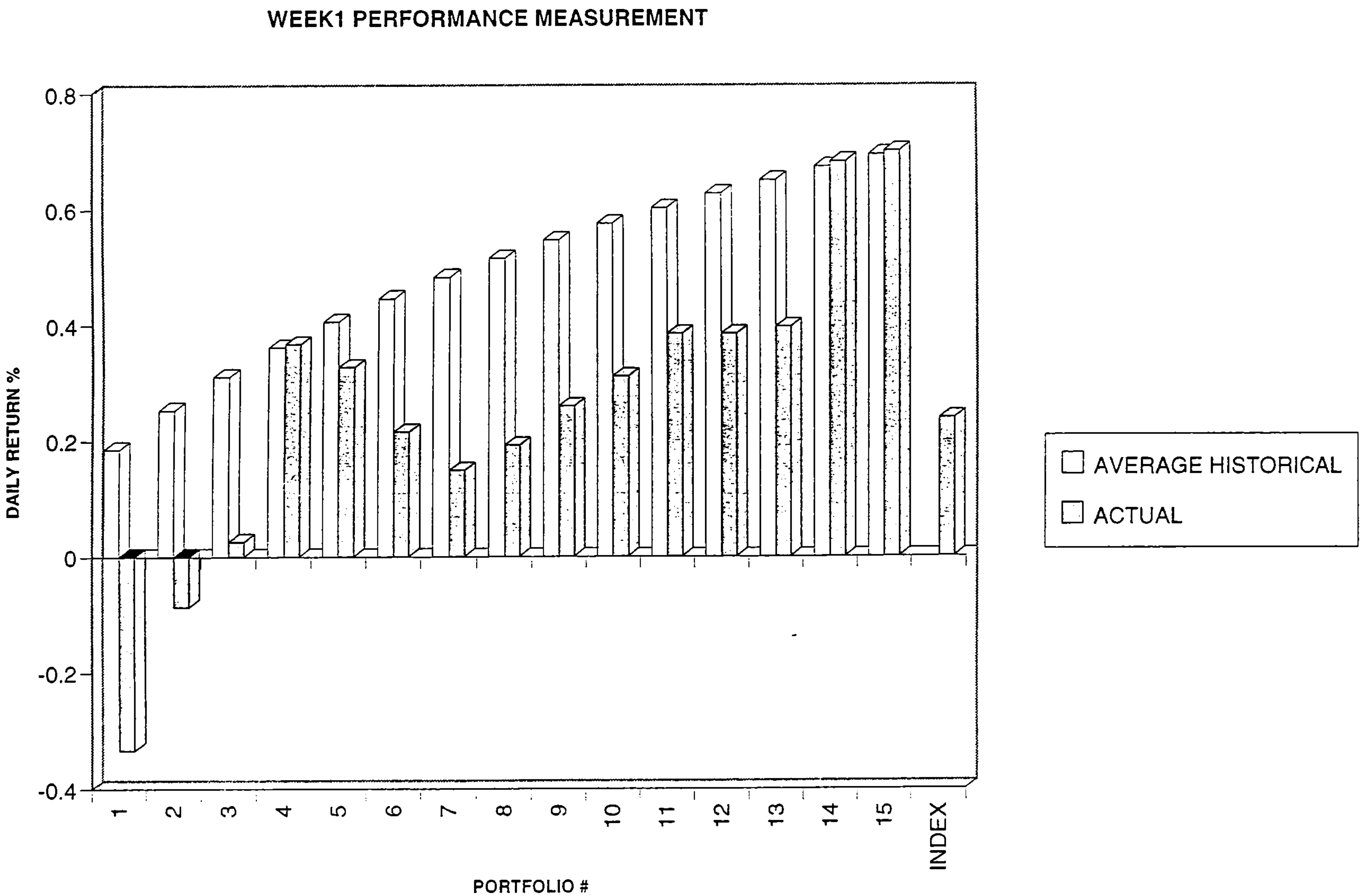

EFFICENT FRONTIER
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Daily Return
0
0.0005
0.001
0.0015
0.002
0.0025
Standard Deviation Daily
Average Daily return
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

6. PERFORMANCE MEASUREMENT

The performance measurement of the stocks are done by comparing the actual rate of return with the average historical rate of return. The comparison is done after one week, two weeks ,three weeks and one month the portfolios are constructed.

