ROUTING AND SPECTRUM ALLOCATION IN STATIC FIBER OPTIC NETWORKS

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By
Pelin Öner
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Routing and Spectrum Allocation in Static Fiber Optic Networks
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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

ROUTING AND SPECTRUM ALLOCATION IN STATIC FIBER OPTIC NETWORKS

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July 2016

The continuous growth of demand on internet requires high speed connection and larger capacity on telecommunication networks. In order to satisfy the day-by-day increasing connection requests, a new modulation technique named Orthogonal Frequency Division Multiplexing (OFDM), which promises faster connection and better usage of optical spectrum has been developed in optical networks. In this thesis, we propose solution methods for the Routing and Spectrum Allocation problem that has emerged with the adaptation of OFDM technology. Our goal is to find the minimum amount of spectrum slots that can be used while routing connection requests from their source to destination and allocating an adequate spectrum to the signals. We provide a new integer linear programming formulation for RSA and an improved formulation for the RSA problem with a predefined set of paths. We also propose two heuristic algorithms, Least Cost Slot Allocation and Iterative Common Path Allocation, and provide computational tests to illustrate the performance of our ILP model and heuristic algorithms.

Keywords: Routing and Spectum Allocation, SLICE, Orthogonal Frequency Division Multiplexing, Optical Networks, Static Fiber Optic, Integer Linear Programming.

ÖZET

STATIK FIBER OPTIK ŞEBEKELERDE ROTALAMA VE SPEKTRUM YERLEŞTIRME

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Internet ağları üzerinde artan talepler telekomünikasyon şebekelerin kapasitelerinin artırılması ve hızlı bağlantı sağlanması ihtiyaçlarını doğurmuştur. Gün geçtikçe artan bağlantı taleplerini karşılayabilmek adına daha hızlı bağlantı sağlayabilen ve optik spektrumun daha etkin kullanılmasına olanak sağlayan "Dik Frekans Bölmeli Çoğullama" tekniği fiber optik şebekelerde uygulanılmaya başlanmıştır. Bu çalışmada, statik fiber optik şebekelerde rotalama ve spektrum yerleştirme probleminin tam sayılı lineer programlama modeli ile çözüm yolları incelenmektedir. Doğrusal amaç fonksiyonu olarak rotalama ve spektrum yerleşimi sırasında kullanılabilecek en az sayıdaki spektrum diliminin belirlenmesi seçilmiştir. Bu tez içerisinde rotalama ve spektrum yerleştirme probleminin en iyi çözümüne ulaşabilen yeni bir tam sayılı lineer programlama modeli ve daha önceden belirlenmiş bir rota kümesi aracılığı ile çözüme ulaşabilen geliştirilmiş bir tam sayılı lineer programlama modeli önerilmiştir. Lineer programlama modellerine ek olarak iki adet sezgisel algoritma geliştirilmiştir. Modellerin ve sezgisel algoritmaların performanslarının ölçülebilmesi için sayısal testler yapılmıştır.

Anahtar sözcükler: Rotalama ve Spektrum Yerleştirme, Dik Frekans Bölmeli Çoğullama, Optik Şebekeler, Statik Fiber Optik, Tamsayılı Lineer Programlama.

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Chapter 1

Introduction

1.1 Optical Networks

The continuous growth of demand on internet with addition of multimedia services and various smart appliances as smart TVs, tablet pcs, mobile phones, cloud systems, smart home appliances as well as increasing business activities relying on online solutions require high speed connection and larger capacity on telecommunications networks. The existing available capacity of data communications cannot meet the demand on internet and it has become clear that copper wire connections will not be efficient enough to mitigate the communication requests of end users. Optical communication systems consisting of bundles of optical fibers and advanced modulation formats provide high speed transmission of data over long distances and at higher bandwidths compared to copper cables. As a result of less attenuation in signals and being immune to electromagnetic interference, optical cables have replaced copper wires in telecommunication networks.

An optical cable is composed of numerous thin optical fibers bundled together. A "Multi Mode Fiber (MMF)" has a core diameter above 10 micrometers and allows multiple modes of light to be propagated through the fiber. A MMF has a high power of transmission but a higher level of attenuation compared to single

mode fibers, which results in MMF to be used for short distance communications with many propagation paths. A "Single Mode Fiber (SMF)" has a core diameter between 8 to 10 micrometers which allows only one mode of light to be propagated through the fiber. Due to fewer number of light pulses sent through the fiber, attenuation between modes of light decreases and signal travels faster and further on the fiber. Therefore SMFs are widely used for links longer than 1000 meters.

An optical fiber acts as a waveguide for light and is composed of three layers: the transparent "core" surrounded by the "cladding" with a lower index of refraction than the core and the "buffer coat" that protects the fiber and reduces crosstalk between fibers. Light is kept in core by a principle called *"total internal reflection"*, where a pulse of light sent within the acceptance cone of the core travels through the fiber without leaking out. Due to total internal reflection, the loss in signal in optical fibers is very small compared to that in copper wires.

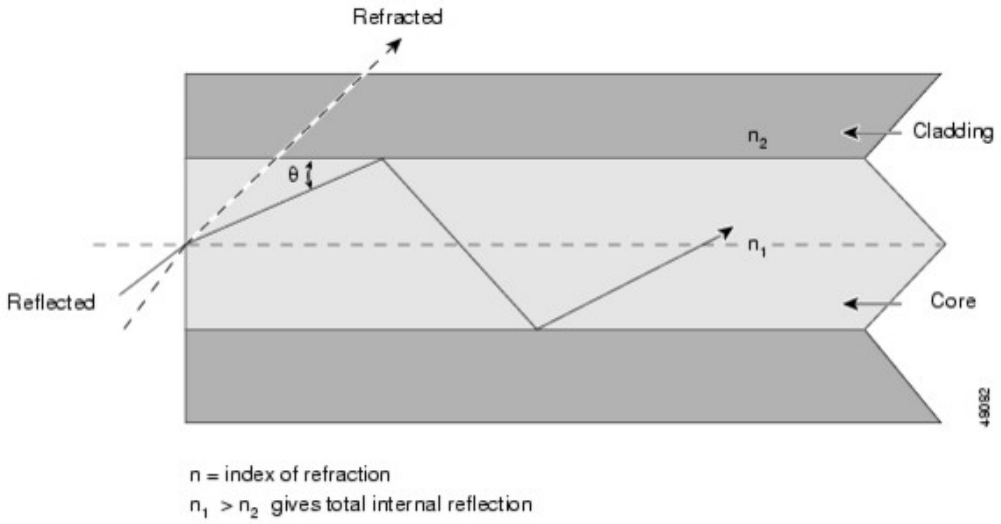


Figure 1.1: Principle of total internal reflection.[1]

1.1.1 Components of an Optical Network

An optical network primarily includes several components consisting of optical cables, multiplexers/de-multiplexers, optical switches and amplifiers as well as transmitters and receivers.

1.1.1.1 Transmitters and Receivers

A transmitter is a light source, generally a laser, which creates an optical signal with a unique wavelength from the original source of data on electronic medium. A receiver acts as a decoder that decrypts the optical signal into its original electronic medium of data such as a voice or a video signal. In "Wavelength Division Multiplexing" technology, the capacity of a single fiber can be increased many times by using several transmitters and receivers, each generating and decoding optical signals at different wavelengths. This enables optical networks to have a much more increased bandwidth compared to conventional copper networks.

1.1.1.2 Multiplexers and Demultiplexers (Mux/Demux)

A multiplexer is an optical bandpass filter that combines multiple colors of light (wavelengths) that are generated by different transmitters into a single fiber. A demultiplexer splits the wavelengths and routes each color to its own receiver at the end of the fiber.

1.1.1.3 Optical Amplifiers

An optical amplifier periodically increases the signal intensity of the light pulse throughout the fiber in order to prevent loss of data. The most common optical amplifier is the Erbium Doped Fiber Amplifier. As the distance travelled by the pulse increases, a reduction in the strength of the signal occurs. The interaction between the Erbium amplifier and a pump laser leads to the emission of light in the C-band spectrum and amplifies the signal.

1.1.1.4 Optical Switches

Optical switches are built of many tiny mirrors and are generally placed at the junction points of the network. They enable directing light between ports without

having to perform an optical-electronic-optical conversion.

