Linear Precoder Design for Simultaneous Information and Energy Transfer Over Two-User MIMO Interference Channels

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Abstract—Communication strategies that utilize wireless media for simultaneous information and power transfer offer a promising perspective for efficient usage of energy resources. With this motivation, we focus on the design of optimal linear precoders for interference channels utilizing such strategies. We formulate the problem of minimizing the total minimum mean-square error while keeping the energy harvested at the energy receivers above given levels. Our framework leads to a non-convex problem formulation. For point-to-point multiple-input multiple-output channels, we provide a characterization of the optimal solutions under a constraint on the number of transmit antennas. For the general interference scenario, we propose two numerical approaches, one for the single antenna information receivers case, and the other for the general case. We also investigate a hybrid signalling scheme, where the transmitter sends a superposition of two signals: a deterministic signal optimized for energy transfer and an information carrying signal optimized for information and energy transfer. It is illustrated that if hybrid signalling is not incorporated into the transmission scheme, interference can be detrimental to the system performance when the number of antennas at the receivers is low.

Index Terms—Wireless power transfer, energy harvesting, simultaneous wireless information and power transfer (SWIPT), interference, linear precoding, MMSE.

I. INTRODUCTION

EFFICIENT usage of energy resources is a growing concern in today's communication systems. Solutions that consider energy harvesting (EH) from radio-frequency signals instead of completely relying on batteries or the power from the grid offer a promising perspective. In these scenarios, wireless media is used for simultaneous information transmission and power transfer in contrast to performing each of these tasks separately. In this paper, we study transmission strategies to accomplish this as efficiently as possible. We focus on the design of optimal linear precoders under the criterion of the minimum mean-square error (MMSE).

Much of the existing research on simultaneous wireless information and power transfer (SWIPT) is conducted with rate as the performance metric. Fundamental trade-offs between the rate and the energy for a single-input single-output point-to-point (P2P) additive white Gaussian noise (AWGN) channel is studied [1]. This framework is extended to AWGN channels with frequency selective fading [2]. Optimal transmission strategies are investigated for broadcast channels [3], [4], relay channels [5], [6] and interference channels [7]–[10]. Optimal power control problems are considered under rate considerations for various single-user and multi-user scenarios [11]. Practical code design solutions are investigated in [12].

Here we adopt an alternative approach and focus on linear precoding at the transmitters and linear filtering at the receivers. Proper design of precoders and filters have been shown to provide significant performance improvements in many different communication scenarios, see for instance [13]–[17] for a limited sample. It is noted that mean-square error filters provide a practical, but still reasonably accurate alternative for estimation of coded data symbols in contrast to maximum likelihood decoding [14]. Investigated scenarios cover a wide range of models, including point-to point channels [13], multiple-access channels [14] relay channels [15] and applications to robust designs [16], [17].

In linear precoder design, mean-square error or signal-to-noise ratio (SNR) based metrics are utilized as the typical performance criteria. Despite the above vast usage of these metrics for various communication scenarios, relatively small number of works that consider such metrics have appeared in the framework of energy harvesting. Most of these works focus on the scenarios where energy is harvested from possibly unreliable resources, but not necessarily from man-made wireless signals [18]–[20]. Contrary to these approaches focusing on the unreliable nature of the energy supply, here we consider another energy harvesting problem and focus on simultaneous transfer of energy and information. There have been a number of works focusing on SNR-based constraints for SWIPT systems, such as single-output multicast channel [21], interference channel [22], [23], and downlink scenario [24]. Unfortunately, these works typically focus single antenna receivers which limits the applicability of the results.

In this paper, we focus on multiple-input multiple-output (MIMO) interference channels. As an expository work, we first
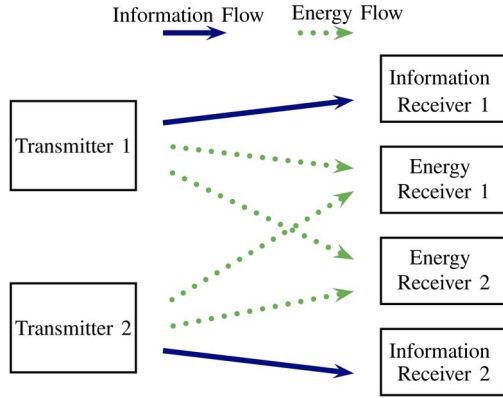
Fig. 1. Simultaneous information and energy transfer in two-user interference channel.

study the point-to-point channel with one information receiver (IR) and one energy receiver (ER). For this set-up, we formulate the problem of finding the optimal linear precoding strategy in order to minimize the MMSE at the information receiver while keeping the energy harvested at the energy receiver above a certain level. This formulation leads to a non-convex problem formulation. Nevertheless, under a constraint on the number of antennas at the transmitter, we provide a characterization of the optimal strategies that reveals the relationship between the channel matrices and the optimal transmission strategies. We also discuss the relationship between the rate maximization problem and the MMSE minimization problem. For the interference channel, we investigate the scenario illustrated in Fig. 1, where the transmitters aim to convey information as reliably as possible to their corresponding IRs while keeping the transferred power to the ERs above given levels. Here the transmitters have two possibly conflicting goals. One of these is keeping the interference at the non-designated IRs as low as possible to be able to transmit information reliably to the designated IRs. The other goal is to send as much power as possible to the ERs (which may be co-located with the IRs) in order to satisfy the EH constraints. We consider weighted sum MMSE as the performance criterion which leads to a non-convex optimization problem formulation. We propose two approaches for joint precoder design, where one of these approaches is developed solely for the single antenna IR case.

We also investigate a power splitting scheme at the transmitter, where the transmitted signal is the superposition of two signals where one of them is chosen to be deterministic and its sole purpose is to transfer power. We show that this scheme allows us to obtain smaller error values especially when the number of receive antennas is low. Contrary to the power splitting strategies for the receivers proposed in [3], here our aim is not to offer a feasible solution for the problem of practical EH receiver design problem. Instead, we illustrate that it is not always optimal to use a sole Gaussian signalling approach at the transmitter, see also [9] for similar discussions in the framework of rate maximization. This observation is important for understanding the fundamental limits of simultaneous information and power transfer. It shows that, even under the assumption of availability of ideal receivers that can simultaneously decode

information and power, the signalling framework should be restructured to go beyond what Gaussian signalling offers.

We compare the performance of our designs with those of a time-division multiple-access (TDMA) approach and a time-division mode switching (TDMS) approach. These comparisons are motivated by the fact that schemes that depend on such mode separations have been considered as practical benchmarks in the interference channel in the context of rate maximization [7]–[10]. We illustrate that our proposed designs outperform the TDMA and TDMS approaches in low to moderate interference scenarios. Nevertheless, we note that in the case of co-located IR and ERs, our designs are based on an ideal receiver structure, i.e., the receiver can harvest energy and decode information simultaneously. It is not clear whether this receiver structure can be realized [3], hence in these scenarios our transmission schemes should be interpreted as designs for the baseline performance.

The rest of the paper is organized as follows. The system model for the interference channel and the joint linear precoder design problem are presented in Section II and Section III, respectively. In Section IV, we present the special case that focuses on the point-to-point channel. In Section V and in Section VI, our joint linear precoder design approach for the interference channel is presented for the single antenna IR case and the general case, respectively. We discuss the hybrid signalling approach in Section VII. The performance of our designs are illustrated in Section VIII. We conclude the paper in Section IX.

The following notation is used throughout the paper. Uppercase and lowercase letters denote matrices, and column/row vectors respectively. The complex conjugate transpose of a matrix A is denoted by $A^\dagger$. The operators $E[.]$, and $\text{tr}[.]$ denote the expectation, and trace operators respectively. $\text{diag}(a)$ denotes the diagonal matrix formed with a as the diagonal elements. I denotes the identity matrix with the suitable dimensions. Positive semi-definite ordering is denoted by $\succeq$, where $A \succeq 0$ denotes a Hermitian positive semi-definite matrix. An optimal value of an optimization variable A is denoted by A^*.

II. SYSTEM MODEL

A. Interference Channel

The multi-antenna transmitters transfer information to information receivers as well as power to energy harvesting receivers according to the following model

$$y_i^I = H_{i1}^I x_1 + H_{i2}^I x_2 + w_i^I, \tag{1}$$

$$y_i^E = H_{i1}^E x_1 + H_{i2}^E x_2 + w_i^E, \tag{2}$$

where $i = 1, 2$. Here $H_{ik}^I \in \mathbb{C}^{n_r \times n_t}$ and $H_{ik}^E \in \mathbb{C}^{n_e \times n_t}$ represent the channel gains from the transmitter k to information receiver i (IR$_i$) and energy receiver i (ER$_i$), respectively $i, k = 1, 2$. This system is illustrated in Fig. 1. Zero-mean complex proper Gaussian $w_i^I \in \mathbb{C}^{n_r \times 1} \sim \mathcal{CN}(0, K_{w_i^I})$, $K_{w_i^I} = E[w_i^I(w_i^I)^\dagger] = \sigma_{w,I,i}^2 I$ and $w_i^E \in \mathbb{C}^{n_e \times 1} \sim \mathcal{CN}(0, , K_{w_i^E})$, $K_{w_i^E} = E[w_i^E(w_i^E)^\dagger] = \sigma_{w,E,i}^2 I$ denote the noise at IR's and ER's channels, respectively.