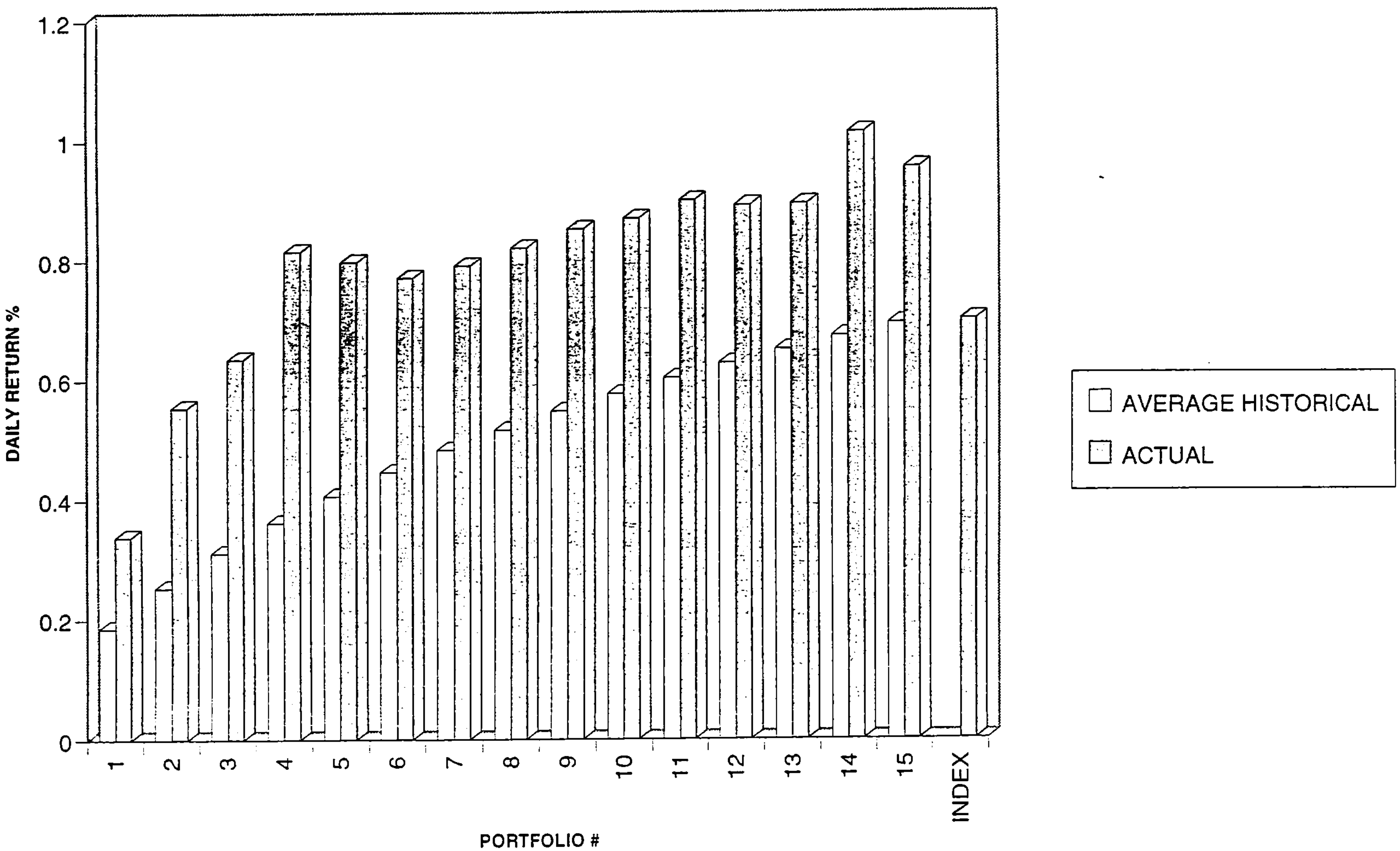
The results show that the portfolios showed the most success during the second week, but their performance begins to decline when we move to third and forth week. (See Figure 6.1, 6.2, 6.3, 6.4.). This means that the method should be used for the short term investments.

Another interesting point is that although portfolio number 4,5 and 6 have inferior performance on the efficient frontier their performance measurement gave better results than most of the other portfolios.

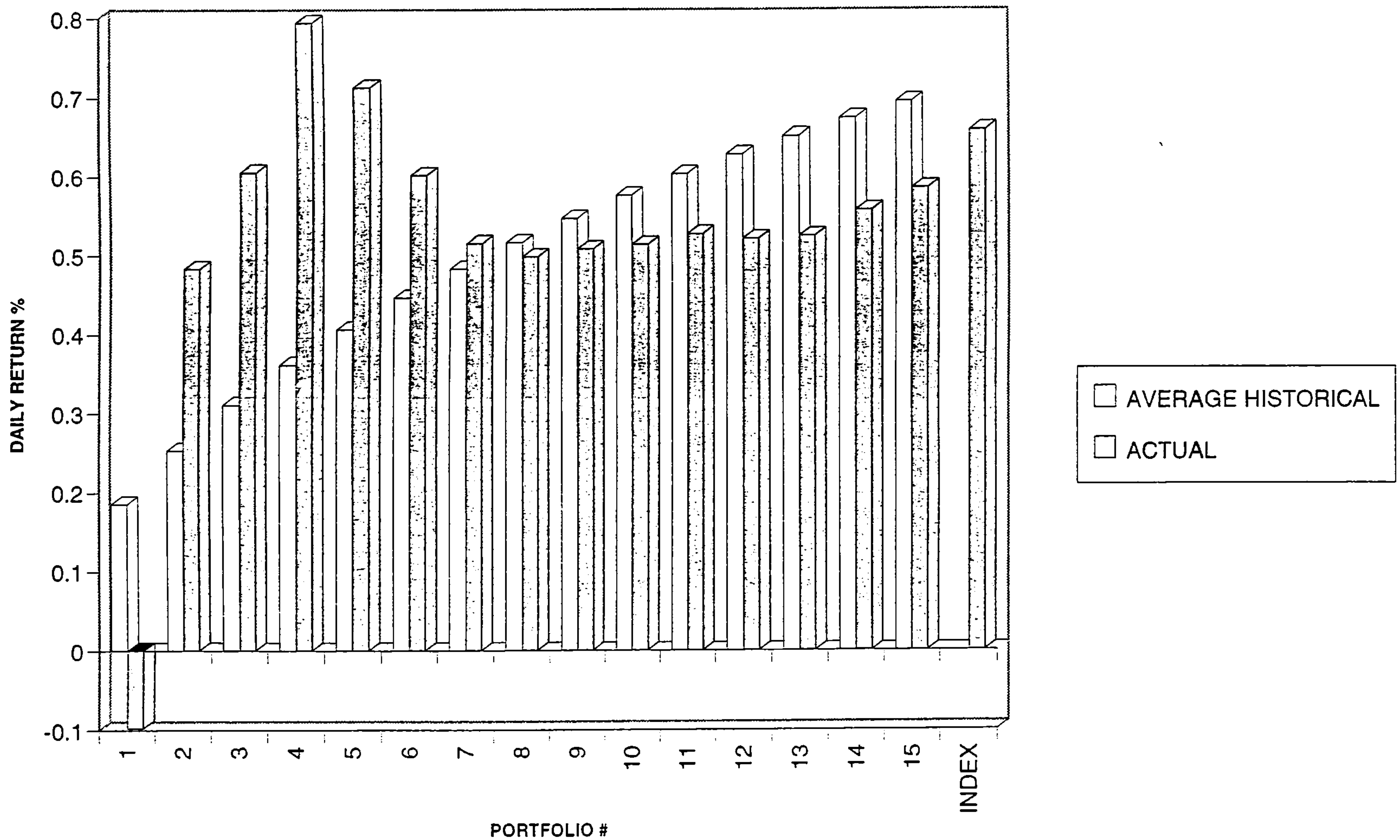


WEEK1 PERFORMANCE MEASUREMENT
DAILY RETURN %
0.8
0.6
0.4
0.2
0
-0.2
-0.4
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
INDEX
PORTFOLIO #
AVERAGE HISTORICAL
ACTUAL

