POWER ALLOCATION STRATEGIES FOR CHANNEL SWITCHING AND WIRELESS LOCALIZATION

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We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

POWER ALLOCATION STRATEGIES FOR CHANNEL SWITCHING AND WIRELESS LOCALIZATION

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Optimal power allocation is an important approach for enhancing performance of both communication and localization systems. In this dissertation, optimal channel switching problems are investigated for average capacity maximization via optimization of power resources in general. In addition, power control games are designed for a wireless localization network including anchor and jammer nodes which compete for the localization performance of target nodes.

First, an optimal channel switching strategy is proposed for average capacity maximization in the presence of average and peak power constraints. Necessary and sufficient conditions are derived in order to determine when the proposed optimal channel switching strategy can or cannot outperform the optimal single channel strategy, which performs no channel switching. Also, it is obtained that the optimal channel switching strategy can be realized by channel switching between at most two different channels. In addition, a low-complexity optimization problem is derived in order to obtain the optimal channel switching strategy. Furthermore, based on some necessary conditions that need to be satisfied by the optimal channel switching solution, an alternative approach is proposed for calculating the optimal channel switching strategy.

Second, the optimal channel switching problem is studied for average capacity maximization in the presence of additive white Gaussian noise channels and channel switching delays. Initially, an optimization problem is formulated for the maximization of the average channel capacity considering channel switching delays and constraints on average and peak powers. Then, an equivalent optimization problem is obtained to facilitate theoretical investigations. The optimal strategy is derived and the corresponding average capacity is specified when channel switching is performed among a given number of channels. Based on this result, it is shown that channel switching among more than two different channels is not optimal. In addition, the maximum average capacity achieved by the optimal channel switching strategy is formulated as a function of the channel switching delay parameter and the average and peak power limits. Then, scenarios under which the optimal strategy corresponds to the exclusive use of a single channel or to channel switching between two channels are described. Furthermore, sufficient conditions are obtained to determine when the optimal single channel strategy outperforms the optimal channel switching strategy.

Third, the optimal channel switching problem is studied for average capacity maximization in the presence of multiple receivers in the communication system. At the beginning, the optimal channel switching problem is proposed for average capacity maximization of the communication between the transmitter and the secondary receiver while fulfilling the minimum average capacity requirement of the primary receiver and considering the average and peak power constraints. Then, an alternative equivalent optimization problem is provided and it is shown that the solution of this optimization problem satisfies the constraints with equality. Based on the alternative optimization problem, it is obtained that the optimal channel switching strategy employs at most three communication links in the presence of multiple available channels in the system. In addition, the optimal strategies are specified in terms of the number of channels employed by the transmitter to communicate with the primary and secondary receivers.

Last, a game theoretic framework is proposed for wireless localization networks that operate in the presence of jammer nodes. In particular, power control games between anchor and jammer nodes are designed for a wireless localization network in which each target node estimates its position based on received signals from anchor nodes while jammer nodes aim to reduce localization performance of target nodes. Two different games are formulated for the considered wireless localization network: In the first game, the average Cramér-Rao lower bound (CRLB) of the target nodes is considered as the performance metric, and it is shown that at least one pure strategy Nash equilibrium exists in the power control game. Also, a method is presented to identify the pure strategy Nash equilibrium, and a sufficient condition is obtained to resolve the uniqueness of the pure Nash equilibrium. In the second game, the worst-case CRLBs for the anchor and jammer nodes are considered, and it is shown that the game admits at least one pure Nash equilibrium.

Keywords: Power allocation, channel switching, capacity, time sharing, switching delay, multiuser, localization, jammer, Nash equilibrium, wireless network.

ÖZET

KANAL DEĞİŞTİRME VE TELSİZ KONUMLANDIRMA İÇİN GÜÇ TAHSİSİ STRATEJİLERİ

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Optimal güç tahsisi, hem iletişim hem de konumlandırma sistemlerinin performansını artırmak için önemli bir yaklaşımdır. Bu tezde, genel olarak güç kaynaklarının optimizasyonu yoluyla ortalama kapasite enbüyütmesi için optimal kanal değiştirme problemleri araştırılmaktadır. Ayrıca, güç kontrol oyunları, hedef düğümlerin konumlandırma performansı için yarışan referans ve karıştırıcı düğümleri içeren bir telsiz konumlandırma ağı için tasarlanmaktadır.

İlk olarak, ortalama ve zirve güç kısıtlarının varlığında ortalama kapasite enbüyütmesi için optimal bir kanal değiştirme stratejisi önerilmektedir. Önerilen optimal kanal değiştirme stratejisinin, herhangi bir kanal geçişi gerçekleştirmeyen optimal tek kanallı stratejiyi geçip geçemeyeceğini belirlemek için gerekli ve yeterli koşullar türetilmektedir. Ayrıca, en iyi kanal değiştirme stratejisinin, en fazla iki farklı kanal arasında kanal değişimi ile gerçekleştirilebileceği elde edilmektedir. Ek olarak, optimal kanal değiştirme stratejisini elde etmek için düşük karmaşıklıklı bir optimizasyon problemi türetilmektedir. Dahası, optimal kanal değiştirme çözümü tarafından karşılanması gereken bazı gerekli koşullara dayanarak, optimal kanal değiştirme stratejisini hesaplamak için alternatif bir yaklaşım önerilmektedir.

İkinci olarak, optimal kanal değiştirme problemi, toplanır beyaz Gauss gürültüsü kanalları ve kanal değiştirme gecikmeleri varlığında ortalama kapasite enbüyütmesi için çalışılmaktadır. Başlangıçta, kanal değiştirme gecikmelerinin ve ortalama ve zirve güçlerdeki kısıtlarının göz önüne alındığı ortalama kanal kapasitesi enbüyütmesi için, bir optimizasyon problemi formülleştirilmektedir. Daha sonra, teorik araştırmaları kolaylaştırmak için eşdeğer bir optimizasyon problemi elde edilmektedir. Optimal strateji türetilmekte ve belirli bir kanal sayısı arasında kanal değiştirme yapıldığında karşılık gelen ortalama kapasite belirlenmektedir. Bu sonuca göre, ikiden fazla farklı kanal arasında kanal değiştirmenin optimal olmadığı gösterilmektedir. Ek olarak, optimal kanal değiştirme stratejisi ile elde edilen maksimum ortalama kapasitesi, kanal değiştirme gecikmesi parametresinin ve ortalama ve zirve güç limitlerinin bir fonksiyonu olarak formülleştirilmektedir. Daha sonra, tek bir kanalın özel kullanımına veya iki kanal arasında kanal değiştirmeye karşılık gelen optimal stratejinin altındaki senaryolar tarif edilmektedir. Dahası, optimal tek kanallı stratejinin optimal kanal değiştirme stratejisinden ne zaman üstün olduğunu belirlemek için yeterli koşullar elde edilmektedir.

Üçüncüsü, optimal kanal değiştirme problemi, iletişim sistemindeki çoklu alıcıların varlığında ortalama kapasite enbüyütmesi için çalışılmaktadır. Başlangıçta, birincil alıcının minimum ortalama kapasite gereksinimini karşılarken ve ortalama ve zirve güç kısıtlarını dikkate alarak, verici ve ikincil alıcı arasındaki iletişimin ortalama kapasite enbüyütmesi için optimal kanal değiştirme problemi önerilmektedir. Daha sonra, alternatif bir eşdeğer optimizasyon problemi sağlanmakta ve bu optimizasyon probleminin çözümünün kısıtları eşitlikle karşıladığı gösterilmektedir. Alternatif optimizasyon problemine dayanarak, optimal kanal değiştirme stratejisinin, sistemdeki birden fazla mevcut kanalın varlığında en fazla üç iletişim bağlantısını kullandığı elde edilmektedir. Ek olarak, optimal stratejiler, verici tarafından birincil ve ikincil alıcılarla iletişim kurmak için kullanılan kanalların sayısı bakımından belirtilmektedir.

Son olarak, karıştırıcı düğümlerin varlığında çalışan telsiz konumlandırma ağları için bir oyun teorisi çerçevesi önerilmektedir. Özellikle, referans ve karıştırıcı düğümleri arasındaki güç kontrol oyunları, karıştırıcı düğümlerin hedef düğümlerin konumlandırma performansını azaltmayı amaçlarken, her bir hedef düğümün referans düğümlerinden alınan sinyallere dayanarak konumunu tahmin ettiği bir telsiz konumlandırma ağı için tasarlanmaktadır. Dikkate alınan telsiz konumlandırma ağı için iki farklı oyun formülleştirilmektedir: İlk oyunda, hedef düğümlerin ortalama Cramér-Rao alt sınırı (CRAS) performans ölçütü olarak kabul edilmekte ve güç kontrol oyununda en az bir saf strateji Nash dengesinin olduğu gösterilmektedir. Ayrıca, saf strateji Nash dengesini tanımlamak için bir yöntem sunulmakta ve saf Nash dengesinin tekliğini çözümlemek için yeterli bir koşul elde edilmektedir. İkinci oyunda, referans ve karıştırıcı düğümleri için en kötü durum CRAS'leri dikkate alınmakta ve oyunun en az bir saf Nash dengesini kabul ettiği gösterilmektedir.

Anahtar sözcükler: Güç tahsisi, kanal değiştirme, kapasite, zaman paylaşımı, değiştirme gecikmesi, çok kullanıcılı, konumlandırma, karıştırıcı, Nash dengesi, telsiz ağ.

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Chapter 1

Introduction

Optimal power allocation has critical importance for improving performance of communication and localization systems. In the following sections, the optimal channel switching for average capacity maximization is introduced to emphasize the significance of optimal resource allocation in communication systems first, and then power control games are motivated to point out the importance of power allocation in wireless localization networks.

1.1 Optimal Channel Switching for Average Capacity Maximization

In recent studies in the literature, benefits of time sharing ("randomization") have been investigated for various detection and estimation problems [1]-[13]. For instance, in the context of noise enhanced detection and estimation, additive "noise" that is realized by time sharing among a certain number of signal levels can be injected into the input of a suboptimal detector or estimator for performance improvement [1]-[5]. Also, error performance of average power constrained communication systems that operate in non-Gaussian channels can be improved by *stochastic signaling*, which involves time sharing among multiple signal values for each information symbol [8, 9]. It is shown that an optimal stochastic signal can be represented by a randomization (time sharing) among no more than three different signal values under second and fourth moment constraints [8]. In a different context, jammer systems can achieve improved jamming performance via time sharing among multiple power levels [6, 11, 14]. In [6], it is shown that a weak jammer should employ on-off time sharing to maximize the average probability of error for a receiver that operates in the presence of noise with a symmetric unimodal density. The optimum power allocation policy for an average power constrained jammer operating over an arbitrary additive noise channel is studied in [14], where the aim is to minimize the detection probability of an instantaneously and fully adaptive receiver that employs the Neyman-Pearson criterion. It is proved that the optimum jamming performance is achieved via time sharing between at most two different power levels, and a necessary and sufficient condition is derived for the improvability of the jamming performance via time sharing of the power compared to a fixed power jamming scheme.

Error performance of some communications systems that operate over additive time-invariant noise channels can also be enhanced via time sharing among multiple detectors, which is called *detector randomization* [3, 10, 15, 16, 17]. In this approach, the receiver employs each detector with a certain time sharing factor (or, probability), and the transmitter adjusts its transmission in coordination with the receiver. In [3], time sharing between two antipodal signal pairs and the corresponding maximum a-posteriori probability (MAP) detectors is studied for an average power constrained binary communication system. Significant performance improvements can be observed as a result of detector randomization in the presence of symmetric Gaussian mixture noise over a range of average power constraint values [3]. In [10], the results in [3] and [9] are extended to an average power constrained M-ary communication system that can employ both detector randomization and stochastic signaling over an additive noise channel with a known distribution. It is obtained that the joint optimization of the transmitted signals and the detectors at the receiver leads to time sharing between at most two MAP detectors corresponding to two deterministic signal constellations. In [12], the benefits of time sharing among multiple detectors are investigated for the downlink of a multiuser communication system and the optimal time sharing strategy is characterized.

In the presence of multiple channels between a transmitter and a receiver, it may be beneficial to perform *channel switching*; that is, to transmit over one channel for a certain fraction of time, and then switch to another channel for the next transmission period [6], [18]–[21]. In [6], the channel switching problem is investigated in the presence of an average power constraint for the optimal detection of binary antipodal signals over a number of channels that are subject to additive unimodal noise. It is proved that the optimal strategy is either to communicate over one channel exclusively, or to switch between two channels with a certain time sharing factor. In [20], the channel switching problem is studied for M-ary communications over additive noise channels (with arbitrary probability distributions) in the presence of time sharing among multiple signal constellations over each channel. It is shown that the optimal strategy that minimizes the average probability of error under an average power constraint corresponds to one of the following approaches: deterministic signaling (i.e., use of one signal constellation) over a single channel; time sharing between two different signal constellations over a single channel; or switching (time sharing) between two channels with deterministic signaling over each channel [20]. With a different perspective, the concept of channel switching is studied for cognitive radio systems in the context of opportunistic spectrum access, where a number of secondary users aim to access the available frequency bands in the spectrum [22]-[25]. In [25], the optimal bandwidth allocation is studied for secondary users in the presence of multiple available primary user bands and under channel switching constraints, and it is shown that secondary users switching among discrete channels can achieve higher capacity than those that switch among consecutive channels.

In a different but related problem, the capacity of the *sum channel* is presented in [26, p.525]. The sum channel is defined as a channel whose input and output alphabets are the unions of those of the original channels; that is, there exist multiple available channels between the transmitter and the receiver but only one channel is used at a given time for each possible symbol in the input alphabet. For example, a sum channel can consist of two binary memoryless channels, and the first two elements of the alphabet, say $\{0, 1\}$, are transmitted over the first channel whereas the last two elements of the alphabet, say $\{2, 3\}$, are transmitted over the second channel. For discrete memoryless channels with capacities $C_1, C_2, \ldots, C_K$, the capacity of the sum channel can be obtained as $\log_2\left(\sum_{i=1}^{K} 2^{C_i}\right)$ [26]. The main difference of the sum channel from the channel switching scenario considered in this dissertation (and those in [6, 20]) is that the alphabet is divided among different channels and each channel is used to transmit a certain subset of the alphabet in the sum channel.

In the literature, optimal resource allocation is commonly employed to enhance the capacity of communication systems. In [27], the optimal dynamic resource allocation for fading broadcast channels is studied for code division, time division, and frequency division in the presence of perfect channel side information at the transmitter and the receivers, and ergodic capacity regions are obtained. In [28], an adaptive resource allocation procedure is presented for multiuser orthogonal frequency division multiplexing (MU-OFDM) systems with the consideration of proportional fairness constraints among users. Optimal and suboptimal algorithms are implemented based on sum capacity maximization while satisfying the minimum required data rate constraint for each user. In [29], optimal joint power and channel allocation strategies are investigated for cognitive radio systems. A near optimal algorithm is presented for the total sum capacity maximization of power-limited secondary users in a centralized cognitive radio network. In [30], capacity maximizing antenna selection is studied for a multiple-input multiple-output (MIMO) system and low-complexity antenna subset selection algorithms are derived. It is shown that near optimal capacity of a full-complexity system is achieved by selecting the number of antennas at the receiver to be at least as large as the number of antennas at the transmitter. In [31], the optimal antenna selection in correlated channels is analyzed for both the transmitter and receiver in order to reduce the number of radio frequency chains. The proposed algorithm results in a near optimal capacity which is achieved without antenna selection. In addition to the capacity, other metrics such as probability of error, probability of detection, and outage probability are considered in various resource allocation problems; e.g., [3]–[13]. For example, in the detector randomization problem, the aim is to minimize the average probability of error of a communication system by optimizing time sharing factors and transmit power (signal) levels corresponding to different detectors at the receiver [3]–[12]. Also, a jammer can maximize the average probability of error or minimize the detection probability of a victim receiver by performing optimal time sharing among multiple power levels [6]–[14]. In [14], the optimal power allocation is performed for an average power constrained jammer to minimize the detection probability of an instantaneously and fully adaptive receiver employing the Neyman-Pearson criterion, and it is shown that the optimal jamming performance is achieved via time sharing between at most two different power levels. In [13], the optimal time sharing of power levels is implemented for minimizing the outage probability in a flat block-fading Gaussian channel under an average power constraint and in the presence of channel distribution information at the transmitter.

Although the optimal channel switching problem is studied thoroughly in terms of average probability of error minimization (e.g., [6, 20, 21]) and in the context of opportunistic spectrum access (e.g., [22]-[25]), no studies in the literature have considered the channel switching problem for maximization of data rates by jointly optimizing time sharing (channel switching) factors and corresponding power levels. In this dissertation, the average Shannon capacity is considered as the main metric since it gives the maximum achievable data rates with low probability of decoding errors. In addition, the data rate targets indicated by the Shannon capacity are achievable in practical communication systems through turbo coding or low density parity check codes [32]. In Chapter 2, we formulate the optimal channel switching problem for average Shannon capacity maximization over Gaussian channels in the presence of average and peak power constraints, and derive necessary and sufficient conditions for the proposed channel switching approach to achieve a higher average capacity than the optimal approach without channel switching [33]. In addition, it is obtained that the optimal solution to the channel switching problem results in channel switching between at most two different channels, and an approach is proposed to obtain the optimal channel switching strategy with low computational complexity. Numerical examples are presented to illustrate the theoretical results.

Some of the practical motivations for studying the channel switching problem for data rate maximization can be summarized as follows: Firstly, the next-generation wireless communication systems are required to support all IP services including high-data-rate multimedia traffic, with bit rate targets as high as 1 Gbit/s for low mobility and 100 Mbit/s for high mobility [34]. Such high data rate requirements make the capacity (usually measured by using Shannon capacity metric [35, 36]) maximization problems (subject to appropriate operating constraints on power and communication reliability) more relevant for next-generation wireless communication systems, rather than focusing on power or bit error minimization (subject to appropriate operating constraints on rate). Secondly, wireless telecommunication technology is currently on the cusp of a major transition from the traditional carefully planned homogenous macro-cell deployment to highly heterogeneous small cell network architectures. These heterogeneous next generation network architectures (alternatively called HetNets) will consist of multiple tiers of irregularly deployed network elements with diverse range of backhaul connection characteristics, signal processing capabilities and electromagnetic radio emission levels. In such a HetNet scenario, it is expected that more than one radio link such as femto-cell connection, macro-cell connection and Wi-Fi connection (with different operating frequency bands, background noise levels and etc.) will be present to use at each mobile user. From an engineering point of view, this dissertation provides some fundamental design insights regarding how to time share (randomize) among available radio links to maximize rates of communication for highly heterogenous wireless environments. Finally, channel switching can be beneficial for secondary users in a cognitive radio system in which there can exist multiple available frequency bands in the spectrum.

In most of the previous studies on optimal channel switching strategies, delays (costs) associated with the channel switching operation are not considered [6, 18]–[33]. However, due to hardware limitations, the channel switching operation takes a certain time in practice. In particular, when switching to a new channel, the parameters at the transmitter and the receiver are set according to the characteristics (i.e., frequency) of the new channel, which induces a channel switching delay and consequently reduces the available time for data transmission [37, 38]. Most of the studies in the literature omit the channel switching overhead (delay) by assuming that it is negligible due to improved hardware technologies. However, the study in [39] shows that the state-of-the-art algorithms related to scheduling in wireless mesh networks experience performance degradation in the presence of the channel switching latency. Similarly, in [40], the channel switching cost is considered in the design of the energy efficient centralized cognitive radio networks, and an energy efficient heuristic scheduler is proposed to allocate each idle frequency to the cognitive radio with the highest energy efficiency at that frequency. In [41], effects of channel switching time and energy on cooperative sensing scheduling are analyzed for cognitive radio networks. In [42], a spectrum aware routing algorithm for multi-hop cognitive radio networks is proposed with the consideration of the channel switching overhead.

Although the channel switching problem has been investigated from various perspectives, no studies in the literature have considered channel switching for average capacity maximization in the presence of channel switching delays. In Chapter 3, the optimal channel switching strategy is proposed for average capacity maximization under power constraints and considering a time delay for each channel switching operation during which data communication cannot be performed [43]. After presenting an optimization theoretic formulation of the proposed problem, an equivalent optimization problem is obtained to facilitate theoretical investigations. It is observed that consideration of channel switching delays leads to significant differences in the formulation and analyses compared to those obtained by omitting the effects of channel switching delays [33]. First, the optimal strategy is obtained and the corresponding average capacity is specified when channel switching is performed among a given number of channels. Based on this result, it is then shown that channel switching among more than two different channels cannot be optimal. Also, the maximum average capacity achieved by the optimal channel switching strategy is formulated for various values of the channel switching delay parameter and the average and peak power limits. In addition, scenarios under which the optimal strategy corresponds to the utilization of a single channel or to channel switching between two channels are described. Furthermore, sufficient conditions are derived to determine when the optimal single channel strategy outperforms the optimal channel switching strategy. Numerical examples are presented for the theoretical results and effects of channel switching delays are investigated.

In [33], the optimal channel switching strategies are investigated for a communication system in which a single transmitter communicates with a single receiver in the presence of the average and peak power constraints. It is obtained that the optimal channel switching strategy corresponds to the exclusive use of a single channel or to channel switching between two channels. In [43], the study in [33] is extended for a communication system where the channel switching delays (costs) are considered due to hardware limitations. It is shown that any channel switching strategy consisting of more than two different channels cannot be optimal.

Although the channel switching problem has been studied for communication between a single transmitter and a single receiver in the presence of average and peak power constraints and in the consideration of channel switching delays, no studies in the literature have considered the channel switching problem in the presence of multiple receivers in the communication system. In Chapter 4, a transmitter communicates with two receivers (classified as primary and secondary) by employing a channel switching strategy among available multiple channels in the system [44]. The aim of the transmitter is to enhance the average capacity of the secondary receiver while satisfying the minimum average capacity requirement for the primary receiver in the presence of average and peak power constraints.[1] Also, due to hardware limitations, the transmitter can establish only one communication link with one of the receivers at a given time by employing one of the communication channels available in the system. It is obtained that if more than one channel is available, then the optimal channel switching strategy which maximizes the average capacity of the secondary receiver consists of no more than 3 communication links. (It is important to note that each channel in the system constitutes two communication links; that is, one for the communication

[1] In this case, the channel switching delays are omitted in order to simplify the system model. However, the main contributions are valid in the presence of switching delays, as well.

between the transmitter and the primary receiver and one for the communication between the transmitter and the secondary receiver.) In addition, with regard to the number of channels employed in the optimal channel switching strategy, it is concluded that the transmitter either communicates with the primary receiver over at most two channels and employs a single channel for the secondary receiver, or communicates with the primary receiver over a single channel and employs at most two channels for the secondary receiver. In addition to the communication system with a single primary receiver, the channel switching problem in this study is also extended for communication systems in which there exist multiple primary receivers, each having a separate minimum average capacity requirement for the communication with the transmitter. Lastly, numerical examples are provided to exemplify the theoretical results.

1.2 Power Control Games for Wireless Localization

In recent years, research communities have developed a significant interest in wireless localization networks, which provide important applications for various systems and services [45, 46]. To name a few, smart inventory tracking systems, location sensitive billing services, and intelligent autonomous transport systems benefit from wireless localization networks [47]. In such a wide variety of applications, accurate and robust position estimation plays a crucial role in terms of efficiency and reliability. In the literature, various theoretical and experimental studies have been conducted in order to analyze wireless position estimation in the context of accuracy requirements and system constraints; e.g., [48, 49].

In a wireless localization network, there exist two types of nodes in general; namely, anchor nodes and target nodes. Anchor nodes have known positions and their location information is available at target nodes. On the other hand, target nodes have unknown positions, and each target node in the network estimates its own position based on received signals from anchor nodes (in the case of self localization [47]). In particular, position estimation of a target node is performed by using various signal parameters extracted from received signals (i.e., waveforms). Commonly employed signal parameters are time-of-arrival (TOA) [50, 51], time-difference-of-arrival (TDOA) [52], angle-of-arrival (AOA) [53], and received signal strength (RSS) [54]. TOA and TDOA are time based parameters which measure the signal propagation time (difference) between nodes. AOA is obtained based on the angle at which the transmitted signal from one node arrives at another node. RSS is another signal parameter which gathers information from power or energy of a signal that travels between anchor and target nodes [48]. Since a signal traveling from an anchor node to a target node experiences multipath fading, shadowing, and path-loss, position estimates of target nodes are subject to errors and uncertainty. As the Cramér-Rao lower bound (CRLB) expresses a lower bound on the variance of any unbiased estimator for a deterministic parameter, it is also considered as a common performance metric for wireless localization networks [55]–[57].

Besides anchor and target nodes, a wireless localization network can contain undesirable jammer nodes, the aim of which is to degrade the localization performance (i.e., accuracy) of the network. In the literature, various studies have been performed on the jamming of wireless localization networks. The jamming and anti-jamming of the global positioning system (GPS) are studied in [58] for various jamming schemes. Similarly, in [59], an adaptive GPS anti-jamming algorithm is proposed. In addition, the optimal power allocation problem is investigated for jammer nodes in a given wireless localization network based on the CRLB metric, and the optimal jamming strategies are obtained in the presence of peak power and total power constraints in [55].

In the literature, various studies have been conducted on power allocation for wireless localization networks [60]–[63]. In [60], the optimal anchor power allocation strategies are investigated together with anchor selection and anchor deployment strategies for the minimization of the squared position error bound (SPEB), which identifies fundamental limits on localization accuracy. The work in [61] provides a robust power allocation framework for network localization in the presence of imperfect knowledge of network parameters. Based on the performance metrics SPEB and the directional position error bound (DPEB), the optimal power allocation problems are formulated in the consideration of limited power resources and it is shown that the proposed problems can be solved via conic programming. In [62], ranging energy optimization problems are investigated for an unsynchronized positioning network based on two-way ranging between a sensor and beacons. In [63], the work in [62] is extended for a positioning network in which the collaborative anchors added to the system help sensors locate themselves.

In the presence of jammer nodes in a wireless localization network, anchor nodes can adapt their power allocation strategies in response to the strategies employed by jammer nodes and enhance the localization performance of the network. On the other hand, jammer nodes can respond by updating their corresponding power allocation strategies in order to degrade the localization performance. These conflicting interests between anchor and jammer nodes can be analyzed by employing game theory as a tool. In the literature, game theoretic frameworks have been applied for investigating power allocation strategies of users in a competitive system. In [64], competitive interactions between a secondary user transmitter-receiver pair and a jammer are analyzed by applying a game-theoretic framework in the presence of interference constraints, power constraints, and incomplete channel gain information. In particular, the strategic power allocation game between the two players is proposed first, and then it is presented that the solution of the game corresponds to Nash equilibria points. In [65], a zero-sum game is modeled between a centralized detection network and a jammer in the presence of complete information. It is obtained that the jammer has no effect on the error probability observed at the fusion center when it employs pure strategies at the Nash equilibrium.

Although there exist research papers that analyze the non-cooperative behavior of system users and jammer nodes in wireless communication networks in terms of successful transmissions under a minimum signal-to-interference-plus-noise ratio (SINR) constraint and error probability [64, 65], no studies in the literature have investigated the interactions between anchor nodes and jammer nodes in a wireless localization network, where target nodes estimate their positions based on signals received from anchor nodes and jammer nodes try to degrade the localization performance of the network. In the field of wireless localization, there exist some recent studies (e.g., [57] and [66]) that analyze the interactions of entities in a wireless localization network. However, no jammer nodes are considered in those studies, which focus on a cooperative localization network where the target nodes share information with each other to improve their position estimates. Therefore, the theoretical analyses presented therein differ from the ones performed in this dissertation, which considers non-cooperative localization where anchor and jammer nodes compete for the localization performance of target nodes.

In Chapter 5, power control games between anchor and jammer nodes are designed based on a game-theoretic framework by employing the CRLB metric [67]. In particular, two different games are formulated for the considered wireless localization network: In the first game, the average CRLB of the target nodes is considered as the performance metric whereas in the second one, the worst-case CRLBs for the anchor and jammer nodes are employed. As a solution approach, Nash equilibria of the games are examined, and it is shown that a pure Nash equilibrium exists in both of the proposed power control games. In addition, for the game in which the anchor and jammer nodes compete according to the average CRLB, a method is presented to obtain a pure strategy Nash equilibrium and a sufficient condition is provided to decide whether the pure strategy Nash equilibrium is unique. Finally, numerical examples are presented to demonstrate the theoretical results.

1.3 Organization of the Dissertation

This dissertation is organized as follows. In Chapter 2, the optimal channel switching strategies are presented for average capacity maximization in the presence of average and peak power constraints. In Chapter 3, the optimal channel switching strategies are designed in the consideration of channel switching costs (delays) together with average and peak power constraints. Then, the study in Chapter 2 is extended in Chapter 4 to multiuser scenarios in a wireless communication system. In Chapter 5, power control games between anchor and jammer nodes are investigated for wireless localization networks. Finally, Chapter 6 concludes this dissertation and provides remarks on future work.

Chapter 2

Optimal Channel Switching Strategy for Average Capacity Maximization

In this chapter, an optimal channel switching strategy is presented for average capacity maximization in the presence of average and peak power constraints [33]. The main contributions of this chapter can be outlined as follows:

- For the first time, the optimal channel switching problem is investigated *for average capacity maximization* in the presence of multiple Gaussian channels and under average and peak power constraints.
- It is shown that the optimal channel switching strategy switches among at most two different channels, and operates at the average power limit.
- Necessary and sufficient conditions are derived to specify when performing channel switching can or cannot provide improvements over the optimal approach without channel switching.
- Optimality conditions are obtained for the proposed channel switching strategy, and an approach with low computational complexity is presented

for calculating the parameters of the optimal strategy.

This chapter is organized as follows: The problem formulation for optimal channel switching is presented in Section 2.1. Section 2.2 investigates the solution of the optimal channel switching problem and provides various theoretical results about the characteristics of the optimal channel switching strategy. In Section 2.3, numerical examples are presented for illustrating the theoretical results, followed by the concluding remarks in Section 2.4.

2.1 Problem Formulation

Consider a communication system in which a transmitter and a receiver are connected via K different channels as illustrated in Fig. 2.1. The channels are modeled as additive Gaussian noise channels with possibly different noise levels and bandwidths. It is assumed that noise is independent across different channels. The transmitter and the receiver can switch (time share) among these K channels in order to enhance the capacity of the communication system. A relay at the transmitter controls the access to the channels in such a way that only one of the channels can be used for information transmission at any given time. It is assumed that the transmitter and the receiver are synchronized and the receiver knows which channel is being utilized [6]. In practical scenarios, this assumption can hold in the presence of a communication protocol that notifies the receiver about the numbers of symbols and the corresponding channels to be employed during data communications. This notification information can be sent in the header of a communications packet [10, 20].

In some communication systems, multiple channels with various bandwidth and noise characteristics can be available between a transmitter and a receiver as in Fig. 2.1. For instance, in a cognitive radio system, primary users are the main owners of the spectrum, and secondary users can utilize the frequency bands of the primary users when they are available [22, 23, 24, 68, 69]. In such a case, the available bands in the spectrum can be considered as the channels in Fig. 2.1, and

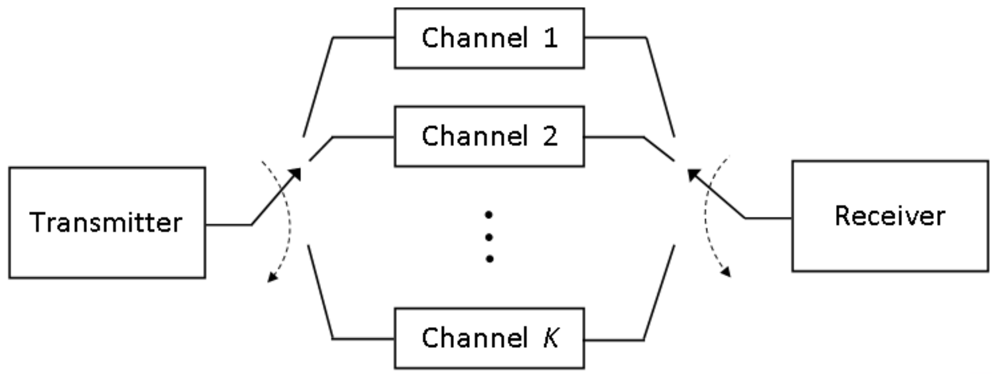

Figure 2.1: Block diagram of a communication system in which transmitter and receiver can switch among K channels.

the aim of a secondary user becomes the maximization of its average capacity by performing optimal channel switching under power constraints that are related to hardware constraints and/or battery life. The motivation for using only one channel at a given time is that the transmitter and the receiver are assumed to have a single RF chain each due to complexity/cost considerations. Then, the transmitter-receiver pair can perform time sharing among different channels (i.e., channel switching) by employing only one channel at a given time. In a similar fashion, the proposed system also has a potential to improve data rates in emerging open-access K-tier heterogeneous wireless networks by allowing users to switch between multiple access points and available frequency bands in the spectrum [70, 71].

Let B_i and $N_i/2$ represent, respectively, the bandwidth and the constant power spectral density level of the additive Gaussian noise corresponding to channel i for $i \in \{1, \ldots, K\}$. Then, the capacity of channel i is given by

$$C_i(P) = B_i \log_2 \left(1 + \frac{P}{N_i B_i}\right) \text{ bits/sec} \tag{2.1}$$

where P denotes the average transmit power [72].

The aim of this study is to obtain the optimal channel switching strategy that maximizes the average capacity of the communication system in Fig. 2.1

under average and peak power constraints. In order to formulate such a problem, channel switching (time sharing) factors, denoted by $\lambda_1, \ldots, \lambda_K$, are defined first, where λ_i is the fraction of time when channel i is used, with $\lambda_i \geq 0$ for $i = 1, \ldots, K$, and $\sum_{i=1}^{K} \lambda_i = 1$.[1] Then, the optimal channel switching problem for average capacity maximization is proposed as follows:

$$\begin{aligned} \max_{\{\lambda_i, P_i\}_{i=1}^{K}} \quad & \sum_{i=1}^{K} \lambda_i \, C_i(P_i) \\ \text{subject to} \quad & \sum_{i=1}^{K} \lambda_i P_i \leq P_{\text{av}} \\ & P_i \in [0, P_{\text{pk}}] \, , \ \forall i \in \{1, \ldots, K\} \\ & \sum_{i=1}^{K} \lambda_i = 1 \, , \ \lambda_i \geq 0 \, , \ \forall i \in \{1, \ldots, K\} \end{aligned} \tag{2.2}$$

where $C_i(P_i)$ is as defined in (2.1) with P_i denoting the average transmit power allocated to channel i, P_{pk} represents the peak power limit, and P_{av} is the average power limit for the transmitter. In practical systems, the average power limit is related to the power consumption and/or the battery life of the transmitter whereas the peak power limit specifies the maximum power level that can be generated by the transmitter circuitry; i.e., it is mainly a hardware constraint. Since there exists a single RF unit at the transmitter, the peak power limit is taken to be the same for each channel. It is assumed that $P_{\text{av}} < P_{\text{pk}}$ holds. From (2.2), it is observed that the design of an optimal channel switching strategy involves the joint optimization of the channel switching factors and the corresponding power levels under average and peak power constraints for the purpose of average capacity maximization.

[1] Channel switching can be implemented in practice by transmitting the first $\lambda_1 N_s$ symbols over channel 1, the next $\lambda_2 N_s$ symbols over channel 2, ..., and the final $\lambda_K N_s$ symbols over channel K, where N_s is the total number of symbols (over which channel statistics do not change), and $\lambda_1, \lambda_2, \ldots, \lambda_K$ are the channel switching factors. In this case, suitable channel coding-decoding algorithms can be employed for each channel to achieve a data rate close to the Shannon capacity of that channel.

2.2 Optimal Channel Switching

In general, it is challenging to find the optimal channel switching strategy by directly solving the optimization problem in (2.2). For this reason, our aim is to obtain a simpler version of the problem in (2.2) and to calculate the optimal channel switching solution in a low-complexity manner. To that end, an alternative optimization problem is obtained first. Let $\{\lambda_i^*, P_i^*\}_{i=1}^K$ denote the optimal channel switching strategy obtained as the solution of (2.2) and define C^* as the corresponding maximum average capacity; that is, $C^* = \sum_{i=1}^K \lambda_i^* \, C_i(P_i^*)$. Then, the following proposition presents an alternative optimization problem, the solution of which achieves the same maximum average capacity as (2.2) does.

Proposition 1: *The solution of the following optimization problem results in the same maximum value that is achieved by the problem in* (2.2)*:*

$$\begin{aligned} \max_{\{\nu_i, P_i\}_{i=1}^K} \quad & \sum_{i=1}^K \nu_i \, C_{\max}(P_i) \\ \text{subject to} \quad & \sum_{i=1}^K \nu_i P_i \leq P_{\text{av}} \\ & P_i \in [0, P_{\text{pk}}] \,, \ \forall i \in \{1, \ldots, K\} \\ & \sum_{i=1}^K \nu_i = 1 \,, \ \nu_i \geq 0 \,, \ \forall i \in \{1, \ldots, K\} \end{aligned} \tag{2.3}$$

where $C_{\max}(P)$ *is defined as*

$$C_{\max}(P) \triangleq \max\{C_1(P), \ldots, C_K(P)\} \ . \tag{2.4}$$

Proof: The proof consists of two steps. Let $\{\nu_i^\star, P_i^\star\}_{i=1}^K$ represent the solution of (2.3) and define $C^\star$ as the corresponding maximum average capacity; that is, $C^\star = \sum_{i=1}^K \nu_i^\star \, C_{\max}(P_i^\star)$. First, it can be observed from (2.2) and (2.3) that $C^\star \geq C^*$ due to the definition in (2.4), where C^* is the maximum average capacity obtained from (2.2). Next, define function $g(i)$ and set S_m as follows:[2]

$$g(i) \triangleq \underset{l\in\{1,\ldots,K\}}{\arg\max}\; C_l(P_i^\star)\,,\;\; \forall i \in \{1,\ldots,K\} \tag{2.5}$$

and

$$S_m \triangleq \{i \in \{1,\ldots,K\} \mid g(i) = m\}\,,\;\; \forall m \in \{1,\ldots,K\}\,. \tag{2.6}$$

Then, the following relations can be obtained for $C^\star$:

$$\begin{aligned} C^\star = \sum_{i=1}^{K} \nu_i^\star\, C_{\max}(P_i^\star) &= \sum_{i=1}^{K} \nu_i^\star\, C_{g(i)}(P_i^\star) && (2.7)\\ &= \sum_{i=1}^{K} \sum_{k\in S_i} \nu_k^\star\, C_i(P_k^\star) && (2.8)\\ &\le \sum_{i=1}^{K} \left(\sum_{k\in S_i} \nu_k^\star\right) C_i\left(\frac{\sum_{k\in S_i} \nu_k^\star P_k^\star}{\sum_{k\in S_i} \nu_k^\star}\right) && (2.9)\\ &= \sum_{i=1}^{K} \bar{\lambda}_i\, C_i(\bar{P}_i) && (2.10) \end{aligned}$$

where $\bar{\lambda}_i$ and $\bar{P}_i$ are defined as

$$\bar{\lambda}_i \triangleq \sum_{k\in S_i} \nu_k^\star \;\;\text{and}\;\; \bar{P}_i \triangleq \frac{\sum_{k\in S_i} \nu_k^\star P_k^\star}{\sum_{k\in S_i} \nu_k^\star}\,. \tag{2.11}$$

for $i \in \{1,\ldots,K\}$. The equalities in (2.7) and (2.8) are obtained from the definitions in (2.5) and (2.6), respectively, and the inequality in (2.9) follows from Jensen's inequality due to the concavity of the capacity function [72, 73]. It is noted from (2.11), based on (2.5) and (2.6), that $\bar{\lambda}_i$'s and $\bar{P}_i$'s satisfy the constraints in (2.2); that is, $\sum_{i=1}^{K} \bar{\lambda}_i\, \bar{P}_i \le P_{\text{av}}$, $\bar{P}_i \in [0, P_{\text{pk}}]$, $\forall i \in \{1,\ldots,K\}$, $\sum_{i=1}^{K} \bar{\lambda}_i = 1$, and $\bar{\lambda}_i \ge 0$, $\forall i \in \{1,\ldots,K\}$. Therefore, the inequality in (2.7)-(2.10), namely, $C^\star \le \sum_{i=1}^{K} \bar{\lambda}_i\, C_i(\bar{P}_i)$, implies that the optimal solution of (2.3) cannot achieve a higher average capacity than that achieved by (2.2); that is, $C^\star \le C^*$. Hence, it is concluded that $C^\star = C^*$ since $C^\star \ge C^*$ must also hold as

[2]In the case of multiple maximizers in (2.5), any maximizing index can be chosen for $g(i)$.

mentioned at the beginning of the proof. ∎

Based on Proposition 1, the maximum average capacity C^* achieved by the optimal channel switching problem in (2.2) can also be obtained by solving the optimization problem in (2.3). Let $\{\nu_i^\star, P_i^\star\}_{i=1}^K$ denote the optimal solution of (2.3). Proposition 1 states that $\sum_{i=1}^K \nu_i^\star C_{\max}(P_i^\star) = C^*$. In addition, the optimal channel switching strategy corresponding to the channel switching problem in (2.2) can be obtained, based on the arguments in the proof of Proposition 1, as follows: Once $\{\nu_i^\star, P_i^\star\}_{i=1}^K$ is calculated from (2.3), the optimal channel switching strategy can be obtained as $\{\lambda_i^*, P_i^*\}_{i=1}^K$, where $\lambda_i^* = \sum_{k\in S_i} \nu_k^\star$ and $P_i^* = (\sum_{k\in S_i} \nu_k^\star P_k^\star)/(\sum_{k\in S_i} \nu_k^\star)$ with S_i being given by (2.6). It should be emphasized that a low-complexity approach is developed in the remainder of this section for solving (2.3); hence, it is useful to obtain the optimal channel switching strategy corresponding to the channel switching problem in (2.2) based on the solution of (2.3).

The significance of Proposition 1 also lies in the fact that the alternative optimization problem in (2.3), which achieves the same maximum average capacity as the original channel switching problem in (2.2), facilitates detailed theoretical investigations of the optimal channel switching strategy, as discussed in the remainder of this section.

Towards the purpose of characterizing the optimal channel switching strategy, the following lemma is presented first, which states that the optimal solutions of (2.2) and (2.3) operate at the average power limit.

Lemma 1: *Let $\{\lambda_i^*, P_i^*\}_{i=1}^K$ and $\{\nu_i^\star, P_i^\star\}_{i=1}^K$ denote the solutions of the optimization problems in* (2.2) *and* (2.3)*, respectively. Then, $\sum_{i=1}^K \lambda_i^* P_i^* = P_{\text{av}}$ and $\sum_{i=1}^K \nu_i^\star P_i^\star = P_{\text{av}}$ hold.*

Proof: The proof is provided for the optimization problem in (2.3) only since the one for (2.2) can easily be obtained based on a similar approach (cf. Proposition 1 in [21]). Suppose that $\{\nu_i, P_i\}_{i=1}^K$ is an optimal solution of the problem in (2.3) such that $\sum_{i=1}^K \nu_i P_i < P_{\text{av}}$. Since $P_{\text{av}} < P_{\text{pk}}$, there exist at least one P_i that is strictly smaller than P_{pk}. Let P_l be one of them. Then, consider an alternative solution $\{\nu_i^{'}, P_i^{'}\}_{i=1}^{K}$, with $\nu_i^{'} = \nu_i$, $\forall i \in \{1, \ldots, K\}$, $P_i^{'} = P_i$, $\forall i \in \{1, \ldots, K\} \setminus \{l\}$, and $P_l^{'} = \min\{P_{\text{pk}}, P_l + (P_{\text{av}} - \sum_{i=1}^{K} \nu_i P_i)/\nu_l\}$. Note that the alternative solution, $\{\nu_i^{'}, P_i^{'}\}_{i=1}^{K}$, achieves a larger average capacity than $\{\nu_i, P_i\}_{i=1}^{K}$ due to the following relation:

$$\sum_{i=1}^{K} \nu_i^{'} C_{\max}(P_i^{'}) = \sum_{\substack{i=1 \\ i \neq l}}^{K} \nu_i^{'} C_{\max}(P_i^{'}) + \nu_l^{'} C_{\max}(P_l^{'}) \tag{2.12}$$

$$> \sum_{\substack{i=1 \\ i \neq l}}^{K} \nu_i C_{\max}(P_i) + \nu_l\, C_{\max}(P_l) \tag{2.13}$$

$$= \sum_{i=1}^{K} \nu_i C_{\max}(P_i) \tag{2.14}$$

where the inequality follows from the facts that $C_{\max}(P)$ is a monotone increasing function of P (please see (2.1) and (2.4))[3], and that $P_l^{'} > P_l$. Therefore, $\{\nu_i, P_i\}_{i=1}^{K}$ cannot be an optimal solution of (2.3), which leads to a contradiction. Hence, any feasible point of the problem in (2.3) which utilizes an average power strictly smaller than P_{av} cannot be optimal; that is, the optimal solution must operate at the average power limit. ■

2.2.1 Optimal Channel Switching versus Optimal Single Channel Strategy

Next, possible improvements that can be achieved via the optimal channel switching strategy over the *optimal single channel* strategy are investigated. The optimal single channel strategy corresponds to the case of no channel switching and the use of the best channel all the time at the average power limit. For that strategy, the achieved maximum capacity can be expressed as $C_{\max}(P_{\text{av}})$, where $C_{\max}$ is as defined in (2.4), and the best channel is the one with the index

[3]Note that the maximum of a set of monotone increasing functions is also monotone increasing.

$\arg\max_{l\in\{1,\ldots,K\}} C_l(P_{\rm av})$.[4] It is noted that when a single channel is used (i.e., no channel switching), it is optimal to utilize all the available power, $P_{\rm av}$ since $C_{\max}(P)$ is a monotone increasing and continuous function of P, as can be verified from (2.1) and (2.4). In the following proposition, a necessary and sufficient condition is presented for the optimal channel switching strategy to have the same performance as the optimal single channel strategy.

Proposition 2: *Suppose that $C_{\max}(P)$ in (2.4) is first-order continuously differentiable in an interval around $P_{\rm av}$. Then, the optimal channel switching and the optimal single channel strategies achieve the same maximum average capacity if and only if*

$$(P - P_{\rm av})\frac{B_{i^*}\log_2 e}{N_{i^*}B_{i^*} + P_{\rm av}} \geq C_{\max}(P) - C_{\max}(P_{\rm av}) \tag{2.15}$$

for all $P \in [0, P_{\rm pk}]$, where $i^* = \arg\max_{i\in\{1,\ldots,K\}} C_i(P_{\rm av})$.