The channel input x_i is formed as $x_i = A_i s_i$, where the zero mean complex proper Gaussian $s_i \in \mathbb{C}^{n \times 1}$, $s_i \sim \mathcal{CN}(0, K_{s_i})$ $K_{s_i} = I$, denotes the data, $A_i \in \mathbb{C}^{n_t \times n}$ denotes the precoding matrix at the ith transmitter. All signals, w_i^I, w_i^E, and s_i, are assumed to be statistically independent. All channel gains are fixed throughout the transmission.

B. MMSE Estimation

The designated information receiver for transmitter i is the receiver i. Hence upon receiving y_i^I, IR_i forms an estimate of s_i. We assume that IRs employ MMSE estimation. Hence the estimate of s_i at IR_i can be expressed as follows [25, Ch. 2]:

$$E\left[s_i | y_i^I\right] = K_{s_i y_i^I} K_{y_i^I}^{-1} y_i^I, \tag{3}$$

where $K_{s_i y_i^I} = E[s_i y_i^{I\dagger}] = K_{s_i} A_i^\dagger H_{ii}^{I\dagger}$, and $K_{y_i^I} = E[y_i^I y_i^{I\dagger}] = H_{ii}^I A_i A_i^\dagger H_{ii}^{I\dagger} + H_{ij}^I A_j A_j^\dagger H_{ij}^{I\dagger} + \sigma_{w,I,i}^2 I$. We note that since $\sigma_{w,I,i}^2 I \succ 0$, we have $K_{y_i} \succ 0$, and hence $K_{y_i}^{-1}$ exists. The MMSE at IR_i can be expressed as follows

$$\varepsilon_i(A_1, A_2) = E\left[\left\| s_i - E\left[s_i | y_i^I\right] \right\|^2\right] \tag{4a}$$

$$= \text{tr}\left[K_{s_i} - K_{s_i y_i^I} K_{y_i^I}^{-1} K_{s_i y_i^I}^\dagger\right] \tag{4b}$$

$$= n - \text{tr}\left[A_i^\dagger H_{ii}^{I\dagger} \left(T_i + \sigma_{w,I,i}^2 I\right)^{-1} H_{ii}^I A_i\right], \tag{4c}$$

where $T_i = H_{ii}^I A_i A_i^\dagger H_{ii}^{I\dagger} + H_{ij}^I A_j A_j^\dagger H_{ij}^{I\dagger}$, $i, j = 1, 2, i \neq j$.

C. Energy Harvesting

The energy harvested at the ER_i can be expressed as [3]

$$\mathcal{J}_i(A_1, A_2) = \beta\left(\text{tr}\left[H_{i1}^E A_1 A_1^\dagger H_{i1}^{E\dagger}\right] + \text{tr}\left[H_{i2}^E A_2 A_2^\dagger H_{i2}^{E\dagger}\right]\right), \tag{5}$$

where $0 \leq \beta \leq 1$ accounts for the possible loss in the energy conversion process. Without loss of generality, we assume that this loss is accounted for while setting desired energy levels and hence we use $\beta = 1$ in our formulations.

III. LINEAR PRECODER DESIGN

We consider the following joint linear precoder design problem which seeks the optimal linear precoders in order to minimize the weighted MMSE at the information receivers while satisfying the energy requirements at the energy receivers:

$$(P1) \quad \min_{A_1, A_2} \quad \alpha_1 \varepsilon_1(A_1, A_2) + \alpha_2 \varepsilon_2(A_1, A_2) \tag{6a}$$

$$\text{s.t.} \quad \mathcal{J}_1(A_1, A_2) \geq \gamma_1, \quad \mathcal{J}_2(A_1, A_2) \geq \gamma_2, \tag{6b}$$

$$\text{tr}\left[A_1 A_1^\dagger\right] \leq P_1, \quad \text{tr}\left[A_2 A_2^\dagger\right] \leq P_2. \tag{6c}$$

Here (6c) represents the power constraints at the transmitters. The scalars α_1 and α_2 represent the error weights for different users. These weights can be used to prioritize one of the IRs in the system, for instance a large α_1/α_2 ratio will give a large

penalty to the estimation error at IR_1, hence will result in strategies that favor IR_1. Alternative formulations for prioritizing different users can be also adopted, see for instance [26] where energy efficiency in an interference channel scenario is studied.

We now discuss the convexity properties of the formulation in Problem $P1$. Using the property $\text{tr}[AB] = \text{tr}[BA]$, the energy harvesting constraints can be equivalently written as

$$\mathcal{J}_i(A_1, A_2) = \text{tr}\left[A_1^\dagger H_{i1}^{E\dagger} H_{i1}^E A_1\right] + \text{tr}\left[A_2^\dagger H_{i2}^{E\dagger} H_{i2}^E A_2\right], \tag{7}$$

which is a quadratic function in (A_1, A_2). Moreover, since $H_{ij}^{E\dagger} H_{ij}^E$, $i, j = 1, 2$, are positive semi-definite, $\mathcal{J}_i(A_1, A_2)$ is a convex quadratic function. Hence the EH constraints form non-convex constraints since they bound convex functions from below. It is worth mentioning that the objective function is also not a convex function of (A_1, A_2), which is true even for the scalar case.

We now illustrate that even when the traditional semi-definite rank relaxations are introduced, the resulting optimization problem is still non-convex. We introduce the following new variables of optimization: $K_i = A_i A_i^\dagger$, $i = 1, 2$. Hence the energy harvested at the ERs can be expressed as linear functions of (K_1, K_2)

$$\mathcal{J}_i^K(K_1, K_2) = \text{tr}\left[H_{i1}^E K_1 H_{i1}^{E\dagger}\right] + \text{tr}\left[H_{i2}^E K_2 H_{i2}^{E\dagger}\right]. \tag{8}$$

Furthermore, it is possible to write the objective function in terms of (K_1, K_2), for instance by using the property $\text{tr}[AB] = \text{tr}[BA]$ on (4c) as follows

$$\varepsilon_i^K(K_1, K_2) = n - \text{tr}\left[H_{ii}^I K_i^\dagger H_{ii}^{I\dagger} \left(T_i^K + \sigma_{w,I,i}^2 I\right)^{-1}\right], \tag{9}$$

where $T_i^K = H_{ii}^I K_i H_{ii}^{I\dagger} + H_{ij}^I K_j H_{ij}^{I\dagger}$ and $i, j = 1, 2, i \neq j$. We note that now K_i should have a decomposition such that $K_i = A_i A_i^\dagger$, $A_i \in \mathbb{C}^{n_t \times n}$, i.e., we have rank constraints on the variables K_i, $\text{rank}(K_i) \leq n$. By lifting these constraints, one may form the following relaxed optimization problem

$$(\bar{P}1) \quad \min_{K_1 \geq 0, K_2 \geq 0} \quad \alpha_1 \varepsilon_1^K(K_1, K_2) + \alpha_2 \varepsilon_2^K(K_1, K_2) \tag{10a}$$

$$\text{s.t.} \quad \mathcal{J}_1^K(K_1, K_2) \geq \gamma_1, \quad \mathcal{J}_2^K(K_1, K_2) \geq \gamma_2, \tag{10b}$$

$$\text{tr}[K_1] \leq P_1, \quad \text{tr}[K_2] \leq P_2. \tag{10c}$$

Hence Problem $\bar{P}1$ forms a semi-definite programming (SDP) relaxation of Problem $P1$. Further information about such rank relaxations can be found in [27]–[29]. Although now all the constraints form convex constraints, this formulation is still non-convex, since the objective function in (10a) is a not a convex function of (K_1, K_2) for the general interference channel scenario. An exception is the case of point-to-point channel, where there is only one user (but the channel may still be a MIMO channel). For this case, by using convexity, we provide a semi-explicit analytic characterization of the solutions in Section IV for the case $n_t \leq n$. Starting with Section V, we focus on the interference channel.