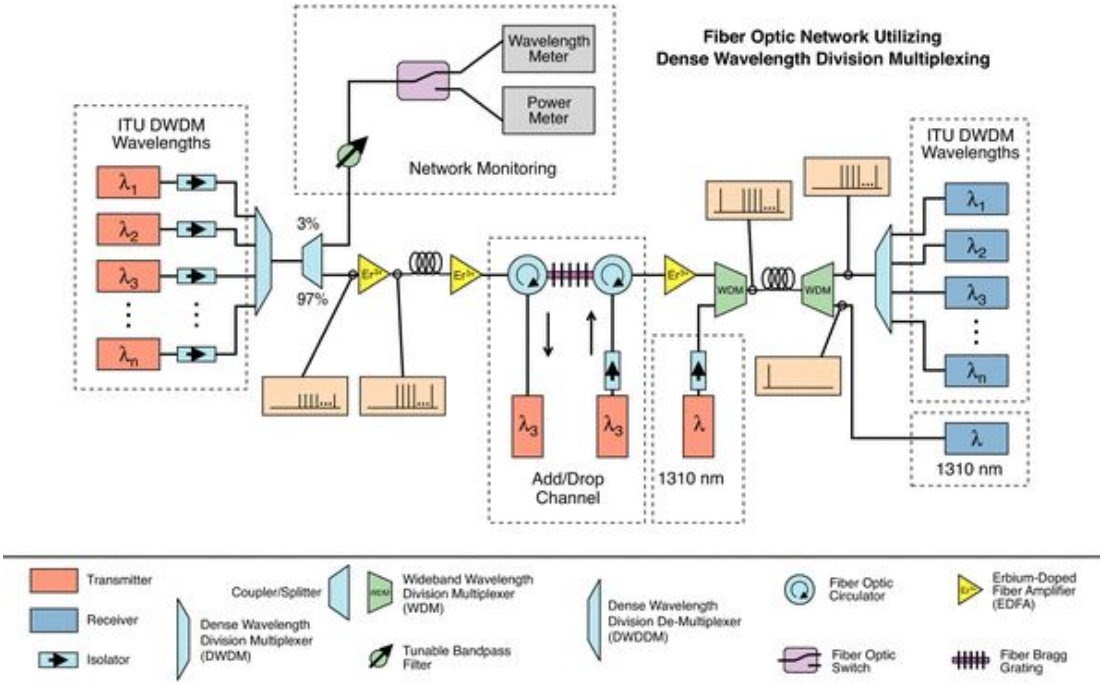


Figure 1.2: Components of a fiber optic network.[2]

With the help of optical network components, voice, video or data signals can be encoded into light pulses and can be transmitted across an optical fiber at many wavelengths simultaneously. Using a technique called *"Wavelength Division Multiplexing (WDM)"*, optical networks can carry data thousands of times more than the copper-based networks.

1.2 Wavelength Division Multiplexing (WDM)

Wavelength Division Multiplexing (WDM) is a multiplexing technique that enables carrying numerous optical signals of separate wavelengths on the same strand of fiber simultaneously. WDM allows bidirectional flow of data and expands the capacity of the optical network without needing any additional fiber strands to carry data. There are two types of WDM systems, namely *"Coarse Wavelength Division Multiplexing (CWDM)"* and *"Dense Wavelength Division*

Multiplexing (DWDM)". The difference between CWDM and DWDM is the channel spacing they use. In Coarse WDM, transmission occurs by using 16 channels with 20 nm spacing in between wavelengths of 1270 nm and 1610 nm. The channel spacing in CWDM spans multiple transmission windows; from O band to L band. In Dense WDM, transmission occurs only on the C band transmission window (optical spectrum range of 1530 nm to 1565 nm), but with a denser channel spacing - typically 100 GHz for 40 channels or 50 GHz for 80 channels. DWDM allows up to 40 or 80 wavelengths with bit rates up to 100 Gb/s to be effectively carried on a single fiber, however *it is likely that bit rates greater than 100 Gb/s will not fit into this scheme.* [3]

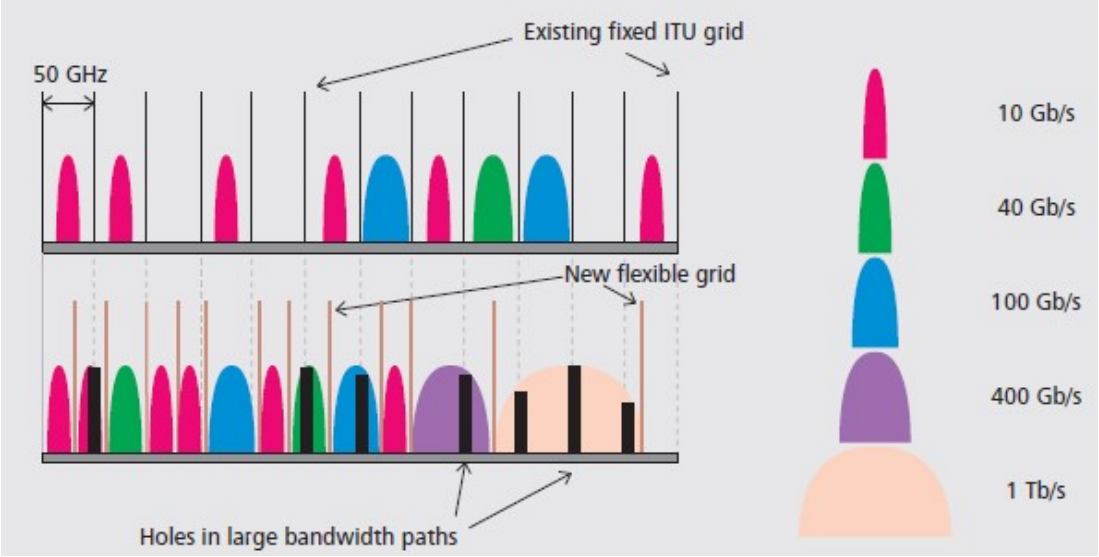


Figure 1.3: Bandwidth allocation in existing fixed ITU grid versus in flexible grid.[3]

Dividing the optical spectrum into fixed wavelength channels leads to some inefficiencies in terms of network utilization efficiency. Signals with larger bit rates than 100 Gb/s overlap with the holes at the grid boundary and therefore cannot be supported by the fixed 50 GHz grid. Another inefficiency occurs when the bit rate of the signal is smaller than the capacity of the channel. The whole channel is reserved to a single signal even when the bit rate is smaller than 100 Gb/s, which results in excess channel width and waste of capacity. In order to enable spectrum savings, a new modulation technique that will enable usage of a flexible grid and also support higher bit rates than 100Gb/s has been developed.

1.3 Orthogonal Frequency Division Multiplexing (OFDM)

Orthogonal Frequency Division Multiplexing (OFDM) is a digital multicarrier modulation technique that enables the usage of sub and super wavelengths for an elastic bandwidth allocation. OFDM uses multiple subcarrier frequencies that can be partially overlapping instead of a single wavelength carrier in WDM. These subcarriers, namely spectrum slots, are densely spaced together but due to the orthogonality property, OFDM signals carried on the subcarriers do not interfere with each other or crosstalk. The capacity of a subcarrier can differ depending on the modulation level used (BPSK, QPSK, 8-QAM), and in [4] two possible candidate capacity schemes are proposed as the *"single slot approach"* with a slot capacity of 12.5 GHz and the *"double sided half slot approach"* with a slot capacity of 6.25 GHz.

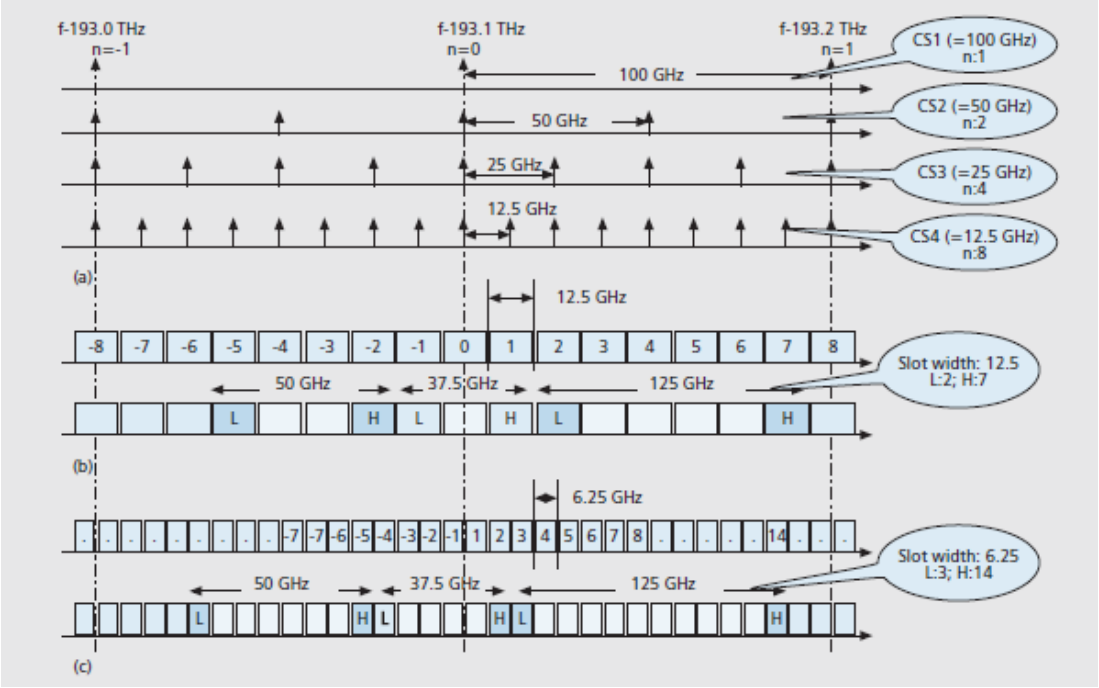


Figure 1.4: Comparison of frequency grid slot divisions. *a)*Current ITU-T DWDM frequency grid. *b)*single slot approach. *c)*double-sided half slot approach.[4]