Proof: The proof consists of the sufficiency and the necessity parts. The sufficiency of the condition in (2.15) can be proved by employing a similar approach to that in the proof of Proposition 3 in [14]. Under the condition in the proposition, the aim is to prove that the optimal channel switching and the optimal single channel strategies achieve the same maximum average capacity; that is, $\sum_{i=1}^{K}\nu_i^\star C_{\max}(P_i^\star) = C_{\max}(P_{\rm av})$, where $\{\nu_i^\star, P_i^\star\}_{i=1}^K$ denotes the solution of (2.3), which achieves the same average capacity as the optimal channel switching strategy corresponding to (2.2) based on Proposition 1. Due to the assumption in the proposition, the first-order derivative of $C_{\max}(P)$ in (2.4) exists in an interval around $P_{\rm av}$ and its value at $P_{\rm av}$ is calculated from (2.1) as

$$C'_{\max}(P_{\rm av}) = \frac{B_{i^*}\log_2 e}{N_{i^*}B_{i^*} + P_{\rm av}} \tag{2.16}$$

where $i^* = \arg\max_{i\in\{1,\ldots,K\}} C_i(P_{\rm av})$. From (2.16), the condition in (2.15) can be expressed as $C_{\max}(P) \leq C_{\max}(P_{\rm av}) + C'_{\max}(P_{\rm av})(P - P_{\rm av})$ for all $P \in [0, P_{\rm pk}]$. Then, for any channel switching strategy denoted as $\{\nu_i, P_i\}_{i=1}^K$, the following

[4] In the case of multiple best channels, any of them can be chosen to achieve $C_{\max}(P_{\rm av})$.

inequalities can be obtained:

$$\sum_{i=1}^{K}\nu_i\,C_{\max}(P_i)\leq C_{\max}(P_{\rm av})+C_{\max}^{'}(P_{\rm av})\left(\sum_{i=1}^{K}\nu_iP_i-P_{\rm av}\right) \tag{2.17}$$

$$\leq C_{\max}(P_{\rm av}) \tag{2.18}$$

where $P_i\in[0,P_{\rm pk}]$ and $\nu_i\geq 0$ for $i\in\{1,\ldots,K\}$, $\sum_{i=1}^{K}\nu_i=1$, and $\sum_{i=1}^{K}\nu_iP_i\leq P_{\rm av}$. It is noted that the inequality in (2.18) is obtained from the facts that $C_{\max}^{'}(P_{\rm av})$ in (2.16) is positive and that $\sum_{i=1}^{K}\nu_iP_i-P_{\rm av}$ is non-positive due to the average power constraint. From (2.17) and (2.18), it is concluded that when the condition in the proposition holds, channel switching can never result in a higher average capacity than the optimal single channel strategy, which achieves a capacity of $C_{\max}(P_{\rm av})$. On the other hand, for $\nu_{i^*}^{\star}=1$, $P_{i^*}^{\star}=P_{\rm av}$, and $\nu_i^{\star}=P_i^{\star}=0$ for all $i\in\{1,\ldots,K\}\setminus\{i^*\}$, where $i^*=\arg\max_{i\in\{1,\ldots,K\}}C_i(P_{\rm av})$, the $\sum_{i=1}^{K}\nu_iC_{\max}(P_i)$ term in (2.17) becomes equal to $C_{\max}(P_{\rm av})$. Since this possible solution satisfies $\sum_{i=1}^{K}\nu_i^{\star}P_i^{\star}=P_{\rm av}$ (cf. Lemma 1) and all the constraints of the optimization problem in (2.3), it is concluded that $\sum_{i=1}^{K}\nu_i^{\star}\,C_{\max}(P_i^{\star})=C_{\max}(P_{\rm av})$ under the condition in the proposition.

The necessity part of the proof is contrapositive. Therefore, the aim is to prove that if

$$(P-P_{\rm av})C_{\max}^{'}(P_{\rm av})<C_{\max}(P)-C_{\max}(P_{\rm av}) \tag{2.19}$$

for some $P\in[0,P_{\rm pk}]$, then the optimal channel switching strategy outperforms the optimal single channel strategy in terms of the maximum average capacity. First, assume that there exists $\tilde{P}\in[0,P_{\rm av}]$ that satisfies the condition in (2.19) and consider the straight line that passes through the points $(\tilde{P},C_{\max}(\tilde{P}))$ and $(P_{\rm av},C_{\max}(P_{\rm av}))$. Let φ denote the slope of this line. From (2.19), the following relation is observed:

$$\varphi\triangleq\frac{C_{\max}(P_{\rm av})-C_{\max}(\tilde{P})}{P_{\rm av}-\tilde{P}}<C_{\max}^{'}(P_{\rm av})\,. \tag{2.20}$$

Due to the assumption in the proposition, the first-order derivative of $C_{\max}(P)$ in

(2.4) is continuous in an interval around $P_{\rm av}$. Therefore, $C_{\max}(P)$ must correspond to the same channel over an interval around $P_{\rm av}$,[5] which implies the concavity of $C_{\max}(P)$ in that interval as the capacity curves are concave. By definition of the concavity around $P_{\rm av}$, there exists a point $P_{\rm av}^{+} \triangleq P_{\rm av} + \epsilon$ for an infinitesimally small positive number ϵ such that

$$\varphi < \frac{C_{\max}(P_{\rm av}) - C_{\max}(P_{\rm av}^{+})}{P_{\rm av} - P_{\rm av}^{+}} < C'_{\max}(P_{\rm av})\,. \tag{2.21}$$

Then, choose a $\tilde{\lambda}$ such that $\tilde{\lambda}\,\tilde{P} + (1-\tilde{\lambda})P_{\rm av}^{+} = P_{\rm av}$ and consider the following relations:

$$\begin{aligned}
&\tilde{\lambda}\,C_{\max}(\tilde{P}) + (1-\tilde{\lambda})C_{\max}(P_{\rm av}^{+}) \\
&> \tilde{\lambda}\,C_{\max}(\tilde{P}) + (1-\tilde{\lambda})\big((P_{\rm av}^{+} - P_{\rm av})\varphi + C_{\max}(P_{\rm av})\big) && (2.22)\\
&= \frac{P_{\rm av}^{+} - P_{\rm av}}{P_{\rm av}^{+} - \tilde{P}}\,C_{\max}(\tilde{P}) + \frac{P_{\rm av} - \tilde{P}}{P_{\rm av}^{+} - \tilde{P}}\,\big((P_{\rm av}^{+} - P_{\rm av})\varphi + C_{\max}(P_{\rm av})\big) && (2.23)\\
&= C_{\max}(P_{\rm av}) && (2.24)
\end{aligned}$$

where the inequality in (2.22) is obtained from (2.21), the equality in (2.23) follows from the definition of $\tilde{\lambda}$, and the final equality is due to the definition of φ in (2.20). Overall, the inequality in (2.22)-(2.24), namely, $\tilde{\lambda}\,C_{\max}(\tilde{P}) + (1-\tilde{\lambda})C_{\max}(P_{\rm av}^{+}) > C_{\max}(P_{\rm av})$, implies that the channel switching strategy (specified by channel switching factors $\tilde{\lambda}$ and $(1-\tilde{\lambda})$ and power levels $\tilde{P}$ and $P_{\rm av}^{+}$) achieves a higher average capacity than the optimal single channel strategy.[6] Since the optimal channel switching strategy always achieves an average capacity that is equal to or larger than the average capacity of any other channel switching strategy, it is concluded that the optimal channel switching strategy outperforms the optimal single channel strategy.

[5]If there multiple channels with the same bandwidths and noise levels, they can be regarded as a single channel (i.e., only one of them should be considered) since there is no advantage of switching between such channels.

[6]Note that the channel switching strategy denoted by channel switching factors $\tilde{\lambda}$ and $(1-\tilde{\lambda})$ and power levels $\tilde{P}$ and $P_{\rm av}^{+}$ must involve switching between two different channels since the inequality $\tilde{\lambda}\,C_{\max}(\tilde{P}) + (1-\tilde{\lambda})C_{\max}(P_{\rm av}^{+}) > C_{\max}(P_{\rm av})$ cannot be satisfied for a single channel due to the concavity of the capacity curves.

Next, assume that there exists $\bar{P} \in (P_{\rm av}, P_{\rm pk}]$ that satisfies the condition in (2.19). Similar to the previous part of the proof, let ϕ denote the slope of the straight line that passes through the points $(\bar{P}, C_{\max}(\bar{P}))$ and $(P_{\rm av}, C_{\max}(P_{\rm av}))$. Then, the following expression is obtained from (2.19):

$$\phi \triangleq \frac{C_{\max}(P_{\rm av}) - C_{\max}(\bar{P})}{P_{\rm av} - \bar{P}} > C^{'}_{\max}(P_{\rm av})\,. \tag{2.25}$$

Similarly, due to the concavity around $P_{\rm av}$, there exists a point $P^{-}_{\rm av} \triangleq P_{\rm av} - \epsilon$ for an infinitesimally small $\epsilon > 0$ such that

$$\phi > \frac{C_{\max}(P_{\rm av}) - C_{\max}(P^{-}_{\rm av})}{P_{\rm av} - P^{-}_{\rm av}} > C^{'}_{\max}(P_{\rm av})\,. \tag{2.26}$$

By choosing a $\bar{\lambda} \in (0, 1)$ such that $\bar{\lambda}\,\bar{P} + (1 - \bar{\lambda})P^{-}_{\rm av} = P_{\rm av}$ and considering the expressions in (2.25) and (2.26), the same approach employed in the previous part of the proof (see (2.22)-(2.24)) can be applied to show that the optimal channel switching strategy outperforms the optimal single channel strategy. Thus, it is concluded that when the condition in Proposition 2 is not satisfied, the optimal single channel strategy achieves a smaller average capacity than the optimal channel switching strategy, which implies that the condition in the proposition is necessary to achieve the same maximum average capacity for both strategies. ■

A more intuitive description of Proposition 2 can be provided as follows: Based on (2.16), the condition in (2.15) is equivalent to having the tangent line to $C_{\max}(P)$ at $P = P_{\rm av}$ lie completely above the $C_{\max}(P)$ curve [14]. If this condition is satisfied, then channel switching, which performs convex combination of different $C_{\max}(P)$ values (as can be noted from (2.3)), cannot achieve an average capacity above $C_{\max}(P_{\rm av})$, which is already achieved by the optimal single channel strategy. Otherwise, a higher average capacity than $C_{\max}(P_{\rm av})$ is obtained via optimal channel switching.

It is also noted from (2.15) and (2.16) that the condition in Proposition 2 corresponds to the subgradient inequality at $P_{\rm av}$. Therefore, the proposition can also be stated as "the optimal channel switching and the optimal single channel strategies achieve the same maximum average capacity if and only if there exists a sub-gradient at P_{av}." In addition, it should be emphasized that although concavity of $C_{\max}(P)$ around $P = P_{\text{av}}$ is a necessary condition for the scenario in Proposition 2 to hold, it is not a sufficient condition in general.

Based on Proposition 2, it can be determined whether the channel switching strategy can improve the average capacity of the system compared to the optimal single channel strategy. For instance, if $C_{\max}(P)$ in (2.4) is first-order continuously differentiable in an interval around P_{av} and the condition in (2.15) is satisfied for all $P \in [0, P_{\text{pk}}]$ in a given system, then it is concluded that the optimal single channel strategy has the same performance as the optimal channel switching strategy; that is, there is no need for channel switching. In that case, the maximum average channel capacity is given by $C_{\max}(P_{\text{av}})$. On the other hand, if there exist some $P \in [0, P_{\text{pk}}]$ for which the condition in (2.15) is not satisfied, then the optimal channel switching strategy is guaranteed to achieve a higher average capacity than $C_{\max}(P_{\text{av}})$.

Remark 1: *As a special case, it can be concluded from Proposition 2 that if the bandwidths of the channels are the same, the optimal strategy is to transmit over the least noisy (best) channel exclusively at the average power limit. In order to make this conclusion, first consider* $C_{\max}(P)$ *in* (2.4)*, which becomes equal to the capacity of the least noisy channel, say channel b, when the channels have the same bandwidth (cf.* (2.1)*); that is,* $C_{\max}(P) \triangleq \max\{C_1(P), \ldots, C_K(P)\} = C_b(P)$*. Then, from* (2.16)*, the condition in* (2.15) *of Proposition 2 is expressed as* $(P - P_{\text{av}})C_b'(P_{\text{av}}) \geq C_b(P) - C_b(P_{\text{av}})$*, which always holds for all* $P \in [0, P_{\text{pk}}]$ *due to the concavity of the capacity function,* $C_b(P)$ *(see* (2.1)*). Hence, Proposition 2 applies in this scenario; that is, the optimal single channel strategy (i.e., the use of the best channel all the time at the average power limit) becomes the optimal solution.*

In Proposition 2, it is assumed that $C_{\max}(P)$ in (2.4) is first-order continuously differentiable in an interval around P_{av}. In order to cover all possible scenarios and to specify the optimal strategy in all cases, the following proposition presents a result for the case of $C_{\max}(P)$ that has a discontinuous first-order derivative at $P = P_{\text{av}}$, which states that the optimal channel switching always outperforms the optimal single channel strategy in this scenario.

Proposition 3: *If the first-order derivative of* $C_{\text{max}}(P)$ *in* (2.4) *is discontinuous at* $P = P_{\text{av}}$*, then the optimal channel switching strategy outperforms the optimal single channel strategy.*

Proof: The aim is to prove that if the condition in Proposition 3 is satisfied, then the channel switching strategy achieves a higher average capacity than the optimal single channel strategy. To that aim, define P_{av}^{+} and P_{av}^{-} as $P_{\text{av}} + \epsilon$ and $P_{\text{av}} - \epsilon$, respectively, where ϵ is an infinitesimally small positive number. The proof consists of two parts.

First, it is proved that if the first-order derivative, $C_{\text{max}}^{'}(P)$, is discontinuous at $P = P_{\text{av}}$, which implies that $C_{\text{max}}^{'}(P_{\text{av}}^{-}) \neq C_{\text{max}}^{'}(P_{\text{av}}^{+})$, then $C_{\text{max}}^{'}(P_{\text{av}}^{-}) < C_{\text{max}}^{'}(P_{\text{av}}^{+})$ holds. Due to the discontinuous first-order derivative assumption, $C_{\text{max}}(P_{\text{av}}^{-})$ and $C_{\text{max}}(P_{\text{av}}^{+})$ must correspond to different channels since the first-order derivative would be continuous otherwise (please see (2.1)). Therefore, let channel i and channel j denote the channels corresponding to the maximum capacities for power levels P_{av}^{-} and P_{av}^{+}, respectively; that is, $C_{\text{max}}(P_{\text{av}}^{-}) = C_i(P_{\text{av}}^{-})$ and $C_{\text{max}}(P_{\text{av}}^{+}) = C_j(P_{\text{av}}^{+})$ for $i \neq j$ where $i = \arg\max_{l\in\{1,\ldots,K\}} C_l(P_{\text{av}}^{-})$ and $j = \arg\max_{l\in\{1,\ldots,K\}} C_l(P_{\text{av}}^{+})$. Also, $C_i(P_{\text{av}}) = C_j(P_{\text{av}})$ and $C_i(P_{\text{av}}^{-}) < C_j(P_{\text{av}}^{+})$ since $C_{\text{max}}(\cdot)$ is a continuous monotone increasing function. Based on Taylor series expansions of $C_i(\cdot)$ and $C_j(\cdot)$ around P_{av}^{+}, $C_i(P_{\text{av}}^{+})$ and $C_j(P_{\text{av}}^{+})$ can be expressed as follows:

$$C_i(P_{\text{av}}^{+}) = C_i(P_{\text{av}}) + C_i^{'}(P_{\text{av}})(P_{\text{av}}^{+} - P_{\text{av}}) + R_i(P_{\text{av}}^{+}) \tag{2.27}$$

$$C_j(P_{\text{av}}^{+}) = C_j(P_{\text{av}}) + C_j^{'}(P_{\text{av}})(P_{\text{av}}^{+} - P_{\text{av}}) + R_j(P_{\text{av}}^{+}) \tag{2.28}$$

where $R_i(P_{\text{av}}^{+})$ and $R_j(P_{\text{av}}^{+})$ are the second-order remainder terms for $C_i(P_{\text{av}}^{+})$ and $C_j(P_{\text{av}}^{+})$, respectively. Based on the remainder theorem, there exist $\kappa \in [P_{\text{av}}, P_{\text{av}}^{+}]$

and $\upsilon \in [P_{\mathrm{av}}, P_{\mathrm{av}}^{+}]$ such that

$$R_i(P_{\mathrm{av}}^{+}) = \frac{C_i^{''}(\kappa)(P_{\mathrm{av}}^{+} - P_{\mathrm{av}})^2}{2} \tag{2.29}$$

$$R_j(P_{\mathrm{av}}^{+}) = \frac{C_j^{''}(\upsilon)(P_{\mathrm{av}}^{+} - P_{\mathrm{av}})^2}{2} \tag{2.30}$$

where $C_i^{''}(\cdot)$ and $C_j^{''}(\cdot)$ are the second-order derivatives of $C_i(\cdot)$ and $C_j(\cdot)$, respectively [74]. The second-order derivatives, which can be calculated from (2.1) as $C_i^{''}(P) = -B_i \log_2 e/(N_i B_i + P)^2$ and $C_j^{''}(P) = -B_j \log_2 e/(N_j B_j + P)^2$, are finite negative numbers for all possible power levels. Since $C_j(P_{\mathrm{av}}^{+}) > C_i(P_{\mathrm{av}}^{+})$ and $C_i(P_{\mathrm{av}}) = C_j(P_{\mathrm{av}})$ as discussed previously, the following inequality can be obtained based on (2.27)-(2.30):

$$C_j^{'}(P_{\mathrm{av}}) - C_i^{'}(P_{\mathrm{av}}) + \frac{(C_j^{''}(\upsilon) - C_i^{''}(\kappa))\,\epsilon}{2} > 0 \tag{2.31}$$

where $\epsilon = P_{\mathrm{av}}^{+} - P_{\mathrm{av}}$ as defined above. As the second-order derivatives are finite and the relation in (2.31) should hold for any infinitesimally small ϵ value, it is concluded that $C_i^{'}(P_{\mathrm{av}}) < C_j^{'}(P_{\mathrm{av}})$. In other words, there is an increase in the first-order derivative of $C_{\max}(P)$ around $P = P_{\mathrm{av}}$, which implies that $C_{\max}^{'}(P_{\mathrm{av}}^{-}) < C_{\max}^{'}(P_{\mathrm{av}}^{+})$.

In the second part, it is proved that when there is an increase in the first-order derivative of $C_{\max}(P)$ around $P = P_{\mathrm{av}}$, the optimal channel switching strategy outperforms the optimal single channel strategy. To that aim, consider a channel switching strategy (not necessarily an optimal one) that performs channel switching between channel i and channel j by employing power levels of P_{av}^{-} and P_{av}^{+}, respectively, with equal channel switching factors; i.e., 0.5 each, where i, j, P_{av}^{-} and P_{av}^{+} are as defined in the previous paragraph. Then, that channel switching strategy achieves an average capacity of $0.5\, C_i(P_{\mathrm{av}}^{-}) + 0.5\, C_j(P_{\mathrm{av}}^{+})$, which can be expressed via Taylor series expansion as follows:

$$\begin{aligned} &0.5 \left(C_i(P_{\mathrm{av}}) + C_i^{'}(P_{\mathrm{av}})(P_{\mathrm{av}}^{-} - P_{\mathrm{av}}) + R_i(P_{\mathrm{av}}^{-}) \right) \\ &+ 0.5 \left(C_j(P_{\mathrm{av}}) + C_j^{'}(P_{\mathrm{av}})(P_{\mathrm{av}}^{+} - P_{\mathrm{av}}) + R_j(P_{\mathrm{av}}^{+}) \right) \end{aligned} \tag{2.32}$$

where $R_j(P_{\mathrm{av}}^{+})$ is as in (2.30) and $R_i(P_{\mathrm{av}}^{-}) = C_i^{''}(\omega)(P_{\mathrm{av}}^{-} - P_{\mathrm{av}})^2/2$ for a $\omega \in [P_{\mathrm{av}}^{-}, P_{\mathrm{av}}]$. Since $C_i(P_{\mathrm{av}}) = C_j(P_{\mathrm{av}}) = C_{\max}(P_{\mathrm{av}})$ as mentioned in the previous paragraph, (2.32) becomes equal to

$$C_{\max}(P_{\mathrm{av}}) + 0.5\,\epsilon \left(C_j^{'}(P_{\mathrm{av}}) - C_i^{'}(P_{\mathrm{av}})\right) + 0.25\,\epsilon^2 \left(C_i^{''}(\omega) + C_j^{''}(\upsilon)\right). \qquad (2.33)$$

Based on the result obtained in the first part of the proof, namely, $C_i^{'}(P_{\mathrm{av}}) < C_j^{'}(P_{\mathrm{av}})$, (2.33) implies that there exists an infinitesimally small $\epsilon > 0$ such that the channel switching strategy achieves a larger average capacity than $C_{\max}(P_{\mathrm{av}})$, which is the capacity achieved by the optimal single channel strategy. Hence, based on the first and the second parts of the proof, it is concluded that the optimal channel switching strategy always provides a larger average capacity than the optimal single channel strategy in the case of a discontinuous first-order derivative of $C_{\max}(P)$ at $P = P_{\mathrm{av}}$. ■

As stated in the proof of Proposition 3, the discontinuities in the first-order derivative of $C_{\max}(P)$ are observed when capacity curves intersect. The capacity curves of two channels, say channel k and channel l, can intersect [27] if one of them has a smaller bandwidth and a lower noise level than the other one; i.e., $B_k < B_l$ and $N_k < N_l$. In such a case, channel k has a higher capacity than channel l for small power levels (i.e., in the power-limited regime) since the capacity expression in (2.1) becomes approximately equal to $(\log_2 e)P/N_k$ and $(\log_2 e)P/N_l$ for channel k and channel l, respectively, when P is close to zero. On the other hand, for high power levels (i.e., in the bandwidth-limited regime), channel l achieves a higher capacity than channel k due to the following reason:

$$\lim_{P\to\infty} \frac{B_l \log_2\left(1 + \frac{P}{N_l B_l}\right)}{B_k \log_2\left(1 + \frac{P}{N_k B_k}\right)} = \frac{B_l}{B_k} > 1\,. \qquad (2.34)$$

Therefore, the capacity curves can intersect in such scenarios. For example, in cognitive radio systems, there can exist multiple available frequency bands in the spectrum with various bandwidths and noise levels. Hence, such scenarios can be encountered in these systems.

Remark 2: *The main reason for the improvements that can be realized via optimal channel switching is related to the fact that the optimal single channel approach can achieve the capacity values specified by* $C_{\max}(P)$ *in* (2.4) *only whereas the upper boundary of the convex hull of* $C_{\max}(P)$ *can also be achieved via optimal channel switching (cf.* (2.3)*). Therefore, the improvements that can be obtained via optimal channel switching over the optimal single channel approach are related to the convexity/concavity properties of* $C_{\max}(P)$*. Even though each capacity function is concave, their maximum is not necessarily concave. Therefore, opportunities can appear for average power values corresponding to convex regions of* $C_{\max}(P)$ *as illustrated in Section 2.3. The proof of Proposition 3 contains the theoretical explanation about this situation by showing that the first-order derivative of* $C_{\max}(P)$ *increases at the intersection point of two capacity curves, which implies that if two capacity functions intersect at a single point, there always exists a convex region around that intersection due to the mathematical expression for the capacity. Hence, improvements may be realized via channel switching around those intersection points.*

2.2.2 Solution of Optimal Channel Switching Problem

When the optimal channel switching strategy is guaranteed to achieve a higher average capacity than the optimal single channel strategy (which can be deduced from Proposition 2 or Proposition 3), the optimization problem in (2.2) or (2.3) needs to be solved in order to calculate the maximum average capacity of the system, which involves a search over a $2K$ dimensional space. However, the following proposition states that the optimal strategy can be obtained by switching between no more than two different channels, and the resulting optimal strategy can be found via a search over a two-dimensional space only.

Proposition 4: *The optimal solution of* (2.2) *results in channel switching between at most two different channels, and the achieved maximum average capacity is calculated as* $\lambda^* C_{\max}(P_1^*) + (1-\lambda^*) C_{\max}(P_2^*)$*, where* P_1^* *and* P_2^* *are the* *solutions of the following problem:*

$$\max_{\substack{P_1 \in (P_{\text{av}}, P_{\text{pk}}] \\ P_2 \in [0, P_{\text{av}}]}} \frac{P_{\text{av}} - P_2}{P_1 - P_2} C_{\max}(P_1) + \frac{P_1 - P_{\text{av}}}{P_1 - P_2} C_{\max}(P_2) \tag{2.35}$$

and λ^* *is given by*

$$\lambda^* = \frac{P_{\text{av}} - P_2^*}{P_1^* - P_2^*} \; . \tag{2.36}$$

Proof: As discussed in Proposition 1 and its proof, the optimization problems in (2.2) and (2.3) achieve the same maximum average capacity and the optimal channel switching strategy corresponding to (2.2) can be obtained from the solution of (2.3). Therefore, the optimization problem in (2.3) is considered, where the convex combinations of $C_{\max}(P_i)$'s and P_i's are the two main functions. The set of all possible pairs of $C_{\max}(P)$ and P is defined as set $\mathcal{U}$; that is, $\mathcal{U} = \{(C_{\max}(P), P), \ \forall P \in [0, P_{\text{pk}}]\}$. The convex hull of $\mathcal{U}$, denoted by $\mathcal{V}$, is guaranteed to contain the optimal solution of (2.3) since $\mathcal{V}$ consists of all the convex combinations of the elements of $\mathcal{U}$ by definition. In addition, it can be shown, similarly to [1], that the optimal solution of (2.3) should be on the boundary of $\mathcal{V}$ since no interior points can be the maximizer of (2.3). Then, Carathéodory's theorem [75, 76] is invoked, which states that any point on the boundary of the convex hull $\mathcal{V}$ of set $\mathcal{U}$ can be represented by a convex combination of at most D points in set $\mathcal{U}$, where D is the dimension of space in which $\mathcal{U}$ and $\mathcal{V}$ reside. Hence, in this scenario (where $\mathcal{U} \subset \mathcal{V} \subset \mathbb{R}^2$), Carathéodory's theorem implies that an optimal solution of (2.3) can be expressed as the convex combination of (i.e., time sharing between) at most two different power levels; that is, $\nu_i \neq 0$ for one or two indices in (2.3). Therefore, the optimal solution of the channel switching problem in (2.2) corresponds to channel switching between at most two different channels.

Based on the previous result, the problem in (2.3) can be expressed as follows:

$$\max_{\lambda, P_1, P_2} \lambda\, C_{\max}(P_1) + (1-\lambda) C_{\max}(P_2) \tag{2.37}$$

$$\text{subject to } \lambda\, P_1 + (1-\lambda) P_2 = P_{\text{av}} \tag{2.38}$$

$$P_1 \in [0, P_{\text{pk}}],\ \ P_2 \in [0, P_{\text{pk}}] \tag{2.39}$$

$$\lambda \in [0, 1] \tag{2.40}$$

where the average power constraint is imposed with equality based on Lemma 1. Then, by substituting the constraints in (2.38)-(2.40) into the objective function and specifying the search space, the optimization problem in (2.35) can be obtained. ■

Once λ^*, P_1^*, and P_2^* are calculated as in Proposition 4, the optimal strategy can be specified as follows:

- **Case-1 (Channel Switching):** If $\lambda^* \in (0, 1)$, the optimal strategy is to switch between channel i and channel j with channel switching (time sharing) factors λ^* and $1 - \lambda^*$ and power levels P_1^* and P_2^*, respectively, where i and j are given by[7]

$$i = \arg\max_{l \in \{1,\dots,K\}} C_l(P_1^*)\ , \tag{2.41}$$

$$j = \arg\max_{l \in \{1,\dots,K\}} C_l(P_2^*)\ . \tag{2.42}$$

- **Case-2 (Single Channel):** If $\lambda^* = 0$, the optimal strategy is to perform communications over channel m all the time with a power level of P_{av}, where m is defined as

$$m = \arg\max_{l \in \{1,\dots,K\}} C_l(P_{\text{av}})\ . \tag{2.43}$$

Note that, in the case of $\lambda^* \in (0, 1)$, $i = j$ is not possible since time sharing of

[7]In the case of multiple maximizers in (2.41) or (2.42), any of them can be chosen for the optimal strategy.

different power levels over the same channel always reduces the capacity due to the convexity of the capacity function in (2.1).

A flowchart is provided in Fig. 2.2 to explain the results obtained in this section. In particular, the optimal strategy can be specified as shown in the flowchart based on the propositions. Depending on the system parameters, either the single channel strategy or the channel switching strategy can be the optimal approach. From Proposition 2 and Proposition 3, the optimal strategy can be classified as *single channel* (case 2) or *channel switching* (case 1) without solving the optimization problem in (2.35): If the first-order derivative of $C_{\max}(P)$ is continuous at $P_{\rm av}$ (i.e., the condition in Proposition 3 does not hold) and the condition in Proposition 2 is satisfied, then the optimal single channel strategy is optimal (i.e., there is no need for channel switching), as shown in Fig. 2.2. In that case, the optimal solution of (2.2) can directly be expressed as $\lambda_{i^*} = 1$, $P_{i^*} = P_{\rm av}$, and $\lambda_j = 0$ for all $j \in \{1, \ldots, K\} \setminus \{i^*\}$, where $i^* = \arg\max_{i \in \{1,\ldots,K\}} C_i(P_{\rm av})$ (cf. (2.43)), and the maximum capacity becomes $C_{\max}(P_{\rm av})$. If the condition in Proposition 3 holds or the condition in Proposition 2 is not satisfied, the optimal strategy is to switch between two different channels, and the optimization problem in Proposition 4 (i.e., (2.35)) can be solved in that case, as illustrated in Fig. 2.2. (As discussed in the next section, the solution of (2.35) can also be obtained based on Proposition 5.)

It is noted that the computational complexity of the optimization problem in (2.35) depends on the number of channels, K, only through $C_{\max}$ in (2.4), and the dimension of the search space is always two irrespective of the number of channels. Therefore, Proposition 4 can provide a significant simplification over the original formulation in (2.2), which requires a search over a $2K$ dimensional space.

2.2.3 Alternative Solution for Optimal Channel Switching

When the optimal strategy involves channel switching, which can be deduced from Proposition 2 and Proposition 3, one way to obtain the solution is to solve

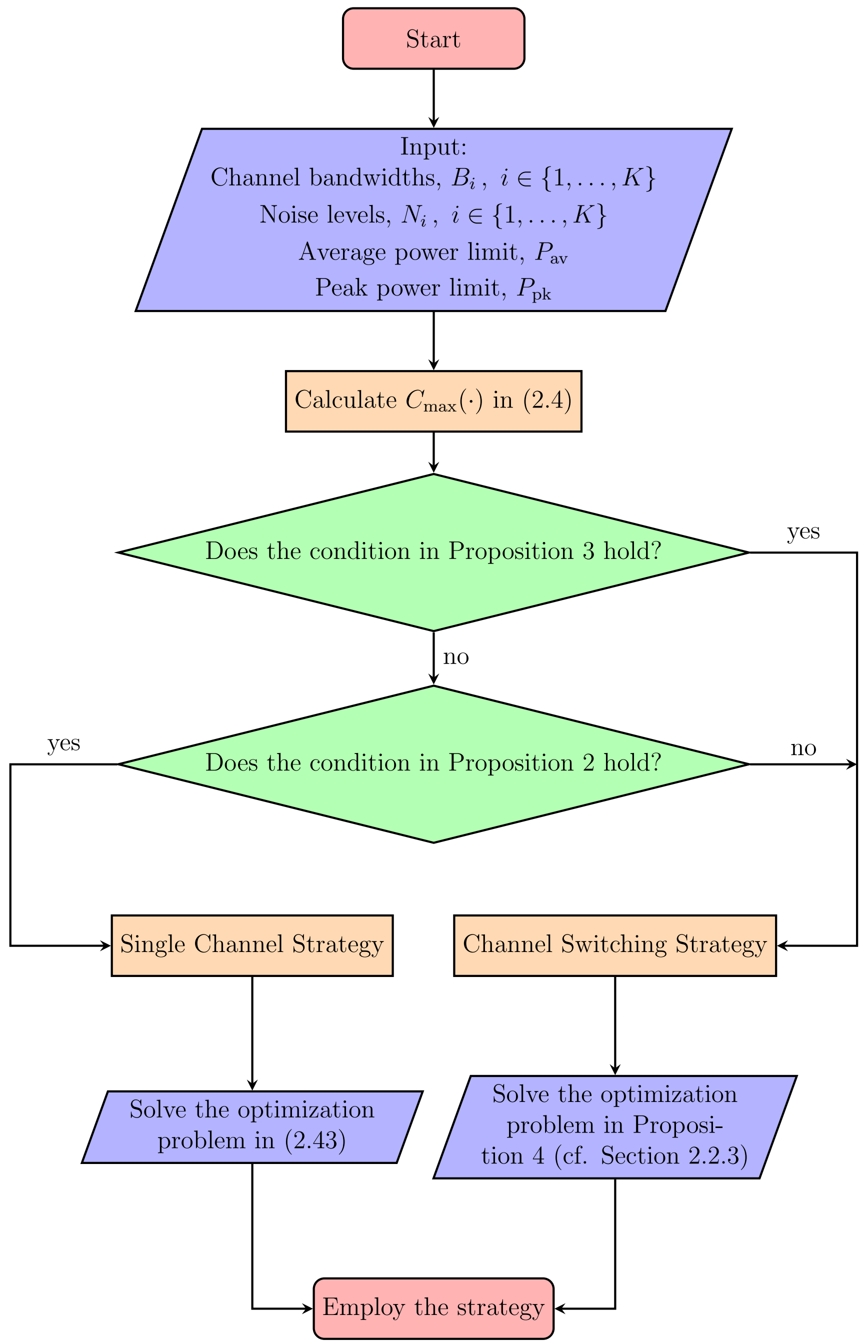

Figure 2.2: A flowchart indicating the outline of the proposed optimal channel switching and optimal single channel approaches.

the optimization problem in (2.35). An alternative approach can be developed based on the following proposition:

Proposition 5: *Consider a scenario in which channel switching between channel k and channel l is optimal. Let P_1^* and P_2^* denote the optimal transmit powers allocated to channel k and channel l, respectively. Then, the optimal solution satisfies at least one of the following conditions:*

(i) $N_k + \frac{P_1^*}{B_k} = N_l + \frac{P_2^*}{B_l}$, *where B_k and $N_k/2$ (B_l and $N_l/2$) are, respectively, the bandwidth and the constant power spectral density level of the additive Gaussian noise corresponding to channel k (channel l).*

(ii) $P_1^* = P_{\rm pk}$ *and* $P_2^* = \frac{P_{\rm av}-\lambda^* P_{\rm pk}}{1-\lambda^*}$, *where* $\lambda^* = (P_{\rm av} - P_2^*)/(P_{\rm pk} - P_2^*)$.

(iii) $P_2^* = P_{\rm pk}$ *and* $P_1^* = \frac{P_{\rm av}-(1-\lambda^*) P_{\rm pk}}{\lambda^*}$, *where* $\lambda^* = (P_{\rm pk} - P_{\rm av})/(P_{\rm pk} - P_1^*)$.

Proof: The results in the proposition can be proved via Karush-Kuhn-Tucker (KKT) conditions [73] based on the optimal channel switching problem formulated in (2.2). To that aim, the Lagrangian [73] for the optimization problem in (2.2) is obtained first:

$$\begin{aligned} L(\boldsymbol{\lambda}, \boldsymbol{P}, \mu, \boldsymbol{\gamma}, \boldsymbol{\beta}, \theta, \boldsymbol{\alpha}) &= -\sum_{i=1}^{K} \lambda_i\, C_i(P_i) + \mu \left(\sum_{i=1}^{K} \lambda_i\, P_i - P_{\rm av} \right) \\ &- \sum_{i=1}^{K} \gamma_i\, P_i + \sum_{i=1}^{K} \beta_i\, (P_i - P_{\rm pk}) + \theta \left(\sum_{i=1}^{K} \lambda_i - 1 \right) - \sum_{i=1}^{K} \alpha_i\, \lambda_i \end{aligned} \tag{2.44}$$

where $\boldsymbol{\lambda} = [\lambda_1 \cdots \lambda_K]$ and $\boldsymbol{P} = [P_1 \cdots P_K]$ are the optimization variables in (2.2), and μ, $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, θ, and $\boldsymbol{\alpha}$ are the KKT multipliers, with $\boldsymbol{\gamma} = [\gamma_1 \cdots \gamma_K]$, $\boldsymbol{\beta} = [\beta_1 \cdots \beta_K]$, and $\boldsymbol{\alpha} = [\alpha_1 \cdots \alpha_K]$. Then, the optimal solution of the problem in (2.2), denoted by $\{\lambda_i^*, P_i^*\}_{i=1}^K$ (equivalently, by $\boldsymbol{\lambda}^*, \boldsymbol{P}^*$), satisfies the following KKT conditions:

- **Stationarity:** $\frac{\partial L(\boldsymbol{\lambda}^*, \boldsymbol{P}^*, \mu, \boldsymbol{\gamma}, \boldsymbol{\beta}, \theta, \boldsymbol{\alpha})}{\partial \lambda_i} = 0$ and $\frac{\partial L(\boldsymbol{\lambda}^*, \boldsymbol{P}^*, \mu, \boldsymbol{\gamma}, \boldsymbol{\beta}, \theta, \boldsymbol{\alpha})}{\partial P_i} = 0$ for $i \in \{1, \ldots, K\}$, where L is as defined in (2.44).

- **Complementary slackness:** $\mu\left(\sum_{i=1}^{K}\lambda_i^*\,P_i^* - P_{\text{av}}\right) = 0$, $\sum_{i=1}^{K}\gamma_i\,P_i^* = 0$, $\sum_{i=1}^{K}\beta_i\,(P_i^* - P_{\text{pk}}) = 0$, and $\sum_{i=1}^{K}\alpha_i\,\lambda_i^* = 0$.

- **Primal and dual feasibility:** $\mu \geq 0$, $\gamma_i \geq 0$, $\beta_i \geq 0$, and $\alpha_i \geq 0$ for $i \in \{1,\ldots,K\}$.

From the stationarity conditions; the following equalities are obtained based on (2.44):

$$C_i(P_i^*) = \mu\,P_i^* + \theta - \alpha_i\,,\ \forall i \in \{1,\ldots,K\}\,, \tag{2.45}$$

$$C_i^{'}(P_i^*) = \mu + \frac{\beta_i - \gamma_i}{\lambda_i^*}\,, \qquad \forall i \in \{1,\ldots,K\}\,. \tag{2.46}$$

Now consider the scenario in the proposition, where channel switching between channel k and channel l is optimal; that is, $\lambda_k^* \neq 0$, $\lambda_l^* \neq 0$, $P_k^* = P_1^* \neq 0$, $P_l^* = P_2^* \neq 0$, and $P_i^* = \lambda_i^* = 0$ for $i \in \{1,\ldots,K\} \setminus \{k,l\}$.[8] Then, $\gamma_k = \gamma_l = 0$ and $\alpha_k = \alpha_l = 0$ can be obtained from the second and fourth complementary slackness conditions. For the optimal power levels, three possible scenarios exist:

- First, it is assumed that $P_1^* < P_{\text{pk}}$ and $P_2^* < P_{\text{pk}}$ hold. Then, $\beta_k = 0$ and $\beta_l = 0$ are satisfied due to the third complementary slackness condition. Combining this result with $\gamma_k = \gamma_l = 0$, $\lambda_k^* \neq 0$, and $\lambda_l^* \neq 0$, the condition in (2.46) can be expressed as $C_k^{'}(P_k^*) = C_l^{'}(P_l^*) = \mu$, which leads to condition (i) in the proposition based on the first-order derivative expression in (2.16).

- Second, it is assumed that $P_1^* = P_{\text{pk}}$ and $P_2^* < P_{\text{pk}}$. Due to Lemma 1, the average power constraint must be satisfied with equality, which leads to $P_2^* = (P_{\text{av}} - \lambda^* P_{\text{pk}})/(1-\lambda^*)$, where $\lambda^* = (P_{\text{av}} - P_2^*)/(P_{\text{pk}} - P_2^*)$. Hence, condition (ii) in the proposition is obtained. Note that in this case $\beta_k \geq 0$ and $\beta_l = 0$, which implies that $C_k^{'}(P_k^*) \geq C_l^{'}(P_l^*)$ based on (2.46).

- For the third scenario, the third condition in the proposition can similarly be obtained under the assumption that $P_1^* < P_{\text{pk}}$ and $P_2^* = P_{\text{pk}}$.

[8] Note that the on-off scheme, in which one power level is equal to zero, cannot be optimal due to the concavity of the capacity curves and the fact that $C_i(0) = 0$, $\forall i \in \{1,\ldots,K\}$.

Finally, it is noted that $P_1^* = P_2^* = P_{\text{pk}}$ is not possible since it violates the average power constraint as $P_{\text{pk}} > P_{\text{av}}$. Therefore, the optimal solution of the channel switching strategy between two channels satisfies at least one of the three conditions in Proposition 5. ■

Proposition 5 presents necessary conditions that need to be satisfied by the optimal channel switching strategy. Based on this proposition, the optimal solution of the problem in (2.2) can also be calculated as described in the following. For the scenario in which one of the power levels is set to P_{pk}, the maximum capacity achieved can be calculated from the second and third conditions in Proposition 5 as follows:

$$\tilde{C}_{\text{av}}(i,j) \triangleq \max_{P_j \in [0, P_{\text{av}}]} \frac{P_{\text{av}} - P_j}{P_{\text{pk}} - P_j} C_i(P_{\text{pk}}) + \frac{P_{\text{pk}} - P_{\text{av}}}{P_{\text{pk}} - P_j} C_j(P_j) \tag{2.47}$$

where $i, j \in \{1, \ldots, K\}$ and $i \neq j$. Since one power level is fixed to P_{pk}, it is sufficient to consider the best channel only for that power level in calculating the maximum average capacity. Hence, a new function, which is a function of a single channel index only, is defined in that respect as follows:

$$\tilde{C}_{\text{av}}(j) \triangleq \max_{P_j \in [0, P_{\text{av}}]} \frac{P_{\text{av}} - P_j}{P_{\text{pk}} - P_j} C_{\max}(P_{\text{pk}}) + \frac{P_{\text{pk}} - P_{\text{av}}}{P_{\text{pk}} - P_j} C_j(P_j) \tag{2.48}$$

where $j \in \{1, \ldots, K\} \setminus \{k^*\}$ with $k^* = \arg\max_{i \in \{1,\ldots,K\}} C_i(P_{\text{pk}})$ and $C_{\max}(P_{\text{pk}}) = C_{k^*}(P_{\text{pk}})$. Then, in the case of channel switching between two channels where one power level is equal to P_{pk}, the maximum achieved capacity can be calculated as follows:

$$\tilde{C}_{\text{av}} = \max_{\substack{j \in \{1,\ldots,K\} \\ j \neq k^*}} \tilde{C}_{\text{av}}(j) \tag{2.49}$$

It should be noted that $\tilde{C}_{\text{av}}$ also includes the maximum capacity that can be achieved by the optimal single channel strategy since $\tilde{C}_{\text{av}}(j)$ in (2.48) reduces to $C_j(P_{\text{av}})$ for $P_j = P_{\text{av}}$ (which is added to the search space for this purpose). For the scenario in which the optimal power levels are below P_{pk}, the first condition in Proposition 5, namely, $N_i + P_i/B_i = N_j + P_j/B_j$, can be employed to obtain the following formulation for the maximum achieved capacity:

$$\bar{C}_{\mathrm{av}}(i,j) \triangleq \max_{P_j \in \left(P_{ij}^{\mathrm{lb}}, P_{ij}^{\mathrm{ub}}\right]} \frac{P_{\mathrm{av}} - P_j}{P_i - P_j} C_i(P_i) + \frac{P_i - P_{\mathrm{av}}}{P_i - P_j} C_j(P_j) \tag{2.50}$$

where $P_{ij}^{\mathrm{lb}} \triangleq \max\left\{0\,, \left(P_{\mathrm{av}}\frac{B_j}{B_i} + B_j\,(N_i - N_j)\right)\right\}$, $P_{ij}^{\mathrm{ub}} \triangleq \min\left\{P_{\mathrm{av}}\,, \left(P_{\mathrm{pk}}\frac{B_j}{B_i} + B_j\,(N_i - N_j)\right)\right\}$, and $P_i = B_i(N_j - N_i) + B_i P_j / B_j$. Note that the search space for P_j (namely, P_{ij}^{lb} and P_{ij}^{ub}) is obtained by the joint consideration of $P_j \in (0, P_{\mathrm{av}}]$ and $P_i = B_i(N_j - N_i) + B_i P_j / B_j \in (P_{\mathrm{av}}, P_{\mathrm{pk}}]$. Then, the maximum capacity that can be achieved by switching between two channels with power levels lower than P_{pk} can be calculated as follows:

$$\bar{C}_{\mathrm{av}} = \max_{\substack{i,j \in \{1,\ldots,K\} \\ i \neq j}} \bar{C}_{\mathrm{av}}(i,j) \tag{2.51}$$

Overall, the solution of the optimal channel switching problem in (2.2) achieves the following maximum average capacity:

$$C_{\mathrm{av}}^{\max} = \max\left\{\tilde{C}_{\mathrm{av}}, \bar{C}_{\mathrm{av}}\right\} \tag{2.52}$$

where $\tilde{C}_{\mathrm{av}}$ and $\bar{C}_{\mathrm{av}}$ are as in (2.49) and (2.51), respectively. Also, the optimal strategy can be obtained as follows: If $\tilde{C}_{\mathrm{av}} = C_{\max}(P_{\mathrm{av}}) \geq \bar{C}_{\mathrm{av}}$, then the optimal solution corresponds to the single channel strategy, which is to transmit over channel m all the time with power level P_{av}, where $m = \arg\max_{i \in \{1,\ldots,K\}} C_i(P_{\mathrm{av}})$. (In fact, based on Proposition 2, the cases in which the single channel strategy is optimal can be determined beforehand, and the efforts in solving (2.48)-(2.52) can be avoided.) If $\tilde{C}_{\mathrm{av}} \geq \bar{C}_{\mathrm{av}}$ and $\tilde{C}_{\mathrm{av}} > C_{\max}(P_{\mathrm{av}})$, the optimal strategy is to switch over channel k^* and channel j^* with power levels P_{pk} and $P_{j^*}^*$ and channel switching factors $(P_{\mathrm{av}} - P_{j^*}^*)/(P_{\mathrm{pk}} - P_{j^*}^*)$ and $(P_{\mathrm{pk}} - P_{\mathrm{av}})/(P_{\mathrm{pk}} - P_{j^*}^*)$, respectively, where $P_{j^*}^*$ denotes the maximizer of the problem in (2.48), $k^* = \arg\max_{i \in \{1,\ldots,K\}} C_i(P_{\mathrm{pk}})$, and $j^* = \arg\max_{j \in \{1,\ldots,K\},\, j \neq k^*} \tilde{C}_{\mathrm{av}}(j)$, with $\tilde{C}_{\mathrm{av}}(j)$ being as defined in (2.48). Finally, if $\bar{C}_{\mathrm{av}} > \tilde{C}_{\mathrm{av}}$, then the optimal strategy is to switch between channel j^* and channel i^* with power levels $P_{j^*}^*$ and $P_{i^*}^* = B_{i^*}(N_{j^*} - N_{i^*}) + B_{i^*} P_{j^*} / B_{j^*}$ and channel switching factors $(P_{i^*} - P_{\mathrm{av}})/(P_{i^*} - P_{j^*})$ and $(P_{\mathrm{av}} - P_{j^*})/(P_{i^*} - P_{j^*})$, respectively, where $P_{j^*}^*$ is the maximizer of the problem in (2.50) and i^* and j^*

denote the maximizers of (2.51).

In order to compare the approach in the previous paragraph (called the second approach) to the one provided in Proposition 4 (called the first approach) in terms of the computational complexity in obtaining the optimal switching solution, the optimization problems in (2.35) and in (2.48)-(2.52) are considered. In the first approach, the problem in (2.35) requires a two-dimensional search over $[0, P_{\text{av}}] \times (P_{\text{av}}, P_{\text{pk}}]$. On the other hand, the main operations in the second approach are related to the optimization problem in (2.48), which requires a one-dimensional search over $[0, P_{\text{av}}]$, and the optimization problem in (2.50), which requires a one-dimensional search over a subset of $[0, P_{\text{av}}]$. It is observed from (2.49) and (2.51) that the problem in (2.48) is solved for $K - 1$ different channel indices and the one in (2.50) is solved for $K(K - 1)$ different channel pairs. Therefore, overall, the second approach involves $K^2 - 1$ one-dimensional searches. In fact, instead of K, a smaller number can be considered in many scenarios when some channels outperform other channels in the sense that they have larger or equal capacities for all possible power values. From (2.1), it is observed that, for channel i and channel j, if $N_i \leq N_j$ and $B_i \geq B_j$, then channel i outperforms channel j for all power values. Therefore, channel j can be excluded from the set of channels for the optimal channel switching solution. Hence, based on this observation, it can be stated that the second approach involves $\tilde{K}^2 - 1$ one-dimensional searches, where $\tilde{K}$ is the number of elements in set C, which is defined as $C = \{i \in \{1, \ldots, K\} \,|\, (N_i < N_j \text{ or } B_i > B_j)\ \forall j \in \{1, \ldots, K\} \setminus \{i\}\}$.[9] Therefore, the computational complexity comparison between the first approach and the second approach depends on the number of channels and their noise levels and bandwidths. In particular, the second (first) approach become more desirable for small (large) values of $\tilde{K}$.

[9]For convenience, it is assumed that the identical channels (the same bandwidth and noise level) are already eliminated.