IV. POINT-TO-POINT CHANNEL

In this section, we consider the scenario where there is only transmitter with one designated IR and one designated ER. We assume that $x_2 = 0$ without loss of generality. Hence transmitter 1 sends data to IR$_1$ as well as power to ER$_1$

$$y_1^I = H_{11}^I x_1 + w_1^I, \tag{11}$$

$$y_1^E = H_{11}^E x_1 + w_1^E, \tag{12}$$

where H_{11}^I, H_{11}^E, w_1^I, w_1^E, and x_1 are as defined in Section II-A. We focus on the case $n_t \leq n$. The MMSE at IR$_1$ can be expressed as

$$\varepsilon_1(A_1, 0) = \mathrm{tr}\left[I - A_1^\dagger H_{11}^{I\dagger}\left(H_{11}^I A_1 A_1^\dagger H_{11}^{I\dagger} + \sigma_{w,I,1}^2 I\right)^{-1} H_{11}^I A_1\right]$$

$$= \mathrm{tr}\left[\left(\left(I + \frac{1}{\sigma_{w,I,1}^2}A_1^\dagger H_{11}^{I\dagger}H_{11}^I A_1\right)^{-1}\right)\right], \tag{13}$$

where (13) follows from Sherman-Morrison-Woodbury identity [30]. The energy harvested at the ER$_1$ can be expressed as

$$\mathcal{J}_1(A_1, 0) = \mathrm{tr}\left[H_{11}^E A_1 A_1^\dagger H_{11}^{E\dagger}\right]. \tag{14}$$

We consider the problem of minimization of the MMSE at the IR$_1$ while satisfying the EH constraint at the ER$_1$

$$(P2) \quad \min_{A_1} \quad \varepsilon(A_1, 0) \tag{15a}$$

$$\text{s.t.} \quad \mathcal{J}_1(A_1, 0) \geq \gamma_1, \quad \mathrm{tr}\left[A_1 A_1^\dagger\right] \leq P_1. \tag{15b}$$

This formulation is non-convex. Nevertheless, for $n_t \leq n$, it is possible to characterize the optimal solutions for (15) by introducing an equivalent formulation that is convex and whose solutions in fact provide optimal solutions for (15).

Utilizing [31, Lem. 2], we express the problem of minimizing (13) equivalently as the problem of minimizing $\mathrm{tr}[(I + (1/\sigma_{w,I,1}^2)H_{11}^I A_1 A_1^\dagger H_{11}^{I\dagger})^{-1}]$. Hence we arrive at the following optimization problem by introducing $K_1 = A_1 A_1^\dagger$

$$(P3) \quad \min_{K_1 \geq 0} \quad \mathrm{tr}\left[\left(\left(I + \frac{1}{\sigma_{w,I,1}^2}H_{11}^I K_1 H_{11}^{I\dagger}\right)^{-1}\right)\right] \tag{16a}$$

$$\text{s.t.} \quad \mathrm{tr}\left[H_{11}^E K_1 H_{11}^{E\dagger}\right] \geq \gamma_1, \quad \mathrm{tr}[K_1] \leq P_1. \tag{16b}$$

This is a convex optimization problem whose convexity can be established as follows: i) The objective function is a convex function of K_1 since $f(X) = \mathrm{tr}[X^{-1}]$ is convex over $X \succ 0$ [32]; ii) The inequality constraints are linear, hence convex. Since this is a convex SDP problem, an optimal solution can be found numerically by using off-the-shelf numerical optimization tools such as SeDuMi, SDPT3 and CVX [33]–[35]. Instead, here we

utilize the solutions of Problem P3 to arrive at a characterization of the solutions of Problem P2.

Theorem 4.1: Let $n_t \leq n$. Assume that there exists a strictly feasible A_1 for Problem P2. Then an optimal solution has the following form:

$$A_1^* = R^{-1/2}V_{\bar{H}}\Lambda^{1/2}, \tag{17}$$

where $\Lambda = \mathrm{diag}(\lambda_i)$ with λ_i given by the following water-filling type solution

$$\lambda_i = \left[\sqrt{\frac{1}{\lambda_{\bar{H},i}}} - \frac{1}{\lambda_{\bar{H},i}}\right]^+, \tag{18}$$

where $[x]^+ = \max(0, x)$. Here $R = \mu_p I - \mu_e H_{11}^{E\dagger}H_{11}^E$, $\mu_p, \mu_e \geq 0$ and $\bar{H} = R^{-1/2}H_{11}^{I\dagger}H_{11}^I R^{-1/2} = V_{\bar{H}}\Lambda_{\bar{H}}V_{\bar{H}}^\dagger$ is the singular value decomposition of $\bar{H}$, where $\Lambda_{\bar{H}} = \mathrm{diag}(\lambda_{\bar{H},i})$, $\lambda_{\bar{H},1} \geq \lambda_{\bar{H},2}, \ldots, \lambda_{\bar{H},m}$, $m = \mathrm{rank}[\bar{H}]$.

The proof relies on the solution of the dual problem for Problem P3 and the fact that it is always possible to find an optimal A_i from an optimal K_i due to $n_t \leq n$. This line of arguments has been successfully adopted to reveal structures of the optimal solutions in a number of scenarios [3], [32], [36]. In particular, we refer the reader to [3] for the solution of rate maximization under EH constraints. Here we omit the proof for the MMSE case for the sake of brevity.

The result reveals that optimal solutions lie in the span of the right singular vectors of the modified channel matrix $H_{11}^I R^{-1/2}$. This result also illustrates the relationship between the rate maximization problem investigated in [3] and the MMSE minimization problem investigated here: the general structure of the solutions are similar where the eigenvectors of the optimal transmit covariance matrix K_1 lie in the span of the singular vectors of a matrix in the form $H_{11}^I R^{-1/2}$, $R = \mu_p I - \mu_e H_{11}^{E\dagger}H_{11}^E$ where μ_p, μ_e may take possibly different values for the MMSE minimization and rate maximization. Another related issue is the use of weighted MMSE criterion as an intermediate step in rate maximization problem, as explored in [37] without the EH constraints. Our result here can be utilized while formulating a similar relationship for SWIPT systems.

As discussed earlier, the linear precoder design problem with MMSE minimization, in general, has a rank constraint: K_1 should have a decomposition such that $K_1 = A_1 A_1^\dagger$, $A_1 \in \mathbb{C}^{n_t \times n}$. On the other hand, in the case of rate maximization, there is no such constraint, the transmit covariance matrix K_1 is the sole variable of interest [3, Problem P2]. Although a solution for an optimal A_1 satisfying rank constraints can be always found without putting any restrictions on the number of transmit antennas in the case of MMSE minimization without the EH constraints [13], [16], whether this is the case under EH constraints is not clear. We note that, even in the case with $n_t \leq n$, the designs optimized for these two metrics (rate and the MMSE) lead to different MMSE performance (with the exception of the case $n_r = 1$, where these metrics lead to equivalent objective functions). This performance gap is illustrated in Section VIII.

V. INTERFERENCE SCENARIO: MISO INFORMATION CHANNEL

We now consider the interference scenario with multiple-input single-output (MISO) information channel, i.e., Problem $P1$ with $n_r = 1$. As discussed in Section III, neither the general problem with multiple antenna IRs nor the single antenna scenario result in convex formulations. Nevertheless, here we propose a method to solve Problem $P1$ for the MISO case using a sequence of convex problems.

In the following we will first consider the relaxed problem, Problem $\bar{P}1$ in (10), and ignore the constraints $K_i = A_i A_i^\dagger$, $i = 1, 2$. We will first focus on finding optimal K_i's, then we will discuss how to find optimal A_i's for Problem $P1$ in Lemma 5.1. Under MISO information channel scenario, the MMSE at IR_i can be specialized to the following expression

$$\varepsilon_i^K(K_1, K_2) = n - \frac{h_{ii}^I K_i h_{ii}^{I^\dagger}}{h_{ii}^I K_i h_{ii}^{I^\dagger} + h_{ij}^I K_j h_{ij}^{I^\dagger} + \sigma_{w,I,i}^2}. \quad (19)$$

Here we have used lower case letters for the channel matrices to emphasize that they can now be represented as row vectors. Energy harvested at the ERs are given as in (8).