With the development of OFDM technology, building a spectrum efficient optical network has become possible. Based on the foundations of OFDM, Spectrum-Sliced Elastic Optical Path Network (SLICE) is a novel elastic optical network scheme that can satisfy the flow of demand in between source and sink nodes by using one or multiple consecutive spectrum slots. OFDM resolves the problems encountered at WDM technology such as waste of capacity at low bit rates or overlapping with the boundaries of the fixed grid at higher bit rates. *Unlike the "rigid" bandwidth of the conventional fixed-bandwidth optical path, an optical path in SLICE expands and contracts" according to the traffic volume and user request, if necessary.* [5] When a traffic demand requires less than a whole single wavelength, OFDM can efficiently distribute this sub-wavelength traffic demands data on numerous subcarriers. When a traffic demand has a super-wavelength requirement, for example a bit rate higher than 100 Gb/s; data can be allocated on as much consecutive subcarriers as needed without requiring a spectral gap or overlapping with a guard band frequency. Numerous benefits of SLICE networks are given in [6], [7],and [8] such as spectrum savings due to expansion and contraction quality, energy efficient network operations, and ease of reachability and placement issues encountered in WDM networks. OFDM based SLICE networks provide faster and efficient traffic allocation compared to WDM networks, however there are also important constraints that need to be considered during allocation of traffic on spectrum slices, which is referred to as the Routing and Spectrum Allocation (RSA) Problem in the literature.

1.4 Routing and Spectrum Allocation Problem

The traditional Routing and Wavelength Allocation (RWA) Problem defined in WDM networks tries to allocate an optical route and fixed wavelengths to each traffic demand present in the network. A demand gets allocated to a full wavelength even if its traffic size is less than 100 Gb/s, which results in inefficient utilization of optical domain. The difference of Routing and Spectrum Allocation problem from the Routing and Wavelength Assignment problem is that RSA tries to route and allocate each traffic demand to as much sub-carriers as needed rather

than capturing unnecessary spectrum slots. Furthermore, in RWA guard-band wavelengths are pre-determined and fixed in the spectrum domain, but in RSA any sub-carrier can be utilized as a guard-band. The presence of non-fixed wavelengths and guard bands complicate the light path construction and wavelength allocation problem, and result in a requirement for a different solution approach than traditional solution techniques for RWA.

Although OFDM distributes the traffic on only as many sub-carriers as needed and leaves the remaining spectrum slots in full wavelength for further occupation by different traffic demands, the sub-carriers used for each respective traffic demand need to be consecutive in order to efficiently modulate and transmit data without loss. In other words, a single traffic can be only distributed on a condensed number of sub-carriers which are bundled together and which have no spectral gap in between utilized sub-carriers. This constraint of using consecutive subcarriers is referred to as the *contiguity constraint*. Secondly, the spectrum slices utilized for a single traffic demand has to be constant throughout the optical path of this traffic. In other words, each single traffic cannot change the wavelength (or subcarriers) it is assigned to and it has to utilize the same indexed sub-carriers throughout all of the edges/connections it passes through. This constraint of using the same index starting slot and utilized sub-carriers is referred to as the *continuity constraint*. Thirdly, there has to be a number of guard band spectrum slots, namely *guard-carrier constraint*, that separate traffic demands which utilize common arcs in their respective optical paths, in order to provide easier optical signal filtering.

There are two versions of RSA problem: static and dynamic. In static, or offline RSA problem, traffic demands' information are known prior to the planning phase of routing and spectrum allocation, and is assumed to be steady until the solution. Each new arriving traffic in the network requires a different routing and solution in offline RSA problem. In dynamic, or online RSA problem, the arrival rate and source-sink allocation of traffic demands are not known beforehand. The scope of this thesis is limited to offline RSA problem and it intends to provide an exact solution approach to find the routing of each traffic demand and the total minimum number of utilized spectrum slices in the static RSA problem, taking

the above mentioned constraints into consideration.

To the best of our knowledge, most of the work done on Routing and Spectrum Allocation Problem in literature takes a predefined set of paths into consideration during the routing phase. However, taking a selected subset of potential routes and limiting the search space within a preset of paths prevents the global optimum to be found. We have encountered only in [9] an ILP formulation that searchs the entire solution space. In this thesis, we aim to provide a different and more efficient ILP formulation which spans all of the routes in a given network and that can solve instances for larger graphs in lower run time values. The rest of this thesis is organized as follows: In Chapter 2, we provide a literature review on existing formulations for Routing and Spectrum Allocation problem. In Chapter 3, we analyze the formulation of Christodoulopoulos et.al's [10] formulation for RSA ILP with predefined set of paths, which is a benchmark for Routing and Spectrum Allocation problem. We propose two lemmas to strengthen their formulation and provide a stronger ILP for RSA with predefined set of paths. Then we define a global ILP that spans all of the solution space, namely "RSA without Predefined Set of Paths" (also referred as $ILP_{currentwork}$). In Chapter 4, we propose two construction heuristic algorithms for the RSA problem and finally in Chapter 5, we provide experimental results for our ILP formulation and heuristic algorithms.

Chapter 2

Literature Review

Christodoulopoulos et al, [10] address Routing and Spectrum Allocation (RSA) problem to determine optimal allocation scheme in OFDM based Spectrum Sliced Elastic (SLICE) Optical Path Networks. They are the first to introduce RSA problem in replacement to the traditional Routing and Wavelength Assignment (RWA) problem. The proposed RSA model tries to minimize the spectrum used to serve the traffic matrix, with the assumptions that the traffic matrix (source-destination pairs and demand sizes) is known and all demands are served, i.e. no demand gets rejected.

There are two formulations in [10], namely (i) RSA ILP and (ii) R+SA. In the ILP formulation, authors define an integer linear model that tries to minimize the maximum number of spectrum slices used in any arc while allocating an adequate spectrum scheme through selection of paths from a pre-computed set of paths for each traffic demand. In sub-problem version of ILP, R+SA, they first form a set P* from the pre-computed set of paths for each demand (R) and pass this information to the second phase of spectrum allocation (SA). For heuristics, the authors propose a Single Demand RSA Heuristic Algorithm and a Simulated Annealing Meta-Heuristic.

The authors evaluate the performance of their proposed algorithms by using

computational tests in MATLAB and LINDO API. They consider two cases for experiments: (i) A small network topology with 6 nodes and (ii) A generic network topology with 14 nodes and 46 directed links (Deutsche Telekom Network). They use two different values of traffic demand: D=4 as low load case and D=30 as high load case. The distribution of the demand width value, Tsd for connection (s,d), is chosen as uniformly distributed between 0 and D. Results show that for small network topology, RSA model finds a solution in an average of 3.25 seconds for low load case and 503.19 seconds for high load case. R+SA formulation finds a solution faster; 2.72 seconds for low load, 6.48 seconds for high load. For realistic network topology, RSA was not able to produce results and R+SA finds the best solutions within 2 hours. The shortcoming of both of these formulations is that a global minimum is not guaranteed as the algorithm does not cover all paths, but uses a set of pre-computed paths.

Further on, Christodoulopoulos, Tomkos and Varvarigos [11] address an extension of the RSA problem by adding modulation level constraint. They introduce Routing, Modulation Level, and Spectrum Allocation (RMLSA) problem which tries to minimize the spectrum used to serve the traffic matrix via choosing an appropriate modulation level with respect to the transmission distance. They break the problem into two sequential sub-problems: namely (i) Routing and Modulation Level (RML) and (ii) Spectrum Allocation (SA). Further on, they propose a sequential heuristic algorithm that serves all the connections in the traffic matrix one by one by using simulated annealing meta-heuristic to obtain orderings.

The authors evaluate the performance of their proposed algorithms by using computational tests in MATLAB and LINDO API. For all algorithms they use 3 pre-computed paths for each demand and two load cases (D=4 for low load traffic and D=30 for high load traffic). They use two network topologies; one consisting of a 6 node small network and another a 14 node generic Deutsche Telekom topology. Computational results show that for the small network topology, RMLSA produces a solution in average 3.25 seconds for low load and 503.19 seconds for high load traffic cases. RML+SA produces a solution in average 2.72 seconds for low load and 6.48 seconds for high load traffic cases. For DT network,

RMLSA could not produce results and RML+SA could produce solutions only after 2 hours.

Wang et al, [12] prove NP-hardness of the static RSA problem and present a different RSA ILP formulation to optimally allocate the sub-carriers (slices) and guard-carriers in SLICE networks in their work. They analyze the upper/lower bounds of the number of slices needed in different topologies and present two heuristic algorithms; (i) Balanced Load Spectrum Allocation (BLSA) and (ii) Shortest Path with maximum Spectrum Reuse (SPSR). Like Christodoulopoulos et al, they try to minimize the maximum number of subcarriers required in any fiber of a SLICE network. They test the upper/lower bounds of their formulation in two cases of Ring Network topology with and without predetermined routing.

