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Use of Characteristic Basis Function Method for Scattering from Terrain Profiles

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Abstract

An integral equation (IE) based solution procedure is presented for the rigorous analysis of scattering from terrain profiles. The procedure uses characteristic basis function method (CBFM), which is hybridized with the forward-backward method (FBM), to reduce the storage requirements of the resultant Method of Moments (MoM) impedance matrix, as well as to accelerate the solution procedure. Numerical results in the form of induced current and scattered field are presented to assess the accuracy and efficiency of the solution procedure.

1. Introduction

Algorithms based on the Method of Moments (MoM) [1] are becoming widely used for the analysis of electromagnetic scattering from rough terrain profiles, because they are accurate and robust. However, possible reductions in the storage of the MoM impedance matrix, and in the computational cost when inverting the MoM impedance matrix, have become a primary concern when applied to electrically large geometries. The use of stationary and nonstationary iterative techniques reduces the computational cost requirements from $O(N^3)$ to $O(N^2)$ with N being the number of surface unknowns. Among them, the stationary forward-backward method (FBM) [2] is one of the most efficient technique for scattering from rough surface problems, and provides accurate solution within few iterations. Using its spectrally accelerated version [3] in terrain propagation problems [4], the storage and computational cost requirements are reduced to $O(N)$. However, when applying the spectrally accelerated forward backward method (SAFBM) to surface profiles with large height variations, serious convergence problems can occur. To circumvent this problem, in this paper we use the Characteristic Basis Function Method (CBFM), proposed in [5], and hybridize it with FBM, to analyze electromagnetic scattering from electrically large terrain profiles. Preliminary results, presented in [6], have been quite promising. The objective of the present paper is to further enhance the technique by accelerating it, and by reducing the storage requirements via the use of an interpolation process.

2. Formulation

To formulate the problem, the impedance boundary condition is used together with an integral equation—specifically, an electric field integral equation (EFIE) for transverse magnetic (TM) polarization, and magnetic field integral equation (MFIE) for transverse electric (TE) polarization—for two-dimensional terrain profiles. Then, by using the MoM procedure with pulse bases functions and point matching, this integral equation is converted into the matrix equation

$$\mathbf{B} = \bar{\mathbf{A}}\ \mathbf{X}, \tag{1}$$

where $\bar{\mathbf{A}}$ is the known $N \times N$ impedance matrix, $\mathbf{B}$ is the $N \times 1$ excitation vector, and $\mathbf{X}$ is the $N \times 1$ unknown solution vector. The CBFM approach starts by partitioning the terrain profile with a total of N unknowns into M blocks. Let N_i be the number of unknowns in block i (i.e., $\sum_{i=1}^{M} N_i = N$). Each block is then extended in both directions by Δ to eliminate the spurious edge effects. Let N_i^e be the number of unknowns in the extended block i. Using the equation

$$\bar{\mathbf{Z}}_e^{(i)} \mathbf{J}_i^{(i)} = \mathbf{R}^{(i)} \text{ for } i = 1, 2, \ldots, M, \tag{2}$$

the primary basis function (PBF) for block i, $\mathbf{J}_i^{(i)}$, is generated, where $\mathbf{Z}_e^{(i)}$ is the coefficient matrix for the extended block i and $\mathbf{R}^{(i)}$ is a subset of $\mathbf{B}$, which includes the rows belonging to block i. To accelerate the efficiency of the method, single iteration FBM is utilized to solve (2) rather than performing a direct inversion. All blocks are treated in a similar fashion, and yield M primary basis functions at the end.

Once the primary basis functions have been constructed, the next step is to generate the secondary basis functions (SBF). However, in contrast to [5], only the mutual coupling between the adjacent blocks are retained by taking advantage of the fact that, in an electrically large terrain, the mutual interactions among the far away blocks are very weak. The coupling is accounted for by using the relation

$$\bar{\mathbf{Z}}_e^{(i)} \mathbf{J}_k^{(i)} = \mathbf{R}_k^{(i)} \text{ for } k = i-1, i+1\,, \tag{3}$$

