GÜLHAN CAN

2017

INVESTIGATING SELF EFFICACY BELIEFS AND ALGEBRAIC KNOWLEDGE OF TURKISH MIDDLE SCHOOL MATHEMATICS TEACHERS BY THE INTERACTION OF AGE GROUPS AND TEACHING DEGREES

A MASTER’S THESIS

BY

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THE PROGRAM OF CURRICULUM AND INSTRUCTION
İHSAN DOĞRAMACI BILKENT UNIVERSITY
ANKARA

FEBRUARY 2017

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The Graduate School of Education

of

İhsan Doğramacı Bilkent University

by

Gülhan Can

In Partial Fulfilment of the Requirements for the Degree of

Master of Arts

in

The Program of Curriculum and Instruction
İhsan Doğramacı Bilkent University
Ankara

February 2017

İHSAN DOĞRAMACI BILKENT UNIVERSITY

GRADUATE SCHOOL OF EDUCATION

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Gülhan Can

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

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ABSTRACT

INVESTIGATING SELF EFFICACY BELIEFS AND ALGEBRAIC KNOWLEDGE OF TURKISH MIDDLE SCHOOL MATHEMATICS TEACHERS BY THE INTERACTION OF AGE GROUPS AND TEACHING DEGREES

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M.A., Program of Curriculum and Instruction
Supervisor: Assoc. Prof. Dr. M. Sencer Çorlu
February 2017

The purpose of the current study was to investigate whether there was a statistically significant relationship between Turkish middle school mathematics teachers' knowledge for teaching algebra, self-efficacy beliefs, age groups, and teaching certification types. Participants of this study were 43 middle school mathematics teachers from 15 randomly selected state schools in a socio-economically low-risk district of Ankara. For the data collection, *mathematical knowledge for teaching patterns, functions, and algebra scale* and *mathematics teaching efficacy beliefs instrument* were used. Data were analysed with multivariate analysis of variance approach. The dependent variables were teachers' patterns, functions, and algebra knowledge and their self-efficacy scores while the independent variables were age groups and certification types (faculty of education certified and alternatively certified). The analysis disclosed that there was no statistically significant difference between two age groups and certification types in mathematical knowledge or self-efficacy beliefs of teachers. Results were discussed with respect to recruitment and placement system in teacher education and quality of professional development programs for in-service teachers.

Key words: mathematical knowledge for teaching, algebra, algebraic knowledge, self-efficacy, middle school.

ÖZET

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN ÖZ YETERLİK SEVİYELERİ VE CEBİRSEL BİLGİLERİNİN YAŞ GRUPLARI VE ÖĞRETMENLİK SERTİFİKASYONLARI ARACILIĞIYLA İNCELENMESİ

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Şubat 2017

Bu çalışmanın amacı; Türkiye'deki ortaokul matematik öğretmenlerinin cebir öğretimi ile ilgili alan bilgileri, öz yeterlik inançları, yaş grupları ve öğretmenlik sertifikasyonları göz önüne alındığında, bu değişkenler arasında istatistiksel olarak anlamlı bir ilişki olup olmadığını araştırmaktır. Katılımcılar, Ankara ilinin sosyo-ekonomik olarak düşük risk taşıyan bir bölgesindeki devlet okulları arasından rasgele seçilen 15 okulda çalışmakta olan 43 orta okul matematik öğretmenidir. Data toplama sürecinde *örüntü, fonksiyon, ve cebir alan bilgisi ölçeği* ile *matematik öğretimi özyeterlik ölçeği* kullanılmıştır. Data, çoklu varyans analizi yaklaşımıyla analiz edilmiştir. Bu çalışmanın bağımsız değikenleri, yaş grubu ve sertifikasyon türü (eğitim fakültesi sertifikalı ve alternatif sertifikalı) iken; bağımlı değişkenleri, öğretmenlerin örüntü, foksiyon, ve cebir alan bilgisi ile öz yeterlik puanlarıdır. Analiz sonuçları, öğretmenlerin matematiksel alan bilgileri ve öz yeterlikleri ile yaş grubu ve sertifikasyon türü arasında istatistiksel olarak anlamlı hiçbir ilişki bulunamadığını göstermiştir. Sonuçlar, Türkiye'deki öğretmen eğitimi, işe alımı ve atama sistemi ile hizmet içi mesleki gelişim programlarının kalitesi göz önüne alınarak tartışılmıştır.

Anahtar Kelimeler: Matematik öğretimi için gereken alan bilgisi, cebir, cebirsel bilgi, öz yeterlik, orta okul.

ACKNOWLEDGEMENTS

I would like to offer my sincerest appreciation to Prof. Dr. Ali Doğramacı, Prof. Dr. Margaret K. Sands, Prof. Dr. Mehmet Baray and to everyone at Bilkent University Graduate School of Education for sharing their experiences and supporting me throughout the program.

I would like to thank my official supervisor Dr. M. Sencer Çorlu for his guidance and encouraging me to be a "teacher as a researcher" all the time. I am also thankful to him for teaching me a life lesson throughout the writing of my thesis.

I would also like to thank the committee members Dr. İlker Kalender and Dr. Niyazi Erdoğan for their suggestions and precious feedbacks about this thesis. Dr. İlker Kalender have been like a co-advisor for me at the very tough times of the writing process of this thesis. He dedicated most of his time in the last stages of this marathon with his always-polite and hope-inspiring wording.

Additionally, I would like to thank all the lovely people at TED Bursa College, my students, my colleagues at the mathematics department, and my administrators Ebru Bekil and Murat Akkuş for their understanding and support. My special thanks are for Pelin Yıldız and Athena Kolukısa for making the life easier for me with their precious help and friendship. I am also grateful to Gizem Ökmen for standing right next to me while travelling between Bursa and Ankara, struggling and finally achieving together.

I would like to offer my acknowledgements to all my friends in CITE program, but especially my classmates Ezgi Çallı, Ayşenur Alp, Çiğdem Özdemir, Tuğba Özcan, Ceren Özbay and Vildan Sertkaya. Ezgi was my workmate from back-to-back translations to data collection processes. Ayşenur was the one who always pushed me out of my nest to act. Without her endless energy and cheer, I would not accomplish my thesis.

The final and most heartfelt thanks are for my family; Zeyhan and Şükran Can for challenging me to be a better person all the time, my brother Cihan Can since he is the most tight-lipped person I have ever met and he believed wholeheartedly to his elder-sister all the time. And I am most grateful to my husband, Şafak Baş, for his endless love, caring, patience and understanding. He has always shared my tears and laughs, without his support I would not overcome writing process of this thesis.

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CHAPTER 1: INTRODUCTION

Teaching is one of the oldest professions in the world. Although there is not a single definition of teaching; it could be represented within its multidimensional process which Shulman (1987) explained in terms of how it begins, proceeds, and ends:

> Teaching necessarily begins with a teacher's understanding of what is to be learned and how it is to be taught. It proceeds through a series of activities during which the students are provided specific instruction and opportunities for learning, though the learning itself ultimately remains the responsibility of the students. Teaching ends with new comprehension by both the teacher and the student. (p.7)

Shulman (1987) was one of those people who made a stand against trivialization of teaching and indicated the crucial need for professionalization of teaching in his works. Subsequently, a fundamental question occurred in the literature about how to professionalize teaching. The essential step was taken by the first attempts of forming a knowledge base teaching structure which aimed to reveal and regularize what teachers know. Since those days, today's educational world is still discussing on the issue that what/how teachers know and which factors affect their teaching.

Teachers' subject matter knowledge is one of the milestones for an effective teaching (Ball, 1988; Ferguson, 1991; Shulman, 1986). However, is it possible to understand and even assess teachers' knowledge to teach? Could affective factors such as beliefs, attitudes, and self-efficacy be linked with teachers' content knowledge? Are there any teacher characteristics that lead excellence in teaching? The present study aims to contribute to a knowledge base on teachers' *mathematical knowledge to teach* without ignoring the complexity of the construct.

Background

In one of the first attempts to form a knowledge base of teaching which aimed to reveal and regularize what teachers know, some researchers followed Piaget's lead about knowledge growth (Shulman, 1987). Those researchers thought that plenty of data about the knowledge and its development could be gathered by means of careful observation. By doing so, the steps starting from a student to become a teacher could have been followed by the teacher/researcher to be able to understand the process of being an expert teacher starting from a learner.

Apart from that idea, several different approaches have been speculated on the definition of teachers' knowledge. *Policy response approach*, *characteristics of teachers approach*, and *teachers' knowledge approach* (which was reviewed in Chapter 2) were widely covered in the literature. However, by the definition of pedagogical content knowledge (PCK) as "*special amalgam of content and pedagogy*", Shulman (1987) has brought a new perspective. The reason behind the wide acceptance of Shulman's definition was that he drew attention to the synthesis of content and pedagogy, instead of focusing only one of these elements. Shulman's teacher knowledge model consisted of mainly three components: *content knowledge* (CK), pedagogical content knowledge (PCK) and curriculum knowledge.

MKT: Mathematical knowledge for teaching

The field of mathematics education was affected by the theories of Shulman the most; and Shulman's work was followed by several researchers. Deborah Ball (Ball & Bass, 2000a, 2000b, 2003) has developed a conceptual framework named, Mathematical Knowledge for Teaching (MKT). Ball, Thames and Phelps (2008) defined MKT as the mathematical knowledge that teachers need in the teaching process. Under the roof of Learning Mathematics for Teaching Project (LMT), researchers developed the MKT instruments which were *number concepts and operations, patterns functions and algebra,* and *geometry* scales for elementary and middle school grades. The current study is notable since it provides one of the first uses of MKT's *patterns functions and algebra* scale in the Turkish context.

Patterns, functions, and algebra

Algebra is one of the core subjects in school mathematics. Teaching and learning of algebra has taken researchers' attention for many years. Mathematics educators have investigated alternative ways for a more effective teaching of algebra. In recent years, one of the most studied alternatives has been the use of patterns and functions to improve conceptual understanding of algebra. National Council of Teachers of Mathematics (NCTM, 2000) emphasized the significance of this approach while describing the algebra strand as *"...systematic experience with patterns can build up to an understanding of the idea of function, and experience with numbers and their properties lays a foundation for later work with symbols and algebraic expressions"* (p. 37). Similar to NCTM's approach, in Turkey, it was stated that different representations of patterns, and especially their symbolic expressions would contribute significantly to the formation of basic concepts of algebra in the amended mathematics curriculum (MEB, 2009). Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) results of Turkey showed that Turkish mathematics curriculum has given weight to operational knowledge and skills mostly instead of conceptual understanding of algebra concepts (Baki & Kartal, 2004).

By this sense, the current study investigates whether there is a statistically significant relationship between teachers' content knowledge of *patterns, functions, and algebra* to teach, their self-efficacy levels, age groups, and teaching certification types according to the pathway that they enter to the teaching profession.

Problem

Previous studies on teachers' knowledge for teaching have indicated a positive relationship between teachers' mathematical knowledge for teaching and students' achievement. The ones that examine teachers' subject matter knowledge in mathematics were usually focused on the knowledge of pre-service teachers (Alpaslan, Işıksal, & Haser, 2014; Baki & Çekmez, 2012; Baki, 2013; Turnuklu & Yeşildere, 2007; Ubuz & Yayan, 2010; Uçar, 2011; Uygan, Tanışlı, & Köse, 2014).

The complexity of teaching requires the consideration of the affective domain, as well as the cognitive dimension of teaching. Considering the increasing interest in self-efficacy beliefs of mathematics teachers in recent years (see Ingvarson, Meiers, & Beavis, 2005; Ross & Bruce, 2007; Swackhomer et al., 2009; Watson, 2006), there is a need to examine in-service mathematics teachers' knowledge and their self-efficacy beliefs together.

Purpose

The purpose of this study is to investigate whether there is a statistically significant relationship between middle school mathematics teachers' subject matter knowledge to teach patterns, functions, and algebra and their self-efficacy beliefs by analyzing the interaction of age group (teachers under 40 and teachers over 40) and teaching certification type (faculty of education certified and alternatively certified).