The authors evaluate the performance of their algorithms by using computational tests in ILOG CPLEX. They have found out that the lower bound for the ILP model on Ring Networks can be achieved by the cut-set (CS) method. They test their ILP model on two Ring networks with 4 and 5 nodes and uniform traffic demand. For the heuristics proposed, they use two different network topologies to compare the performance of the algorithms with the ILP model; 6 nodes for a small network and 14 nodes for a large network. The computational results of their simulations show that for uniform network demands, heuristic algorithms produce close solutions to the optimal ILP model. However, there is no information on the computation times to find the optimal solution for the ILP model and/or heuristics.

Velasco et al, [13] approach to the problem of routing and spectrum allocation by eliminating the complexity that the contiguity constraint gives by defining a concept of channels for the representation of contiguous spectral resources. They introduce Channel Assignment (CA) as an equivalent of Wavelength Assignment (WA) approach in WDM, which tries to minimize the number of rejected demands based on the assignment of demands on channels using a pre-computed set of channels as input. The authors propose two ILP formulations for the CA problem: (i) a link-path formulation and (ii) a node-link formulation. They also formulate a relaxed version of the RSA problem which does not consider spectrum continuity

constraint, in order to obtain lower bounds.

The authors evaluate the performance of their algorithms by using 5 different network topologies; Ring9 (9 nodes, 9 edges), Brasil network (10 nodes, 12 edges), Abilene network (12 nodes, 15 edges), Spanish Telefonica (21 nodes, 35 links) and Deutsche Telekom network (14 nodes, 23 edges). They create 36 demands with a uniform distribution between 1 and 4 slices. CA model finds a solution in over 6 hours both for the link-path approach and the node-link approach.

Klinkowski and Walkowiak [14], formulate the RSA problem with an objective of minimizing the number of frequency slots that are assigned to at least one demand in the network. Similar to [10], they use a set of predefined paths to solve the problem. They also propose a novel heuristic Adaptive Frequency Assignment-Collision Avoidance, namely AFA-CA, which estimates the number of frequency slots that might be allocated to a link taking all candidate paths into account and then adaptively selects demands to be assigned on the spectrum such that the less congested path and lowest indexed frequency slots are picked for each demand. They test their ILP formulation on a small 6 node 16 links network with different combinations of frequency slot requirement distribution-demand number pairs and AFA-CA algorithm on a small 6 node 16 links, NSFNET(15 node 46 links) and UBN24 (24 node 86 links) networks. Results show that the ILP formulation has a run time value ranging from 13 seconds for a demand number of 30 demands with frequency slot requirements of Uniform(0,5) and to 12747 seconds for a demand number of 15 with frequency slot requirements of Uniform(0,30). The experiment shows that as the number of frequency slot requirements increases, ILP formulation of [14] produces longer run time values. For their proposed heuristic AFA-CA, the results show that the heuristic has a run time value of about 1 seconds and has an optimality gap ranging from 2.49% to 7.02%. The authors state that the main difficulty for their ILP formulation comes from the huge number of constraints which occurs when large number of frequency slots are required by a demand in some network scenarios. No further analysis on run time values in larger networks are present in [14].

Paul [9] proposes an ILP model to optimally solve the RSA problem with the

objective of finding the minimum number of spectrum slots used while allocating demand in OFDM based optical networks. The first formulation, namely ILP1, spans all network and tries to allocate all demands by finding the optimal routing scheme and spectrum allocation. The second formulation proposed by the author, namely ILP2, takes a pre-determined routing scheme for the demands and allocates spectrum slots according to these schemes. The author then compares his formulations with Christodoulopoulos et al's ILP formulation (CHR) and conducts performance analyses. Performance analyses are conducted using ILOG CPLEX and consist of 8, 12, 15, 18 and 20 commodities and 3 pre-computed paths for each commodity. Topologies of 8, 12, 15 node networks and 14 node Deutsche Telekom network are used. The results of [9]'s experiments show that ILP1 formulation can process up to 15 demands in 15 node networks, with an average run time value of 824,31 seconds. In real-life network topology, the author uses Deutsche Telekom(14 nodes, 23 edges) network and the results for ILP1 show that the formulation can handle up to 20 demands with an average run time value of 896,02 seconds.

In this thesis, we propose a different and more efficient ILP formulation that spans all possible routes and finds a global optimum to the RSA problem. The performance analysis of our proposed ILP formulation, $ILP_{currentwork}$, show that our formulation can solve instances for up to 50 demands in 8,14 and 50 node networks, up to 100 demands in 25 node networks and up to 30 demands in 100 node networks for complete graphs. For real-life network topology, we use Deutsche Telekom(14 nodes, 23 edges) network similar to [10] and [9], and our formulation can handle up to 25 demands with an average run time value of 646,58 seconds. The complete test results for varying sparsity, total demand numbers and number of nodes in the graph can be seen in Chapter 5. When compared to Christodoulopoulos et.al [10], Klinkowski and Walkowiak [14] and Paul [9] experiments, the results of our experiments show that our $ILP_{currentwork}$ formulation can solve instances for larger graphs, for larger frequency slot distributions (we have taken frequency slot distributions from Uniform(0,50)) and has lower run time values.

Chapter 3

Routing and Spectrum Allocation Modelling

In this chapter, we analyze the ILP formulation for Routing and Spectrum Allocation in [10] that acts as a benchmark for RSA problem and define two lemmas to strengthen the model. Then from our proposed formulation, we derive another ILP model that considers every path during routing and allocation phase and provides a global solution to the RSA problem. To the best of our knowledge, such a global formulation is only proposed by [9] in the literature, and we aim to provide a more efficient formulation that can solve instances for larger networks.

In the static RSA problem, a connection request, i.e. a demand d, which originates from a source node $u(d)$ has to be transferred to its destination, the sink node $v(d)$, by traversing an optical route that connects $u(d)$ to $v(d)$. In order to be sent as a light pulse from its source node to sink node, each demand has to be assigned to a number of frequency slots (or spectrum slots) in the optical spectrum such that there is no crosstalk between demands that share an edge in their optical routes. Each demand has a bandwidth which denotes the length of the signal and is translated into the size of the demand as a number of spectrum slots by dividing the bandwidth of the demand to the capacity of each subcarrier, assuming that the capacity of each subcarrier is constant and fixed. In order to

formulate the RSA problem as an ILP, we assume that we have a graph consisting of the hubs in the network and we assume that the graph is connected. We assume that the nodes in the graph represent the hubs in the network and the edges that connect nodes represent physical fiber optic cables. As fiber optic cables can carry bidirectional information, a single link of cable represents an edge in the graph. We assume that the total connection requests form the demand set D, and the information of all demands in the set D are known prior to the planning phase of the RSA problem. Within the context of this assumptions, the objective of the static RSA problem is to find the minimum number of spectrum slots required to route and assign spectrum slots to each demand in the demand set D. We assume that no demand gets rejected, i.e. all of the demands are allocated to a route and adequate spectrum slots.

In order to find the number of spectrum slots in the visible light spectrum, we use the formula $\lambda = \frac{c}{F}$ where λ is the wavelength, c is the speed of the light and F is the frequency. Taking the visible light's spectrum as 400nm to 700nm, taking $\lambda 1 = 400nm$ and $\lambda 2 = 700nm$ and solving for corresponding F's result that the frequency of the visible light is between 430 THz and 750 THz, which is an approximately 320000 GHz interval. Taking each spectrum slot with a 12.5 GHz capacity results that the visible spectrum has $\frac{320000}{12.5} = 25600$ slots. As to completely occupy 25600 slots in the spectrum require much more amount of traffic requests with huge demand sizes than the RSA problem can be solved for, we assume that the edges are uncapacitated.

The important constraints to be considered in Routing and Spectrum Allocation problem are *continuity*, *contiguity*, *guard-carrier* and *non-overlapping spectrum* constraints. *Continuity* constraint requires that each demand has a steady spectrum slot allocation throughout its path, i.e. has the same starting slot allocation at each of the edges it uses in its path. *Contiguity* constraint requires that each demand is allocated on spectrum slots that are bundled together and have no spectral gaps in between. *Non-overlapping spectrum* constraint requires that for demands sharing an edge, a spectrum slot can only be allocated to a single demand in order to prevent crosstalk (the direction of the demands are not important as an edge can carry bidirectional information and crosstalk can occur

without depending on the direction). *Guard-carrier* constraint requires that for demands sharing an edge, the spectrum allocation of these demands have to be separated by a number of slots, referred to as the guard carrier.

3.1 Christodoulopoulos et.al's RSA ILP formulation

The joint formulation of Christodoulopoulos, Tomkos and Varvarigos [10] considers demand allocation in OFDM based Optical Spectrum SLICE Network with pre-computed paths. The Integer Linear Programming (ILP) formulation considers an offline network with a given traffic matrix consisting of source, demand and requested transmission rates; and tries to minimize the utilized spectrum while serving the connections through adequate spectrum allocation, with the constraint that no spectrum overlapping is allowed and each traffic demand is assigned consecutive spectrum slots *(contiguity)*.

The assumptions of the formulation are as follows:

1. The spectral granularity of the transmitters and WXCs is one subcarrier corresponding to F GHz of spectrum.

2. The capacity of a subcarrier is equal to C Gbps and is taken as constant without taking the modulation level into consideration.