where $\mathbf{J}_k^{(i)}$ is the k^{th} secondary basis for block i, and $\mathbf{R}_k^{(i)}$ is the excitation vector resulting from the mutual coupling between block i and block k. Note that the two end-blocks only have single secondary basis functions; hence, the total number of secondary basis functions is $2M-2$. The excitation vector $\mathbf{R}_k^{(i)}$, used in (3) is computed by using the relation

$$\mathbf{R}_k^{(i)} = \ -\bar{\mathbf{Z}}^{(i,k)} \mathbf{J}_k^{(k)}, \tag{4}$$

where $\bar{\mathbf{Z}}^{(i,k)}$ is the impedance matrix formed by selecting the testing location at the extended block i, with the source location being the block k. However, extended block i shares some of the unknowns with block k. Let $N_{i,k}^{(c)}$ be the number of such unknowns. Then, by eliminating these source locations, the sizes of $\bar{\mathbf{Z}}^{(i,k)}$ and $\mathbf{J}_k^{(k)}$ become $N_i^e \times (N_k - N_{i,k}^{(c)})$ and $(N_k - N_{i,k}^{(c)}) \times 1$, respectively. The total number of basis functions constructed is then $3M-2$, comprising M primary basis functions and $2M-2$ secondary basis functions.

In the next step, the solution to the entire problem is expressed as a linear combination of characteristic basis functions:

$$[X]_{N\times 1} = \sum_{k=1}^{2} \alpha_k^{(1)} \begin{bmatrix} [\mathbf{J}_k^{(1)}] \\ [0] \\ . \\ . \\ [0] \end{bmatrix} + \sum_{k=1}^{3} \alpha_k^{(2)} \begin{bmatrix} [0] \\ \left[\mathbf{J}_k^{(2)}\right] \\ . \\ . \\ [0] \end{bmatrix} + \cdots + \sum_{k=1}^{2} \alpha_k^{(M)} \begin{bmatrix} [0] \\ [0] \\ . \\ . \\ \left[\mathbf{J}_k^{(M)}\right] \end{bmatrix}. \tag{5}$$

Here, $\alpha_k^{(i)}$ is the unknown complex expansion coefficient for the k^{th} basis function of block i. Substituting (5) into (1), one can show that solution can be written in the form

$$\sum_{k=1}^{2} \alpha_k^{(1)} \mathbf{v}_k^{(1)} + \sum_{k=1}^{3} \alpha_k^{(2)} \mathbf{v}_k^{(2)} + \cdots + \sum_{k=1}^{3} \alpha_k^{(M-1)} \mathbf{v}_k^{(M-1)} + \sum_{k=1}^{2} \alpha_k^{(M)} \mathbf{v}_k^{(M)} = [B]_{NX1}, \tag{6}$$

where

$$\mathbf{v}_k^{(i)} = [[\bar{\mathbf{A}}_{1,i}][\mathbf{J}_k^{(i)}]\ [\bar{\mathbf{A}}_{2,i}][\mathbf{J}_k^{(i)}] \ldots [\bar{\mathbf{A}}_{M,i}][\mathbf{J}_k^{(i)}]]^T. \tag{7}$$

To solve (6), the inner product of both sides is taken with the Hermitian of each $\mathbf{v}_k^{(i)}$ constructed above, to generate the reduced matrix whose size is $(3M-2)\times(3M-2)$. The solution of this matrix yields the unknown expansion coefficients for the characteristic bases (CBs).

The main memory-intensive and time consuming steps in the CBFM are the storage of the $\mathbf{v}$ vectors as well as the generation of the reduced matrix (RM), regardless of its hybridization with FBM. In this work, a significant improvement in reduction of both of the above is achieved via the use of an interpolation process. Because the $\mathbf{v}$ vectors given in (7) represent fields, it is observed that for relatively long distances their amplitudes vary only slightly and their phase variation is almost linear. Therefore, during the generation of (7), elements of $\mathbf{v}$ can be divided into groups such that each group contains 20, 50 or in some cases even 100 elements. One can decide how to form these small groups and the number of elements in each group by looking at the roughness of the surface, which governs the number of elements in each group. Within each group, the phase is assumed to vary linearly, while the amplitude is assumed to be uniform. Consequently, two elements at the middle of each group are chosen, the phase difference between these elements is computed, and the value of the remaining elements in the group are determined. Thus, if we define a reduction factor k, then $1/k$ becomes the number of elements in these small groups divided by two. Although this interpolation technique requires a modest amount of pre-processing, it is relatively easy to implement, and it can accelerate the method by a factor of 10 in many cases investigated.

