bols) and with a simple zero-dimensional model (disconnected symbols), taking into account both the main capacitance and the capacitance of the bottom patch to ground. The agreement obtained is clearly good over a broad frequency range, thus demonstrating the effectiveness of the proposed model.

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Inductorless realisation of Chua oscillator

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Indexing terms: Chua circuit, Chaos

An inductorless realisation of a Chua oscillator, which exhibits chaotic behaviour is presented. This new realisation consists of the Wien bridge oscillator, coupled in parallel with the same nonlinear resistor used in the standard realisation of a Chua oscillator. This new circuit is shown experimentally to also exhibit similar chaotic behaviour.

Introduction: Nonlinear systems may exhibit many types of complex behaviour such as chaos, and this complexity has attracted many scientists from various fields (e.g. physics, mathematics, engineering etc.) to study such systems. Since it has been shown that simple electronic circuits may exhibit chaotic behaviour, the study of chaos in nonlinear electronic circuits received a great deal of attention in the last decade. (See IEEE Trans. Circuits Syst., special issue on chaos in nonlinear electronical circuits, Part 1, October and November 1993, and Part 2, October 1993). Among such electronic circuits, the Chua oscillator, shown in Fig. 1, received a great deal of attention since it is quite simple, well-studied and can easily be realised in the laboratory using standard electronic components.

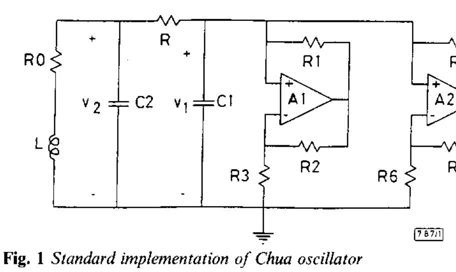

Fig. 1 *Standard implementation of Chua oscillator*

In Fig. 1, the operational amplifiers and the associated resistances ($R_1,...,R_6$) are used to realise the three-segment piecewise linear resistance called a Chua diode, see [1]. The resistor R_0 represents the internal resistance of the inductor L and is used in the simulations, but not in the actual implementation [1]. For $R_0 = 0$ and $R_0 > 0$, this circuit is called a Chua circuit and a Chua oscillator, respectively [2]. The resistor R is a potentiometer and can be used to tune the circuit for observing chaotic behaviour.

For most of the electronic circuits, the inductor is a less desirable circuit element. There are various reasons for this. For example, inductors are less standard compared to other circuit elements and must be prepared separately in most applications, they are not as ideal as other circuit elements, and in terms of spatial dimensions they are larger than the other circuit elements, unless the inductance is rather small. We present an inductorless realisation of a Chua oscillator. We replace the parallel/series combination of R_0, L and C_2 by a Wien-bridge oscillator type circuit and show that this circuit may exhibit chaotic behaviour similar to that of the Chua oscillator.

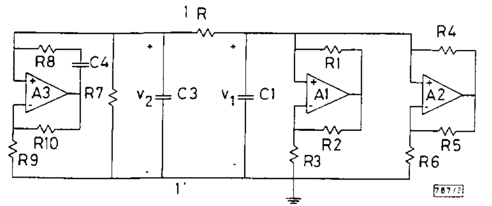

Fig. 2 *New implementation of Chua oscillator*

New realisation of Chua oscillator: We consider the circuit shown in Fig. 2. Compared with Fig. 1, the parallel/series combination of R_0, L and C_2 has been replaced by a Wien-bridge oscillator type circuit. The impedance Z_1 of the parallel/series combination of R_0, L and C_2 is given by

$$Z_1 = \frac{sL + R_0}{s^2LC_2 + sC_2R_0 + 1} \qquad (1)$$

By using the condition

$$R_7R_{10} = R_8R_9 \qquad (2)$$

and assuming that the operational amplifier A3 is ideal and operates in the linear region, it can easily be shown that the impedance Z_2 of the corresponding circuit in Fig. 2 (i.e. to the left of $1 - 1'$) is given as

$$Z_2 = \frac{sC_4R_7R_8 + R_7}{s^2C_3C_4R_7R_8 + sC_3R_7 + 1} \qquad (3)$$

Since the denominator of eqn. 3 is not in the form $s^2 + \omega_0^2$, the Wien bridge is not in the oscillatory mode. It can easily be shown that the relevant oscillation condition for this circuit is $R_9(C_4R_8 + C_3R_7) = C_4R_7R_{10}$, and this condition can not be satisfied when eqn. 2 holds. Furthermore, eqn. 2 is satisfied when $R_7 = R_9$, $R_8 = R_{10}$.

