Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 50-1 (2017) 5492–5497

Adaptive Control Allocation for Over-Actuated Systems with Actuator Saturation ⋆

S. S. Tohidi * **Y. Yildiz** * **I. Kolmanovsky** **

* *Mechanical Engineering Department, Bilkent University, Ankara 06800, Turkey, (e-mail: {shahabaldin, yyildiz} @ bilkent.edu.tr)*
** *Aerospace Engineering Department, University of Michigan, Ann Arbor MI 48109, USA (e-mail: ilya@umich.edu)*

Abstract: This paper proposes an adaptive control allocation approach for over-actuated systems with actuator saturation. The methodology can tolerate actuator loss of effectiveness without utilizing the control input matrix estimation, eliminating the need for persistence of excitation. Closed loop reference model adaptive controller is used for identifying adaptive parameters, which provides improved performance without introducing undesired oscillations. The modular design of the proposed control allocation method improves the flexibility to develop the outer loop controller and the control allocation strategy separately. The ADMIRE model is used as an over-actuated system, to demonstrate the effectiveness of the proposed method using simulation results.

Keywords: Adaptive control, Control allocation, Actuator constraint, Sliding mode control.

1. INTRODUCTION

Control allocation (CA) methodologies can be used to distribute control signals among redundant actuators. CA can also be used to redistribute the control inputs in the event of an actuator fault or loss of effectiveness. Surveys on control allocation methodologies and various methods of reconfigurable fault tolerant control can be found in Johansen et al. (2013) and Zhang (2008), respectively. Two main control allocation approaches that are used for fault tolerance applications are optimization based control allocation and adaptive control allocation.

Error minimization as an optimization based control allocation method is used in Tjønnås et al. (2010) to improve the performance of steering in faulty automotive vehicles considering faults as asphalt conditions. In another study by Podder et al. (2001), thruster force is allocated among faulty redundant thrusters using control minimization. The study by Sadeghzadeh et al. (2012) shows the experimental results under different propeller faults on a modified quad-rotor helicopter. This method is implemented in various other over-actuated systems to tolerate faults, but in all of them, the control input matrix is either estimated or assumed to be known (Casavola et al. (2010); Liu et al. (2015); Wang et al. (2013); Liu C. et al. (2012); Doman et al. (2002); Reish et al. (2013); Tohidi et al. (2016b)).

Lower computational complexity of adaptive control allocation methods is one of their benefits in comparison with optimization based control allocation methods. However, guaranteeing persistent excitation conditions in adaptive methods is necessary for accurate parameter estimation. In Casavola et al. (2010), faults are estimated adaptively using a recursive least square method and an online dither generation methodology is proposed to guarantee the persistence of excitation of signals. The control allocation problem is considered as a gain scheduling problem in Liu Y. et al. (2012) and the gains are estimated adaptively. However, the allocation problem is coupled with the model reference adaptive controller design i.e., the structure is not modular. An adaptive fault tolerant controller is proposed in Liu et al. (2008) to tolerate the actuator lock-in-place failures, but this method does not have modular structure. Useful information about faults can be inferred using fault detection and isolation methodologies for control allocation (see Davidson et al. (2001)). In Cristofaro et al. (2014, 2016), an unknown input observer (UIO) is applied to identify actuator and effector faults. In Alwi et al. (2008), sliding mode controller is coupled with pseudo inverse method to design a fault tolerant controller, but the faults are assumed to be estimated and actuator constraints are not considered. Adaptive control allocation without utilizing fault estimation is proposed in Tohidi et al. (2016a), but in that work actuator saturations are not considered.

A study on control allocation that considers actuator constraints is conducted in Durham (1993) by using direct allocation method. Optimization based control allocation is one of the most common method of accounting for actuator constraints. Optimization based control allocation is used in various papers like Petersen et al. (2006), Johansen et al. (2013), Yildiz et al. (2010, 2011a, 2011b) and Acosta et al. (2015). Convexification of a non-convex attainable region in the control allocation setting is investigated in

⋆ Author Yildiray Yildiz would like to thank the Scientific and Technological Research Council of Turkey (TUBITAK) for its financial support through the 2232 Reintegration Scholarship Program.

2405-8963 © 2017, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.1088

Johansen et al. (2008). An optimal iterative method to force allocated signals to satisfy their constraints is proposed in Tohidi et al. (2016b). In Tjønnås et al. (2008), the unknown parameters are estimated adaptively, and are used in an optimization based control allocation which considers actuator constraints. The proposed adaptive law guarantees the parameter convergence which leads to finding a global optimal solution if the persistence of excitation assumptions are satisfied.

This paper proposes an adaptive control allocation method for systems with actuator constraints. The method does not need fault estimation, so it does not require persistence of excitation or additional sensors to determine actuator effectiveness. Adaptive parameters are estimated rapidly without causing excessive oscillations with the help of the adaptive method that utilizes closed loop reference models (Gibson (2014)). In addition, a sliding mode control is designed to control the outer loop.

This paper is organized as follow. Section 2 presents the faulty over-actuated system where actuator faults are modeled as loss of effectiveness. The adaptive control allocation which considers actuator constraints is presented in Section 3. Section 4 presents the sliding mode controller design. The ADMIRE model is used in Section 5 to illustrate the effectiveness of the proposed methodology in the simulation environment. Finally, Section 6 concludes the paper.