3. Numerical Results

The hybrid CBFM-FBM technique, together with the aforementioned interpolation process, has been tested to solve the scattered fields form terrain profiles with 20,000 and 50,000 unknowns. For both problems, the incident wave is from an isotropic radiator placed at a height of 25λ above the terrain, and it transmits a transmit power of 25 Watts. The horizontal distance is shown by x, and the source is located at $x=0$. The operating frequency is chosen to be 300 MHz so the free-space wavelength $\lambda = 1$ m. In the MoM procedure, the pulse width for rectangular basis functions is chosen to be $\Delta x = \frac{\lambda}{10}$.

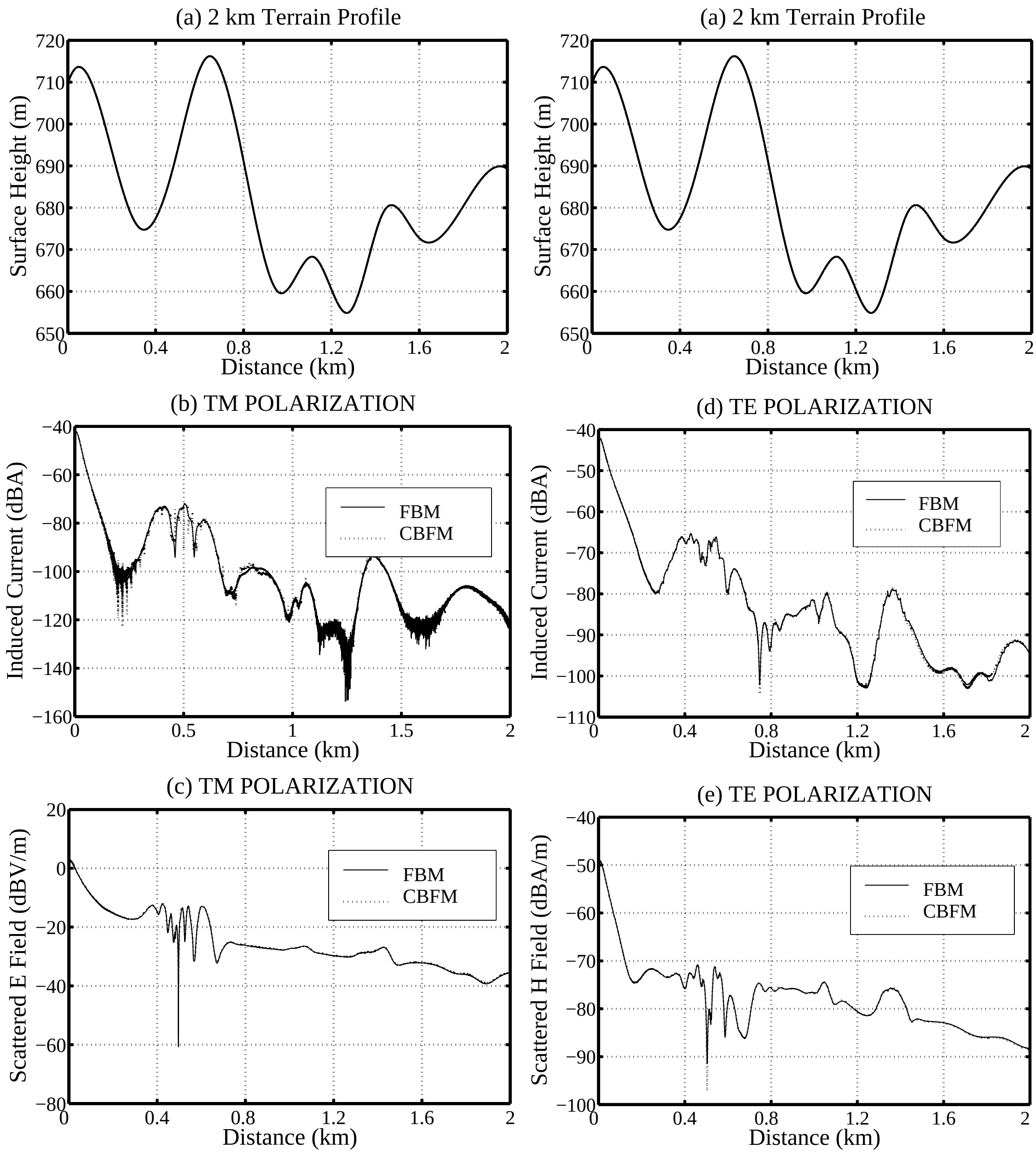

Figure 1. Scattering from a 2000λ Terrain Profile.

For both 20,000 and 50,000 unknown problems, the profile is partitioned into blocks of 50 and 100, and they are extended by 1λ to reduce the spurious truncation effects. An extension of the blocks by more than 1λ does not improve the accuracy of the method, though, it obviously increases the computational cost. As mentioned before, during the generation of the CBs, the FBM is used with a single iteration. Note that, at this point, an accurate solution of the matrix equation is not very important, and one can determine the CBs via a simple matrix inversion as well. However, in that case, the computational cost increases, but the accuracy of the overall solution does not improve to any consequence.