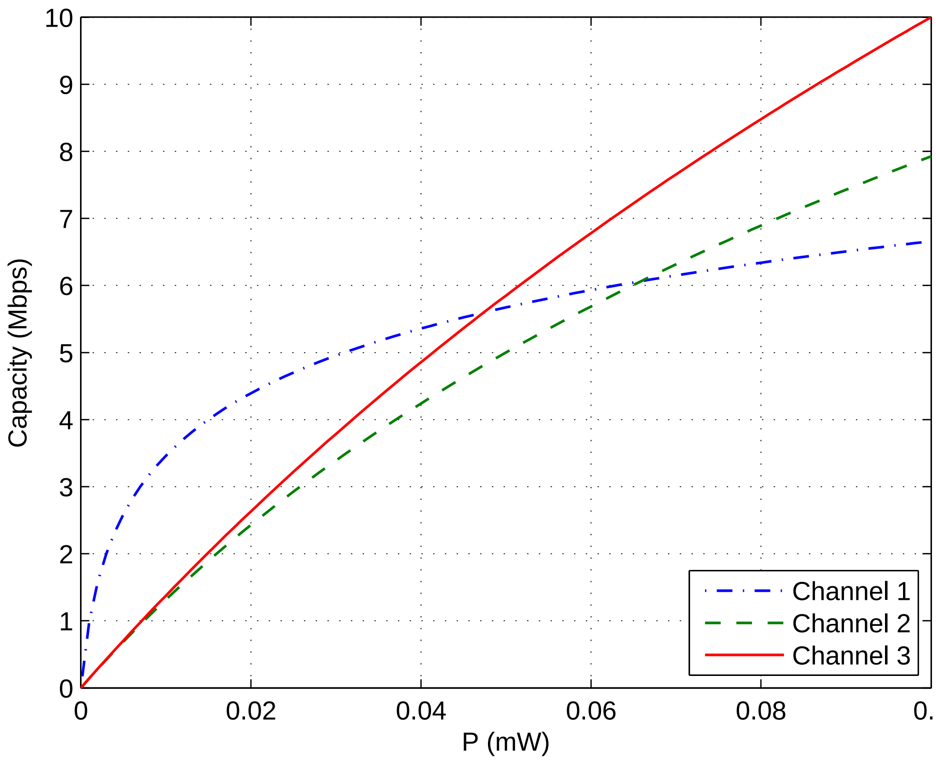

Figure 2.3: Capacity of each channel versus power, where $B_1 = 1\,\text{MHz}$, $B_2 = 5\,\text{MHz}$, $B_3 = 10\,\text{MHz}$, $N_1 = 10^{-12}\,\text{W/Hz}$, $N_2 = 10^{-11}\,\text{W/Hz}$, and $N_3 = 10^{-11}\,\text{W/Hz}$.

2.3 Numerical Results

In this section, numerical examples are provided in order to investigate the proposed optimal channel switching strategy and to compare it against the optimal single channel strategy. First, consider a scenario with $K = 3$ channels and the following bandwidths and noise levels (cf. (2.1)): $B_1 = 1\,\text{MHz}$, $B_2 = 5\,\text{MHz}$, $B_3 = 10\,\text{MHz}$, $N_1 = 10^{-12}\,\text{W/Hz}$, $N_2 = 10^{-11}\,\text{W/Hz}$, and $N_3 = 10^{-11}\,\text{W/Hz}$. Suppose that the peak power limit in (2.2) is set to $P_{\text{pk}} = 0.1\,\text{mW}$. In Fig. 2.3, the capacity of each channel is plotted as a function of power based on the capacity formula in (2.1).

For the scenario in Fig. 2.3, the proposed optimal channel switching strategy and the optimal single channel strategy are calculated for various average power limits (P_{av}), and the achieved maximum average capacities are plotted in Fig. 2.4 versus P_{av}. Also, the shaded area in the figure indicates the achievable rates (average capacities) via channel switching that are higher than those achieved by the optimal single channel strategy. As discussed in the previous section, the optimal single channel strategy achieves a capacity of $C_{\max}(P_{\text{av}})$, which is $C_{\max}(P_{\text{av}}) = \max\{C_1(P_{\text{av}}), C_2(P_{\text{av}}), C_3(P_{\text{av}})\}$ in the considered scenario. It is observed from Fig. 2.3 and Fig. 2.4 that $C_{\max}(P_{\text{av}}) = C_1(P_{\text{av}})$ for $P_{\text{av}} \in (0, 0.048)\,\text{mW}$ and $C_{\max}(P_{\text{av}}) = C_3(P_{\text{av}})$ for $P_{\text{av}} \in [0.048, 0.1]\,\text{mW}$; that is, channel 1 is the best channel up to $P_{\text{av}} = 0.048\,\text{mW}$, and channel 3 is the best after that power level. From Fig. 2.4, it is also noted that the proposed optimal channel switching strategy outperforms the optimal single channel strategy for $P_{\text{av}} \in [0.0196, 0.1]\,\text{mW}$, and the two strategies have the same performance for $P_{\text{av}} < 0.0196\,\text{mW}$. These regions can also be obtained by checking the necessary and sufficient condition in Proposition 2 (see (2.15)), which is satisfied for all $P \in [0, 0.1]\,\text{mW}$ for $P_{\text{av}} < 0.0196\,\text{mW}$, and is not satisfied for some $P \in [0, 0.1]\,\text{mW}$ for $P_{\text{av}} \in [0.0196, 0.1]\,\text{mW}$. In addition, in accordance with Proposition 3, it is observed that the optimal channel switching strategy outperforms the optimal single channel strategy at $P_{\text{av}} = 0.048\,\text{mW}$, which corresponds to a discontinuity point for the first-order derivative of $C_{\max}(P)$.

In order to provide a detailed investigation of the optimal channel switching strategy, Table 2.1 presents the optimal channel switching solutions for various values of the average power limit, P_{av}. In the table, the optimal solution is represented by parameters λ^*, P_1^*, P_2^*, i, and j, meaning that channel i is used with channel switching factor λ^* and power P_1^*, and channel j is used with channel switching factor $1 - \lambda^*$ and power P_2^*. It is observed from the table that the optimal solution reduces to the optimal single channel strategy for $P_{\text{av}} = 0.01\,\text{mW}$ (in which case channel 1 is used all the time), and it involves switching between channel 1 and channel 3 for larger values of P_{av}. This observation is also consistent with Fig. 2.4, which illustrates improvements via channel switching for $P_{\text{av}} > 0.0196\,\text{mW}$. It is also observed from the table that the optimal channel switching

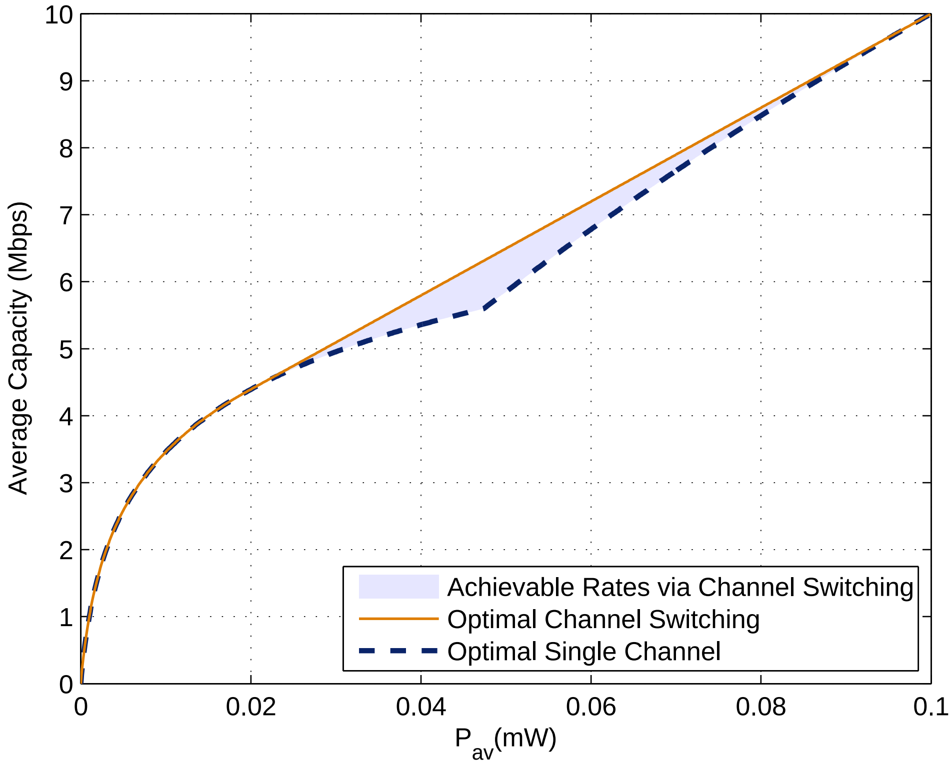

Figure 2.4: Average capacity versus average power limit for the optimal channel switching and the optimal single channel strategies for the scenario in Fig. 2.3, where $P_{\mathrm{pk}} = 0.1\,\mathrm{mW}$. The shaded area indicates the achievable rates via channel switching that are higher than those achieved by the optimal single channel strategy.

Table 2.1: Optimal strategy for the scenario in Fig. 2.3, which employs channel i and channel j with channel switching factors λ^* and $(1-\lambda^*)$ and power levels P_1^* and P_2^*, respectively.

P_{av} (mW)	λ^*	P_1^*	i	$(1-\lambda^*)$	P_2^*	j
0.01	–	–	–	1	0.01	1
0.02	0.005	0.1000	3	0.995	0.0196	1
0.03	0.129	0.1000	3	0.871	0.0196	1
0.04	0.254	0.1000	3	0.746	0.0196	1
0.05	0.378	0.1000	3	0.622	0.0196	1
0.06	0.503	0.1000	3	0.498	0.0196	1
0.07	0.627	0.1000	3	0.373	0.0196	1
0.08	0.751	0.1000	3	0.249	0.0196	1
0.09	0.876	0.1000	3	0.124	0.0196	1

solution for $P_{\text{av}} > 0.0196\,\text{mW}$ satisfies condition (ii) in Proposition 5 since $P_1^* = P_{\text{pk}} = 0.1$, $P_2^* = (P_{\text{av}} - \lambda^* P_{\text{pk}})/(1-\lambda^*) = 0.0196\,\text{mW}$, and $\lambda^* = (P_{\text{av}} - P_2^*)/(P_{\text{pk}} - P_2^*)$. In addition, as stated in Lemma 1, the optimal solutions always operate at the average power limits.

For the scenario in Fig. 2.3, the average capacity versus the peak power limit curves are presented for the optimal channel switching and the optimal single channel strategies in Fig. 2.5, where the average power limit is set to $P_{\text{av}} = 0.04\,\text{mW}$. From the figure, it is observed that the average capacity for the optimal single channel strategy does not depend on the P_{pk} value since this strategy achieves an average capacity of $C_{\max}(P_{\text{av}})$ and $P_{\text{pk}} > P_{\text{av}} = 0.04\,\text{mW}$ in this scenario. On the other hand, increased P_{pk} can improve the average capacity for the optimal channel switching strategy as observed from the figure. The intuition behind this increase can be deduced from Fig. 2.3 and Table 2.2. In particular, as observed from Table 2.2, when the peak power limit is larger than $0.048\,\text{mW}$, which is the discontinuity point for the first-order derivative of $C_{\max}$, the optimal channel switching strategy performs time sharing (switching) between channel 1 and channel 3, where channel 3 is operated at the peak power limit, P_{pk}.

Next, a scenario with $K = 4$ channels is considered, where the bandwidths and the noise levels of channels are specified as $B_1 = 0.5\,\text{MHz}$, $B_2 = 2.0\,\text{MHz}$, $B_3 = 2.5\,\text{MHz}$, $B_4 = 5.0\,\text{MHz}$, $N_1 = 10^{-12}\,\text{W/Hz}$, $N_2 = 1.5 \times 10^{-11}\,\text{W/Hz}$,

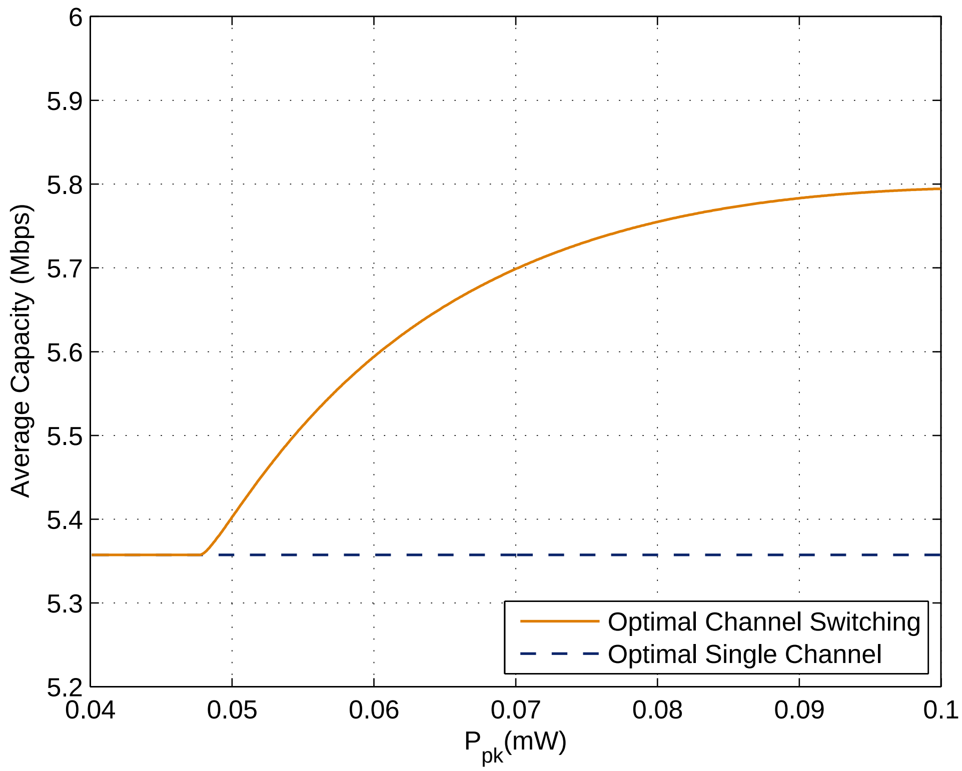

Figure 2.5: Average capacity versus peak power limit for the optimal channel switching and the optimal single channel strategies for the scenario in Fig. 2.3, where $P_{\text{av}} = 0.04\,\text{mW}$.

Table 2.2: Optimal strategy for the scenario in Fig. 2.3, which employs channel i and channel j with channel switching factors λ^* and $(1-\lambda^*)$ and power levels P_1^* and P_2^*, respectively.

P_{pk} (mW)	λ^*	P_1^*	i	$(1-\lambda^*)$	P_2^*	j
0.045	–	–	–	1.000	0.0400	1
0.050	0.465	0.050	3	0.535	0.0313	1
0.055	0.488	0.055	3	0.512	0.0257	1
0.060	0.455	0.060	3	0.545	0.0233	1
0.065	0.419	0.065	3	0.581	0.0220	1
0.070	0.387	0.070	3	0.613	0.0211	1
0.075	0.357	0.075	3	0.643	0.0206	1
0.080	0.331	0.080	3	0.669	0.0202	1
0.085	0.309	0.085	3	0.691	0.0199	1
0.090	0.289	0.090	3	0.711	0.0197	1
0.095	0.271	0.095	3	0.729	0.0196	1
0.100	0.254	0.100	3	0.746	0.0196	1

$N_3 = 2.0 \times 10^{-11}$ W/Hz, and $N_4 = 2.5 \times 10^{-11}$ W/Hz. Also, the peak power limit is set to $P_{\text{pk}} = 0.25$ mW. In Fig. 2.6, the capacity of each channel is plotted versus the transmit power.

In Fig. 2.7, the average capacity versus P_{av} curves are presented for the proposed optimal channel switching strategy and the optimal single channel strategy. In addition, the shaded area in the figure indicates the achievable rates via channel switching that are higher than those achieved by the optimal single channel strategy. From Fig. 2.7, it is observed that the optimal channel switching strategy outperforms the optimal single channel strategy for $P_{\text{av}} \in (0.031, 0.187)$ mW. Also, it can be deduced from Fig. 2.6 and Fig. 2.7 that channel 3 is not employed in any strategy since $C_{\max}(P) \neq C_3(P)$ for $P \in [0, 0.25]$ mW. In Table 2.3, the optimal strategies are presented for the scenario in Fig. 2.6 for various values of P_{av}. As observed from the table, the optimal strategy corresponds to the optimal single channel strategy for small and large values of P_{av} and it corresponds to channel switching between channel 1 and channel 4 for medium range of P_{av} values. These observations are in accordance with Fig. 2.7. In addition, it is important to emphasize that the channels employed in the optimal channel switching strategy for a given value of P_{av} may not correspond to the channel

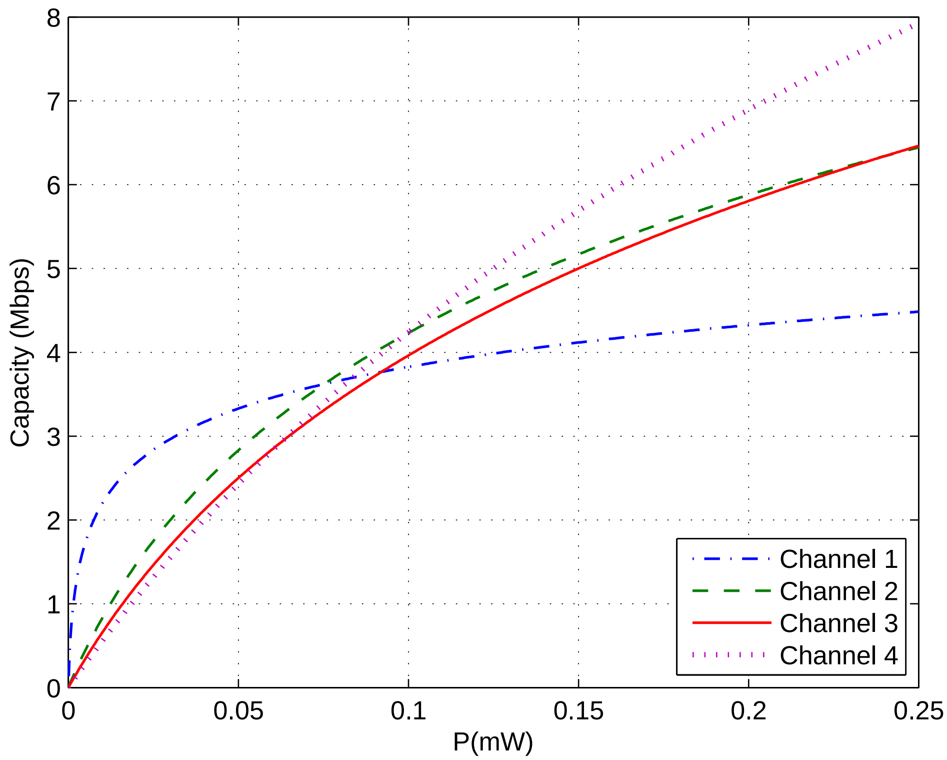

Figure 2.6: Capacity of each channel versus power, $B_1 = 0.5\,\mathrm{MHz}$, $B_2 = 2.0\,\mathrm{MHz}$, $B_3 = 2.5\,\mathrm{MHz}$, $B_4 = 5.0\,\mathrm{MHz}$, $N_1 = 10^{-12}\,\mathrm{W/Hz}$, $N_2 = 1.5 \times 10^{-11}\,\mathrm{W/Hz}$, $N_3 = 2.0 \times 10^{-11}\,\mathrm{W/Hz}$, $N_4 = 2.5 \times 10^{-11}\,\mathrm{W/Hz}$, and $P_{\mathrm{pk}} = 0.25\,\mathrm{mW}$.

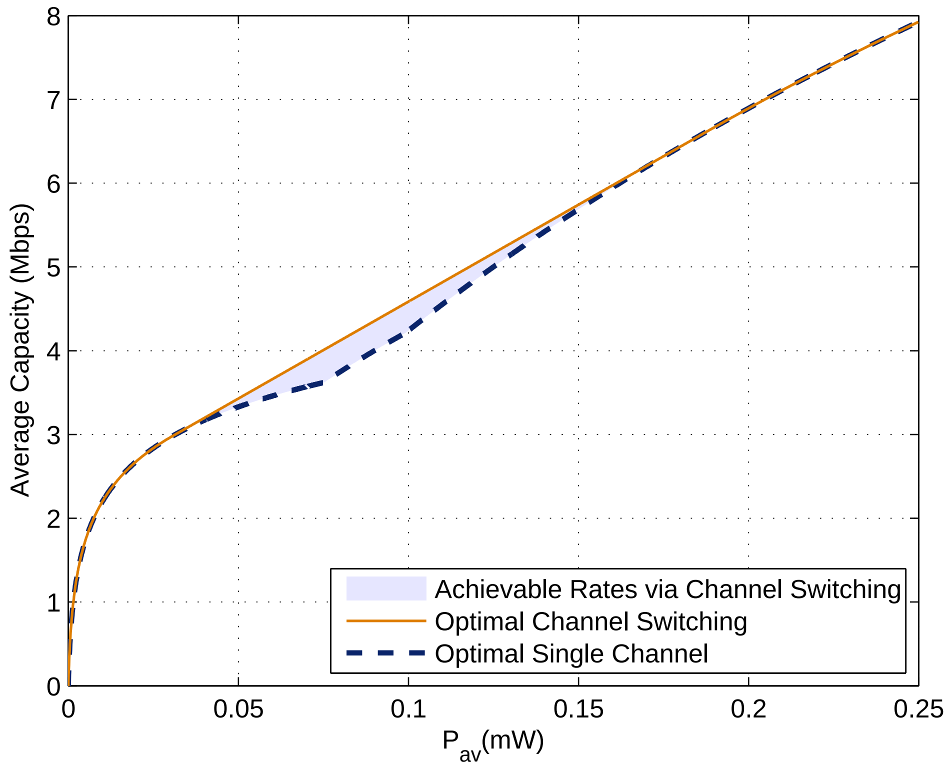

Figure 2.7: Average capacity versus average power limit for the optimal channel switching and the optimal single channel approaches for $P_{\text{pk}} = 0.25\,\text{mW}$. The shaded area indicates the achievable rates via channel switching that are higher than those achieved by the optimal single channel strategy.

Table 2.3: Optimal strategy for the scenario in Fig. 2.6, which employs channel i and channel j with channel switching factors λ^* and $(1-\lambda^*)$ and power levels P_1^* and P_2^*, respectively.

P_{av} (mW)	λ^*	P_1^*	i	$(1-\lambda^*)$	P_2^*	j
0.025	–	–	–	1.0000	0.0250	1
0.050	0.1236	0.1868	4	0.8764	0.0307	1
0.075	0.2838	0.1868	4	0.7162	0.0307	1
0.100	0.4439	0.1868	4	0.5561	0.0307	1
0.125	0.6041	0.1868	4	0.3959	0.0307	1
0.150	0.7643	0.1868	4	0.2357	0.0307	1
0.175	0.9244	0.1868	4	0.0756	0.0307	1
0.200	–	–	–	1.0000	0.2000	4
0.225	–	–	–	1.0000	0.2250	4

used in the optimal single channel strategy for the same P_{av} value. For example, as can be observed from Fig. 2.6 and Fig. 2.7, channel 2 is not employed in the optimal channel switching strategy for $P_{\text{av}} \in (0.031, 0.187)\,\text{mW}$ (channel 1 and channel 4 are employed); however, it is the optimal channel for the optimal single channel strategy for $P_{\text{av}} \in [0.075, 0.099]\,\text{mW}$ as $C_{\max}(P_{\text{av}}) = C_2(P_{\text{av}})$. This is mainly due to the fact that the optimal single channel approach achieves the capacity value specified by $C_{\max}(P_{\text{av}})$ whereas the upper boundary of the convex hull of $C_{\max}(P)$ is achieved via the optimal channel switching approach.

Finally, for the scenario in Fig. 2.6, P_{av} is set to $P_{\text{av}} = 0.07\,\text{mW}$, and the effects of the peak power limit, P_{pk}, are investigated. In Fig. 2.8, the average capacity is plotted versus P_{pk} for the optimal channel switching and optimal single channel strategies. It is observed that the optimal single channel strategy achieves a constant capacity of $C_{\max}(P_{\text{av}})$ for all P_{pk} values, where $P_{\text{pk}} \in (0.07, 0.25]\,\text{mW}$. On the other hand, for the optimal channel switching strategy, improvements in the average capacity are observed for when P_{pk} is larger than $0.075\,\text{mW}$. It is also noted that the behavioral changes in the average capacity curve for the optimal channel switching strategy occurs at $0.075\,\text{mW}$ and $0.099\,\text{mW}$, which correspond to the discontinuity points for the first-order derivative of $C_{\max}$, as can be observed from Fig. 2.6. Similar to Table 2.2, Table 2.4 presents the solutions corresponding to the optimal strategy for various values of P_{pk}. From the table, it is observed that the optimal strategy changes with respect to the peak power

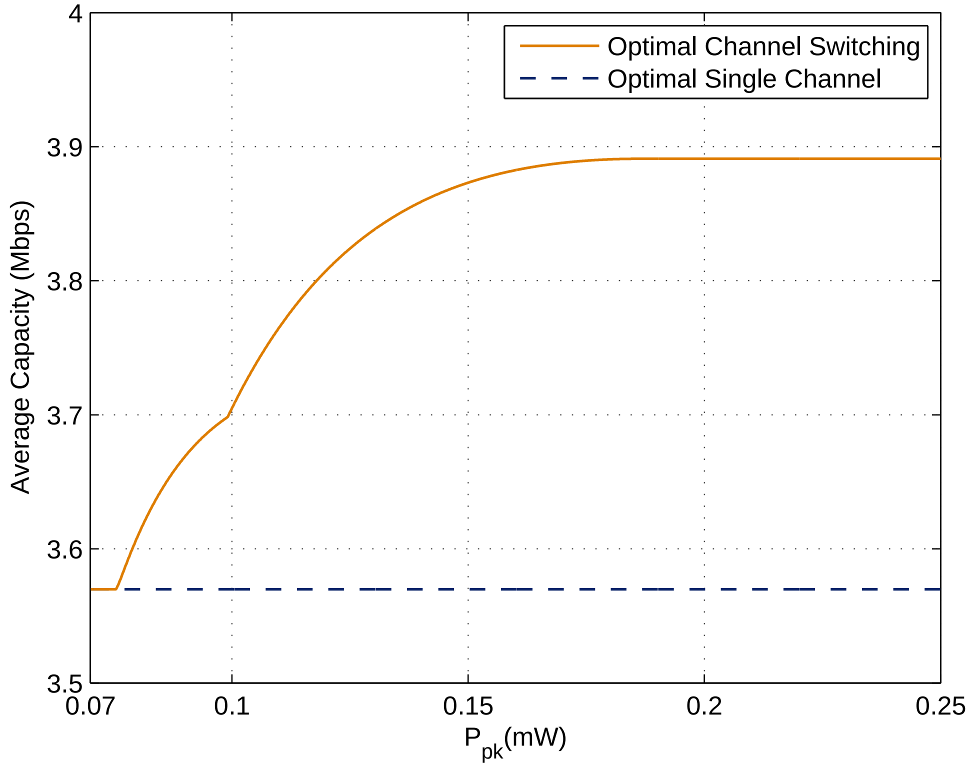

Figure 2.8: Average capacity versus peak power limit for the optimal channel switching and the optimal single channel strategies for the scenario in Fig. 2.6, where $P_{\text{av}} = 0.07\,\text{mW}$.

limit. In addition, it can be shown that the solutions of the optimal channel switching strategy satisfy condition (i) in Proposition 5 for $P_{\text{pk}} \geq 0.1868\,\text{mW}$ and condition (ii) for $P_{\text{pk}} \in (0.075, 0.1868)\,\text{mW}$.

Based on the numerical examples, an intuitive explanation can be provided about the benefits of channel switching and why the optimal channel switching strategy involves switching between no more than two channels. In the absence of channel switching, the maximum capacity is given by $C_{\max}(P_{\text{av}})$, whereas via channel switching, the upper boundary of the convex hull of $C_{\max}(P_{\text{av}})$ can also be achieved (see, e.g., Fig. 2.4). Since the upper boundary of the convex hull can always be formed by a convex combination of two different points, no more than two different channels are needed to achieve the optimal capacity. Finally, it is

Table 2.4: Optimal strategy for the scenario in Fig. 2.6, which employs channel i and channel j with channel switching factors λ^* and $(1 - \lambda^*)$ and power levels P_1^* and P_2^*, respectively.

P_{pk} (mW)	λ^*	P_1^*	i	$(1 - \lambda^*)$	P_2^*	j
0.075	–	–	–	1.0000	0.0700	1
0.080	0.6540	0.0800	2	0.3460	0.0511	1
0.085	0.6173	0.0850	2	0.3827	0.0458	1
0.090	0.5745	0.0900	2	0.4255	0.0430	1
0.095	0.5336	0.0950	2	0.4664	0.0414	1
0.100	0.5008	0.1000	4	0.4992	0.0399	1
0.125	0.4009	0.1250	4	0.5991	0.0332	1
0.150	0.3260	0.1500	4	0.6740	0.0313	1
0.175	0.2723	0.1750	4	0.7277	0.0307	1
0.200	0.2518	0.1868	4	0.7482	0.0307	1
0.225	0.2518	0.1868	4	0.7482	0.0307	1
0.250	0.2518	0.1868	4	0.7482	0.0307	1

important to note that the optimal solution to the channel switching problem in (2.3) may not be unique in general; that is, in some cases, two different channel switching strategies or a channel switching strategy and a single channel strategy can be the optimal solutions.

2.4 Concluding Remarks

In this study, the optimal channel switching strategy has been proposed for average capacity maximization in the presence of average and peak power constraints. Necessary and sufficient conditions have been derived for specifying whether the proposed optimal channel switching strategy can or cannot outperform the optimal single channel strategy. In addition, the optimal channel switching solution has been shown to be realized by channel switching between at most two different channels, and a low-complexity optimization problem has been formulated to calculate the optimal channel switching solution. Furthermore, based on the necessary conditions that need to be satisfied by the optimal channel switching solution, an alternative approach has been proposed for calculating the optimal channel switching strategy. Numerical examples have been investigated and intuitive explanations about the benefits of channel switching have been provided. Although Gaussian channels have been considered in this study, the results can also be applied to block frequency-flat fading channels in the presence of Gaussian noise when the channel state information is available at the transmitter and the receiver. In that scenario, the proposed channel switching strategy can be adopted for each channel state. As future work, performance improvements that can be achieved by performing both channel switching at each channel state and adaptation over varying channel states can be considered. Another future work involves the consideration of channel switching costs (delays) in the design of optimal channel switching strategies.

Chapter 3

Average Capacity Maximization via Channel Switching in the Presence of Additive White Gaussian Noise Channels and Switching Delays

In this chapter, the optimal channel switching problem is proposed for average capacity maximization in the presence of not only average and peak power constraints but also channel switching delays (costs) [43]. The major contributions of this chapter can be summarized as follows:

- The channel switching problem for average capacity maximization in the presence of channel switching delays is studied for the first time in the literature.
- An alternative optimization problem, which facilitates theoretical investigations, is formulated in terms of the number of channels employed in the channel switching process.

- When the channel switching is to be performed among a certain number of channels, the optimal strategy and the corresponding average capacity are derived.

- It is shown that channel switching among more than two different channels is not optimal, and an expression for the maximum average capacity of the optimal channel switching strategy is presented.

- Conditions are specified for the cases in which the optimal strategy corresponds to the exclusive use of a single channel or to channel switching between two channels.

This chapter is organized as follows: Section 3.1 presents the system model and the problem formulation for optimal channel switching in the presence of channel switching delays. In Section 3.2, the solution of the optimal channel switching problem including switching delays is investigated and theoretical results are provided about the characteristics of the optimal channel switching strategy. Numerical examples are presented in Section 3.3 for validating the theoretical results, and the presented results are extended in various directions in Section 3.4. Finally, concluding remarks are provided in Section 3.5.

3.1 System Model and Problem Formulation

Consider a communication system in which K different channels are available in the communication link between a transmitter and a receiver. The channels are assumed to introduce independent additive Gaussian noise with constant spectral density levels over the channel bandwidths.[1] It is assumed that the spectral density levels and the bandwidths of the channels can be different in general. The transmitter and the receiver can switch among these K channels in order

[1]The additive Gaussian channel is an accurate model in the presence of thermal noise. In addition, it can also be employed in the presence of interference and jamming if they can be approximated by a Gaussian distribution; e.g., multiuser interference due to a large number of users with similar power levels and Gaussian jamming [77]-[79].

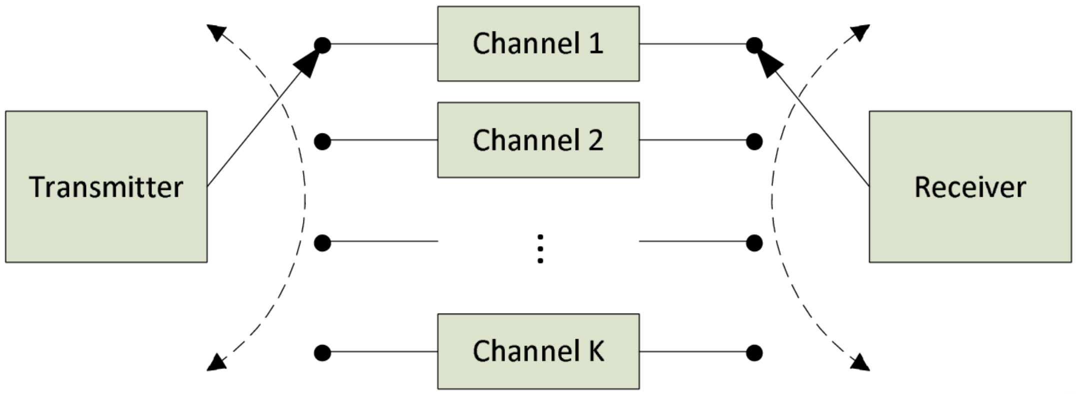

Figure 3.1: Block diagram of a communication system in which transmitter and receiver can switch among K channels.

to enhance the capacity of the communication system. At any given time, only one channel can be utilized for the transmission and the transmitter informs the receiver about which channel is occupied for the given time so that the transmitter and the receiver are synchronized [6], [33]. Fig. 3.1 illustrates the system with K different channels with possibly various bandwidths and noise levels. In practice, the transmitter can perform communication over one channel for a certain fraction of time; then, it switches to another channel and continues communication for another fraction of time, and so on. This scenario is applicable for cognitive radio systems in which a secondary user utilizes multiple available frequency bands that are not in use by primary users [68, 69]. Hence, secondary users can improve their average channel capacity by employing the channel switching strategy proposed in this study.

The main motivation behind the use of a single channel at a time is to realize a system with low cost/complexity. Since the channels considered in the system model in Fig. 3.1 have different center frequencies which can be dispersed over a wide range of frequencies in general (e.g., in cognitive radio systems [68, 69]), simultaneous utilization of multiple channels requires either multiple RF units (one for each channel) at the transmitter and the receiver, or single RF units that operate over the whole possible range of frequencies (i.e., over a very wide bandwidth)[2]. Therefore, simultaneous utilization of multiple channels leads to high complexity/cost compared to the use of one channel at a time. In the latter case, the single RF units at the transmitter and the receiver can be designed for a relatively narrowband scenario, and only one channel is used at a time by tuning the filters and amplifiers in the RF units and adjusting the upconversion/downconversion frequency according to the employed channel [80, 81].

In fact, if the frequency bands of two channels are adjacent to each other, they can be treated as a single channel with a larger bandwidth if the total bandwidth is within the operating range of the RF components. Hence, the theoretical analysis in this study is also valid for scenarios in which two (multiple) such frequency bands (channels) are used simultaneously. In that case, all the theoretical results would hold by updating the definitions of the channels.

In the considered system model, before data communication commences, the transmitter determines a channel switching strategy that will be employed during a time duration of T_d seconds and informs the receiver about the channels to be utilized and the respective utilization times according to that strategy. It is assumed that the channel characteristics do not change during T_d seconds. To start data communication, the transmitter and the receiver set their parameters for the first channel to be utilized (i.e., they switch to the same channel), and this process is assumed to take a time duration T_{cs} seconds, which is called the *channel switching delay (cost)*. During T_{cs} seconds, there is no data communication and consequently no power is transmitted. Then, data transmission starts and lasts for a certain time duration based on the employed strategy. Next, the transmitter and the receiver switch to the second channel to be utilized, which again takes T_{cs} seconds, and then data communication occurs over that channel for a specified time. The process continues in this manner according to the employed channel switching strategy, which may utilize a subset of all channels in general. For the next period of T_d seconds, the optimal channel switching strategy is calculated again according to the new channel characteristics, and communication continues in the same fashion as described above.

[2]In this case, very high rates would be required for analog-to-digital converters, which would lead to increased cost and high power consumption.

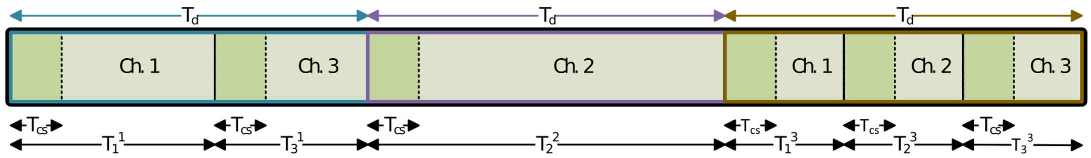

Figure 3.2: A sample time frame structure of a communication system in which transmitter and receiver can switch among 4 channels.

In Fig. 3.2, a sample time frame structure is presented for channel switching over 4 channels. In this case, the transmitter and the receiver communicate during $3T_d$ seconds. In first T_d seconds, the channel switching strategy is to communicate over channel 1 and channel 3 for T_1^1 and T_3^1 seconds, respectively, where $T_1^1 + T_3^1 = T_d$. Before the data transmission over each channel, there exists a channel switching time (cost) of T_{cs} seconds, which is required for the transmitter and the receiver to set their parameters for communication over the desired channel. During the second T_d seconds, the communication is performed over only channel 2 for a time duration of T_2^2 seconds, where $T_2^2 = T_d$, and there is no channel switching to another channel in this case. Finally, channels 1, 2 and 3 are utilized for the communication in the last T_d seconds. It is important to note that it is not necessary to utilize all the channels in a given channel switching strategy. For example, channel 4 is not utilized in any of the channel switching strategies in Fig. 3.2.

Let B_i and $N_i/2$ denote, respectively, the bandwidth and the constant power spectral density level of the additive Gaussian noise for channel i, where $i \in \{1, \ldots, K\}$. Then, the capacity of channel i is expressed as

$$C_i(P) = B_i \log_2 \left(1 + \frac{P}{N_i B_i}\right) \text{ bits/sec} \tag{3.1}$$

where P represents the average transmit power [72].

The main aim of this study is to characterize the optimal channel switching strategy that maximizes the average capacity of the communication system in Fig. 3.1 under average and peak power constraints and in the presence of channel switching delays. To that aim, channel time-sharing (channel switching) factors are expressed as $\lambda_1 \triangleq \frac{T_1}{T_d}, \ldots, \lambda_K \triangleq \frac{T_K}{T_d}$, where T_i denotes the amount of time allocated for channel i and T_d is the duration over which the channel switching strategy is employed. In addition, $\varepsilon \triangleq \frac{T_{cs}}{T_d}$ is defined as the *channel switching delay factor*, and $(\lambda_i - \varepsilon)\mathbb{I}_{\{\lambda_i > 0\}}$ represents the fraction of time when channel i is used for communication, where $\mathbb{I}_{\{\lambda_i > 0\}}$ denotes the indicator function, which is equal to 1 if $\lambda_i > 0$ and 0 otherwise. Then, the following optimal channel switching problem is proposed for capacity maximization in the presence of channel switching delays:

$$
\begin{aligned}
\max_{\{\lambda_i, P_i\}_{i=1}^K} \quad & \sum_{i=1}^{K} \mathbb{I}_{\{\lambda_i > 0\}} \, (\lambda_i - \varepsilon) \, C_i(P_i) \\
\text{subject to} \quad & \sum_{i=1}^{K} \mathbb{I}_{\{\lambda_i > 0\}} \, (\lambda_i - \varepsilon) P_i \leq P_{\text{av}} \, , \\
& P_i \in [0, P_{\text{pk}}] \, , \ \forall i \in \{1, \ldots, K\} \, , \\
& \sum_{i=1}^{K} \lambda_i = 1 \, , \ \lambda_i \in \{0\} \cup [\varepsilon, 1] \, , \ \forall i \in \{1, \ldots, K\}
\end{aligned}
\tag{3.2}
$$

where $C_i(P_i)$ is as in (3.1), P_i is the average transmit power allocated to channel i, P_{pk} denotes the peak power limit, and P_{av} represents the average power limit for the transmitter. It is assumed that $P_{\text{av}} < P_{\text{pk}}$ and $0 < \varepsilon < 1$. From (3.2), it is noted that due to the channel switching delay, a channel can be utilized only if its time-sharing factor is larger than or equal to the channel switching delay factor, ε. In addition, ε fractions are subtracted from both the average capacity and the average power terms since no data transmission occurs during channel switching. It should be emphasized that the objective function in (3.2) is referred to as the "average" capacity due to the averaging operation over time, considering the use of different channels and the channel switching delays.

For convenience, the symbols that are frequently used throughout the study are summarized in Table 3.1.

Table 3.1: Symbols and their definitions

Symbol	Definition
K	Number of channels in the system
B_i	Bandwidth of channel i
N_i	Noise power spectral density level for channel i
ε	Channel switching delay factor
P_{pk}	Peak power limit
P_{av}	Average power limit
P_i	Average transmit power allocated to channel i
$C_i(P)$	Capacity of channel i for average power P

3.2 Optimal Channel Switching With Switching Delays

In its current form, the optimization problem in (3.2) is difficult to solve in general since it requires a search over a $2K$ dimensional space. Therefore, our aim is to derive an equivalent formulation of the problem in (3.2), which leads to a low-complexity solution for the optimal channel switching strategy. To achieve such a formulation, the optimization problem in (3.2) is first converted into another problem, the solution of which achieves the same maximum average capacity as (3.2) does. In the following proposition, this alternative optimization problem is presented.

Proposition 1: *Define set A as $A = \{1, \ldots, K\}$ and let $\mathcal{P}(A)$ denote the power set of set A. Then, the solution of the following optimization problem results in the same maximum value that is achieved by the problem in* (3.2)*:*

$$\begin{aligned}
&\max_{\tilde{K}\in A}\ \max_{S\in B^{\tilde{K}}}\ \max_{\{\nu_{s_i},P_{s_i}\}_{i=1}^{\tilde{K}}}\ \sum_{i=1}^{\tilde{K}}(\nu_{s_i}-\varepsilon)\,C_{s_i}(P_{s_i})\\
&\text{subject to}\ \sum_{i=1}^{\tilde{K}}(\nu_{s_i}-\varepsilon)P_{s_i}\leq P_{\text{av}}\\
&\qquad\qquad P_{s_i}\in[0,P_{\text{pk}}]\,,\ \forall i\in\{1,\ldots,\tilde{K}\}\\
&\qquad\qquad \sum_{i=1}^{\tilde{K}}\nu_{s_i}=1\,,\ \nu_{s_i}\geq\varepsilon\,,\ \forall i\in\{1,\ldots,\tilde{K}\}
\end{aligned}\tag{3.3}$$

where s_i represents the ith element of set S, and $B^{\tilde{K}}$ is defined as

$$B^{\tilde{K}} \triangleq \{\chi \in \mathcal{P}(A) \mid |\chi| = \tilde{K}\} \tag{3.4}$$

for $\tilde{K} \in \{1, \ldots, K\}$, with $|\chi|$ denoting the cardinality of set χ.

Proof: Let $\{\lambda_i^*, P_i^*\}_{i=1}^K$ represent the solution of (3.2) and define C^* as the maximum average capacity achieved by the optimization problem in (3.2); that is,

$$C^* = \sum_{i=1}^{K} \mathbb{I}_{\{\lambda_i^*>0\}} \left(\lambda_i^* - \varepsilon\right) C_i(P_i^*) \ . \tag{3.5}$$

Also, define a set as

$$M \triangleq \{l \in \{1, \ldots, K\} \mid \lambda_l^* > 0\} \tag{3.6}$$

which consists of the channel indices with nonzero (positive) time-sharing factors. Next, consider the following transformation:

$$\nu_{m_i}^* = \lambda_{m_i}^*, \ \bar{P}_{m_i}^* = P_{m_i}^* \ , \ \forall i \in \{1, \ldots, |M|\} \tag{3.7}$$

where m_i represents the ith element of M, and $|M|$ is the cardinality of set M. Then, the following relations can be obtained for C^*:

$$\begin{aligned} C^* &= \sum_{i=1}^{K} \mathbb{I}_{\{\lambda_i^*>0\}} \left(\lambda_i^* - \varepsilon\right) C_i(P_i^*) \\ &= \sum_{m \in M} \left(\lambda_m^* - \varepsilon\right) C_m(P_m^*) \qquad (3.8) \\ &= \sum_{i=1}^{|M|} (\nu_{m_i}^* - \varepsilon)\, C_{m_i}(\bar{P}_{m_i}^*) \qquad (3.9) \end{aligned}$$

where the equalities in (3.8) and (3.9) are obtained from the definitions in (3.6) and (3.7), respectively. Next, define $\tilde{K}^*$ as $\tilde{K}^* \triangleq |M|$ and S^* as $S^* \triangleq M$. Then, the relation in (3.9) implies that the optimization problem in (3.3) achieves C^*

for $\tilde{K}^*$, S^*, and $\{\nu^*_{s^*_i}, \bar{P}^*_{s^*_i}\}_{i=1}^{\tilde{K}^*}$ (see (3.7)), where s^*_i denotes the ith element of S^*.[3] Hence, (3.3) is guaranteed to yield the maximum average capacity achieved by the optimization problem in (3.2), that is, $C^* \leq C^\diamond$, where $C^\diamond$ represents the maximum average capacity achieved by (3.3).

Next, suppose that $\tilde{K}^\diamond$, $S^\diamond$, and $\{\nu^\diamond_{s^\diamond_i}, \bar{P}^\diamond_{s^\diamond_i}\}_{i=1}^{\tilde{K}^\diamond}$ denote the solution of the optimization problem in (3.3), where $s^\diamond_i$ denotes the ith element of $S^\diamond$. Consider the following functions that map the solution set of the problem in (3.3) to the possible solution set of the problem in (3.2):

$$\lambda_i^\diamond = \begin{cases} \nu_i^\diamond, & \text{if } i \in S^\diamond \\ 0, & \text{otherwise} \end{cases}, \ \forall i \in \{1, \ldots, K\} \tag{3.10}$$

$$P_i^\diamond = \begin{cases} \bar{P}_i^\diamond, & \text{if } i \in S^\diamond \\ 0, & \text{otherwise} \end{cases}, \ \forall i \in \{1, \ldots, K\} \tag{3.11}$$

Then, the following relations can be written for $C^\diamond$:

$$\begin{aligned} C^\diamond &= \sum_{i=1}^{\tilde{K}^\diamond} (\nu^\diamond_{s^\diamond_i} - \varepsilon)\, C_{s^\diamond_i}(\bar{P}^\diamond_{s^\diamond_i}) \\ &= \sum_{m \in S^\diamond} (\nu^\diamond_m - \varepsilon)\, C_m(\bar{P}^\diamond_m) & (3.12) \\ &= \sum_{m \in S^\diamond} (\lambda^\diamond_m - \varepsilon)\, C_m(P^\diamond_m) & (3.13) \\ &= \sum_{i=1}^{K} \mathbb{I}_{\{i \in S^\diamond\}}\, (\lambda^\diamond_i - \varepsilon)\, C_i(P^\diamond_i) & (3.14) \\ &= \sum_{i=1}^{K} \mathbb{I}_{\{\lambda^\diamond_i > 0\}}\, (\lambda^\diamond_i - \varepsilon)\, C_i(P^\diamond_i) & (3.15) \end{aligned}$$

where the equality in (3.12) is due to the definition of set S^* (see (3.3)), the equalities in (3.13) and (3.14) follow from the mapping functions in (3.10) and (3.11), and (3.15) is obtained from the fact that $\lambda_i^\diamond > 0$ only for $i \in S^\diamond$. Based on the transformations defined in (3.10) and (3.11), $\{\lambda_i^\diamond, P_i^\diamond\}_{i=1}^K$ satisfies the constraints

[3]Note that the constraints in (3.3) are satisfied for $\tilde{K}^*$, S^*, and $\{\nu^*_{s^*_i}, \bar{P}^*_{s^*_i}\}_{i=1}^{\tilde{K}^*}$.

in (3.2) and the relation in (3.12)-(3.15) implies that (3.2) yields the average capacity of $C^{\diamond}$ for $\{\lambda_i^{\diamond}, P_i^{\diamond}\}_{i=1}^{K}$; hence, it is concluded that $C^{\diamond} \leq C^*$. Overall, it is concluded that $C^{\diamond} = C^*$ must hold in order to satisfy both $C^* \leq C^{\diamond}$ and $C^{\diamond} \leq C^*$. ■

In the optimization problem in (3.3), parameter $\tilde{K}$ indicates the number of employed channels in a channel switching strategy; that is, the optimization is performed for all possible numbers of employed channels explicitly. In this way, the indicator functions in (3.2) are removed. Since there exist K available channels in the system, the optimization problem in (3.3) requires a search over all possible values of $\tilde{K} \in A$, where $A = \{1, \ldots, K\}$. For each $\tilde{K}$, set $B^{\tilde{K}}$ in (3.4) consists of the sets that are subsets of set A with $\tilde{K}$ elements; that is, $B^{\tilde{K}}$ corresponds to all possible $\tilde{K}$ combinations of K different channels. Hence, $B^{\tilde{K}}$ consists of $\binom{K}{\tilde{K}}$ sets. For example, if $K = 3$ and $\tilde{K} = 2$, then $B^{\tilde{K}} = \{\{1,2\},\{1,3\},\{2,3\}\}$. For each element of $B^{\tilde{K}}$, which is denoted by S in (3.3), the optimization is performed over $\{\nu_{s_i}, P_{s_i}\}_{i=1}^{\tilde{K}}$, where s_i selects the ith channel in S and ν_{s_i} and P_{s_i} denote, respectively, the time-sharing factor and the average transmit power allocated to channel s_i; i.e., the ith employed (selected) channel.

