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Large Tullock contests

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Abstract
We characterize the equilibrium effort function of a large Tullock contest game with heterogeneous agents under mild conditions on the contest success function and effort cost function. Later, writing the equilibrium total effort explicitly under a uniform type distribution, we identify the effort-maximizing large Tullock contest. It is shown that the contest designer needs to increase the curvature of the effective effort function, thereby encouraging high-type agents to exert even higher efforts, as the curvature of the effort cost function increases or the support of the type distribution gets narrower.

Keywords Large game · Tullock contests · Asymmetric contest · Contest design · Effort-maximizing contests

JEL Classification C72 · D47 · D74

1 Introduction

There are a large number of contestants in various competitive settings such as college admissions, online innovation contests, intra-firm opinion challenges, and e-sports competitions. To study such strategic environments, there has been an increased interest in the formal analysis of games with a large number of players in recent years (see a recent review by Gradwohl and Kalai (2021) and references therein). This paper contributes to that strand of literature by studying a large Tullock contest game with heterogeneous agents.

A contest game is used to award or allocate a valuable prize to some agents who exert costly and irreversible efforts to increase their winning probabilities or prize

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shares. The applications include election, job search, litigation, lobbying, R&D, sports competition, and warfare. It is said that a contest success function (CSF) determines each agent's winning probability or prize share as a function of all agents' efforts. Defined as the ratio of an agent's effective effort to the aggregate effective effort, a Tullock CSF is the most commonly used function in the literature (see Tullock 1980).[1] The interested reader is referred to Corchón (2007) and Konrad (2009) for an extensive overview of the literature.

In our paper, there is a continuum of different types of agents who (i) compete in a one-shot contest game and (ii) are differentiated by their utilities received from the prize they earn. An agent's type is his private information, whereas the type distribution is common knowledge. The overall prize is distributed according to a Tullock CSF. First, we implicitly characterize the equilibrium for a general type distribution and without specifying the effective effort and effort cost functions. Second, under specific assumptions on those functions, we provide an explicit characterization of equilibrium efforts for a general type distribution. Observing in the explicit characterization that the equilibrium efforts do not respond monotonically to changes in the curvatures of the assumed functions inspires us to conduct an effort-maximizing contest design analysis. Along these lines, third, we identify the optimal contest that maximizes the equilibrium total effort within the family of Tullock CSFs under uniform distribution.

Our analyses show that the optimal contest (i) depends on the curvature of the effort cost function and (ii) does not depend on the absolute values of the lower and upper bounds of the type distribution, but depends on the ratio of those bounds. More precisely, we report that the contest designer needs to increase the curvature of the effective effort function with an aim to encourage high-type agents to exert even higher efforts when the curvature of the effort cost function increases (which makes high efforts more costly) or when the lowest-type to highest-type ratio increases (which decreases the level of heterogeneity among contestants). These insights will be particularly useful for future research focusing on the design of optimal contests with a large number of contestants.

Olszewski and Siegel (2016, 2020) are closely related to our paper. These authors studied large all-pay contests with heterogeneous agents. Olszewski and Siegel (2016) showed that the equilibrium outcomes of such large contests can be approximated by the outcomes of mechanisms that implement the assortative allocation in a framework with a single agent who has a continuum of possible types. Later, building on their earlier work, Olszewski and Siegel (2020) characterized the prize structures that maximize agents' aggregate performance in equilibrium in large all-pay contests. Their main result is that the optimal contest awards many different prizes, with gradually decreasing values, in the case when agents have concave prize valuations and convex effort costs. It is worth noting that although our analysis of effort-maximizing contests aims to answer a similar question, our results are not comparable to those of Olszewski and Siegel (2016). This is because they analyzed the optimal prize distribution for a specific CSF (e.g., all-pay auction), whereas here we analyze the optimal CSF (within

[1] See Skaperdas (1996) for an axiomatic foundation of Tullock contests.

the Tullock-family) for a given overall prize, while what portion of that prize will be awarded to which ranking is determined in the equilibrium as a function of exerted efforts.

In two recent working papers, Lahkar and Mukherjee (2022) and Lahkar and Sultana (2022) utilized a similar idea of analyzing equilibrium behavior in a large contest game with a Tullock CSF. Differently from our analysis, the authors assumed a finite number of agent types who differ in effort cost parameters and also considered type-specific bias parameters in their CSF. Lahkar and Mukherjee (2022) further analyzed the optimal bias parameters that need to be implemented to maximize the equilibrium total effort, whereas our focus is on the optimization of the curvature of the effective effort function.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the equilibrium and provides the characterization results. Section 4 includes the effort-maximizing contest design analysis. Section 5 concludes.