Research questions

The research questions of this study are stated in the following:

- Is there any statistically significant difference on the average between self-efficacy beliefs and patterns functions and algebra knowledge of Turkish middle school mathematics teachers in terms of their age group (*teachers under 40* and *teachers over 40*)?

- Is there any statistically significant difference on the average between self-efficacy beliefs and patterns functions and algebra knowledge of Turkish middle school mathematics teachers with different teaching certification (*faculty of education certified* and *alternatively certified*)?

- Are the self-efficacy beliefs and patterns functions and algebra knowledge of Turkish middle school mathematics teachers on the average affected by the interaction of age groups (under 40 or over 40) and teaching certification types?

Intellectual merit and broader impact

The current study has the potential to make contributions to the findings related with in-service mathematics teachers' content knowledge and self-efficacy beliefs in Turkey. The instruments and methodology that is used in this study could be used in different investigations with various grade level teachers and in other regions of Turkey.

Additionally, this study aims to support policymakers in Turkey by suggesting professional development programs for in-service teachers in which content knowledge and affective factors are handled together. The teachers who attended some of the professional development programs such as Cognitively Guided Instruction (Carpenter et al, 1996) and Multi-tier Program Development (Clark & Lesh, 2003) usually integrated their knowledge in their instructions and this situation contributed on students' achievement (Carpenter et al, 1989; Fenema et al, 1993).

Definition of key terms

Common Content Knowledge (CCK): In Ball's teacher knowledge model, it is one of the three subheadings under the frame of subject matter knowledge. It refers to the common mathematical knowledge which is not unique to the teaching profession.

Content knowledge (CK): Content knowledge refers to the body of information that teachers teach and that students are expected to learn in a given subject or content area, such as English language, arts, mathematics, science, or social studies. Content knowledge generally refers to the facts, concepts, theories, and principles that are taught and learned, rather than to related skills—such as reading, writing, or researching— which students also learn in academic courses.

Curriculum knowledge: It refers to the effective use of curriculum materials. It also refers the knowledge that teachers not only have in their own subject area (such as mathematics), but also have in other disciplines (such as natural sciences or social sciences).

Horizon content knowledge: It refers to the mathematical knowledge which will continue to progress throughout the curriculum.

Knowledge of content and curriculum (KCC): It refers to the interaction between mathematical knowledge and mathematics curriculum.

Knowledge of content and students (KCS): It refers to the interaction between mathematical knowledge and mathematical perceptions of students.

Knowledge of content and teaching (KCT): It could be considered as a synthesis of mathematical knowledge and teaching methods.

LMT: Learning Mathematics for Teaching

MKT (Mathematical Knowledge for Teaching): Ball, Thames, and Phelps (2008) define MKT as "mathematical knowledge that a teacher needs to teach".

MoNE: Ministry of National Education

MTEBI: Mathematics Teaching Efficacy Beliefs Instrument

MTLT: Mathematics Teaching and Learning to Teach

NCTM: National Council of Teachers of Mathematics

OECD: Organization for Economic Co-operation and Development

PFA: Patterns Functions and Algebra

PISA: Programme for International Student Assessment

Pedagogical Content Knowledge (PCK): It refers to the knowledge that synthesize pedagogy and content knowledge in the same framework. It also refers to the knowledge of how to organize and present mathematical concepts according to students' different interests and abilities through in-class instructions.

SII: Study of Instructional Improvement

Specialized Content Knowledge (SCK): In Ball's model of teacher knowledge, it is one of the three subheadings under the frame of subject matter knowledge. It refers to the knowledge of methods and techniques which are unique to the teaching profession. For instance, the mathematical knowledge those mathematics teachers have.

STEBI: Science Teaching Efficacy Beliefs Instrument

TALIS: The OECD Teaching and Learning International Survey

TIMSS: Trends in International Mathematics and Science Study

CHAPTER 2: LITERATURE REVIEW

Introduction

The present chapter provides detailed analysis of the theoretical background and the existing research findings related to the research questions of the present study. In the first section of the chapter, the studies related with identifying teachers' knowledge to teach is presented with an extended focus of mathematical knowledge to teach. In the second section, particularly the research which based on teaching and learning of algebra and changing views on school algebra is given. Additionally, a new trend for a more effective algebra teaching-the approach to teach patterns, functions and algebra associated- is discussed. Then, in the third section, the studies on teachers' self-efficacy beliefs are analyzed.

Identifying teachers' knowledge

As Putnam, Heaton, Prawat, and Remillard (1992) claimed "...the desired learning environments can result only from knowledgeable teachers" (p. 225-226). Although many people would agree that teacher quality was one of the weightiest agent in student learning (Ferguson, 1991), there was not a common consensus on the definition of *teachers' knowledge*. In the last few decades, policymakers and academes placed a particular importance on identifying *teachers' knowledge*. Therefore, some approaches have defined teachers' knowledge in terms of *policy response approach*, *characteristics of teachers approach*, *teachers' knowledge approach*, and *mathematical knowledge for teaching* which have been extended forms of Shulman's model (1986, 1987).

Approaches to define mathematics teachers' knowledge

According to the *policy response approach*, some documents such as Curriculum and Evaluation Standards for School Mathematics (1989) and Principles and Standards for School Mathematics (NCTM, 2000) were used to describe mathematics teachers' knowledge. In those documents, some principles and standards for effective mathematics teaching was portrayed like knowing well about the subject area taught, students' needs as learners, and teaching methods in a broad sense (NCTM, 2000). Though it seems right, the fact remains that notedly most of the previous works in *policy response approach* were not structured according to research results, but "policy deliberations" (Ball et al., 2001, p.441).

The deficiencies criticized about *policy response approach*, indicated the need of some further studies which were based on research results. *Characteristics of teachers approach* was one of these approaches which used some statistics in its studies. The statistics in those works were mostly provided by quantitative data like teachers' certification types, the quantity of college mathematics courses taken, having minor and major degree in mathematics (For further info: Tutak, 2009, pp. 29-30).

On the other hand, throughout *teachers' knowledge approach*, the main point was *knowledge* of teachers rather than the teacher characteristics as mentioned above in *characteristics of teachers approach* (Tutak, 2009). Many researchers like Shulman (1986, 1987), Fenstermacher (1994), Wilson et al. (1987), and Grossman et al. (1989) turned up with different types of teacher knowledge in their studies in terms of *teachers' knowledge approach*. Subsequently, the complicated and multidimensional structure of knowledge of teaching was tenable once more in the literature.

Mathematical knowledge for teaching (MKT): An extended form of Shulman's model

On the way of understanding teachers' knowledge and its components, Shulman's model of teachers' knowledge was one of the widely-accepted frames in the academes. In his model, he analyzed teachers' knowledge under three main headings as *content knowledge*, *pedagogical content knowledge*, and *curricular knowledge* (Shulman, 1986, 1987). Despite a few opposing ideas (like Fenstermacher, 1994), Shulman provided an extensively recognized framework for teaching profession.

In these knowledge types, content knowledge (CK) had the same meaning as *subject matter knowledge* which Shulman (1986) defined as "*the amount and organization of knowledge per se in the mind of teacher*" (p.9). CK included mathematics knowledge for classroom and mathematical explanations (Tutak, 2009).

Pedagogical content knowledge (PCK) was defined as "*a special amalgam*" which combines subject matter knowledge and pedagogy for an effective teaching (Shulman, 1986). It was underlined that PCK was not just about pedagogy or pedagogical skills, but a part of teachers' content knowledge (p. 9). Mathematical representations and student conceptions could be counted in PCK.

Lastly, curricular (or curriculum) knowledge was expressed in three components which are *alternative curriculum materials* (such as texts, software, visual displays), *lateral* and *vertical* aspects of curriculum. By lateral aspects of the curriculum, a teacher could enrich classes by utilizing from different disciplines; and by vertical aspects of the curriculum, a teacher could have a solid grasp of different grades' curriculum knowledge on the same subject area.

Since this model was introduced by Shulman, center of attention in those knowledge types had been content knowledge in most studies on teacher education area. Research on this area showed that restricted content knowledge caused difficulties in the process of training pre-service teachers (Brown & Borko, 1992). Furthermore, in-service teachers' insufficient content knowledge influenced their teaching methods in a negative way as well (Carpenter, Fennema, Peterson, & Carrey, 1988; Leinhardt & Smith, 1985).

In this sense, Ball and her team started investigating teaching mathematics as a profession in a project named Mathematics Teaching and Learning to Teach (MTLT). The main question that the researchers have been looking for was what is needed by teachers through the teaching/learning process of mathematics. During the project, the researchers observed classroom studies, interviewed with teachers, students, and even parents; and in time they developed some scales to measure teachers' existing content knowledge which is necessary to teach effectively. The researchers called their works as "a practice-based theory of mathematical knowledge for teaching" (Ball et al., 2008, p. 395).

Mathematical Knowledge for Teaching (MKT) has been regarded as one of the most promising frameworks of teacher knowledge (Morris, Hiebert, & Spitzer, 2009). Ball, Thames and Phelps (2008) defined MKT as the mathematical knowledge that teachers need in teaching process. Such a kind of mathematical knowledge was different from the knowledge that other professions needed in their fields. The perspective that Ball et al. had about teachers' knowledge was not different from what Shulman had explained before--but an extension of it (Ball, Thames & Phelps, 2008). See Figure 1 for a representation of six main domains of MKT.

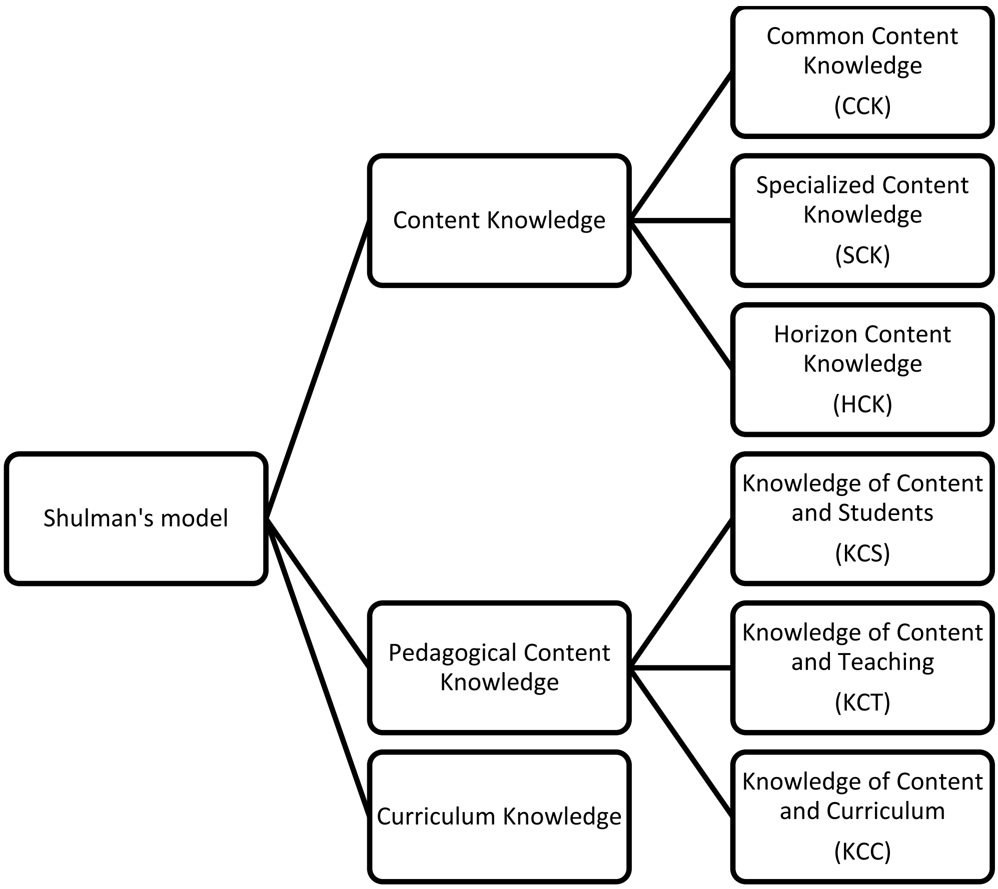

Figure 1. A representation of MKT model as an extension of Shulman's model

According to MKT approach, there have been three different subheadings under the frame of subject matter knowledge (or CK): common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). At a closer look to those knowledge types, CCK represents the common mathematical knowledge which is not unique to teaching profession; while SCK represents the mathematical knowledge unique to the teaching profession. Horizon content knowledge, however, indicates the mathematical knowledge which continues to progress throughout the curriculum (Ball et al., 2008).