The optimization problem in (3.3) is not only more convenient than the one in (3.2), which involves indicator functions, but also leads to simpler formulations of the optimal channel switching problem. To that end, the following proposition provides a scaled and more compact version of the optimization problem in (3.3), the solution of which achieves the same maximum average capacity as (3.2) and (3.3) do.

Proposition 2: *The optimization problem in* (3.3) *can be expressed in the* *form of the following optimization problem:*

$$\max_{\tilde{K}\in A}\max_{S\in B^{\tilde{K}}}\max_{\{\mu_{s_i},P_{s_i}\}_{i=1}^{\tilde{K}}}\left(1-\tilde{K}\varepsilon\right)\sum_{i=1}^{\tilde{K}}\mu_{s_i}C_{s_i}(P_{s_i})$$

$$\text{subject to }\sum_{i=1}^{\tilde{K}}\mu_{s_i}P_{s_i}\leq\frac{P_{\text{av}}}{\left(1-\tilde{K}\varepsilon\right)}$$

$$P_{s_i}\in[0,P_{\text{pk}}]\,,\ \forall i\in\{1,\ldots,\tilde{K}\}$$

$$\sum_{i=1}^{\tilde{K}}\mu_{s_i}=1\,,\ \mu_{s_i}\geq 0\,,\ \forall i\in\{1,\ldots,\tilde{K}\}$$

$$\tilde{K}<\frac{1}{\varepsilon}\tag{3.16}$$

where A, $B^{\tilde{K}}$, and s_i are as defined in Proposition 1.

Proof: Consider the optimization problem in (3.3) and define new variables γ_{s_i} as $\gamma_{s_i}\triangleq\nu_{s_i}-\varepsilon\,,\ \forall i\in\{1,\ldots,\tilde{K}\}$. Then, the problem in (3.3) can be written as follows:

$$\max_{\tilde{K}\in A}\max_{S\in B^{\tilde{K}}}\max_{\{\gamma_{s_i},P_{s_i}\}_{i=1}^{\tilde{K}}}\sum_{i=1}^{\tilde{K}}\gamma_{s_i}C_{s_i}(P_{s_i})\tag{3.17}$$

$$\text{subject to }\sum_{i=1}^{\tilde{K}}\gamma_{s_i}P_{s_i}\leq P_{\text{av}}\tag{3.18}$$

$$P_{s_i}\in[0,P_{\text{pk}}]\,,\ \forall i\in\{1,\ldots,\tilde{K}\}\tag{3.19}$$

$$\sum_{i=1}^{\tilde{K}}\gamma_{s_i}=1-\tilde{K}\varepsilon\,,\ \gamma_{s_i}\geq 0\,,\ \forall i\in\{1,\ldots,\tilde{K}\}\tag{3.20}$$

It is noted from (3.20) that $1-\tilde{K}\varepsilon\geq 0$ should be satisfied since $\sum_{i=1}^{\tilde{K}}\gamma_{s_i}=1-\tilde{K}\varepsilon$ and $\gamma_{s_i}\geq 0\,,\ \forall i\in\{1,\ldots,\tilde{K}\}$. Suppose that $\tilde{K}^{\diamond}$, $S^{\diamond}$, and $\{\gamma^{\diamond}_{s^{\diamond}_i},P^{\diamond}_{s^{\diamond}_i}\}_{i=1}^{\tilde{K}^{\diamond}}$ denote the solution of (3.17)-(3.20) such that $1-\tilde{K}^{\diamond}\varepsilon=0$. Then, based on the constraint in (3.20), $\gamma^{\diamond}_{s^{\diamond}_i}=0\,,\ \forall i\in\{1,\ldots,\tilde{K}\}$, and consequently $\sum_{i=1}^{\tilde{K}^{\diamond}}\gamma^{\diamond}_{s_i}C_{s_i}(P^{\diamond}_{s_i})=0$. Also, $\tilde{K}^{\diamond}$ satisfies $\tilde{K}^{\diamond}>1$ since $K^{\diamond}=1/\varepsilon$ and $0<\varepsilon<1$ by assumption. Hence, more than one channel is available for channel switching. Now, consider an alternative solution, denoted by $\tilde{K}^{*}$, S^{*}, and $\{\gamma^{*}_{s^{*}_i},P^{*}_{s^{*}_i}\}_{i=1}^{\tilde{K}^{*}}$, where $\tilde{K}^{*}=1$,

$S^* = \{1\}$, $\gamma^*_{s_1} = 1-\varepsilon$, and $P^*_{s_1} = \min\left\{\frac{P_{\rm av}}{(1-\varepsilon)}, P_{\rm pk}\right\}$. Then, the alternative solution achieves an average capacity of $\sum_{i=1}^{\tilde{K}^*} \gamma^*_{s_i} C_{s_i}(P^*_{s_i}) = (1-\varepsilon)\, C_1\big(\min\left\{\frac{P_{\rm av}}{(1-\varepsilon)}, P_{\rm pk}\right\}\big)$, which is positive; hence, larger than the one achieved by $\tilde{K}^\diamond$, $S^\diamond$, and $\{\gamma^\diamond_{s_i^\diamond}, P^\diamond_{s_i^\diamond}\}_{i=1}^{\tilde{K}^\diamond}$. Therefore, $\tilde{K}^\diamond$, $S^\diamond$, and $\{\gamma^\diamond_{s_i^\diamond}, P^\diamond_{s_i^\diamond}\}_{i=1}^{\tilde{K}^\diamond}$ with $1-\tilde{K}^\diamond\varepsilon = 0$ cannot be optimal, which contradicts with the initial assumption. Hence, the solution of (3.17) must satisfy $1-\tilde{K}\varepsilon > 0$. Based on this inequality, μ_{s_i} is defined as follows:

$$\mu_{s_i} \triangleq \gamma_{s_i}/(1-\tilde{K}\varepsilon) \tag{3.21}$$

for $i \in \{1, \ldots, \tilde{K}\}$. Thus, the optimization problem in (3.16) can be obtained by substituting the new variables defined in (3.21) into the optimization problem in (3.17)-(3.20). ■

The optimization problem in (3.16) can be separated into two optimization problems based on the value of $\tilde{K}$ as follows:

- **Case-1 (Single Channel):** For the case in which a single channel is employed for communication, that is, $\tilde{K} = 1$, the optimization problem in (3.16) can be stated as follows:

$$\begin{aligned} \max_{S\in B^1} \max_{\mu_{s_1}, P_{s_1}} \quad & (1-\varepsilon)\,\mu_{s_1} C_{s_1}(P_{s_1}) \\ \text{subject to } & \mu_{s_1} P_{s_1} \leq \frac{P_{\rm av}}{(1-\varepsilon)} \\ & P_{s_1} \in [0, P_{\rm pk}] \\ & \mu_{s_1} = 1\,,\ \mu_{s_1} \geq 0 \\ & \varepsilon < 1 \end{aligned} \tag{3.22}$$

  where $B^1 = \{\{1\}, \{2\}, \ldots, \{K\}\}$ and s_1 denotes the (first) element of S. The optimization problem in (3.22) achieves the maximum average capacity that can be obtained by employing a single channel during data communication. This approach corresponds to the case of no channel switching and is easily solvable by using simple algebra. Let $C_{\rm scs}$ denote the solution of (3.22). Then, the achieved maximum capacity via the optimal *single* *channel strategy* can be expressed as

$$C_{\text{scs}} = \max_{l \in \{1,\ldots,K\}} \ (1-\varepsilon)\, C_l \left(\min\left\{ \frac{P_{\text{av}}}{(1-\varepsilon)}, P_{\text{pk}} \right\} \right) \tag{3.23}$$

and the channel index m employed in this strategy can be obtained as

$$m = \arg\max_{l \in \{1,\ldots,K\}} C_l \left(\min\left\{ \frac{P_{\text{av}}}{(1-\varepsilon)}, P_{\text{pk}} \right\} \right). \tag{3.24}$$

In the optimal single channel strategy, it is optimal to use all the available and attainable power, $\min\left\{\frac{P_{\text{av}}}{(1-\varepsilon)}, P_{\text{pk}}\right\}$ over a single channel since $C_i(P)$ in (3.1) is a monotone increasing and continuous function.

- **Case-2 (Channel Switching):** Consider the optimization problem in (3.16) in the presence of channel switching; that is, $\tilde{K} \geq 2$. Then, the following optimization problem is obtained:

$$\begin{aligned} C_{\text{css}} = \max_{\tilde{K} \in A \setminus \{1\}} \max_{S \in B^{\tilde{K}}} \max_{\{\mu_{s_i}, P_{s_i}\}_{i=1}^{\tilde{K}}} & \left(1 - \tilde{K}\varepsilon\right) \sum_{i=1}^{\tilde{K}} \mu_{s_i}\, C_{s_i}(P_{s_i}) \\ \text{subject to } & \sum_{i=1}^{\tilde{K}} \mu_{s_i} P_{s_i} \leq \frac{P_{\text{av}}}{\left(1 - \tilde{K}\varepsilon\right)} \\ & P_{s_i} \in [0, P_{\text{pk}}]\,,\ \forall i \in \{1,\ldots,\tilde{K}\} \\ & \sum_{i=1}^{\tilde{K}} \mu_{s_i} = 1\,,\ \mu_{s_i} \geq 0\,,\ \forall i \in \{1,\ldots,\tilde{K}\} \\ & \tilde{K} < \frac{1}{\varepsilon} \end{aligned} \tag{3.25}$$

The solution of the optimization problem in (3.25) results in the maximum average capacity that can be achieved by employing at least two different channels. In general, it is difficult to obtain the solution of (3.25). Therefore, further analysis is performed in the remainder of this study to obtain the optimal solution of (3.25) with low computational complexity.

Based on Case-1 and Case-2, the solution of (3.16) corresponds to either the single channel strategy or the channel switching strategy. Let $C_{\rm scs}$ and $C_{\rm css}$ denote the solutions of the optimization problems in (3.23) and (3.25), respectively. Then, the solution of (3.16) can be calculated as

$$\max\left\{C_{\rm scs}, C_{\rm css}\right\}. \tag{3.26}$$

As discussed in Case-1, the optimal single channel strategy has a simple closed-form solution. However, it is difficult to solve the channel switching problem in the form of (3.25). Therefore, the following proposition is presented to simplify the optimization problem in (3.25).

Proposition 3: *Assume that $\bar{K} \geq 2$ channels are employed in the channel switching strategy and $\varepsilon < 1/\bar{K}$ holds. Then, the maximum average capacity achieved via the optimal channel switching strategy over $\bar{K}$ channels can be expressed as*

$$\psi(\bar{K}) = \begin{cases} \max\limits_{\substack{\tilde{P}_1 \in \left[\frac{P_{\rm av}}{1-\bar{K}\varepsilon}, P_{\rm pk}\right] \\ \tilde{P}_2 \in \left[0, \frac{P_{\rm av}}{1-\bar{K}\varepsilon}\right)}} \left(1-\bar{K}\varepsilon\right) \left(\frac{\frac{P_{\rm av}}{1-\bar{K}\varepsilon} - \tilde{P}_2}{\tilde{P}_1 - \tilde{P}_2} C_{\max}(\tilde{P}_1) \right. \\ \qquad\qquad \left. + \frac{\tilde{P}_1 - \frac{P_{\rm av}}{1-\bar{K}\varepsilon}}{\tilde{P}_1 - \tilde{P}_2} C_{\max}(\tilde{P}_2) \right), & \text{if } \frac{P_{\rm av}}{1-\bar{K}\varepsilon} < P_{\rm pk} \\ \left(1-\bar{K}\varepsilon\right) C_{\max}(P_{\rm pk}), & \text{otherwise} \end{cases} \tag{3.27}$$

where $C_{\max}(P)$ is defined as

$$C_{\max}(P) \triangleq \max\{C_1(P), \ldots, C_K(P)\}\,. \tag{3.28}$$

Proof: Under the assumption in the proposition, the optimization problem in (3.25) can be expressed for $\bar{K}$ channels as follows:

$$\begin{aligned}
&\max_{S\in B^{\bar{K}}} \max_{\{\mu_{s_i},P_{s_i}\}_{i=1}^{\bar{K}}} \left(1-\bar{K}\varepsilon\right)\sum_{i=1}^{\bar{K}} \mu_{s_i}\, C_{s_i}(P_{s_i}) \\
&\text{subject to } \sum_{i=1}^{\bar{K}} \mu_{s_i} P_{s_i} \leq \frac{P_{\text{av}}}{\left(1-\bar{K}\varepsilon\right)} \\
&\qquad P_{s_i} \in [0, P_{\text{pk}}]\,,\ \forall i \in \{1,\ldots,\bar{K}\} \\
&\qquad \sum_{i=1}^{\bar{K}} \mu_{s_i} = 1\,,\ \mu_{s_i} \geq 0\,,\ \forall i \in \{1,\ldots,\bar{K}\}
\end{aligned} \tag{3.29}$$

Then, based on a similar approach to that in Proposition 1 of [33], the problem in (3.29) can be stated as

$$\begin{aligned}
&\max_{S\in B^{\bar{K}}} \max_{\{\mu_{s_i},P_{s_i}\}_{i=1}^{\bar{K}}} \left(1-\bar{K}\varepsilon\right)\sum_{i=1}^{\bar{K}} \mu_{s_i}\, C^{S}_{\max}(P_{s_i}) \\
&\text{subject to } \sum_{i=1}^{\bar{K}} \mu_{s_i} P_{s_i} \leq \frac{P_{\text{av}}}{\left(1-\bar{K}\varepsilon\right)} \\
&\qquad P_{s_i} \in [0, P_{\text{pk}}]\,,\ \forall i \in \{1,\ldots,\bar{K}\} \\
&\qquad \sum_{i=1}^{\bar{K}} \mu_{s_i} = 1\,,\ \mu_{s_i} \geq 0\,,\ \forall i \in \{1,\ldots,\bar{K}\}
\end{aligned} \tag{3.30}$$

where $C^{S}_{\max}(P)$ is defined as

$$C^{S}_{\max}(P) \triangleq \max_{m\in S} C_m(P)\ . \tag{3.31}$$

That is, since the optimal solution involves the use of the best channel (among the given set of channels) for each power level (cf. (3.31)), the problem in (3.29) can be solved based on (3.30).

It is noted from (3.30) that, for each S, the aim is to find the optimal $\{\mu_{s_i}, P_{s_i}\}_{i=1}^{\bar{K}}$ for maximizing the convex combination of the $C^{S}_{\max}(P_{s_i})$ terms subject to the constraints on the average and peak powers. This formulation for each S has the same form as the problem formulation in eqn. (3) of [33]; hence, similar to Proposition 4 in [33], it can be shown that the optimal $\{\mu_{s_i}, P_{s_i}\}_{i=1}^{\bar{K}}$ has at most two nonzero μ_{s_i} for each S (i.e., channel switching between at most two different channels is optimal for each S). Therefore, the problem in (3.30) can be expressed as follows:

$$\max_{S\in B^{\bar{K}}} \max_{\mu,\tilde{P}_1,\tilde{P}_2} (1-\bar{K}\varepsilon)\left(\mu\, C^S_{\max}(\tilde{P}_1) + (1-\mu) C^S_{\max}(\tilde{P}_2)\right) \tag{3.32}$$

$$\text{subject to } \mu\,\tilde{P}_1 + (1-\mu)\tilde{P}_2 \leq \frac{P_{\text{av}}}{1-\bar{K}\varepsilon} \tag{3.33}$$

$$\tilde{P}_1 \in [0, P_{\text{pk}}],\ \ \tilde{P}_2 \in [0, P_{\text{pk}}] \tag{3.34}$$

$$\mu \in [0,1] \tag{3.35}$$

where $\tilde{P}_1$ and $\tilde{P}_2$ denote the average transmit powers allocated to channel i and channel j, respectively, with $i = \arg\max_{l\in S} C_l(\tilde{P}_1)$ and $j = \arg\max_{l\in S} C_l(\tilde{P}_2)$.

It is noted that $C^S_{\max}$ in (3.32) is maximized with respect to set S, and S does not depend on the other parameters, μ, $\tilde{P}_1$, and $\tilde{P}_2$. Therefore, the maximization with respect to S can be considered first for simplifying the problem in (3.32)-(3.35). For that purpose, the following expressions are obtained for $C^S_{\max}$:

$$\max_{S\in B^{\bar{K}}} C^S_{\max}(P) = \max_{S\in B^{\bar{K}}} \max_{m\in S} C_m(P) \tag{3.36}$$

$$= \max_{l\in\{1,\dots,K\}} C_l(P) \tag{3.37}$$

$$= C_{\max}(P) \tag{3.38}$$

where (3.36) follows from the definition of $C^S_{\max}$ in (3.31), (3.37) is obtained based on the definition of $B^{\bar{K}}$ in (3.4), and finally (3.38) is due to (3.28). Based on (3.36)-(3.38), the problem in (3.32)-(3.35) can be stated as follows:

$$\max_{\mu,\tilde{P}_1,\tilde{P}_2} (1-\bar{K}\varepsilon)\left(\mu\, C_{\max}(\tilde{P}_1) + (1-\mu) C_{\max}(\tilde{P}_2)\right) \tag{3.39}$$

$$\text{subject to } \mu\,\tilde{P}_1 + (1-\mu)\tilde{P}_2 \leq \frac{P_{\text{av}}}{1-\bar{K}\varepsilon} \tag{3.40}$$

$$\tilde{P}_1 \in [0, P_{\text{pk}}],\ \ \tilde{P}_2 \in [0, P_{\text{pk}}] \tag{3.41}$$

$$\mu \in [0,1] \tag{3.42}$$

where $\tilde{P}_1$ and $\tilde{P}_2$ denote the average transmit powers allocated to channel i and channel j, respectively, with $i = \arg\max_{l\in\{1,\ldots,K\}} C_l(\tilde{P}_1)$ and $j = \arg\max_{l\in\{1,\ldots,K\}} C_l(\tilde{P}_2)$.

Next, consider the optimization problem in (3.39)-(3.42) for $\frac{P_{\rm av}}{\left(1-\bar{K}\varepsilon\right)} < P_{\rm pk}$. Similarly to Lemma 1 in [33], it is obtained that the optimal μ, $\tilde{P}_1$, and $\tilde{P}_2$ satisfy the average power constraint with equality; that is, $\mu\,\tilde{P}_1+(1-\mu)\tilde{P}_2 = \frac{P_{\rm av}}{1-\bar{K}\varepsilon}$. Then, by considering (3.40) as an equality constraint and substituting the constraints in (3.40)-(3.42) into the objective function and specifying the search space, it is obtained that the achieved capacity for $\frac{P_{\rm av}}{1-\bar{K}\varepsilon} < P_{\rm pk}$ can be calculated by solving the optimization problem in (3.27). Otherwise, i.e., if $\frac{P_{\rm av}}{1-\bar{K}\varepsilon} \geq P_{\rm pk}$, then the solution of the optimization problem in (3.39)-(3.42) can easily be obtained as $\left(1-\bar{K}\varepsilon\right) C_{\max}(P_{\rm pk})$. ■

Remark 1: *For the case of* $P_{\rm av}/(1-\bar{K}\varepsilon) \geq P_{\rm pk}$ *in* (3.27)*, the average capacity of* $\left(1-\bar{K}\varepsilon\right) C_{\max}(P_{\rm pk})$ *can be achieved by the following approach: First, switching to the best channel that achieves the maximum capacity for power level* $P_{\rm pk}$ *and transmitting at power level* $P_{\rm pk}$ *over that channel*[4] *for a time fraction of* $\left(1-\bar{K}\varepsilon\right)$*; then, switching among any* $(\bar{K}-1)$ *channels, except for the best channel, without transmitting any power (i.e., by only consuming a time fraction of* ε *for each channel). As will be proved towards the end of this section, it is always better to employ a single channel and not to perform channel switching in the case of* $P_{\rm av}/(1-\bar{K}\varepsilon) \geq P_{\rm pk}$*. Hence, the solution of the optimal channel switching problem in* (3.16) *does not correspond to* $\left(1-\bar{K}\varepsilon\right) C_{\max}(P_{\rm pk})$ *for* $\bar{K} \geq 2$*. Therefore, the approach in this remark is optimal only under the condition that* $\bar{K} \geq 2$ *channels are employed, but not optimal for the overall problem in* (3.16)*.*

Proposition 3 provides a significant simplification for the solution of the optimization problem in (3.25) and leads to the following formulation for the optimal

[4]In the case of multiple channels that achieve the maximum capacity for power level $P_{\rm pk}$, any of them can be chosen as the best channel.

channel switching strategy (Case-2):

$$\max_{\tilde{K}\in A\setminus\{1\}} \quad \psi(\tilde{K})$$
$$\text{subject to } \tilde{K} < \frac{1}{\varepsilon} \tag{3.43}$$

where $\psi(\tilde{K})$ is as in (3.27). Compared to (3.25), the problem in (3.43) has significantly lower computational complexity since its search space is only two-dimensional for each feasible $\tilde{K}$ (see (3.27)) whereas a search over a $2\tilde{K}$ dimensional space is required in (3.25) for each $(\tilde{K}, S)$ pair.

Towards the aim of specifying the solution of (3.43), the following lemma is presented first, which states a useful inequality for $C_{\max}(\cdot)$ in (3.28).

Lemma 1: *Let $C_{\max}(P/\alpha)$ and $C_{\max}(P/\beta)$ denote the capacities of the best channels for power levels P/α and P/β, respectively, where $C_{\max}$ is as in* (3.28)*, α, $\beta \in (0,1)$ and $P > 0$. Then, the following inequality holds for $\alpha > \beta$:*

$$\alpha\, C_{\max}\left(\frac{P}{\alpha}\right) > \beta\, C_{\max}\left(\frac{P}{\beta}\right) \tag{3.44}$$

Proof: Let channel i and channel j denote the channels corresponding to the maximum capacities for power levels P/α and P/β, respectively; that is, $C_{\max}(P/\alpha) = C_i(P/\alpha)$ and $C_{\max}(P/\beta) = C_j(P/\beta)$ where $i = \arg\max_{l\in\{1,\ldots,K\}} C_l(P/\alpha)$ and $j = \arg\max_{l\in\{1,\ldots,K\}} C_l(P/\beta)$.

First, consider the case of $i = j$. Then, $C_{\max}(P/\alpha) = C_i(P/\alpha)$ and $C_{\max}(P/\beta) = C_i(P/\beta)$. Since the capacity curves are strictly concave and $C_i(P) = 0$ for $P = 0$, $\forall i \in \{1,\ldots,K\}$ (cf. (3.1)), the following relation can be obtained based on the definition of concavity:

$$\frac{\beta}{\alpha} C_i\left(\frac{P}{\beta}\right) + \left(1 - \frac{\beta}{\alpha}\right) C_i\left(0\right) < C_i\left(\frac{P}{\alpha}\right) \tag{3.45}$$

where $\beta/\alpha < 1$ as the statement in the lemma is for $\alpha > \beta$ and α, $\beta \in (0,1)$. Thus, it is obtained from (3.45) that $\beta\, C_{\max}\left(P/\beta\right) < \alpha\, C_{\max}\left(P/\alpha\right)$ as claimed in the lemma.

Next, consider the case of $i \neq j$. Since $C_{\max}(P/\alpha) = C_i(P/\alpha)$, $C_{\max}(P/\beta) = C_j(P/\beta)$, and C_i and C_j are monotone increasing and continuous functions, then there exists a single point $P/\gamma \in (P/\alpha, P/\beta)$ for $\beta < \gamma < \alpha$ at which the capacity curves of channel i and channel j intersect; that is, $C_i(P/\gamma) = C_j(P/\gamma)$. Now considering the capacity of channel j for power levels P/γ and P/β, it can be shown that $\beta\, C_j\,(P/\beta) < \gamma\, C_j\,(P/\gamma)$ based on a similar approach to that in (3.45). Similarly, for channel i, the following relation is obtained: $\gamma\, C_i\,(P/\gamma) < \alpha\, C_i\,(P/\alpha)$. Since $C_i(P/\gamma) = C_j(P/\gamma)$, these two inequalities imply that $\beta\, C_j\,(P/\beta) < \alpha\, C_i\,(P/\alpha)$, which is equivalent to $\beta\, C_{\max}\,(P/\beta) < \alpha\, C_{\max}\,(P/\alpha)$ as claimed in the lemma. ■

It is noted that although $C_{\max}$ in (3.28) is not a concave function in general (cf. Fig. 3.3), the inequality in (3.44) always holds due to the fact that the capacity curve for each channel is nonnegative, concave, monotone increasing, and continuous.

In the following proposition, a general solution for (3.43) is provided, and it is shown that the optimal channel switching strategy (Case-2) corresponds to switching between two of the channels.

Proposition 4: *The optimal channel switching strategy (Case-2) is to switch between two channels; that is, switching among more than two channels is not optimal. In addition, the maximum average capacity C_{css} achieved by the optimal channel switching strategy, which is obtained as the solution of* (3.43), *can be* *expressed as follows:*

$$C_{\text{css}} = \begin{cases} 0, & \text{if } \varepsilon \geq \frac{1}{2} \\ (1-2\varepsilon)C_{\max}(P_{\text{pk}}), & \text{if } \varepsilon < \frac{1}{2} \text{ and } \frac{P_{\text{av}}}{1-2\varepsilon} \geq P_{\text{pk}} \\ \max\limits_{\substack{\tilde{P}_1 \in \left[\frac{P_{\text{av}}}{1-2\varepsilon}, P_{\text{pk}}\right] \\ \tilde{P}_2 \in \left[0, \frac{P_{\text{av}}}{1-2\varepsilon}\right)}} (1-2\varepsilon)\left(\frac{\frac{P_{\text{av}}}{1-2\varepsilon} - \tilde{P}_2}{\tilde{P}_1 - \tilde{P}_2} C_{\max}(\tilde{P}_1) \right. & \\ \left. + \frac{\tilde{P}_1 - \frac{P_{\text{av}}}{1-2\varepsilon}}{\tilde{P}_1 - \tilde{P}_2} C_{\max}(\tilde{P}_2)\right), & \text{otherwise} \end{cases} \tag{3.46}$$

Proof: The aim is to prove that the statement in the proposition holds for all the cases specified in (3.46). Firstly, for $\varepsilon \geq \frac{1}{2}$, the constraint in (3.43) cannot be satisfied for any $\tilde{K}$, and consequently, channel switching is not feasible in this case. Therefore, if $\varepsilon \geq \frac{1}{2}$, the maximum average capacity via channel switching can be specified as $C_{\text{css}} = 0$.[5] Secondly, if $\varepsilon < \frac{1}{2}$ and $P_{\text{av}}/(1-2\varepsilon) \geq P_{\text{pk}}$, then the maximum average capacity achieved by performing optimal channel switching between two channels is obtained based on (3.27) as $\psi(2) = (1-2\varepsilon)C_{\max}(P_{\text{pk}})$. On the other hand, for optimal channel switching among $M > 2$ channels, the following arguments can be provided. Since $P_{\text{av}}/(1-2\varepsilon) \geq P_{\text{pk}}$ in this case, it is obtained that $P_{\text{av}}/(1-M\varepsilon) > P_{\text{pk}}$ for $M > 2$ and $M < 1/\varepsilon$, which is the constraint in (3.43). Then, it follows from (3.27) that $\psi(M) = (1-M\varepsilon)C_{\max}(P_{\text{pk}})$ for $M > 2$. Since $\psi(2) > \psi(M)$ for $M > 2$, it is concluded for $\varepsilon < \frac{1}{2}$ and $P_{\text{av}}/(1-2\varepsilon) \geq P_{\text{pk}}$ that the optimal channel switching strategy with two channels achieves a higher average capacity than the optimal channel switching strategies with more than two channels, and that the maximum average capacity achieved by the optimal channel switching strategy with two channels is equal to $C_{\text{css}} = (1-2\varepsilon)C_{\max}(P_{\text{pk}})$, as specified in (3.46). Finally, if $\varepsilon < \frac{1}{2}$ and $P_{\text{av}}/(1-2\varepsilon) < P_{\text{pk}}$, the maximum average capacity for the channel switching strategy with $\bar{K}$ channels

[5] In this case, the solution of the optimization problem in (3.16) corresponds to the optimal single channel strategy (Case-1).

can be obtained based on (3.27) as follows:[6]

$$\psi(\bar{K}) = \max_{\substack{\tilde{P}_1 \in \left[\frac{P_{\rm av}}{1-\bar{K}\varepsilon}, P_{\rm pk}\right] \\ \tilde{P}_2 \in \left[0, \frac{P_{\rm av}}{1-\bar{K}\varepsilon}\right)}} \left(1-\bar{K}\varepsilon\right) \left(\frac{\frac{P_{\rm av}}{1-\bar{K}\varepsilon} - \tilde{P}_2}{\tilde{P}_1 - \tilde{P}_2} C_{\max}(\tilde{P}_1) + \frac{\tilde{P}_1 - \frac{P_{\rm av}}{1-\bar{K}\varepsilon}}{\tilde{P}_1 - \tilde{P}_2} C_{\max}(\tilde{P}_2) \right) \tag{3.47}$$

Define $\bar{P}_1$ and $\bar{P}_2$ as $\bar{P}_1 \triangleq \left(1-\bar{K}\varepsilon\right) \tilde{P}_1$ and $\bar{P}_2 \triangleq \left(1-\bar{K}\varepsilon\right) \tilde{P}_2$. Then, $\psi(\bar{K})$ in (3.47) can be expressed as follows:

$$\psi(\bar{K}) = \max_{\substack{\bar{P}_1 \in [P_{\rm av}, (1-\bar{K}\varepsilon)P_{\rm pk}] \\ \bar{P}_2 \in [0, P_{\rm av})}} \left(1-\bar{K}\varepsilon\right) \left(\frac{P_{\rm av} - \bar{P}_2}{\bar{P}_1 - \bar{P}_2} \times C_{\max}\left(\frac{\bar{P}_1}{1-\bar{K}\varepsilon}\right) + \frac{\bar{P}_1 - P_{\rm av}}{\bar{P}_1 - \bar{P}_2} C_{\max}\left(\frac{\bar{P}_2}{1-\bar{K}\varepsilon}\right) \right). \tag{3.48}$$

For the optimal channel switching strategy with two channels, the maximum average capacity is given by

$$\psi(2) = \max_{\substack{\bar{P}_1 \in [P_{\rm av}, (1-2\varepsilon)P_{\rm pk}] \\ \bar{P}_2 \in [0, P_{\rm av})}} (1-2\varepsilon) \left(\frac{P_{\rm av} - \bar{P}_2}{\bar{P}_1 - \bar{P}_2} C_{\max}\left(\frac{\bar{P}_1}{1-2\varepsilon}\right) + \frac{\bar{P}_1 - P_{\rm av}}{\bar{P}_1 - \bar{P}_2} C_{\max}\left(\frac{\bar{P}_2}{1-2\varepsilon}\right) \right). \tag{3.49}$$

The aim is to prove that $\psi(2) > \psi(M)$ for $M > 2$, where $\psi(M)$ denotes the maximum average capacity achieved by optimal channel switching among $M > 2$ channels. To that aim, define a new optimization problem identical to (3.49) except that the search space for $\bar{P}_1$ is $[P_{\rm av}, (1-M\varepsilon)P_{\rm pk}]$ instead of $[P_{\rm av}, (1-2\varepsilon)P_{\rm pk}]$.

[6] The equation in (3.47) is valid if $P_{\rm av}/(1-\bar{K}\varepsilon) < P_{\rm pk}$. Otherwise, it is easy to prove that $\psi(2) > \psi(M)$ for $M > 2$ based on Lemma 1 since $\psi(2) \geq (1-2\varepsilon)C_{\max}(P_{\rm av}/(1-2\varepsilon)) > (1-M\varepsilon)C_{\max}(P_{\rm av}/(1-M\varepsilon)) \geq (1-M\varepsilon)C_{\max}(P_{\rm pk}) = \psi(M)$.

Let ξ denote the solution of this problem, which can be stated as follows:

$$\xi = \max_{\substack{\bar{P}_1 \in [P_{\text{av}}, (1-M\varepsilon)P_{\text{pk}}] \\ \bar{P}_2 \in [0, P_{\text{av}})}} (1-2\varepsilon) \left(\frac{P_{\text{av}} - \bar{P}_2}{\bar{P}_1 - \bar{P}_2} C_{\max}\left(\frac{\bar{P}_1}{1-2\varepsilon}\right) + \frac{\bar{P}_1 - P_{\text{av}}}{\bar{P}_1 - \bar{P}_2} C_{\max}\left(\frac{\bar{P}_2}{1-2\varepsilon}\right) \right). \tag{3.50}$$

The optimization problem in (3.50) is the same as the problem in (3.49) except that the search space for $\bar{P}_1$ in (3.50) is a subset of that in (3.49). Therefore, it is obtained that $\psi(2) \geq \xi$. Also, the following relations can be derived for $M > 2$ based on (3.44) in Lemma 1:

$$(1-2\varepsilon)\ C_{\max}\left(\frac{\bar{P}_1}{1-2\varepsilon}\right) > (1-M\varepsilon)\ C_{\max}\left(\frac{\bar{P}_1}{1-M\varepsilon}\right), \quad \forall \bar{P}_1 \in [P_{\text{av}}, (1-M\varepsilon)P_{\text{pk}}] \tag{3.51}$$

$$(1-2\varepsilon)\ C_{\max}\left(\frac{\bar{P}_2}{1-2\varepsilon}\right) \geq (1-M\varepsilon)\ C_{\max}\left(\frac{\bar{P}_2}{1-M\varepsilon}\right), \quad \forall \bar{P}_2 \in [0, P_{\text{av}}) \tag{3.52}$$

where the equality sign in (3.52) is included to cover the case of $\bar{P}_2 = 0$. Based on (3.51), (3.52), and the fact that the search spaces of the optimization problems in (3.48) and (3.50) are the same for $\bar{K} = M$, it is obtained that $\xi > \psi(M)$ for $M > 2$. (Note that $\frac{(P_{\text{av}} - \bar{P}_2)}{(\bar{P}_1 - \bar{P}_2)} > 0$ and $\frac{(\bar{P}_1 - P_{\text{av}})}{(\bar{P}_1 - \bar{P}_2)} \geq 0$.) Therefore, it is concluded that $\psi(2) > \psi(M)$ for $M > 2$ since $\psi(2) \geq \xi$ as shown previously. Hence, in accordance with (3.46), it is shown that $C_{\text{css}} = \psi(2)$ for $\varepsilon < \frac{1}{2}$ and $P_{\text{av}}/(1-2\varepsilon) < P_{\text{pk}}$, where $\psi(\cdot)$ is as defined in (3.27) (cf. (3.49)). To sum up, the optimal channel switching strategy is to switch between two channels and the achieved maximum average capacity can be obtained as in (3.46). ■

Based on Proposition 4, the optimal channel switching strategy can be specified in various scenarios. For the first scenario in (3.46), i.e., for $\varepsilon \geq 1/2$, $C_{\text{css}} = 0$ since channel switching is not feasible, as noted from the constraint in (3.43). For $\varepsilon < 1/2$ and $P_{\text{av}}/(1-2\varepsilon) \geq P_{\text{pk}}$, the solution of the optimal channel switching problem is to transmit at power level P_{pk} over the best channel (that achieves the maximum capacity for power level P_{pk}) for a time fraction of $(1-2\varepsilon)$, then switching to another channel and not transmitting any power (i.e., by consuming a time fraction of ε), which results in $C_{\text{css}} = (1-2\varepsilon)C_{\max}(P_{\text{pk}})$ (see Remark 1). Finally, for $\varepsilon < 1/2$ and $P_{\text{av}}/(1-2\varepsilon) < P_{\text{pk}}$, the achieved maximum average capacity can be calculated based on (3.39) as $C_{\text{css}} = (1-2\varepsilon)(\mu^* C_{\max}(\tilde{P}_1^*) + (1-\mu^*)C_{\max}(\tilde{P}_2^*))$, where $\tilde{P}_1^*$ and $\tilde{P}_2^*$ are the optimizers of the maximization problem in (3.46),

$$\mu^* = \left(\frac{P_{\text{av}}}{1-2\varepsilon} - \tilde{P}_2^*\right) \Big/ \left(\tilde{P}_1^* - \tilde{P}_2^*\right), \tag{3.53}$$

and the optimal channel switching strategy is to switch between channel i and channel j with power levels $\tilde{P}_1^*$ and $\tilde{P}_2^*$, respectively, where i and j are given by[7]

$$i = \underset{l\in\{1,\dots,K\}}{\arg\max}\; C_l(\tilde{P}_1^*) \tag{3.54}$$

$$j = \underset{l\in\{1,\dots,K\}}{\arg\max}\; C_l(\tilde{P}_2^*)\,. \tag{3.55}$$

Remark 2: *It is important to note that* μ^* *in* (3.53) *and* $1-\mu^*$ *do not directly correspond to the time-sharing factors defined in the optimization problem in* (3.2). *In terms of the notation of the optimization problem in* (3.2), *the optimal time-sharing factors, denoted by* λ_i^* *and* λ_j^*, *for the optimal channel switching strategy between channel* i *and channel* j *can be obtained based on the transformations in Proposition 1 and Proposition 2 as*

$$\lambda_i^* = (1-2\varepsilon)\mu^* + \varepsilon \tag{3.56}$$

$$\lambda_j^* = (1-2\varepsilon)(1-\mu^*) + \varepsilon \tag{3.57}$$

where μ^* *is as defined in* (3.53). *Since the optimal channel switching strategy is to switch between two channels as stated in Proposition 4,* $\lambda_k^* = 0$ *for* $k \in \{1,\dots,K\} \setminus \{i,j\}$.

Next the solutions of the optimal single channel strategy in (3.23) and the

[7]In the case of multiple maximizers in (3.54) or (3.55), any of them can be chosen for the optimal strategy.

optimal channel switching strategy in (3.46) are considered together. Overall, the optimal strategy corresponds to one of them, which achieves the higher average capacity, as expressed in (3.26).

- If $\varepsilon \geq 1/2$, then the optimal single channel strategy outperforms the optimal channel switching strategy since $C_{\rm scs}$ in (3.23) always satisfies $C_{\rm scs} > 0$ whereas $C_{\rm css} = 0$ in this case.

- If $\varepsilon < 1/2$ and $P_{\rm av}/(1-2\varepsilon) \geq P_{\rm pk}$, then the following expressions can be obtained for $C_{\rm scs}$:

$$C_{\rm scs} = (1-\varepsilon)\left(C_{\max}\left(\frac{P_{\rm av}}{1-\varepsilon}\right)\mathbb{I}_{\left\{\frac{P_{\rm av}}{1-\varepsilon}<P_{\rm pk}\right\}} + C_{\max}(P_{\rm pk})\mathbb{I}_{\left\{\frac{P_{\rm av}}{1-\varepsilon}\geq P_{\rm pk}\right\}}\right) \tag{3.58}$$

$$> (1-2\varepsilon)\left(C_{\max}\left(\frac{P_{\rm av}}{1-2\varepsilon}\right)\mathbb{I}_{\left\{\frac{P_{\rm av}}{1-\varepsilon}<P_{\rm pk}\right\}} + C_{\max}(P_{\rm pk})\mathbb{I}_{\left\{\frac{P_{\rm av}}{1-\varepsilon}\geq P_{\rm pk}\right\}}\right) \tag{3.59}$$

$$\geq (1-2\varepsilon)\left(C_{\max}(P_{\rm pk})\mathbb{I}_{\left\{\frac{P_{\rm av}}{1-\varepsilon}<P_{\rm pk}\right\}} + C_{\max}(P_{\rm pk})\mathbb{I}_{\left\{\frac{P_{\rm av}}{1-\varepsilon}\geq P_{\rm pk}\right\}}\right) \tag{3.60}$$

$$= (1-2\varepsilon)\, C_{\max}(P_{\rm pk}) \tag{3.61}$$

  where the equality in (3.58) is obtained from (3.23), the inequality in (3.59) follows from (3.44) in Lemma 1, the relation in (3.60) is due to the condition $P_{\rm av}/(1-2\varepsilon) \geq P_{\rm pk}$ and the monotone increasing property of $C_{\max}$ in (3.28), and the final expression in (3.61) follows from the definition of the indicator function. From (3.58)-(3.61), is obtained that $C_{\rm scs} > (1-2\varepsilon)C_{\max}(P_{\rm pk}) = C_{\rm css}$; that is, the optimal single channel strategy achieves a higher average capacity than the optimal channel switching strategy for $\varepsilon < 1/2$ and $P_{\rm av}/(1-2\varepsilon) \geq P_{\rm pk}$.

- Finally, for the case of $\varepsilon < 1/2$ and $P_{\rm av}/(1-2\varepsilon) < P_{\rm pk}$, the optimal strategy is either the single channel strategy or the channel switching strategy, and the achieved maximum average capacity is expressed as

$$C_{\rm av}^{\max} = \max\{C_{\rm scs}, C_{\rm css}\} \tag{3.62}$$

  where $C_{\rm scs}$ is as in (3.23) and $C_{\rm css}$ can be calculated as specified in (3.46), namely,

$$\max_{\substack{\tilde{P}_1 \in \left[\frac{P_{\rm av}}{1-2\varepsilon}, P_{\rm pk}\right] \\ \tilde{P}_2 \in \left[0, \frac{P_{\rm av}}{1-2\varepsilon}\right)}} (1-2\varepsilon) \left(\frac{\frac{P_{\rm av}}{1-2\varepsilon} - \tilde{P}_2}{\tilde{P}_1 - \tilde{P}_2} C_{\rm max}(\tilde{P}_1) + \frac{\tilde{P}_1 - \frac{P_{\rm av}}{1-2\varepsilon}}{\tilde{P}_1 - \tilde{P}_2} C_{\rm max}(\tilde{P}_2) \right). \quad (3.63)$$

Remark 3: *The fact that the optimal single channel strategy outperforms the optimal channel switching strategy for* $\varepsilon \geq 1/2$ *is valid not only for the capacity metric in* (3.1) *but also for any performance metric that is a nonnegative function of the average transmit power. Similarly, the result in Proposition 3 can be extended for any performance metric that is a continuous and bounded function of the transmit power* P *for* $P \in [0, P_{\rm pk}]$*. On the other hand, in the proof of Proposition 4, additional properties of nonnegativity, monotonicity, and concavity are also employed since Lemma 1 is utilized. For example, the capacity of a discrete memoryless channel (not necessarily Gaussian) with average transmit power constraint* P *is a nondecreasing, concave, and continuous function of* P *[82].*

For the case of $\varepsilon < 1/2$ and $P_{\rm av}/(1-2\varepsilon) < P_{\rm pk}$, the following result can be obtained in a similar fashion to Proposition 2 of [33], which presents a sufficient condition for the optimal single channel strategy to achieve a higher average capacity than the optimal channel switching strategy.

Proposition 5: *Suppose that* $\varepsilon < 1/2$ *and* $P_{\rm av}/(1-2\varepsilon) < P_{\rm pk}$ *hold, and* $C_{\rm max}(P)$ *in* (3.28) *is first-order continuously differentiable in an interval around* $P_{\rm av}/(1-2\varepsilon)$*. Then, the optimal single channel strategy outperforms the optimal channel switching strategy in terms of the maximum average capacity if*

$$(P - P_{\rm av}) \frac{B_{i^*} \log_2 e}{(1-2\varepsilon) N_{i^*} B_{i^*} + P_{\rm av}} \geq C_{\rm max}\left(\frac{P}{1-2\varepsilon}\right) - C_{\rm max}\left(\frac{P_{\rm av}}{1-2\varepsilon}\right) \quad (3.64)$$

for all $P \in [0, (1-2\varepsilon)P_{\rm pk}]$*, where* $i^* = \arg\max_{i \in \{1,\dots,K\}} C_i\left(\frac{P_{\rm av}}{1-2\varepsilon}\right)$.