3. A guardband of G subcarriers seperates adjacent spectrum paths.

4. Serving a connection i that requires T_i subcarriers is translated to finding a starting subcarrier frequency f_i after which it can use T_i contiguous subcarriers.

5. Assuming a constant subcarrier capacity C, a bandwidth demand of B_i can be mapped to a demand of T_i subcarriers as $T_i = \lceil \frac{B_i}{C} \rceil$.

3.1.1 Parameters and Variables

The network topology of the RSA problem is denoted by a connected graph $\mathcal{G}$ =(V,E), where V is the set of nodes and E is the set of edges (i.e. fiber links) connecting the nodes. Given the graph $\mathcal{G}$ and the demand set D, the aim of this formulation is to choose a route from the source node to the destination node for each demand, from the pre-determined route set P_d, and an allocation in the optical spectrum (i.e. indices of frequency slots) that is constant in all edges the demand traverses in its chosen path. A demand $d = (u(d), v(d))$ is a connection request which originates from node $u(d)$ and terminates at node $v(d)$. In the static RSA problem, the demands' information –source node, sink node and demand size– is known prior to the planning phase. $T(d)$ denotes the number of subcarriers required (i.e. the demand size) for the communication d between source $u(d)$ and destination $v(d)$; such that $T(d) \geq 0, \forall d \in D$. For each commodity $d \in D$, k paths are pre-calculated, i.e. $|P(d)| = k$.

Sets and Parameters:

P_d: The set of k candidate paths for connection d.

$\mathcal{P}$=$\cup_d P_d$: The total set of candidate paths for the demand set D.

G: The amount of subcarriers required as guardband in between demands that use the same edge in their paths, and is taken as constant.

$T(d)$: The number of subcarriers required for the communication between source u and destination v.

Variables:

X_p: 0 if path p is not utilized , and 1 if p is utilized.

$S(d)$: Integer variable that denotes the starting frequency for connection $d = (u(d), v(d))$. Assuming $T_{total} = \sum_d T_{(d)}$, 0≤ S(d)≤ T_{total}.

W_{d_k,d_t}: Boolean variable that equals 1 if the starting frequency of connection d_k is smaller than the starting frequency of connection d_t (i.e. $S_{d_k} < S_{d_t}$) and 0 otherwise.

c: Maximum utilized spectrum slot number.

3.1.2 Routing and Spectrum Allocation Modelling

$$minimize \qquad c \tag{3.1}$$

subject to :

$$\sum_{p \in P_d} X_p = 1 \qquad \forall d \in D \tag{3.2}$$

$$c \geq S(d) + T(d) \qquad \forall d \in D \tag{3.3}$$

If $\exists p \in P_{d_k}$ and $\exists q \in P_{d_t}$ such that $p \cap q \neq \emptyset$, then for each d_t , $d_k \in D$ and every $p, q \in \mathcal{P}$ the following equations (3.4-3.8) are employed:

$$W_{d_k,d_t} + W_{d_t,d_k} = 1 \tag{3.4}$$

$$S(d_k) - S(d_t) \leq T_{total} * W_{d_t,d_k} \tag{3.5}$$

$$S(d_t) - S(d_k) \leq T_{total} * W_{d_k,d_t} \tag{3.6}$$

$$S(d_t) + T(d_t) + G - S(d_k) \leq (T_{total} + G) * [1 - W_{d_t,d_k} + 2 - X_p - X_q] \tag{3.7}$$

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G) * [1 - W_{d_k,d_t} + 2 - X_p - X_q] \tag{3.8}$$

$$X_p \in \{0, 1\} \qquad p \in \mathcal{P} \tag{3.9}$$

$$S(d) \geq 0 \qquad d \in D \tag{3.10}$$

$$W_{d_t,d_k} \in \{0, 1\} \qquad d_t, d_k \in D \tag{3.11}$$

3.1.3 Analysis of Christodoulopoulos et.al's Formulation

Number of variables and constraints

There are $|\mathcal{P}|$ binary X variables, $|D|$ continuous S variables and $\frac{|D|*|D-1|}{2}$ binary W variables. In total there are $|\mathcal{P}| + |\mathcal{D}|+ \frac{|D|*|D-1|}{2}$ variables in the formulation.

For each value of d, there is one constraint for (3.2) and in total, the number of constraints for (3.2) are $|D|$. In the same manner, there are $|D|$ constraints for (3.3). For constraints (3.4), (3.5) and (3.6) there are $\frac{|D|*|D-1|}{2}$ constraints for each of the respective equations. For constraints (3.7) and (3.8) there are $|P_{d_k}| * |P_{d_t}|$ constraints for each demand pair d_k,d_t. As each demand has k precalculated paths, i.e. as $|P_d| = k$; $\forall d \in D$, there are $k^2 * \frac{|D|*|D-1|}{2}$ constraints in total for each equation. Therefore, the total number of constraints in the formulation are $2 * |D| + \frac{3}{2} * |D| * |D-1| + k^2 * |D| * |D-1|$.

The overall complexity for variables are $O(|D|^2 + |\mathcal{P}|)$ and $O(|D|^2 * k^2)$ for constraints.

Analysis of the constraints

Given a set of pre-computed paths for each connection request d, equation (3.2) ensures that only one of these paths would be chosen for each of the connection requests. Equation (3.3) computes the maximum index of spectrum slot that each connection terminates at. Together with the equation (3.3), objective function minimizes the maximum number of spectrum slots used.

Maintaining a single starting slot index throughout all links that each demand uses, $S(d)$: $\forall d \in D$, satisfies the *continuity* constraint of the RSA problem. In order to satisfy the *contiguity* and *non-overlapping spectrum* constraints, the demands that share at least one common link in their respective paths have to be allocated on the spectrum such that each spectrum slice is allocated to at most one demand. In order to ensure this, equations (3.4)-(3.8) are employed for all demands that share at least one common arc in their respective paths: $d_t, d_k \in D$ such that $\exists p \in P_{d_k}$ and $\exists q \in P_{d_t}$ where $p \cap q \neq \emptyset$. Equations (3.4)-(3.6) ensure that for every demand pair that shares a common fiber arc in their paths, one of these demands should be placed before the other in the optical spectrum. Note that if either one (or both) of X_p and X_q is 0, this means that the overlapping of these demands on these paths is not important, therefore sorting of the demands is not necessary. Therefore, we have to look at the case where both X_p and X_q are 1. Assuming that d_t and d_k share at least one arc in their paths, lets place demand d_t in front of demand d_k, which means that $W_{d_t,d_k} = 1$, to analyze these constraints.

Taking $W_{d_t,d_k} = 1$ ensures that $W_{d_k,d_t} = 0$ in our case by equation (3.4). As W_{d_t,d_k} is 1 and W_{d_k,d_t} is 0, constraint (3.6) ensures that the starting frequency slot of demand d_t is smaller than demand d_k, which means that d_t is placed before d_k. Constraint (3.5) is deactivated when W_{d_t,d_k} is 1; $S(d_k) - S(d_t) \leq T_{total}$ is trivially satisfied as we have $S(d) \leq T_{total}$ $\forall d \in D$.

Contiguity constraint is satisfied by equations (3.7) and (3.8). Constraint (3.7) becomes:

$$S(d_t) + T(d_t) + G - S(d_k) \leq (T_{total} + G) * [1 - 1 + 2 - 1 - 1]$$

since both X_p and X_q are 1 as they utilize paths p and q; and W_{d_t,d_k} is 1. Therefore the right hand side of the equation is 0, yielding in

$$S(d_t) + T(d_t) + G \leq S(d_k)$$

This constraint ensures that demand d_k is placed at least a number of the demand size of d_t plus a guardband slices after demand d_t.

Similarly, Constraint (3.8) becomes:

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G)$$

which is reduced to

$$S(d_k) + T(d_k) \leq S(d_t) + T_{total}$$

Likewise constraint (3.5), this constraint also ensures that the demand d_k stays within the boundaries of the total sum of the demand sizes.

Lemma 1: Equations (3.5) and (3.6) are implied by equations (3.7) and (3.8) and can be eliminated from the model.

Proof: *Recall - Equation (3.7)*

$$S(d_t) + T(d_t) + G - S(d_k) \leq (T_{total} + G) * [1 - W_{d_t,d_k} + 2 - X_p - X_q]$$

For $W_{d_t,d_k} = 1$ and $X_p = 1$ and $X_q = 1$, equation (3.7) becomes

$$S(d_t) + T(d_t) + G - S(d_k) \leq (T_{total} + G) * [1 - 1 + 2 - 1 - 1]$$

$$S(d_t) + T(d_t) + G - S(d_k) \leq (T_{total} + G) * [0]$$

$$S(d_t) + T(d_t) + G - S(d_k) \leq 0$$

$$S(d_t) + T(d_t) + G \leq S(d_k)$$

$$S(d_k) \geq S(d_t) + T(d_t) + G$$

As $T_{d_t} \geq 0$ and $G \geq 0$, the above equation implies that;

$$S(d_k) \geq S(d_t) \tag{3.12}$$

Recall - Equation (3.6):

$$S(d_t) - S(d_k) \leq T_{total} * W_{d_k,d_t}$$

As we already know that $W_{d_k,d_t} = 0$ holds when $W_{d_t,d_k} = 1$ by equation (3.4), equation (3.6) becomes;

$$S(d_t) - S(d_k) \leq T_{total} * 0$$

$$S(d_t) - S(d_k) \leq 0$$

$$S(d_t) \leq S(d_k)$$

$$S(d_k) \geq S(d_t)$$

As the above inequality is already suggested by equation (3.7) via (3.12), equation (3.6) can be eliminated from the model.