2 The model

Consider a continuum of agents competing in a large contest game. Each agent is of measure zero. Each agent is characterized by his privately-known type t. Let T be the random variable for the type distribution with a cumulative distribution function $F : \Omega \to [0, 1]$ where $\Omega \subset \mathbb{R}_+$ denotes the type space. We assume that Ω is bounded. Each agent chooses an effort level $x \in \mathbb{R}_+$ in the contest, such that the collection of these efforts yields the effort profile for all types of agents, denoted by $X : \Omega \to \mathbb{R}_+$. It is assumed that X is Lebesgue measurable and integrable.

Let $g : \mathbb{R}_+ \to \mathbb{R}_+$ denote the effective effort function, such that when an agent exerts an effort of x, his success in the large contest is proportional to $g(x)$. It is assumed that g is a twice-differentiable and strictly increasing function with $g(0) = 0$. For any given effort profile X, let

$$A(X) = \mathbb{E}\left[g(X(T))\right] = \int_{\Omega} g(X(t))dF(t)$$

be defined as the aggregate effective effort. Considering an overall prize of $V > 0$, an agent who exerts an effort of x is awarded a prize of

$$\frac{g(x)}{A(X)}V.$$

Note that $\frac{g(x)}{A(X)}$ is of the Tullock CSF form (see Tullock 1980), since it is the ratio of the effective effort to the aggregate effective effort.

The utility of an agent of type t from choosing an effort of x given the effort profile X can be written as

$$U(t, x, X) = t\frac{g(x)}{A(X)}V - c(x)$$

where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is a twice-differentiable, strictly increasing, and convex effort cost function. This utility function is similar to the quasi-linear form used by Olszewski and Siegel (2016, 2020), with the only difference being the CSF employed.

We denote this large Tullock contest game by Γ.

Assuming that $H(x) \equiv \frac{c'(x)}{g'(x)}$ is well-defined for any $x \in \mathbb{R}_+$, the following assumption regulates the strategic framework in our model.

Assumption 1 For any $x \in \mathbb{R}_+$, if $g''(x) \geq 0$, then

$$H(x) < \frac{c''(x)}{g''(x)}.$$

This assumption implies that if effective effort function is convex, it should be 'less convex' than effort cost function at any effort level.[2] It is worth noting that (Acemoglu and Jensen 2013) argued that a similar assumption implies the *uniform local solvability condition* in a rent-seeking contest, which provides monotone comparative static results in aggregative games. As the authors further noted, similar curvature conditions also appeared in the industrial organization literature, e.g., in the analysis of price discrimination (see Schmalensee 1981). In our context, Assumption 1 is equivalent to assuming that marginal cost of marginal effective effort is a strictly increasing function, which we later utilize to verify the existence and uniqueness of a best response for some agent types.

3 The equilibrium analysis

In this section, we first implicitly characterize the equilibrium of our large Tullock contest game with heterogeneous agents for a general type distribution and without specifying the effective effort and effort cost functions (see Proposition 1). Second, under specific assumptions on those functions, we provide an explicit characterization of equilibrium efforts for a general type distribution (see Proposition 2).

Proposition 1 *Under Assumption* 1, *there exits a unique equilibrium effort profile in* Γ. *The equilibrium effort profile can be written as*

$$X^*(t) = \begin{cases} H^{-1}\left(\frac{tV}{A(X^*)}\right) & \text{if } t \geq \frac{H(0)A(X^*)}{V} \\ 0 & \text{if otherwise} \end{cases}$$

where $A(X^*)$ *is implicitly characterized by*

$$A(X^*) = \int_{\Omega} g(X^*(t))dF(t).$$

[2] Note that if effective effort function is strictly concave, the same implication trivally follows.