2. PROBLEM STATEMENT

Consider the following plant dynamics

$$\dot{x} = Ax + B_u u = Ax + B_v B u \tag{1}$$

where $x \in R^n$ is the state vector, $u = [u_1, ..., u_m]^T \in R^m$ is the constrained control input vector with amplitude limits $u_i \in [-u_{max_i}, u_{max_i}]$ and rate limits $\dot{u}_i \in [-\dot{u}_{max_i}, \dot{u}_{max_i}]$, $A \in R^{n\times n}$ is the known state matrix and $B_u \in R^{n\times m}$ is the known control input matrix which is decomposed into a product of matrices $B_v \in R^{n\times r}$ and $B \in R^{r\times m}$ (see Harkegard et al. (2005)). Since the system has redundant actuators, $\text{Rank}(B_u) = r < m$. To model the actuator effectiveness uncertainty, a diagonal matrix $\Lambda \in R^{m\times m}$ with uncertain positive elements is added to the system dynamics as follows

$$\dot{x} = Ax + B_v B \Lambda u. \tag{2}$$

Let $v \in R^r$ denote the virtual control input produced by an outer loop controller. The control allocation problem is to achieve

$$\dot{x} = Ax + B_v v. \tag{3}$$

Conventional control allocation methods do not apply since Λ is an uncertain matrix. In addition, matrix identification methods may not be used since it may be hard to realize the persistent excitation conditions in real applications. The following assumptions guarantee the controllability of the system.

Assumption 1. A and B_u are known matrices and the system (A, B_u) is controllable.

Assumption 2. $\text{Rank}(B_v B\Lambda) = \text{Rank}(B_v B)$.

Remark 1. Assumption 2 guarantees that the pair $(A, B_v B\Lambda)$ is controllable.

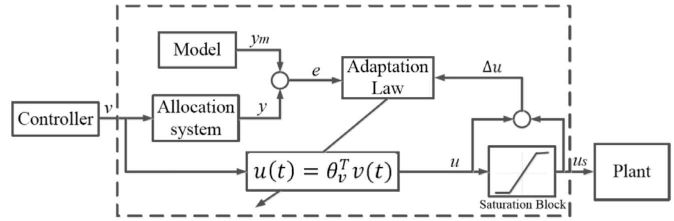

Fig. 1. Block diagram of the proposed adaptive control allocation method.

Remark 2. If the actuators are unconstrained, q actuators can fail completely where $q \le m - r, u \in R^m, v \in R^r$, without loosing controllability.

3. ADAPTIVE CONTROL ALLOCATION

In this section, we develop the proposed adaptive control allocation method for the over-actuated system with actuator rate and amplitude saturation. Towards this end, we first transform the control allocation problem into a conventional model reference adaptive control problem and then develop the corresponding adaptive laws. The block diagram of the proposed adaptive control allocation is presented in Fig. 1.

Consider the following dynamics for a variable y

$$\dot{y} = A_m y + B\Lambda u - v \tag{4}$$

where $A_m \in R^{r\times r}$ is a stable matrix and a reference model is given as

$$\dot{y}_m = A_m y_m. \tag{5}$$

Defining the control input as a mapping from v to u,

$$u = \theta_v^T v \tag{6}$$

where $\theta_v \in R^{r\times m}$ represents the adaptive parameter matrix to be determined, and substituting (6) into (4), we obtain

$$\dot{y} = A_m y + (B\Lambda\theta_v^T - I)v. \tag{7}$$

To consider rate and amplitude saturations, the output of the control allocation system is defined as (see Leonessa et al. (2009))

$$\begin{aligned} &\sigma_i(t) \equiv \begin{cases} 0 \;\; if \;\; |u_i(t)| = u_{max,i} \;\; and \;\; u_i(t)\dot{u}_i(t) > 0 \\ 1 \;\; otherwise \end{cases} \\ &\dot{u}_{s,i}(t) \equiv \dot{u}_i(t)\sigma_i^*(t), \quad \sigma_i^*(t) \equiv min\{\sigma_i(t), \frac{\dot{u}_{max,i}}{|\dot{u}_i(t)|}\} \end{aligned} \tag{8}$$

where $u_s = [u_{s,1} u_{s,2} ... u_{s,m}]^T \in R^m$ is the allocated control input for the actuators. In the light of (8), (4) can be represented as

$$\dot{y} = A_m y + B\Lambda u_s - v. \tag{9}$$

Defining $\Delta u \equiv u_s - \theta_v^T v$, (9) is written as

$$\dot{y} = A_m y + (B\Lambda\theta_v^T - I)v + B\Lambda\Delta u. \tag{10}$$

It is assumed that there exists a θ_v^* such that $B\Lambda{\theta_v^*}^T = I$. Defining $e \equiv y - y_m$ and using (5) and (10), it is obtained that

$$\dot{e} = A_m e + B\Lambda\tilde{\theta}_v^T v + B\Lambda\Delta u. \tag{11}$$

Consider the following equation (Karason et al. (1994))

$$\dot{e}_\Delta = A_m e_\Delta + k_\Delta(t)\Delta u, \quad e_\Delta(t_0) = 0, \tag{12}$$

with $k_\Delta(t)$ a time-varying matrix, and let $e_u = e - e_\Delta$. The derivative of e_u is obtained as