In addition, the other three subheadings placed in MKT model, could be classified under the main heading of pedagogical content knowledge: Knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum. With those definitions, KCS could be considered as the interaction of mathematical knowledge and students' mathematical perceptions. In a similar way, KCT could be thought as a synthesis of mathematical knowledge and teaching methods. Lastly, knowledge of content and curriculum represents the interaction of mathematical knowledge and mathematics curriculum (Hill & Ball, 2004; Hill, Schilling & Ball, 2004; Hill, Rowan & Ball, 2005).

Mathematical knowledge for teaching patterns, functions and algebra (PFA)

Beginning in 2000s, it came into prominence to have valid and reliable instruments to understand the factors which affect mathematics teachers' content knowledge--. With this respect, some projects followed each other like *Mathematics Teaching and Learning to Teach (MTLT), Study of Instructional Improvement (SII),* and *Learning Mathematics for Teaching (LMT)* as a sister project of SII. Throughout those projects, the researchers developed items by using theory, research, curriculum materials, student work, and their own experiences, and piloted them with the contributions of almost 5000 participants. It might be stated as an indicator to the importance of these projects since the works were supported and even funded by some of the leading establishments in education area such as University of California Office of the President (UCOP), the National Science Foundation, U.S. Department of Education, and Atlantic Philanthropies (For further info check out LMT website: http://www.umich.edu/~lmtweb/about.html).

In the process of developing items, Ball et al. designed problems which refer to common student mistakes so that teachers might have needed to evaluate an exceptional student work, to explain mathematical reasoning of a procedure or to determine on any appropriate definitions for the grade level taught. Some research supported these attempts by suggesting a method- analyzing student work- which could be a path to a deeper content knowledge for teachers (Franke & Kazemi, 2001; Kazemi & Franke, 2004). In line with this understanding, the researchers have matured the items from elementary-grade level forms to middle-school forms: *number and operations content knowledge* (grades 6-8), *patterns, functions and algebra content knowledge* (grades 6-8), and *geometry content knowledge* (grades 3-8).

In the recent times, researchers have utilized from MKT items in other countries (Blömeke & Delaney, 2012). The current study has the distinction of being the first in Turkey in terms of using PFA (patterns, functions, and algebra) scale of MKT items. The MKT PFA items were used to understand in-service mathematics teachers' content knowledge through the study.

Learning and teaching algebra: Changing views on school algebra

Algebra has been seen as a *science of equation solving* for many long years since Al-Khwarizmi invented it in the ninth century (Kieran, 2004). However, in 17th and 18th centuries, mathematics educators used algebra as means to manipulate symbols mostly. When it was 1960s, school algebra was studied in a more general way for increasing skills and structure of memory by cognitive behaviorists.

Freudenthal (1977) was one of the researchers who have moved the perspective of equation solving to *algebraic thinking*. That kind of portraying melted the ability to recognize relations and equation solving procedures in the same pot. Within this perspective, a new window opened for the studies which focused on *students' meaning-making* (Kaput, 1989; Kirshner, 2001; Wagner & Kieran, 1989). Later on, with their graphical, tabular, and symbolic representations, functions became a critical organ while elaborating the scope of algebra teaching (Schwartz & Yerushalmy, 1992) despite some counter-views (see, e.g., Lee, 1997).

A revolutionary shift was *algebraic reasoning* which was an extension of algebraic thinking and meaning-making that Kieran (2014) described as "*a consideration of the thinking process that precede –and eventually accompany- activity with algebraic symbols, such as the expression of general rules with words, actions and gestures*" (p.28).

After the idea of algebraic reasoning has been unfolded, it brought to mind a question: Could we make algebra more attainable for all school-age children starting from primary level to the higher grades? The *Algebra Strand* in the *Principles and Standards for School Mathematics,* published by *the National Council of Teachers of Mathematics* (2000) was a very descriptive answer to this question:

> The Algebra Standard emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change. …By viewing algebra as a strand in the curriculum from prekindergarten on, teachers can help students build a solid foundation of understanding and experience as a preparation for more-sophisticated work in algebra in the middle grades and high school. (p. 37)

By this way, the topic has been moved to the next point: How students learn algebra, and characterization of algebraic activities. Kieran (1996) gathered those activities under three headings: *generational*, *transformational*, and *global/meta-level*. Hence, the activities which include forming expressions and equations by recognizing geometric and arithmetic patterns, multiple representations of functions, and solving equations could be counted as generational activities according to Kieran. While transformational activities contain miscellaneous symbol manipulation like operations with polynomial expressions, factoring, substituting; global/meta-level activities embodied problem solving, modeling, working with generalizable patterns, justifying and proving, making predictions and conjectures, studying change in functional situations, looking for relationships and structures (Kieran, 2014). An overview for some major characterization of algebraic activities that raised up in terms of algebra teaching, see Figure 2 (based on Kaput, 1995; Kieran, 1996; Kieran, 2014; NCTM, 1998; Usiskin, 1988). The bolt fonts were used to indicate the common elements throughout the provided approaches which refer four organizing themes of NCTM for school algebra. Apart from those, several other researchers like Bell (1996), Mason et al. (2005) and Sfard (2008) studied on the aspects of teaching algebra as well.

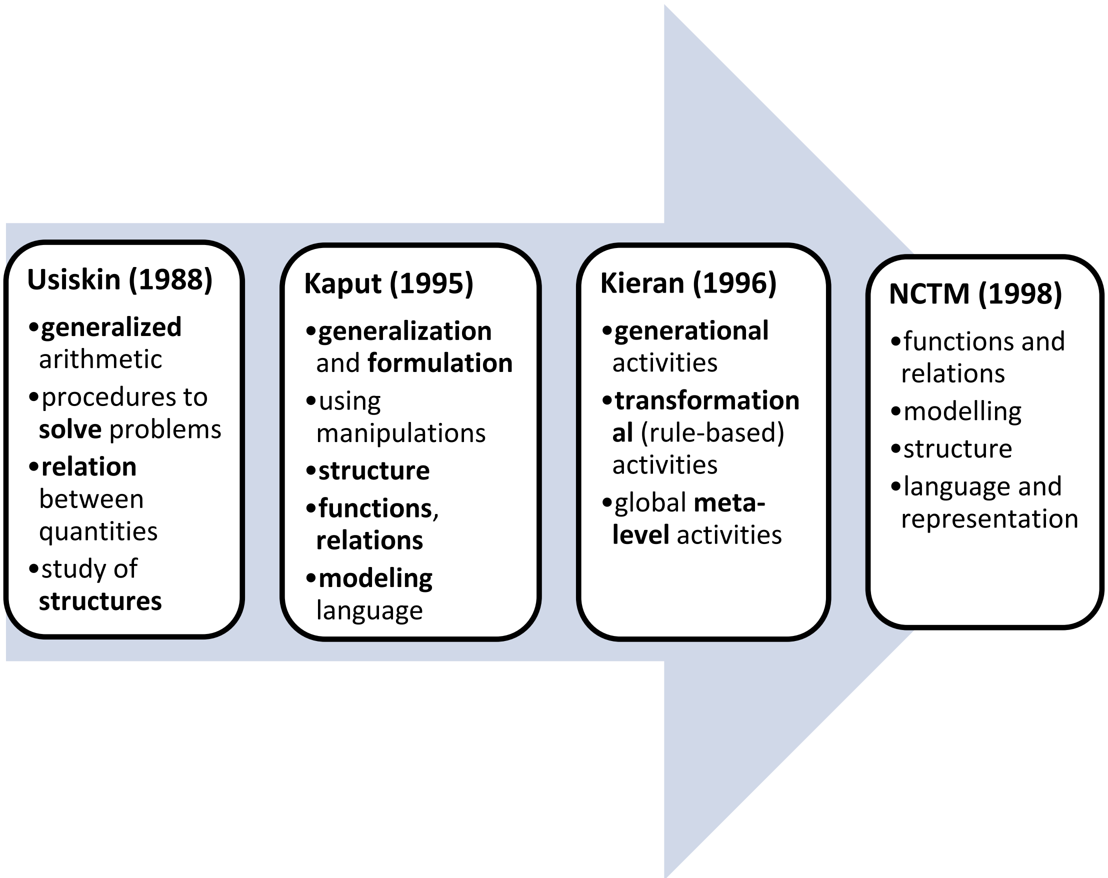

Figure 2. An overview for characterization of algebraic activities

Studies about learning and teaching of school algebra

Algebra has been seen one of the most troublesome learning areas of mathematics, which has propelled mathematics educators to investigate alternative ways for a more effective teaching of algebra. Although there has been numerous research related teaching and learning of algebra and following regulations in recent years, international evaluation programs such as PISA and TIMSS have brought out that difficulties which students had experienced were still going on (Kieran, 2007).

In this sense, research on learning algebra has pointed out the lack of conceptual understanding. Many students have had difficulty to perceive algebra in real-life context. Instead, algebra was seen as a series of rules about simplifying algebraic expressions, using notations and symbols to solve equations by most of the pupils (Kaput, 1999).

Many students tend to find/calculate an exact answer (or number) when they encounter a mathematical problem (Kieran, 2014). Acquiring expressions like x+2, or 5x-3y as accurate responses, failing to analyze structure of a situation and represent it by using algebra, and skipping some transformations that are applied to both sides of an equation were some other findings about lack of students' conceptualizing (p.29). Studies implemented in Turkey in the same field have also indicated similar shortcomings experienced by students (See Dede & Argün, 2003; Erbaş, Çetinkaya, & Ersoy, 2009). According to Willoughby (1999), the reason behind this issue was trying to teach algebra suddenly and in an abstract way. Instead, Willoughby (1999) suggested the transition from the concrete to the abstract for an effective algebra teaching.

Looking through the literature, it could be seen that approaches in school algebra have evolved towards utilizing from the concepts of *patterns* and *functions* in recent years. Cathcart, Pothier, Vance and Bezuk (2003) offered that teachers should analyze pattern(s) with their students and help them with recognizing similar patterns in class activities so that algebraic thinking process could be encouraged. Even number sense and mathematical exploration could be promoted by using patterns (Reys et al., 1998) which develops extremely important skills in algebra teaching such as recognizing patterns, generalizing, comprehending mathematical order and structures (Burns et al., 2000). Similarly, English and Warren (1999) have claimed the concept of *variable* should be taught starting from pattern-based exercises.

Concordantly, Kabael and Tanışlı (2010) have offered using daily life examples to recognize functional relations in early and later grades.

Despite a few opposing ideas that patterns were not useful and effective tools in algebra teaching (Orton & Orton, 1996); many researchers, mathematics teachers, and policymakers have agreed on starting from patterns in early grades, proceeding with ability to think by functional relations in later grades is located in the heart of algebraic reasoning (Blanton & Kaput, 2004; Carraher & Martinez, 2007; Driscoll & Moyer, 2001; Mor, Noss, Hoyles, Kahn & Simpson, 2006; Warren & Cooper, 2005; Usiskin, 1997). As an example of these studies which conducted to test the point at issue, an experimental study has carried out with seventh grade students in a state school located in a big city in Central Anatolia Region of Turkey (See Palabıyık & Ispir, 2011). Throughout the study, control group students were taught algebra by conventional teaching techniques while experiment group students were taught algebra by pattern-based instructions. At the end of 24 weeks, the responses of the two groups were analyzed by using the instruments *Conceptual Algebra Test*- which was designed by Concepts in Secondary Mathematics and Science Team (CSMST) (Hart, Brown, Kerslake, Küchemann & Ruddock, 1985) and translated to Turkish by Akkuş (2004)- and *Computational Algebra Test*- which was designed by Akkuş (2004). As a result, experiment group students' conceptual algebra test scores were significantly higher than the control group scores; while there was no statistically significant difference between the two groups' computational algebra test scores (p. 114). When it is looked through the studies on the use of patterns and following concepts such as functions and algebra, the results revealed that the pattern-based education could have a positive effect to provide a conceptual understanding of algebra for middle school students. However, this approach should be supported by different teaching methods at later stages. Those interventions could provide an appropriate environment to fulfill the learning of conceptual and procedural knowledge in algebraic terms (Palabıyık & İspir, 2011).