Proof An agent of type t maximizes $U(t, x, X)$ by choosing his own effort level x as a response to a given effort profile X. The respective first-order condition can be written as

$$\frac{\partial}{\partial x}U(t,x,X) = t\frac{g'(x)}{A(X)}V - c'(x) = 0,$$

which is

$$\frac{tV}{A(X)} - \frac{c'(x)}{g'(x)} = 0 \tag{1}$$

A necessary condition for an *interior* best response of a type t agent is to solve this equation. Under Assumption 1, we find that $H(x)$ is a strictly increasing function, since

$$\frac{\partial H(x)}{\partial x} = \frac{\partial}{\partial x}\frac{c'(x)}{g'(x)} = \frac{c''(x)g'(x) - c'(x)g''(x)}{(g'(x))^2} > 0.$$

Given that each agent is of measure zero, the choice of x does not influence the aggregate effective effort, so that $\frac{tV}{A(X)}$ is independent of x. As such, it follows that there can exist at most one solution to equation (1) for any given effort profile X. Denoting that solution by x^*, we verify that it is indeed a best response, maximizing the agent's utility, since the left-hand-side of equation (1) is positive for any $x < x^*$ (i.e., increasing utility) and negative for any $x > x^*$ (i.e., decreasing utility). Then, we obtain $x^* = H^{-1}\left(\frac{tV}{A(X)}\right)$, also noting that H^{-1} is well-defined, since H is a strictly increasing function.

It must be noted, however, that if $\frac{tV}{A(X)}$ is not in H's image, there may not exist a solution to the first-order condition above. This may occur for low-type agents when the given effort profile X leads to a sufficiently high aggregate effective effort such that $\frac{tV}{A(X)} < H(0)$. This implies that $\frac{\partial}{\partial x}U(t,x,X)$ is always negative for such agents, further implying that their best response is to exert zero effort. It may also occur for high-type agents when the aggregate effective effort is sufficiently low such that $\frac{tV}{A(X)} \geq \lim_{x\to\infty} H(x)$. This implies that $\frac{\partial}{\partial x}U(t,x,X)$ is always positive for such agents, further implying that they always prefer exerting a higher effort level, so that their best response does not exist. Accordingly, the best response of a type t agent to any given effort profile X can be written as 0 if $t < \frac{H(0)A(X)}{V}$, as $\emptyset$ if $t \geq \frac{A(X)\lim_{x\to\infty} H(x)}{V}$, and as $H^{-1}\left(\frac{tV}{A(X)}\right)$ if otherwise.

It is known that if X^* is the equilibrium effort profile, it must be that $X^*(t)$ is equal to the best response of a type t agent to X^*. Thus, given our best response characterization above, we can write

$$A(X^*) = \int_{\Omega} g(X^*(t))dF(t) = \int_{\Omega^+} g\left(H^{-1}\left(\frac{tV}{A(X^*)}\right)\right)dF(t) \tag{2}$$

where Ω^+ is the set of all agent types with $t \geq \frac{H(0)A(X^*)}{V}$. Note that the right-hand-side of equation (2) is well-defined and finite because (i) Ω is bounded and (ii) the case of no best response cannot be a part of X^*, since $t < \frac{A(X^*)\lim_{x\to\infty} H(x)}{V}$ for any $t \in \Omega$ as $A(X^*) \to \infty$.[3]

The fact that H is an increasing function implies that H^{-1} and $g \circ H^{-1}$ are also increasing functions. Then, given that $\frac{tV}{A(X^*)}$ decreases in $A(X^*)$, we find that the right-hand-side of Eq. (2) is decreasing in $A(X^*)$ as well. Since it is trivial that the left-hand-side of the same equation is strictly increasing in $A(X^*)$, with values between 0 and ∞, there is a unique $A(X^*)$ satisfying that equation. For that aggregate effective effort value, we find that there exists a unique effort profile X^* such that $X^*(t) = 0$ for low-type agents with $t \notin \Omega^+$ and $X^*(t) = H^{-1}\left(\frac{tV}{A(X^*)}\right)$ for the remaining agents with $t \in \Omega^+$. This completes the proof. □

The equilibrium effort profile X^* suggests that low-type agents can be fully discouraged (i.e., exerting zero effort) in an equilibrium if the aggregate effective effort $A(X^*)$ turns out to be too high due to the efforts exerted by high-type agents.

Thus far, we implicitly characterized the equilibrium effort profile for any type distribution and any form of g and c functions under the aforementioned assumptions. It can be seen that the equilibrium effort profile X^* is an increasing function of type t and the overall prize V, since H and H^{-1} are known to be increasing functions. Now, to have a clearer understanding of equilibrium efforts, we turn to the analysis of equilibrium under specific functions while preserving the generality of the type distribution.

The next proposition explicitly characterizes the equilibrium efforts for a model in which $g(x) = x^\gamma$ and $c(x) = x^\theta$ where (i) $\theta \geq 1$ and (ii) $0 < \gamma < \theta$. The assumption (i) implies that the effort cost function is convex, whereas the assumption (ii) implies our Assumption 1.