$$\dot{e}_u = A_m e_u + B\Lambda\tilde{\theta}_v^T v + \kappa\Delta u, \tag{13}$$

where $\kappa = B\Lambda - k_\Delta(t) \in R^{r\times m}$. Let $\Gamma = \Gamma^T \in R^{r\times r} > 0$, $\bar{\Gamma} = \bar{\Gamma}^T \in R^{r\times r} > 0$ and consider a Lyapunov function candidate

$$V = e_u^T P e_u + tr(\tilde{\theta}_v^T \Gamma^{-1}\tilde{\theta}_v \Lambda) + tr(\kappa^T \bar{\Gamma}^{-1}\kappa), \tag{14}$$

where tr refers to the trace operation and P is the positive definite symmetric matrix solution of the Lyapunov equation $A_m^T P + PA_m = -Q$, where Q is a symmetric positive definite matrix. The derivative of the Lyapunov function candidate can be calculated as

$$\begin{aligned}\dot{V} &= e_u^T(A_m P + PA_m)e_u + 2e_u^T PB\Lambda\tilde{\theta}_v^T v\\ &+2tr(\tilde{\theta}_v^T\Gamma^{-1}\dot{\tilde{\theta}}_v\Lambda) + 2\Delta u^T\kappa^T Pe_u + 2tr(\kappa^T\bar{\Gamma}^{-1}\dot{\kappa})\\ &= -e_u^T Qe_u + 2e_u^T PB\Lambda\tilde{\theta}_v^T v + 2tr(\tilde{\theta}_v^T\Gamma^{-1}\dot{\tilde{\theta}}_v\Lambda)\\ &+2\Delta u^T\kappa^T Pe_u + 2tr(\kappa^T\bar{\Gamma}^{-1}\dot{\kappa}).\end{aligned} \tag{15}$$

Using the property of the trace operation $a^T b = tr(ba^T)$ where a and b are vectors, (15) can be rewritten as

$$\begin{aligned}\dot{V} &= -e_u^T Qe_u + 2tr\bigg(\tilde{\theta}_v^T\Big(ve_u^T PB + \Gamma^{-1}\dot{\tilde{\theta}}_v\Big)\Lambda\bigg)\\ &+2tr\bigg(\kappa^T\Big(Pe_u\Delta u^T + \bar{\Gamma}^{-1}\dot{\kappa}\Big)\bigg).\end{aligned} \tag{16}$$

Using the following adaptive laws

$$\begin{aligned}\dot{\theta}_v &= \Gamma\mathrm{Proj}\big(\theta_v, -ve_u^T PB\big),\\ \dot{\kappa} &= \bar{\Gamma}\mathrm{Proj}\big(\kappa, -Pe_u\Delta u^T\big),\end{aligned} \tag{17}$$

where "Proj" refers to the projection operator (see Lavretsky et al. (2011)), it is obtained that

$$\begin{aligned}\dot{V} &= -e_u^T Qe_u\\ &+2tr\bigg(\tilde{\theta}_v^T\Big(ve_u^T PB + \mathrm{Proj}\big(\theta_v, -ve_u^T PB\big)\Big)\Lambda\bigg)\\ &+2tr\bigg(\kappa^T\Big(Pe_u\Delta u^T + \mathrm{Proj}\big(\kappa, -Pe_u\Delta u^T\big)\Big)\bigg).\end{aligned} \tag{18}$$

Defining $Y = -ve_u^T PB$ and $X = -Pe_u\Delta u^T$, and using the property of the projection operator given in Lavretsky et al. (2011):

$$\begin{aligned}&tr\bigg(\tilde{\theta}_v^T\big(-Y + \mathrm{Proj}(\theta_v, Y)\big)\Lambda\bigg) \le 0\\ &tr\bigg(\kappa^T\big(-X + \mathrm{Proj}(\kappa, X)\big)\bigg) \le 0\end{aligned} \tag{19}$$

we obtain that $\dot{V} \le 0$.

Remark 3. A negative semi-definite Lyapunov function derivative ensures that the error signal e_u and the adaptive parameters $\tilde{\theta}_v$ and κ are bounded. Assuming that v is bounded, (6) implies that u is bounded and therefore Δu is bounded. Therefore, since A_m is Hurwitz, it can be shown, using (10)-(13), that all the signals in the control allocation system are bounded.

To obtain fast convergence without introducing excessive oscillations, the open loop reference model (5) is modified to obtain the following closed loop reference model (Gibson et al. (2012)),

$$\dot{y}_m = A_m y_m - L(y - y_m) \tag{20}$$

where $A_m \in R^{r\times r}$ is Hurwitz and $L = -\ell I_r, \ell > 0$ and $I_r \in R^{r\times r}$ is an identity matrix. Defining $\bar{A}_m = A_m + L$, assuming this is a Hurwitz matrix for an appropriate selection of L, and subtracting (20) from (10), it is obtained that

$$\dot{e} = \bar{A}_m e + B\Lambda\tilde{\theta}_v^T v + B\Lambda\Delta u. \tag{21}$$

Consider the following differential equation

$$\dot{e}_\Delta = \bar{A}_m e_\Delta + k_\Delta\Delta u, \quad e_\Delta(t_0) = 0. \tag{22}$$

Letting $e_u \equiv e - e_\Delta$, the derivative of e_u is obtained as

$$\dot{e}_u = \bar{A}_m e_u + B\Lambda\tilde{\theta}_v^T v + \kappa\Delta u, \tag{23}$$

where $\kappa = B\Lambda - k_\Delta \in R^{r\times m}$. Using the Lyapunov function (14), where P is the symmetric positive definite matrix solution of the following Lyapunov equation

$$\bar{A}_m^T P + P\bar{A}_m = -Q, \tag{24}$$

the derivative of the Lyapunov function can be obtained as given in (16). Using the adaptive laws (17) and a similar procedure as above, it can be shown that all the signals in the control allocation system are bounded.