However, assessment and evaluations upon TIMSS and PISA results of Turkey showed that Turkish mathematics curriculum has given weight to operational knowledge and skills mostly (Baki & Kartal, 2004). This engrossing situation has been tried to adjust by some revisions and reforms in Turkish mathematics curriculum in 2005-2006 education year. Similar to NCTM Algebra Strand mentioned in the previous part; Ministry of National Education (MoNE) in Turkey stated that different representations of patterns, and especially their symbolic expressions would contribute significantly to the formation of basic concepts of algebra in the amended mathematics curriculum (MEB, 2009a).

At this point, issues that need to be considered are: Despite all this work all around the world, the international evaluation results (such as PISA and TIMSS) have been indicating almost same problems for years. The reason behind the case might be lying on teachers' knowledge, perceptions, and attitudes. However, there is not enough teacher-oriented research about algebra teaching. According to Doerr (2004), teacher-oriented research (which has been and will be conducted) should be addressed in three fields: teachers' content knowledge and pedagogical content knowledge, teachers' conceptualizing of algebra, and teachers' learning to teach algebra.

Regarding with this sense, teacher training and professional development play a critical role for change. Guskey (1986) has offered a model which supports teachers' growth by staff development, changing teachers' in-class practices to be able to change learning outcomes in a positive way, and relatedly to range up teachers' attitudes and beliefs. For the general view of this process, see Figure 3.

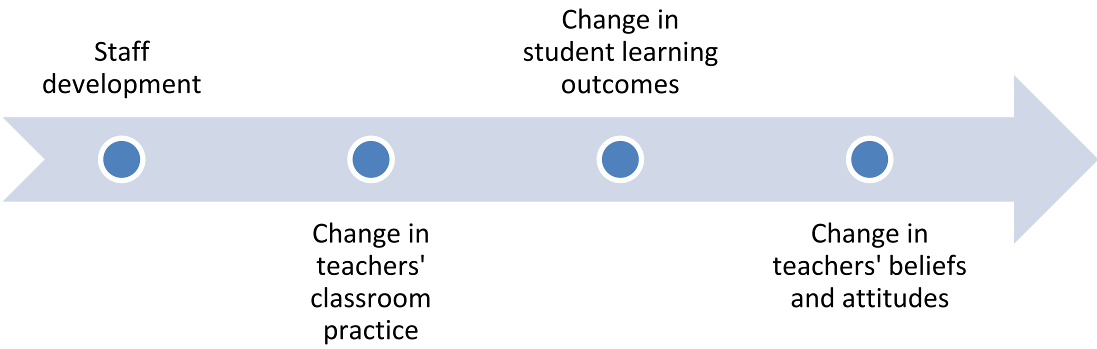

Figure 3. Guskey's model of teacher change (based on Guskey, 1986)

Under the roof of professional development for teachers, specifically learning/teaching of algebra strand was studied in a project named *Transformative Teaching in the Early Years Mathematics* (TTEYM) to support the implementation of the new patterns and algebra strand in Australia (Warren, 2009). The mathematical focus of TTEYM originated in *patterns, equivalence and equations*, and *functions* while the participant teachers were collaborating and achieving learning experiences. The results demonstrated "*a pathway of change guiding the novice learner to become an expert*" in terms of content and pedagogical knowledge of the Patterns and Algebra strand (pp.34-35).

In a similar vein, professional development programs might be conducted in Turkish context as well. Additionally, studies associated with teachers' content knowledge (or pedagogical content knowledge) could be enriched by the use of affective factors such as teachers' beliefs, attitudes, and self-efficacy for the sake of completeness in professional development programs (Warren, 2009). By this sense, the current study has investigated participant teachers' self-efficacy beliefs in addition to mathematical content knowledge.

Teachers' self-efficacy beliefs

In recent years, there has been a spreading paradigm about the impacts of affective factors in an educational context (e.g. Bandura, 1997; Ernest, 1989; Philipp, 2007; Thompson, 1992). Affective domain (McLeod, 1992) had a bearing on beliefs, attitudes, and self-efficacy which indicated a form of expression of one's own internal state like embodying feelings, emotions, beliefs, attitudes, morals, values, and ethics (DeBellis & Goldin, 2006). Among these factors, as a continuum of feelings of emotions which were seen short-lived but highly-charged, beliefs were considered a "more cognitive and stable in nature" according to Philippou and Christou (2002). Hence, attitudes were seen as "manifestations of beliefs" (Liljedahl, 2005). However, self-efficacy was placed between beliefs and attitudes (Liljedahl & Osterlee, 2014). Specifically, Woolfolk and Hoy (1990) described teachers' self-efficacy as beliefs of a teacher with the perception of the ability to engage his/her students through the desirable learning outcomes.

The answer to the question that why self-efficacy has become of interest in the latest studies and in the design and evaluation of professional development programs could be listed as follows from the literature: Efficacious teachers have been linked with higher student achievement scores (Anderson, Grene, & Loeven, 1988; Cannon & Sharmann, 1996; Ross, Hogaboam-Grey, & Hannay, 2001), higher student motivation (Midgley, Feldlaufer, & Eccles, 1989), higher levels of flexibility and exploration in teaching (Allinder, 1994; Guskey, 1988), resilient in classroom difficulties and uncomplaining with student mistakes (Aston & Webb, 1986), and more liable with struggling students (Meijer & Foster, 1988; Podell & Soodak, 1993).

Based on the self-efficacy definition of Bandura (1986); for teachers, mastery experiences in subject area taught could be regarded as a factor on self-efficacy beliefs of teachers. Specifically, while teachers might feel qualified with sufficient ability to solve on highest degree mathematical problems; they might feel insufficient in terms of engaging students or giving instructions (Stevens, Aguirre-Munoz, Harris, Higgings, & Liu, 2013). The justification of examining both content knowledge and self-efficacy beliefs of the teachers at the same time lies on this rationale in the actual study.

In U.S., a professional development based study was conducted by considering a similar rationale that combining MKT items and a self-efficacy instrument to examine the outcomes of the professional development program. The participants of the study were West Texas Middle School math teachers which divided in two groups in terms of their less and more mathematical background. Since coursework beyond algebra was not taught at middle grades in general, the deciding factor was teachers' achieving algebra at college level in this study. The participants attended in the professional development program across two summers. The activities in the program were related with teachers' self-efficacy beliefs (and implicitly their mathematical background). The results of this study indicated that teachers with less mathematical background showed higher self-efficacy than those with more background. On the other hand, teachers with more background had a tendency to benefit more in professional development activities associated with MKT items (Stevens, Aguirre-Munoz, Harris, Higgings, & Liu, 2013).

Studies in this area were mostly carried out by pre-service teachers rather than in-service teachers --mostly with lack of content knowledge analysis in the same context. One of the reasons behind this situation could be associated with Hoy's finding that self-efficacy is more moldable in early careers of teachers (Hoy, 2004). Some studies with pre-service teachers' self-efficacy beliefs in Turkey revealed that seniors had the higher scores than the rest of pre-service teachers (Çakıroğlu & Işıksal, 2009). Fortunately, no statistically significant difference was found between male and female teachers' self-efficacy levels in terms of teaching math and science (Bursal, 2010). Additionally, evaluation of TALIS data revealed that Turkish mathematics teachers' self-efficacy beliefs were at a similar degree with the OECD average (see Corlu, Erdogan, & Sahin, 2011).

Lastly, Swars et al. (2009) emphasized a significant detail that for the teachers whose self-efficacy beliefs were bounded with conventional teacher-centered approaches, it would be challenging to adapt with constructivist philosophies which lie on the ground of many curriculum reforms recently. So, by remembering all the positive effects stated previously about efficacious teachers, educational programs should be associated with "appropriate pedagogical beliefs" (Liljedahl & Osterlee, 2014, p. 586).

CHAPTER 3: METHOD

Introduction

The current research has investigated the impact of teachers' *age* (teachers under 40 or teachers over 40) and teachers' *certification ty*pe (faculty of education certified or alternatively certified) on middle school mathematics teachers' subject matter knowledge (particularly mathematical knowledge to teach *patterns, functions, and algebra*) and their self-efficacy levels. The research design, pilot study, sampling procedure, data collection and analysis process were included in this chapter.

Research design

For the current study, a non-experimental quantitative research design was used. In a quantitative study, researchers "explain the causes of changes in social facts, primarily through objective measurement and quantitative analysis" (Firestone, 1987, p. 16). Particularly, in a non-experimental quantitative research, there could be more than one variable need to be studied that cannot be manipulated since they are naturally existing attributes or it would be unethical to manipulate them (Belli, 2009). In this sense, quantitative research dwells on proving or disproving a hypothesis in terms of participants' responses (Arghode, 2012). The hypothesis testing procedure in this study was based on Huck's 9-step version of hypothesis testing which could be outlined as in the following:

1. State the null hypothesis (H_0),
2. State the alternative hypothesis (H_a),
3. Specify the desired level of significance (α),
4. Specify the minimally important effect size,

5. Specify the desired level of power,
6. Determine the proper size of the sample(s),
7. Collect and analyze the sample data,
8. Refer to a criterion for assessing the sample evidence,
9. Make a decision to discard/retain H_0 (Huck, 2011, p. 165).

After developing a hypothesis about the circumstance, alternative hypothesis evolved with the help of pilot study and theoretical framework. Alongside, the procedures were determined within distinguishing the variables and participants to collect data. Finally, the data was analyzed and the findings were declared in the present study.

Pilot study

Briggs and Coleman (2007) attracted notice on the importance of piloting process before conducting any research. On the one hand, piloting was considered as a need for the sake of intended research since: "Careful and appropriate piloting of research instruments will weed out inappropriate, poorly worded or irrelevant items, highlight design problems, and provide feedback on how easy or difficult the questionnaire was to complete" (p. 130).

By means of, researcher could improve quality of the instrument meant to be used, determine the needed logistics (time, budget, response rate etc.), and finalize the research questions and research plan (Cohen, Manion & Morrison, 2005). On the other hand, possible failure of a pilot study was exemplified since it might induce extra work for the researcher, suspense, and even non-response on the part of participant, researcher, or both (Briggs & Coleman, 2007).

Considering the mentioned benefits, the researcher has conducted a piloting process. The pilot study included some of the items from *Patterns, Functions, and Algebra* (PFA) scale and *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBI). For the pilot study, Google Forms -a free online tool to conduct surveys/questionnaires- was used to gather responses of participants. Afterwards, the responses were transposed to a spreadsheet via Google Forms.

To initiate the piloting process of this study, ten pre-service and nine in-service teachers were invited via e-mails, phone calls, and face-to-face meetings. The reason behind inviting not only in-service but also pre-service teachers was to be able to gather data as much as possible. 11 of the invited teachers accepted to take part in the study. After accumulating feedbacks from the participant mathematics teachers, the researcher improved back-to-back translation from English to Turkish, and corrected the wording of some items. The needed time to complete items of the instruments was estimated and the confirmation of the instrument was approved by means of the piloting process.

One of the advantages that piloting provide was to approximate the required sample size for the current study (Teijlingen & Hundley, 2001). A priori power analysis (Cohen, 1988) is typically used in estimating sufficient sample sizes to achieve adequate power. A power analysis software called G*Power3 (Faul, Erdfelder, & Buchner, 2007) was used to estimate minimum sample size required for the actual study. When means and standard deviations were entered into G*Power3, the program estimates an effect size (Cohen's *d*). Since there has not been a similar study reporting an effect size in Turkish context to refer, a predominantly accepted large effect size (Cohen's *d* = 0.75) was used in this study. Hence the required sample size was found to be 69 in order to be 95% sure ($\alpha = 0.05$) that there would be a statistically significant difference in the model represented in this study.