Given L, C_2 and R_0, if we choose $R_7,...,R_{10}$, C_3 and C_4 so that eqn. 2 and

$$R_7 = R_0 \qquad C_3 = C_2 \qquad C_4R_7R_8 = L \qquad (4)$$

are satisfied, then by comparing eqn. 1 and eqn. 3 we find that $Z_1 = Z_2$. Hence, we expect that the circuits in [1, 2] behave similarly. Since the Chua oscillator in Fig. 1 exhibits chaotic behaviour (e.g. double scroll attractor), we expect similar behaviour for the circuit in Fig. 2. We show that this is indeed the case.

Experimental results: For the Chua oscillator, chaotic behaviour is observed for the following set of element values: $R_0 = 12.5\Omega$, $R_1 = 3.3k\Omega$, $R_2 = R_3 = 22k\Omega$, $R_4 = 2.2k\Omega$, $R_5 = R_6 = 220\Omega$, $C_1 = 10nF$, $C_2 = 100nF$, $L = 18mH$, $R = 1730\Omega$, and A1 and A2 are operational amplifiers (AD712 or equivalent). These values are taken from [1]. For this circuit, chaotic behaviour can be observed over a relatively large range of R.

For the circuit of Fig. 2, we used the same values for the resistors $R_1,...,R_6$ to realise the same three-segment picewise linear resistor. To determine the remaining element values, we first used the values of [1] and eqn. 4. However, by using the resulting element values, we did not observe chaotic behaviour in our experiments. For the following set of element values: $R_7 = R_9 = 120\Omega$, $R_8 = R_{10} = 680\Omega$, $C_1 = 1nF$, $C_3 = 15nF$ and $C_4 = 33nF$, we obtained various forms of chaotic behaviour for different values of R in the range $1269\Omega \le R \le 1600\Omega$. In Figs. 3 and 4, the waveforms of v_1 and v_2 and the v_2 against v_1 characteristics are given for $R = 1600$ and 1494Ω, respectively. All element values are standard and have a 10% tolerance, except for R, which is a (2 kΩ) potentiometer, and A1, A2 and A3, which are operational amplifiers (AD 712 or equivalent).

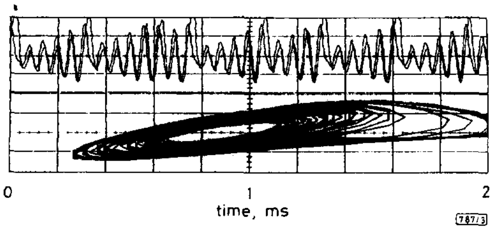

Fig. 3 *$V_2 - V_1$ characteristics for $R = 1600\Omega$*

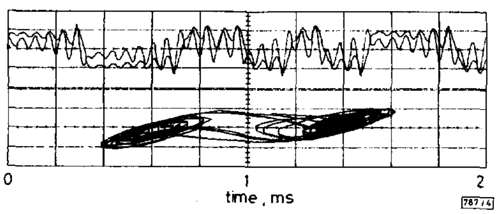

Fig. 4 *$V_2 - V_1$ characteristics for $R = 1494\Omega$*

To obtain these figures, we first observed the $v_2 - v_1$ characteristics on an analogue oscilloscope in the X – Y mode. The same figure is then obtained on a digitising oscilloscope (HP 54502A). After storing the screen image in the memory of the oscilloscope, the screen is printed on a plotter (HP 7475A) by using an HB-IB bus. Since the important characteristics indicating chaotic motion are those of $v_2 - v_1$, we do not label the signals v_1 and v_2.

We make the following comments:

(i) Figs. 3 and 4 indicate different types of strange attractor. The attractor shown in Fig. 4 is known as a double-scroll attractor, which is a characteristic of the Chua oscillator.

(ii) In our experiments, we observed chaotic behaviour for various values of C_1, C_3 and C_4 in the range $C_1 \leq 1.5$nF, $C_3 \leq 33$nF and $C_4 \leq 47$nF.

(iii) In the experiments described above, the Wien bridge oscillator is not used in the oscillatory mode. It is possible to obtain chaotic behaviour in the proposed circuit when the Wien bridge is used in the oscillatory mode. This makes the proposed circuit very flexible for obtaining different chaotic behaviours.