Proof: For $\varepsilon < 1/2$ and $P_{\rm av}/(1-2\varepsilon) < P_{\rm pk}$, the optimal single channel strategy achieves an average capacity of $C_{\rm scs} = (1-\varepsilon) C_{\rm max}\left(\frac{P_{\rm av}}{1-\varepsilon}\right)$, which is obtained from (3.23) since $\frac{P_{\rm av}}{1-\varepsilon} < \frac{P_{\rm av}}{1-2\varepsilon}$ and $\frac{P_{\rm av}}{1-2\varepsilon} < P_{\rm pk}$. Also, the maximum average capacity C_{css} obtained by the optimal channel switching strategy can be calculated from (3.63) in this case. The aim is to prove that under the assumptions in the proposition, if the condition in (3.64) holds, then the optimal single channel strategy achieves a higher average capacity than the optimal channel switching strategy; that is, $C_{\mathrm{scs}} > C_{\mathrm{css}}$. The assumption in the proposition states that the first-order derivative of $C_{\max}(P)$ in (3.28) exists in an interval around $\frac{P_{\mathrm{av}}}{1-2\varepsilon}$. Then its derivative at $\frac{P_{\mathrm{av}}}{1-2\varepsilon}$ can be obtained from (3.1) as follows:

$$C_{\max}^{'}\left(\frac{P_{\mathrm{av}}}{1-2\varepsilon}\right) = \frac{(1-2\varepsilon)B_{i^*}\log_2 e}{(1-2\varepsilon)N_{i^*}B_{i^*} + P_{\mathrm{av}}} \tag{3.65}$$

where $i^* = \arg\max_{i\in\{1,\ldots,K\}} C_i(\frac{P_{\mathrm{av}}}{1-2\varepsilon})$. From (3.65) and the definition of $\bar{P} \triangleq \frac{P}{1-2\varepsilon}$, the condition in (3.64) can be expressed in the following form:

$$C_{\max}(\bar{P}) \le C_{\max}\left(\frac{P_{\mathrm{av}}}{1-2\varepsilon}\right) + \left(\bar{P} - \frac{P_{\mathrm{av}}}{1-2\varepsilon}\right) C_{\max}^{'}\left(\frac{P_{\mathrm{av}}}{1-2\varepsilon}\right), \ \forall \bar{P} \in [0, P_{\mathrm{pk}}]. \tag{3.66}$$

It is noted that the problem for C_{css} in (3.63) can be expressed similarly to (3.39)-(3.42) as follows:

$$\max_{\mu,\tilde{P}_1,\tilde{P}_2} \ (1-2\varepsilon)(\mu\, C_{\max}(\tilde{P}_1) + (1-\mu)C_{\max}(\tilde{P}_2)) \tag{3.67}$$

$$\text{subject to } \mu\,\tilde{P}_1 + (1-\mu)\tilde{P}_2 = \frac{P_{\mathrm{av}}}{1-2\varepsilon} \tag{3.68}$$

$$\tilde{P}_1 \in [0, P_{\mathrm{pk}}],\ \ \tilde{P}_2 \in [0, P_{\mathrm{pk}}] \tag{3.69}$$

$$\mu \in [0, 1] \tag{3.70}$$

Then, for the solution of the channel switching strategy in (3.67)-(3.70) denoted as μ^*, $\tilde{P}_1^*$, and $\tilde{P}_2^*$, the following expressions can be obtained:

$$\begin{aligned}
C_{\text{css}} &= (1-2\varepsilon)(\mu^*\, C_{\max}(\tilde{P}_1^*) + (1-\mu^*)C_{\max}(\tilde{P}_2^*)) && (3.71)\\
&\le (1-2\varepsilon)\Bigg(\left(\mu^*\,\tilde{P}_1^* + (1-\mu^*)\,\tilde{P}_2^* - \frac{P_{\text{av}}}{1-2\varepsilon}\right)\\
&\qquad \times C'_{\max}\left(\frac{P_{\text{av}}}{1-2\varepsilon}\right) + C_{\max}\left(\frac{P_{\text{av}}}{1-2\varepsilon}\right)\Bigg) && (3.72)\\
&= (1-2\varepsilon)\, C_{\max}\left(\frac{P_{\text{av}}}{1-2\varepsilon}\right) && (3.73)\\
&< (1-\varepsilon)\, C_{\max}\left(\frac{P_{\text{av}}}{1-\varepsilon}\right) && (3.74)\\
&= C_{\text{scs}} && (3.75)
\end{aligned}$$

where $\tilde{P}_1^*, \tilde{P}_2^* \in [0, P_{\text{pk}}]$ and $\mu^* \ge 0$. The equality in (3.71) follows from (3.67)-(3.70), and the inequality in (3.72) is obtained based on (3.66). The equality in (3.73) holds since μ^*, $\tilde{P}_1^*$, and $\tilde{P}_2^*$ satisfy the average power constraint in (3.68); that is, $\mu^*\,\tilde{P}_1^* + (1-\mu^*)\tilde{P}_2^* = \frac{P_{\text{av}}}{1-2\varepsilon}$, and since $C'_{\max}(\frac{P_{\text{av}}}{1-2\varepsilon})$ is finite. Finally, (3.74) is obtained due to (3.44) in Lemma 1, which results in the maximum average capacity achieved via the optimal single channel strategy as noted in (3.75). From (3.71)-(3.75), it is concluded that the optimal single channel strategy outperforms the optimal channel switching strategy in terms of the maximum average capacity if the assumptions and the condition in the proposition hold. ■

Based on Proposition 5, if the condition in (3.64) is satisfied for the case of $\varepsilon < 1/2$ and $P_{\text{av}}/(1-2\varepsilon) < P_{\text{pk}}$, and $C_{\max}(P)$ in (3.28) is first-order continuously differentiable in an interval around $\frac{P_{\text{av}}}{1-2\varepsilon}$, then the optimal strategy corresponds to the optimal single channel strategy and there is no need for channel switching. Otherwise, the optimal strategy cannot be directly determined and it requires the comparison of the average capacities obtained by the optimal single channel and the optimal channel switching strategies.

Remark 4: *Overall, the solution of the optimal channel switching problem in the presence of switching delays can be specified as follows:*

- *If $\varepsilon \ge 1/2$ or if $\varepsilon < 1/2$ and $P_{\text{av}}/(1-2\varepsilon) \ge P_{\text{pk}}$, then the optimal strategy* *is to transmit over a single channel, which has the maximum capacity for power level* $\min\left\{\frac{P_{\rm av}}{(1-\varepsilon)}, P_{\rm pk}\right\}$ *(see* (3.23) *and* (3.24)*).*

- *If* $\varepsilon < 1/2$ *and* $P_{\rm av}/(1-2\varepsilon) < P_{\rm pk}$*,*
  - *if* $C_{\max}(P)$ *in* (3.28) *is first-order continuously differentiable in an interval around* $P_{\rm av}/(1-2\varepsilon)$ *and the condition in* (3.64) *holds, then the optimal strategy is to transmit over a single channel, which has the maximum capacity for power level* $P_{\rm av}/(1-\varepsilon)$*.*
  - *otherwise, depending on which one achieves a higher average capacity, the optimal solution is either transmission over a single channel that has the maximum capacity for power level* $\min\left\{\frac{P_{\rm av}}{(1-\varepsilon)}, P_{\rm pk}\right\}$ *or channel switching between channel* i *and channel* j *with time-sharing factors* $\lambda_i^* = (1-2\varepsilon)\left(\frac{P_{\rm av}}{1-2\varepsilon} - \tilde{P}_2^*\right)/(\tilde{P}_1^* - \tilde{P}_2^*) + \varepsilon$ *and* $\lambda_j^* = 1 - \lambda_i^* = (1-2\varepsilon)\left(\tilde{P}_1^* - \frac{P_{\rm av}}{1-2\varepsilon}\right)/(\tilde{P}_1^* - \tilde{P}_2^*) + \varepsilon$ *(see Remark 2) and power levels* $P_i^* = \tilde{P}_1^*$ *and* $P_j^* = \tilde{P}_2^*$*, respectively, where* i *and* j *are given by* (3.54) *and* (3.55)*, and* $\tilde{P}_1^*$ *and* $\tilde{P}_2^*$ *are the optimizers of* (3.63)*.*

3.3 Numerical Results

In this section, numerical examples are presented to investigate the effects of the channel switching delay on the proposed optimal channel switching strategy, and to compare performance of the optimal channel switching and optimal single channel strategies in terms of average capacity maximization. Consider a scenario with $K = 3$ channels where the bandwidths and the noise levels (cf. (3.1)) are given by $B_1 = 1\,\text{MHz}$, $B_2 = 5\,\text{MHz}$, $B_3 = 10\,\text{MHz}$, $N_1 = 10^{-12}\,\text{W/Hz}$, $N_2 = 10^{-11}\,\text{W/Hz}$, and $N_3 = 10^{-11}\,\text{W/Hz}$. Suppose that the peak power constraint and the channel switching delay factor in (3.2) are set to $P_{\rm pk} = 0.1\,\text{mW}$ and $\varepsilon = 0.1$, respectively. In Fig. 3.3, the capacity of each channel is plotted versus power based on the capacity formula in (3.1).

For the scenario in Fig. 3.3, the proposed optimal channel switching strategies and the optimal single channel strategy are calculated for various average power

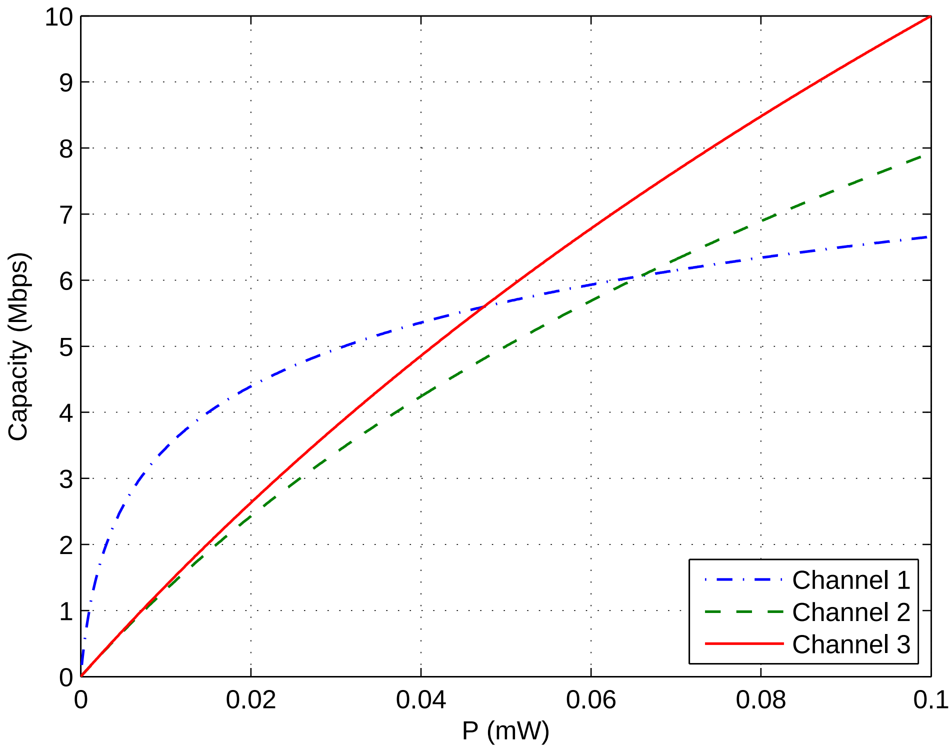

Figure 3.3: Capacity of each channel versus power, where $B_1 = 1\,\mathrm{MHz}$, $B_2 = 5\,\mathrm{MHz}$, $B_3 = 10\,\mathrm{MHz}$, $N_1 = 10^{-12}\,\mathrm{W/Hz}$, $N_2 = 10^{-11}\,\mathrm{W/Hz}$, and $N_3 = 10^{-11}\,\mathrm{W/Hz}$.

limits ($P_{\rm av}$), and the achieved maximum average capacities are plotted versus $P_{\rm av}$ in Fig. 3.4. As discussed in the previous section, the optimal single channel strategy achieves a capacity of $(1-\varepsilon)\,C_{\max}(\phi)$, where $\phi \triangleq \min\left\{\frac{P_{\rm av}}{(1-\varepsilon)}, P_{\rm pk}\right\}$ and $C_{\max}(\phi) = \max\{C_1(\phi), C_2(\phi), C_3(\phi)\}$ in the considered scenario. It is observed from Fig. 3.3 and Fig. 3.4 that $C_{\max}(\phi) = C_1(\phi)$ for $P_{\rm av} \in (0, 0.0426)$ mW and $C_{\max}(\phi) = C_3(\phi)$ for $P_{\rm av} \in [0.0426, 0.1]$ mW; that is, channel 1 is the best channel up to $P_{\rm av} = 0.0426$ mW, and channel 3 is the best after that power level. Among the optimal channel switching strategies discussed in the previous section, it can be observed from Fig. 3.4 that the optimal channel switching strategy with two channels outperforms the optimal channel switching strategy with three channels for all $P_{\rm av} \in [0, 0.1]$ mW in accordance with Proposition 4. Overall, the optimal strategy is to employ the optimal channel switching strategy with two channels for $P_{\rm av} \in (0.0332, 0.0582)$ mW and the optimal single channel strategy for $P_{\rm av} \in [0, 0.0332] \cup [0.0582, 0.1]$ mW. From (3.46) in Proposition 4, the behaviour of the optimal channel switching strategy with two channels in Fig. 3.4 can be explained as follows: For $P_{\rm av}/(1-2\varepsilon) \geq P_{\rm pk}$; that is, for $P_{\rm av} \geq 0.08$ mW, $C_{\rm css}$ in (3.46) is given by $(1-2\varepsilon)C_{\max}(P_{\rm pk}) = 0.8C_{\max}(0.1)$. On the other hand, for $P_{\rm av} < 0.08$ mW, $C_{\rm css}$ is calculated from the third expression in (3.46). In a similar fashion, based on (3.27) in Proposition 3, the optimal channel switching strategy with three channels achieves an average capacity of $(1-3\varepsilon)C_{\max}(P_{\rm pk}) = 0.7C_{\max}(0.1)$ for $P_{\rm av} \geq 0.07$ mW and yields the average capacity obtained from the first expression in (3.27) for $P_{\rm av} < 0.07$ mW. In addition, in accordance with Proposition 5, the optimal strategy is the optimal single channel strategy for $P_{\rm av} \in [0, 0.0176]$ mW since the condition in (3.64) holds for $P_{\rm av} \in [0, 0.0176]$ mW.

In order to investigate the optimal strategy in Fig. 3.4 in more detail, Table 3.2 presents the solutions of the optimal strategy for various values of the average power limit, $P_{\rm av}$. In the table, the optimal solution is represented by parameters λ^*, P_1^*, P_2^*, i, and j, meaning that channel i is used with time-sharing factor λ^* and power P_1^*, and channel j is employed with time-sharing factor $1-\lambda^*$ and power P_2^*. From Table 3.2, it is observed that the optimal channel switching strategy with two channels is the optimal strategy for $P_{\rm av} = 0.04$ mW and

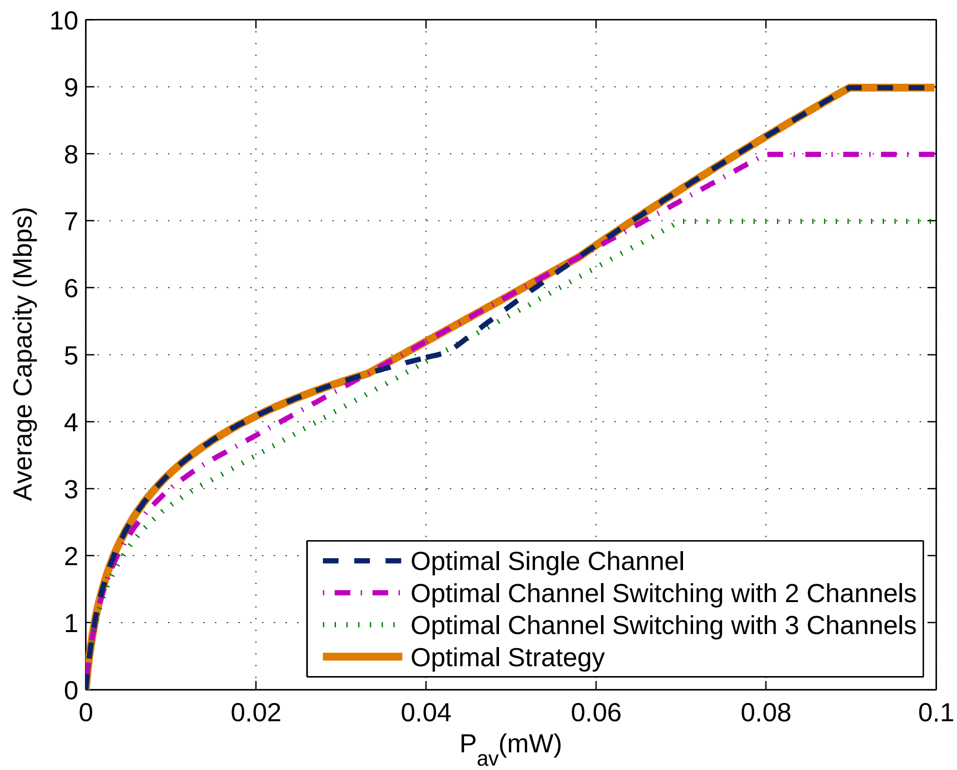

Figure 3.4: Average capacity versus average power limit for the optimal channel switching and the optimal single channel strategies for the scenario in Fig. 3.3, where $P_{\mathrm{pk}} = 0.1\,\mathrm{mW}$.

Table 3.2: Optimal strategy for the scenario in Fig. 3.3, which employs channel i and channel j with time-sharing factors λ^* and $(1-\lambda^*)$ and power levels P_1^* and P_2^*, respectively.

P_{av} (mW)	λ^*	P_1^*	i	$(1-\lambda^*)$	P_2^*	j
0.01	–	–	–	1	0.0111	1
0.02	–	–	–	1	0.0222	1
0.03	–	–	–	1	0.0333	1
0.04	0.4026	0.1	3	0.5974	0.0196	1
0.05	0.527	0.1	3	0.473	0.0196	1
0.06	–	–	–	1	0.0667	3
0.07	–	–	–	1	0.0778	3
0.08	–	–	–	1	0.0889	3
0.09	–	–	–	1	0.1	3
0.099	–	–	–	1	0.1	3

$P_{\text{av}} = 0.05\,\text{mW}$, where switching between channel 1 and channel 3 is performed. For the other P_{av} values in Table 3.2, it is optimal to employ the optimal single channel strategy which achieves higher average capacities than the optimal channel switching strategy.

To provide benchmarks on the performance of the proposed optimal channel switching strategy, two scenarios are considered: In the first one, the optimal channel switching strategy is performed in the absence of channel switching delays (i.e., $\varepsilon = 0$), which leads to an upper performance limit. In the second one, a lower performance limit is obtained by designing the "optimal" channel switching strategy without the consideration of channel switching delays (i.e., assuming that ε is zero even though it is not). This scenario corresponds to the use of the approach in [33] (which is optimal for $\varepsilon = 0$) in the presence of channel switching delays. Fig. 3.5 presents the average capacities achieved in these two scenarios, together with that achieved by the proposed optimal strategy obtained from (3.2) for the system in Fig. 3.3, where $P_{\text{pk}} = 0.1\,\text{mW}$ and $\varepsilon = 0.1$. For the calculation of the average capacities achieved by the "optimal" strategy without the consideration of channel switching delays, the problem in [33] is solved first, and then the obtained solution is substituted into the objective function in (3.2). Namely, if λ^*, P_1^*, and P_2^* denote the solution of [33], the maximum average

Table 3.3: Optimal strategy for the scenario in Fig. 3.3, which employs channel i and channel j with time-sharing factors λ^* and $(1-\lambda^*)$ and power levels P_1^* and P_2^*, respectively.

ε	λ^*	P_1^*	i	$(1-\lambda^*)$	P_2^*	j
0.05	0.4526	0.1	3	0.5474	0.0196	1
0.1	0.527	0.1	3	0.473	0.0196	1
0.2	–	–	–	1	0.0625	3
0.3	–	–	–	1	0.0714	3
0.4	–	–	–	1	0.0833	3
0.5	–	–	–	1	0.1	3
0.6	–	–	–	1	0.1	3
0.7	–	–	–	1	0.1	3
0.8	–	–	–	1	0.1	3
0.9	–	–	–	1	0.1	3

capacity obtained via the strategy in which the delays are neglected is given by $\max\{\lambda^*-\varepsilon,0\}C_{\max}(P_1^*)+\max\{1-\lambda^*-\varepsilon,0\}C_{\max}(P_2^*)$. On the other hand, the maximum average capacity achieved by the optimal channel switching strategy in the absence of channel switching delays (i.e., for $\varepsilon=0$) can be expressed as $\lambda^*C_{\max}(P_1^*)+(1-\lambda^*)C_{\max}(P_2^*)$. Based on these strategies, it is observed from Fig. 3.5 that the optimal strategy in the absence of channel switching delays outperforms the other strategies; hence, presents an upper limit, as expected. In addition, the delay-ignorant strategy (i.e., assuming no delays) cannot achieve a higher average capacity than that achieved by (3.2) (i.e., the proposed approach) due to the inefficient use of the average power and the optimization of the channel switching factors and power levels based on an unrealistic setting. On the other hand, the proposed optimal strategy obtained from (3.2) takes into account the fact that no data transmission occurs during channel switching and consequently no power is transmitted. Therefore, it optimizes the channel switching factors and power levels by using the average power efficiently. It is also noted that the abrupt behavioral changes in the average capacity curve of the delay-ignorant strategy occurs due to the change in the number of channels employed in the strategy and the decrease in the efficiency of average power usage.

Based on the scenario in Fig. 3.3, the maximum average capacities for the

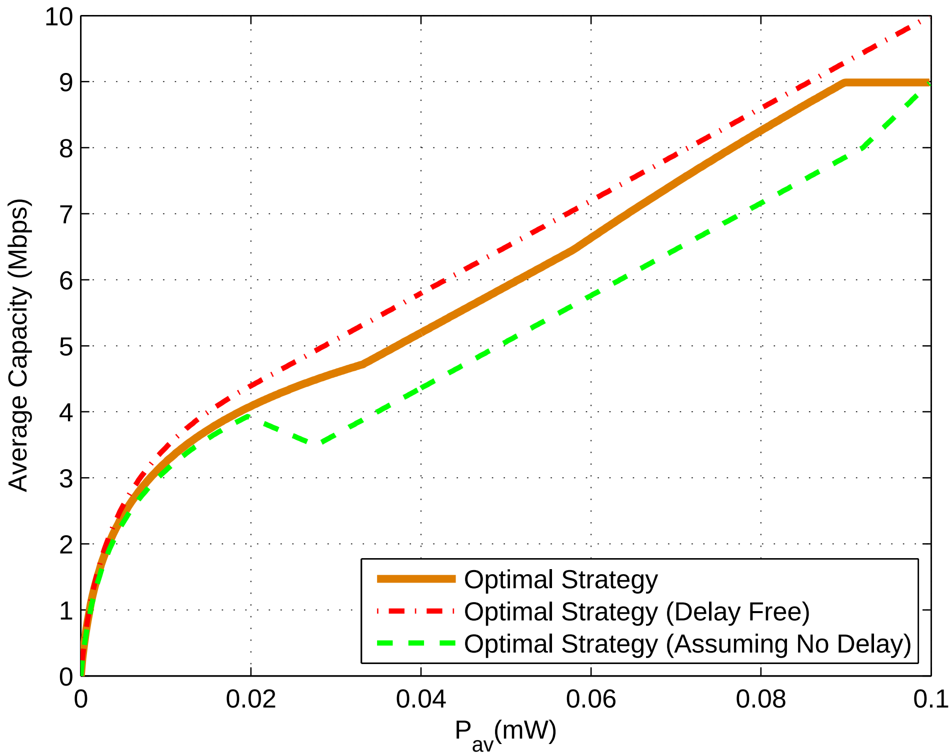

Figure 3.5: Average capacity versus average power limit for the optimal strategy in the absence of channel switching delays ($\varepsilon = 0$) and the optimal strategy without considering channel switching delays ($\varepsilon = 0.1$), together with the proposed optimal strategy for the scenario in Fig. 3.3, where $P_{\mathrm{pk}} = 0.1\,\mathrm{mW}$ and $\varepsilon = 0.1$.

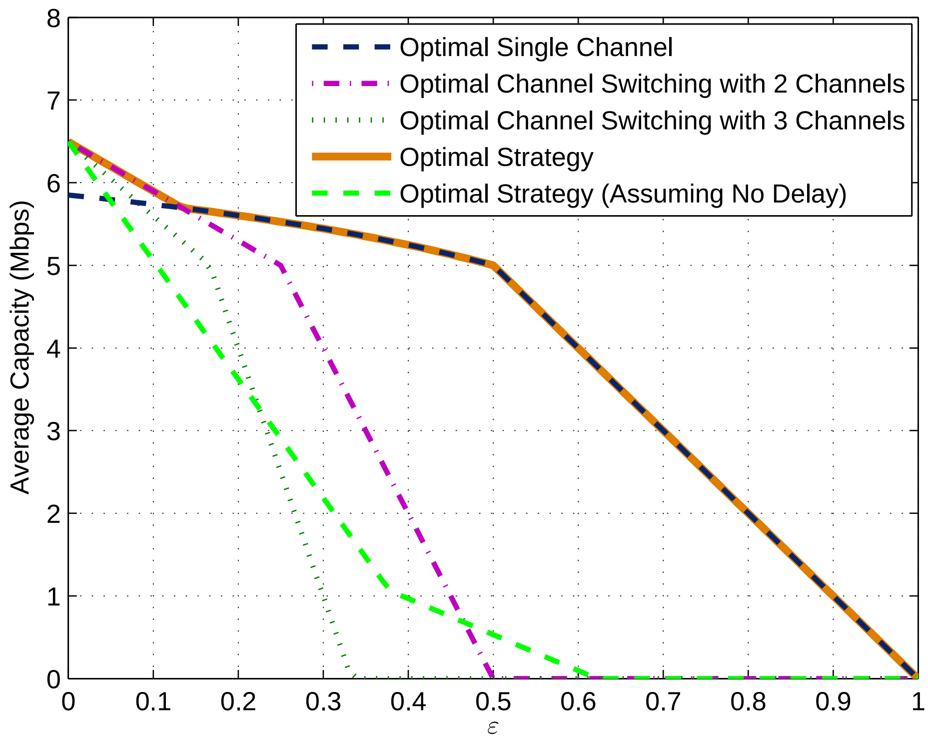

Figure 3.6: Average capacity versus channel switching delay factor for various optimal strategies for the scenario in Fig. 3.3, where $P_{\mathrm{av}} = 0.05\,\mathrm{mW}$ and $P_{\mathrm{pk}} = 0.1\,\mathrm{mW}$.

strategies specified in Fig. 3.4 are plotted versus the channel switching delay factor (ε) in Fig. 3.6 to investigate the effects of the channel switching delay factor on the average capacity. The average power limit and the peak power constraint in (3.2) are set to $P_{\text{av}} = 0.05\,\text{mW}$ and $P_{\text{pk}} = 0.1\,\text{mW}$, respectively. From Fig. 3.6, it is noted that, in accordance with Proposition 4, the optimal channel switching strategy with two channels achieves a higher average capacity than the optimal channel switching strategy with three channels whenever channel switching is feasible; i.e., $\varepsilon < 0.5$. For $\varepsilon \in (0, 0.134)$, the optimal strategy is the optimal channel switching strategy between two channels, whereas the optimal single channel strategy is the overall optimal for $\varepsilon \in [0.134, 1)$. It is important to note that the behavioral change in the average capacity curve of the optimal strategy at $\varepsilon = 0.5$ is observed due to the peak power constraint in (3.2). Since $P_{\text{av}}/(1-\varepsilon) \geq P_{\text{pk}}$ for $\varepsilon \geq 0.5$, the optimal strategy achieves an average capacity of $(1-\varepsilon)C_{\max}(P_{\text{pk}})$ by allocating all the available and attainable power to a single channel and transmitting over this single channel. For comparison purposes, Fig. 3.6 also presents the average capacity achieved by the "optimal" strategy which assumes no channel switching delays and optimizes the parameters accordingly [33]. It is noted that this strategy is outperformed by the proposed optimal strategy, which takes into account the channel switching delays. Fig. 3.6 clearly points out that the consideration of channel switching delays in the strategy design becomes more crucial for improved average capacity as the channel switching delay factor increases.

Similar to Table 3.2, Table 3.3 presents the solutions corresponding to the optimal strategy for various values of the channel switching delay factor. For $\varepsilon = 0.05$ and $\varepsilon = 0.1$, it is observed that the optimal strategy is to switch between channel 1 and channel 3. For the other ε values satisfying $\varepsilon \geq 0.134$ in Table 3.3, the optimal strategy is to transmit over channel 3 exclusively with power level $P_2^* = P_{\text{av}}/(1-\varepsilon)$ for $\varepsilon < 0.5$ and $P_2^* = P_{\text{pk}}$ otherwise.

In order to investigate whether channel switching can provide any benefits for practical modulation schemes, consider the achievable capacity of the discrete-input continuous-output memoryless channel (DCMC) with 64-QAM signaling in the presence of additive white Gaussian noise [83, eqn. (23.23)] for the scenario in Fig. 3.3. As an example, for $P_{\text{av}} = 0.04\,\text{mW}$ and $\varepsilon = 0.1$, the calculations show that when the optimal strategy for $P_{\rm av} = 0.04\,{\rm mW}$ in Table 3.2 (that is, $\lambda^* = 0.4026$, $P_1^* = 0.1\,{\rm mW}$, $P_2^* = 0.0196\,{\rm mW}$, $i = 3$, and $j = 1$) is employed for the achievable capacity of 64-QAM [83], an average achievable capacity of 5.033 Mbps is obtained whereas the optimal single channel approach yields an achievable capacity of 4.5819 Mbps. Hence, it is observed that it is possible to achieve performance improvements via channel switching also for practical modulation schemes.[8]

3.4 Extensions

In this study, the optimal channel switching problem is investigated for a single user. In the presence of multiple users, the results in this study can be extended in various directions. First, if orthogonal resource allocation is employed such that each user utilizes a different channel at a given time, then the results in this study would still hold. In such a scenario, a central unit can provide coordination by informing each user about the available channels for that user in each time frame. Secondly, if users are allowed to employ the same channels and possible interference to a user is modeled by a Gaussian noise process, then the channel switching problem in (3.2) can be extended for nonorthogonal resource allocation, as well. In this case, when a user wishes to commence communications over the available channels, it first performs spectrum sensing and determines the interference level in each channel. Then, the capacity of each channel is given by

$$C_i(P) = B_i \log_2\left(1 + \frac{P}{2B_i(N_i/2 + I_i)}\right) \text{ bits/sec} \tag{3.76}$$

where I_i is the spectral density level of the interference (due to the other users) in channel i and the other parameters are as defined for (3.2). When the channel switching problem in (3.2) is solved based on the capacity expression in (3.76),

[8]It is noted that this performance improvement is achieved without performing specific optimization for the achievable capacity function corresponding to a practical modulation scheme, which can be implemented to obtain further improvements.

the solution corresponds to the optimal channel switching strategy in the presence of multiuser interference. Since the structure of this new problem is the same as that of the original problem (cf. (3.1)), all the theoretical results apply to this scenario, as well. An example application for this scenario is a cognitive radio system with the underlay approach, where a secondary user utilizes the channels of primary users as long as it does not cause significant performance degradation for primary users [84, Ch. 2]. In that case, the secondary user performs channel (spectrum) sensing and determines the presence of primary users and the corresponding interference levels. Then, the proposed optimal channel switching strategy can be obtained as described above.

In case of non-orthogonal multiple access, fairness should be considered to satisfy certain average capacity requirements for all users. One way of achieving fairness is related to the limitation of power levels over different channels so that interference to users is limited; hence, no significant capacity degradations are observed. In other words, for each user, the maximum amount of power that can be transmitted over each channel can be determined according to a fairness criterion, which is set by a central unit. To provide a generic analysis that covers various fairness strategies, let $\hat{P}_i$ represent the maximum power that can be transmitted over channel i. When a user wants to start communications over the available channels, it designs the optimal channel switching strategy as follows (cf. (3.2)):

$$\begin{aligned}
\max_{\{\lambda_i, P_i\}_{i=1}^K} \quad & \sum_{i=1}^K \mathbb{I}_{\{\lambda_i>0\}}\,(\lambda_i-\varepsilon)\,C_i(P_i) \\
\text{subject to} \quad & \sum_{i=1}^K \mathbb{I}_{\{\lambda_i>0\}}\,(\lambda_i-\varepsilon)P_i \le P_{\text{av}}\,, \\
& P_i \in [0, \min\{\hat{P}_i, P_{\text{pk}}\}]\,,\ \forall i\in\{1,\dots,K\}\,, \\
& \sum_{i=1}^K \lambda_i = 1\,,\ \lambda_i\in\{0\}\cup[\varepsilon,1]\,,\ \forall i\in\{1,\dots,K\}
\end{aligned} \tag{3.77}$$

where $C_i(P_i)$ is as in (3.76), $\hat{P}_i$ is the power limit for channel i, and the other parameters are as in (3.2). In this way, fairness among various users can be achieved by adjusting the power limits of each user over different channels.

The results in this study can be extended for the problem in (3.77) as follows: Similar to Proposition 1, an alternative optimization problem to (3.77) can be obtained as in (3.3) by updating the definition of $C_{s_i}(\cdot)$ and replacing the peak power constraints with $P_{s_i} \in [0, \min\{\hat{P}_{s_i}, P_{\text{pk}}\}]$, $\forall i \in \{1, \ldots, \tilde{K}\}$. It can be shown based on similar arguments to those in the proof of Proposition 1 that the alternative problem achieves the same maximum average capacity as (3.77). Next, define the following function:

$$\hat{C}_i(P) \triangleq \begin{cases} C_i(P), & \text{if } P \leq \min\{\hat{P}_i, P_{\text{pk}}\} \\ 0, & \text{otherwise} \end{cases} \tag{3.78}$$

for $i \in \{1, \ldots, K\}$, where $C_i(P)$ is as in (3.76). Based on a similar approach to that in Proposition 2, the alternative optimization problem can be expressed as in (3.16) by replacing $C_{s_i}(P_{s_i})$ and $P_{s_i} \in [0, P_{\text{pk}}]$ in (3.16) with $\hat{C}_{s_i}(P_{s_i})$ in (3.78) and $P_{s_i} \in [0, \min\{\hat{P}_{s_i}, P_{\text{pk}}\}]$, respectively. Then, the resulting optimization problem can be separated into two optimization problems in a similar fashion:

Case-1 (Single Channel): In this case, the following optimization problem can be obtained:

$$\begin{aligned} \max_{S \in B^1} \max_{\mu_{s_1}, P_{s_1}} \ & (1-\varepsilon)\, \mu_{s_1} \hat{C}_{s_1}(P_{s_1}) \\ \text{subject to } & \mu_{s_1} P_{s_1} \leq \frac{P_{\text{av}}}{(1-\varepsilon)} \\ & P_{s_1} \in [0, \min\{\hat{P}_{s_1}, P_{\text{pk}}\}] \\ & \mu_{s_1} = 1\,,\ \mu_{s_1} \geq 0 \\ & \varepsilon < 1 \end{aligned} \tag{3.79}$$

where the parameters are as defined in (3.22). Let $\hat{C}_{\text{scs}}$ denote the solution of (3.79). Then, $\hat{C}_{\text{scs}}$ can be expressed as

$$\hat{C}_{\text{scs}} = \max_{l \in \{1, \ldots, K\}} (1-\varepsilon)\, C_l \left(\min \left\{ \frac{P_{\text{av}}}{(1-\varepsilon)}, \min\{\hat{P}_l, P_{\text{pk}}\} \right\} \right) \tag{3.80}$$

and the channel index m employed in this strategy can be obtained as

$$\hat{m} = \underset{l \in \{1,\ldots,K\}}{\arg\max} \, C_l \left(\min \left\{ \frac{P_{\rm av}}{(1-\varepsilon)}, \min\{\hat{P}_l, P_{\rm pk}\} \right\} \right). \tag{3.81}$$

Case-2 (Channel Switching): In this case, the following optimization problem can be obtained:

$$\begin{aligned}
\hat{C}_{\rm css} = \max_{\tilde{K} \in A \setminus \{1\}} \max_{S \in B^{\tilde{K}}} \max_{\{\mu_{s_i}, P_{s_i}\}_{i=1}^{\tilde{K}}} & \left(1 - \tilde{K}\varepsilon\right) \sum_{i=1}^{\tilde{K}} \mu_{s_i} \hat{C}_{s_i}(P_{s_i}) \\
\text{subject to } & \sum_{i=1}^{\tilde{K}} \mu_{s_i} P_{s_i} \leq \frac{P_{\rm av}}{\left(1 - \tilde{K}\varepsilon\right)} \\
& P_{s_i} \in [0, \min\{\hat{P}_{s_i}, P_{\rm pk}\}], \ \forall i \in \{1, \ldots, \tilde{K}\} \\
& \sum_{i=1}^{\tilde{K}} \mu_{s_i} = 1, \ \mu_{s_i} \geq 0, \ \forall i \in \{1, \ldots, \tilde{K}\} \\
& \tilde{K} < \frac{1}{\varepsilon}
\end{aligned} \tag{3.82}$$

where the parameters are as in (3.25). Based on Case-1 and Case-2, the solution can be calculated as $\max\left\{\hat{C}_{\rm scs}, \hat{C}_{\rm css}\right\}$.

For the optimization problem in (3.82), the statement in Proposition 3 can be extended as follows: Assume that $\bar{K} \geq 2$ channels are employed in the channel switching strategy and $\varepsilon < 1/\bar{K}$ holds. Also, $P_{\rm max}$ is defined as $P_{\rm max} = \max_{i \in \{1,\ldots,K\}} \min\{\hat{P}_i, P_{\rm pk}\}$. Then, the maximum average capacity achieved via the optimal channel switching strategy over $\bar{K}$ channels can be expressed as

$$\psi(\bar{K}) = \begin{cases} \displaystyle\max_{\substack{\tilde{P}_1 \in \left[\frac{P_{\rm av}}{1-\bar{K}\varepsilon}, P_{\rm max}\right] \\ \tilde{P}_2 \in \left[0, \frac{P_{\rm av}}{1-\bar{K}\varepsilon}\right)}} \left(1 - \bar{K}\varepsilon\right) \left(\frac{\frac{P_{\rm av}}{1-\bar{K}\varepsilon} - \tilde{P}_2}{\tilde{P}_1 - \tilde{P}_2} \hat{C}_{\rm max}(\tilde{P}_1) \right. \\ \qquad \left. + \frac{\tilde{P}_1 - \frac{P_{\rm av}}{1-\bar{K}\varepsilon}}{\tilde{P}_1 - \tilde{P}_2} \hat{C}_{\rm max}(\tilde{P}_2) \right), \ \text{if } \frac{P_{\rm av}}{1-\bar{K}\varepsilon} < \hat{P} \\ \left(1 - \bar{K}\varepsilon\right) \hat{C}_{\rm max}(\hat{P}), \qquad \text{otherwise} \end{cases} \tag{3.83}$$

where $\hat{C}_{\max}(P)$ is defined as

$$\hat{C}_{\max}(P) \triangleq \max\{\hat{C}_1(P), \ldots, \hat{C}_K(P)\} \tag{3.84}$$

and $\hat{P}$ is given by

$$\hat{P} \triangleq \arg\max_{P\in[0,P_{\max}]} \hat{C}_{\max}(P)\,. \tag{3.85}$$

The solution of the optimization problem in (3.82) can be obtained from (3.43) where $\psi(\tilde{K})$ is as in (3.83). In addition, the statement in Lemma 1 also holds for positive $\hat{C}_{\max}(\cdot)$; i.e., it holds if P/α and P/β satisfy $P/\alpha, P/\beta \in [0, P_{\max}]$. Then, the optimal channel switching strategy is to switch between two channels and the maximum average capacity $\hat{C}_{\rm css}$ achieved by the optimal channel switching strategy can be expressed, similar to Proposition 4, as follows:

$$\hat{C}_{\rm css} = \begin{cases} 0, & \text{if } \varepsilon \geq \frac{1}{2} \\ (1-2\varepsilon)C_{\max}(\hat{P}), & \text{if } \varepsilon < \frac{1}{2} \text{ and } \frac{P_{\rm av}}{1-2\varepsilon} \geq \hat{P} \\ \displaystyle\max_{\substack{\tilde{P}_1\in\left[\frac{P_{\rm av}}{1-2\varepsilon}, P_{\max}\right] \\ \tilde{P}_2\in\left[0, \frac{P_{\rm av}}{1-2\varepsilon}\right)}} (1-2\varepsilon)\left(\frac{\frac{P_{\rm av}}{1-2\varepsilon}-\tilde{P}_2}{\tilde{P}_1-\tilde{P}_2}\, C_{\max}(\tilde{P}_1) \right. & \\ \left. \quad +\frac{\tilde{P}_1-\frac{P_{\rm av}}{1-2\varepsilon}}{\tilde{P}_1-\tilde{P}_2}\, C_{\max}(\tilde{P}_2)\right), \text{ otherwise} & \end{cases} \tag{3.86}$$

Based on (3.80) and (3.86), it can be obtained that the optimal strategy corresponds to the optimal single channel strategy if $\varepsilon \geq 1/2$ or if $\varepsilon < 1/2$ and $P_{\rm av}/(1-2\varepsilon) \geq \hat{P}$. Otherwise, the optimal strategy is either the single channel strategy or the channel switching strategy based on the comparison of the average capacities obtained from (3.80) and (3.86). Overall, it is concluded that in the presence of generic power limits for different channels for each user (due to a fairness criterion), the results in this study are still valid with slight modifications in the optimization problems and the statements in the propositions.

Another way of providing fairness can be realized via the joint optimization of the multiuser system. In that case, the aim is to maximize the sum of the average capacities of the users under constraints on the average capacity of each user (to guarantee a certain average capacity for all users), the average power, and the peak powers. In general, it is quite difficult to obtain the solution of this joint optimization problem. Theoretical and numerical investigations of this problem are considered as an important direction for future work.

3.5 Concluding Remarks

In this study, the optimal channel switching problem has been investigated for average capacity maximization in the presence of channel switching delays. First, an equivalent formulation of the optimal channel switching problem has been obtained to facilitate theoretical investigations. Then, the optimal strategy has been obtained and the corresponding average capacity has been specified when channel switching is performed among a given number of channels. Based on this result (and Lemma 1), it has been shown that optimal channel switching does not involve more than two different channels, and the resulting maximum average capacity has been formulated for various values of the channel switching delay parameter and the average and peak power limits. Then, the scenarios under which the optimal strategy corresponds to the exclusive use of a single channel or to channel switching between two channels have been specified. Furthermore, sufficient conditions have been obtained to determine when the optimal single channel strategy outperforms optimal channel switching. Via numerical examples, the theoretical results and the effects of channel switching delays have been illustrated.

The capacity metric in (3.1) specifies the maximum data rates, which can be achieved in practice via turbo coding or low density parity check codes [32]. The results in this study can also be extended for any other performance metric that is a nonnegative, concave, monotone increasing, bounded, and continuous function of the transmit power. For example, considering a certain modulation/demodulation scheme, the average number of correctly received symbols can be defined as an alternative performance metric. Since, in Gaussian channels, the probability of correct decision is a concave function of the transmit power for many modulation types (for all modulation types at high signal-to-noise ratios) [85], it can be shown that the average number of correctly received symbols becomes a nonnegative, concave, monotone increasing, bounded, and continuous function of the transmit power. Therefore, it can be shown that the results in Propositions 1–4 and Lemma 1 hold for such a scenario, as well, and Proposition 5 can also be extended.

Chapter 4

Optimal Channel Switching in Multiuser Systems under Average Capacity Constraints

In this chapter, the optimal channel switching problem is investigated for average capacity maximization in the presence of multiple receivers in the communication system [44]. The key contributions of this chapter can be highlighted as follows:

- For the first time in the literature, the channel switching problem is studied for average capacity maximization in the presence of multiple receivers in a communication system where the transmitter communicates with the primary and secondary receivers in order to improve the average capacity of the secondary receiver under the average and peak power constraints and the minimum average capacity requirement for the primary receiver.
- It is obtained that the optimal channel switching strategy includes no more than 3 communication links in the presence of multiple available communication channels in the system.
- It is shown that the optimal channel switching strategy corresponds to one of the following strategies:

- The transmitter performs communication with the primary receiver over at most two channels and employs a single channel for the secondary receiver.
- The transmitter communicates with the primary receiver over a single channel and at most two channels are occupied for the communication to the secondary receiver.

- A low-complexity solution to the channel switching problem is provided, which requires the comparison of the average capacities obtained by two optimization problems, each having significantly lower computational complexity than the original channel switching problem.
- As an extension, the channel switching problem is reformulated in the consideration of multiple primary receivers and their corresponding minimum average capacity requirements.

This chapter is organized as follows: Section 4.1 introduces the system model and the problem formulation for optimal channel switching in multiuser systems. Section 4.2 presents the optimal channel switching strategies for a communication system in which a transmitter communicates with the primary and secondary receivers. Section 4.3 extends the theoretical results in Section 4.2 for the case that the communication system includes multiple primary receivers, each having an individual minimum average capacity requirement. Numerical results are presented in Section 4.4 and concluding remarks are provided in Section 4.5.

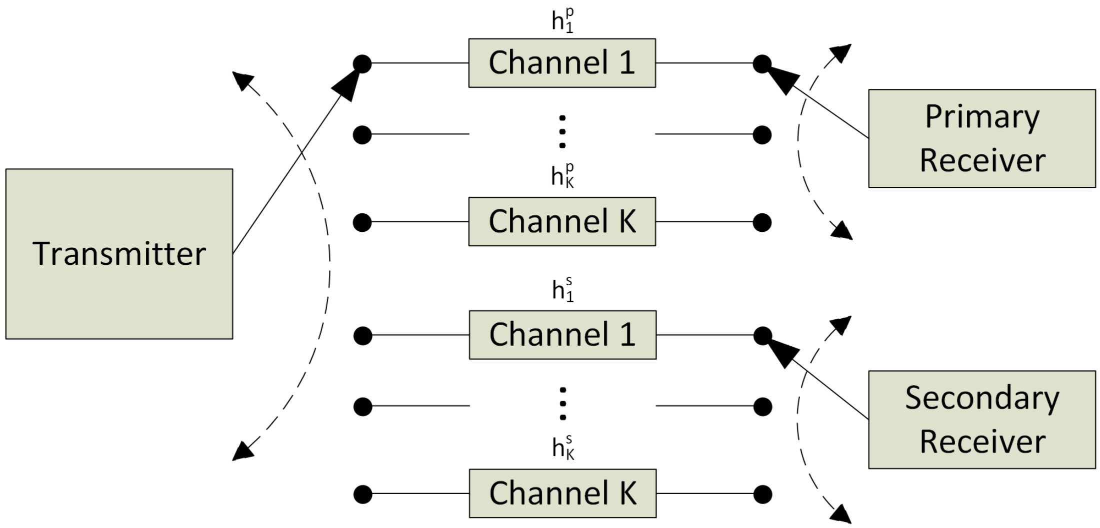

Figure 4.1: Block diagram of a communication system in which transmitter communicates with primary and secondary receivers via channel switching among K channels (frequency bands). It is noted that the the channel coefficients can be different for the same channels

4.1 System Model and Problem Formulation

Consider a communication system in which K different channels (frequency bands) are available for a transmitter to communicate with two receivers classified as primary and secondary.[1] It is assumed that, due to hardware constraints, the transmitter can establish only one communication link with one of the receivers at a given time by performing communication over one of the channels [33, 43]. The reason for this assumption is that the transmitter and the receivers are assumed to have a single RF chain each due to complexity and cost considerations. The restriction caused by this assumption simplifies the circuit and antenna design at transmitters and receivers while reducing the hardware costs by allowing to employ a single RF chain to transmit/receive data. The transmitter can switch (time share) among these K channels to improve the average capacity of the secondary receiver while satisfying the minimum average capacity requirement

[1]Extensions to multiple receivers are presented in Section 4.3. Also, the terms, primary and secondary, used in the study have different meanings from the ones used in the cognitive radio literature where primary users are licensed users and secondary users are unlicensed users that are allowed to access the spectrum when primary users are not active.

for the primary receiver. The channels are modeled as statistically independent flat-fading additive Gaussian noise channels with constant power spectral density levels over the channel bandwidths. Also, the channel state information (CSI) is assumed to be available at both the transmitter and the associated receiver, and the channels can have different bandwidths and constant spectral density levels in general. Fig. 4.1 illustrates the system model with K different channels (frequency bands), where the transmitter communicates with one primary and one secondary receiver via channel switching (i.e., time sharing). In practice, the transmitter can initiate communication with the primary receiver and communicate over one channel for a certain fraction of time. Then, it switches to another channel and communicates with the primary receiver over that channel for another fraction of time. The similar process continues for the remaining channels. Later, the transmitter establishes communication with the secondary receiver and it applies the same procedure as employed for the primary receiver; that is, for a certain fraction of time, it communicates with the secondary transmitter over one channel and it switches to the remaining channels in order and communicates over those channels for certain fractions of time. It is important to emphasize that the receivers are classified as primary and secondary in the study since the transmitter primarily satisfies the minimum average capacity requirement for the primary receiver and then performs communication with the secondary receiver to enhance the average capacity of the communication with the secondary receiver. This scenario is applicable to wireless sensor networks in which child nodes can employ the channel switching strategy in order to improve their average capacity while fulfilling the minimum average capacity constraint of the parent node. Also, it can be stated that the channel switching strategy may improve the energy efficiency of the communication system by requiring a lower average power to achieve the same average channel capacity achieved by the conventional methods [86, 87].

Let B_i and $N_i/2$ denote, respectively, the bandwidth and the constant power spectral density level of the additive Gaussian noise for channel i, where $i \in \{1, \ldots, K\}$, and let h_i^k represent the complex channel gain for channel i between the transmitter and receiver k, where $k \in \{\mathrm{p}, \mathrm{s}\}$ denotes the label for either the primary or the secondary receiver. Then, the capacity of channel i between the transmitter and receiver k is expressed as

$$C_i^k(P) = B_i \log_2 \left(1 + \frac{|h_i^k|^2 P}{N_i B_i} \right) \text{ bits/sec} \tag{4.1}$$

where P represents the average transmit power [72].