Participants

This study was conducted with middle school mathematics teachers working at different state schools in Çankaya district of Ankara, Turkey. The schools were selected randomly from official database of Ministry of National Education (MoNE). In the database, 51 state middle schools were listed for Çankaya district of Ankara. With the help of the websites of the schools, the number of mathematics teachers for each school was noted. As the next step, the average number of mathematics teachers was computed as 4.52. In this sense, 15 schools were determined adequate to achieve the predetermined sample size. After enumerating each of the 51 middle school, 15 schools were selected by using random number generator software.

In the selected schools, there were 75 mathematics teachers in total. All mathematics teachers in departments were kindly asked to participate in the study on a voluntary basis. After all, 39 of the mathematics teachers responded to the instruments. Since in pilot study it was decided as 69, the researcher visited randomly selected five more schools and achieved 43 participants at all. There were two concerns to continue the data gathering process: One was the time restriction since the MoNE permission was just for one academic year. The other concern was another study which conducted in the same region within 51 state schools by a different MKT instrument at the same period (See Çallı, 2015). To avoid coincidence, random number generator software was used a few more times carefully. So, the participant teachers of these two studies were set to be different from each other. Accordingly, the response rate of the current study was designated as 57.33%.

The participants of the study (N = 43) included 28 female and 15 male middle school mathematics teachers. The teachers had 20.88 years of teaching experience on the average with the standard deviation 10. The range of teaching experience differed from 6 years to 37 years.

Almost half of the participant teachers (n = 23) had their bachelor's degrees (Bs) from education faculties as Elementary Mathematics Education (Bs. EME) and Secondary Mathematics Education (Bs. SME). The other half (n = 20) had their bachelor's degrees from mathematics departments of science faculties. To be appointed as a teacher, graduates of science faculties have had to cover some pedagogy courses/credits since 1997 (Gürşimşek, Kaptan, & Erkan, 1997). Table 1 shows the numerical distribution of the participant teachers' college level (Elementary or Secondary) with respect to alma mater.

Table 1
Numerical distribution of the participants' college level

	Elementary Level	Secondary Level	Total
Education Faculty	14	9	23
Science Faculty	-	20	20
Total	14	29	43

Throughout the study, the mathematics department graduates would be assumed to have had an education at secondary level rather than elementary level. Additionally, their certification would be considered as *alternative certification*. In return, the certification type would be called *faculty of education certification* for the graduates of education faculties.

Besides of the certification types, the participants who had an advanced degree (Master's or Ph.D.) were viewed throughout the sample. There had been four teachers who had their master's degrees and no Ph.D. degree was found. Some demographical information is given about four of those teachers in Table 2.

Table 2
Demographic information of the participants with MA degree

	Age interval	Gender	Bachelor's degree	Experience years
Participant 1	More than 50	Female	Bs. SME	32
Participant 2	31-40	Female	Bs. M	12
Participant 3	31-40	Female	Bs. EME	9
Participant 4	31-40	Female	Bs. M	6

Moreover, the distribution of the participant teachers' certification types is represented in terms of age intervals in Table 3. There had been presented five age intervals as 18-25, 26-30, 31-40, 41-50, and more than 50 in demographic form of the instrument of this study. However, there were no participants at 18-25 age-intervals.

Besides the age intervals were approved to be collected under two sections as *40 and less* which is described as *teachers under 40* and *more than 40* is described as *teachers over 40* through the current study. The reason behind to choose age 40 as a critical point was based on a resolution of Council of Higher Education (Yüksek Öğretim Kurulu, YÖK) about faculty of education teacher education undergraduate programs in 1998. One of the related resolutions according to this program was splitting up undergraduate mathematics teacher education programs as elementary and secondary level in mathematics teacher education (YÖK, 1998). On this basis, teachers who were graduates of education faculty and under 40 were evaluated as people affected by this change in the general sense and raised as middle school mathematics teachers specifically. On the other hand, another date close to the mentioned resolution was 1997. Graduates of science faculties have had to cover some pedagogy courses/credits since 1997 to be appointed as a teacher, as mentioned above (Gürşimşek, Kaptan, & Erkan, 1997). Additionally, a quite similar research with the same context conducted in Çankaya district of Ankara has chosen age 40 as a critical point and revealed some statistically significant difference between those two age groups (See Çallı, 2015).

Table 3
Distribution of the participant teachers' certification types in terms of age intervals

	Teachers under 40		Teachers over 40		
	26-30	31-40	41-50	More than 50	Total
Faculty of education certification	3	10	3	7	23
Alternative certification	-	5	7	8	20
Total	3	15	10	15	43

Instrumentation

Patterns functions and algebra knowledge of mathematics teachers

One of the two dependent variables of this study is middle school mathematics teachers' mathematical knowledge to teach (MKT) within a subdomain: patterns, functions, and algebra. To measure teachers' MKT; Hill, Schilling and Ball (2004) developed a scale at University of Michigan. The validity of the items in the scale was also studied by different specialists (Ball et al. 2008; Hill et al., 2004). After all, the instrument was further developed by Hill (2007) for *Learning Mathematics for Teaching* (LMT) Project. The *SII/LMT* (Study of Instructional Improvement/LMT) instrument was used in the current study to measure the participant teachers' MKT.

The measurement instrument of LMT is a widely-accepted one in mathematics education community on account of its reliability and validity (Tutak, 2009). Another reason that highlights LMT instrument is the variety of mathematic topics covered to measure teachers' knowledge to teach. Those topics are classified under three categories: Number and operations, patterns functions and algebra, and geometry.

Patterns, functions and algebra (PFA) scale -which was designed to particularly measure middle school mathematics teachers' MKT- was used in the actual study. The PFA scale involved 15 items in multiple choice styles. Along with some items' annexes, there were 33 items in total. The PFA scale was used in the original form without eliminating any items. Identifying and evaluating students' perceptions, recognizing alternative methods, setting algebraic expressions in real life contexts, interpretation of figures, tables and graphs, modeling, reasoning and justification were the main concerns throughout the PFA scale items. Table 4 represents a short description for each item in the PFA scale. As a whole the PFA instrument could not be added to this dissertation because of the copyright issues[1]. Instead, some of the released items were represented in Appendix 1 to provide a general overview for readers.

Table 4
Item descriptions of patterns functions and algebra scale

Item number	Description
1	Formulization of a linear function
2	Solving word problems for a given real life context
3	Recognizing a non-linear function
4a	Constructing an algebraic expression: area of rectangle (x+x)(3+x)
4b	Constructing an algebraic expression: area of rectangle 2x(3+x)
4c	Constructing an algebraic expression: area of rectangle 2(3+x)x
4d	Constructing an algebraic expression: area of rectangle 3x(x + x)
5	Solving algebraic equations: 2(x+3)=12
6a	Modeling y=2x+3: birthday cards
6b	Modeling y=2x+3: magazines
6c	Modeling y=2x+3: baseball cards
7	Solving 2x squared = 6x
8a	Definition of corresponding sets: 1-4; 1,4,9,16.

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

Table 4 (cont'd)
Item descriptions of patterns functions and algebra scale

8b	Definition of corresponding sets: 1-4; 1-4
8c	Definition of corresponding sets: A-F; 1-6
8d	Definition of corresponding sets: A-D; 1,2
8e	Definition of corresponding sets: A-C; 1-6
9	Evaluating student's explanation for square formula
10a	Justification of a-(b+c): substitute
10b	Justification of a-(b+c): not equal a-b+c
10c	Justification of a-(b+c): product of -1
10d	Justification of a-(b+c): adding inverse
11	Why vertical line slope undefined
12a	Evaluating student predictions for a function: constant
12b	Evaluating student predictions for a function: linear
12c	Evaluating student predictions for a function: quadratic
13a	Real number statements: l-xl=x
13b	Real number statements: -x<=0
13c	Real number statements: -x squared
13d	Real number statements: -(x/-x)=1
13e	Real number statements: (-x) tenth
14	Anticipate solution(s) for equations
15	Interpretation of *velocity-time* graph

When processing the data, responses for each item were coded as 0 for wrong answers and 1 for right answers by using the answer key. *PFA_total* scores for each participant teacher were calculated by adding those 0's and 1's. Consequently, the possible range for *PFA_total* variable was from 0 to 33.

Mathematics teaching efficacy beliefs

Another instrument used in the current study was a Turkish version of *Mathematics Teaching Efficacy Beliefs Instrument* (MTEBI) (Enochs, Smith & Huinker, 2000) which was adapted into Turkish by Bursal (2010). MTEBI was a later-version of *Science Teaching Efficacy Beliefs Instrument* (STEBI) (Enochs & Riggs, 1990) which was specifically customized to measure mathematics teachers' self-efficacy levels. The instrument used in this study (see Appendix 2) comprises 13 five-point Likert-type items (1: strongly disagree, 2: disagree, 3: neutral, 4: agree, 5: strongly agree) with five positively worded and eight negatively worded items. Negatively worded items were recoded before the statistical analysis in SPSS. Thus, the score range for mathematics teaching efficacy beliefs was from 1 to 5; individual scores of the participants were calculated by averaging the responses in each measure. Subsequently, the second dependent variable was described as *SE_average*.

Data collection and variables

At the beginning of data collection process, a proposal of the actual study was submitted to Provincial Directorate for National Education of Ankara (Ankara İl Millî Eğitim Müdürlüğü). In nearly two-week time, the proposal was approved. Thus, the researcher got a written permission to conduct the instruments in different state schools in Çankaya district of Ankara.

In order to collect data, the researcher went to the 15 randomly selected middle schools. The permission from MoNE was provided to school administration first. Then, the researcher briefly explained the aim of the study and asked for permission to have a face to face meeting with mathematics teachers at the school. Almost all administrators of the schools expressed their concerns about confidentiality. After assuring the administrators that the aim of the study was not either ranking of the participant schools or the participant teachers, the researcher had a chance to meet with mathematics teachers at the school. However, it was not always possible to reach all the mathematics teachers at once. For this reason, the researcher usually visited the participant schools for several times.

At the participant schools, some teachers refused to participate to the study directly. While some others refused with the reason that they did not want to be assessed by the PFA questions and/or by the researcher. For the rest who chose to participate to the study, the data collection instruments (demographic information form, MTEBI, and PFA) were given on paper throughout face to face meetings.

In order to address the research questions, the two dependent variables were defined as *PFA_total* and *SE_average* while the independent variables were *AgeD* and *TeachingDegreeD*. The dependent and independent variables of the present study are explained in the following in conjunction with the data scales:

PFA_total was measured in a ratio scale and stood for each participant’s total score in MKT patterns functions and algebra instrument. The possible range for *PFA_total* variable was from 0 to 33.

SE_average was measured in a ratio scale and expressed the average self-efficacy scores of the participant teachers in MTEBI instrument. The possible range for *SE_average* variable was from 1 to 5.

TeachingDegreeD (the dichotomous version of teaching degree intervals) was set in a nominal scale which denoted bachelor's degrees of the participant teachers. If a participant was a graduate of an education faculty, this data was noted as 1; while the rest (graduates from mathematics departments of science faculties) was coded as 0 since they were defined as alternatively-certified teachers.

AgeD (the dichotomous version of age intervals) was set in an ordinal scale. The participants whose ages were 40 or less represented by 0 in the data set; while those whose ages were more than 40 represented by 1.

Reliability and validity

Internal consistency of the current research was evaluated by use of Cronbach's *alpha* which is appropriate to use for dichotomously scored items and Likert type surveys (Huck, 2011). On this basis, the reliability of the PFA scale was estimated as Cronbach's *alpha* value of .636. Additionally, Cronbach's *alpha* was found to be .627 for the score reliability of MTEBI.

For the validity of the study, pilot study evaluation and expert reviews (mathematics education professors' responses) were considered as affirmation. Approving that reliability confines validity, the square root of the Cronbach's alpha coefficients as 0.80 (PFA) and 0.79 (MTEBI) were estimations for the upper limits of validity (Angoff, 1988). Nevertheless, there has been none of the qualitative components such as observation of classroom activities or interviews to assess the validity of the scale results.

Data analysis

In the current study, multivariate analysis of variance (MANOVA) was carried out to explain whether there were any meaningful relations between the dependent and independent variables. The assumptions to conduct MANOVA (see Appendix 3) was analyzed through the following steps: The participant teachers' mathematical knowledge to teach patterns functions and algebra (*PFA_total*) and their self-efficacy levels on their own teaching (*SE_average*) were the two dependent variables of the study; for which skewness, kurtosis and graphical representations were tested. Standardized z-values and univariate normality were screened out. To check multivariate normality, Mahalanobis distances and graphical representations were used. *Pearson's product-moment correlation coefficient r* was used to understand the correlation between the dependent variables. Finally, homogeneity of variance and covariance matrices and multicollinearity were checked.