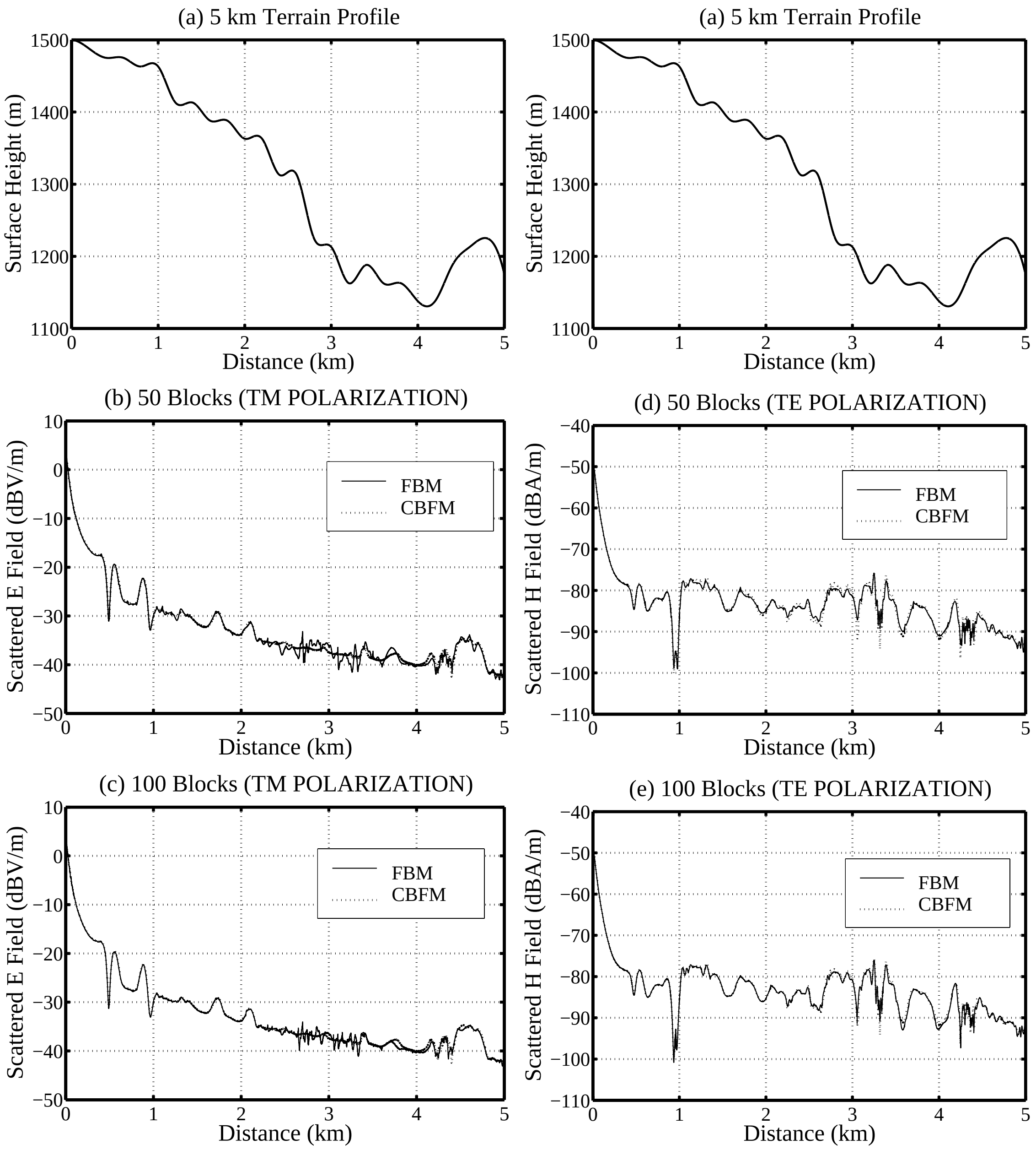

Figure 2. Scattering from a 5000λ Terrain Profile.

In the implementation of the interpolation process for the terrain profiles with 20,000 and 50,000 unknowns, groups of 20 elements (i.e., a distance of 2λ) are formed uniformly through the terrain. As a result k becomes $1/10$. Figure 1 shows the $2{,}000\lambda$ (i.e., 20,000 unknowns) terrain profile, induced currents on the profile and the scattered fields when $M = 100$ for both TM and TE polarizations. The same procedure is followed for the $5{,}000\lambda$ (i.e., 50,000 unknowns) terrain profile and the scattered fields are shown in Figure 2 for $M = 50$ and $M = 100$.

The CPU times required to derive the aforementioned results have been compared with that for a single iteration of the conventional FBM solver, and are presented in Table 1. Table 2 shows the CPU times required to construct the reduced matrix in some detail. The generation of this matrix entails the generation of the $\mathbf{v}$ vector and the computations of the inner products.

Table 1. CPU times in seconds.

N	M	PBF+SBF	RM	Total	FBM (1 Iteration)
20,000	100	49	1,339	1,388	1,152
20,000	50	84	427	511	1,152
50,000	100	248	4,682	4,930	7,242
50,000	50	479	1,514	1,993	7,242
N	M	$O(1/M)$			$O(N^2)$

As seen from both Tables, the number of blocks, M, dominates the efficiency of the method. The accuracy increases with an increase in M (see Figure 2), but the CPU time also increases. Note in the limiting case the CBFM recovers the conventional MoM. For a large M, the generation of the reduced matrix is still the most time-consuming step in the method, even when the interpolation process is used, and ways to further reduce this time is currently