To find a convergence set for e and $\tilde{\theta}_v$, it is necessary to define the following parameters (Gibson et al. (2012))

$$\sigma \equiv -\max_i(\mathrm{Real}(\lambda_i(A_m))), \tag{25}$$

$$s \equiv -\min_i(\lambda_i(A_m + A_m^T)/2), \quad a \equiv ||A_m||. \tag{26}$$

Lemma 1. (Gibson et al. (2012)). Using the definitions (2 5) - (26) and by considering $Q = I_r$ in (24) where I_r is an identity matrix of dimension $r\times r$, P satisfies the following properties

$$||P|| \le \frac{m^2}{\sigma + 2\ell}, \; m = \frac{3}{2}(1 + 4\frac{a}{\sigma})^{(r-1)}, \tag{27}$$

$$\lambda_{min}(P) \ge \frac{1}{2(s+\ell)}. \tag{28}$$

An upper bound for V is obtained as in (Gibson et al. (2012))

$$\begin{aligned}V &= e_u^T Pe_u + tr(\tilde{\theta}_v^T\Gamma^{-1}\tilde{\theta}_v\Lambda) + tr(\kappa^T\bar{\Gamma}^{-1}\kappa)\\ &\le ||e_u||^2||P|| + tr(\tilde{\theta}_v^T\Gamma^{-1}\tilde{\theta}_v\Lambda) + tr(\kappa^T\bar{\Gamma}^{-1}\kappa)\\ &= ||e_u||^2||P|| + (1/\gamma)tr(\tilde{\theta}_v^T\tilde{\theta}_v\Lambda) + (1/\bar{\gamma})tr(\kappa^T\kappa)\\ &\le ||e_u||^2||P|| + (1/\gamma)||\tilde{\theta}_v||^2||\Lambda|| + (1/\bar{\gamma})||\kappa||^2\\ &\le ||e_u||^2||P|| + (1/\gamma)\tilde{\theta}_{max}^2 + (1/\bar{\gamma})\kappa_{max}^2\end{aligned} \tag{29}$$

where $\Gamma^{-1} = (1/\gamma)I_r$, $\bar{\Gamma}^{-1} = (1/\bar{\gamma})I_r$ and $\Lambda = diag(\lambda_1, ..., \lambda_m)$, $0 \le \lambda_i \le 1$. Thus we have

$$\frac{V}{||P||} - \frac{\tilde{\theta}_{max}^2}{\gamma||P||} - \frac{\kappa_{max}^2}{\bar{\gamma}||P||} \le ||e_u||^2. \tag{30}$$

Using (18) and (19), $\dot{V} \le -e_u^T e_u \le -||e_u||^2$, in addition, by using (30), we have

$$\dot{V} \le -\frac{V}{||P||} + \frac{\tilde{\theta}_{max}^2}{\gamma||P||} + \frac{\kappa_{max}^2}{\bar{\gamma}||P||} = -\alpha_1 V + \alpha_2 \tag{31}$$

where $\alpha_1 = \frac{\sigma+2\ell}{m^2}$ and $\alpha_2 = \frac{\sigma+2\ell}{m^2}\Big(\frac{\tilde{\theta}_{max}^2}{\gamma} + \frac{\kappa_{max}^2}{\bar{\gamma}}\Big)$. By using the Gronwall-Bellman inequality, (31) can be rewritten as

$$V \le \big(V(0) - \frac{\alpha_2}{\alpha_1}\big)e^{-\alpha_1 t} + \frac{\alpha_2}{\alpha_1}. \tag{32}$$

Using $e^T Pe \le V \le \big(V(0) - \frac{\alpha_2}{\alpha_1}\big)e^{-\alpha_1 t} + \frac{\alpha_2}{\alpha_1}$ and taking the limit of left and right hand sides, we have

$$\lim_{t\to\infty} e_u^T Pe_u \le \frac{\alpha_2}{\alpha_1} = \frac{\tilde{\theta}_{max}^2}{\gamma} + \frac{\kappa_{max}^2}{\bar{\gamma}}. \tag{33}$$

By using the following inequality

$$\lambda_{min}(P)||e_u||^2 \le e_u^T Pe_u \le \lambda_{max}(P)||e_u||^2 \tag{34}$$

and (28), we have

$$\frac{1}{2(s+\ell)}||e_u||^2 \le \lambda_{min}(P)||e_u||^2 \le e_u^T Pe_u. \tag{35}$$

By using (33) and taking the limit of both sides of (35),

$$\lim_{t\to\infty} ||e_u||^2 \leq 2(s+\ell)\Big(\frac{\tilde{\theta}^2_{max}}{\gamma} + \frac{\kappa^2_{max}}{\bar{\gamma}}\Big) \tag{36}$$