The main objective of this study is to determine the optimal channel switching strategy that maximizes the average capacity of the communication between the transmitter and the secondary receiver while ensuring the minimum average capacity constraint for the primary receiver with the consideration of average and peak power constraints. To provide a mathematical formulation, time-sharing (channel switching) factors are defined as $\lambda_1^{\text{p}}, \ldots, \lambda_K^{\text{p}}, \lambda_1^{\text{s}}, \ldots, \lambda_K^{\text{s}}$, where λ_i^{p} and λ_i^{s} denote the fractions of time when channel i is utilized by the transmitter for communication with the primary receiver and the secondary receiver, respectively. Then, the following optimal channel switching problem is proposed for average capacity maximization of the link between the transmitter and the secondary receiver under a minimum average capacity constraint of the primary receiver:

$$\max_{\{\lambda_i^{\text{p}}, \lambda_i^{\text{s}}, P_i^{\text{p}}, P_i^{\text{s}}\}_{i=1}^K} \sum_{i=1}^K \lambda_i^{\text{s}} \, C_i^{\text{s}}(P_i^{\text{s}}) \tag{4.2a}$$

$$\text{subject to } \sum_{i=1}^K \lambda_i^{\text{p}} \, C_i^{\text{p}}(P_i^{\text{p}}) \geq C_{\text{req}} \tag{4.2b}$$

$$\sum_{i=1}^K (\lambda_i^{\text{p}} \, P_i^{\text{p}} + \lambda_i^{\text{s}} \, P_i^{\text{s}}) \leq P_{\text{av}} \, ,$$

$$P_i^{\text{p}}, P_i^{\text{s}} \in [0, P_{\text{pk}}] \, , \; \forall i \in \{1, \ldots, K\} \tag{4.2c}$$

$$\sum_{i=1}^K (\lambda_i^{\text{p}} + \lambda_i^{\text{s}}) = 1 \, ,$$

$$\lambda_i^{\text{p}}, \lambda_i^{\text{s}} \in [0, 1] \, , \; \forall i \in \{1, \ldots, K\} \tag{4.2d}$$

where $C_i^k(P_i)$ for $k \in \{\text{p}, \text{s}\}$ is as in (4.1), P_i^{p} and P_i^{s} represent the average transmit powers allocated to channel i in order to communicate with the primary and secondary receivers, respectively, C_{req} is the minimum average capacity requirement for the primary receiver, P_{pk} denotes the peak power limit, and P_{av} represents the average power limit for the transmitter. The average power limit can be associated with the power consumption and/or the batery life at the transmitter. On the other hand, the peak power constraint refers to the maximum power level that can be produced by the transmitter circuitry (i.e., a hardware constraint). It is assumed that $P_{\text{av}} < P_{\text{pk}}$ and $C_{\text{req}} > 0$. It is also important to note that there exists a total of $2K$ communication links in the system since each of the K channels (frequency bands) can be used for communicating with the primary receiver or secondary receiver.

4.2 Optimal Channel Switching for Communication between the Transmitter and the Secondary Receiver

Since the optimization problem in (4.2) requires a search over a $4K$ dimensional space in general, it is hard to obtain the solution of the problem in its current form. Therefore, the aim is to convert the optimization problem in (4.2) into a tractable equivalent optimization problem, the solution of which is the same as that of (4.2). The following optimization problem represents such an alternative optimization problem.

Proposition 1: *The following optimization problem results in the same maximum average capacity for the secondary receiver as the original optimization* *problem in* (4.2)*:*

$$
\begin{aligned}
\max_{\{\lambda_i^{\mathrm{p}},\lambda_i^{\mathrm{s}},P_i^{\mathrm{p}},P_i^{\mathrm{s}}\}_{i=1}^K} \quad & \sum_{i=1}^{K} \lambda_i^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(P_i^{\mathrm{s}}) && \text{(4.3a)}\\
\text{subject to} \quad & \sum_{i=1}^{K} \lambda_i^{\mathrm{p}}\, C_{\max}^{\mathrm{p}}(P_i^{\mathrm{p}}) \geq C_{\mathrm{req}} && \text{(4.3b)}\\
& \sum_{i=1}^{K} (\lambda_i^{\mathrm{p}}\, P_i^{\mathrm{p}} + \lambda_i^{\mathrm{s}}\, P_i^{\mathrm{s}}) \leq P_{\mathrm{av}}\,, \\
& P_i^{\mathrm{p}}, P_i^{\mathrm{s}} \in [0, P_{\mathrm{pk}}]\,,\ \forall i \in \{1,\ldots,K\} && \text{(4.3c)}\\
& \sum_{i=1}^{K} (\lambda_i^{\mathrm{p}} + \lambda_i^{\mathrm{s}}) = 1\,, \\
& \lambda_i^{\mathrm{p}}, \lambda_i^{\mathrm{s}} \in [0, 1]\,,\ \forall i \in \{1,\ldots,K\} && \text{(4.3d)}
\end{aligned}
$$

where $C_{\max}^{k}(P)$ *is defined as*

$$C_{\max}^{k}(P) \triangleq \max\{C_1^{k}(P), \ldots, C_K^{k}(P)\} \tag{4.4}$$

for $k \in \{\mathrm{p}, \mathrm{s}\}$.

Proof: Let $\{\tilde{\lambda}_i^{\mathrm{p}}, \tilde{\lambda}_i^{\mathrm{s}}, \tilde{P}_i^{\mathrm{p}}, \tilde{P}_i^{\mathrm{s}}\}_{i=1}^K$ denote the solution of the optimization problem in (4.2) and C^* denote the corresponding maximum average capacity. Then, the achieved maximum average capacity for the communication between the transmitter and the secondary receiver can be written as $C^* = \sum_{i=1}^{K} \tilde{\lambda}_i^{\mathrm{s}}\, C_i^{\mathrm{s}}(\tilde{P}_i^{\mathrm{s}})$. From the definition of $C_{\max}^{k}$ in (4.4), the following relation is obtained:

$$C^* = \sum_{i=1}^{K} \tilde{\lambda}_i^{\mathrm{s}}\, C_i^{\mathrm{s}}(\tilde{P}_i^{\mathrm{s}}) \leq \sum_{i=1}^{K} \tilde{\lambda}_i^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\tilde{P}_i^{\mathrm{s}}). \tag{4.5}$$

It is noted that $\{\tilde{\lambda}_i^{\mathrm{p}}, \tilde{\lambda}_i^{\mathrm{s}}, \tilde{P}_i^{\mathrm{p}}, \tilde{P}_i^{\mathrm{s}}\}_{i=1}^K$ satisfies the constraints in (4.3). Therefore, it is deduced that the problem in (4.3) can achieve the maximum average capacity obtained by the problem in (4.2); that is, $C^* \leq C^\star$, where $C^\star$ denotes the maximum average capacity according to (4.3). Next, consider the solution of the optimization problem in (4.3). The maximum average capacity obtained by (4.3) can be expressed as $C^\star = \sum_{i=1}^{K} \bar{\lambda}_i^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\bar{P}_i^{\mathrm{s}})$, where $\{\bar{\lambda}_i^{\mathrm{p}}, \bar{\lambda}_i^{\mathrm{s}}, \bar{P}_i^{\mathrm{p}}, \bar{P}_i^{\mathrm{s}}\}_{i=1}^K$ denotes the solution of (4.3). Now, define functions $g^{(k)}(i)$ for $k \in \{\mathrm{p}, \mathrm{s}\}$ and sets $S_m^{(k)}$ for $k \in \{\mathrm{p}, \mathrm{s}\}$ as follows:[2]

$$g^{(k)}(i) \triangleq \underset{l \in \{1,\ldots,K\}}{\arg\max}\; C_l(\bar{P}_i^k)\,,\ \forall i \in \{1,\ldots,K\} \tag{4.6}$$

and

$$S_m^{(k)} \triangleq \{i \in \{1,\ldots,K\} \mid g^{(k)}(i) = m\}\,,\ \forall m \in \{1,\ldots,K\}. \tag{4.7}$$

Then, the following relations can be obtained for $k \in \{\mathrm{p}, \mathrm{s}\}$:

$$\begin{aligned}
\sum_{i=1}^{K} \bar{\lambda}_i^k\, C_{\max}^k(\bar{P}_i^k) &= \sum_{i=1}^{K} \bar{\lambda}_i^k\, C_{g^{(k)}(i)}^k(\bar{P}_i^k) && (4.8)\\
&= \sum_{i=1}^{K} \sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k\, C_i^k(\bar{P}_n^k) && (4.9)\\
&\le \sum_{i=1}^{K} \left(\sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k \right) C_i^k \left(\frac{\sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k \bar{P}_n^k}{\sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k} \right) && (4.10)\\
&= \sum_{i=1}^{K} \hat{\lambda}_i^k\, C_i^k(\hat{P}_i^k) && (4.11)
\end{aligned}$$

where $\hat{\lambda}_i^k$ and $\hat{P}_i^k$ are defined as

$$\hat{\lambda}_i^k \triangleq \sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k \quad \text{and} \quad \hat{P}_i^k \triangleq \frac{\sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k \bar{P}_n^k}{\sum_{n \in S_i^{(k)}} \bar{\lambda}_n^k} \tag{4.12}$$

for $i \in \{1,\ldots,K\}$. The equalities in (4.8) and (4.9) are obtained from the definitions in (4.6) and (4.7), respectively, and the inequality in (4.10) follows from Jensen's inequality due to the concavity of the capacity function [72, 73]. Based on the inequality in (4.8)–(4.11), it is obtained that $\hat{\lambda}_i^{\mathrm{p}}$'s and $\hat{P}_i^{\mathrm{p}}$'s satisfy the minimum average capacity requirement in (4.2); that is, $\sum_{i=1}^{K} \hat{\lambda}_i^{\mathrm{p}}\, C_i^{\mathrm{p}}(\hat{P}_i^{\mathrm{p}}) \ge C_{\mathrm{req}}$ since $\sum_{i=1}^{K} \hat{\lambda}_i^{\mathrm{p}}\, C_i^{\mathrm{p}}(\hat{P}_i^{\mathrm{p}}) \ge \sum_{i=1}^{K} \bar{\lambda}_i^{\mathrm{p}}\, C_{\max}^{\mathrm{p}}(\bar{P}_i^{\mathrm{p}})$ and $\sum_{i=1}^{K} \bar{\lambda}_i^{\mathrm{p}}\, C_{\max}^{\mathrm{p}}(\bar{P}_i^{\mathrm{p}}) \ge C_{\mathrm{req}}$. Also, it is noted from (4.12), based on (4.6) and (4.7), that $\hat{\lambda}_i^k$'s and $\hat{P}_i^k$'s for $k \in \{\mathrm{p}, \mathrm{s}\}$

[2] In the case of multiple maximizers in (4.6), any maximizing index can be chosen for $g^{(k)}(i)$.

satisfy the other constraints in (4.2); that is, $\sum_{i=1}^{K}(\hat{\lambda}_i^{\rm p}\,\hat{P}_i^{\rm p}+\hat{\lambda}_i^{\rm s}\,\hat{P}_i^{\rm s}) \leq P_{\rm av}$, $\hat{P}_i^{\rm p}, \hat{P}_i^{\rm s} \in [0, P_{\rm pk}]$, $\forall i \in \{1,\ldots,K\}$, $\sum_{i=1}^{K}(\hat{\lambda}_i^{\rm p} + \hat{\lambda}_i^{\rm s}) = 1$, and $\hat{\lambda}_i^{\rm p}, \hat{\lambda}_i^{\rm s} \geq 0$, $\forall i \in \{1,\ldots,K\}$. Therefore, the inequality in (4.8)–(4.11), namely, $C^\star \leq \sum_{i=1}^{K}\hat{\lambda}_i\,C_i(\hat{P}_i)$, implies that the optimal solution of (4.3) cannot achieve a higher average capacity than that achieved by (4.2); that is, $C^\star \leq C^*$. Hence, it is concluded that $C^\star = C^*$ since $C^\star \geq C^*$ must also hold as mentioned at the beginning of the proof. ■

Based on Proposition 1, the solution of the original problem in (4.2) can be obtained from the optimization problem in (4.3), which is more tractable than the one in (4.2), as investigated in the following. Proposition 1 also implies that an optimal strategy always utilizes the best channel for a given power level, as noted from (4.3a), (4.3b), and (4.4), which is intuitive due to the monotone increasing nature of the capacity expression.

As a first step towards characterizing the solution of (4.3), the following proposition provides a useful statement that the constraints in (4.3b) and (4.3c) always hold with equality.

Proposition 2: *The solution of the optimization problem in* (4.3) *satisfies the constraints in* (4.3b) *and* (4.3c) *with equality; that is,* $\sum_{i=1}^{K}\bar{\lambda}_i^{\rm p}\,C_{\rm max}^{\rm p}(\bar{P}_i^{\rm p}) = C_{\rm req}$ *and* $\sum_{i=1}^{K}\bar{\lambda}_i^{\rm p}\,\bar{P}_i^{\rm p} + \bar{\lambda}_i^{\rm s}\,\bar{P}_i^{\rm s} = P_{\rm av}$*, where* $\{\bar{\lambda}_i^{\rm p}, \bar{\lambda}_i^{\rm s}, \bar{P}_i^{\rm p}, \bar{P}_i^{\rm s}\}_{i=1}^{K}$ *denotes the solution of* (4.3).

Proof: Assume that $\{\bar{\lambda}_i^{\rm p}, \bar{\lambda}_i^{\rm s}, \bar{P}_i^{\rm p}, \bar{P}_i^{\rm s}\}_{i=1}^{K}$ is the solution of (4.3) such that $\sum_{i=1}^{K}(\bar{\lambda}_i^{\rm p}\,\bar{P}_i^{\rm p} + \bar{\lambda}_i^{\rm s}\,\bar{P}_i^{\rm s}) < P_{\rm av}$. Then, the following cases are considered[3]:

- If $\bar{\lambda}_i^{\rm s} = 0$, $\forall i \in \{1,\ldots,K\}$, then there exists at least one $\bar{P}_i^{\rm p}$ such that $\bar{P}_i^{\rm p} < P_{\rm pk}$ since $\sum_{i=1}^{K}\lambda_i^{\rm p}\,P_i^{\rm p} \leq P_{\rm av}$ and $\sum_{i=1}^{K}\lambda_i^{\rm p} = 1$ due to the constraints in (4.3c) and (4.3d), respectively, and $P_{\rm av} < P_{\rm pk}$ by the assumption for (4.2). Let $\bar{P}_l^{\rm p}$ denote one of them. Then, consider an alternative solution

[3]In this case, it is assumed that multiple channels are available for communication; that is, $K > 1$. In the case of a single channel available for communication (i.e., $K = 1$), a similar approach can be employed.

$\{\hat{\lambda}_i^{\mathrm{p}}, \hat{\lambda}_i^{\mathrm{s}}, \hat{P}_i^{\mathrm{p}}, \hat{P}_i^{\mathrm{s}}\}_{i=1}^{K}$, where

$$\hat{P}_l^{\mathrm{p}} = \min\left\{P_{\mathrm{pk}}, \bar{P}_l^{\mathrm{p}} + \left(P_{\mathrm{av}} - \sum_{i=1}^{K} \bar{\lambda}_i^{\mathrm{p}} \bar{P}_i^{\mathrm{p}}\right) / \bar{\lambda}_l^{\mathrm{p}}\right\}, \tag{4.13}$$

$$\hat{\lambda}_l^{\mathrm{p}} = \frac{\bar{\lambda}_l^{\mathrm{p}} \, C_{\max}^{\mathrm{p}}(\bar{P}_l^{\mathrm{p}})}{C_{\max}^{\mathrm{p}}(\hat{P}_l^{\mathrm{p}})}, \tag{4.14}$$

$$\hat{\lambda}_i^{\mathrm{p}} = \bar{\lambda}_i^{\mathrm{p}}\,,\ \forall i \in \{1, \ldots, K\} \setminus \{l\}, \tag{4.15}$$

$$\hat{P}_i^{\mathrm{p}} = \bar{P}_i^{\mathrm{p}}\,,\ \forall i \in \{1, \ldots, K\} \setminus \{l\}, \tag{4.16}$$

$$\hat{\lambda}_1^{\mathrm{s}} = \bar{\lambda}_l^{\mathrm{p}} - \hat{\lambda}_l^{\mathrm{p}}, \tag{4.17}$$

$$\hat{P}_1^{\mathrm{s}} = \hat{P}_l^{\mathrm{p}}, \tag{4.18}$$

$$\hat{\lambda}_i^{\mathrm{s}} = \bar{\lambda}_i^{\mathrm{s}}\,,\ \forall i \in \{2, \ldots, K\}, \tag{4.19}$$

$$\hat{P}_i^{\mathrm{s}} = \bar{P}_i^{\mathrm{s}}\,,\ \forall i \in \{2, \ldots, K\}. \tag{4.20}$$

The solution $\{\bar{\lambda}_i^{\mathrm{p}}, \bar{\lambda}_i^{\mathrm{s}}, \bar{P}_i^{\mathrm{p}}, \bar{P}_i^{\mathrm{s}}\}_{i=1}^{K}$ achieves an average capacity of $\bar{C}^s = 0$ due to $\bar{\lambda}_i^{\mathrm{s}} = 0\,,\ \forall i \in \{1, \ldots, K\}$. On the other hand, the alternative solution satisfies the constraints in (4.3) and achieves a larger capacity; that is $\hat{C}^s = \hat{\lambda}_1^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\hat{P}_1^{\mathrm{s}}) > 0$ since $\hat{\lambda}_1^{\mathrm{s}} > 0$ and $\hat{P}_1^{\mathrm{s}} > 0$. Therefore, $\{\bar{\lambda}_i^{\mathrm{p}}, \bar{\lambda}_i^{\mathrm{s}}, \bar{P}_i^{\mathrm{p}}, \bar{P}_i^{\mathrm{s}}\}_{i=1}^{K}$ cannot be optimal if $\bar{\lambda}_i^{\mathrm{s}} = 0\,,\ \forall i \in \{1, \ldots, K\}$, which contradicts with the assumption at the beginning of the proof.

- For the case that $\bar{\lambda}_i^{\mathrm{s}} > 0\,,\ \exists i \in \{1, \ldots, K\}$, define a set as

$$M \triangleq \{i \in \{1, \ldots, K\} \mid \bar{\lambda}_i^{\mathrm{s}} > 0\}\,. \tag{4.21}$$

Next, consider the following cases:

  - If $\bar{P}_k^{\mathrm{s}} = P_{\mathrm{pk}}\,,\ \forall k \in M$, then there exists at least one $\bar{P}_i^{\mathrm{p}}$ that satisfies $\bar{P}_i^{\mathrm{p}} < P_{\mathrm{pk}}$ since the constraints in (4.3c) and (4.3d) hold. Let $\bar{P}_l^{\mathrm{p}}$ represent one of them and consider an alternative solution $\{\hat{\lambda}_i^{\mathrm{p}}, \hat{\lambda}_i^{\mathrm{s}}, \hat{P}_i^{\mathrm{p}}, \hat{P}_i^{\mathrm{s}}\}_{i=1}^{K}$, where $\hat{P}_l^{\mathrm{p}}$, $\hat{\lambda}_l^{\mathrm{p}}$, $\hat{\lambda}_i^{\mathrm{p}}$ for all $i \in \{1, \ldots, K\} \setminus \{l\}$, $\hat{P}_i^{\mathrm{p}}$ for all $i \in \{1, \ldots, K\} \setminus \{l\}$, $\hat{\lambda}_1^{\mathrm{s}}$, and $\hat{P}_1^{\mathrm{s}}$ are as in (4.13)-(4.18) and the remaining terms are as follows:

$$\hat{\lambda}_2^{\mathrm{s}} = \sum_{k \in M} \bar{\lambda}_k^{\mathrm{s}}, \tag{4.22}$$

$$\hat{P}_2^{\mathrm{s}} = P_{\mathrm{pk}}, \tag{4.23}$$

$$\hat{\lambda}_i^{\mathrm{s}} = 0\,,\ \forall i \in \{3, \ldots, K\}, \tag{4.24}$$

$$\hat{P}_i^{\mathrm{s}} = 0\,,\ \forall i \in \{3, \ldots, K\}. \tag{4.25}$$

The achieved average capacity by $\{\bar{\lambda}_i^{\mathrm{p}}, \bar{\lambda}_i^{\mathrm{s}}, \bar{P}_i^{\mathrm{p}}, \bar{P}_i^{\mathrm{s}}\}_{i=1}^{K}$ is $\bar{C}^s = \sum_{i=1}^{K} \bar{\lambda}_i^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\bar{P}_i^{\mathrm{s}})$, which is lower than that achieved by the alternative solution due to the following relation:

$$\begin{aligned}
\bar{C}^s = \sum_{i=1}^{K} \bar{\lambda}_i^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\bar{P}_i^{\mathrm{s}}) &= \sum_{k \in M} \bar{\lambda}_k^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(P_{\mathrm{pk}}) && (4.26)\\
&< \sum_{k \in M} \bar{\lambda}_k^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(P_{\mathrm{pk}}) \\
&\quad + \hat{\lambda}_1^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\hat{P}_1^{\mathrm{s}}) && (4.27)\\
&= \sum_{i=1}^{K} \hat{\lambda}_i^{\mathrm{s}}\, C_{\max}^{\mathrm{s}}(\hat{P}_i^{\mathrm{s}}) && (4.28)\\
&= \hat{C}^s && (4.29)
\end{aligned}$$

where (4.26) follows from the condition that $\bar{P}_k^{\mathrm{s}} = P_{\mathrm{pk}}\,,\ \forall k \in M$, the inequality in (4.27) is due to $\hat{\lambda}_1^{\mathrm{s}} > 0$ and $\hat{P}_1^{\mathrm{s}} > 0$, (4.28) is obtained based on (4.13)-(4.18) and (4.22)-(4.25), and finally $\hat{C}^s$ in (4.28) denotes the achieved average capacity by the alternative solution. Based on (4.26)-(4.29), it is obtained that $\bar{C}^s < \hat{C}^s$. Therefore, $\{\bar{\lambda}_i^{\mathrm{p}}, \bar{\lambda}_i^{\mathrm{s}}, \bar{P}_i^{\mathrm{p}}, \bar{P}_i^{\mathrm{s}}\}_{i=1}^{K}$ cannot be optimal and consequently the assumption at the beginning of the proof must be false if $\bar{P}_k^{\mathrm{s}} = P_{\mathrm{pk}}\,,\ \forall k \in M$ for the case that $\bar{\lambda}_i^{\mathrm{s}} > 0\,,\ \exists i \in \{1, \ldots, K\}$.

– If $\bar{P}_k^{\mathrm{s}} < P_{\mathrm{pk}}\,,\ \exists k \in M$, then based on a similar approach to that in Lemma 1 of [33], an alternative solution $\{\hat{\lambda}_i^{\mathrm{p}}, \hat{\lambda}_i^{\mathrm{s}}, \hat{P}_i^{\mathrm{p}}, \hat{P}_i^{\mathrm{s}}\}_{i=1}^{K}$ can be expressed as

$$\hat{\lambda}^{\mathrm{p}}_i = \bar{\lambda}^{\mathrm{p}}_i \,, \ \forall i \in \{1, \ldots, K\}, \tag{4.30}$$

$$\hat{P}^{\mathrm{p}}_i = \bar{P}^{\mathrm{p}}_i \,, \ \forall i \in \{1, \ldots, K\}, \tag{4.31}$$

$$\hat{P}^{\mathrm{s}}_l = \min\left\{ P_{\mathrm{pk}}, \bar{P}^{\mathrm{s}}_l + \left(P_{\mathrm{av}} - \sum_{i=1}^{K} \bar{\lambda}^{\mathrm{s}}_i \bar{P}^{\mathrm{s}}_i \right) / \bar{\lambda}^{\mathrm{s}}_l \right\}, \tag{4.32}$$

$$\hat{P}^{\mathrm{s}}_i = \bar{P}^{\mathrm{s}}_i \,, \ \forall i \in \{1, \ldots, K\} \setminus \{l\}, \tag{4.33}$$

$$\hat{\lambda}^{\mathrm{s}}_i = \bar{\lambda}^{\mathrm{s}}_i \,, \ \forall i \in \{1, \ldots, K\} \tag{4.34}$$

where $\bar{P}^{\mathrm{s}}_l$ is one of the power levels that satisfies $\bar{P}^{\mathrm{s}}_l < P_{\mathrm{pk}}$. Since $\hat{P}^{\mathrm{s}}_l > \bar{P}^{\mathrm{s}}_l$ and $C^{\mathrm{s}}_{\max}(P)$ in (4.4) is a monotone increasing function of P, it is obtained that the alternative solution achieves a larger average capacity than $\{\bar{\lambda}^{\mathrm{p}}_i, \bar{\lambda}^{\mathrm{s}}_i, \bar{P}^{\mathrm{p}}_i, \bar{P}^{\mathrm{s}}_i\}_{i=1}^{K}$ does. Therefore, the assumption at the beginning of the proof must not be true.

Based on the cases specified above, it is concluded by contradiction that the solution of the optimization problem in (4.3) satisfies the constraint in (4.3c) with equality; that is, $\sum_{i=1}^{K} \bar{\lambda}^{\mathrm{p}}_i \bar{P}^{\mathrm{p}}_i + \bar{\lambda}^{\mathrm{s}}_i \bar{P}^{\mathrm{s}}_i = P_{\mathrm{av}}$.

In the second part of the proof, the aim is to prove that the solution of (4.3) satisfies the constraint in (4.3b) with equality. Assume that $\{\bar{\lambda}^{\mathrm{p}}_i, \bar{\lambda}^{\mathrm{s}}_i, \bar{P}^{\mathrm{p}}_i, \bar{P}^{\mathrm{s}}_i\}_{i=1}^{K}$ is the solution of (4.3) such that $\sum_{i=1}^{K} \bar{\lambda}^{\mathrm{p}}_i C^{\mathrm{p}}_{\max}(\bar{P}^{\mathrm{p}}_i) > C_{\mathrm{req}}$. Since $C_{\mathrm{req}} > 0$ by assumption, there exists at least one $\{\bar{\lambda}^{\mathrm{p}}_i, \bar{P}^{\mathrm{p}}_i\}$ pair such that $\bar{\lambda}^{\mathrm{p}}_i > 0$ and $\bar{P}^{\mathrm{p}}_i > 0$. Let $\{\bar{\lambda}^{\mathrm{p}}_l, \bar{P}^{\mathrm{p}}_l\}$ denote one of them. Then, there exists a non-negative $\hat{P}^{\mathrm{p}}_l < \bar{P}^{\mathrm{p}}_l$ such that $\sum_{i=1}^{K} \hat{\lambda}^{\mathrm{p}}_i C^{\mathrm{p}}_{\max}(\hat{P}^{\mathrm{p}}_i) \geq C_{\mathrm{req}}$, where $\hat{\lambda}^{\mathrm{p}}_i = \bar{\lambda}^{\mathrm{p}}_i$ for all $i \in \{1, \ldots, K\}$ and $\hat{P}^{\mathrm{p}}_i = \bar{P}^{\mathrm{p}}_i$ for all $i \in \{1, \ldots, K\} \setminus \{l\}$ since $C^{\mathrm{p}}_{\max}(P)$ is a monotone increasing and continuous function of P.

- If $\bar{\lambda}^{\mathrm{s}}_i = 0\,, \ \forall i \in \{1, \ldots, K\}$, then consider an alternative solution

$\{\hat{\lambda}_i^{\mathrm{p}}, \hat{\lambda}_i^{\mathrm{s}}, \hat{P}_i^{\mathrm{p}}, \hat{P}_i^{\mathrm{s}}\}_{i=1}^{K}$, where

$$\hat{\lambda}_1^{\mathrm{s}} = \bar{\lambda}_l^{\mathrm{p}}, \tag{4.35}$$

$$\hat{P}_1^{\mathrm{s}} = \bar{P}_l^{\mathrm{p}} - \hat{P}_l^{\mathrm{p}}, \tag{4.36}$$

$$\hat{\lambda}_i^{\mathrm{s}} = \bar{\lambda}_i^{\mathrm{s}}\,,\ \forall i \in \{2, \ldots, K\}, \tag{4.37}$$

$$\hat{P}_i^{\mathrm{s}} = \bar{P}_i^{\mathrm{s}}\,,\ \forall i \in \{2, \ldots, K\}. \tag{4.38}$$

- For the case that $\bar{\lambda}_i^{\mathrm{s}} > 0\,,\ \exists i \in \{1, \ldots, K\}$, define a set as

$$M \triangleq \{i \in \{1, \ldots, K\} \mid \bar{\lambda}_i^{\mathrm{s}} > 0\}. \tag{4.39}$$

  Next, consider the following cases:

  – If $\bar{P}_k^{\mathrm{s}} = P_{\mathrm{pk}}\,,\ \forall k \in M$, then consider an alternative solution $\{\hat{\lambda}_i^{\mathrm{p}}, \hat{\lambda}_i^{\mathrm{s}}, \hat{P}_i^{\mathrm{p}}, \hat{P}_i^{\mathrm{s}}\}_{i=1}^{K}$, where $\hat{\lambda}_1^{\mathrm{s}}$ and $\hat{P}_1^{\mathrm{s}}$ are as in (4.36) and (4.37), respectively, and

$$\hat{\lambda}_2^{\mathrm{s}} = \sum_{k \in M} \bar{\lambda}_k^{\mathrm{s}}, \tag{4.40}$$

$$\hat{P}_2^{\mathrm{s}} = P_{\mathrm{pk}}, \tag{4.41}$$

$$\hat{\lambda}_i^{\mathrm{s}} = 0\,,\ \forall i \in \{3, \ldots, K\}, \tag{4.42}$$

$$\hat{P}_i^{\mathrm{s}} = 0\,,\ \forall i \in \{3, \ldots, K\}. \tag{4.43}$$

  – If $\bar{P}_k^{\mathrm{s}} < P_{\mathrm{pk}}\,,\ \exists k \in M$, then based on a similar approach to that in Lemma 1 of [33], an alternative solution $\{\hat{\lambda}_i^{\mathrm{p}}, \hat{\lambda}_i^{\mathrm{s}}, \hat{P}_i^{\mathrm{p}}, \hat{P}_i^{\mathrm{s}}\}_{i=1}^{K}$ can be expressed as

$$\hat{P}_l^{\mathrm{s}} = \min\{P_{\mathrm{pk}}, \bar{P}_l^{\mathrm{s}} + \bar{\lambda}_l^{\mathrm{p}}(\bar{P}_l^{\mathrm{p}} - \hat{P}_l^{\mathrm{p}})/\bar{\lambda}_l^{\mathrm{s}}\}, \tag{4.44}$$

$$\hat{P}_i^{\mathrm{s}} = \bar{P}_i^{\mathrm{s}}\,,\ \forall i \in \{1, \ldots, K\} \setminus \{l\}, \tag{4.45}$$

$$\hat{\lambda}_i^{\mathrm{s}} = \bar{\lambda}_i^{\mathrm{s}}\,,\ \forall i \in \{1, \ldots, K\} \tag{4.46}$$

  where $\bar{P}_l^{\mathrm{s}}$ is one of the power levels that satisfies $\bar{P}_l^{\mathrm{s}} < P_{\mathrm{pk}}$.

Similar to the first part of the proof, all alternative solutions specified for the cases above achieve a larger average capacity than $\{\bar{\lambda}_i^{\rm p}, \bar{\lambda}_i^{\rm s}, \bar{P}_i^{\rm p}, \bar{P}_i^{\rm s}\}_{i=1}^K$ does. Therefore, it is proved by contradiction that the solution satisfies the constraint in (4.3b) with equality; that is, $\sum_{i=1}^K \bar{\lambda}_i^{\rm p} C_{\max}^{\rm p}(\bar{P}_i^{\rm p}) = C_{\rm req}$. ■

Even though Proposition 2 states that the constraints in (4.3b) and (4.3c) are satisfied with equality, it is still difficult to solve the optimization problem in (4.3). Therefore, the following proposition is presented in order to provide a further simplification for the optimization problem in (4.3).

Proposition 3: *The optimal channel switching strategy based on the optimization problem in* (4.3) *employs at most* $\min\{3, 2K\}$ *communication links.*

Proof: If $K \leq 1$, then the assertion in Proposition 3 holds obviously. Otherwise, (if $K > 1$), then consider the following transformations:

$$\nu_i = \begin{cases} \lambda_i^{\rm p}, & \text{if } i \leq K \\ \lambda_m^{\rm s}, & \text{if } i > K \end{cases}, \ \forall i \in \{1, \ldots, 2K\} \tag{4.47}$$

$$P_i = \begin{cases} P_i^{\rm p}, & \text{if } i \leq K \\ P_m^{\rm s}, & \text{if } i > K \end{cases}, \ \forall i \in \{1, \ldots, 2K\} \tag{4.48}$$

where $m \triangleq i - K$. Also, define the following functions:

$$C_{\max,i}^{\rm p}(P) = \begin{cases} C_{\max}^{\rm p}(P), & \text{if } i \leq K \\ 0, & \text{if } i > K \end{cases}, \ \forall i \in \{1, \ldots, 2K\} \tag{4.49}$$

$$C_{\max,i}^{\rm s}(P) = \begin{cases} 0, & \text{if } i \leq K \\ C_{\max}^{\rm s}(P), & \text{if } i > K \end{cases}, \ \forall i \in \{1, \ldots, 2K\} \tag{4.50}$$

for all $P \in [0, P_{\rm pk}]$. Based on the transformations in (4.47) and (4.48) and the functions in (4.49) and (4.50), the optimization problem in (4.3) can be written in the following form:

$$\max_{\{\nu_i, P_i\}_{i=1}^{2K}} \sum_{i=1}^{2K} \nu_i \, C^{\rm s}_{\max,i}(P_i) \tag{4.51a}$$

$$\text{subject to} \sum_{i=1}^{2K} \nu_i \, C^{\rm p}_{\max,i}(P_i) \geq C_{\rm req} \tag{4.51b}$$

$$\sum_{i=1}^{2K} \nu_i \, P_i \leq P_{\rm av} \,, \;\; P_i \in [0, P_{\rm pk}] \,, \;\; \forall i \in \{1, \ldots, 2K\} \tag{4.51c}$$

$$\sum_{i=1}^{2K} \nu_i = 1 \,, \;\; \nu_i \in [0,1] \,, \;\; \forall i \in \{1, \ldots, 2K\} \tag{4.51d}$$

Next, define the following sets:

$$\mathcal{V} = \Bigg\{ \left(\sum_{i=1}^{2K} \nu_i \, C^{\rm p}_{\max,i}(P_i), \sum_{i=1}^{2K} \nu_i \, C^{\rm s}_{\max,i}(P_i), \sum_{i=1}^{2K} \nu_i \, P_i \right) \in \mathbb{R}^3 \\ \Bigg| \sum_{i=1}^{2K} \nu_i = 1 \,, \;\; \nu_i \in [0,1] \,, \;\; P_i \in [0, P_{\rm pk}] \,, \;\; \forall i \in \{1, \ldots, 2K\} \Bigg\} \tag{4.52}$$

$$\mathcal{W} = \left\{ w = \{u_1, \ldots, u_{2K}\} \,\middle|\, u_i \in \mathcal{U}_i \,, \;\; \forall i \in \{1, \ldots, 2K\} \right\} \tag{4.53}$$

where

$$\mathcal{U}_i = \left\{ \left(P, C^{\rm p}_{\max,i}(P), C^{\rm s}_{\max,i}(P)\right) \in \mathbb{R}^3 \,\middle|\, P \in [0, P_{\rm pk}] \right\} , \;\; \forall i \in \{1, \ldots, 2K\} \,. \tag{4.54}$$

It is noted that set $\mathcal{V}$ includes the solution of the optimization problem in (4.51) by definition. Let $\mathcal{W}_i$ represent the ith element of set $\mathcal{W}$, which is also a set. Then, set $\mathcal{V}$ is equal to the union of the convex hulls of set $\mathcal{W}_i \,, \;\forall i \in \{1, \ldots, |\mathcal{W}|\}$; that is, $\mathcal{V} = \bigcup_{i=1}^{|\mathcal{W}|} \text{conv}(\mathcal{W}_i)$. Therefore, $\bigcup_{i=1}^{|\mathcal{W}|} \text{conv}(\mathcal{W}_i)$ also includes the solution of the optimization problem in (4.51). The definition of union implies that the solution of (4.51) is an element of $\text{conv}(\mathcal{W}_i)$ for some $i \in \{1, \ldots, |\mathcal{W}|\}$. Without loss of generality, let l be one of them. Since the optimization problem in (4.51) is a maximization problem, the solution of (4.51) resides on the boundary of the convex hull of set $\mathcal{W}_l$. Then, by Carathéodory's theorem [75, 76], any point on the boundary of the convex hull of set $\mathcal{W}_l$ can be represented by a convex combination of at most d points in set $\mathcal{W}_l$, where d is the dimension of space in which $\mathcal{W}_l$ resides. Since $\mathcal{W}_l \subset \mathbb{R}^3$ and the optimal solution of (4.51) corresponds to a point on the boundary of conv($\mathcal{W}_l$), the optimal channel switching strategy employs at most 3 communication links. ■

Based on Proposition 3 and the study in [33], the optimal channel switching strategy can be investigated as follows: Let $\bar{C}_{\text{req}}$ denote the achieved maximum average capacity for the communication between the transmitter and the primary receiver when there is no secondary receiver in the system. Then, $\bar{C}_{\text{req}}$ can be calculated as follows:

$$\bar{C}_{\text{req}} = \max_{\{\lambda_i^{\text{p}}, P_i^{\text{p}}\}_{i=1}^K} \sum_{i=1}^{K} \lambda_i^{\text{p}}\, C_{\max}^{\text{p}}(P_i^{\text{p}}) \tag{4.55a}$$

$$\text{subject to} \sum_{i=1}^{K} \lambda_i^{\text{p}}\, P_i^{\text{p}} = P_{\text{av}}\,,$$

$$P_i^{\text{p}} \in [0, P_{\text{pk}}]\,,\ \forall i \in \{1, \dots, K\} \tag{4.55b}$$

$$\sum_{i=1}^{K} \lambda_i^{\text{p}} = 1,\ \lambda_i^{\text{p}} \in [0, 1],\ \forall i \in \{1, \dots, K\} \tag{4.55c}$$

If the maximum average capacity achieved by the optimization problem in (4.55) is strictly lower than the minimum average capacity requirement for the primary receiver (i.e., $\bar{C}_{\text{req}} < C_{\text{req}}$), then there is no possible channel switching strategy for the problem in (4.2) since the optimization problem in (4.3) is infeasible. If $\bar{C}_{\text{req}} = C_{\text{req}}$, the optimal channel switching strategy corresponds to switching between at most two channels between the transmitter and the primary receiver based on the optimization problem in (4.2) and Proposition 4 in [33]. In that case, no communication is performed between the transmitter and the secondary receiver. Therefore, the achieved maximum average capacity is $C^\star = 0$. Finally, if $\bar{C}_{\text{req}} > C_{\text{req}}$, then the optimal channel switching strategy corresponds to one of the following two strategies:

- **Strategy-1 (Communicate with the primary receiver over at most**

two channels and employ single channel for the secondary receiver): In this strategy, the transmitter employs one or two channels to satisfy the minimum average capacity requirement of the primary receiver and uses only one channel in order to maximize the average capacity of the communication to the secondary receiver. The achieved maximum average capacity for the communication to the secondary receiver, denoted by $C_{\text{str},1}$, can be calculated as follows:

$$C_{\text{str},1} = \max_{\lambda_1,\lambda_2,\lambda_3,P_1,P_2,P_3} \lambda_1 \, C^{\text{s}}_{\max}(P_1) \tag{4.56a}$$

$$\text{subject to } \lambda_2 \, C^{\text{p}}_{\max}(P_2) + \lambda_3 \, C^{\text{p}}_{\max}(P_3) = C_{\text{req}} \tag{4.56b}$$

$$\lambda_1 \, P_1 + \lambda_2 \, P_2 + \lambda_3 \, P_3 = P_{\text{av}} \,,$$

$$P_1, P_2, P_3 \in [0, P_{\text{pk}}] \,, \tag{4.56c}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1, \;\; \lambda_1, \lambda_2, \lambda_3 \in [0, 1] \tag{4.56d}$$

- **Strategy-2 (Communicate with the secondary receiver over at most two channels and employ single channel for the primary receiver):** In this case, the transmitter maximizes the average capacity of the communication to the secondary receiver by employing one or two channels while meeting the minimum average capacity requirement for the primary receiver via communication over a single channel. In this case, the achieved average capacity can be expressed as

$$C_{\text{str},2} = \max_{\lambda_1,\lambda_2,\lambda_3,P_1,P_2,P_3} \lambda_1 \, C^{\text{s}}_{\max}(P_1) + \lambda_2 \, C^{\text{s}}_{\max}(P_2) \tag{4.57a}$$

$$\text{subject to } \lambda_3 \, C^{\text{p}}_{\max}(P_3) = C_{\text{req}} \tag{4.57b}$$

$$\lambda_1 \, P_1 + \lambda_2 \, P_2 + \lambda_3 \, P_3 = P_{\text{av}} \,,$$

$$P_1, P_2, P_3 \in [0, P_{\text{pk}}] \,, \tag{4.57c}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1, \;\; \lambda_1, \lambda_2, \lambda_3 \in [0, 1] \tag{4.57d}$$

Based on Strategy 1 and Strategy 2, the maximum average capacity for the communication between the transmitter and the secondary receiver, which is the solution of (4.2), can be calculated as

$$C^{\star} = \max\{C_{\mathrm{str},1}, C_{\mathrm{str},2}\} \tag{4.58}$$

where $C_{\mathrm{str},1}$ and $C_{\mathrm{str},2}$ are as in (4.56) and (4.57), respectively.

It is important to note that the optimization problems in (4.56) and (4.57) have significantly lower computational complexity compared to the original optimization problem in (4.2) since each of (4.56) and (4.57) requires a search only over a 3 dimensional space[4] whereas a search over a $4K$ dimensional space is required for the problem in (4.2), where $K > 1$.

4.3 Optimal Channel Switching in the Presence of Multiple Primary Receivers

In the presence of multiple primary receivers, each having an individual minimum average capacity requirement, the optimization problem in (4.2) can be extended

[4]Note that the search space dimensions of the optimization problems in (4.56) and (4.57) are obtained by substituting the equality constraints in (4.56b)-(4.56d) and (4.57b)-(4.57d) into the objective functions in (4.56a) and (4.57a), respectively.

as follows:

$$\max_{\left\{\lambda_i^{\rm s}, P_i^{\rm s}, \left\{\lambda_i^{{\rm p}_j}, P_i^{{\rm p}_j}\right\}_{j=1}^{N}\right\}_{i=1}^{K}} \sum_{i=1}^{K} \lambda_i^{\rm s}\, C_i^{\rm s}(P_i^{\rm s}) \tag{4.59a}$$

$$\text{subject to } \sum_{i=1}^{K} \lambda_i^{{\rm p}_j}\, C_i^{{\rm p}_j}(P_i^{{\rm p}_j}) \geq C_{\rm req}^{j}\,,\ \forall j \in \{1, \ldots, N\}, \tag{4.59b}$$

$$\sum_{i=1}^{K} \left(\lambda_i^{\rm s}\, P_i^{\rm s} + \sum_{j=1}^{N} \lambda_i^{{\rm p}_j}\, P_i^{{\rm p}_j} \right) \leq P_{\rm av}, \tag{4.59c}$$

$$P_i^{\rm s}, P_i^{{\rm p}_j} \in [0, P_{\rm pk}],\ \forall i \in \{1, \ldots, K\},\ \forall j \in \{1, \ldots, N\}, \tag{4.59d}$$

$$\sum_{i=1}^{K} \left(\lambda_i^{\rm s} + \sum_{j=1}^{N} \lambda_i^{{\rm p}_j} \right) = 1, \tag{4.59e}$$

$$\lambda_i^{\rm s}, \lambda_i^{{\rm p}_j} \in [0, 1],\ \forall i \in \{1, \ldots, K\},\ \forall j \in \{1, \ldots, N\}, \tag{4.59f}$$

where $\lambda_i^{{\rm p}_j}$ and $P_i^{{\rm p}_j}$ denote, respectively, the time-sharing factor and the average transmit power allocated to channel i for the communication between the transmitter and the jth primary receiver, N is the number of primary receivers in the system, $C_i^{{\rm p}_j}(P)$ as defined in (4.1), $C_{\rm req}^{j}$ is the minimum average capacity requirement for the jth primary receiver, and the other parameters are as in (4.2).

It is noted that the optimization problem in (4.2) is a special case of (4.59) when there exists only one primary receiver; that is, when $N = 1$. Therefore, it is in general more difficult to solve the optimization problem in (4.59) since it requires a search over a $2K(N + 1)$ dimensional space, which is higher than $4K$, corresponding to (4.2), for $N > 1$. However, the results obtained for the problem in (4.2) can be extended for (4.59), as explained in the following remark.

Remark 1: *Based on a similar approach to that in Proposition 1, it can be shown that an alternative optimization problem to the problem in* (4.59) *can be obtained. Also, the approach in Proposition 2 also holds for the optimization problem in* (4.59) *and consequently the solution of* (4.59) *satisfies the constraints in* (4.59b) *and* (4.59c) *with equality. Moreover, similar to the proof in Proposition 3, it can be stated for the optimization problem in* (4.59) *that the optimal channel switching strategy based on* (4.59) *employs at most* $\min\{N+2, K(N+1)\}$

communication links in the system, where $K(N+1)$ links are available in total. Specifically, the optimal channel switching strategy can be realized by switching among at most $(N+2)$ communication links in the presence of multiple available channels in the system; that is, $K > 1$.

It is concluded from Remark 1 that the solution of (4.59) can be calculated by solving a total of $(N+1)$ optimization problems, each requiring a search over a $2(N+2)$ dimensional space, and then choosing the maximum among the obtained average capacities. Hence, the optimal channel switching strategy based on the optimization problem in (4.59) can be obtained with low computational complexity.

For complexity comparisons, assume that there exist finitely many possible values of λ_i^k and P_i^k for each $k \in \{\mathrm{p}, \mathrm{s}\}$ and $i \in \{1, \dots, K\}$, where $\lambda_i^k \in [0, 1]$ and $P_i^k \in [0, P_{\mathrm{pk}}]$ for all $k \in \{\mathrm{p}, \mathrm{s}\}$ and $i \in \{1, \dots, K\}$. Let N_λ denote the number of different λ values for $\lambda \in [0, 1]$ and N_P represent the number of different P values for $P \in [0, P_{\mathrm{pk}}]$. Then, the original optimization problem in (4.2) has a computational complexity of $\mathcal{O}(N_\lambda^{2K} \times N_P^{2K})$. On the other hand, the complexity of each optimization problem in (4.56) and (4.57) is in the order of $\mathcal{O}(N_\lambda^3 \times N_P^3)$. Therefore, in the presence of multiple available channels, instead of solving the original optimization problem in (4.2) with a complexity of $\mathcal{O}(N_\lambda^{2K} \times N_P^{2K})$ where $K > 1$, the solution of (4.2) can be obtained with a lower computational complexity by solving two optimization problems in (4.56) and (4.57), each having a computational complexity of $\mathcal{O}(N_\lambda^3 \times N_P^3)$. In the presence of N primary receivers in the communication system, the complexity of the optimization problem in (4.59) is $\mathcal{O}(N_\lambda^{K(N+1)} \times N_P^{K(N+1)})$. However, the solution of (4.59) can be calculated with a lower complexity by solving $N+1$ optimization problems, each having a computational complexity of $\mathcal{O}(N_\lambda^{N+2} \times N_P^{N+2})$.

4.4 Numerical Results

In this section, several numerical examples are presented to investigate the performance of the proposed strategies and to illustrate the optimal strategy for various values of the average power limit and the minimum average capacity requirement for the primary receiver. To that aim, a communication system is considered with $K = 5$ channels, the bandwidths and the noise levels of which are given by $B_1 = 1\,\text{MHz}$, $B_2 = 3\,\text{MHz}$, $B_3 = 4\,\text{MHz}$, $B_4 = 5\,\text{MHz}$, $B_5 = 10\,\text{MHz}$, and $N_1 = N_2 = N_3 = N_4 = N_5 = 10^{-12}\,\text{W/Hz}$ (cf. (4.1)). It is assumed that all the channels are available for the transmitter and can be used to communicate with both the primary and secondary receivers. Also, for these channels, the channel power gains from the transmitter to the primary and secondary receivers are given by $|h_1^{\text{p}}|^2 = 1$, $|h_2^{\text{p}}|^2 = 0.1$, $|h_3^{\text{p}}|^2 = 0.1$, $|h_4^{\text{p}}|^2 = 0.1$, $|h_5^{\text{p}}|^2 = 0.05$, $|h_1^{\text{s}}|^2 = 1$, $|h_2^{\text{s}}|^2 = 0.1$, $|h_3^{\text{s}}|^2 = 0.1$, $|h_4^{\text{s}}|^2 = 0.1$, and $|h_5^{\text{s}}|^2 = 0.1$. In this scenario, the peak power constraint in (4.2) is set to $P_{\text{pk}} = 0.1\,\text{mW}$. The capacity of each link available for the transmitter to communicate with the primary and secondary receivers is plotted as a function of power in Fig. 4.2 and Fig. 4.3, respectively.