We now consider Problem $\bar{P}1$. We recall that although the EH and power constraints form convex constraints, Problem $\bar{P}1$ is not convex. As seen in (19), the individual terms $\varepsilon_i^K(K_1, K_2)$, $i = 1, 2$ in the objective function are linear fractional functions in (K_1, K_2), hence they are not convex. In order to solve this non-convex optimization problem, we propose the following approach which utilizes the additional variable κ

$$(\bar{P}1_{SO}) \quad \min_{\kappa_L \leq \kappa \leq \kappa_U} \quad \min_{K_1 \succeq 0, K_2 \succeq 0} \quad \alpha_1 \varepsilon_1^K(K_1, K_2) + \alpha_2 \kappa \quad (20a)$$
$$\text{s.t.} \quad \varepsilon_2^K(K_1, K_2) \leq \kappa, \quad (20b)$$

subject to (10b) and (10c). Here $\kappa_L = n - 1$ and $\kappa_U = n$, which are the lower and upper bounds on the MMSE for a single-output antenna system with signal power $\text{tr}(K_{s_2}) = n$. We note that $(\bar{P}1_{SO})$ with the additional variable κ is equivalent to the Problem $\bar{P}1$; further discussions on this type of transformations can be found in [32, Ch. 4]. Here (20b) can be equivalently written as a linear inequality constraint

$$-h_{22}^I K_2 h_{22}^{I^\dagger} + \bar{\kappa} \left(h_{21}^I K_1 h_{21}^{I^\dagger} + \sigma_{w,I,2}^2 \right) \leq 0 \quad (21)$$

where $\bar{\kappa} \doteq (n - \kappa)/(1 - n + \kappa)$.

Let us now consider the inner minimization problem in Problem $\bar{P}1_{SO}$. After straightforward algebraic manipulations, it can be expresssed as follows for a given κ

$$\max_{K_1, K_2} \quad \frac{h_{11}^I K_1 h_{11}^{I^\dagger}}{h_{12}^I K_2 h_{12}^{I^\dagger} + \sigma_{w,I,1}^2} \quad (22)$$

subject to (10b), (10c), and (21). The objective function is still in linear fractional form, hence the problem is not convex. To obtain a convex formulation, we utilize Charnes-Cooper transform [38] and define the following new variables:

$$t = \left(h_{12}^I K_2 h_{12}^{I^\dagger} + \sigma_{w,I,1}^2 \right)^{-1}, \quad (23)$$
$$\bar{K}_1 = t K_1, \quad \bar{K}_2 = t K_2. \quad (24)$$

Rewriting the optimization problem in (22) in terms of these variables, we arrive at the following formulation

$$\max_{K_1 \succeq 0, K_2 \succeq 0, t \geq 0} \quad h_{11}^I \bar{K}_1 h_{11}^{I^\dagger}$$

$$\text{s.t.} \quad h_{12}^I \bar{K}_2 h_{12}^{I^\dagger} + t\sigma_{w,I,1}^2 = 1,$$
$$-h_{22}^I \bar{K}_2 h_{22}^{I^\dagger} + \bar{\kappa} \left(h_{21}^I \bar{K}_1 h_{21}^{I^\dagger} + t\sigma_{w,I,2}^2 \right) \leq 0,$$
$$\mathcal{J}_1(\bar{K}_1, \bar{K}_2) \geq t\gamma_1, \quad \mathcal{J}_2(\bar{K}_1, \bar{K}_2) \geq t\gamma_2,$$
$$\text{tr}[\bar{K}_1] \leq tP_1, \quad \text{tr}[\bar{K}_2] \leq tP_2. \quad (25)$$

We observe that under mild conditions it is possible to construct optimal precoders (A_1^*, A_2^*) from an optimal $(\bar{K}_1^*, \bar{K}_2^*)$:

Lemma 5.1: Let $n \geq 2$. Assume that the optimization problem in (25) and its dual are solvable. Then an optimal solution in terms of (A_1^*, A_2^*, t^*) can be always formed from an optimal solution $(\bar{K}_1^*, \bar{K}_2^*, t^*)$.

The proof is given in Appendix A. Now a solution to Problem $P1$ under the MISO scenario can be found by solving Problem $\bar{P}1_{SO}$. The solution to Problem $\bar{P}1_{SO}$ will be found using a line search over κ and the solution of inner optimization problem, i.e., (22) or equivalently (25). By Lemma 5.1, optimal precoders (A_1^*, A_2^*) can be found from an optimal solution of (25). The optimization problem in (25) is convex in $(\bar{K}_1, \bar{K}_2, t)$, hence it can be solved by using available solvers, such as [33]–[35]. We note that the inner optimization problem formulates a scenario which may be of independent interest. Here the error performance for one of the users is optimized under a performance guarantee for the other user.

VI. INTERFERENCE SCENARIO: MIMO INFORMATION CHANNEL

We now consider the general MIMO information channel scenario. We propose an alternating minimization technique for the solution of Problem $P1$. We first consider the fixed receiver estimator case in Section VI-A. In Section VI-B, we utilize this scenario to provide linear precoder designs for the MMSE receiver case.

A. Fixed Estimator at the Receiver

Let B_i be the estimator at IR_i. Hence the mean-square error at IR_i can be expressed as follows:

$$\varepsilon_i^F = E\left[\left\| s_i - B_i y_i^I \right\|^2 \right],$$
$$= \text{tr}[K_{s_i}] - \text{tr}\left[K_{s_i y_i^I} B_i^\dagger \right] - \text{tr}\left[B_i K_{s_i y_i^I}^\dagger \right] + \text{tr}\left[B_i K_{y_i^I} B_i^\dagger \right],$$
$$= n - \text{tr}\left[A_i^\dagger H_{ii}^{I^\dagger} B_i^\dagger \right] - \text{tr}\left[B_i H_{ii}^I A_i \right] + \text{tr}\left[B_i H_{ii}^I A_i A_i^\dagger H_{ii}^{I^\dagger} B_i^\dagger \right]$$
$$+ \text{tr}\left[B_i H_{ij}^I A_j A_j^\dagger H_{ij}^{I^\dagger} B_i^\dagger \right] + \sigma_{w,I,i}^2 \text{tr}\left[B_i B_i^\dagger \right].$$

where $i, j = 1, 2$, $i \neq j$. Hence, for fixed receiver filters, the problem of finding the optimal linear precoders in order to

minimize weighted sum of the estimation errors can be formulated as follows

$$\min_{A_1, A_2} \quad \alpha_1 \varepsilon_1^F + \alpha_2 \varepsilon_2^F \tag{26}$$

subject to (6b) and (6c). We note that the objective and the constraint functions are quadratic functions in (A_1, A_2), hence this is a quadratically constrained quadratic programming (QCQP) problem. In general, QCQP problems are known to be NP hard even for the formulations in which objective function is convex and there is only one vector optimization variable [39] as opposed to the more involved case of two matrix variables here.

As discussed earlier, the EH constraints are not convex in (A_1, A_2). To deal with these constraints, we introduce new variables $Z_i = A_i A_i^\dagger$, $i = 1, 2$. (Here we refrain from using the notation $K_i = A_i A_i^\dagger$ to avoid confusion with the previous formulations in Sections IV and V where it is possible to write the whole optimization problem in terms of K_i.) Ignoring the constant terms, we rewrite the part of the error that depends on (A_1, A_2, Z_1, Z_2) as follows

$$\varepsilon_i^z(A_1, A_2, Z_1, Z_2) = \text{tr}\left[B_i H_{ii}^I Z_i H_{ii}^{I\dagger} B_i^\dagger\right] + \text{tr}\left[B_i H_{ij}^I Z_j H_{ij}^{I\dagger} B_i^\dagger\right]$$
$$- 2\text{Re}\left(\text{tr}\left[A_i^\dagger H_{ii}^{I\dagger} B_i^\dagger\right]\right),$$

where $\text{Re}[z]$ denotes the real part of $z \in \mathbb{C}$. Hence the optimization problem in (26) can be reformulated as follows:

$$\min_{\substack{A_1, A_2, \\ Z_1, Z_2}} \quad \alpha_1 \varepsilon_1^z(A_1, A_2, Z_1, Z_2) + \alpha_2 \varepsilon_2^z(A_1, A_2, Z_1, Z_2) \tag{27a}$$

$$\text{s.t.} \quad \mathcal{J}_1^K(Z_1, Z_2) \geq \gamma_1, \quad \mathcal{J}_2^K(Z_1, Z_2) \geq \gamma_2, \tag{27b}$$

$$\text{tr}[Z_1] \leq P_1, \quad \text{tr}[Z_2] \leq P_2, \tag{27c}$$

$$Z_1 = A_1 A_1^\dagger, \quad Z_2 = A_2 A_2^\dagger. \tag{27d}$$

The constraints in (27d) are not convex, since they represent equality constraints involving a convex function of the variables. Except the constraints in (27d), all constraints are now linear functions of the variables (A_1, A_2, Z_1, Z_2), hence this formulation would constitute a convex problem if the equality constraints in (27d) were not there. We relax these as follows:

$$Z_i \succeq A_1 A_1^\dagger, \quad Z_2 \succeq A_2 A_2^\dagger. \tag{28}$$

By using Schur complement [32, A.5.5], one can equivalently write the expressions in (28) as linear matrix inequalities

$$S_1 = \begin{bmatrix} I & A_1^\dagger \\ A_1 & Z_1 \end{bmatrix} \succeq 0, \quad S_2 = \begin{bmatrix} I & A_2^\dagger \\ A_2 & Z_2 \end{bmatrix} \succeq 0. \tag{29}$$

Hence the relaxed version of the problem in (27) can be expressed as

$$\min_{\substack{A_1, A_2, \\ Z_1 \succeq 0, Z_2 \succeq 0}} \quad \alpha_1 \varepsilon_1^z(A_1, A_2, Z_1, Z_2) + \alpha_2 \varepsilon_2^z(A_1, A_2, Z_1, Z_2)$$

$$\text{s.t.} \quad \text{(27b), (27c), (29).} \tag{30}$$

This is a convex optimization problem, hence it can be solved efficiently by standard numerical optimization tools. We observe that since the optimization in the formulation in (30) is done over a larger set than the formulation in (27), solution of (30) provides a lower bound for the solution of (27). The next theorem shows that a stronger result is true.