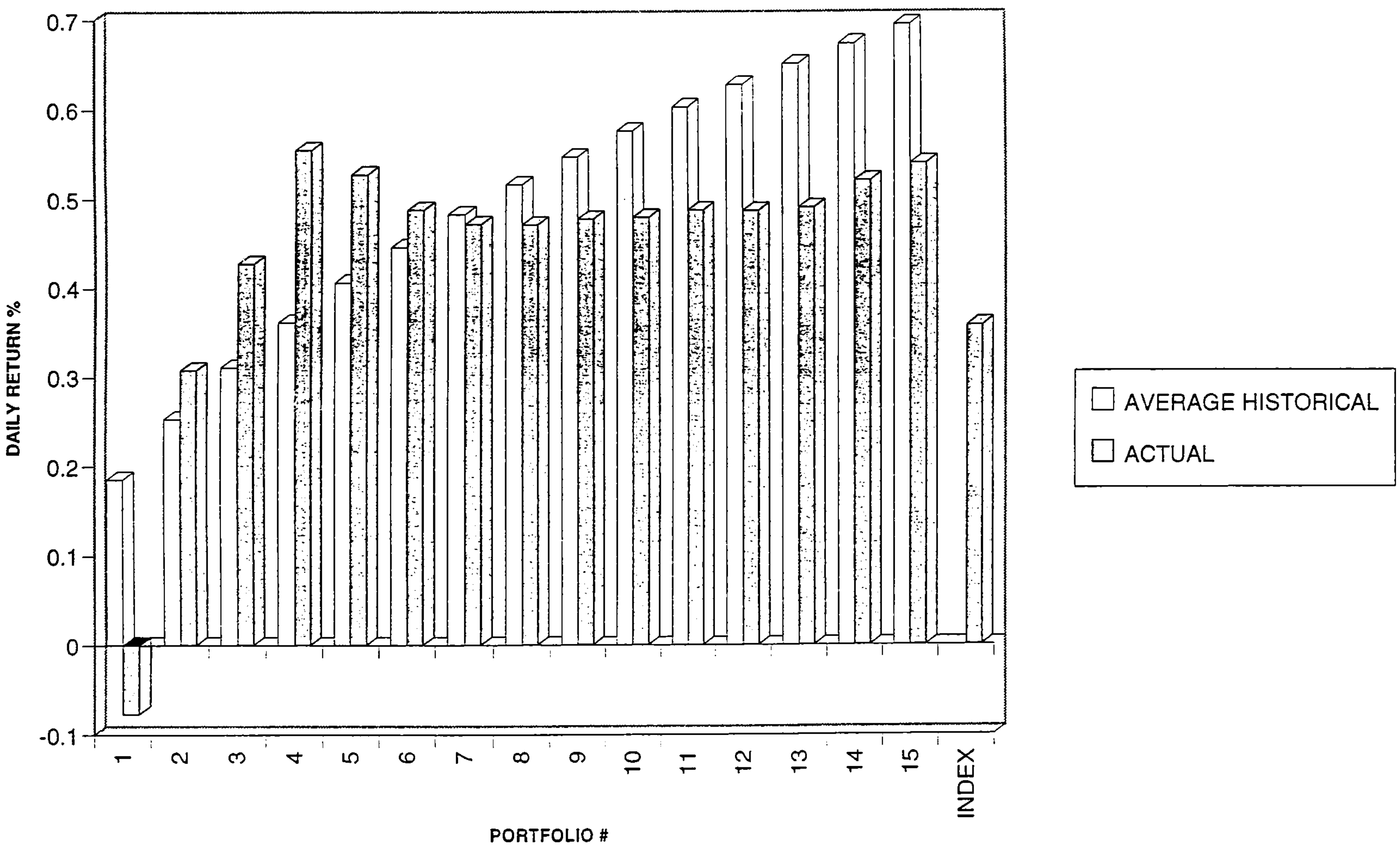
WEEK 2 PERFORMANCE MEASUREMENT



WEEK 3 PERFORMANCE MEASUREMENT



WEEK 4 PERFORMANCE MEASUREMENT



7. CONCLUSION

In this study it is tried to find the portfolios containing minimum expected deviation of return around the mean for a given return by using risk diversification. As seen from the performance measurement, that is comparing the actual rate of returns with the historical average rate of returns, it is not realistic to expect for the method to give a 100% success. First of all it should be kept in mind that this method has its limiting assumptions as stated above. Secondly the approach can be expected to give better results for stable economies, which is not the case in Turkey. Thirdly the speculative moves, and insider trading in Turkey are problems that can decrease the performance of the method.

Considering these points an investor should combine and revise a methodology that would also include the fundamental analysis and technical analysis as well. Also the investor should know that, the main objective of this method is to diversify risk not to maximize profits. If the method will be used the data should always be updated and better computer approaches must be searched for the time problem.

Potential investors should also keep in mind that, investing stocks is a risky investment and may cause in big losses although every approach is used.

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APPENDIX A

FINDINGS: Returns of Portfolios, Standard Deviation of Portfolios,
and Weights of stocks in each portfolio.