CHAPTER 4: RESULTS

Introduction

In the current study, results from the data analysis of Multivariate Analysis of Variance (MANOVA) was demonstrated to explain whether there were any meaningful relations among the dependent and independent variables. The following research questions were examined throughout the investigation:

- Is there any statistically significant difference on the average between self-efficacy beliefs and patterns functions and algebra knowledge of Turkish middle school mathematics teachers in terms of their age group (*teachers under 40* and *teachers over 40*)?

- Is there any statistically significant difference on the average between self-efficacy beliefs and patterns functions and algebra knowledge of Turkish middle school mathematics teachers with different teaching certification (*faculty of education certified* and *alternatively certified*)?

- Are the self-efficacy beliefs and patterns functions and algebra knowledge of Turkish middle school mathematics teachers on the average affected by the interaction of age groups (under 40 or over 40) and teaching certification types?

Descriptive analysis of data

Patterns, functions and algebra scores (PFA)

Participant middle school teachers' *PFA_total* scores was one of the two dependent variables in the study. The potential range of the scores was from 0 to 33. However, the participants' scores differed from 12 to 30 in the actual data. The mode score was 25 with 9 participants out of 42 (A participant teacher did not respond on PFA items).

The mean scores of the participants' total correct answers on the 33 PFA items, the standard deviations, and the number of participants in each age category which was split up as *40 and below 40* and *above 40* dichotomously is represented in Table 5 according to the participants' bachelor's degree which differ as *faculty of science* (with alternative teaching certification) and *faculty of education*. The highest mean score could be seen in the category of faculty of science graduate teachers who were younger than 40 years old (*M* = 25.20 points, *SD* = 3.19 points). Besides, the lowest mean score was in the category of faculty of education graduate teachers who were older than 40 years old (*M* = 22.20 points, *SD* = 4.83 points).

Table 5
Descriptive statistics for *PFA_total* scores

Age	Teaching Degree	N	Mean	SD
Age ≤ 40	Faculty of Science	5	25.20	3.19
	Faculty of Education	13	24.77	3.06
	Total	18	24.89	3.01
Age > 40	Faculty of Science	15	23.60	3.48
	Faculty of Education	10	22.20	4.83
	Total	25	23.04	4.04
Total	Faculty of Science	20	24.00	3.40
	Faculty of Education	23	23.65	4.04
	Total	43	23.81	3.72

The participant teachers' percentages of correct answers for each item in patterns, functions and algebra (*PFA*) scale is shown in Table 6. The range of the percentages varied from 12% and 100%. The item with lowest percentage was described as "*Evaluating student predictions for a function: quadratic*" and the second lowest items were described as "*Constructing an algebraic expression: area of rectangle 2(3+x)x*", "*Evaluating student's explanation for square formula*", and "*Justification of a-(b+c): substitute*" with the percentage of 17%. On the other hand the items with the highest percentage were "*Constructing an algebraic expression: area of rectangle 2x(3+x)*", "*Justification of a-(b+c): product of -1*", and "*Real number statements: -x<=0*".

Table 6
Percentages of correct answers for PFA items

Item number	Description	% correct answer
1	Formulization of a linear function	93
2	Solving word problems for a given real life context	98
3	Recognizing a non-linear function	40
4a	Constructing an algebraic expression: area of rectangle (x+x)(3+x)	93
4b	Constructing an algebraic expression: area of rectangle 2x(3+x)	100
4c	Constructing an algebraic expression: area of rectangle 2(3+x)x	17
4d	Constructing an algebraic expression: area of rectangle 3x(x + x)	98
5	Solving algebraic equations: 2(x+3)=12	95
6a	Modeling y=2x+3: birthday cards	79
6b	Modeling y=2x+3: magazines	76
6c	Modeling y=2x+3: baseball cards	88
7	Solving 2x squared = 6x	67
8a	Definition of corresponding sets: 1-4; 1,4,9,16.	93
8b	Definition of corresponding sets: 1-4; 1-4	93
8c	Definition of corresponding sets: A-F; 1-6	93
8d	Definition of corresponding sets: A-D; 1,2	71
8e	Definition of corresponding sets: A-C; 1-6	95
9	Evaluating student's explanation for square formula	17
10a	Justification of a-(b+c): substitute	17
10b	Justification of a-(b+c): not equal a-b+c	62
10c	Justification of a-(b+c): product of -1	100

Table 6 (cont'd)
Percentages of correct answers for PFA items

10d	Justification of a-(b+c): adding inverse	81
11	Why vertical line slope undefined	60
12a	Evaluating student predictions for a function: constant	57
12b	Evaluating student predictions for a function: linear	79
12c	Evaluating student predictions for a function: quadratic	12
13a	Real number statements: l-xl=x	38
13b	Real number statements: -x<=0	100
13c	Real number statements: -x squared	83
13d	Real number statements: -(x/-x)=1	93
13e	Real number statements: (-x) tenth	90
14	Anticipate solution(s) for equations	95
15	Interpretation of velocity-time graph	40

Self-efficacy belief scores (SE)

Participant middle school teachers' *SE_average* scores was the other dependent variable in the study. The range of the scores was from 1 to 5. However, the participants' scores differed from 3.8 to 5 in the actual data. The mean scores of the participants' self-efficacy beliefs on the 13 Likert type MTEBI items, the standard deviations, and the number of participants in each age category which was split up as *40 and below 40* and *above 40* dichotomously is represented in Table 7 according to the participants' bachelor's degree which differ as *faculty of science* (with alternative teaching certification) and *faculty of education*. The highest mean score could be seen in the category of faculty of science graduate teachers who were younger than 40 years old (*M*=4.48 points, *SD*= 0.44 points). This situation is in a parallel manner with the previous section, since the same group had the highest mean scores in PFA items. Besides, the lowest mean for self-efficacy average score was in the category of faculty of science graduate teachers who were older than 40 years old (*M* = 4.29 points, *SD* = 0.37 points).

Table 7
Descriptive statistics for *SE_average* scores

Age	Teaching Degree	N	Mean	SD
Age ≤ 40	Faculty of Science	5	4.48	0.44
	Faculty of Education	13	4.36	0.32
	Total	18	4.39	0.35
Age > 40	Faculty of Science	14	4.29	0.37
	Faculty of Education	10	4.40	0.32
	Total	24	4.34	0.34
Total	Faculty of Science	19	4.34	0.38
	Faculty of Education	23	4.37	0.31
	Total	42	4.36	0.34

Table 8 reveals the frequency of the responses for each MTEBI item. Since one of the participant teachers skipped (or ignored) to take the self-efficacy belief instrument, the total number is stated as 42 in the table. The participant middle school mathematics teachers demonstrated high self-efficacy belief scores with a mode of five for ten of the 13 items. The lowest self-efficacy belief score was the item "*If I had a choice, I would not want my math class to be observed and evaluated* *by the inspector"*, which was negatively worded but recoded as *SE10*: *If I had a choice, I would want my math class to be observed and evaluated by the inspector* while processing the current data.

Table 8
Frequency of responses for each MTEBI item

Item	1	2	3	4	5	Total
SE1	0	2	0	19	**21**	42
SE2	0	0	0	19	**23**	42
SE3	1	1	1	**20**	19	42
SE4	2	0	5	**22**	13	42
SE5	1	1	0	17	**23**	42
SE6	0	0	0	9	**33**	42
SE7	1	1	2	**23**	15	42
SE8	0	0	0	20	**22**	42
SE9	1	0	0	11	**30**	42
SE10	4	8	10	8	**12**	42
SE11	1	2	0	17	**22**	42
SE12	1	0	1	15	**25**	42
SE13	0	0	3	13	**26**	42

Note. 1: strongly disagree, 2: disagree, 3: neutral, 4: agree, 5: strongly agree

Bivariate correlations

The correlations between all the variables stated in the actual study -the two dependent (*PFA_total* and *SE_average*) and the independent variables (*AgeD* and *TeachingDegreeD*)-were analyzed using point-biserial correlation coefficient and represented in Table 9. It was observed that teachers' age groups were negatively correlated to their teaching degrees at the level of $r = -.32$ ($p < .05$).

Table 9
Bivariate correlation matrix for the variables

	Age	TeachingDegreeD	PFA_total	SE_average
AgeD	1	-.32*	-.25	-.08
TeachingDegreeD		1	-.05	.05
PFA_total			1	.02
SE_average				1

Note. *Correlation is significant at the .05 level

Inferential analysis of data

Analysis for the combined dependent variables

The research questions were addressed by conducting a multivariate analysis of variance (MANOVA) for the dependent variables: Self efficacy average scores *(SE_average)* and mathematical knowledge to teach patterns, functions and algebra total scores (PFA_total) with the dichotomous independent variables: Age group *(AgeD)* and teaching certification type *(TeachingDegreeD).*

The interaction effect of teachers' age groups and their teaching degrees (*AgeD* TeachingDegreeD)* was analyzed by the use of Wilk's criterion. Wilk's λ = .97, $F_{calculated}$ (2, 37) = 0.51, p = .61, and the F_{critical}(2, 37)=3.25 were calculated at α = .05 level. Since $F_{calculated}$ value was less than $F_{critical}$ value, the interaction effect was not stated as statistically significant. A measure of effect size, *partial* $\eta 2$ (eta-squared), was calculated by using Wilk's λ:

$$\eta_i^2 = 1 - \lambda_i$$

Hence, the effect size was measured with partial η^2 = .03 for the interaction effect. For an easier interpretation, this value was converted into Cohen's d as 0.35 standard deviations according to Cohen's (1988) conversion formulas, cited by DeCoster (2009).

In the current study, the interaction effect was not found statistically significant. Therefore, main effects were checked. The combined dependent variables were not statistically significantly affected by the independent variable *AgeD,* as Wilk's λ = .91, F (2, 37) =1.79, p= .18, partial η^2= .09, Cohen's d = 0.63 standard deviations. These values revealed that being age group did not contribute significantly toward discriminating the teachers' average self-efficacy scores and mathematical knowledge to teach PFA total scores. 0.63 standard deviation difference was calculated between the combined competencies of teachers in terms of their age groups.

According to the analysis conducted, there had been no statistically significant difference found between the faculty of education certified teachers and alternatively certified teachers in terms of their self-efficacy and mathematical knowledge level on patterns functions and algebra subdomain. *TeachingDegreeD* did not figure out any statistically significant difference to interpret the dependent variables of the current research with the values calculated as Wilk's $\lambda = .99$, $F(2, 37) = 0.19$, $p = .83$, partial $\eta 2 = .01$, Cohen's $d = 0.20$ standard deviations.

CHAPTER 5: DISCUSSION

Introduction

The current research has investigated the impact of teachers’ *age* (teachers under 40 or teachers over 40) and teachers’ *teaching certification types* (faculty of education certified or alternatively certified) on middle school mathematics teachers’ specialized content knowledge (particularly mathematical knowledge to teach *patterns, functions, and algebra*) and their self-efficacy levels. In this chapter, major findings which gained from the current study were demonstrated through mathematical knowledge for teaching and self-efficacy perception of teachers related with Turkish context. Additionally, implications for practice and further research were discussed. Conclusively, the limitations of the study were represented at the end of this chapter.

Major findings

For the current study, the major findings could be outlined as in the following:

- There is no statistically significant difference between middle school mathematics teachers’ self-efficacy beliefs in terms of their age groups.
- Faculty of education certified middle school mathematics teachers do not have higher or less self-efficacy beliefs than the alternatively certified middle school mathematics teachers.
- No statistically significant difference was observed between middle school mathematics teachers’ mathematical knowledge to teach patterns functions and algebra with respect to their age group.

- Faculty of education certified middle school mathematics teachers do not have higher or less mathematical knowledge to teach patterns functions and algebra than the alternatively certified middle school mathematics teachers.