Theorem 6.1: Let $n \geq 2$ where $s_i \in \mathbb{C}^{n \times 1}$, $i = 1, 2$. Let (30) be solvable. Then the optimum error values for (26) and the relaxed problem in (30) are equal and can be attained. Moreover, an optimal solution for (26) can be constructed from an optimal solution of (30).

The proof is given in Appendix B. This result shows that one can guarantee to find the optimal value for (27) (equivalently (26)) using (30) under solvability of the relaxed problem. Furthermore, a solution to the original problem (a solution satisfying (27d)) can be constructed from an optimal solution for the relaxed problem. Hence although fixed receivers problem is non-convex, Thm. 6.1 guarantees that it can be efficiently solved using a convex problem.

To find a solution for (27) from an optimal solution for (30), following approach is adopted. Let $V^* = (A_1^*, A_2^*, Z_1^*, Z_2^*)$ denote an optimal solution to (30). We output A_1^*, A_2^* as a solution to (27) if the following condition is satisfied,

$$\mathcal{J}_i(A_1^*, A_2^*) \geq \gamma_i, \quad i = 1, 2. \tag{31}$$

We note that it is guaranteed that the transmit power conditions are satisfied, and the error values are non-increasing if Z_i^* is replaced with $A_i^* A_i^{*\dagger}$, due to the conditions $Z_i \succeq A_i A_i^\dagger$, $i = 1, 2$. Together with the optimality of V^* for (30), (A_1^*, A_2^*) is optimal for (27). If (31) is violated, a rank constrained solution for (27) is generated using [27, Algorithm RED], [28, Algorithm 1]. Details can be found in Appendix B.

B. MMSE Estimator at the Receiver

We now consider the case where MMSE estimators are employed at the receivers. In order to solve the resulting non-convex problem, i.e. Problem $P1$, we propose a block coordinate-descent method where we take turns in fixing the precoder matrices and the estimators.

For fixed linear precoders (A_1, A_2), the problem of finding the MMSE estimators is the classical MMSE estimation problem, and the optimal B_i's are given by (3), [25, Ch. 2],

$$B_i = K_{s_i y_i^I} K_{y_i^I}^{-1}, \tag{32}$$

$$= A_i^\dagger H_{ii}^{I\dagger} \left(T_i + \sigma_{w,I,i}^2 I\right)^{-1}, \tag{33}$$

where $T_i = H_{ii}^I A_i A_i^\dagger H_{ii}^{I\dagger} + H_{ij}^I A_j A_j^\dagger H_{ij}^{I\dagger}$, and $i, j = 1, 2$, $i \neq j$ as before. To find linear precoders for fixed receivers, we solve the problem in (30). To initialize the algorithm, we solve the following problem which maximizes the energy harvested

$$\max_{A_1, A_2} \quad \mathcal{J}_1(A_1, A_2) + \mathcal{J}_2(A_1, A_2)$$

$$\text{s.t.} \quad \text{(6b), (6c).} \tag{34}$$

The resulting method is summarized in Algorithm 1.

Algorithm 1 Algorithm for Problem P1

Initialize:
    Solve (34) for (A_1^0, A_2^0)
    if ((34) is infeasible) **then**
        // *Problem P1 is infeasible.*
        Set $\varepsilon_1 = \text{tr}[K_{s_1}]$, $\varepsilon_2 = \text{tr}[K_{s_2}]$.
        Quit Algorithm 1.
    end if
    Using (A_1^0, A_2^0), solve (33) for (B_1^0, B_2^0).
    Set $i = 1$.
repeat
    Using (B_1^{i-1}, B_2^{i-1}), solve (30) for $(A_1^i, A_2^i, Z_1^i, Z_2^i)$.
    if (31) is not satisfied **then**
        Generate new (A_1^i, A_2^i) using [27, Algorithm RED].
    end if
    Using (A_1^i, A_2^i), solve (33) for (B_1^i, B_2^i).
    Using (A_1^i, A_2^i) and (4c), find $(\varepsilon_1^i, \varepsilon_2^i)$.
until $(\alpha_1 \varepsilon_1^{i-1} + \alpha_2 \varepsilon_2^{i-1} - (\alpha_1 \varepsilon_1^i + \alpha_2 \varepsilon_2^i) \leq \epsilon)$ // *The stopping criterion is met.*
Output: (A_1, A_2), $(\varepsilon_1^i, \varepsilon_2^i)$.

We now discuss the convergence of this method. At each fixed (A_1, A_2) step, the estimators are found optimally according to (33). As shown in Theorem 6.1, at each step where the receiver filters (B_1, B_2) are fixed, the problem can be optimally solved using a convex problem whenever the original problem is feasible. Hence the objective function is guaranteed to decrease under each iteration. Since the error is bounded from below, Algorithm 1 is guaranteed to converge. We note that due to non-convexity of Problem P1, the optimality of the proposed solutions obtained by Algorithm 1 cannot be guaranteed. Hence they give achievable, but possibly sub-optimal solutions. Nevertheless, for the MISO case, our numerical experiments illustrate that the optimal error values provided by Algorithm 1 coincides with the values provided by the approach discussed in Section V, which is designed specifically for the MISO case and reduces the problem to a line search. Hence with its consistent results in the MISO case, and the general convergence guarantee, Algorithm 1 offers a promising design framework for the joint linear precoder design problem.

We note that each step in the algorithm can be done in polynomial complexity, which includes the solution of the SDP [29], [40], finding a rank constrained solution whose complexity is dominated by the complexity of the solution of a system of linear equations [27, Algorithm RED-step (c)] and finding the estimators through (33) or by solving the corresponding system of linear equations. Required number of iterations is discussed in more detail in Section VIII.

VII. HYBRID TRANSMISSION STRATEGIES

Here we propose a power splitting scheme at the transmitter, where the transmitted signal is the superposition of two signals one of which is chosen to be a deterministic signal. In Section VIII, we illustrate that this hybrid scheme allows us to obtain significant improvements over sole Gaussian signalling, see also [9] for similar discussions for rate maximization. These results suggest that, even under the assumption of receivers that can simultaneously decode information and harvest energy, transmission strategies should be restructured in order to go beyond what Gaussian signalling offers.

We consider the following scheme as the transmission strategy

$$x_i = A_i s_i + g_i, \tag{35}$$

where $g_i \in \mathbb{C}^{n_t \times 1}$ is a deterministic signal that is known at the transmitters and the receivers. The sole purpose of g_i is to transfer energy, whereas the purpose of $A_i s_i$ is to transfer information and also possibly energy. The IRs will be able to perform the MMSE estimation after removing the known interference g_i's. Hence here g_i's do not degrade the performance of the unintended IRs as opposed to the case of using all the power for sending information signal. On the other hand, g_i uses some of the power that could have been allocated to the information signal at the transmitter i, which may degrade the error performance at IR$_i$.

To find the optimal power allocation trade-off between the energy components (g_i) and information carrying components $(A_i s_i)$, along with the optimal waveforms (g_1, g_2) and the precoders (A_1, A_2), we formulate an optimization problem similar to Problem P1. The power constraint at transmitter i takes the following form

$$\text{tr}\left[g_i g_i^\dagger\right] + \text{tr}\left[A_i A_i^\dagger\right] \leq P_i, \quad i = 1, 2. \tag{36}$$

The energy harvesting constraints can be expressed as follows

$$\mathcal{J}_i(A_1, A_2) + \mathcal{J}_i(g_1, g_2) \geq \gamma_i, \quad i = 1, 2. \tag{37}$$

The resulting optimal precoder design problem is the following

$$(\text{P1}_{GD}) \quad \min_{g_1, g_2, A_1, A_2,} \quad \alpha_1 \varepsilon_1(A_1, A_2) + \alpha_2 \varepsilon_2(A_1, A_2)$$

$$\text{s.t.} \quad (36), (37). \tag{38}$$

We note that Problem P1 can be considered as a special case of Problem P1$_{GD}$ with $g_1 = 0$, $g_2 = 0$. Here the MMSE estimators are again given by (33) since the estimation is done after removing the known interference g_i. We note that g_i's can be calculated by the IRs using the channel state information of the nodes in the system, which is also used for implementing the MMSE estimation.