Proposition 2 *Given $g(x) = x^\gamma$ and $c(x) = x^\theta$ where $\theta \geq 1$ and $0 < \gamma < \theta$, the unique equilibrium effort profile in Γ can be written as*

$$X^*(t) = t^{\frac{1}{\theta-\gamma}} \left(\frac{\gamma V}{\theta \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]} \right)^{\frac{1}{\theta}},$$

which yields the aggregate effective effort

$$A(X^*) = \left(\frac{\gamma V}{\theta} \right)^{\frac{\gamma}{\theta}} \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]^{\frac{\theta-\gamma}{\theta}}.$$

Proof Given the power-form functions, we have

[3] If $\lim_{x\to\infty} H(x) \to \infty$, any agent of type t has a finite best response to any effort profile X. However, as shown in the current proof, such an assumption is not needed for the existence of an equilibrium effort profile.

$$H(x) = \frac{\theta}{\gamma} x^{\theta-\gamma}$$

so that

$$H^{-1}(x) = \left(\frac{x\gamma}{\theta}\right)^{\frac{1}{\theta-\gamma}}.$$

Note that since $H(0) = 0$, the case of zero best response does not emerge under the assumed functions. That is, any agent of type $t > 0$ exerts a positive effort in equilibrium.

Then, $A(X^*)$ can be implicitly written as

$$\begin{aligned} A(X^*) = \int_\Omega g\left(H^{-1}\left(\frac{tV}{A(X^*)}\right)\right) dF(t) &= \int_\Omega g\left(\left(\frac{\gamma}{\theta}\frac{tV}{A(X^*)}\right)^{\frac{1}{\theta-\gamma}}\right) dF(t) \\ &= \int_\Omega \left(\frac{\gamma}{\theta}\frac{tV}{A(X^*)}\right)^{\frac{\gamma}{\theta-\gamma}} dF(t) \\ &= \left(\frac{\gamma}{\theta}\frac{V}{A(X^*)}\right)^{\frac{\gamma}{\theta-\gamma}} \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]. \end{aligned}$$

From here we find

$$A(X^*) = \left(\frac{\gamma V}{\theta}\right)^{\frac{\gamma}{\theta}} \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]^{\frac{\theta-\gamma}{\theta}}.$$

Now, we can write $X^*(t)$ explicitly as

$$\begin{aligned} X^*(t) = H^{-1}\left(\frac{tV}{A(X^*)}\right) &= \left(\frac{\gamma}{\theta}\frac{tV}{A(X^*)}\right)^{\frac{1}{\theta-\gamma}} \\ &= t^{\frac{1}{\theta-\gamma}} \left(\frac{\gamma V}{\theta}\right)^{\frac{1}{\theta-\gamma}} \left(\frac{\gamma V}{\theta}\right)^{-\frac{\gamma}{\theta(\theta-\gamma)}} \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]^{-\frac{1}{\theta}} \\ &= t^{\frac{1}{\theta-\gamma}} \left(\frac{\gamma V}{\theta \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]}\right)^{\frac{1}{\theta}}. \end{aligned}$$

This completes the proof. □

The explicit characterization of equilibrium efforts above reveals how the effective effort and effort cost functions influence the equilibrium behavior. Yet, it is seen that the equilibrium efforts do not respond monotonically to changes in the curvature of those functions, regulated by γ and θ, respectively. This observation raises the question of which large Tullock contest maximizes the equilibrium efforts. The next section is interested in this question.

4 Effort-maximizing Tullock contests

This section is devoted to the analysis of the optimal large Tullock contest game. The optimal contest is defined as the one that maximizes the equilibrium total effort (see Nti 2004; Szymanski 2003; Franke et al. 2013; Olszewski and Siegel 2016; Çağlayan et al. 2022 among others).

Given the power-form g and c functions specified earlier, the equilibrium total effort can be explicitly written as

$$\int_{\Omega} X^*(t)dF(t) = \mathbb{E}[T^{\frac{1}{\theta-\gamma}}]\left(\frac{\gamma V}{\theta \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]}\right)^{\frac{1}{\theta}}.$$

Our aim is to identify an optimal $\gamma^* \in (0, \theta)$ for any given $\theta \geq 1$ such that this equilibrium total effort value is maximized.[4] However, an explicit solution cannot be provided here unless the type distribution is known. As such, from this point onward, it is assumed that agent types are distributed uniformly, $T \sim U[a, b]$, where $b > a \geq 0$.