Findings related to teachers' self-efficacy

The study revealed that age groups of the teachers have no statistically significant effect on teachers' self-efficacy beliefs. This situation could be explained by some findings about more experienced teachers' lack of motivation after many years in teaching profession (Beijaard, Verloop, & Vermunt, 2000) and/or less experienced teachers' lack of experience to evaluate different teaching scenarios and classroom dynamics effectively (Hoy & Spero, 2005). However, such a comment might be incomplete and inadequate since the self-efficacy average scores of the teachers have been found sufficiently high for both groups. Therefore, the reason behind high self-efficacy scores for less experienced teachers could be expanded to the area that they are more capable of following contemporary educational issues which helps them to be more comfortable about their instructions and teaching in general (Cavas, Cavas, Karaoglan, & Kısla, 2009). While, the situation about more experienced teachers could be explained with the idea that experienced teachers do not seem to realign their efficacy beliefs once established (Çallı, 2015).

Another predictor to examine teachers' self-efficacy levels in the current study was bachelor's degree of mathematics teachers in terms of their alma mater. The reason for choosing such an independent variable was to examine whether pedagogical methods provide more efficiency rather than college level mathematical courses for teachers. That is why the faculty of education certified teachers' and alternatively certified teachers' self-efficacy beliefs were also checked in the present study. Appleton (1995) and Palmer (2001) emphasized the importance of pedagogical methods that teachers had and how they performed them in educational environments. Unlike, there has been not found any statistically significant difference between the two groups of teachers with different teaching degrees in the current study. This could be explained by the lack of qualitative components of this study since the researcher neither observed any classroom activities to see how the teachers perform in their classes related with the reported self-efficacy beliefs; nor interviewed with the participant teachers.

However, Turkish teachers are not much happy with being inspected in a broad sense (Toremen & Dos, 2009). The lowest scored specific MTEBI item (*SE10*) which was described as "*If I had a choice I would (not) want my math class to be observed and evaluated by the inspector*" might be addressing this idea. But still, Çallı (2015) who conducted a very similar research design in the same context and used the same MTEBI items, suggested to interpret this particular item carefully, since it might be related with a past experience or reluctance rather than self-efficacy beliefs of a teacher.

Findings related to teachers' mathematical knowledge for teaching

In the related literature, it was stated that the previous research mostly agreed on the issue that number of mathematics courses taken in college was not found efficient enough to understand teachers' mathematical knowledge (Ball, Hill, & Bass, 2005). Instead of focusing on mathematical background, teaching methods and techniques would be more relevant according to that result. In the current study, there was found no statistically significant difference in terms of mathematical knowledge to teach

PFA between the graduates of science faculty who were considered as more and in-depth mathematics courses they were exposed, and the graduates of education faculty who were considered as more and in-depth pedagogical courses they were taught.

This situation might be explained by the view that suggests conceptual understanding for both groups of teachers. In other words, without conceptual understanding, it might lead inadequate and tenuous performing especially while teaching with lower grades. According to Ball et al., after graduating from the college, novice teachers have experienced many difficulties to adjust the teaching models and strategies they have learned to be relevant to teach at middle level. For this reason, teachers need to "unpack" (Ball, 1988) what they have learned before to be able to design more favorable mathematical models for their classes.

On the other hand, Ball, Lubienski, and Mevborn (2001) deliberated that taking more advanced math courses at college level undermines the conceptual understanding while promoting procedural knowledge. The current study supports this idea in terms of the participant teachers' lowest scores on PFA items which were mostly addressing the lack of global/meta-level activities of algebra teaching that Kieran (1996) had suggested. With a closer look, the description of those items "***Evaluating** student predictions for a function: quadratic*", "***Constructing** an algebraic expression: area of rectangle 2(3+x)x*", "***Evaluating** student's explanation for square formula*", and "***Justification** of a-(b+c): substitute*" refers to lack of conceptual understanding; while the items with highest percentages indicated procedural knowledge mostly. Moreover, this might be one of the reasons that lie in the ground of the low mathematics scores of Turkish students in national and international evaluation exams.

Apart from these, the recent study has the importance to be one of the first studies which conducted MKT items in Turkish context. Previously, Çallı (2015) has worked with MKT number concepts and operations scale which was designed by LMT researchers in the same context. Use of MKT patterns, functions and algebra scale is the first in Turkish context as Çallı also offered in her dissertation for further research (p.70).

Implications for practice

Good teaching requires a deep understanding of the subject area taught including teachers' knowing well about their students and the social, political, cultural context of the environment that they work in (Ball & McDiarmid, 1990). The major finding of the current study speculates the deficiencies in conceptual understanding of middle school mathematics teachers and their misleading perceptions about self-efficacy. Even in the process of data collection, most of the teachers were uncomfortable since they thought that the researcher would assess them. Some of the teachers directly rejected the researcher by reasoning "she was no right to assess or judge such an experienced teacher". Many of the teachers expressed their concerns about confidentially of the work despite providing all legal permits and documents. However, they were suggested that instruments were available via internet so they could have answered the questions with no name and send the outputs in the virtual environment. All teachers ignored that option and the researcher visited the schools for several times to gather data. According to the researcher's observations in that process, many teachers were also uncomfortable with the use of any technological tools; the ones who accepted to participate in the research preferred paper-pencil instead.

In Turkey, professional development courses are much about legal procedures and general issues for all teachers-not subject area specific mostly. The number of hours spent in such a professional development process would not be a meaningful predictor to measure how teachers have improved their instructional skills and self-efficacy. Instead, professional development should set a sight on content knowledge and pedagogical skill which is designed to reveal the teachers' potential and to improve teachers' self-efficacy (Ingvarson et al., 2005; Watson, 2006). Current evidence proposed that self-efficacy of mathematics teachers was positively influenced by professional development (e.g. Ingvarson, Meiers, & Beavis, 2005; Watson, 2006; Ross & Bruce, 2007; Swockhammer et al., 2009).

Besides all, further and sustainable development should be considered. In Turkish context, the professional development should be content-related, long-term which spread over with practices and careful follow-up processes, and compulsory for all teachers. The rationale of compulsory professional development that I suggest, is the finding that more than 60% of in-service Turkish teachers have reported no need for a professional development (MoNE, 2008), although the situation required just the opposite (e.g. Çallı, 2015). As an example; results of a comprehensive and long-term professional development experience proved not only the increase of teachers' self-efficacy; but its shifting for post professional development levels and continuance for the subsequent six years (Watson, 2006).

In the young Turkish Republic, Atatürk (1925), the first president of Turkey, who is known as *Head Teacher*, explained the crucial importance of teaching profession by saying "*Teachers are the one and only people who save nations*". Unfortunately, since those days, the teaching profession has suffered serious loss of status in the society in Turkey. The Turkish education community and policymakers should act on this essential problem as soon as possible. The recruitment and placement process of teachers should be improved in terms of the desirable characteristics of 21st century skills. The research conducted by the International Baccalaureate Organization (IBO) showed how important to have talented and enthusiastic teachers through offering 21st century skills such as critical thinking, creativity, communication, and collaboration for children (e.g. Bergeron & Dean, 2013). As Judith Fabian, the chief academic officer in IBO states: "*IB teachers are themselves lifelong learners who aim to develop the same keen interest in their students*".

Implications for further research

The present study was conducted as a non-experimental quantitative research. By adding qualitative components such as observing classroom activities, videotaping, interviewing with students and teachers, a mixed method could be constructed. Hence the mathematical knowledge of teachers and students, the self-efficacy levels in terms of *doing math* for teachers and students could be examined whether there is any statistically significant relationship between teachers' learning and teaching; and students' attitudes and achievement. Furthermore, number concepts and operations scale (NCOP) and patterns functions and algebra scale (PFA) of LMT has been used in Turkish context, but still geometry scale of MKT could be used in the same context. The MKT instruments could be used in other regions of Turkey, and not only in state schools, but also in private colleges. Additionally, MKT items could be used for different grades such as elementary or secondary. Apart from self-efficacy instruments, (and/or) other measures could be used for teachers' attitudes, values, and beliefs. Finally, technological components of teaching PFA (related with designing activities, instructions, methods, and strategies) could be added to measure teachers' mathematical knowledge.

Limitations

Responding on the instruments -especially for PFA scale- took much time for the participant middle school mathematics teachers. Since none of the teachers agreed on responding the instruments via internet, it took much time and effort for the researcher as well. Despite, the response rate was lower than expected which might be a barrier for more accurate results. As Hill (2010) reported that when teachers were paid $50 to complete the survey, they got higher response rate for their study. So instead of voluntary basis, if it had been paid to the teachers, the response rate of the study might be higher. Other option might be that kind of surveys could be sponsored by government, and/or educational institutions to get more desirable results.

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APPENDICES

APPENDIX 1: Learning mathematics for teaching - sample released items

LEARNING MATHEMATICS FOR TEACHING

MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) MEASURES

MATHEMATICS RELEASED ITEMS 2008

University of Michigan, Ann Arbor

610 E. University #1600

Ann Arbor, MI 48109-1259

(734) 647-5233

www.sitemaker.umich.edu/lmt

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

December 26, 2008

Dear Colleague:

Thank you for your interest in our survey items measuring mathematical knowledge for teaching. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement* (SII). SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and understand how such knowledge affects program implementation. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (over 5000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President[2], we piloted these items with K-6 teachers engaged in mathematics professional development. This work developed into a sister project to SII, *Learning Mathematics for Teaching* (LMT). With funding from the National Science Foundation, LMT has taken over instrument development from SII, developing and piloting geometry and middle school items.

We have publicly released a small set of items from our projects' efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We

[2] Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.

encourage their use in such contexts. However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge. In other words, one cannot calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

> Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.sitemaker.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

Deborah Loewenberg Ball

Dean, School of Education

William H. Payne Collegiate Professor of Education

University of Michigan

Heather Hill

Associate Professor

Harvard Graduate School

Released Sample Items:

MIDDLE SCHOOL CONTENT KNOWLEDGE ITEMS (2008)

18. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e., a (b + c) = ab + ac]? (Mark APPLIES, DOES NOT APPLY, or I'M NOT SURE for each.)

	Applies	Does not apply	I'm not sure
a) Adding $\frac{3}{4}+\frac{5}{4}$	1	2	3
b) Solving 2x – 5 = 8 for x	1	2	3
c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$	1	2	3
d) Adding 34 + 25 using this method: $\begin{array}{r} 34 \\ \underline{+25} \\ 59 \end{array}$	1	2	3

19. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and

a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes <u>closest</u> to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

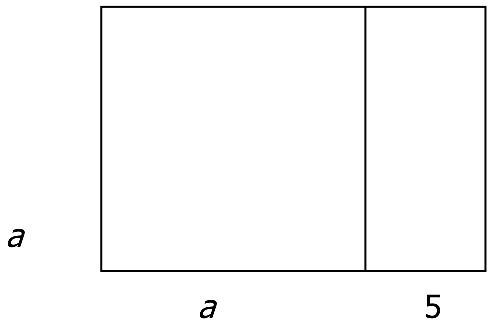
b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

c) They're equal because of the associative property. We know that a – (b + c) equals (a – b) – c which equals a – b – c.

d) They're equivalent because what you do to one side you must always do to the other.

e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

20. Ms. Whitley was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression correctly represents or does not correctly represent the area of the figure. (Mark REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE for each.)



	Correctly represents	Does not correctly represent	I'm not sure
a) $a^2 + 5$	1	2	3
b) $(a + 5)^2$	1	2	3
c) $a^2 + 5a$	1	2	3
d) $(a + 5)a$	1	2	3
e) $2a + 5$	1	2	3
f) $4a + 10$	1	2	3

21. Ms. Hurlburt was teaching a lesson on solving problems with an inequality in them. She assigned the following problem.

$$-x < 9$$

Marcie solved this problem by reversing the inequality sign when dividing by

-1, so that $x > -9$. Another student asked why one reverses the inequality when dividing by a negative number; Ms. Hurlburt asked the other students to explain. Which student gave the best explanation of why this method works? (Mark ONE answer.)

a) Because the opposite of x is less than 9.

b) Because to solve this, you add a positive x to both sides of the inequality.

c) Because $-x < 9$ cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.

e) Because this method is a shortcut for moving both the x and 9 across the inequality. This gives the same answer as Marcie's, but in different form: $-9 < x$.