We adopt the two step approach in Section VI to solve Problem P1$_{GD}$. For the fixed estimator step where the receivers use fixed filters B_i, $i = 1, 2$, the following problem is considered

$$\min_{A_1, A_2, g_1, g_2} \quad \alpha_1 \varepsilon_1^F + \alpha_2 \varepsilon_2^F \tag{39}$$

subject to (36) and (37). Introducing the variables $G_i = g_i g_i^\dagger$, $Z_i = A_i A_i^\dagger$, (36) can be expressed as follows

$$\text{tr}[G_i] + \text{tr}[Z_i] \leq P_i, \quad i = 1, 2. \tag{40}$$

The energy harvesting constraints can be written as follows

$$\mathcal{J}_i^K(Z_1, Z_2) + \mathcal{J}_i^K(G_1, G_2) \geq \gamma_i, \quad i = 1, 2. \quad (41)$$

Hence (39) can be expressed as follows

$$\min_{\substack{A_1, A_2, Z_1, Z_2, \\ g_1, g_2, G_1, G_2}} \quad \alpha_1 \varepsilon_1^F + \alpha_2 \varepsilon_2^F$$

$$\text{s.t.} \quad (40), (41)$$

$$G_i = g_i g_i^\dagger, \quad Z_i = A_i A_i^\dagger, \quad i = 1, 2. \quad (42)$$

Using the relaxation in (28), (or equivalently (29)) and lifting the rank constraint on G_i's, we relax this problem as follows

$$\min_{\substack{A_1, A_2, \\ Z_1 \geq 0, Z_2 \geq 0, \\ G_1 \geq 0, G_2 \geq 0}} \quad \alpha_1 \varepsilon_1^z(A_1, A_2, Z_1, Z_2) + \alpha_2 \varepsilon_2^z(A_1, A_2, Z_1, Z_2) \quad (43)$$

subject to (29), (40), (41). The following relationship holds between the solutions of the original fixed estimator problem (39) and the relaxed problem (43):

Theorem 7.1: Let $n \geq 2$. Let (43) be solvable. Then (39) has the same optimal value with (43). An optimal solution for (39) can be constructed from a solution of (43).

The proof is given in Appendix C. The above result shows that one can effectively solve (43) instead of (39) for the fixed estimator problem. Now the two step procedure in Algorithm 1 can be modified by replacing the step that solves (30) with a step that solves (43) to find designs for the MMSE receiver problem with hybrid signalling in (38). Similar to the sole Gaussian signalling case, the error is non-increasing at each iteration, hence the modified algorithm is guaranteed to converge.

VIII. NUMERICAL RESULTS

We now illustrate the performance of our designs and the trade-off between the error and the energy harvested through numerical results. The error performance is reported as the weighted sum of normalized MMSEs as follows:

$$\bar{\varepsilon} = \frac{\alpha_1 \varepsilon_1 + \alpha_2 \varepsilon_2}{\varepsilon_0}, \quad (44)$$

where $\varepsilon_0 = \text{tr}[K_{s_1}] = \text{tr}[K_{s_2}] = n$. We choose $\alpha_1 = \alpha_2 = 1$. When the problem is not feasible, i.e. the EH constraints cannot be satisfied under the given power constraints, the transmission does not occur; hence the error values are set to $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$. We assume that the energy and information receivers are co-located, $H_{ij}^I = H_{ij}^E = H_{ij}$, $i, j = 1, 2$. We generate the channel matrices independently with i.i.d. complex proper zero-mean Gaussian components with variance $\sigma_H^2 = 1$. We report the average results for 100 channel realizations. We set $\epsilon = 10^{-5}\varepsilon_0$ and SNR = 10 dB, where SNR is defined as σ_H^2/σ_w^2 with $\sigma_{w,I,i}^2 = \sigma_{w,E,i}^2 = \sigma_w^2$, $i = 1, 2$. The EH constraints are set as $\gamma_1 = \gamma_2 = \gamma$ (Watts). We assume that the system parameters, including the power constraints and the EH constraints, are scaled to the proper ranges. Discussions on the admissible values can be found in [3], [8]. The convex optimization problems including (30), (34), and (43) are solved using [33]–[35]. Convergence behavior of Algorithm 1 is further discussed at the end of this section.

We label the transmission strategies as follows: TX_G is the proposed design for Problem P1 (Gaussian signalling) found by the approach in Section VI. TX_{GD} is for the hybrid signalling framework in Problem P1$_{GD}$ (Gaussian+deterministic signalling) found by the approach in Section VII. We also compare the performance of our joint design strategy with that of individual design where transmitters decide on their transmission strategies independently without any cooperation (TX_{IND}). Here each transmitter assumes there is no interference and aims to minimize the MMSE at its designated IR under the EH constraint at its designated ER.

We also compare the performance of our designs with that of TDMA and TDMS of [7]. In both schemes, transmission interval is divided into two time slots. In TDMA, for $0 \leq t_a \leq 1$ fraction of the time, the system operates in (I, E) mode: Receiver 1 operates in the information decoding (ID) mode whereas Receiver 2 operates in the EH mode. In the remaining $1 - t_a$ fraction of the time, the operating mode is (E, I) where the roles of the receivers are swapped. In TDMS, for $0 \leq t_s \leq 1$ fraction of time, both receivers operate in the EH mode (E, E). In the remaining $1 - t_s$ fraction of time, both receivers operate in the ID mode (I, I). As previously mentioned, these comparisons are motivated by the fact that schemes that depend on such mode separations have been considered as practical benchmarks in the context of rate maximization [7]–[10]. Here we adopt these schemes to the MMSE minimization problem. For TDMA, transmitters adopt deterministic signalling when their intended receiver operates in the EH mode. For optimization over the parameter t_a, we adopt a line search over the set $S_{t_a} = \{0.05 \, k : k = 0, \dots, 20\}$. We note that Algorithm 1 can be used to find the transmission strategy for the transmitter serving to both IR and ER receiver, for instance transmitter 1 in (I, E) mode. Nevertheless, for the sake of reduced computation time, we have preferred to solve the relaxed problem in (16). Hence TDMA curves we present here are lower bounds on TDMA performance on S_{t_a}. For TDMS, optimum time-sharing parameter t_s is found using a convex optimization approach similar to [7, Sec. IV-A].

We first study the MMSE performance difference between the rate maximization and the mean-square error minimization problems. Our aim is to illustrate that these two metrics, (rate and the MMSE) are although closely related, lead to different MMSE performance trade-offs. Without the EH constraints, it is known that the MMSE minimization and rate maximization problems have different optimal solutions, see for instance [28, Table 3.1]. The difference in the form of optimal solutions under EH constraints can be seen by comparing Thm. 4.1 here and [3, Thm. 3.1]. Here we present a numerical qualification of the resulting performance difference. We focus on the P2P case with $n_t \leq n$ so that the results can be fully attributed to the difference between the optimal solutions without any reference to possibly sub-optimal approaches we will have to refer to in the case of interference channel and the rank constrained scenarios. Hence, for the MMSE minimization, we consider the formulation in (15). For the rate maximization problem, we consider the following problem

$$\max_{K_1 \geq 0} \quad \log\left(\left\| I + \frac{1}{\sigma_{w,I,1}^2} H_{11}^I K_1 H_{11}^{I\dagger} \right\|\right) \quad (45)$$
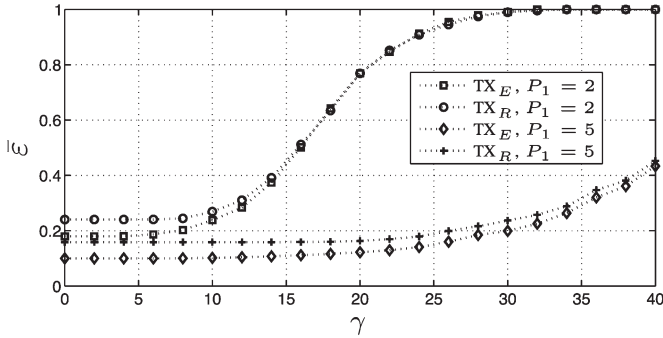
Fig. 2. MMSE versus energy harvesting requirements, comparison of the MMSE performance of the rate maximization and the MMSE minimization schemes.
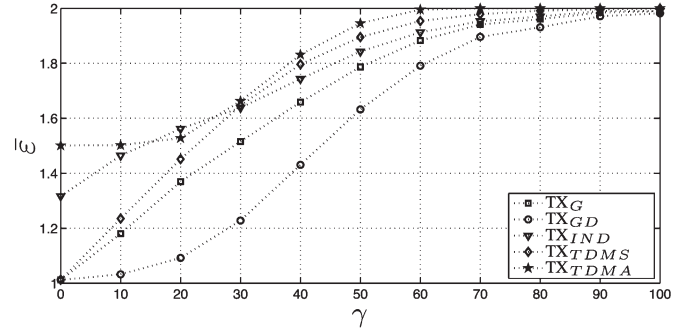