Likewise, for equations (3.8) and (3.5), we can apply the same steps.

Recall - Equation (3.8):

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G) * [1 - W_{d_k,d_t} + 2 - X_p - X_q]$$

For $W_{d_k,d_t} = 0$ and $X_p = 1$ and $X_q = 1$, equation (3.8) becomes

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G) * [1 - 0 + 2 - 1 - 1]$$

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G) * [1]$$

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G)$$

$$S(d_k) \leq S(d_t) + T_{total} - T(d_k)$$

As $T(d) \geq 0 \ \forall d \in D$; $T_{total} = \sum_d T_{(d)} \geq 0$ and $T_{total} \geq T(d_k) \geq 0$, $\forall (d_k) \in D$. Therefore the following always holds: $T_{total} - T(d_k) \geq 0$ and $T_{total} - T(d_k) \leq T_{total}$.

Hence $S(d_t) + T_{total} - T(d_k) \leq S(d_t) + T_{total}$ and this implies that

$$S(d_k) \leq S(d_t) + T_{total} - T(d_k) \leq S(d_t) + T_{total}$$

$$S(d_k) \leq S(d_t) + T_{total} \tag{3.13}$$

Recall - Equation (3.5):

$$S(d_k) - S(d_t) \leq T_{total} * W(d_t)(d_k)$$

As we already know that $W(d_t)(d_k) = 1$, equation (3.5) becomes;

$$S(d_k) - S(d_t) \leq T_{total} * (1)$$

$$S(d_k) - S(d_t) \leq T_{total}$$

$$S(d_k) \leq S(d_t) + T_{total}$$

As the above inequality is already suggested by equation (3.8) via (3.13), equation (3.5) can be eliminated from the model.

The same steps apply if we take $W_{d_k,d_t} = 1$ and $W_{d_t,d_k} = 0$, but in that case equation (3.7) will imply equation (3.5); and (3.6) will be deactivated and also be implied by (3.8).

Taking a multiplier of $(T_{total} + G)$ in equations (3.4) and (3.5) creates an upperbound for the number of spectrum slots to be used when there are at most 2 demands that share a common link in their paths; however this bound is not enough for congested cases when there are 3 or more demands that share a link in their paths as it diminishes the total amount of guardband required in between demands.

Lemma 2: The multiplier on the right hand side of equations (3.7) and (3.8) must be an upperbound for the slots of the last demand assigned on a link; and should be replaced by $\left[T_{total} + \sum_{d=1}^{|D|} G\right]$, where $T_{total} = \sum_{d \in D} T(d)$.

Proof:

Assume that there are n demands $D = \{d_{t_i} : i = 1...n\}$ that all share a common link in their paths ($\exists p_i \in P_{d_{t_i}}$ $\forall i \in \{1...n\}$ such that $p_1 \cap p_2 \cap ... \cap p_n \neq \emptyset$). Taking the most congested case into consideration (i.e. when there is exactly one arc, say (j, k), present in the graph and all d_{t_i}, $i \in \{1...n\}$ have exactly one path P_i that uses (j, k)), we have to make sure that all demands can be allocated to as many spectrum slots as needed. As all demands use arc (j, k), we have

$X_{P_i} = 1\ \forall i \in \{1...n\}$. Take any two demands d_t, d_k from the demand set D and let $W_{d_t,d_k} = 1$, meaning demand d_t is placed before d_k.

From equation (3.4), we have $W_{d_k,d_t} = 0$. Equation (3.7) becomes

$$S(d_t) + T(d_t) + G - S(d_k) \leq 0$$

$$S(d_k) \geq S(d_t) + T(d_t) + G \tag{3.14}$$

and Equation (3.8) becomes

$$S(d_k) + T(d_k) + G - S(d_t) \leq (T_{total} + G)$$

$$S(d_k) + T(d_k) + G \leq S(d_t) + (T_{total} + G) \tag{3.15}$$

For any demand $d_k \in D$, we have $(n-1)$ constraint pairs of (3.14) and (3.15). Equation (3.14) assigns the starting slot index of d_k, $S(d_k)$, as greater than the size of demand d_t+ guardband; and demand d_k is placed at the end of all present demands on the common arc by taking the largest $S(d_k)$ value among all Equations (3.14) for demand d_k. Note that the same applies to all demands that are placed before d_k.

From equation (3.15), demand d_k has to be assigned such that the endpoint of d_k is within the boundary of $S(d_t) + T_{total}$. The lower bound of all right hand side values of (3.15) occurs at $S(d_0) + T_{total}$, where $S(d_0)$ denotes the starting slot of the first assigned demand (which is the minimum among all $S(d_i)$'s). Note that the right hand side value of this constraint applies to all demands in the same way, since $S(d_0) + T_{total} \leq S(d_i) + T_{total}$ as we know $S(d_0) \leq S(d_i)\ \forall i \in D$. Hence we can say that $S(d_i) + T(d_i) + G \leq S(d_0) + (T_{total} + G)$ is the constraint that defines the upperbound for all $S(d_i)$ values.

Let the multiplier of equations (3.7) and (3.8) be $\mathcal{X}$ and let $\mathcal{X}$=$T_{total} + G$. Regardless of the sorting of the demands, this occurs as an upperbound for all $S(d_i)$ and becomes problematic for the last demand assigned. In order to make sure that the last demand is still in the spectrum, we have to analyze whether $\mathcal{X}$=$T_{total} + G$ is an adequate upperbound. Let d_k be the last demand to be assigned on the spectrum. We have $(n-1)$ constraint couples of

$$S(d_k) \geq S(d_t) + T(d_t) + G$$

$$S(d_k) + T(d_k) + G \leq S(d_t) + \mathcal{X}$$

for $t \in D/\{k\}$. The largest of the first constraint's right hand side occurs at the last demand before k, say t, and the lowest of the right hand side of the second equation occurs at $S(d_0) + \mathcal{X} - T(d_k) - G$ (when $S(d_k)$ is left alone on the left hand side). In order for $S(d_k)$ to have a value, we have to satisfy

$$S(d_t) + T(d_t) + G = S(d_0) + \mathcal{X} - T(d_k) - G \tag{3.16}$$

Solving for $\mathcal{X}$ results

$$\mathcal{X} = S(d_t) + T(d_t) + T(d_k) - S(d_0) + 2G \tag{3.17}$$

We know that d_t is the demand right before d_k. For d_t we have constraints

$$S(d_t) \geq S(d'_t) + T(d'_t) + G$$

and

$$S(d_t) + T(d_t) + G \leq S(d_0) + \mathcal{X}$$

where d'_t is the last demand before d_t. In order for any demand t to be assigned, equation (3.16) must be satisfied. Hence whenever d_t has a solution, we have $S(d_t) = S(d'_t) + T(d'_t) + G$. When we substitute $S(d_t)$ into equation (3.17), we get

$$\mathcal{X} = S(d'_t) + T(d'_t) + G + T(d_t) + T(d_k) - S(d_0) + 2G$$

In a similar fashion when we solve for all $S(d'_t)$ iteratively, we get the equation

$$\mathcal{X} = [S(d_0)+T(d_0)+...+T(d''_t)+(n-3)*G]+T(d'_t)+G+T(d_t)+T(d_k)-S(d_0)+2G$$

which reduces to

$$\mathcal{X} = S(d_0) + \sum_{i=1}^{n} T(d_i) + \sum_{i=1}^{n} G - S(d_0)$$

and finally

$$\mathcal{X} = \sum_{i=1}^{n} T(d_i) + \sum_{i=1}^{n} G \tag{3.18}$$

As we can see, $\mathcal{X} = T_{total} + G$ does not appear as an adequate upperbound, and has to be updated as $\sum_{d \in D} T(d) + \sum_{i=1}^{|D|} G$.

3.2 Routing and Spectrum Allocation with Predefined Set of Paths

As we have found out in the previous section, equations (3.5) and (3.6) can be eliminated from Christodoulopoulos et al's formulation of RSA ILP, and the upperbound for equations (3.7) and (3.8) should be updated as $\mathcal{X} = \sum_{i=1}^{|D|} T(d_i) + \sum_{i=1}^{|D|} G$. Within the context of the same assumptions, the new formulation will have $|D| * |D-1|$ constraints less than the latter and hence will be faster in terms of computational time.

Hence, the new model for the RSA within the context of same assumptions can be formulated as the following:

Sets and Parameters:

P_d: The set of k candidate paths for connection d.

$\mathcal{P}$=$\cup_d P_d$: The total set of candidate paths for the demand set D, $d \in D$.

G: The amount of subcarriers required as guardband in between demands that use the same edge in their paths, and is taken as constant.