APPENDIX 2: MTEBI items used in the current study

Lütfen aşağıdaki her önermeye ne oranda katılıp katılmadığınızı belirtmek için her öneri için seçeneklerden bir tanesini işaretleyiniz. Tüm önermeleri değerlendirdiğinizi kontrol etmeyi unutmayınız!

	Kesinlikle Katılmıyorum	Katılmıyorum	Çekimser	Katılıyorum	Kesinlikle Katılıyorum
1. Matematik dersini öğretmek için devamlı daha iyi yöntemler bulacağım.	☐	☐	☐	☐	☐
2. Ne kadar çabalarsam çabalayayım, matematik dersini diğer dersleri öğrettiğim kadar iyi öğretemeyeceğim.	☐	☐	☐	☐	☐
3. Matematiksel kavramları etkili biçimde nasıl öğreteceğimi biliyorum.	☐	☐	☐	☐	☐
4. Matematikle ilgili sınıf etkinliklerini takip etmekte çok etkili olamayacağım.	☐	☐	☐	☐	☐
5. Matematik dersini genelde yetersiz öğreteceğim.	☐	☐	☐	☐	☐
6.İlköğretim matematik dersini etkili öğretmeye yetebilecek derecede matematiksel kavramları anlıyorum.	☐	☐	☐	☐	☐
7. Matematiksel işlemlerin nasıl sonuca ulaştığını öğrencilere açıklamak için sınıf etkinliklerini kullanmakta zorlanacağım.	☐	☐	☐	☐	☐
8. Öğrencilerin matematik ile ilgili sorularını genelde cevaplandırabileceğim.	☐	☐	☐	☐	☐
9. Matematik dersini öğretebilmek için gerekli becerilere sahip olabileceğimden emin değilim.	☐	☐	☐	☐	☐

10. Seçme şansım olursa, matematik dersimin müfettiş tarafından gözlenip değerlendirilmesini istemiyorum.	☐	☐	☐	☐	☐
11. Ne zaman bir öğrencim bir matematiksel kavramı anlamakta zorlansa, kavramı daha iyi anlamasına yardım etmek için çoğunlukla ne yapmam gerektiğini bilemeyeceğim.	☐	☐	☐	☐	☐
12. Matematik dersini anlatırken çoğunlukla öğrencilerin soru sormasını cesaretlendireceğim.	☐	☐	☐	☐	☐
13. Öğrencilerin matematiğe ilgilerini çekmek için ne yapmam gerektiğini bilemiyorum.	☐	☐	☐	☐	☐

APPENDIX 3: Assumptions for the statistical analysis of data

In the following section assumptions to conduct MANOVA was explained. The nine assumptions were checked by the guide of "*Two-way MANOVA in SPSS statistics*" (Lund & Lund, 2015). No assumptions were violated, so MANOVA was used to analyze the data in the current study.

Assumption #1: The **dependent variables** of the current study were **continuous**: *PFA_total* was measured in a ratio scale with the possible range from 0 to 33. *SE_average* was measured in a ratio scale with the possible range from 1 to 5.

Assumption#2: The **independent variables** of the current study consist of two **categorical** and **independent** groups. *AgeD*: 'teachers under 40' and 'teachers over 40'; *TeachingDegreeD*: 'faculty of education certified' and 'alternatively certified'.

Assumption#3: No relationships were observed in each group or between groups in the current study. **Independence of observations** assumption was not violated.

Assumption#4: **Adequate sample size**. The response rate of the current study is 57.33% with 43 participants. However, the preliminary decided sample size was 69 (The concerns and reasons were explained in detail during Chapter 3). The assumption was not violated but the larger sample size is better to get more accurate results.

Assumption#5: There is **no univariate** or **multivariate outliers** (See Figure 4 and Figure 5).

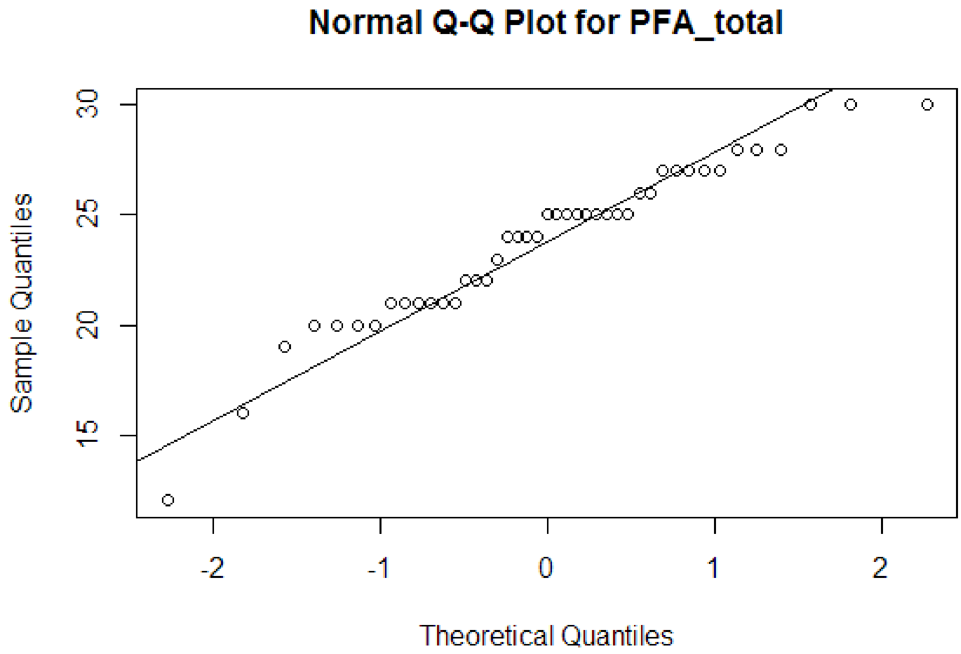


Figure 4. Normal Q-Q plot for PFA_total

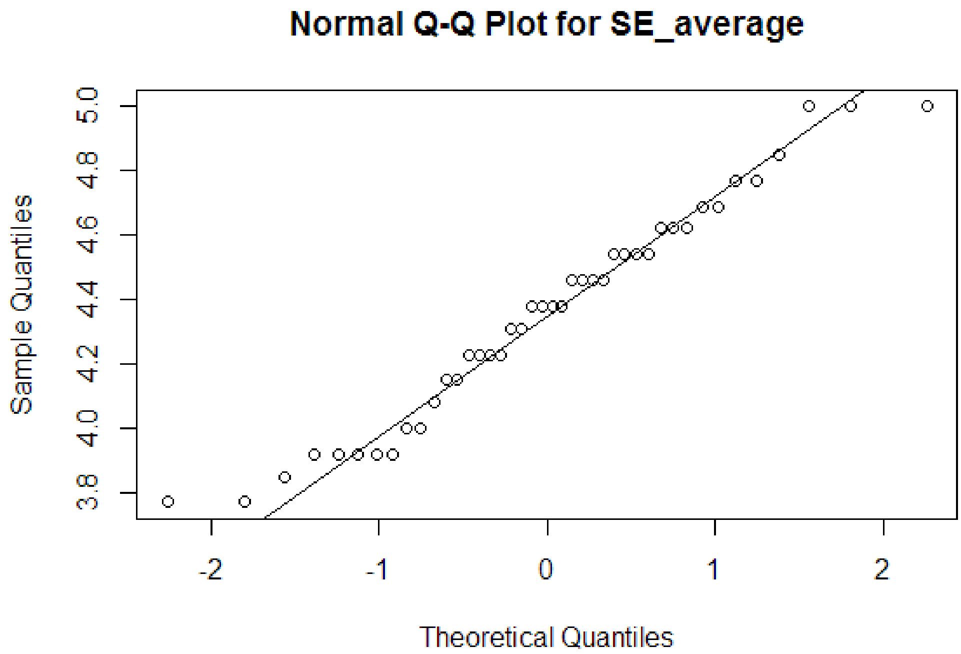


Figure 5. Normal Q-Q plot for SE_average

Assumption#6: **Univariate normality & Multivariate Normality**

Table 10
Descriptive statistics for the dependent variables

Statistic	PFA_total	SE_average
N	43	42
Mean	23.81	4.36
Median	25.00	4.38
Standard Deviation	3.72	0.34
Skewness	0.68	-0.66
Kurtosis	-0.17	0.64
Minimum	12.00	3.77
Maximum	30.00	5.00

Table 11
Descriptive statistics for the standardized z values

	N	Range	Minimum	Maximum	Mean	SD
Z score PFA_total	43	1.4794	-3.178000	1.664000	0.00	1.00
Z score SE_average	42	1.4612	-1.723	1.873	0.00	1.00

Note. SD =Standard deviation, Range = Q3- Q1

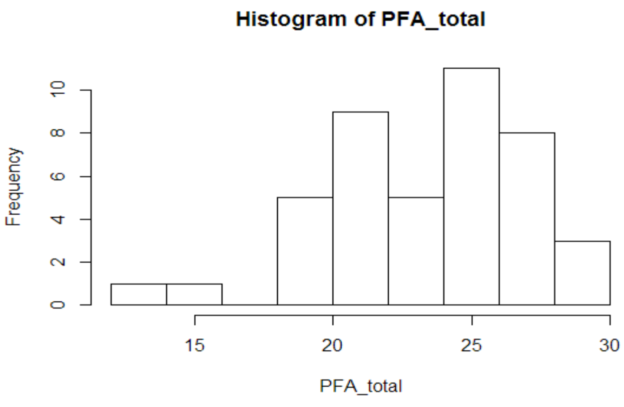


Figure 6. Frequency histogram for dependent variable PFA_total

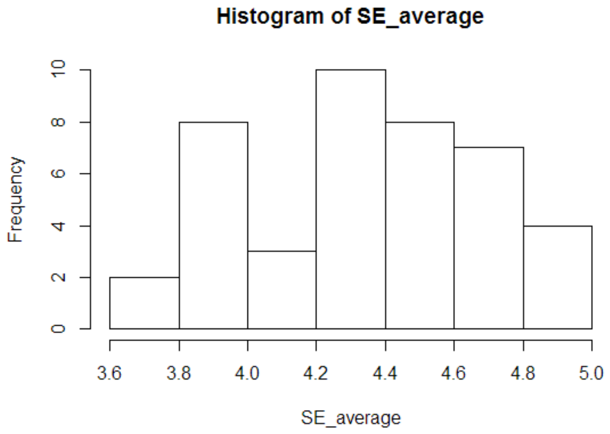


Figure 7. Frequency histogram for dependent variable SE_average

Mahalanobis distance was calculated as 7.67 at maximum (critical χ^2 value for df =2 was 13.82). No multivariate outliers were existed.

Shapiro-Wilks: Doesn't work well if several values in the data set are the same. Works best for data sets with < 50, but can be used with larger data sets.

Assumption#7: **Relationship between dependent variables**

In the current study, the participant teachers' self-efficacy belief average scores were correlated to their PFA_total scores at the level of $r = .02$ ($p < .05$), in the interval low to moderate as expected for the MANOVA (Leech et al., 2008).

Assumption#8: **Homogeneity of covariance**

Homogeneity of covariance assumption across groups was checked: Box's M = 10,051, F (9, 2143.01) = 0.979, p = .46 in the 2x2 MANOVA. The values revealed that there was no statistically significant difference between the covariance matrices (which tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups). Subsequently, the homogeneity of covariance assumption was not violated and it was appropriate to use Wilk's lambda test. To see the test results, check Table 12.

Table 12
Multivariate analysis of variance of combined dependent variables

Effect	Wilk's λ	F	Hyp df	Error df	p	Par. η 2	Observed power
Intercept	.01	3432.80	2	37	<.05	.995	1.000
AgeD	.91	1.79	2	37	.18	.09	.35
TeachingDegreeD	.99	0.19	2	37	.83	.01	.08
AgeD*TeachingDegreeD	.97	0.51	2	37	.61	.03	.13

Assumption#9: There is no **multicollinearity.** Since two independent variables exist in the current study, multicollinearity was not violated. Variance Inflation Factor (VIF) is 1 which is the lower bound for VIF.