In order to investigate the effect of the average power limit on the performance of the optimal channel switching strategies, the minimum average capacity constraint for the primary receiver in (4.2) is set to $C_{\text{req}} = 5\,\text{Mbps}$ first. Then, by considering the channel links in Fig. 4.2 and Fig. 4.3, the optimal average capacities are obtained for different average power limits (P_{av}) based on Strategy 1 in (4.56) and Strategy 2 in (4.57), and the achieved maximum average capacities are presented in Fig. 4.4. From Fig. 4.4, it is observed that $C^{\star} = 0$ for $P_{\text{av}} < 0.031\,\text{mW}$ since there is no feasible solution of the optimization problem in (4.2) for $C_{\text{req}} = 5\,\text{Mbps}$ and $P_{\text{av}} < 0.031\,\text{mW}$. On the other hand, for $P_{\text{av}} \geq 0.031\,\text{mW}$, the optimal channel switching strategy can be obtained based on (4.56) and (4.57), and it corresponds to Strategy 1 for all $P_{\text{av}} \geq 0.031\,\text{mW}$ since Strategy 1 outperforms Strategy 2 in terms of the achievable maximum average capacity for the communication to the secondary receiver. Therefore, the optimal strategy for the transmitter is to communicate with the primary receiver

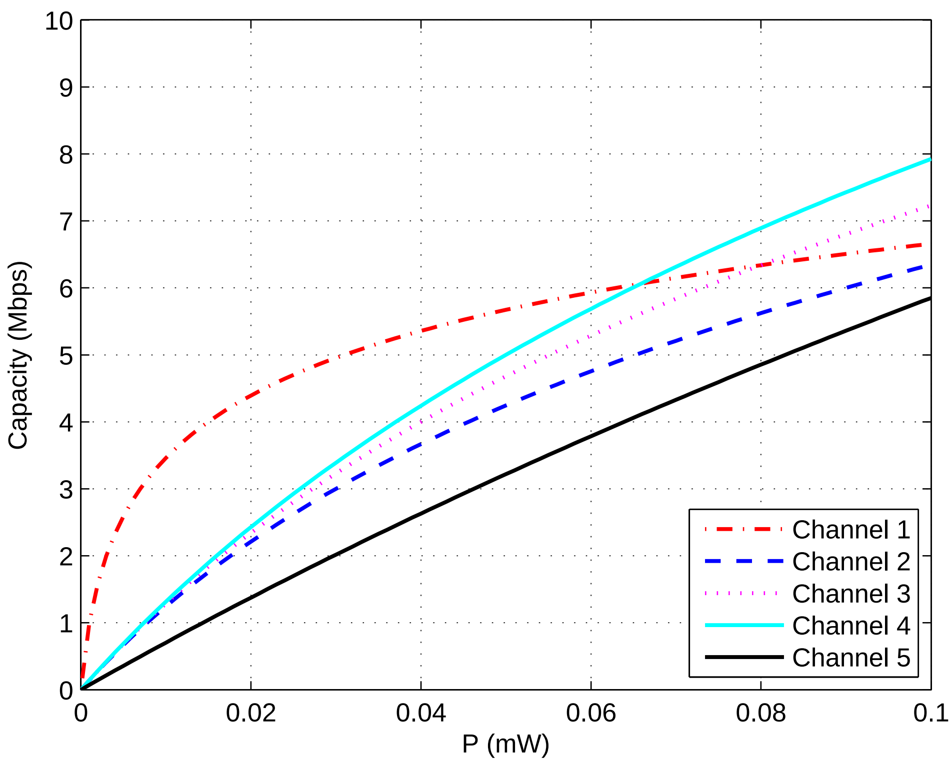

Figure 4.2: Capacity of each link versus power for the communication between the transmitter and the primary receiver, where $B_1 = 1\,\text{MHz}$, $B_2 = 3\,\text{MHz}$, $B_3 = 4\,\text{MHz}$, $B_4 = 5\,\text{MHz}$, $B_5 = 10\,\text{MHz}$, $N_1 = N_2 = N_3 = N_4 = N_5 = 10^{-12}\,\text{W/Hz}$, $|h_1^{\text{p}}|^2 = 1$, $|h_2^{\text{p}}|^2 = 0.1$, $|h_3^{\text{p}}|^2 = 0.1$, $|h_4^{\text{p}}|^2 = 0.1$, and $|h_5^{\text{p}}|^2 = 0.05$.

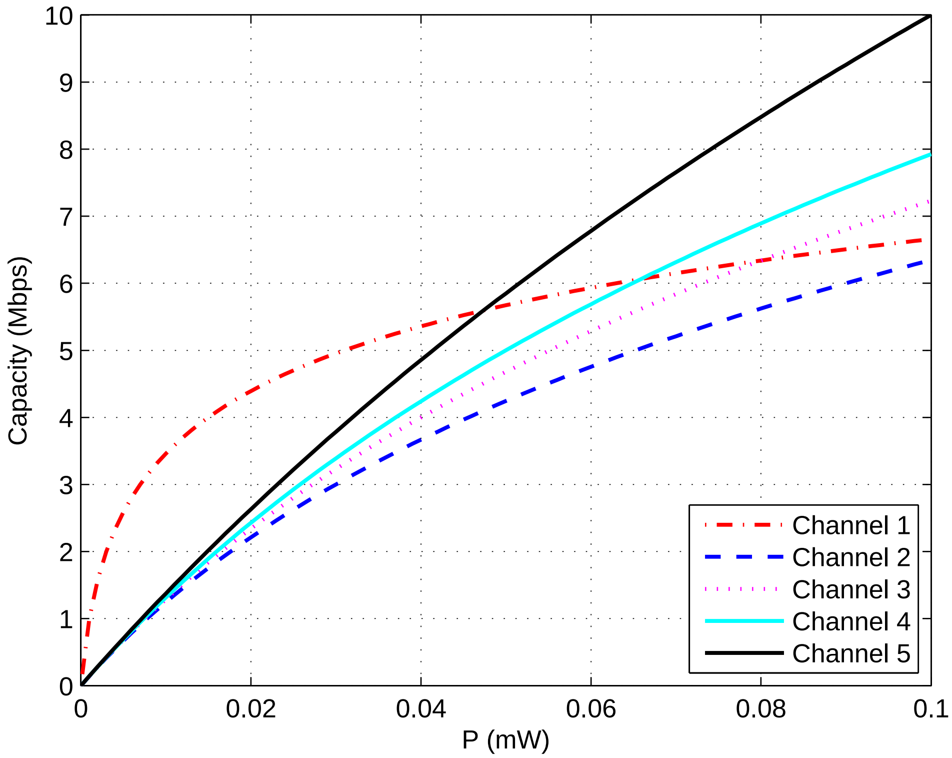

Figure 4.3: Capacity of each link versus power for the communication between the transmitter and the secondary receiver, where $B_1 = 1\,\text{MHz}$, $B_2 = 3\,\text{MHz}$, $B_3 = 4\,\text{MHz}$, $B_4 = 5\,\text{MHz}$, $B_5 = 10\,\text{MHz}$, $N_1 = N_2 = N_3 = N_4 = N_5 = 10^{-12}\,\text{W/Hz}$, $|h_1^{\text{s}}|^2 = 1$, $|h_2^{\text{s}}|^2 = 0.1$, $|h_3^{\text{s}}|^2 = 0.1$, $|h_4^{\text{s}}|^2 = 0.1$, and $|h_5^{\text{s}}|^2 = 0.1$.

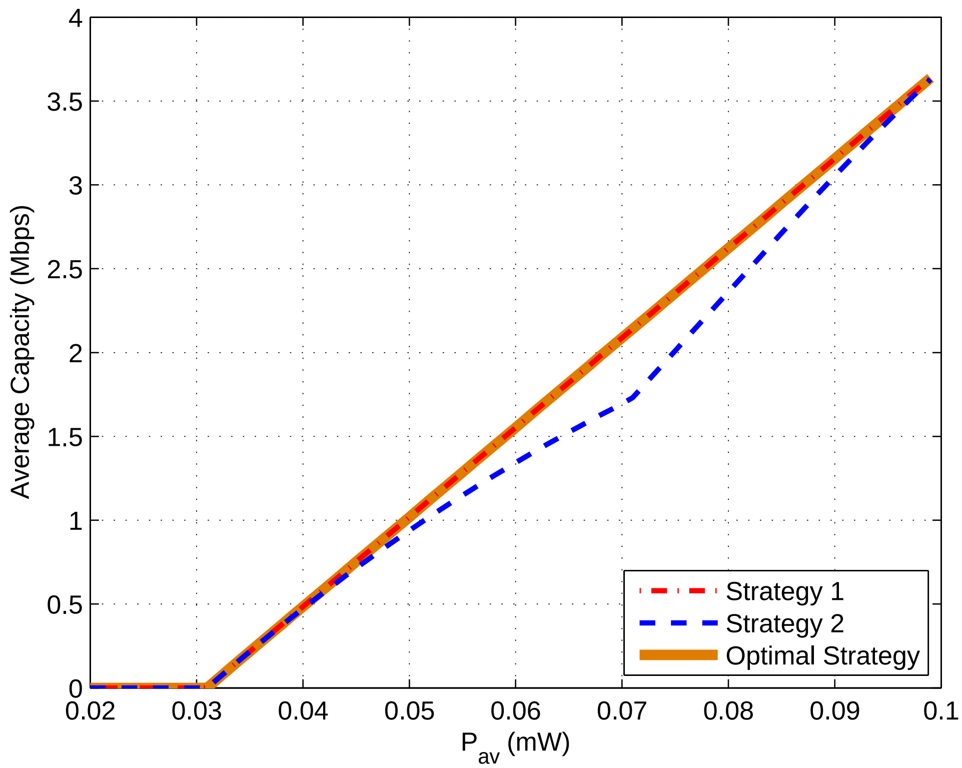

Figure 4.4: Average capacity versus average power limit for Strategy 1, Strategy 2, and the optimal channel switching strategy for the scenario in Fig. 4.2 and Fig. 4.3, where $C_{\text{req}} = 5\,\text{Mbps}$.

over at most two channels and to employ a single channel for the secondary receiver. It is also noted that the solutions of the optimization problem in (4.2) for different values of $P_{\text{av}} \geq 0.031\,\text{mW}$ satisfy the average power and minimum average capacity requirement constraints with equality as Proposition 2 states.

To analyze the optimal strategy in Fig. 4.4 in more detail, Table 4.1 presents the solutions of the optimal strategy for various values of the average power limit, P_{av}. In the table, the optimal solution is represented by parameters λ_1^k, λ_2^k, P_1^k, P_2^k, i^k, and j^k for all $k \in \{\text{p}, \text{s}\}$, meaning that channel i^k is used with time-sharing factor λ_1^k and power P_1^k, and channel j^k is employed with time-sharing factor λ_2^k and power P_2^k to communicate with the primary receiver for $k = \text{p}$ and with the secondary receiver for $k = \mathrm{s}$. It is deduced from Table 4.1 that there is no possible channel switching strategy for $P_{\mathrm{av}} = 0.01\,\mathrm{mW}$, $P_{\mathrm{av}} = 0.02\,\mathrm{mW}$, and $P_{\mathrm{av}} = 0.03\,\mathrm{mW}$. On the other hand, for the other P_{av} values in Table 4.1, the optimal strategy for the average capacity maximization of the secondary receiver is to communicate with the primary receiver over channel 1 and channel 4 via channel switching, and to employ channel 5 exclusively to communicate with the secondary receiver.

In Fig. 4.5, the maximum average capacities for the strategies stated in Fig. 4.4 are plotted versus the minimum average capacity requirement, C_{req}, based on the scenario in Fig. 4.2 and Fig. 4.3. The average power limit in (4.2) is set to $P_{\mathrm{av}} = 0.05\,\mathrm{mW}$. From Fig. 4.5, it is obtained that Strategy 2 is the optimal strategy for $C_{\mathrm{req}} \in (0, 2.6]\,\mathrm{Mbps}$ whereas Strategy 1 is optimal for $C_{\mathrm{req}} \in [3.9, 5.8]\,\mathrm{Mbps}$. On the other hand, for $C_{\mathrm{req}} \in (2.6, 3.9)\,\mathrm{Mbps}$, both Strategy 1 and Strategy 2 are optimal since the communication is performed over a single channel for each of primary and secondary receivers. Also, it is noted that there is no optimal strategy for $C_{\mathrm{req}} > 5.8\,\mathrm{Mbps}$ since $\bar{C}_{\mathrm{req}}$ in (4.55) cannot achieve a capacity equal to or higher than C_{req}; that is, $\bar{C}_{\mathrm{req}} < C_{\mathrm{req}}$.

Similar to Table 4.1, the solutions of the optimal strategies for various values of the minimum average capacity requirement of the primary receiver, C_{req}, are presented in Table 4.2. It is noted from Table 4.2 that the optimal strategy for the values satisfying $C_{\mathrm{req}} \leq 2.5\,\mathrm{Mbps}$ corresponds to the exclusive use of channel 1 for the primary receiver and to channel switching between channel 1 and channel 5 for the secondary receiver whereas for the values of C_{req} with $C_{\mathrm{req}} \geq 4.0\,\mathrm{Mbps}$ and $C_{\mathrm{req}} \leq 5.5\,\mathrm{Mbps}$, it corresponds to switching between channel 1 and channel 4 for the primary receiver and to the use of channel 5 only for the secondary receiver. Also, for $C_{\mathrm{req}} = 3.0\,\mathrm{Mbps}$ and $C_{\mathrm{req}} = 3.5\,\mathrm{Mbps}$, the optimal strategy is to employ channel 1 and channel 5 for the primary and secondary receivers, respectively. In this case, it is observed that both Strategy 1 and Strategy 2 are optimal. Lastly, there is no optimal channel switching strategy for $C_{\mathrm{req}} = 6.0\,\mathrm{Mbps}$.

Table 4.1: Optimal strategy for the scenario in Fig. 4.2 and Fig. 4.3, which employs channel i^k and channel j^k with time-sharing factors λ_1^k and λ_2^k and power levels P_1^k and P_2^k, respectively, to communicate with the primary receiver ($k = \mathrm{p}$) and the secondary receiver ($k = \mathrm{s}$).

P_{av} (mW)	λ_1^{p}	P_1^{p}	i^{p}	λ_2^{p}	P_2^{p}	j^{p}	λ_1^{s}	P_1^{s}	i^{s}	λ_2^{s}	P_2^{s}	j^{s}
0.01	–	–	–	–	–	–	–	–	–	–	–	–
0.02	–	–	–	–	–	–	–	–	–	–	–	–
0.03	–	–	–	–	–	–	–	–	–	–	–	–
0.04	0.8963	0.0331	1	0.0552	0.1	4	0.0484	0.1	5	–	–	–
0.05	0.7469	0.0331	1	0.1512	0.1	4	0.1019	0.1	5	–	–	–
0.06	0.5975	0.0331	1	0.2471	0.1	4	0.1553	0.1	5	–	–	–
0.07	0.4482	0.0331	1	0.3431	0.1	4	0.2088	0.1	5	–	–	–
0.08	0.2988	0.0331	1	0.439	0.1	4	0.2622	0.1	5	–	–	–
0.09	0.1494	0.0331	1	0.535	0.1	4	0.3156	0.1	5	–	–	–

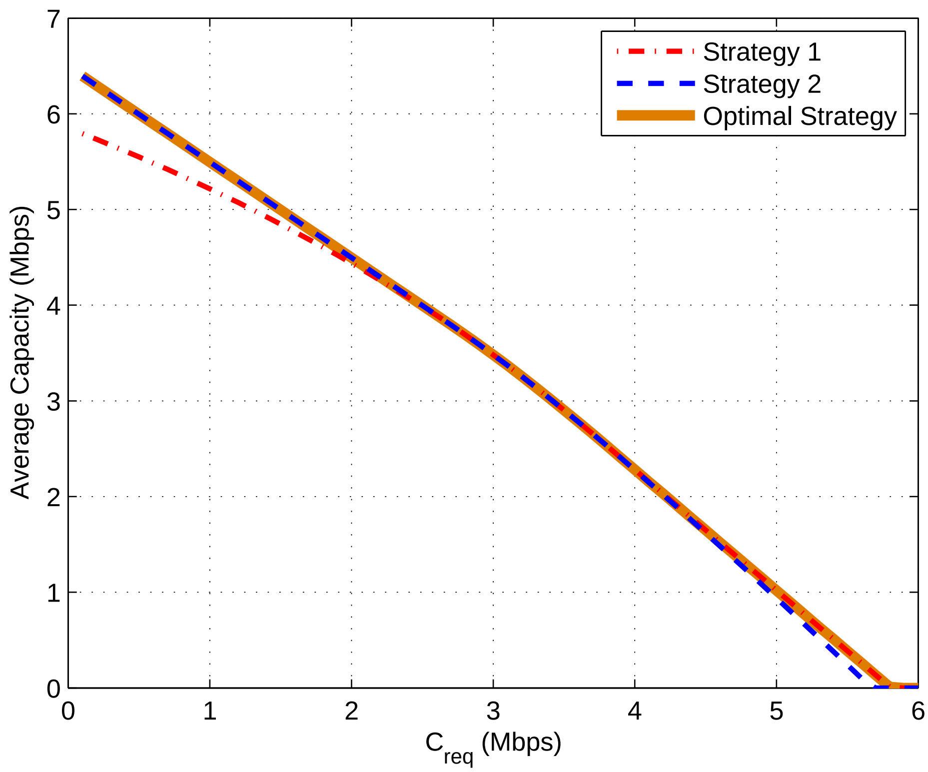

Figure 4.5: Average capacity versus minimum average capacity requirement for Strategy 1, Strategy 2, and the optimal channel switching strategy for the scenario in Fig. 4.2 and Fig. 4.3, where $P_{\text{av}} = 0.05\,\text{mW}$.

Table 4.2: Optimal strategy for the scenario in Fig. 4.2 and Fig. 4.3, which employs channel i^k and channel j^k with time-sharing factors λ_1^k and λ_2^k and power levels P_1^k and P_2^k, respectively, to communicate with the primary receiver ($k = \mathrm{p}$) and the secondary receiver ($k = \mathrm{s}$).

C_{req} (Mbps)	λ_1^{p}	P_1^{p}	i^{p}	λ_2^{p}	P_2^{p}	j^{p}	λ_1^{s}	P_1^{s}	i^{s}	λ_2^{s}	P_2^{s}	j^{s}
0.5	0.1146	0.0196	1	–	–	–	0.5071	0.0196	1	0.3783	0.1	5
1	0.2292	0.0196	1	–	–	–	0.3925	0.0196	1	0.3783	0.1	5
1.5	0.3438	0.0196	1	–	–	–	0.2779	0.0196	1	0.3783	0.1	5
2	0.4584	0.0196	1	–	–	–	0.1633	0.0196	1	0.3783	0.1	5
2.5	0.573	0.0196	1	–	–	–	0.0487	0.0196	1	0.3783	0.1	5
3	0.6518	0.0233	1	–	–	–	0.3482	0.1	5	–	–	–
3.5	0.7096	0.0295	1	–	–	–	0.2904	0.1	5	–	–	–
4	0.7469	0.0331	1	0.025	0.1	4	0.2281	0.1	5	–	–	–
4.5	0.7469	0.0331	1	0.0881	0.1	4	0.165	0.1	5	–	–	–
5	0.7469	0.0331	1	0.1512	0.1	4	0.1019	0.1	5	–	–	–
5.5	0.7469	0.0331	1	0.2143	0.1	4	0.0388	0.1	5	–	–	–
6	–	–	–	–	–	–	–	–	–	–	–	–

4.5 Concluding Remarks

In this study, the optimal channel switching problem has been investigated for average capacity maximization in the presence of multiple receivers in a communication system where multiple AWGN channels are available for a transmitter to communicate with the receivers. First, the optimal channel switching problem has been presented for the communication of a transmitter with the primary and secondary receivers in the presence of the minimum average capacity requirement of the primary receiver and the average and peak power constraints. Then, an equivalent optimization problem has been proposed and it has been proved that the solution of this problem satisfies the constraints in equality. Based on the proposed optimization problems, it has been shown that the optimal channel switching strategy does not involve more than three communication links when multiple channels are available in the communication system. Furthermore, the possible optimal channel switching scenarios have been specified in terms of the number of channels required for the transmitter to communicate with the primary and secondary receivers in order to achieve the maximum average capacity of the communication to the secondary receiver while fulfilling the minimum average capacity requirement of the primary receiver. Numerical examples have been provided to illustrate the theoretical results and to demonstrate the benefits of channel switching.

Chapter 5

Power Control Games between Anchor and Jammer Nodes in Wireless Localization Networks

In this chapter, power control games between anchor and jammer nodes are analyzed for a wireless localization network in which each target node estimates its own position based on received signals from anchor nodes and jammer nodes try to degrade the localization performance of target nodes [67]. The main contributions of this chapter can be summarized as follows:

- A game theoretic formulation is developed between anchor and jammer nodes in a wireless localization network for the first time in the literature.
- Two types of power control games between anchor and jammer nodes are proposed based on the average CRLB and the worst-case CRLBs for the anchor and jammer nodes.
- In a game-theoretic framework, the Nash equilibria of the proposed games are analyzed and it is shown that both of the games have at least one pure strategy Nash equilibrium.

- For the game that employs the average CRLB as a performance metric, an approach is developed to obtain the pure strategy Nash equilibrium and a sufficient condition is derived to determine whether the obtained Nash equilibrium is a unique pure strategy Nash equilibrium.

This chapter is organized as follows: Section 5.1 describes the wireless localization network and introduces the network parameters. Section 5.2 first presents the proposed game formulations, and then provides detailed theoretical analyses. Numerical results are described in Section 5.3, which is followed by the concluding remarks in Section 5.4.

5.1 System Model

Consider a wireless localization network with N_A anchor nodes and N_T target nodes at locations $\boldsymbol{y}_i \in \mathbb{R}^2$ for $i \in \{1, \ldots, N_A\}$ and $\boldsymbol{x}_i \in \mathbb{R}^2$ for $i \in \{1, \ldots, N_T\}$, respectively. Each target node in the system estimates its position based on received signals from the anchor nodes, the locations of which are known by the target nodes (i.e., the target nodes perform self-positioning [47]). Besides the anchor and target nodes, there exist N_J jammer nodes located at $\boldsymbol{z}_i \in \mathbb{R}^2$ for $i \in \{1, \ldots, N_J\}$ in the system. Contrary to the anchor nodes, the aim of the jammer nodes is to reduce the localization performance of the target nodes. In accordance with the common approach in the literature [55], [88]–[90], it is assumed that the jammer nodes transmit zero-mean white Gaussian noise in order to distort the signals observed by the target nodes. The reasons behind the use of a Gaussian noise model can be explained as follows: In wireless localization systems, when the knowledge of the ranging signals sent from the anchor nodes to the target nodes is unavailable to the jammer nodes, the jammer nodes can continuously transmit noise to degrade the localization performance of the target nodes [55]. In the literature, it is shown that the Gaussian noise is the worst-case noise for generic wireless networks modeled with additive noise that is independent of the transmit signals [91]–[92]. (In particular, the Gaussian distribution corresponds to the worst-case scenario among all possible noise distributions in terms of some metrics such as the mutual information and the mean squared error since it minimizes the mutual information between the input and the output when the input is Gaussian, and maximizes the mean squared error of estimating the input given the output for an additive noise channel with a Gaussian input [93].) Therefore, the jammer nodes are expected to transmit Gaussian noise for efficient jamming [55]. Also, a non-cooperative localization scenario is considered; that is, the target nodes do not receive any signals from each other for localization purposes.

Let $\mathcal{A}_i$ denote the connectivity set for target node i, which is defined as $\mathcal{A}_i \triangleq \{j \in \{1, \ldots, N_A\} \mid \text{anchor node } j \text{ is connected to target node } i\}$ for $i \in \{1, \ldots, N_T\}$. Then, corresponding to the transmission from anchor node j, the received signal at target node i can be expressed as

$$r_{ij}(t) = \sum_{k=1}^{L_{ij}} \alpha_{ij}^k \sqrt{P_{ij}^A}\, s(t - \tau_{ij}^k) + \sum_{l=1}^{N_J} \gamma_{il} \sqrt{P_l^J}\, \nu_{ilj}(t) + n_{ij}(t) \tag{5.1}$$

for $t \in [0, T_{\text{obs}}]$, $i \in \{1, \ldots, N_T\}$, and $j \in \mathcal{A}_i$, where T_{obs} is the observation time, L_{ij} is the number of paths between anchor node j and target node i, α_{ij}^k and τ_{ij}^k represent, respectively, the amplitude and the delay of the kth multipath component between anchor node j and target node i, P_{ij}^A is the transmit power of the signal sent from anchor node j to target node i, and γ_{il} represents the channel coefficient between jammer node l and target node i, which has a transmit power of P_l^J [55]. Also, during the reception from anchor node j, $n_{ij}(t)$ denotes the measurement noise at target node i and $\nu_{ilj}(t)$ represents the jammer noise at target node i generated by jammer node l. It is assumed that the transmit signal $s(t)$ is a known signal with unit energy, and the measurement noise $n_{ij}(t)$ and the jammer noise $\nu_{ilj}(t)$ are independent zero-mean white Gaussian random processes, where the spectral density levels of $n_{ij}(t)$ and $\nu_{il}(t)$ are equal to $N_0/2$ and one, respectively [55]. In addition, for each target node, $n_{ij}(t)$'s are independent for $j \in \mathcal{A}_i$, and $v_{ilj}(t)$'s are independent for $l \in \{1, \ldots, N_J\}$ and $j \in \mathcal{A}_i$.[1] The delay τ_{ij}^k is expressed as $\tau_{ij}^k \triangleq (\|\boldsymbol{y}_j - \boldsymbol{x}_i\| + b_{ij}^k)/c$, where b_{ij}^k denotes the non-negative range bias and c is the speed of propagation.

[1]As in [55], it is assumed that the anchor nodes transmit at different time intervals to prevent interference at the target nodes [48], and during those time intervals, the channel coefficient between a jammer node and a target node is assumed to be constant.

5.2 Power Control Games Between Anchor and Jammer Nodes

In this section, the aim is to design and analyze power control games between anchor and jammer nodes. In the proposed setting, the anchor nodes set their power levels in order to maximize the localization performance of the target nodes whereas the jammer nodes try to minimize the localization performance via power allocation. The localization performance is quantified by the average CRLB for the target nodes, which is the metric according to which the anchor and jammer nodes compete. In other words, the anchor nodes (jammer nodes) try to minimize (maximize) the average CRLB for the target nodes to improve (deteriorate) the localization performance of the system. The use of the CRLB as the performance metric can be justified based on the following arguments: As investigated in [94], the ML location estimator becomes asymptotically unbiased and efficient for sufficiently large SNRs and/or effective bandwidths, and consequently, it achieves a mean-squared error (MSE) close to the CRLB. For other cases, the CRLB may not provide a tight bound for MSEs of ML estimators [95, 96]. Therefore, the CRLBs obtained based on the optimal power strategies of the anchor and jammer nodes provide performance bounds for the MSEs of the target nodes. Another reason for the use of the CRLB metric is that it leads to compact closed form expressions for the optimization problems and consequently facilitates theoretical analyses, which lead to intuitive explanations of power control games between anchor and jammer nodes. (Performance optimization based on the CRLB has been considered in various studies in the literature such as [55], [57], [97].)

To obtain the formulation of the proposed problem, the CRLB expression for the target nodes is presented as a utility function first, and then the game model is proposed.

5.2.1 CRLB for Location Estimation of Target Nodes

To provide the CRLB expression for target node i, the unknown parameters related to target node i are defined as [55]

$$\boldsymbol{\theta}_i \triangleq \left[\boldsymbol{x}_i^T \; \boldsymbol{b}_{i\mathcal{A}_i(1)}^T \; \cdots \; \boldsymbol{b}_{i\mathcal{A}_i(|\mathcal{A}_i|)}^T \; \boldsymbol{\alpha}_{i\mathcal{A}_i(1)}^T \; \cdots \; \boldsymbol{\alpha}_{i\mathcal{A}_i(|\mathcal{A}_i|)}^T\right]^T \tag{5.2}$$

where $\mathcal{A}_i(j)$ represents the jth element of set $\mathcal{A}_i$, $|\mathcal{A}_i|$ denotes the cardinality of set $\mathcal{A}_i$, $\boldsymbol{\alpha}_{ij} = \left[\alpha_{ij}^1 \cdots \alpha_{ij}^{L_{ij}}\right]^T$, and $\boldsymbol{b}_{ij}$ is defined as

$$\boldsymbol{b}_{ij} = \begin{cases} \left[b_{ij}^2 \; \cdots \; b_{ij}^{L_{ij}}\right]^T, & \text{if } j \in \mathcal{A}_i^L \\ \left[b_{ij}^1 \; \cdots \; b_{ij}^{L_{ij}}\right]^T, & \text{if } j \in \mathcal{A}_i^{NL} \end{cases} \tag{5.3}$$

with $\mathcal{A}_i^L$ and $\mathcal{A}_i^{NL}$ representing the sets of anchors nodes that are in the line-of-sight (LOS) and non-line-of-sight (NLOS) of target node i, respectively [55]. Then, the CRLB for estimating the location of target node i is given by

$$\mathbb{E}\{\|\hat{\boldsymbol{x}}_i - \boldsymbol{x}_i\|^2\} \geq \text{tr}\left\{\left[\boldsymbol{F}_i^{-1}\right]_{2\times 2}\right\} \triangleq \text{CRLB}_i \tag{5.4}$$

where $\hat{\boldsymbol{x}}_i$ denotes an unbiased estimate of the location of target node i, tr represents the trace operator, and $\boldsymbol{F}_i$ is the Fisher information matrix for vector $\boldsymbol{\theta}_i$ in (5.2). From [48] and [55], $\left[\boldsymbol{F}_i^{-1}\right]_{2\times 2}$ can be expressed as

$$\left[\boldsymbol{F}_i^{-1}\right]_{2\times 2} = \boldsymbol{J}_i\left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)^{-1} \tag{5.5}$$

where $\boldsymbol{J}_i\left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)$ denotes the equivalent Fisher information matrix, which is calculated as

$$\boldsymbol{J}_i\left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right) = \sum_{j\in\mathcal{A}_i^L} \frac{P_{ij}^A \lambda_{ij}}{N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J} \boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T \tag{5.6}$$

with

$$\lambda_{ij} \triangleq \frac{4\pi^2|\alpha_{ij}^1|^2 \int_{-\infty}^{\infty} f^2|S(f)|^2 df}{c^2}(1-\xi_j) \ , \tag{5.7}$$

$$\boldsymbol{a}_i \triangleq \left[|\gamma_{i1}|^2 \ \cdots \ |\gamma_{iN_J}|^2\right]^T \ , \tag{5.8}$$

$$\boldsymbol{p}_i^A \triangleq \left[P_{i\mathcal{A}_i(1)}^A \ \cdots \ P_{i\mathcal{A}_i(|\mathcal{A}_i|)}^A\right]^T \ , \tag{5.9}$$

$$\boldsymbol{p}^J \triangleq \left[P_1^J \ \cdots \ P_{N_J}^J\right]^T \ , \tag{5.10}$$

$$\boldsymbol{\phi}_{ij} \triangleq [\cos\varphi_{ij} \ \sin\varphi_{ij}]^T \ . \tag{5.11}$$

In (5.7), $S(f)$ denotes the Fourier transform of $s(t)$, and the path-overlap coefficient ξ_j is a number that satisfies $0 \leq \xi_j \leq 1$ [61]. Also, in (5.11), φ_{ij} corresponds to the angle between target node i and anchor node j.

5.2.2 Power Control Game Model

Let $\mathcal{G} = \langle \mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}} \rangle$ denote the power control game between anchor nodes (i.e., Player A) and jammer nodes (i.e., Player J), where $\mathcal{N} = \{A, J\}$ is the index set for the players, S_i is the strategy set for player i, and u_i is the utility function of player i. For the anchor nodes, strategy set S_A is defined as

$$S_A \triangleq \left\{\boldsymbol{p}^A \in \mathbb{R}^K \mid \boldsymbol{1}^T\boldsymbol{p}^A \leq P_T^A \ \wedge \ 0 \leq \boldsymbol{e}_i^T\boldsymbol{p}^A \leq P_{\text{peak}}^A \ , \ \forall i \in \{1,\ldots,K\}\right\} \tag{5.12}$$

with

$$\boldsymbol{p}^A \triangleq \left[\left(\boldsymbol{p}_1^A\right)^T \ \cdots \ \left(\boldsymbol{p}_{N_T}^A\right)^T\right]^T \tag{5.13}$$

where $\boldsymbol{p}_i^A$ is as defined in (5.9), $\boldsymbol{1}$ is the vector of ones, $\boldsymbol{e}_i$ is the unit vector whose ith element is one, K is the dimension of $\boldsymbol{p}^A$, P_T^A is the total available power of the anchor nodes, and P_{peak}^A is the maximum allowed and attainable power (peak power) for the anchor nodes. Similarly, strategy set S_J for the jammer nodes is defined as

$$S_J \triangleq \left\{\boldsymbol{p}^J \in \mathbb{R}^{N_J} \mid \boldsymbol{1}^T\boldsymbol{p}^J \leq P_T^J \ \wedge \ 0 \leq \boldsymbol{e}_i^T\boldsymbol{p}^J \leq P_{\text{peak}}^J \ , \ \forall i \in \{1,\ldots,N_J\}\right\} \tag{5.14}$$

where $\boldsymbol{p}^J$ is as defined in (5.10), P_T^J is the total available power of the jammer nodes, and P_{peak}^J is the maximum allowed and attainable power (peak power) for the jammer nodes.

Let $\boldsymbol{p}^A$ and $\boldsymbol{p}^J$ denote strategies of player A and player J, respectively. Then, a strategy (action) profile of the game can be denoted as $(\boldsymbol{p}^A, \boldsymbol{p}^J) \in S$, where $\boldsymbol{p}^A \in S_A$, $\boldsymbol{p}^J \in S_J$, and $S = S_A \times S_J$. For a given action profile, the utility functions of player A and player J are defined as

$$u_A(\boldsymbol{p}^A, \boldsymbol{p}^J) = -\frac{1}{N_T}\sum_{i=1}^{N_T} \text{tr}\Big\{\boldsymbol{J}_i\left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)^{-1}\Big\}\,, \tag{5.15}$$

$$u_J(\boldsymbol{p}^A, \boldsymbol{p}^J) = \frac{1}{N_T}\sum_{i=1}^{N_T} \text{tr}\Big\{\boldsymbol{J}_i\left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)^{-1}\Big\}\,. \tag{5.16}$$

Namely, the average CRLB of the target nodes is employed in the utility functions (see (5.4) and (5.5)). Since $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ and $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ satisfy that $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J) + u_J(\boldsymbol{p}^A, \boldsymbol{p}^J) = 0 \ \ \forall \boldsymbol{p}^A \in S_A \wedge \forall \boldsymbol{p}^J \in S_J$, it is noted that the power control game between player A and player J corresponds to a two-player zero-sum game.

5.2.3 Nash Equilibrium in Power Control Game

The Nash equilibrium is one of the solution approaches that is commonly used for game theoretic problems [98]. In the game-theoretic notation, a strategy profile of game $\mathcal{G}$, denoted as $(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J)$, is a Nash equilibrium if

$$u_A(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J) \geq u_A(\boldsymbol{p}^A, \boldsymbol{p}_\star^J)\,, \ \forall \boldsymbol{p}^A \in S_A\,, \tag{5.17}$$

$$u_J(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J) \geq u_J(\boldsymbol{p}_\star^A, \boldsymbol{p}^J)\,, \ \forall \boldsymbol{p}^J \in S_J\,. \tag{5.18}$$

At a Nash equilibrium, no player can improve its utility by changing its strategy unilaterally. In other words, given the power levels of player J (player A), player A (player J) does not have any incentive to deviate from its power strategy at a Nash equilibrium. Such an equilibrium does not necessarily exist in infinite games. However, power control game $\mathcal{G}$ admits a pure Nash equilibrium as the following proposition states.

Proposition 1: *A pure Nash equilibrium exists in power control game $\mathcal{G}$.*

Proof: The aim in the proof is to show that the game has at least one pure-strategy Nash equilibrium. For that reason, it is first noted that power control game $\mathcal{G}$ in strategic form $\langle \mathcal{N}, (S_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}} \rangle$ admits at least one pure Nash equilibrium if the following conditions are satisfied [99]:

- Strategy set S_i is compact and convex for all $i \in \mathcal{N}$, where $\mathcal{N} = \{A, J\}$.
- $u_i(\boldsymbol{p}^A, \boldsymbol{p}^J)$ is a continuous function in the profile of strategies $(\boldsymbol{p}^A, \boldsymbol{p}^J) \in S$ for all $i \in \mathcal{N}$.
- $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ and $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ are quasi-concave functions in $\boldsymbol{p}^A$ and $\boldsymbol{p}^J$, respectively.

Since set S_A in (5.12) and set S_J in (5.14) are closed and bounded, it can easily be shown that the sets in (5.12) and (5.14) are compact and convex, which satisfies the first condition. Also, $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.15) is a concave function of $\boldsymbol{p}^A$ based on the proof in [100] and $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.16) is a linear (and concave) function of $\boldsymbol{p}^J$ based on [97]. Consequently, (5.15) and (5.16) are continuous and quasi-concave functions, for which the second and the third conditions hold. Therefore, it is concluded that at least one Nash equilibrium exists in power control game $\mathcal{G}$. ■

Based on Proposition 1, the proposed power control game has at least one Nash equilibrium. In order to analyze the Nash equilibrium, first, best response strategies of player A and J are discussed and then, a fixed point equation is obtained.

For a given power strategy of player J (i.e., power levels of jammer nodes), the best response function of player A can be expressed as

$$\begin{aligned}\boldsymbol{p}^A_{\mathrm{BR}} &= \mathrm{BR}_A(\boldsymbol{p}^J) \\ &\triangleq \underset{\boldsymbol{p}^A \in S_A}{\arg\max} \; -\frac{1}{N_T}\sum_{i=1}^{N_T} \mathrm{tr}\Big\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}^A_i, \boldsymbol{p}^J\right)^{-1} \Big\}. \end{aligned} \tag{5.19}$$

On the other hand, for a given power strategy of player A, the best response function of player J is given as

$$\begin{aligned}\boldsymbol{p}^J_{\mathrm{BR}} &= \mathrm{BR}_J(\boldsymbol{p}^A) \\ &\triangleq \underset{\boldsymbol{p}^J \in S_J}{\arg\max} \; \frac{1}{N_T}\sum_{i=1}^{N_T} \mathrm{tr}\Big\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}^A_i, \boldsymbol{p}^J\right)^{-1} \Big\}. \end{aligned} \tag{5.20}$$

Let $\mathrm{BR} = (\mathrm{BR}_A, \mathrm{BR}_J) : S = S_A \times S_J \to S$ be a mapping of a function (correspondence) $BR(\boldsymbol{p})$, where $\boldsymbol{p} = (\boldsymbol{p}^A, \boldsymbol{p}^J) \in S$ is a strategy profile of the power control game, and $\mathrm{BR_A}$ and $\mathrm{BR_J}$ are as in (5.19) and (5.20), respectively. Based on the definition of the Nash equilibrium, the following fixed point equation holds for the Nash equilibrium:

$$\boldsymbol{p}^\star = \mathrm{BR}(\boldsymbol{p}^\star)\,. \tag{5.21}$$

In addition, the utility function in (5.15) is a concave function of $\boldsymbol{p}^A$ and the utility function in (5.16) is a linear (and concave) function of $\boldsymbol{p}^J$. Based on the utility functions in (5.15) and (5.16), the game between player A and player J is called convex-concave game [101], [73]. In a convex-concave game, the Nash equilibrium becomes the saddle-point equilibrium, and if there exist multiple Nash equilibria, the value of the game is unique for every Nash equilibrium. Therefore, the pure Nash equilibrium of power control game $\mathcal{G}$ can be obtained as stated in the following proposition.

Proposition 2: *Let* $\boldsymbol{p}^\star = (\boldsymbol{p}^A_\star, \boldsymbol{p}^J_\star)$ *denote the Nash equilibrium of power* *control game $\mathcal{G}$ in pure strategies. Then, $\boldsymbol{p}^\star$ satisfies the following equation:*

$$u_J(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J) = -u_A(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J) = \min_{\boldsymbol{p}^A \in S_A} \max_{\boldsymbol{p}^J \in S_J} \frac{1}{N_T} \sum_{i=1}^{N_T} \mathrm{tr}\left\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)^{-1} \right\} \quad (5.22)$$

Proof: Since power control game $\mathcal{G}$ is a two-player zero-sum game and $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.15) is a concave function of $\boldsymbol{p}^A$ and $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.16) is a linear (and concave) function of $\boldsymbol{p}^J$, the following equality holds by von Neumann's Minimax Theorem [101, 102]:

$$\min_{\boldsymbol{p}^A \in S_A} \max_{\boldsymbol{p}^J \in S_J} \frac{1}{N_T} \sum_{i=1}^{N_T} \mathrm{tr}\left\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)^{-1} \right\} = \max_{\boldsymbol{p}^J \in S_J} \min_{\boldsymbol{p}^A \in S_A} \frac{1}{N_T} \sum_{i=1}^{N_T} \mathrm{tr}\left\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J\right)^{-1} \right\}. \quad (5.23)$$

In addition, $\boldsymbol{p}^\star = (\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J)$ satisfying the equality in (5.23) is a Nash equilibrium of power control game $\mathcal{G}$. ■

Proposition 1 states that power control game $\mathcal{G}$ admits at least one Nash equilibrium in pure strategies. In order to further analyze the equilibrium in power control game $\mathcal{G}$, the uniqueness of the Nash equilibrium is investigated in the consideration of pure strategies. The following proposition provides a sufficient condition for the uniqueness of the pure strategy Nash equilibrium.

Proposition 3: *Suppose that the Fisher information matrix in* (5.6) *is positive definite.*[2] *Then, power control game $\mathcal{G}$ has a unique Nash equilibrium in pure strategies if all the elements of $\boldsymbol{w} \triangleq \sum_{i=1}^{N_T} r_i \boldsymbol{a}_i^T$ are different, where r_i is defined as*

$$r_i \triangleq \mathrm{tr}\left\{ \left[\sum_{j \in \mathcal{A}_i^L} P_{ij}^A \lambda_{ij} \boldsymbol{\phi}_{ij} \boldsymbol{\phi}_{ij}^T \right]^{-1} \right\}. \quad (5.24)$$

[2]The Fisher information matrix is always positive semidefinite by definition. The assumption in the proposition corresponds to practical scenarios with a sufficient number of anchor nodes and guarantees the invertibility of the Fisher information matrix.

Proof: In order to prove that the Nash equilibrium of power control game $\mathcal{G}$ is unique when the condition in Proposition 3 is satisfied, it is first shown that $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.15) is a strictly concave function of $\boldsymbol{p}^A$ for a fixed $\boldsymbol{p}^J$. To that aim, choose arbitrary $\tilde{\boldsymbol{p}}^A \in S_A$ and $\bar{\boldsymbol{p}}^A \in S_A$ with $\tilde{\boldsymbol{p}}^A \neq \bar{\boldsymbol{p}}^A$. Then, the following relations can be obtained for any $\alpha \in (0, 1)$:

$$
\begin{aligned}
& u_A(\alpha\tilde{\boldsymbol{p}}^A + (1-\alpha)\bar{\boldsymbol{p}}^A, \boldsymbol{p}^J) \\
&= -\frac{1}{N_T}\sum_{i=1}^{N_T} \mathrm{tr}\Big\{ \boldsymbol{J}_i\left(\boldsymbol{x}_i, \alpha\tilde{\boldsymbol{p}}_i^A + (1-\alpha)\bar{\boldsymbol{p}}_i^A, \boldsymbol{p}^J\right)^{-1} \Big\} && (5.25) \\
&= -\frac{1}{N_T}\sum_{i=1}^{N_T} \mathrm{tr}\left\{ \left[\sum_{j\in\mathcal{A}_i^L} \frac{(\alpha\tilde{P}_{ij}^A + (1-\alpha)\bar{P}_{ij})\lambda_{ij}}{N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J} \boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T \right]^{-1} \right\} && (5.26) \\
&= -\frac{1}{N_T}\sum_{i=1}^{N_T} \mathrm{tr}\Bigg\{ \Bigg[\alpha \sum_{j\in\mathcal{A}_i^L} \frac{\tilde{P}_{ij}^A\lambda_{ij}}{N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J} \boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T \\
&\qquad + (1-\alpha) \sum_{j\in\mathcal{A}_i^L} \frac{\bar{P}_{ij}^A\lambda_{ij}}{N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J} \boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T \Bigg]^{-1} \Bigg\} && (5.27) \\
&> -\frac{1}{N_T}\sum_{i=1}^{N_T} \alpha\mathrm{tr}\left\{ \left[\sum_{j\in\mathcal{A}_i^L} \frac{\tilde{P}_{ij}^A\lambda_{ij}}{N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J} \boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T \right]^{-1} \right\} \\
&\qquad + (1-\alpha)\mathrm{tr}\left\{ \left[\sum_{j\in\mathcal{A}_i^L} \frac{\bar{P}_{ij}^A\lambda_{ij}}{N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J} \boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T \right]^{-1} \right\} && (5.28) \\
&= \alpha u_A(\tilde{\boldsymbol{p}}^A, \boldsymbol{p}^J) + (1-\alpha) u_A(\bar{\boldsymbol{p}}^A, \boldsymbol{p}^J) && (5.29)
\end{aligned}
$$

where the equalities in (5.25) and (5.26) are due to the definitions in (5.15) and (5.6), respectively, and the inequality in (5.28) follows from the fact that $\mathrm{tr}\{\boldsymbol{X}^{-1}\}$ is a strictly convex function of $\boldsymbol{X}$ if $\boldsymbol{X}$ is a symmetric positive definite matrix [103]. It is noted that $\alpha \in (0, 1)$, $\boldsymbol{\phi}_{ij}\boldsymbol{\phi}_{ij}^T$ is a symmetric positive semidefinite matrix, and $(\tilde{P}_{ij}^A\lambda_{ij})/(N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J)$ and $(\bar{P}_{ij}^A\lambda_{ij})/(N_0/2 + \boldsymbol{a}_i^T\,\boldsymbol{p}^J)$ are always non-negative for all $i \in \{1, \ldots, N_T\}$ and $j \in \mathcal{A}_i^L$. Based on the relations in (5.25)–(5.29), it is proved that $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.15) is a strictly concave function of $\boldsymbol{p}^A$ for a fixed $\boldsymbol{p}^J$.

Next, it is obtained that there exists a unique maximizer of $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in

(5.16) for a given $\boldsymbol{p}^A$ when the condition in Proposition 3 is satisfied. To that aim, consider the best response function of player J in (5.20). Based on a similar approach to that in [97], the solution of the optimization problem in (5.20) can be expressed as

$$p^J_{\mathrm{BR}}(h(j)) = \min\left\{P^J_T - \sum_{l=1}^{j-1} p^J_{\mathrm{BR}}(h(l)), P^J_{\mathrm{peak}}\right\} \tag{5.30}$$

for $j = 1, \ldots, N_J$, where $h(j)$ denotes the index of the jth largest element of vector $\boldsymbol{w}$ defined in Proposition 3, $p^J_{\mathrm{BR}}(h(j))$ represents the $h(j)$th element of p^J_{BR}, and $\sum_{l=1}^{0}(\cdot)$ is defined as zero. For the condition that all the elements of $\boldsymbol{w}$ are different, index vector $\boldsymbol{h} \triangleq [h(1)\, h(2) \cdots h(N_J)]$ becomes unique and consequently the solution in (5.30) turns into a unique maximizer of $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ for a given $\boldsymbol{p}^A$. Therefore, based on the properties of game $\mathcal{G}$ presented in the proof of Proposition 1 and the statements proved above, it is concluded that if the condition in Proposition 3 is satisfied, then the Nash equilibrium of power control game $\mathcal{G}$ is unique. ∎

It is important to note that the Nash equilibrium obtained by (5.22) based on Proposition 2 may not be unique. However, Proposition 3 provides a sufficient condition to check that the obtained Nash equilibrium is a unique equilibrium of power control game $\mathcal{G}$. If the condition in Proposition 3 is satisfied for a given Nash equilibrium, then there exists a unique equilibrium of game $\mathcal{G}$. Otherwise, the Nash equilibrium may or may not be unique. The condition in Proposition 3 depends on various system parameters such as the power strategy and the locations of the anchor nodes, the properties of the signal transmitted from the anchor nodes, the multipath components between the anchor nodes and the target nodes, and the channel coefficients between the jammer nodes and the target nodes.