Conclusions: We have presented an inductorless realisation of a Chua oscillator. We replaced the parallel combination of inductor (with its internal resistance) and capacitor by a Wien-bridge oscillator. We showed by laboratory experiments that with appropriate element values, this new circuit exhibits varied chaotic behaviour including a double scroll attractor. Since this new circuit does not contain an inductor, its practical realisation will occupy less volume than the circuit given in Fig. 1. Hence this new circuit is more suitable for integrated circuit applications. Furthermore, this circuit has more parameters that can be varied, which makes it more flexible for obtaining different chaotic behaviours.

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Algorithm for superstate matrix from trellis diagram

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Indexing term: Trellis coded modulation

The authors discuss an algorithm for constructing an $s^2 \times s^2$ superstate matrix from the s-state trellis diagram and an $s \times s$ reduced superstate matrix directly from the $s^2 \times s^2$ superstate matrix for trellis codes possessing symmetry properties. The algorithm can also be applied to an MTCM scheme, and is especially convenient for trellis codes with a large number of states and/or parallel transitions.

Introduction: The analytical performance of the trellis-coded modulation (TCM) scheme is widely derived by using upper bounds based on the generalised transfer function $T(I)$. To find $T(I)$, we must first construct an $s^2 \times s^2$ superstate (transition) matrix $\mathbf{A}(I)$ from the s-state trellis diagram [1]. However, for codes with a large number of states and a fully-connected trellis, the procedures become quite complicated and tedious. For codes with a large number of parallel transitions, such as multiple trellis-coded modulation (MTCM) schemes, the matter becomes even worse.

The purpose of this Letter is to provide an algorithm in which the procedure of constructing a superstate matrix for codes with a large number of states and/or parallel transitions is reduced and, as a result, make it easier to evaluate the performance of a TCM scheme.

Inputs of algorithm: The algorithm requires the following inputs: the number of states (s), the size of the M-PSK signal constellation (M), the number of parallel transitions (N_p), the number of input bits (b) and the multiplicity of the code (k^*). The algorithm also needs two input arrays consisting of a description of the trellis diagram and the labels of the $s^2 \times s^2$ matrix $\mathbf{A}(I)$, which are denoted by *trellis*[*s*][*s*][N_p][$b + k^*$] and *sm_label*[$s \times s$][2], respectively. As an example, consider a two-state rate 2/4 ($k^* = 2$) 4-PSK MTCM scheme. The trellis diagram for the example is given in [1]. For this case, $s = 2$, $M = 4$, $Np = 2$, $b = 2$ and $k^* = 2$. The array *trellis* and the array *sm_label* should be given as follows:

```
static char trellis[2][2][2][4] = {
    { { {0,0,0,0}, {0,1,2,2}},      "state 0 → state 0"
      { {1,0,0,2}, {1,1,2,0}} },    "state 0 → state 1"
    { { {0,0,1,3}, {0,1,3,1}},      "state 1 → state 0"
      { {1,0,1,1}, {1,1,3,3}} },    "state 1 → state 1"
};
static int sm_label[4][2] = {
        {0,0}, {1,1}, {0,1}, {1,0}
};
```

On the basis of the trellis diagram, if there are no possible transitions between states (e.g. a half-connected trellis), character 'X' is assigned to the corresponding positions of the array *trellis*.

Algorithm for constructing matrix $A(I)$: The following algorithm is for the special case when $k^* = 2$:

(i) *flag* = 0;
for *row* = 0 to ($s \times s - 1$) and *column* = 0 to ($s \times s - 1$) do steps (ii) to (ix);

(ii) select a pair of transitions from the labels of the matrix $\mathbf{A}(I)$:
GPS = *sm_label*[*row*][0]; GNS = *sm_label*[*row*][0];
BPS = *sm_label*[*column*][1]; BNS = *sm_label*[*column*][1];

(iii) for $i_1 = 0$ to ($N_p - 1$) and $i_2 = 0$ to ($N_p - 1$) do steps (iv) to (vi);

(iv) set $d_H = 0$:
for $j_1 = 0$ to ($b - 1$) do
bit_G = *trellis*[*GPS*][*GNS*][i_1][j_1]; *bit_B* = *trellis*[*BPS*][*BNS*][i_2][j_1];
if *bit_G* or *bit_B* is character 'X', then *sm_table*[*row*][*column*] = 'X':
else if *bit_G* is not equal to *bit_B* then *dh* = *dh* + 1;