Given that $F : [a, b] \to [0, 1]$ is such that $F(t) = \frac{t-a}{b-a}$ and $dF(t) = \frac{1}{b-a}dt$, we have

$$\mathbb{E}[T^\alpha] = \int_a^b t^\alpha \frac{1}{b-a} dt = \frac{b^{\alpha+1} - a^{\alpha+1}}{(b-a)(\alpha+1)}.$$

Thus, under the uniform distribution, the equilibrium total effort can be written as

$$\mathbb{E}[T^{\frac{1}{\theta-\gamma}}]\left(\frac{\gamma V}{\theta \mathbb{E}[T^{\frac{\gamma}{\theta-\gamma}}]}\right)^{\frac{1}{\theta}} = \frac{b^{\frac{\theta-\gamma+1}{\theta-\gamma}} - a^{\frac{\theta-\gamma+1}{\theta-\gamma}}}{\frac{\theta-\gamma+1}{\theta-\gamma}(b-a)}\left(\frac{b^{\frac{\theta}{\theta-\gamma}} - a^{\frac{\theta}{\theta-\gamma}}}{\frac{\theta}{\theta-\gamma}(b-a)}\right)^{-\frac{1}{\theta}}\left(\frac{\gamma V}{\theta}\right)^{\frac{1}{\theta}}.$$

Let $a_0 = \frac{a}{b}$. Then, the equilibrium total effort becomes

$$\frac{1 - a_0^{\frac{\theta-\gamma+1}{\theta-\gamma}}}{\frac{\theta-\gamma+1}{\theta-\gamma}(1-a_0)}\left(\frac{1 - a_0^{\frac{\theta}{\theta-\gamma}}}{\frac{\theta}{\theta-\gamma}(1-a_0)}\right)^{-\frac{1}{\theta}}\left(\frac{\gamma V b}{\theta}\right)^{\frac{1}{\theta}}.$$

Notice that γ^* that maximizes the equation above is independent of the lower and upper bounds of the type distribution, i.e., a and b. This means that for any two different uniform distributions over $[a, b]$ and $[ka, kb]$ for some $k > 0$, the optimal contest is characterized by the same γ^*.

Now, assume that $a_0 = 0$. The equilibrium total effort reduces to

[4] The optimal selection of a contest success function has been studied earlier in contest games with a finite number of players. For examples, see Dasgupta and Nti (1998) and Skaperdas (1996).

$$\frac{\theta-\gamma}{\theta-\gamma+1}\left(\frac{\gamma Vb}{\theta-\gamma}\right)^{\frac{1}{\theta}}.$$

Its derivative with respect to γ is

$$\frac{\theta-2\gamma+1}{\gamma(\theta-\gamma+1)^2}\left(\frac{\gamma Vb}{\theta-\gamma}\right)^{\frac{1}{\theta}}$$

after some algebraic operations. Therefore, the equilibrium total effort is maximized when $\gamma^*=\frac{\theta+1}{2}$. When $\theta=1$, it turns out that an optimal $\gamma^*<\theta$ does not exist. This is because when the effort cost function is linear, the equilibrium total effort always increases as γ increases within the (0,1) interval. For a strictly convex effort cost function, however, the optimal γ^* is found to be the middle point between 1 and θ. These results hold for any uniform distribution over $[0, b]$ with $b>0$.

If $a_0>0$, then after taking the derivative of the logarithm of the equilibrium total effort with respect to γ and setting it equal to 0, we obtain the following first-order condition:

$$\frac{\log a_0}{\theta-\gamma}\left(\frac{1}{1-a_0^{\frac{\theta-\gamma+1}{\theta-\gamma}}}-\frac{1}{1-a_0^{\frac{\theta}{\theta-\gamma}}}\right)=\frac{\theta-2\gamma+1}{\gamma(\theta-\gamma+1)}.$$

The optimal γ^* can be implicitly written as the solution to this equation. However, since such an optimal γ^* cannot be found explicitly, we now proceed to a numerical analysis. To that end, we consider *(a)* a selection of values for $\theta\in\{2,3,4,5,6\}$ and analyze the optimal γ^* for any value of $a_0\in[0,1)$, and *(b)* a selection of values for $a_0\in\{0,0.2,0.4,0.6,0.8\}$ and analyze the optimal γ^* for any value of $\theta\in(1,6]$. The respective graphs are illustrated in Fig. 1.[5] A few observations are in order: (i) for any $a_0\in[0,1)$ and $\theta\in(1,6]$, we have $\gamma^*\in\left[\frac{\theta+1}{2},\theta\right)$; (ii) the optimal γ^* increases as a_0 increases; (iii) as $a_0\to1$, we have $\gamma^*\to\theta$; (iv) a unit change in a_0 leads to a bigger change in γ^* for larger values of θ; (v) the optimal γ^* increases as θ increases; and (vi) a unit change in θ leads to a bigger change in γ^* for larger values of a_0.[6]