Portfolio Number:	**1**	**2**	**3**	**4**
Standard Deviation:	0.002023	0.00	0.000528	0.000372
Daily Return:	**%0.1852**	**%0.253**	**%0.3108**	**%0.36139**
Yearly return:	*%60*	*%90*	*%120*	*%150*
STOCKS				
ADANA	0.00	0.00	0.00	0.00
KENT	0.00	0.00	0.00	0.00
TATKO	0.00	0.00	0.00	0.00
UCAK	0.00	0.00	0.00	0.00
BANVT	0.00	0.01	0.16	0.23
BOLUC	0.00	0.00	0.00	0.00
CIMSA	0.00	0.00	0.00	0.00
CMENT	0.00	0.00	0.00	0.00
DENCM	0.00	0.00	0.00	0.00
EGSER	0.00	0.00	0.00	0.00
KARTN	0.00	0.00	0.00	0.00
LUKSK	0.00	0.00	0.00	0.00
MIGRS	0.00	0.00	0.00	0.00
PETKM	0.00	0.00	0.00	0.00
AKSA	0.00	0.00	0.00	0.00
ASIGO	0.00	0.00	0.00	0.00
ERCYS	0.00	0.00	0.00	0.00
KORDS	0.00	0.00	0.00	0.00
SARKY	0.00	0.00	0.00	0.00
SMENS	0.00	0.00	0.00	0.00
TRKCM	0.00	0.00	0.00	0.08
UNYEC	0.00	0.00	0.00	0.00
USAS	0.00	0.00	0.00	0.00
AKBNK	0.00	0.00	0.02	0.06
DUROF	0.00	0.00	0.00	0.00
EGBRA	0.00	0.00	0.00	0.00
GOODY	0.02	0.31	0.24	0.15
GUNEY	0.00	0.00	0.00	0.00
IZMDC	0.00	0.00	0.00	0.00
KRUMA	0.00	0.00	0.02	0.06
KUTPO	0.00	0.00	0.00	0.00
PNUN	0.00	0.00	0.00	0.00
RABAK	0.00	0.14	0.18	0.18
THYAD	0.00	0.00	0.00	0.00
TIRE	0.00	0.00	0.00	0.00
TSKB	0.98	0.54	0.38	0.24
YASAS	0.00	0.00	0.00	0.00

Portfolio Number:	**5**	**6**	**7**	**8**
Standard Deviation:	0.000295	0.00	0.000249	0.000248
Daily Return:	**%0.40618**	**%0.4460**	**%0.4829**	**%0.5160**
Yearly return:	***%180***	***%210***	***%240***	***%270***
STOCKS				
ADANA	0.00	0.00	0.00	0.00
KENT	0.08	0.17	0.24	0.26
TATKO	0.00	0.00	0.00	0.01
UCAK	0.00	0.00	0.00	0.00
BANVT	0.24	0.23	0.22	0.22
BOLUC	0.00	0.00	0.00	0.00
CIMSA	0.02	0.03	0.04	0.04
CMENT	0.00	0.00	0.01	0.01
DENCM	0.00	0.00	0.00	0.00
EGSER	0.00	0.00	0.00	0.00
KARTN	0.00	0.00	0.00	0.00
LUKSK	0.00	0.00	0.00	0.01
MIGRS	0.00	0.02	0.04	0.05
PETKM	0.00	0.00	0.00	0.00
AKSA	0.01	0.01	0.02	0.01
ASIGO	0.00	0.00	0.00	0.00
ERCYS	0.00	0.00	0.00	0.00
KORDS	0.00	0.00	0.00	0.00
SARKY	0.00	0.00	0.00	0.00
SMENS	0.00	0.00	0.00	0.00
TRKCM	0.08	0.06	0.04	0.04
UNYEC	0.00	0.00	0.00	0.00
USAS	0.00	0.00	0.00	0.00
AKBNK	0.06	0.05	0.04	0.04
DUROF	0.00	0.00	0.00	0.00
EGBRA	0.01	0.02	0.02	0.02
GOODY	0.10	0.06	0.02	0.01
GUNEY	0.00	0.00	0.01	0.02
IZMDC	0.00	0.00	0.00	0.00
KRUMA	0.06	0.05	0.05	0.04
KUTPO	0.00	0.00	0.00	0.00
PNUN	0.00	0.00	0.00	0.00
RABAK	0.17	0.15	0.14	0.14
THYAD	0.00	0.00	0.00	0.00
TIRE	0.00	0.00	0.00	0.00
TSKB	0.18	0.14	0.11	0.09
YASAS	0.00	0.00	0.00	0.00

Portfolio Number:	**9**	**10**	**11**	**12**
Standard Deviation:	0.000273	0.000312	0.000363	0.000426
Daily Return:	**%0.5470**	**%0.5759**	**%0.6026**	**%0.6276**
Yearly return:	***%300***	***%330***	***%360***	***%390***

STOCKS				
ADANA	0.00	0.00	0.00	0.00
KENT	0.27	0.27	0.27	0.27
TATKO	0.02	0.03	0.04	0.04
UCAK	0.00	0.00	0.00	0.01
BANVT	0.22	0.21	0.21	0.21
BOLUC	0.00	0.00	0.00	0.00
CIMSA	0.04	0.03	0.03	0.02
CMENT	0.01	0.00	0.00	0.00
DENCM	0.00	0.00	0.00	0.00
EGSER	0.00	0.00	0.00	0.00
KARTN	0.00	0.00	0.00	0.00
LUKSK	0.02	0.04	0.05	0.05
MIGRS	0.05	0.05	0.05	0.05
PETKM	0.00	0.00	0.00	0.00
AKSA	0.00	0.00	0.00	0.00
ASIGO	0.00	0.00	0.00	0.00
ERCYS	0.00	0.00	0.00	0.00
KORDS	0.00	0.00	0.00	0.00
SARKY	0.00	0.00	0.00	0.00
SMENS	0.00	0.00	0.00	0.00
TRKCM	0.03	0.03	0.03	0.02
UNYEC	0.00	0.00	0.00	0.00
USAS	0.00	0.00	0.00	0.00
AKBNK	0.03	0.03	0.03	0.03
DUROF	0.00	0.01	0.01	0.01
EGBRA	0.02	0.02	0.02	0.02
GOODY	0.00	0.00	0.00	0.00
GUNEY	0.02	0.02	0.02	0.02
IZMDC	0.00	0.00	0.00	0.00
KRUMA	0.04	0.04	0.04	0.04
KUTPO	0.00	0.00	0.00	0.00
PNUN	0.00	0.00	0.00	0.00
RABAK	0.13	0.13	0.13	0.13
THYAD	0.00	0.00	0.00	0.00
TIRE	0.00	0.00	0.00	0.00
TSKB	0.08	0.07	0.06	0.06
YASAS	0.00	0.00	0.00	0.00