Fig. 4. MMSE versus energy harvesting requirements, $n = 2$, $n_t = 4$, $n_r = n_e = 1$, $(P_1, P_2) = (2, 18)$.
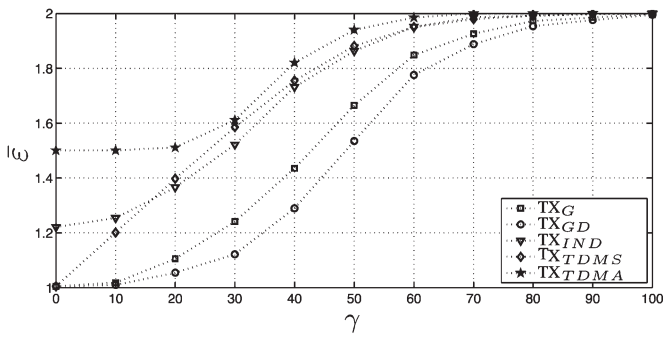

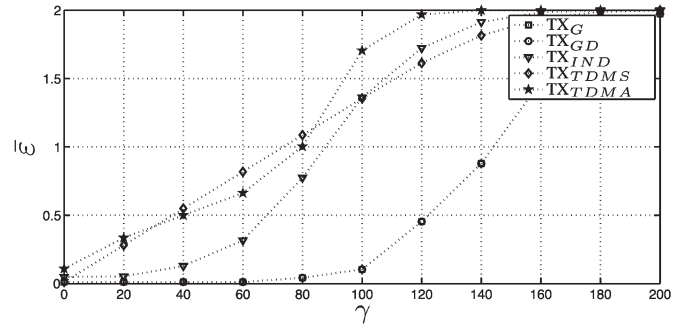Fig. 3. MMSE versus energy harvesting requirements, $n = 2$, $n_t = 4$, $n_r = n_e = 1$, $(P_1, P_2) = (10, 10)$.

Fig. 5. MMSE versus energy harvesting requirements, $n = 2$, $n_t = 4$, $n_r = n_e = 4$, $(P_1, P_2) = (10, 10)$.

Fig. 6. MMSE versus energy harvesting requirements, $n = 2$, $n_t = 4$, $n_r = n_e = 4$, $(P_1, P_2) = (2, 18)$.

under the conditions $\mathrm{tr}[H_{11}^E K_1 H_{11}^{E^\dagger}] \geq \gamma_1$ and $\mathrm{tr}[K_1] \leq P_1$. This problem is studied in [3]. Both of these problems are convex and are solved using [33]–[35]. The MMSE performance of the transmission schemes using the resulting optimum transmit covariance matrices K_1 are presented in Fig. 2 averaged over different channel realizations. Here TX_E and TX_R correspond to the performance of the solution of (16) and (45), respectively. We have $n_t = n_i = n_e = n = 4$. We observe that the relative difference is substantial under small and moderate EH constraints. To quantify this, let us define the relative performance difference ratio $r = 100(\varepsilon_{TX_R} - \varepsilon_{TX_E})/\varepsilon_{TX_E}$, where ε_{TX_S} is the error associated with transmission strategy $S \in \{R, E\}$. For instance, for the EH constraint $\gamma = 8$, the relative performance difference is $r \approx 20\%$ and $r \approx 58\%$ for $P_1 = 2$ and $P_1 = 5$, respectively. With more demanding EH constraints there is little room for error minimization or rate maximization, hence the performance gap gets smaller.

We now consider the MISO interference scenario with $n = 2$, $n_t = 4$, $n_r = n_e = 1$. Figs. 3 and 4 show the error versus the EH constraint curves for $(P_1, P_2) = (10, 10)$ and $(P_1, P_2) = (2, 18)$, respectively. We observe that as expected, for all power budget pairs, error increases as the EH requirements become more demanding. TDMA and TDMS approaches exhibit significantly weak performance compared to TX_G and TX_{GD}, especially for low to moderate EH values. This effect is particularly prominent for the unbalanced power budget case (Fig. 4). These observations confirm the need for the design of novel transmission strategies. The plots also illustrate that the joint design schemes TX_G and TX_{GD} perform substantially better than the
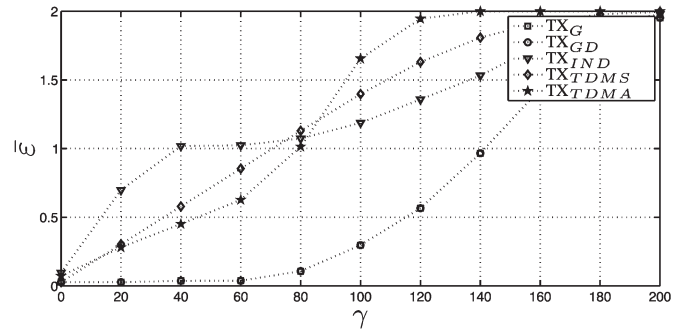
independent design scheme TX_{IND}. Comparing the performance of TX_G and TX_{GD} for fixed transmission power budget, we observe that significant gains can be obtained by adopting the hybrid scheme. Comparing the results in Figs. 3 and 4 we observe that it is possible to obtain lower error values when the power budget pairs are more balanced, i.e., both of the users have equal or close transmission power budgets. This is consistent with the fact that channel conditions are symmetric and the EH demands are equal.

We now consider the MIMO channel case with $n = 2$, $n_t = 4$, $n_r = n_e = 4$. Figs. 5 and 6 show the error versus the EH constraint curves for $(P_1, P_2) = (10, 10)$ and $(P_1, P_2) = (2, 18)$, respectively. Compared to the previous MISO scenarios, for all transmission strategies, it is observed that it is possible to obtain lower values of error for a given EH constraint. This performance improvement is consistent with the higher number of degrees of freedom offered by the multiple antennas at the
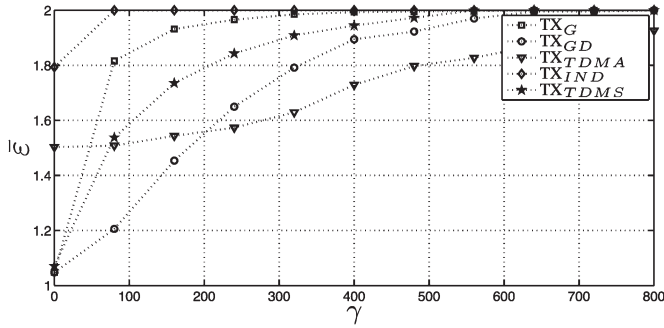
Fig. 7. MMSE versus energy harvesting requirements. High interference scenario.

Fig. 8. Convergence behavior of Algorithm 1.

TABLE I
AVERAGE NUMBER OF ITERATIONS FOR ALGORITHM 1

	TX_G	TX_{GD}
$n_r = n_e = 1$, $(P_1, P_2) = (10, 10)$	5.14	15.45
$n_r = n_e = 1$, $(P_1, P_2) = (2, 18)$	5.89	15.44
$n_r = n_e = 4$, $(P_1, P_2) = (10, 10)$	74.92	75.58
$n_r = n_e = 4$, $(P_1, P_2) = (2, 18)$	105.59	109.37

receivers. Due to these extra degrees of freedom, the transmitters can better shape their transmissions so that the interference to the unintended information receivers can be kept low. This also contributes to the decreasing performance difference between TX_G and TX_{GD} as the number of antennas increases; TX_{GD} does not offer any significant gains over TX_G for $n_r = 4$. The extra antennas at the ERs also allow the receivers to harvest the energy in the signals that can arrive at the receiver through these extra paths, so higher values of energy can be harvested.