under study. It should be noted, that even now a significant acceleration has been achieved in comparison to the FBM, and the solution procedure is faster than even one iteration of FBM in all cases. Note that, typically, 2–6 iterations are necessary for the conventional FBM to converge.

Table 2. CPU times of reduced matrix (RM) generation, in seconds.

N	M	k	$\mathbf{v}$ Vector Generation	Inner Product	Total
20000	100	1/10	124	1215	1339
20000	50	1/10	126	301	427
50000	100	1/10	772	3910	4682
50000	50	1/10	769	745	1514
N	M	k	$O(N^2 \times k)$	$O(N \times M^2)$	

Finally two other important points should be mentioned. First, although, SAFBM is faster than the CBFM, the former fails to converge for many terrain profiles. Second, the reduced matrix should be solved by a direct inversion such as the LU decomposition or Gaussian elimination. However, since the number of unknowns has been reduced significantly from the original level, the required CPU time is negligible and, hence, this time is not included in Table 2.

4. Conclusion

This paper has presented a hybrid approach that combines the CBFM with a single iteration of FBM to derive an efficient but accurate procedure for the analysis of scattering of electromagnetic waves from terrain profiles. The algorithm yields accurate results for both the TM and TE polarizations. A simple interpolation process is used to speed up the solution so that it is significantly faster than a single iteration of FBM. Further acceleration of the method, as well as comparison of the numerical results with the measurements for real terrain profiles are currently under investigation and will be reported separately.

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