$T(d)$: The number of subcarriers required for the communication d between source $u(d)$ and destination $v(d)$.

Variables:

X_p: 0 if path p is not utilized , and 1 if p is utilized;

$S(d)$: Integer variable that denotes the starting frequency for connection d;

W_{d_k,d_t}: Boolean variable that equals 1 if the starting frequency of connection d_k is smaller than the starting frequency of connection d_t (i.e. $S_{d_k} < S_{d_t}$) and 0 otherwise;

c: Maximum utilized spectrum slot number.

$$minimize \qquad c \tag{3.19}$$

subject to :

$$\sum_{p \in P_d} X_p = 1 \qquad \forall d \in D \tag{3.20}$$

$$c \geq S(d) + T(d) \qquad \forall d \in D \tag{3.21}$$

If $\exists p \in P_{d_k}$ and $\exists q \in P_{d_t}$ such that $p \cap q \neq \emptyset$, then for each d_t , $d_k \in D$ and every $p, q \in \mathcal{P}$ equations (3.22-3.24) are employed:

$$W_{d_k,d_t} + W_{d_t,d_k} = 1 \tag{3.22}$$

$$S(d_t) + T(d_t) + G - S(d_k) \leq \mathcal{X} * [1 - W_{d_t,d_k} + 2 - X_p - X_q] \tag{3.23}$$

$$S(d_k) + T(d_k) + G - S(d_t) \leq \mathcal{X} * [1 - W_{d_k,d_t} + 2 - X_p - X_q] \tag{3.24}$$

$$X_p \in \{0, 1\} \qquad p \in \mathcal{P} \tag{3.25}$$

$$S(d) \geq 0 \qquad d \in D \tag{3.26}$$

$$W_{d_t,d_k} \in \{0, 1\} \qquad d_t, d_k \in D \tag{3.27}$$

where $T_{total} = \sum_{d \in D} T(d)$ and $\mathcal{X} = \sum_{d \in D} T(d) + \sum_{i=1}^{|D|} G$;

3.3 Routing and Spectrum Allocation without Predefined Set of Paths

In the previous section, we have found an improved model for the offline routing and spectrum allocation problem with a set of predetermined routing for each demand. Although [10] have shown that their version of RSA problem can reach to near optimal solutions, this model does not guarantee a global optimum as it excludes many feasible paths that can generate optimal solutions. A new model that spans every feasible path to find the global optimum for routing and spectrum

slot selection within the context of the same assumptions as the previous sections can be formulated as follows:

We assume that we are given a connected graph $\mathcal{G}$ =(V,E),where V is the set of nodes and E is the set of edges, a bandwidth demand of d corresponds to a connection request with source $u(d)$ and sink $v(d)$ and D is the set of all connection requests. The aim of this formulation is to provide a route from the source node *u(d)* to the destination node *v(d)* of each demand and an allocation in the optical spectrum (i.e. indices of frequency slots) that is constant in all edges the demand traverses in its path. In this formulation, no pre-calculated routes are present. A route for each demand is determined by flow balance equations in the formulation.

Sets and Parameters:

D: Set of connection requests, i.e. demands.

$\mathcal{A}$=$\{(i,j) \cup (j,i) : \{i,j\} \in E\}$: The arc set generated from the edge set E of graph $\mathcal{G}$.

G: The amount of subcarriers required as guardband in between demands that use the same edge in their paths, and is taken as constant.

$T(d)$: The number of subcarriers required for the communication d between source $u(d)$ and destination $v(d)$.

Variables:

X_{ijd}: boolean arc utilization variable that equals 0 if demand d does not use arc (i,j); and 1 if d uses arc (i,j).

$S(d)$: Integer variable that denotes the starting frequency for connection d.

W_{d_k,d_t}: Boolean variable that equals 1 if the starting frequency of connection d_k is smaller than the starting frequency of connection d_t (i.e. $S_{d_k} < S_{d_t}$) and 0 otherwise.

c: Maximum utilized spectrum slot number.

Model

$$minimize \qquad c \tag{3.28}$$

subject to :

$$\sum_{j:(i,j)\in\mathcal{A}} X_{ijd} - \sum_{j:(j,i)\in\mathcal{A}} X_{jid} = 1 \qquad \forall d \in D : \text{i=u(d)} \tag{3.29}$$

$$\sum_{j:(i,j)\in\mathcal{A}} X_{ijd} - \sum_{j:(j,i)\in\mathcal{A}} X_{jid} = 0 \qquad \forall d \in D : \text{i} \neq \text{u(d)}, \text{i} \neq \text{v(d)} \tag{3.30}$$

$$\sum_{j:(i,j)\in\mathcal{A}} X_{ijd} - \sum_{j:(j,i)\in\mathcal{A}} X_{jid} = -1 \qquad \forall d \in D : \text{i=v(d)} \tag{3.31}$$

$$c \geq S(d) + T(d) \qquad \forall d \in D \tag{3.32}$$

$$W_{d_k,d_t} + W_{d_t,d_k} \leq 1 \qquad \forall d_k, d_t \in D : d_k \neq d_t \tag{3.33}$$

$$W_{d_k,d_t} + W_{d_t,d_k} \geq [(X_{ijd_t} + X_{jid_t}) + (X_{ijd_k} + X_{jid_k}) - 1] \qquad \forall (i,j),(j,i) \in \mathcal{A}\ \forall d_k, d_t \in D : d_t \neq d_k \tag{3.34}$$

$$S(d_t) + T(d_t) + G \leq S(d_k) + \mathcal{X} * [1 - W_{d_t,d_k}] \qquad \forall d_k, d_t \in D : d_t \neq d_k \tag{3.35}$$

$$S(d_k) + T(d_k) + G \leq S(d_t) + \mathcal{X} * [1 - W_{d_k,d_t}] \qquad \forall d_k, d_t \in D : d_t \neq d_k \tag{3.36}$$

$$X_{ijd} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, \forall d \in D \tag{3.37}$$

$$S(d) \geq 0 \qquad \forall d \in D \tag{3.38}$$

$$W_{d_t,d_k} \in \{0,1\} \qquad \forall d_t, d_k \in D \tag{3.39}$$

where $T_{total} = \sum_{d\in D} T(d)$ and $\mathcal{X} = \sum_{d\in D} T(d) + \sum_{i=1}^{|D|} G$.

Analysis of RSA without Pre-Defined Set of Paths

Equations (3.29)-(3.31) are flow balance equations which ensure that each demand will have exactly one path. Equation (3.32) along with the objective function (3.28) is used to find the minimum of the maximum number of spectrum slots used among all demands. Equations (3.33)-(3.34) are used to sort

the demands: Equation (3.34) ensures that for demands sharing an edge $\{i, j\}$, $W_{d_k,d_t} + W_{d_t,d_k} \geq 1$ (as $X_{ijd_t} + X_{jid_t} = 1$ and $X_{ijd_k} + X_{jid_k} = 1$, and it is trivially ≥ 0 if no two demands share this arc. Equation (3.33) sets an upperbound; if two demands do not share an edge, Equation (3.33) results in either ≥ 0 if either one of the demands uses edge $\{i, j\}$ and the other one does not; or ≥ -1 if none of the demands uses edge $\{i, j\}$– in which case sorting of these demands is not necessary. If two demands share an edge, Equation (3.33) combined with Equation (3.34) results in $W_{d_k,d_t} + W_{d_t,d_k} = 1$ exactly, and force the demands to be sorted.

Equations (3.35)-(3.36) are contiguity constraints that are used for defining the starting slot index for each demand. For demands that do not share an edge (i.e. both $W_{d_k,d_t} = 0$ and $W_{d_t,d_k} = 0$), starting slot indices can be any value within the boundaries. For demands that share an edge, as Equations (3.33) and (3.34) ensure that only one of W_{d_k,d_t} and W_{d_t,d_k} will be 1, the demand that is sorted second in the spectrum will be assigned a spectrum slot index greater than the sum of the index of the first demand and its size plus the number of guardian frequencies. Continuity requirement of RSA problem is satisfied by allocating only one starting slot index $S(d)$ for each demand and maintaining it throughout the path.

Number of Variables and Complexity

There are $|A| * |D|$ binary X variables, $|D|$ continuous S variables and $\frac{|D|*|D-1|}{2}$ binary W variables. In total there are $|D| * [|A| + 1 + \frac{|D-1|}{2}]$ variables in the formulation.

For each value of d, there is one constraint for (3.29) and for (3.31) and there are $|V| - 2$ constraints for (3.30). In total, the number of constraints for (3.29)-(3.31) are $|D| * |V|$. In the same manner, there are $|D|$ constraints for (3.32). For (3.33) there are $\frac{|D|*|D-1|}{2}$ constraints. For (3.34) one edge is considered at each equation for each demand pair and in total $|E|$ edges are considered for each demand. In general there are $|E| * \frac{|D|*|D-1|}{2}$ constraints for (3.34). For (3.35) and (3.36) there are $\frac{|D|*|D-1|}{2}$ constraints. Therefore, the total number of constraints in the formulation are $|D| * [|V| + 1 + \frac{|D|-1}{2} * [3 + |E|]]$.