Portfolio Number:	**13**	**14**	**15**
Standard Deviation:	0.000491	0.000599	0.000709
Daily Return:	**%0.651**	**%0.67341**	**%0.694471**
Yearly return:	**%420**	**%450**	**%480**
STOCKS			
ADANA	0.00	0.00	0.00
KENT	0.27	0.30	0.29
TATKO	0.04	0.07	0.07
UCAK	0.02	0.04	0.05
BANVT	0.21	0.20	0.20
BOLUC	0.00	0.00	0.00
CIMSA	0.02	0.00	0.00
CMENT	0.00	0.00	0.00
DENCM	0.00	0.02	0.02
EGSER	0.00	0.00	0.00
KARTN	0.00	0.00	0.00
LUKSK	0.06	0.09	0.10
MIGRS	0.05	0.06	0.06
PETKM	0.00	0.01	0.02
AKSA	0.00	0.00	0.00
ASIGO	0.00	0.01	0.01
ERCYS	0.00	0.00	0.00
KORDS	0.00	0.00	0.00
SARKY	0.00	0.00	0.00
SMENS	0.00	0.00	0.00
TRKCM	0.01	0.00	0.00
UNYEC	0.00	0.00	0.00
USAS	0.00	0.00	0.00
AKBNK	0.02	0.00	0.00
DUROF	0.02	0.03	0.03
EGBRA	0.02	0.00	0.00
GOODY	0.00	0.00	0.00
GUNEY	0.02	0.02	0.02
IZMDC	0.00	0.00	0.00
KRUMA	0.04	0.04	0.03
KUTPO	0.00	0.00	0.00
PNUN	0.00	0.00	0.00
RABAK	0.13	0.10	0.10
THYAD	0.00	0.00	0.00
TIRE	0.00	0.00	0.00
TSKB	0.05	0.00	0.00
YASAS	0.00	0.00	0.00

APPENDIX B

(THE COVARIANCE MATRIX)

APPENDIX B

	ADANA	*KENT*	*TATKO*	*UCAK*	*BANVT*	*BOLUC*	*CIMSA*	*CMENT*
ADANA	0.002779	0.000185	0.001375	0.001572	-1.68E-05	0.00193	0.000983	0.00175
KENT	0.000185	0.000872	0.000103	0.000202	-7.14E-05	0.000345	9.19E-05	0.000112
TATKO	0.001375	0.000103	0.002835	0.001439	-0.00012	0.001281	0.000792	0.001256
UCAK	0.001572	0.000202	0.001439	0.003589	-0.00015	0.001844	0.000811	0.001365
BANVT	-1.7E-05	-7.1E-05	-0.00012	-0.00015	0.001493	-0.00012	-0.00014	-2.26E-05
BOLUC	0.00193	0.000345	0.001281	0.001844	-0.00012	0.003007	0.000876	0.001425
CIMSA	0.000983	9.19E-05	0.000792	0.000811	-0.00014	0.000876	0.001851	0.00072
CMENT	0.00175	0.000112	0.001256	0.001365	-2.3E-05	0.001425	0.00072	0.002446
DENCM	0.001671	0.000242	0.001385	0.002154	-0.00014	0.001718	0.000773	0.001208
EGSER	0.001778	0.000303	0.001494	0.001628	-0.00018	0.00163	0.000718	0.001338
KARTN	0.001542	0.000185	0.001274	0.001535	-0.00012	0.00145	0.000822	0.001058
LUKSK	0.000527	0.000125	0.000269	0.000684	-8.5E-05	0.000668	0.000463	0.000347
MIGRS	0.000695	8.77E-05	0.000622	0.000864	-0.00032	0.000585	0.000679	0.000597
PETKM	0.001798	0.000354	0.001351	0.001519	-2.2E-05	0.001761	0.000792	0.001114
AKSA	-0.0012	-6.5E-05	-0.00107	-0.00118	0.000113	-0.00105	-0.00089	-0.00098
ASIGO	0.001503	0.000268	0.000791	0.001712	-0.00021	0.001711	0.000312	0.000872
ERCYS	0.001561	9.93E-05	0.001147	0.001462	-0.0001	0.001333	0.000994	0.001165
KORDS	0.00191	9.64E-05	0.001264	0.001802	-0.00018	0.001808	0.000959	0.001464
SARKY	0.001557	0.000317	0.001217	0.001583	-0.00031	0.00163	0.000774	0.001379
SMENS	0.00171	0.000219	0.001534	0.001627	-0.00023	0.001794	0.000927	0.001324
TRKCM	0.000936	0.000182	0.000816	0.001141	-0.00011	0.000884	0.000452	0.000689
UNYEC	0.001573	0.00016	0.000896	0.001346	-6.7E-05	0.001541	0.000762	0.001288
USAS	0.001701	0.000203	0.001907	0.001869	-0.00012	0.001842	0.001073	0.001474
AKBNK	-0.00051	-0.00018	-0.00068	-0.00084	8.93E-05	-0.00055	-0.00047	-0.00045
DUROF	0.00126	0.000156	0.001387	0.001287	-7.3E-05	0.000945	0.000714	0.001108
EGBRA	0.00139	-1.3E-05	0.000987	0.000899	-0.00012	0.001179	0.000801	0.000991
GOODY	0.000817	0.000228	0.000706	0.000661	-0.00016	0.000897	0.000588	0.000519
GUNEY	0.001129	-6.4E-06	0.000835	0.000953	-0.00014	0.001057	0.000794	0.000741
IZMDC	0.001999	0.00026	0.001683	0.001734	-0.00019	0.002008	0.000979	0.001411
KRUMA	-4.5E-05	0.000148	-0.00021	7.27E-05	2.39E-05	-2.4E-05	0.000218	3.75E-05
KUTPO	0.001966	0.000213	0.001678	0.00263	7.67E-06	0.002052	0.000841	0.001557
PNUN	0.002002	0.000489	0.00151	0.002142	-0.00016	0.00208	0.001001	0.001655
RABAK	3.21E-05	-3E-05	-0.00016	5.39E-05	-2.1E-05	0.000121	6.95E-05	-5.4E-05
THYAD	0.001923	0.000473	0.001706	0.001713	-6E-05	0.002095	0.000977	0.001344
TIRE	0.001949	0.000304	0.001559	0.002017	-3.6E-05	0.002038	0.001032	0.00147
TSKB	0.000679	3.35E-05	0.000528	0.000799	-0.00017	0.000625	0.000336	0.000444
YASAS	0.001897	0.000149	0.001482	0.002013	3.97E-06	0.002032	0.000943	0.001512