Therefore for the initial conditions $e_u(0)$ and $||\tilde{\theta}_v(0)|| \leq \tilde{\theta}_{max}$, e_u and $\tilde{\theta}_v$ are uniformly bounded for $\forall t \geq 0$ and system trajectories converge to the following sets

$$\begin{aligned} E = \{(e_u, \tilde{\theta}_v) :\ & ||e_u||^2 \leq 2(s+\ell)\Big(\frac{\tilde{\theta}^2_{max}}{\gamma} + \frac{\kappa^2_{max}}{\bar{\gamma}}\Big), \\ & ||\tilde{\theta}_v|| \leq \tilde{\theta}_{max}\}, \\ E_e = \{(e, \tilde{\theta}_v) :\ & ||e||^2 \leq 2(s+\ell)\Big(\frac{\tilde{\theta}^2_{max}}{\gamma} + \frac{\kappa^2_{max}}{\bar{\gamma}}\Big) \\ & + \frac{a}{b}|k_\Delta|\Delta_{max}, ||\tilde{\theta}_v|| \leq \tilde{\theta}_{max}\} \end{aligned} \tag{37}$$

where $a, b \in R_{>0}$ satisfying $||e^{A_m t}|| \leq ae^{-bt}$ and $\Delta_{max} \equiv \sup_{0\leq t\leq\infty}(||\Delta u(t)||)$.

4. OUTER LOOP CONTROLLER

The stability of the inner loop controller is dependent on the boundedness of the virtual control v (see Remark 3), thus the outer loop controller should guarantee the boundedness of v, independently from the stability of the control allocation. Substituting (6) into (2), we rewrite the plant dynamics as

$$\begin{aligned} \dot{x} &= Ax + B_v B\Lambda u = Ax + B_v B\Lambda\theta_v^T v \\ &= Ax + B_v(B\Lambda\theta_v^{*T} + B\Lambda\tilde{\theta}_v^T)v. \end{aligned} \tag{38}$$

Substituting the ideal value of θ_v^* in (38), (using $B\Lambda\theta_v^{*T} = I$), we obtain that

$$\dot{x} = Ax + B_v(I + B\Lambda\tilde{\theta}_v^T)v. \tag{39}$$

Since the projection algorithm is used in the adaptive laws for the control allocation, we know that $\tilde{\theta}_v$ is bounded, regardless of any stability condition. Defining $F(t) \equiv B\Lambda\tilde{\theta}_v^T$, (39) can be rewritten as

$$\dot{x} = Ax + B_v(I + F(t))v, \tag{40}$$

where $F(t) \in R^{r\times r}$ is a matrix with bounded elements.

Lemma 2. There exists a constant $\bar{F}$, which can be computed explicitly, such that $||F(t)|| \leq \bar{F}, \forall t \in R_{\geq 0}$.

Proof. Using $F(t) = B\Lambda\tilde{\theta}_v^T$, it is obtained that $||F(t)|| \leq ||B||\sqrt{m}\tilde{\theta}_{max} = \bar{F}$, where $||\tilde{\theta}_v|| \leq \tilde{\theta}_{max}$ and $\tilde{\theta}_{max}$ can be calculated using $||\tilde{\theta}_v||_F \leq ||\theta_v||_F + ||\theta_v^*||_F$ as

$$\begin{aligned} ||\tilde{\theta}_v||_F &\leq \sqrt{\sum_{i,j}\theta^2_{max_{i,j}}} + \sqrt{\sum_{i,j}\theta^{*2}_{max_{i,j}}} \\ &\leq \sqrt{\sum_{i,j}\big(\theta^*_{max_{i,j}}\sqrt{(1+\varepsilon_{i,j})}\big)^2} + \sqrt{\sum_{i,j}\theta^{*2}_{max_{i,j}}}. \end{aligned}$$

Assuming that $\varepsilon_{1,1} = \cdots = \varepsilon_{r,m}$, then

$$\begin{aligned} ||\tilde{\theta}_v||_F &\leq \sqrt{\sum_{i,j}\theta^{*2}_{max_{i,j}}}(1+\sqrt{1+\varepsilon_{1,1}}) \\ &= ||\theta^*_{max}||_F(1+\sqrt{1+\varepsilon_{1,1}}), \end{aligned}$$

and the result follows.

Assume that (40) can be decomposed into two subsystems given as

$$\dot{x}_1 = A_1x_1 + A_2x_2 \tag{41}$$

$$\dot{x}_2 = A_3x_1 + A_4x_2 + B'_v(I + F(t))v \tag{42}$$

where $x_1 \in R^{n-r}$, $x_2 \in R^r$. Assuming that $B'_v \in R^{r\times r}$ is invertible, (42) is a square system which is suitable for the application of sliding mode control (Slotine et al. (1991)).

If A_1 is stable, showing that the states x_2 are bounded will be enough for the boundedness of x_1.

Remark 4. The assumptions on B_v and A_1 are commonly justified in several aerospace applications. These assumptions will be checked in a realistic implementation scenario in the simulations section.