Based on these observations, it is known that a Tullock CSF with a higher γ should be implemented to maximize total equilibrium effort when the curvature of the effort cost function increases. This occurs when there is an increase in θ, after which sufficiently low efforts become less costly and sufficiently high efforts become more costly. As such, since the equilibrium effort profile is an increasing function of t, high-type agents are discouraged but sufficiently low-type agents are encouraged in equilibrium by an increase in θ. As the equilibrium effort profile

[5] In fact, we conducted our numerical analysis for a larger set of parameter values and observed that the results we report here are robust in that larger set. We choose to not report all for space limitations and reader friendliness.

[6] The reader can also observe in Fig. 1 the aforementioned result that for $a_0=0$ and $\theta>1$, the optimal γ^* is at the middle point between 1 and θ.

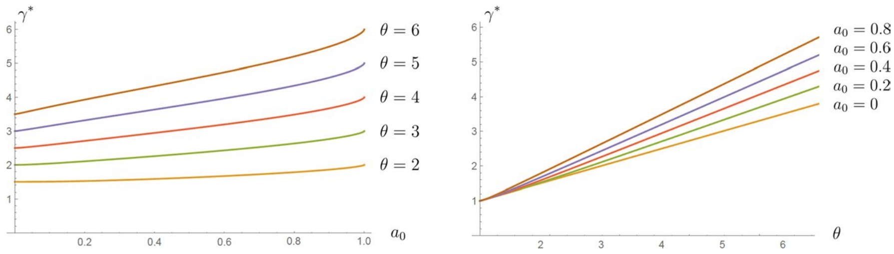

Fig. 1 The Optimal γ^* for Different Values of a_0 and θ

suggests that high-type agents tend to increase their equilibrium efforts when high efforts become more effective, the optimal contest is realized for a relatively higher γ value, which aims to re-motivate high-type agents to exert higher efforts.

In a similar manner, the contest designer chooses to implement a Tullock CSF with a higher γ as the support of the type distribution becomes narrower. Note that a narrower support can be captured by an increase in a_0, indicating a smaller distance between the lower bound a and the upper bound b (i.e., less heterogeneous types). This can be interpreted as follows: For example, for a given value of b, if there is an increase in a, then there would be a more intensive competition, which in turn discourages all contestants, thereby reducing the equilibrium effort for any agent of type $t \in [a, b]$. Once again, it is the designer's aim to re-motivate high-type agents by choosing a higher γ value.

The effect of the support size on γ^* can be analyzed from a different perspective. When agent types are distributed over $[0, b]$ for some $b > 0$, it is known that $\gamma^* = \frac{\theta+1}{2}$, so that the same γ^* is optimal when $b \to 0$. However, when agent types are distributed over $[1, b]$ for some $b > 1$, our numerical analysis shows that $\gamma^* \to \theta$ when $b \to 1$. That is, although the support of the type distribution seemingly gets narrower in both cases, γ^* converges to the other extreme in the latter case. This can be explained as follows: As noted earlier, the optimal γ^* is independent of the lower and upper bounds of the type distribution, hence of the distance between those bounds, but it depends on their ratio a_0. In the former case, $a_0 = 0$, which means that there is always an extremely big difference between the lowest- and highest-type agents for any $b > 0$, whereas in the latter case, $a_0 \to 1$ as $b \to 1$, which means that the same difference gets smaller and smaller as b decreases.

5 Conclusion

We showed that a unique equilibrium of a large Tullock contest game with heterogeneous agents exists under mild conditions on the contest success function and effort cost function. This result does not depend on agents' type distribution. Later, we turned our attention to the effort-maximizing contest design, which required an explicit form for the equilibrium total effort function. Accordingly, assuming a uniform type distribution, we characterized the optimal CSF within the Tullock-family and showed that the optimal contest depends on the curvature of the cost function and the boundaries of the type distribution. We conducted a numerical comparative static analysis to gain further insights into this dependence. We then reported that the contest designer needs to increase the curvature of the effective effort function with an aim to encourage high-type agents to exert even higher efforts when the curvature of the effort cost function increases (which makes high efforts more costly) or when the lowest-type to highest-type ratio increases (which decreases the level of heterogeneity among contestants).

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