	DENCM	EGSER	KARTN	LUKSK	MIGRS	PETKM	AKSA	ASIGO
ADANA	0.001671	0.001778	0.001542	0.000527	0.000695	0.001798	0.001199	0.001503
KENT	0.000242	0.000303	0.000185	0.000125	8.77E-05	0.000354	6.47E-05	0.000268
TATKO	0.001385	0.001494	0.001274	0.000269	0.000622	0.001351	0.001069	0.000791
UCAK	0.002154	0.001628	0.001535	0.000684	0.000864	0.001519	0.001183	0.001712
BANVT	-0.00014	-0.00018	-0.00012	-8.51E-05	-0.00032	-2.24E-05	-0.00011	-0.00021
BOLUC	0.001718	0.00163	0.00145	0.000668	0.000585	0.001761	0.001052	0.001711
CIMSA	0.000773	0.000718	0.000822	0.000463	0.000679	0.000792	0.000889	0.000312
CMENT	0.001208	0.001338	0.001058	0.000347	0.000597	0.001114	0.000983	0.000872
DENCM	0.003683	0.001764	0.001629	0.000718	0.000755	0.001818	0.000974	0.001228
EGSER	0.001764	0.003099	0.001607	0.000461	0.000832	0.001602	0.00125	0.001253
KARTN	0.001629	0.001607	0.002849	0.000251	0.000959	0.001458	0.001304	0.001507
LUKSK	0.000718	0.000461	0.000251	0.003865	0.000312	0.000573	0.00048	0.000285
MIGRS	0.000755	0.000832	0.000959	0.000312	0.002648	0.000625	0.000804	0.000584
PETKM	0.001818	0.001602	0.001458	0.000573	0.000625	0.005156	0.001069	0.001678
AKSA	-0.00097	-0.00125	-0.0013	-0.00048	-0.0008	-0.00107	-0.00242	-0.00084
ASIGO	0.001228	0.001253	0.001507	0.000285	0.000584	0.001678	0.000838	0.016299
ERCYS	0.001514	0.001301	0.001606	0.000639	0.000784	0.001418	0.001343	0.001123
KORDS	0.001655	0.001808	0.001706	0.000402	0.001069	0.001609	0.001462	0.001552
SARKY	0.00153	0.001449	0.001268	0.000564	0.000519	0.001446	0.000874	0.001433
SMENS	0.001569	0.001785	0.001538	0.000667	0.000897	0.001652	0.001478	0.001418
TRKCM	0.000958	0.000895	0.00093	9.18E-05	0.000722	0.000982	0.000806	0.000655
UNYEC	0.001173	0.001203	0.000905	0.000288	0.000451	0.001254	0.000807	0.001139
USAS	0.001892	0.001784	0.001571	0.000619	0.000857	0.001658	0.001164	0.001519
AKBNK	-0.00057	-0.00063	-0.00074	-0.00039	-0.00034	-0.00061	-0.00052	-0.0013
DUROF	0.001433	0.000992	0.001009	8.66E-05	0.000572	0.001075	0.000981	0.000694
EGBRA	0.000992	0.001147	0.001062	0.000111	0.000798	0.001074	0.001191	0.000859
GOODY	0.00062	0.000814	0.000663	0.000432	0.000612	0.000948	0.000446	0.000386
GUNEY	0.001053	0.001366	0.001409	0.000421	0.001087	0.001195	0.001189	0.00113
IZMDC	0.001594	0.001777	0.001528	0.000656	0.000584	0.001848	0.001254	0.002032
KRUMA	-6.4E-05	-0.00011	6.52E-05	0.000378	-2.6E-05	0.000104	3.44E-05	-0.00013
KUTPO	0.002441	0.001899	0.001619	0.000827	0.000901	0.001459	0.000956	0.001681
PNUN	0.002358	0.002015	0.00163	0.000548	0.000995	0.002418	0.001214	0.002064
RABAK	1.46E-05	8.39E-05	5.55E-05	0.000133	-4E-05	-8.5E-05	1.13E-05	-1.9E-05
THYAD	0.002142	0.001648	0.001555	0.000723	0.00077	0.003941	0.001063	0.001506
TIRE	0.002199	0.001876	0.001714	0.000658	0.000784	0.002002	0.001016	0.001867
TSKB	0.000841	0.000629	0.000605	0.00055	0.000344	0.000938	0.000328	0.000896
YASAS	0.001915	0.001788	0.001603	0.000911	0.00082	0.001798	0.001321	0.001783