Each individual scalar equation in (42) can be written as

$$\dot{x}_{2i} = h_i(x) + \sum_{j=1}^{r} b_{ij}(x)v_j \quad i = 1, ..., r, \quad j = 1, ..., r. \tag{43}$$

Defining $s_i \equiv x_{2i} - x_{2d_i}$, where x_{2d_i} is the desired trajectory for x_{2i}, it can be shown (Slotine et al. (1991)) that the following control input satisfies the sliding conditions

$$v = {B'_v}^{-1}(x_{2d} - h(x) - k\mathrm{sgn}(s)), \tag{44}$$

where $x_{2d} \in R^r, h(x) \in R^r$ and $k\mathrm{sgn}(s) \in R^r$ is a vector consisting of components $k_i\mathrm{sgn}(s_i)$. It is noted that $k_i \in R$ must be chosen such that

$$(1-\bar{F})k_i + \sum_{j\neq i}\bar{F}k_j = \sum_{j=1}^{r}\bar{F}|x_{2d_i} - h_j| + \eta_i,\ i = 1, ..., r \tag{45}$$

where $\eta_i \in R$ is the positive scalar used in the sliding condition given as $\frac{1}{2}\frac{d}{dt}s_i^2 \leq -\eta_i|s_i|$.

Chattering is an undesirable result of the controller discontinuity near the switching surface given in (44). To avoid it, a typical approach is to smooth out the discontinuity by changing the term $\mathrm{sgn}(s)$ to $\mathrm{sat}(s/\Phi)$ as:

$$v = {B'_v}^{-1}(x_{2d} - h(x) + k\mathrm{sat}(s/\Phi)), \tag{46}$$

where Φ is the boundary layer thickness and sat(.) is the saturation function. The sliding mode controller (46) guarantees that the boundary layer is attractive and invariant.

It is shown above that the state vector x_2 is bounded. Using (41) and assuming a stable A_1, it is concluded that x_1 is also bounded. Since x_1 and x_2 are bounded, $h(x) = A_3x_1 + A_4x_2$ is bounded and therefore v can be shown to be bounded using (46). This shows that the requirements for the stability of the control allocation system are satisfied.

Remark 5. The boundedness of $F(t)$ in (42) originates from the employment of the projection algorithm in the adaptive control allocation algorithm, which holds true regardless of the stability of the control allocation. Therefore, the boundedness of the virtual control signal v is independent from the stability of the control allocation.

Remark 6. It is noted that in this section, the effects of actuator saturation on the outer loop are not considered. It will be seen in the simulation section that the designed sliding mode controller is robust to these additional disturbances emerging from the control allocation, for the simulated cases. The effect of these disturbances on stability will be rigorously analyzed in future work.

5. APPLICATION EXAMPLE

5.1 ADMIRE MODEL

The Aerodata Model in Research Environment (ADMIRE) which is an over-actuated aircraft model is used to demonstrate the effectiveness of the adaptive control allocation in the presence of actuator faults. The linearized ADMIRE model is provided in Yildiz et al. (2010) and Harkegard et al. (2005), which is also given below:

$$\begin{aligned} &x = [\alpha\ \beta\ p\ q\ r]^T, y = [p\ q\ r]^T, u = [u_c\ u_{re}\ u_{le}\ u_r]^T, \\ &\dot{x} = Ax + B_u u = Ax + B_v v, \\ &v = Bu, \quad B_u = B_v B, \quad B_v = \left[\, 0_{3\times 2}, I_{3\times 3} \,\right]^T, \end{aligned} \tag{47}$$

where α, β, p, q and r are the angle of attack, sideslip angle, roll rate, pitch rate and yaw rate, respectively. u represents the control surface deflections vector which consists of canard wings, right and left elevons and the rudder. State and control matrices are given as:

$$A = \begin{bmatrix} -0.5432 & 0.0137 & 0 & 0.9778 & 0 \\ 0 & -0.1179 & 0.2215 & 0 & -0.9661 \\ 0 & -10.5123 & -0.9967 & 0 & 0.6176 \\ 2.6221 & -0.0030 & 0 & -0.5057 & 0 \\ 0 & 0.7075 & -0.0939 & 0 & -0.2127 \end{bmatrix}, \tag{48}$$

$$B = \begin{bmatrix} 0 & -4.2423 & 4.2423 & 1.4871 \\ 1.6532 & -1.2735 & -1.2735 & 0.0024 \\ 0 & -0.2805 & 0.2805 & -0.8823 \end{bmatrix}. \tag{49}$$

The position and rate limits of the control surfaces are given as $u_c, u_{re}, u_{le}, u_r \in [-45, 45] \times \frac{\pi}{180}(rad)$ and $\dot{u}_c, \dot{u}_{re}, \dot{u}_{le}, \dot{u}_r \in [-30, 30] \times \frac{\pi}{180}(rad/sec)$. It is noted that the control surfaces influence on derivatives of the first two states i.e. $\dot{\alpha}$ and $\dot{\beta}$ is neglected so that control allocation implementation becomes possible. In addition, to represent actuator loss of effectiveness, a diagonal matrix Λ is introduced. The system given in (47)-(49) can be written as two subsystems:

$$\begin{aligned} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} &= \begin{bmatrix} -0.5432 & 0.0137 \\ 0 & -0.1179 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} 0 & 0.9778 & 0 \\ 0.2215 & 0 & -0.9661 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \\ \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} &= \begin{bmatrix} 0 & -10.5123 \\ 2.6221 & -0.0030 \\ 0 & 0.7075 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} -0.9967 & 0 & 0.6176 \\ 0 & -0.5057 & 0 \\ -0.0939 & 0 & -0.2127 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \\ &\quad + B_v'(I + F(t))v \end{aligned}$$

where B_v' is the identity matrix and $(I + F(t))v = B\Lambda u$. This two subsystem representation makes it possible to implement the sliding mode controller design discussed in the previous section. We use a closed loop reference model (20), where $l = 4$ and A_m selected as $A_m = \mathrm{diag}(-0.2, -0.1, -0.1)$. Fault occurs at time $t = 10(sec)$ when the effectiveness of the actuators becomes 70%. To determine the k vector, the condition given in (45) is employed.