In the presence of multiple Nash equilibria, the anchor and jammer nodes may choose the desired Nash equilibrium depending on the conditions and constraints in the specific application. Although the average CRLB of the target nodes (i.e., the value of the game) is the same for all Nash equilibria based on Proposition 2, the anchor and jammer nodes may prefer one Nash equilibrium over the others for the efficient use of limited resources in the wireless localization network.

5.2.4 Power Control Game Based on Minimum and Maximum CRLB

Instead of employing the average CRLB as the performance metric, it is also possible to use the worst-case CRLBs for the anchor and jammer nodes as the performance metrics. In particular, from the viewpoint of the anchor nodes, the target node with the maximum CRLB (i.e., with the worst localization accuracy) can be considered with the aim of minimizing the maximum CRLB (so that a certain level of localization accuracy can be achieved by all the target nodes). Similarly, the jammer nodes can aim to maximize the minimum CRLB of the target nodes in order to degrade the localization performance of the system. For this setting, define a new game $\bar{\mathcal{G}}$ which has the same players and the same strategy sets for the players as $\mathcal{G}$ does, except for the utility functions. For a given action profile, the utility functions of player A and player J in game $\bar{\mathcal{G}}$ are given by

$$u_A(\boldsymbol{p}^A, \boldsymbol{p}^J) = -\max_{i \in 1,\ldots,N_T} \operatorname{tr}\Big\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J \right)^{-1} \Big\}, \tag{5.31}$$

$$u_J(\boldsymbol{p}^A, \boldsymbol{p}^J) = \min_{i \in 1,\ldots,N_T} \operatorname{tr}\Big\{ \boldsymbol{J}_i \left(\boldsymbol{x}_i, \boldsymbol{p}_i^A, \boldsymbol{p}^J \right)^{-1} \Big\}. \tag{5.32}$$

As it can be noted from the utility functions for player A and player J in (5.31) and (5.32), the power control game based on these utility functions is not a zero-sum game; that is, $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J) + u_J(\boldsymbol{p}^A, \boldsymbol{p}^J) \neq 0$, $\exists \boldsymbol{p}^A \in S_A \wedge \exists \boldsymbol{p}^J \in S_J$.

The utility functions in this scenario do not facilitate detailed theoretical analyses as in the case of the average CRLB based utility functions. However, the existence of a pure Nash equilibrium is still guaranteed based on the following result.

Proposition 4: *There exists at least one pure Nash equilibrium in game* $\bar{\mathcal{G}}$.

Proof: Game $\bar{\mathcal{G}}$ admits at least one pure Nash equilibrium if the conditions presented in the proof of Proposition 1 are satisfied. Game $\bar{\mathcal{G}}$ satisfies the first condition since game $\bar{\mathcal{G}}$ has the same strategy sets for the players as $\mathcal{G}$ does. Also, $u_A(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.31) and $u_J(\boldsymbol{p}^A, \boldsymbol{p}^J)$ in (5.32) are concave functions of $\boldsymbol{p}^A$ and $\boldsymbol{p}^J$, respectively, since the minimum (maximum) of concave (convex) functions is also concave (convex). Therefore, game $\bar{\mathcal{G}}$ also satisfies the second and third conditions. Consequently, based on the similar approach employed in the proof of Proposition 1, it can be shown that at least one pure-strategy Nash equilibrium exists in game $\bar{\mathcal{G}}$. ■

5.3 Numerical Results

In this section, numerical examples are provided in order to corroborate the theoretical results obtained in the previous section. To that aim, consider a wireless localization network in which four anchor nodes, three target nodes, and three jammer nodes are located as in Fig. 5.1. For the sake of simplicity, it is assumed that each target node has LOS connections to all of the anchor nodes. Also, the free space propagation model is considered; that is, λ_{ij} in (5.7) is equal to $\lambda_{ij} = 100N_0\|\boldsymbol{x}_i - \boldsymbol{y}_j\|^{-2}/2$ [61]. In addition, $|\gamma_{ij}|^2$ is given by $\|\boldsymbol{x}_i - \boldsymbol{z}_j\|^{-2}/2$ and N_0 is set to 2 [55].

In Fig. 5.2, the average CRLBs of the three target nodes (i.e., the values of the game) are plotted versus the total available power of the anchor nodes (i.e., P_T^A) for various peak powers of the anchor nodes when $P_T^J = 20$, $P_{\text{peak}}^J = 10$, and the anchor nodes and the jammer nodes operate at the Nash equilibrium. From the figure, it is observed that as the total power of the anchor nodes increases, the average CRLB obtained in the Nash equilibrium reduces since more strategies become available for the anchor nodes as P_T^A increases. Also, it can be deduced from the figure that for lower values of the total power of the anchor nodes (e.g., $P_T^A < 5$), the average CRLBs of the target nodes are the same for different values of P_{peak}^A due to the dominant effect of the total power constraint on the game value. On the other hand, for higher values of the total power of the anchor

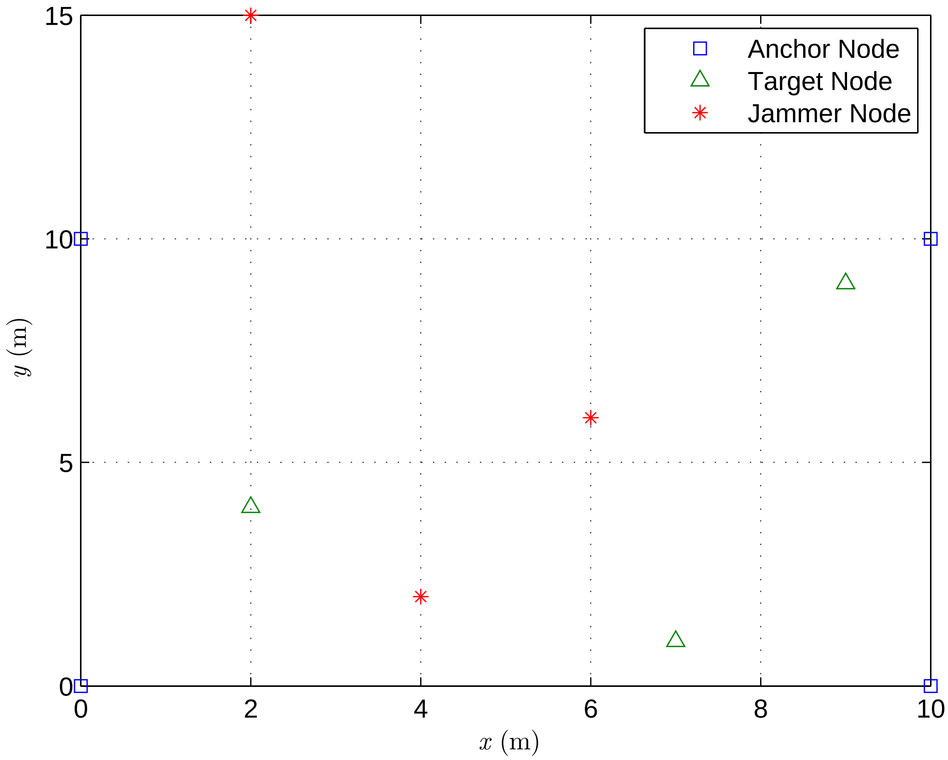

Figure 5.1: The simulated network including four anchor nodes positioned at [00], [10 0], [0 10], and [10 10]m., three jammer nodes positioned at [2 15], [4 2], and [6 6]m., and three target nodes positioned at [2 4], [7 1], and [9 9]m.

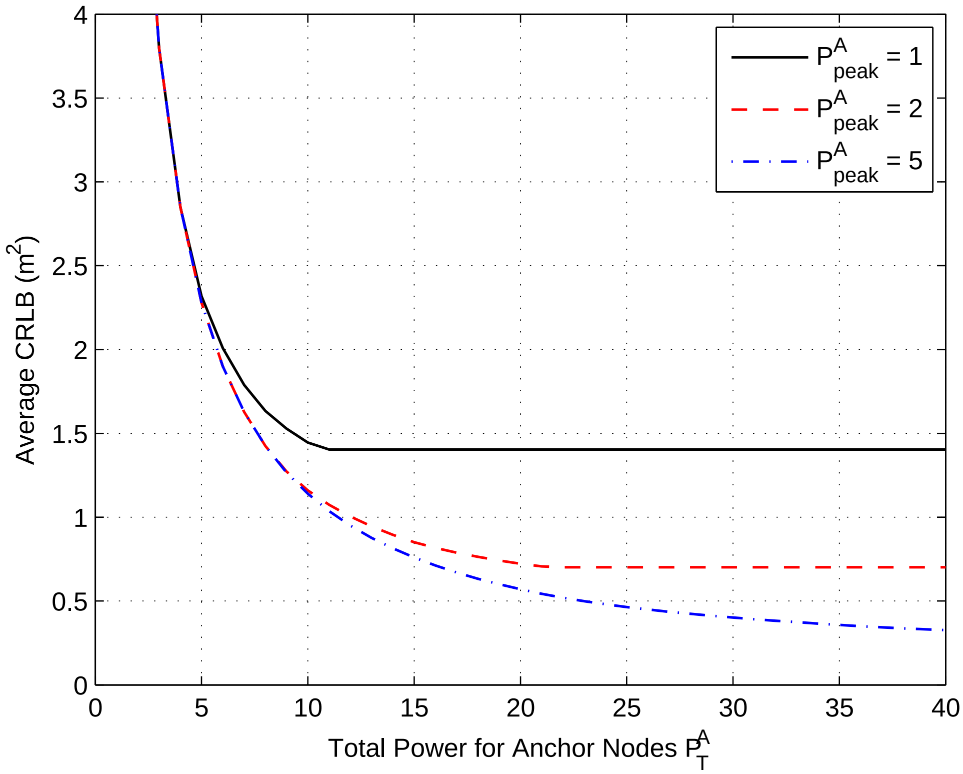

Figure 5.2: Average CRLB of the target nodes versus total power of the anchor nodes for the scenario in Fig. 5.1, where $P_T^J = 20$, $P_{\text{peak}}^J = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\mathcal{G}$.

nodes (e.g., $P_T^A \geq 12$ for $P_{\text{peak}}^A = 1$), the average CRLB of the localization system does not change since the peak power constraint of the anchor nodes limits the use of total power available for the anchor nodes.

In order to observe the effects of the peak power constraint of the anchor nodes on the average CRLB of the target nodes, the average CRLBs of the target nodes are plotted in Fig. 5.3 versus the peak power of the anchor nodes for various values of the total power of the anchor nodes when $P_T^J = 20$ and $P_{\text{peak}}^J = 10$. From Fig. 5.3, similar observations to those for Fig. 5.2 are obtained. It is also stated that the average CRLBs for different values of the total power of the anchor nodes are the same when the peak power of the anchor nodes is below a

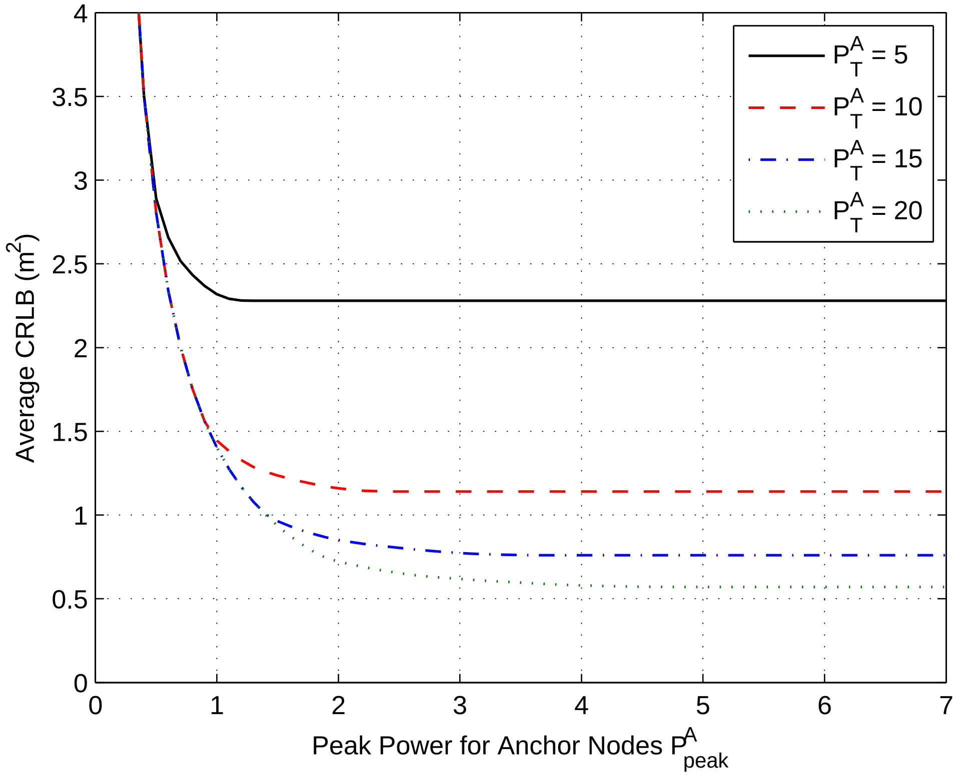

Figure 5.3: Average CRLB of the target nodes versus peak power of the anchor nodes for the scenario in Fig. 5.1, where $P_T^J = 20$, $P_{\text{peak}}^J = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\mathcal{G}$.

certain value since the peak power constraint of the anchor nodes becomes more dominant than the total power constraint in that case.

Similar to Fig. 5.2 and Fig. 5.3, the average CRLBs are plotted versus the total power of the jammer nodes for various values of the peak power and versus the peak power of the jammer nodes for different values of the total power of the jammer nodes in Fig. 5.4 and Fig. 5.5, respectively, where $P_T^A = 20$ and $P_{\text{peak}}^A = 10$. Unlike the trends in Fig. 5.2 and Fig. 5.3, the average CRLBs obtained in the Nash equilibria increase as the total power of the jammer nodes and the peak power of the jammer nodes increase in Fig. 5.4 and Fig. 5.5, respectively, since the aim of the jammer nodes is to reduce the localization performance;

that is, to increase the average CRLB. Similarly, from Fig. 5.4 and Fig. 5.5, the results related to the dominance of the constraints for different total power and peak power levels of the jammer nodes can be deduced. It is important to note that the slope of the curves in Fig. 5.4 and Fig. 5.5 changes due to the peak power and total power constraints. As an example, consider the case (i.e., the red line) in Fig. 5.4, where $P^J_{\text{peak}} = 10$, $P^A_T = 20$, and $P^A_{\text{peak}} = 10$. The slope of the curve changes when $P^J_T = 10$, $P^J_T = 20$, and $P^J_T = 30$. The reason for that can be expressed as follows: For $P^J_T \leq 10$, only one jammer node with the highest impact on the system transmits noise based on the optimization problem in (5.20). For $10 < P^J_T \leq 20$, two jammer nodes are active in the system; that is, the jammer node with the highest impact on the system transmits noise at peak power (i.e., $P^J_{\text{peak}} = 10$) whereas the other jammer node with the second highest impact on the system transmits noise such that the total power of the two nodes is equal to the total power constraint of the jammer nodes. Similarly, for $20 < P^J_T \leq 30$, all the jammer nodes operate. Due to the peak power constraint (i.e., $P^J_{\text{peak}} = 10$ for each jammer node), the power strategies of the jammer nodes remain the same for $P^J_T > 30$. On the other hand, for the cases in Fig. 5.5, a similar process can be considered in the reverse direction. Namely, all the jammer nodes transmit noise for a lower peak power of the jammer nodes and the number of active jammer nodes in the system decreases gradually as the peak power for the jammer nodes increases.

Table 5.1 presents the Nash equilibrium strategies of the anchor and jammer nodes, which are located as in Fig. 5.1, for various peak power and total power constraints of the anchor and jammer nodes. It is important to note that in Table 5.1, the Nash equilibrium strategy of the anchor nodes (i.e., player A) denoted by $\bar{\boldsymbol{p}}^A_\star$ corresponds to the reshaped version of $\boldsymbol{p}^A_\star$ in (5.17) and (5.18) for the purpose of a clear presentation. Namely, $\boldsymbol{p}^A$ is assumed to be defined as $\boldsymbol{p}^A \triangleq \left[\boldsymbol{p}^A_1 \ \cdots \ \boldsymbol{p}^A_{N_T}\right]^T$ instead of the one in (5.13). Table 5.1 provides the strategies for the anchor node and the jammer node for one Nash equilibrium obtained in each case based on the peak power and total power constraints. The results in Table 5.1 agree with Proposition 1 on that power control game $\mathcal{G}$ admits at least one pure Nash equilibrium for each case as one Nash equilibrium is provided for

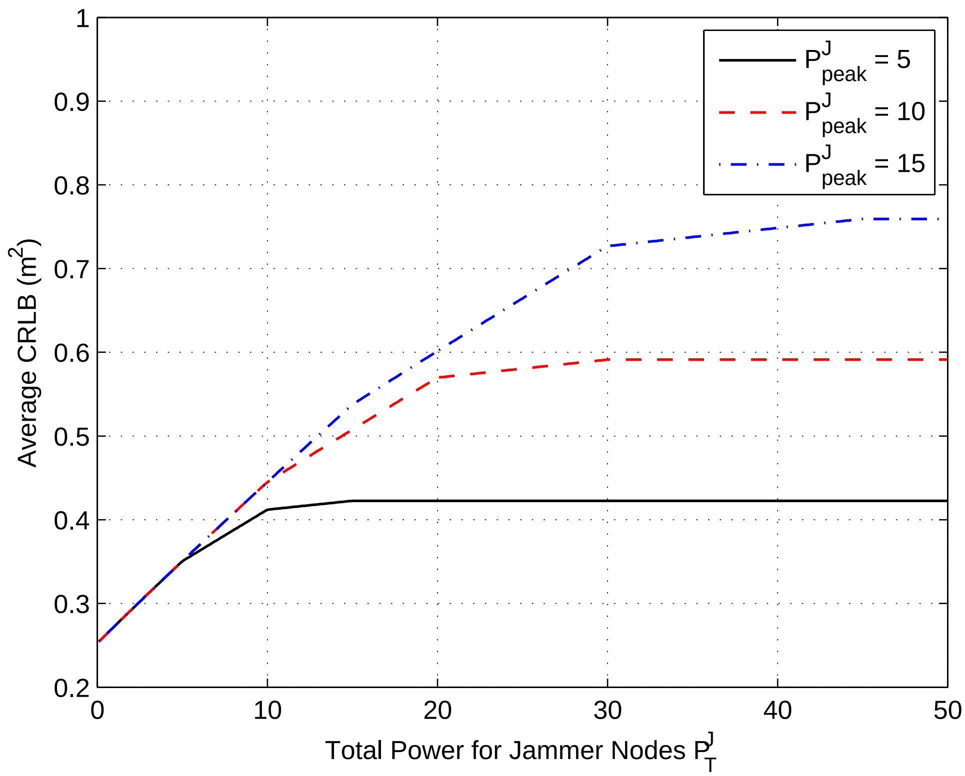

Figure 5.4: Average CRLB of the target nodes versus total power of the jammer nodes for the scenario in Fig. 5.1, where $P_T^A = 20$, $P_{\text{peak}}^A = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\mathcal{G}$.

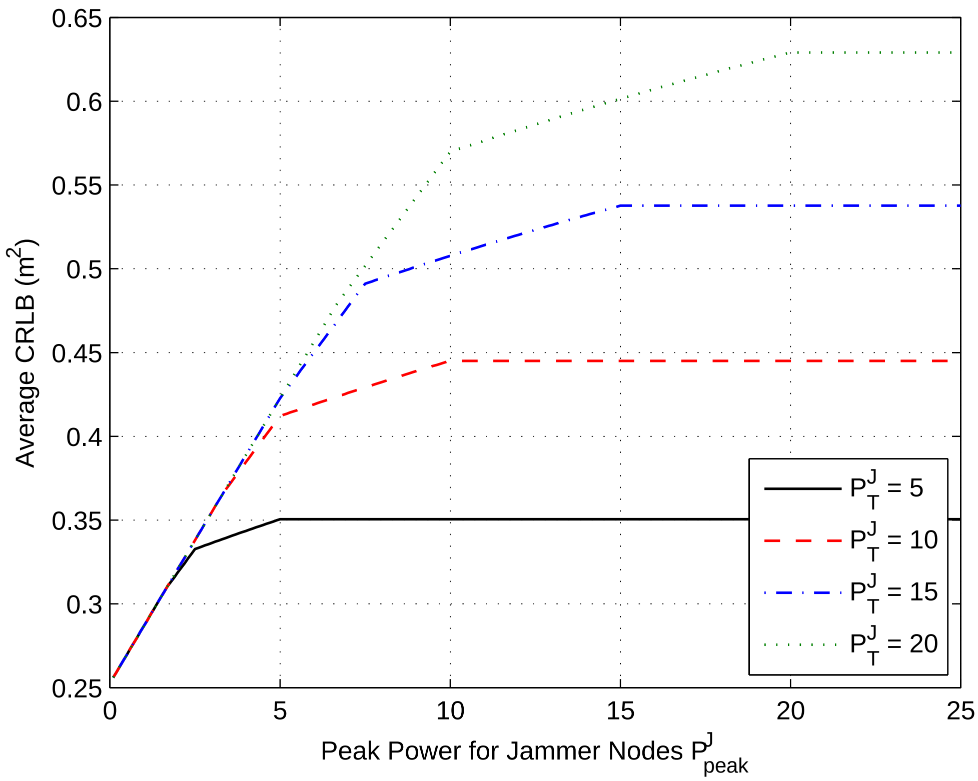

Figure 5.5: Average CRLB of the target nodes versus peak power of the jammer nodes for the scenario in Fig. 5.1, where $P_T^A = 20$, $P_{\text{peak}}^A = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\mathcal{G}$.

each case in Table 5.1. Also, it is obtained that $u_J(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J) = -u_A(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J)$ for each case, as Proposition 2 states. In addition, each obtained pure Nash equilibrium in Table 5.1 is a unique pure Nash equilibrium based on Proposition 3 since all the elements of $\boldsymbol{w}$ presented in Table 5.1 are different in each case.

In order to investigate that power control game $\mathcal{G}$ can have multiple pure Nash equilibria for some given peak power and total power constraints, consider a wireless localization network including four anchor nodes, three target nodes, and three jammer nodes which are located as in Fig. 5.6. In Table 5.2, the Nash equilibria strategies of the anchor nodes and the jammer nodes in Fig. 5.6 are provided for certain peak power and total power constraints. It is obtained from Table 5.2 that there exist multiple pure Nash equilibria for some peak power and total power constraints of the anchor nodes and the jammer nodes (e.g., $P_T^A = 15$, $P_{\text{peak}}^A = 10$, $P_T^J = 15$, and $P_{\text{peak}}^J = 10$). Also, the value of the game is unique for every Nash equilibrium as Proposition 2 states. In addition, based on Proposition 3, it can be argued that some of the elements of $\boldsymbol{w}$ provided in Table 5.2 must be the same since power control game $\mathcal{G}$ has multiple pure strategy Nash equilibria for that case, which complies with the results in Table 5.2.

At this point, it would be useful to mention that the conventional iterative algorithm based on best response dynamics is employed in the numerical examples to obtain the Nash equilibrium. In the best response dynamics, one player chooses an arbitrary strategy first and then the other player plays the best response to the opponent's current best strategy. At each round, each player employs the best response to the current strategy of the opponent iteratively and the algorithm terminates when no players have an incentive to deviate from their previous strategies, which corresponds to a Nash equilibrium in the game. When the condition in Proposition 3 is satisfied, the obtained Nash equilibrium is guaranteed to be unique. On the other hand, when that condition is not satisfied, that is, when some elements of $\boldsymbol{w}$ are identical, the power levels of the corresponding jammer nodes can be redistributed and the resulting strategies for the anchor and jammer nodes are checked to determine if another Nash equilibrium is achieved. In order to verify that the resulting strategies constitute a different Nash equilibrium, the

Table 5.1: Various strategies obtained for the scenario in Fig. 5.1 when the anchor nodes and the jammer nodes are at a Nash equilibrium in power control game $\mathcal{G}$.

$\begin{bmatrix} P_T^A \\ P_{\text{peak}}^A \\ P_T^J \\ P_{\text{peak}}^J \end{bmatrix}$	$\bar{\boldsymbol{p}}_\star^A$	$\boldsymbol{p}_\star^J$	$\boldsymbol{w}^T$	$u_J(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J)$ $(-u_A(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J))$
$\begin{bmatrix} 20 \\ 10 \\ 20 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 2.3908 & 4.2860 & 0.8796 & 0 \\ 0 & 1.6703 & 0 & 5.0111 \\ 0 & 2.5912 & 2.5912 & 0.5797 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 0.0065 \\ 0.0572 \\ 0.0371 \end{bmatrix}$	0.5698
$\begin{bmatrix} 10 \\ 10 \\ 20 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 1.1954 & 2.1430 & 0.4398 & 0 \\ 0 & 0.8352 & 0 & 2.5056 \\ 0 & 1.2956 & 1.2956 & 0.2898 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 0.0129 \\ 0.1145 \\ 0.0743 \end{bmatrix}$	1.1396
$\begin{bmatrix} 20 \\ 10 \\ 10 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 2.4470 & 4.3868 & 0.9002 & 0 \\ 0 & 1.7309 & 0 & 5.1928 \\ 0 & 2.4024 & 2.4024 & 0.5375 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.0066 \\ 0.0560 \\ 0.0378 \end{bmatrix}$	0.4450
$\begin{bmatrix} 20 \\ 1 \\ 20 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 0.0155 \\ 0.1420 \\ 0.0905 \end{bmatrix}$	1.4031
$\begin{bmatrix} 20 \\ 10 \\ 20 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 2.3341 & 4.1844 & 0.8586 & 0 \\ 0 & 1.6473 & 0 & 4.9420 \\ 0 & 2.7133 & 2.7133 & 0.6070 \end{bmatrix}$	$\begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$	$\begin{bmatrix} 0.0064 \\ 0.0581 \\ 0.0368 \end{bmatrix}$	0.4564

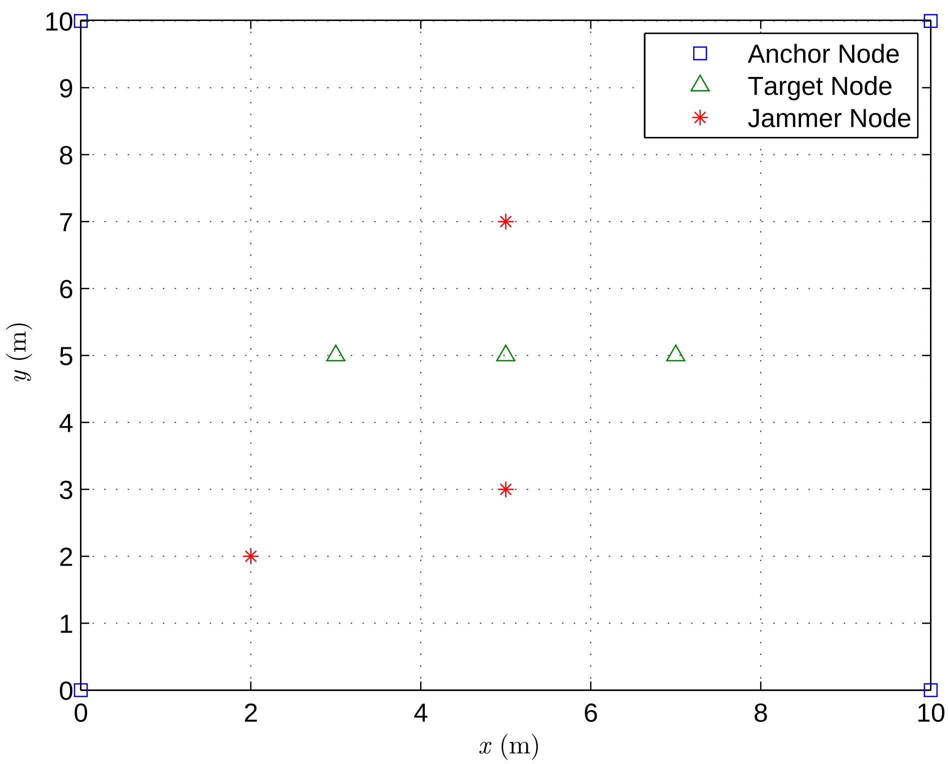

Figure 5.6: The simulated network including four anchor nodes positioned at [0 0], [10 0], [10 10], and [0 10]m., three jammer nodes positioned at [5 3], [5 7], and [2 2]m., and three target nodes positioned at [3 5], [5 5], and [7 5]m.

Table 5.2: Various strategies obtained for the scenario in Fig. 5.6 when the anchor nodes and the jammer nodes are at a Nash equilibrium in power control game $\mathcal{G}$.

$\begin{bmatrix} P_T^A \\ P_{\text{peak}}^A \\ P_T^J \\ P_{\text{peak}}^J \end{bmatrix}$	$\bar{\boldsymbol{p}}_\star^A$	$\boldsymbol{p}_\star^J$	$\boldsymbol{w}^T$	$u_J(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J)$ $\left(-u_A(\boldsymbol{p}_\star^A, \boldsymbol{p}_\star^J)\right)$
$\begin{bmatrix} 15 \\ 10 \\ 15 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 2.2220 & 0 & 0 & 2.2220 \\ 1.5280 & 1.5280 & 1.5280 & 1.5280 \\ 0 & 2.2220 & 2.2220 & 0 \end{bmatrix}$	$\begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.1801 \\ 0.1801 \\ 0.0690 \end{bmatrix}$	1.2715
$\begin{bmatrix} 15 \\ 10 \\ 15 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 2.2220 & 0 & 0 & 2.2220 \\ 1.5280 & 1.5280 & 1.5280 & 1.5280 \\ 0 & 2.2220 & 2.2220 & 0 \end{bmatrix}$	$\begin{bmatrix} 7.5 \\ 7.5 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.1801 \\ 0.1801 \\ 0.0690 \end{bmatrix}$	1.2715
$\begin{bmatrix} 15 \\ 10 \\ 15 \\ 10 \end{bmatrix}$	$\begin{bmatrix} 2.2220 & 0 & 0 & 2.2220 \\ 1.5280 & 1.5280 & 1.5280 & 1.5280 \\ 0 & 2.2220 & 2.2220 & 0 \end{bmatrix}$	$\begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 0.1801 \\ 0.1801 \\ 0.0690 \end{bmatrix}$	1.2715

best response strategy of the anchor nodes to the resulting strategy of the jammer nodes is determined first based on the best response function of the anchor nodes in (5.19). Then, if the obtained strategy of the anchor nodes does not differ from the strategy of the anchor nodes in the previous Nash equilibrium, it is concluded that the resulting strategies for the anchor and jammer nodes obtained by redistributing the power levels of the jammer nodes correspond to another Nash equilibrium. Otherwise, if the strategies of the anchor nodes do not match, the resulting strategies cannot be considered as a Nash equilibrium and other possible strategies of the jammer nodes produced based on redistribution of the power levels may be examined to find another Nash equilibrium. In this way, multiple Nash equilibria can be obtained, as in Table 5.2.

To analyze power control game $\bar{\mathcal{G}}$ in which the utility functions of the players are based on the minimum and maximum CRLBs instead of the average CRLB (see Section 5.2.4), consider the wireless localization network in Fig. 5.1. In Fig. 5.7, the minimum and maximum CRLBs of the target nodes are plotted versus the total available power of the anchor nodes for various values of the peak power constraint of the anchor nodes when $P_T^J = 20$ and $P_{\text{peak}}^J = 10$. It is noted that for low values of the total power constraint of the anchor nodes, the utility functions of the anchor nodes and the jammer nodes become equal in magnitude; that is, the sum of the utility functions of the players is equal to zero. On the other hand, the utility functions of the anchor nodes and the jammer nodes are not equal for higher values of the total power constraint of the anchor nodes. Then, in Fig. 5.8, the minimum and maximum CRLBs of the target nodes are plotted versus the peak power of the anchor nodes when the anchor nodes and the jammer nodes operate at the Nash equilibrium, $P_T^J = 20$, and $P_{\text{peak}}^J = 10$. Unlike the previous figure, the utility functions of the players in game $\bar{\mathcal{G}}$ differ in magnitude for low values of the peak power of the anchor nodes. On the other hand, for high values of the peak power of the anchor nodes, the sum of the utility functions of the anchor and jammer nodes becomes zero. It is also important to emphasize that as the total power of the anchor nodes increases, the CRLBs (i.e., the minimum of targets' CRLBs for the jammer nodes and the maximum of targets' CRLBs for the anchor nodes) obtained in the Nash equilibrium reduce. Similar plots to those in Fig. 5.7 and Fig. 5.8 are presented in Fig. 5.9 and Fig. 5.10 for the jammer nodes considering various values of the total power and peak power constraints of the jammer nodes when $P_T^A = 20$ and $P_{\text{peak}}^A = 10$. From Fig. 5.9 and Fig. 5.10, it is noticed that multiple Nash equilibria can be observed for power control game $\bar{\mathcal{G}}$ in some cases and the magnitude of the utilities obtained in those Nash equilibria points can get the values represented in the shaded regions of Fig. 5.9 and Fig. 5.10. However, for some values of the constraints, the Nash equilibria may be unique (e.g., for high values of the total power of the jammer nodes). Lastly, the results in the figures comply with the statement in Proposition 4 that power control game $\bar{\mathcal{G}}$ has at least one pure Nash equilibrium.

5.4 Concluding Remarks

In this chapter, interactions between anchor and jammer nodes have been analyzed for a wireless localization network. Based on a game-theoretic framework, two types of power control games between anchor and jammer nodes have been investigated by employing the average CRLB and the worst-case CRLBs of the target nodes (from the viewpoints of the anchor and jammer nodes) as performance metrics. It has been proved that both games have at least one pure strategy Nash equilibrium. This implies that there exist deterministic power allocation strategies for the anchor and jammer nodes that lead to one or more Nash equilibria in both games. In addition, an approach has been presented in order to figure out the Nash equilibrium of the game which employs the average CRLB as the performance metric, and a sufficient condition has been provided to determine the uniqueness of the Nash equilibrium. The theoretical investigations have been illustrated via numerical examples. As an interesting direction for future work, uncertainty on various parameters of anchor and jammer nodes can be incorporated into the game models, and different game models such as stochastic and repetitive games can be considered for the localization performance of target nodes in the presence of jammer nodes in a wireless localization network.

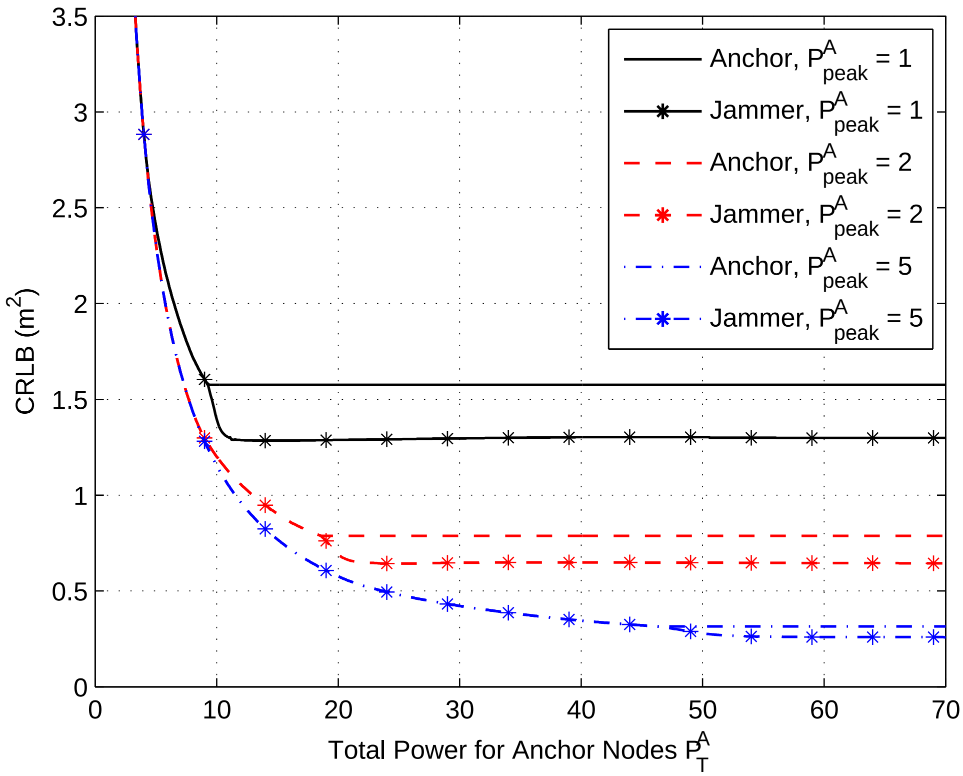

Figure 5.7: Minimum and maximum CRLBs (i.e., absolute utility values for the jammer and anchor nodes, respectively) of the target nodes versus total power of the anchor nodes for the scenario in Fig. 5.1, where $P_T^J = 20$, $P_{\text{peak}}^J = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\bar{\mathcal{G}}$.

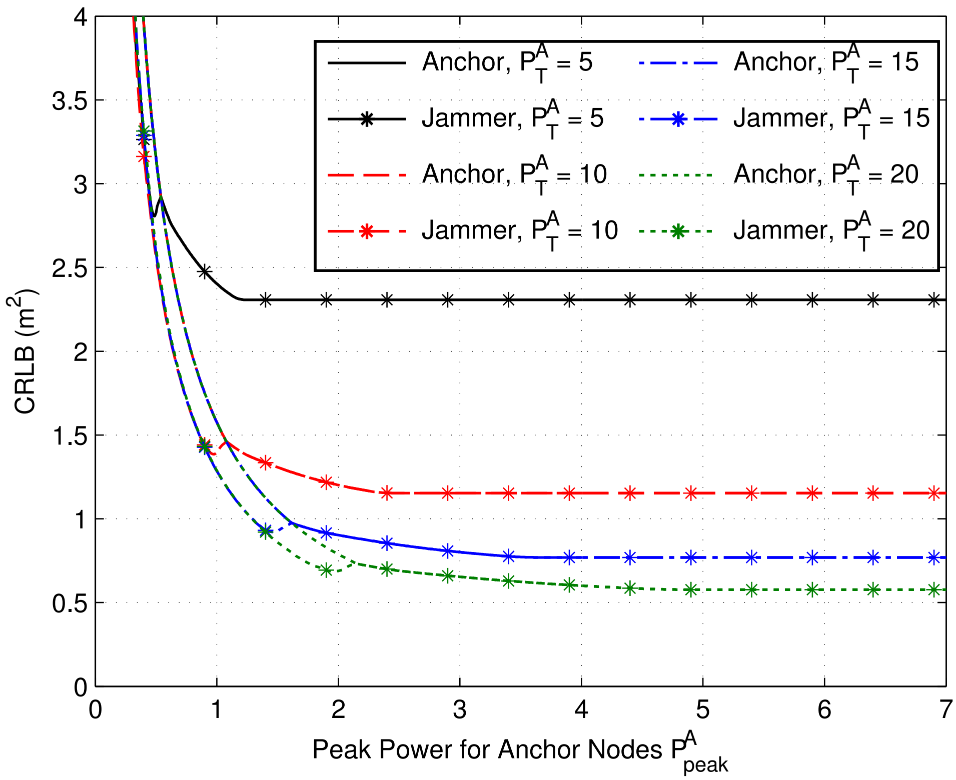

Figure 5.8: Minimum and maximum CRLBs (i.e., absolute utility values for the jammer and anchor nodes, respectively) of the target nodes versus peak power of the anchor nodes for the scenario in Fig. 5.1, where $P_T^J = 20$, $P_{\text{peak}}^J = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\bar{\mathcal{G}}$.

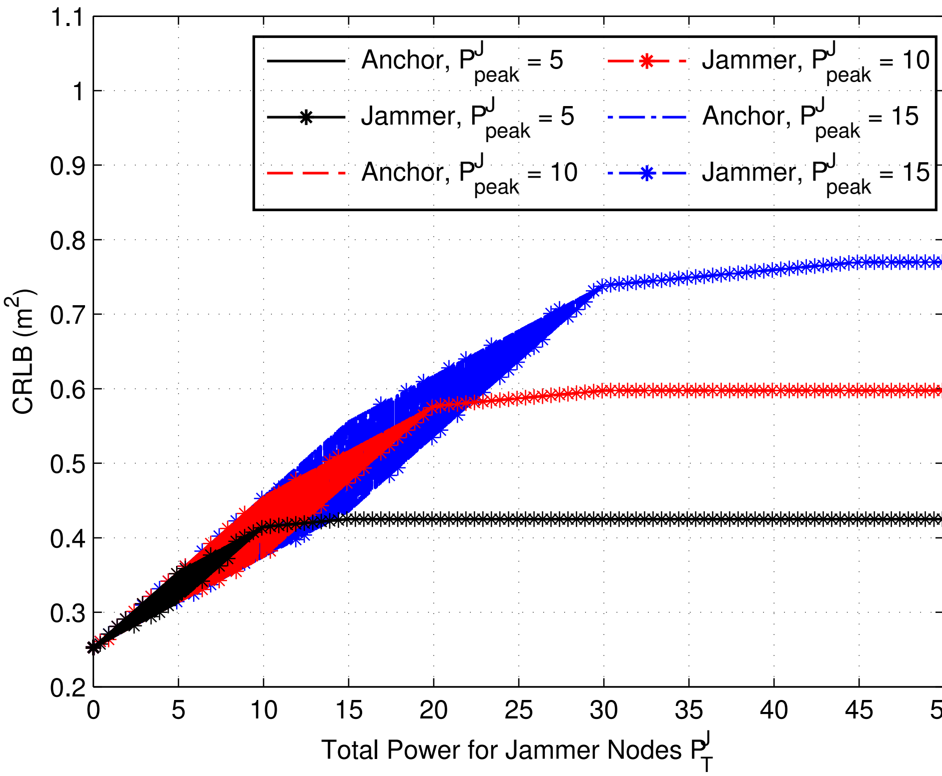

Figure 5.9: Minimum and maximum CRLBs (i.e., absolute utility values for the jammer and anchor nodes, respectively) of the target nodes versus total power of the jammer nodes for the scenario in Fig. 5.1, where $P_T^A = 20$, $P_{\text{peak}}^A = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\bar{\mathcal{G}}$.

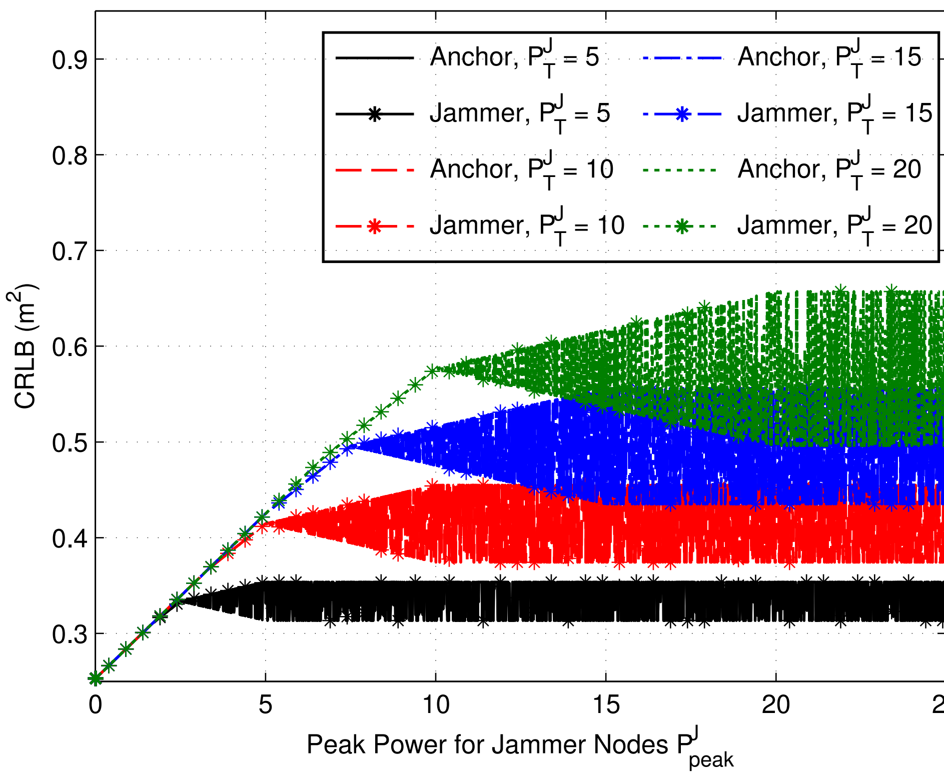

Figure 5.10: Minimum and maximum CRLBs (i.e., absolute utility values for the jammer and anchor nodes, respectively) of the target nodes versus peak power of the jammer nodes for the scenario in Fig. 5.1, where $P_T^A = 20$, $P_{\text{peak}}^A = 10$, and the anchor nodes and the jammer nodes operate at Nash equilibrium in power control game $\bar{\mathcal{G}}$.

Chapter 6

Conclusions and Future Work

In this dissertation, power allocation strategies have been considered for both channel switching and wireless localization problems. In the first part of the dissertation, optimal channel switching approaches have been designed for average capacity maximization. In Chapter 2, the optimal channel switching problem has been presented in the presence of average and peak power constraints. It has been shown that the optimal channel switching strategy can be obtained by switching between no more than two different channels. Also, a low-complexity optimization problem has been investigated to obtain the optimal channel switching solution. In addition, necessary and sufficient conditions have been provided to determine whether the optimal channel switching strategy can or cannot outperform the optimal single channel strategy. Then, in Chapter 3, the study in Chapter 2 has been extended for a communication system where channel switching delays (costs) are considered. It has been proved that the optimal strategy corresponds to switching between at most two channels. Sufficient conditions have been specified for the scenarios in which the optimal strategy corresponds to the exclusive use of a single channel or to channel switching between two channels. In Chapter 4, the optimal channel switching strategies have been investigated for average capacity maximization in the consideration of multiple receivers in a communication system. The optimal channel switching problem has been formulated for a transmitter which communicates with primary and secondary receivers in the consideration of the minimum average capacity requirement of the primary receiver and the average and peak power constraints. It has been investigated that the optimal channel switching strategy can be realized by channel switching between at most three communication links in the presence of multiple available channels in the communication system. The possible optimal channel switching scenarios have been discussed in terms of the number of channels employed by the transmitter to communicate with the primary and secondary receivers while fulfilling the capacity requirement of the primary receiver. In the second part of the dissertation, power control games have been analyzed for wireless localization networks with not only anchor and jammer nodes but also jammer nodes. In Chapter 5, a game-theoretic framework has been developed to examine the interactions between anchor and jammer nodes in terms of equilibria. Power control games between anchor and jammer nodes have been designed and it has been shown that the designed games admit at least one pure strategy Nash equilibrium. Also, an approach has been developed to obtain the pure strategy Nash equilibrium and a sufficient condition has been formulated to specify whether the obtained Nash equilibrium is a unique pure strategy Nash equilibrium. In each chapter, numerical examples have been provided to illustrate the theoretical investigations.

As future work for the first part of the dissertation, power adaptation over varying channel states can be considered in the design of channel switching strategies for average capacity maximization. Also, optimal channel switching strategies can be designed in the presence of multiple secondary receivers in the communication system. For the second part of the dissertation, a future research direction can be to design power control games between anchor and jammer nodes in the presence of imperfect knowledge among the entities and to characterize the equilibria of such games in terms of existence, uniqueness and stability.

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