	ERCYS	KORDS	SARKY	SMENS	TRKCM	UNYEC	USAS	AKBNK
ADANA	0.001561	0.00191	0.001557	0.00171	0.000936	0.001573	0.001701	0.00051
KENT	9.93E-05	9.64E-05	0.000317	0.000219	0.000182	0.00016	0.000203	0.000177
TATKO	0.001147	0.001264	0.001217	0.001534	0.000816	0.000896	0.001907	0.000685
UCAK	0.001462	0.001802	0.001583	0.001627	0.001141	0.001346	0.001869	0.000839
BANVT	-0.0001	-0.00018	-0.00031	-0.00023	-0.00011	-6.65E-05	-0.00012	-8.93E-05
BOLUC	0.001333	0.001808	0.00163	0.001794	0.000884	0.001541	0.001842	0.000546
CIMSA	0.000994	0.000959	0.000774	0.000927	0.000452	0.000762	0.001073	0.000467
CMENT	0.001165	0.001464	0.001379	0.001324	0.000689	0.001288	0.001474	0.000449
DENCM	0.001514	0.001655	0.00153	0.001569	0.000958	0.001173	0.001892	0.000574
EGSER	0.001301	0.001808	0.001449	0.001785	0.000895	0.001203	0.001784	0.000627
KARTN	0.001606	0.001706	0.001268	0.001538	0.00093	0.000905	0.001571	0.000737
LUKSK	0.000639	0.000402	0.000564	0.000667	9.18E-05	0.000288	0.000619	0.000386
MIGRS	0.000784	0.001069	0.000519	0.000897	0.000722	0.000451	0.000857	0.00034
PETKM	0.001418	0.001609	0.001446	0.001652	0.000982	0.001254	0.001658	0.000609
AKSA	-0.00134	-0.00146	-0.00087	-0.00148	-0.00081	-0.00081	-0.00116	-0.00052
ASIGO	0.001123	0.001552	0.001433	0.001418	0.000655	0.001139	0.001519	0.001299
ERCYS	0.002929	0.001806	0.001293	0.001556	0.000801	0.001003	0.001528	0.000788
KORDS	0.001806	0.003727	0.001582	0.00198	0.001163	0.00116	0.001709	0.000883
SARKY	0.001293	0.001582	0.002846	0.001756	0.000813	0.001054	0.001741	0.000648
SMENS	0.001556	0.00198	0.001756	0.003328	0.000719	0.001185	0.001874	0.000913
TRKCM	0.000801	0.001163	0.000813	0.000719	0.001735	0.000883	0.000906	0.000444
UNYEC	0.001003	0.00116	0.001054	0.001185	0.000883	0.002444	0.001093	0.000594
USAS	0.001528	0.001709	0.001741	0.001874	0.000906	0.001093	0.003803	0.00087
AKBNK	-0.00079	-0.00088	-0.00065	-0.00091	-0.00044	-0.00059	-0.00087	-0.00268
DUROF	0.000982	0.001256	0.001172	0.001267	0.000707	0.000792	0.001344	0.000178
EGBRA	0.001475	0.001474	0.000798	0.001245	0.000753	0.00099	0.00106	0.000542
GOODY	0.000686	0.000957	0.00085	0.000865	0.000641	0.000846	0.000874	0.000532
GUNEY	0.001762	0.001632	0.001078	0.001315	0.000569	0.000726	0.001096	0.000859
IZMDC	0.001459	0.001853	0.001868	0.001825	0.001013	0.001472	0.0022	0.000978
KRUMA	0.000216	1.12E-06	9.4E-05	-2.3E-05	-3.4E-05	-2.3E-05	2.93E-05	0.000232
KUTPO	0.001191	0.001689	0.001615	0.001791	0.001142	0.001651	0.002131	0.000672
PNUN	0.001689	0.001996	0.001821	0.001874	0.001195	0.001553	0.002091	0.000932
RABAK	-0.00034	-0.00011	-5.2E-05	-7E-06	6.24E-05	9.83E-05	-9.3E-05	-0.00022
THYAD	0.001406	0.001738	0.00143	0.00193	0.001158	0.001437	0.001979	0.000597
TIRE	0.001471	0.002006	0.001794	0.002082	0.001019	0.001348	0.002263	0.000709
TSKB	0.000561	0.000774	0.0009	0.000906	0.000346	0.000507	0.000755	0.000438
YASAS	0.001594	0.001893	0.00192	0.00209	0.000887	0.001392	0.001986	0.000748