We study the effect of the level of cross-interference on the trade-offs in Fig. 7. To quantify the level of cross-interference, a scaling parameter μ is used where the cross-channel matrices are scaled as μH_{12} and μH_{21}. We set $n = 2$, $n_t = 2$, $n_r = n_e = 1$, $(P_1, P_2) = (10, 10)$ and $\mu = 4$. We observe that TDMA can outperform both TX_G and TX_{GD} and TDMS can outperform TX_G. (Since there are no rank constraints, here TDMA performance is the true performance on the set S_{t_a} rather than a lower bound.) We note that the superior performance of TDMA is not a characteristics specific to the energy harvesting problem. Under heavy interference, better MMSE values can be obtained by the TDMA approach compared to the optimized signalling, even when there are no EH constraints. This can be seen, for instance, by considering the scalar channel case and letting the cross-link powers go to infinity. On the other hand, TDMS cannot outperform TX_{GD}. This is an analytical property of TDMS and TX_{GD}: Let $0 \leq t_s \leq 1$ be the fraction of time spent in energy transfer in TDMS. Then any error value that is achievable by TDMS using t_s will be also achievable by TX_{GD} by using t_s fraction of the power available for energy transfer with deterministic signalling.

The convergence behavior of Algorithm 1 is illustrated in Fig. 8. The MMSE versus iteration index curves are presented for $P_1 = P_2 = 10$ for $n = 2$, $n_t = 4$, $n_r = n_e$ for TX_G and TX_{GD} for $\gamma = 50$ and $\gamma = 100$, for $n_r = 1$ and $n_r = 4$, respectively. For each case, curves for three different channel realizations are plotted. In Table I, we also give the average number of iterations for the scenarios presented in Figs. 3–6, where the stopping tolerance is $\epsilon = 10^{-5} \epsilon_0$. The convergence is observed to be pretty rapid, especially in the $n_r = 1$ scenario. In general, the EH constraints also affect the number of iterations. For $n_r = 1$, $P_1 = P_2 = 10$, TX_G shows no significant dependence on γ, whereas the average number of iterations increases as γ increases for TX_{GD}. For instance, for TX_{GD} approximately 8 and 17 iterations are needed on average for $\gamma = 20$ and

$\gamma = 60$. For $n_r = 4$, $P_1 = P_2 = 10$, the general behaviour of both schemes are the same; the average number of iterations decreases as γ increases. For instance, approximately 90 and 60 iterations are needed on average for $\gamma = 40$ and $\gamma = 160$.

IX. CONCLUSION

We have considered the problem of linear precoder design with the aim of minimizing the sum MMSE in MIMO interference channels with energy harvesting constraints. In the case where there is only one user, i.e. for the P2P channel, the problem reduces to a convex problem under a constraint on the transmit antennas. For this case, we have provided a characterization of the optimum solutions. For the general interference scenario, the problem leads to a non-convex formulation for the solution of which we have proposed an efficient numerical approach. We have also investigated a hybrid signalling scheme, where the transmitters send a superposition of two signals: a deterministic signal optimized for energy transfer and an information carrying signal optimized for information and energy transfer. It is illustrated that hybrid signalling offers significant gains over sole Gaussian signalling when the number of antennas at the receivers are relatively small.

APPENDIX A
PROOF OF LEMMA 5.1

Let us consider the inner optimization problem in (25) obtained by fixing t, which is a SDP problem with six constraints (other than positive semi-definiteness constraints). By [28, Thm. 3.2], there exists an optimal solution which satisfies $\text{rank}(\bar{K}_i^*)^2 + \text{rank}(\bar{K}_2^*)^2 \leq 6$. Hence $\text{rank}(\bar{K}_i^*) = \text{rank}(K_i^*) \leq 2$. Hence under the condition $n \geq 2$, an $A_i^* \in \mathbb{C}^{n_t \times n}$ satisfying $K_i = A_i^* A_i^{*\dagger}$ can be always formed. (We note that the condition $n \geq 2$ is merely a sufficient condition that guarantees existence of an admissible A_i.) Such a solution can be constructed

using [27, Algorithm RED], [28, Algorithm 1]. We finish the proof by noting that since this above argument is true for an arbitrary t, it is also true for an optimal t^*. We note that if the optimal solutions have rank smaller than n, during linear precoding only rank(K_i) of the data streams will be sent. This is similar to water-filling type solutions for MMSE minimization under sole transmission power constraints, see for instance [16] for further discussions.

APPENDIX B
PROOF OF THM. 6.1

The proof relies on considering the individual optimization problems in (26) obtained by fixing A_1 or A_2, and the results on semi-definite programming relaxations of QCQP problems with one matrix variable. Let $(A_1^*, A_2^*, Z_1^*, Z_2^*)$ be an optimal solution of (30). Let us consider (26) with fixed A_2^*, Z_2^*

$$\min_{A_1} \quad f_1(A_1)$$

$$\text{s.t.} \quad \text{tr}\left[A_1^\dagger H_{i1}^{E\dagger} H_{i1}^E A_1\right] \geq \bar{\gamma}_i, \quad i = 1, 2$$

$$\text{tr}\left[A_1 A_1^\dagger\right] \leq P_1, \tag{46}$$

where

$$f_1(A_1) = \text{tr}\left[A_1^\dagger C_1 A_1\right] - 2\alpha_1 \text{Re}\left[\text{tr}\left[A_1^\dagger H_{11}^{I\dagger} B_1^\dagger\right]\right],$$

$$C_1 = \alpha_1 H_{11}^{I\dagger} B_1^\dagger B_1 H_{11}^I + \alpha_2 H_{21}^{I\dagger} B_2^\dagger B_2 H_{21}^I$$

and $\bar{\gamma}_i = \gamma_i - \text{tr}[H_{i2}^{E\dagger} H_{i2}^E Z_2^*]$, $i = 1, 2$. Hence for fixed A_2, (26) is a QCQP problem with a matrix variable and three constraints.

We observe that (30) for fixed A_2^*, Z_2^* can be alternatively written in terms of the positive semi-definite variable S_1 instead of A_1, Z_1. Hence (30) for fixed A_2, Z_2 is in fact the SDP relaxation of (46). (One may refer to [27, 2.7] for the general form of the SDP relaxation of a QCQP problem with matrix variables.) By [27, Thm 2.2], (46) and its SDP relaxation have the same optimal value if the relaxation is solvable and the number of constraints is equal to or smaller than $2n$. Here the SDP relaxation of (46) is guaranteed to be solvable, since the bi-variate relaxation (30) is assumed to be solvable, and A_2^*, Z_2^* is an optimal solution. We observe that the dual of (30) (for fixed A_2^*) is strictly feasible since the regularity condition in [27, 2.10] holds. (This is due to the fact that the matrix associated with the power constraints, identity, is positive definite.) Hence together with the feasibility of (46), this implies solvability of SDP relaxation [27, Cor. 2.1].

One can utilize the same arguments for the optimization over A_2 for fixed A_1^* and its relaxation. We also note that the optimum value for (26) can be found by first optimizing over one variable, and treating the other one fixed, and then optimizing over the second variable. First part of Thm. 6.1 follows from these observations and the above arguments. For the second part, we observe the following: An optimal solution for (26) can be constructed from a solution of (30) using [27, Algorithm RED] (or similarly [28, Algorithm 1]) on S_1 and S_2 and by considering the sub-problems for fixed (A_2, Z_2) and (A_1, Z_1). Due to

[27, Lemma 2.1], desired solution for A_i is given by the lower left $n_t \times n$ matrix of the rank-constrained S_i. We note that by construction these algorithms guarantee these sub-matrices satisfy the constraints of the original problem and do not degrade the objective function.

APPENDIX C
PROOF OF THM. 7.1

The proof adopts the same arguments in the proof of Thm. 6.1 in Appendix B. Here we highlight the main differences. Let (30) be solvable, and $V^* = \{A_1^*, A_2^*, Z_1^*, Z_2^*, G_1^*, G_2^*\}$ be an optimal solution. We consider the following feasibility problem over G_1 when the other variables are kept fixed

$$\min_{G_1 \succeq 0} \quad 0 \tag{47}$$

subject to $\text{tr}[G_1^\dagger] \leq \bar{P}_1$ and $\mathcal{J}_i^K(G_1, 0) \geq \bar{\gamma}_i$, $i = 1, 2$. Here $\bar{\gamma}_1$, $\bar{\gamma}_2$, and $\bar{P}_1$ are the modified values of the constraints found by using the optimum values of the variables other than G_1^*, i.e., $V^* \setminus \{G_1^*\}$. This is a homogeneous QCQP problem with three constraints. We note that (47) is solvable, since (43) is solvable. Hence by [28, Thm 3.2], [27, Thm 2.1], there exists a solution for which rank(G_1) ≤ 1. This solution can be constructed by [27, Algorithm RED], [28, Algorithm 1]. By considering the sub-problems obtained by fixing the other variables and utilizing Theorem 6.1, the result follows.

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