5.2 Simulation results

Fig. 2 shows that the states α and β remain bounded while p, q and r follow their desired references. The variables p, q and r continue to track their desired values after the fault occurs at $t = 10(sec)$. Fig. 3 and Fig. 4 show the virtual control input realizations and the control surface deflections, respectively. It is seen that the virtual control inputs are successfully tracked and the corresponding control surface deflections remain bounded with smooth variations. Also, it is seen in Fig. 4 that the closed loop system remains stable although actuators saturate both in position and rate.

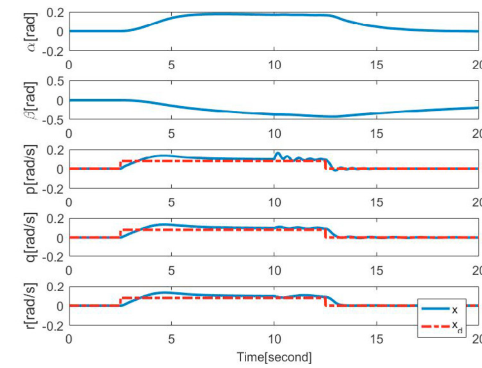

Fig. 2. System states and reference tracking.

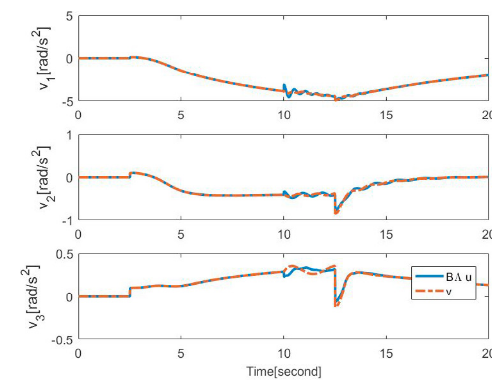

Fig. 3. Virtual control signal v tracking.

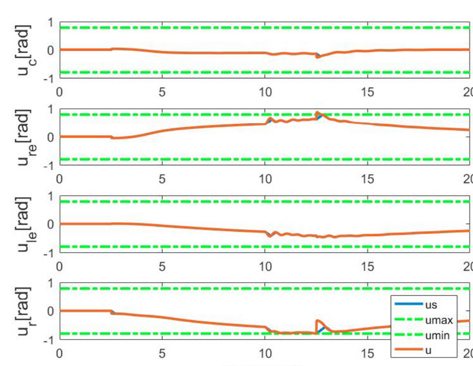

Fig. 4. Control surface deflections.

6. SUMMARY

An adaptive control allocation scheme which is able to tolerate actuator loss of effectiveness in over-actuated systems is proposed in this paper. The adaptive control allocation is designed based on model reference adaptive control and therefore does not need exact parameter identification and persistent excitation. It is shown that the adaptive CA maintains its stability in the presence of actuator saturation. Closed loop reference model ideas were employed to improve the performance of the adaptive control allocation without causing excessive oscillations. The modular structure of the proposed CA allows designing the controller without considering control allocation. The main controller used in the outer loop is a sliding mode controller. The simulations performed using the proposed

CA show that the realization of the virtual control signals by the CA is achieved successfully while all the system signals remain bounded.

REFERENCES

Acosta, D.M., Yildiz, Y., Craun, R.W., Beard, S.D., Leonard, M.W., Hardy, G.H., and Weinstein, M. (2015). Piloted Evaluation of a Control Allocation Technique to Recover from Pilot-Induced Oscillations. *J. Aircraft*, 52, 130–140.

Alwi, H., and Edwards, C. (2008). Fault tolerant control using sliding modes with on-line control allocation. *Automatica*, 44, 1859–1866.

Casavola, A., and Garone, E. (2010). Fault-tolerant adaptive control allocation schemes for over-actuated systems. *Int. J. Robust Nonlinear Control*, 20, 1958–1980.

Cristofaro, A., and Johansen, T.A. (2014). Fault tolerant control allocation using unknown input observers. *Automatica*, 50, 1891–1897.

Cristofaro, A., and Johansen, T.A. (2016). An unknown input observer based control allocation scheme for icing diagnosis and accommodation in overactuated UAVs. *European Control Conference*, 2171–2178.

Davidson, J.B., Lallman, F.J., and Bundick, W.T. (2001). Real-time adaptive control allocation applied to a high performance aircraft. *5th SIAM Conference on Control and Its Applications*.

Doman, D.B., and Ngo, A.D. (2002). Dynamic inversion-based adaptive/reconfigurable control of the X-33 on ascent. *J. Guidance, Control and Dynamics*, 25, 275–284.