	DUROF	EGBRA	GOODY	GUNEY	IZMDC	KRUMA	KUTPO	PNUN
ADANA	0.00126	0.00139	0.000817	0.001129	0.001999	-4.55E-05	0.001966	0.002002
KENT	0.000156	-1.3E-05	0.000228	-6.36E-06	0.00026	0.000148	0.000213	0.000489
TATKO	0.001387	0.000987	0.000706	0.000835	0.001683	-0.00021	0.001678	0.00151
UCAK	0.001287	0.000899	0.000661	0.000953	0.001734	7.27E-05	0.00263	0.002142
BANVT	-7.30E-05	-0.00012	-0.00016	-0.00014	-0.00019	2.39E-05	7.67E-06	-0.00016
BOLUC	0.000945	0.001179	0.000897	0.001057	0.002008	-2.42E-05	0.002052	0.00208
CIMSA	0.000714	0.000801	0.000588	0.000794	0.000979	0.000218	0.000841	0.001001
CMENT	0.001108	0.000991	0.000519	0.000741	0.001411	3.75E-05	0.001557	0.001655
DENCM	0.001433	0.000992	0.00062	0.001053	0.001594	-6.4E-05	0.002441	0.002358
EGSER	0.000992	0.001147	0.000814	0.001366	0.001777	-0.00011	0.001899	0.002015
KARTN	0.001009	0.001062	0.000663	0.001409	0.001528	6.52E-05	0.001619	0.00163
LUKSK	8.66E-05	0.000111	0.000432	0.000421	0.000656	0.000378	0.000827	0.000548
MIGRS	0.000572	0.000798	0.000612	0.001087	0.000584	-2.61E-05	0.000901	0.000995
PETKM	0.001075	0.001074	0.000948	0.001195	0.001848	0.000104	0.001459	0.002418
AKSA	-0.00098	-0.00119	-0.00045	-0.00119	-0.00125	-3.4E-05	-0.00096	-0.00121
ASIGO	0.000694	0.000859	0.000386	0.00113	0.002032	-0.00013	0.001681	0.002064
ERCYS	0.000982	0.001475	0.000686	0.001762	0.001459	0.000216	0.001191	0.001689
KORDS	0.001256	0.001474	0.000957	0.001632	0.001853	1.12E-06	0.001689	0.001996
SARKY	0.001172	0.000798	0.00085	0.001078	0.001868	9.4E-05	0.001615	0.001821
SMENS	0.001267	0.001245	0.000865	0.001315	0.001825	-2.34E-05	0.001791	0.001874
TRKCM	0.000707	0.000753	0.000641	0.000569	0.001013	-3.41E-05	0.001142	0.001195
UNYEC	0.000792	0.00099	0.000846	0.000726	0.001472	-2.33E-05	0.001651	0.001553
USAS	0.001344	0.00106	0.000874	0.001096	0.0022	2.93E-05	0.002131	0.002091
AKBNK	-0.00018	-0.00054	-0.00053	-0.00086	-0.00098	-0.00023	-0.00067	-0.00093
DUROF	0.005796	0.000627	0.000796	0.00024	0.001352	-4.81E-05	0.001275	0.001263
EGBRA	0.000627	0.002858	0.000645	0.001472	0.001239	-6.18E-05	0.000998	0.00123
GOODY	0.000796	0.000645	0.002052	0.000848	0.001039	0.000221	0.000871	0.000987
GUNEY	0.00024	0.001472	0.000848	0.00336	0.001213	-9.03E-05	0.000987	0.00122
IZMDC	0.001352	0.001239	0.001039	0.001213	0.003561	0.000102	0.001942	0.002138
KRUMA	-4.8E-05	-6.2E-05	0.000221	-9E-05	0.000102	0.002731	-2.46E-05	-2.30E-06
KUTPO	0.001275	0.000998	0.000871	0.000987	0.001942	-2.5E-05	0.003895	0.002327
PNUN	0.001263	0.00123	0.000987	0.00122	0.002138	-2.3E-06	0.002327	0.004302
RABAK	4.18E-05	4.55E-05	6.56E-05	-0.00023	-0.00021	0.000311	0.000176	0.000181
THYAD	0.001306	0.001526	0.000836	0.001037	0.00187	-0.00015	0.001844	0.002474
TIRE	0.00145	0.001062	0.001104	0.001194	0.002	1.1E-06	0.00249	0.002505
TSKB	0.000672	0.000365	0.000533	0.000489	0.000831	0.000298	0.000677	0.000717
YASAS	0.001464	0.001181	0.001025	0.001402	0.002198	-4E-06	0.002369	0.002361

	RABAK	THYAD	TIRE	TSKB	YASAS
ADANA	3.21E-05	0.001923	0.001949	0.000679	0.001897
KENT	-3.05E-05	0.000473	0.000304	3.35E-05	0.000149
TATKO	-0.00016	0.001706	0.001559	0.000528	0.001482
UCAK	5.39E-05	0.001713	0.002017	0.000799	0.002013
BANVT	-2.05E-05	-6.01E-05	-3.59E-05	-0.00017	3.97E-06
BOLUC	0.000121	0.002095	0.002038	0.000625	0.002032
CIMSA	6.95E-05	0.000977	0.001032	0.000336	0.000943
CMENT	-5.42E-05	0.001344	0.00147	0.000444	0.001512
DENCM	1.46E-05	0.002142	0.002199	0.000841	0.001915
EGSER	8.39E-05	0.001648	0.001876	0.000629	0.001788
KARTN	5.55E-05	0.001555	0.001714	0.000605	0.001603
LUKSK	0.000133	0.000723	0.000658	0.00055	0.000911
MIGRS	-3.95E-05	0.00077	0.000784	0.000344	0.00082
PETKM	-8.5E-05	0.003941	0.002002	0.000938	0.001798
AKSA	-1.1E-05	-0.00106	-0.00102	-0.00033	-0.00132
ASIGO	-1.89E-05	0.001506	0.001867	0.000896	0.001783
ERCYS	-0.00034	0.001406	0.001471	0.000561	0.001594
KORDS	-0.00011	0.001738	0.002006	0.000774	0.001893
SARKY	-5.21E-05	0.00143	0.001794	0.0009	0.00192
SMENS	-7.01E-06	0.00193	0.002082	0.000906	0.00209
TRKCM	6.24E-05	0.001158	0.001019	0.000346	0.000887
UNYEC	9.83E-05	0.001437	0.001348	0.000507	0.001392
USAS	-9.33E-05	0.001979	0.002263	0.000755	0.001986
AKBNK	0.00022	-0.0006	-0.00071	-0.00044	-0.00075
DUROF	4.18E-05	0.001306	0.00145	0.000672	0.001464
EGBRA	4.55E-05	0.001526	0.001062	0.000365	0.001181
GOODY	6.56E-05	0.000836	0.001104	0.000533	0.001025
GUNEY	-0.00023	0.001037	0.001194	0.000489	0.001402
IZMDC	-0.00021	0.00187	0.002	0.000831	0.002198
KRUMA	0.000311	-0.00015	1.10E-06	0.000298	-4.03E-06
KUTPO	0.000176	0.001844	0.00249	0.000677	0.002369
PNUN	0.000181	0.002474	0.002505	0.000717	0.002361
RABAK	0.001984	-2.18E-06	0.00018	-0.00011	8.20E-05
THYAD	-2.2E-06	0.005652	0.002132	0.001094	0.001908
TIRE	0.00018	0.002132	0.003817	0.000728	0.002339
TSKB	-0.00011	0.001094	0.000728	0.002071	0.000741
YASAS	8.2E-05	0.001908	0.002339	0.000741	0.003588