Durham, W.C. (1993). Constrained Control Allocation. *J. Guidance, Control and Dynamics*, 16, 717–725.

Gibson, T.E. (2014). *Closed-loop reference model adaptive control: with application to very flexible aircraft.* Ph.D. dissertation, Dept. Elect. Eng., M.I.T., Cambridge, MA.

Gibson, T.E., Annaswamy, A.M., and Lavretsky, E. (2012). Adaptive systems with closed-loop reference models: stability, robustness and transient performance. *arXiv ePrints*, arXiv:1201.4897.

Harkegard, O., and Glad, S.T. (2005). Resolving actuator redundancy - Optimal control vs. control allocation. *Automatica*, 41, 137–144.

Johansen T.A., and Fossen, T.I. (2013). Control allocation-a survey. *Automatica*, 49, 1087–1103.

Johansen, T.A., Fuglseth, T.P., Tndel, P., and Fossen, T.I. (2008). Optimal Constrained Control Allocation in Marine Surface Vessels with Rudders. *Control Engineering Practice*, 16, 457–464.

Karason, S.P., and Anasswamy, A.M. (1994). Adaptive control in the presence of input constraints. *IEEE Transaction on Automatic Control*, 39, 2325–2330.

Lavretsky, E., and Gibson, T.E. (2011). Projection operator in adaptive systems. *arXiv ePrints*, arXiv:1112.4232.

Leonessa, A., Haddad, W.M., Hayakawa, T., and More, Y. (2009). Adaptive control for nonlinear uncertain systems with actuator amplitude and rate saturation constraints. *International Journal of Adaptive Control and Signal Processing*, 23, 73–96.

Liu, C., Jiang, B.,Song, X., and Zhang, S. (2015). Fault-tolerant control allocation for over-actuated discrete-time systems. *J. Franklin Institute*, 352, 2297–2313.

Liu, C., Zhu, X., and Jiang, B. (2012). Fault tolerant control based on adaptive control allocation with a multiple effectors aircraft application. *American Control Conference (ACC)*, 2533–2538.

Liu, Y., and Crespo, Y. (2012). Adaptive control allocation in the presence of actuator failures. *Journal of Control Science and Engineering*, 2012.

Liu, Y., Tang, X., Tao, G., and Joshi, S.M. (2008). Adaptive compensation of aircraft actuation failures using an engine differential model. *IEEE Trans. Control Syst. Technol.*, 16, 971–982.

Petersen, J.A.M., and Bodson, M. (2006). Constrained Quadratic Programming Techniques for Control Allocation. *IEEE Trans. Control Syst. Technol.*, 14, 91–98.

Podder, T.K., and Sarkar, N. (2001). Fault-tolerant control of an autonomous underwater vehicle under thruster redundancy. *Robotics and Autonomous Systems*, 34, 39–52.

Reish, B., and Chowdhary, G. (2013). Concurrent learning adaptive control in presence of uncertain control allocation matrix. *Guidance, Navigation and Control and Co-located Conferences.*

Sadeghzadeh, I., Chamseddine, A., Zhang, Y., and Theilliol, D. (2012). Control allocation and re-Allocation for a modified quadrotor helicopter against actuator faults. *8th IFAC Symposium on Fault Detection*, 8, 247–252.

Slotine, J.J.E., and Li, W. (1991). *Applied Nonlinear Control.* Prentice-Hall.

Tjønnås, J., and Johansen, T.A. (2008). Adaptive control allocation. *Automatica*, 44, 2754–2765.

Tjønnås, J., and Johansen, T.A. (2010). Stabilization of automotive vehicles using active steering and adaptive brake control allocation. *IEEE Trans. Control System Technology*, 18, 545–558.

Tohidi, S.S., Yildiz, Y. and, Kolmanovsky, I. (2016a). Fault Tolerant Control for Over-Actuated Systems: An Adaptive Correction Approach. *American Control Conference (ACC)*, 2530–2535.

Tohidi, S.S., Khaki Sedigh, A., and, Buzorgnia, D. (2016b). Fault tolerant control design using adaptive control allocation based on the pseudo inverse along the null space. *International Journal of Robust and Nonlinear Control*, 26, 3541–3557.

Wang, M.,Yang, J.,Qin, G., and Yan, Y. (2013). Adaptive fault-tolerant control with control allocation for flight systems with severe actuator failures and input saturation. *American Control Conference (ACC)*, 5134–5139.

Yildiz, Y., and Kolmanovsky, I.V. (2010). A control allocation technique to recover from pilot-induced oscillations (CAPIO) due to actuator rate limiting. *American Control Conference (ACC)*, 516–523.

Yildiz, Y., Kolmanovsky, I.V., and Acosta, D. (2011a). A Control Allocation System for Automatic Detection and Compensation of Phase Shift due to Actuator Rate Limiting. *American Control Conference (ACC)*, 444–449.

Yildiz, Y., and Kolmanovsky, I. (2011b). Stability Properties and Cross-Coupling Performance of the Control Allocation Scheme CAPIO. *J. Guidance, Control and Dynamics*, 1190–1196.

Zhang, Y. (2008). Bibliographical review on reconfigurable fault-tolerant control systems. *Annual reviews in control*, 32, 229–252.