The overall complexity for constraints are $O(|D|^2 * |E| + |D| * |V|)$ and $O(|D|^2 + |D| * |A|)$ for variables.

Chapter 4

Heuristic Algorithms

In the previous chapter, we have formulated a mathematical model that spans all of the feasible region and yields in a global optimum solution for Routing and Spectrum Allocation problem. Although a global optimum is guaranteed in this model, for large networks and high traffic loads, solution times can increase exponentially which results in impracticality in real life. To find near optimal solutions in a fast manner, heuristic algorithms are used as substitutes for ILP models. In this chapter, we propose two construction algorithms for the RSA problem, namely *Least Cost Slot Allocation (LCS)* and *Iterative Common Path Allocation (ICPA).*

4.1 Least Cost Slot Allocation (LCS)

Given an undirected graph $\mathcal{G}$=(V, E); where V is the set of nodes and E is the set of edges connecting nodes, and a demand set D with $d \in D$ such that $d = (u(d), v(d))$ –$u(d)$ is the source node, $v(d)$ is the sink node and $T(d)$ is the number of required slots (demand size)– the aim of this heuristic algorithm is to compute a route and starting slot index allocation for each demand in a fast manner. To build the algorithm, an implementation of Dijkstra's Single Source

Shortest Path algorithm is used. Least Cost Slot Allocation (LCS) Algorithm is an adaptation of Shortest Path algorithm in an undirected graph with weighted edges to solve Routing and Spectrum Allocation by picking the least cost shortest path from source node to sink node and by defining a starting slot index for each demand given, iteratively. We assume that edges are uncapacitated. Ordering of the demands is an important topic as different orderings yield in different solutions for heuristic algorithms. In the LCS algorithm that we propose, we order the demands according to their demand size in a non-increasing manner.

The scope of this algorithm is to provide a near-optimal feasible solution in a tractable time frame, however an optimal solution is not guaranteed. LCS does not consider slot by slot inspection during spectrum slot index allocation phase, which results in gaps from optimality. We propose LCS as a starting point for a feasible allocation in the optical spectrum. By using the path and starting slot allocation results from LCS as an initial solution input for ILP model, we aim to decrease the solution time of the ILP formulation. The algorithm is described as follows:

Least Cost Slot Allocation Algorithm

1. Start with sorting the demands according to their demand size, T(d) in a descending order. Pick the demand with the highest number of slot requirements first. If ties occur, break ties arbitrarily. For each demand d, denote the starting slot index of d as S(d) and let $S(d) = 0 \ \forall d \in D$ as initial slot indices.

2. Construct a VxV cost matrix C, with 0 initial cost for edges $e \in E$ and a very large number M for edges not present in the graph.

3. Pick the first demand, say d, in the sorted list and find the shortest path from the source $u(d)$ to sink $v(d)$ node of demand d. Let $Path(d)$ be such a path. Calculate the new costs of the edges used in the path $(\forall e \in Path(d))$ as follows: $c'_e = c_e + T(d) + G$; where c'_e is the new cost of the edge $e \in Path(d)$ after allocating demand d, T(d) is the demand size and G is the number of required guard carrier slots. Update

the cost matrix as: $c_{\{i,j\}}$= $\max_{e \in Path(d)} \{c'_e\}$; $\forall \{i,j\} \in Path(d)$. Update S(d) as $S(d)$= $\max_{e \in Path(d)} \{c'_e\} - T(d)$.

4. Taking the new cost matrix into consideration, repeat step 3 for all demands with respect to the sorting. Stop when all demands are allocated.

Algorithm 1 Least Cost Slot Allocation Algorithm

```
BEGIN
INITIALIZE Sets: D := {d : (u(d), v(d))}, Parameters: T(d), G, S(d) = 0,
Matrices: C
SET SD := Array of sorted demands according to T(d), decreasingly.
SET C := { c_{i,j} = 0| ∀{i,j} ∈ E : c_{i,j} = M otherwise }
SET Path(d) := Shortest path of d from source u(d) to sink v(d)
SET i ← 1
while i ≤ Size[SD] do
    d=SD[i]
    Calculate Path(d).
    Calculate c'_e = c_e + T(d) + G for each e ∈ Path(d).
    Update c_{i,j} = max_{e∈Path(d)} {c'_e} for each {i,j} ∈ Path(d)
    S(d) = max_{e∈Path(d)} {c'_e} − T(d)
    i ← i + 1
end while
```

4.2 Iterative Common Path Allocation (ICPA)

Given an undirected graph $\mathcal{G}$=(V, E); where V is the set of nodes and E is the set of edges connecting nodes, and a demand set D with $d \in D$ such that $d = (u(d), v(d))$ –$u(d)$ is the source node, $v(d)$ is the sink node and $T(d)$ is the number of required slots (demand size)– the aim of ICPA heuristic algorithm is to compute a route and starting slot index allocation for each demand iteratively, similarly to LCS. Iterative Common Path Allocation (ICPA) algorithm is a multi-phase method that is a combination of Dijkstra's Single Source Shortest Path algorithm with Taboo search. ICPA aims to find the minimum amount of spectrum slots needed to allocate all demands in the network, by computing shortest paths for each demand and trying to reduce any congestion that may occur on the most

loaded arcs by computing alternative paths for the demands that use those arcs in their shortest paths iteratively. Unlike LCS where assignment is done according to the demand sizes, ICPA assigns demands to arcs in order to decrease overlapping edges in the shortest paths of any demand pair from the demand set.

The first phase of the Iterative Common Path Allocation (ICPA) algorithm is an adaptation of the Shortest Path algorithm in an undirected graph $\mathcal{G}$ where weight of all edges are equal to 1. In the first phase, shortest paths for all demands, $Path(d)$; $\forall d \in D$, are calculated and the edges that occur most frequently in $\bigcup_{d \in D} Path(d)$ are chosen to be assigned a demand. For any such edge chosen (say edge l), the demand with maximum number of arcs in its path (say demand t), such that $l \in Path(t)$, is picked to be assigned to its path. Any other demand $k \in D \setminus \{t\}$, such that path of demand k shares a common edge with the path of the demand picked first t, is removed from the demand set D and moved to the Taboo set $\mathcal{T}$ in order to be assigned on an alternative path later in the second phase of the algorithm. Demand t and any demand k with the attributes described before is removed from the set D and with the remaining set, the first phase is iteratively conducted until there are no demands left in D to be assigned on the spectrum. This ensures that the demands assigned in the first phase have no edges in common. Any demand that has a common edge in its shortest path with the demands assigned on spectrum is transferred to the second phase in order to be assigned on alternative paths, so that the congestion on any edge is reduced to minimum. In the second phase, demands in the Taboo set $\mathcal{T}$ are transferred to D and a similar search as the first phase begins, but this time search is conducted by finding the least cost shortest path in graph $\mathcal{G}$ with weighted edges, whose weights come from the amount of spectrum slots utilized in the first phase. By searching for the least cost shortest paths and recomputing these paths for the remaining demands in the demand set until all demands get allocated, alternative paths for demands in Taboo state are found.

The scope of this algorithm is to provide an improved solution to LCS in a tractable time frame, however an optimal solution is again not guaranteed. Similar to LCS, ICPA does not consider slot by slot inspection during spectrum

slot index allocation phase, which results in gaps from optimality. We propose ICPA as an improved method to find a feasible allocation in the optical spectrum. The details of the algorithm is described as follows:

Iterative Common Path Allocation Algorithm

1. Construct a demand set D consisting of all demands to be allocated in the spectrum. For each demand $d \in D$, denote the starting slot index of d as S(d) and initialize $S(d) = 0$, $\forall d \in D$. Find the shortest path on graph $\mathcal{G}$ with weight of all edges equal to 1, from source node $u(d)$ to sink node $v(d)$ for all d in the demand set D. Construct a path matrix $\mathcal{P}$ from the shortest paths of each demand and a cost matrix C with 0 initial cost for edges $e \in E$ and a very large number M for edges not present in the graph.

2. Find the edge which occurs most in $\mathcal{P}$. For the edge chosen, say edge $\{i, j\}$, find the demand with the most number of edges in its shortest path. Let $Path(d)$ be such a path for demand d such that $\{i, j\} \in Path(d)$. If ties occur, pick the demand with the maximum demand size. Allocate d on its path $Path(d)$ and calculate the new costs of the edges used in the path $(\forall e \in Path(d))$ as follows: $c'_e = c_e + T(d) + G$; where c'_e is the new cost of the edge $e \in Path(d)$ after allocating demand d, T(d) is the demand size and G is the number of required guard carrier slots. Update the costs of all of the edges $\{i, j\}$ present in the $Path(d)$ as $c_{\{i,j\}} = \max_{e \in Path(d)} \{c'_e\}$. Update $S(d) = \max_{e \in Path(d)} \{c'_e\} - T(d)$. Set the demand set $D = D \setminus \{d\}$.

3. $\forall \{i, j\} \in Path(d)$, find the demands that use edges $\{i, j\}$ in their paths and move these demands to the Taboo set, $\mathcal{T}$. Update D as $D = D \setminus \mathcal{T}$.

4. Repeat steps 2 and 3 until D is empty, then move to Phase 2.

5. Phase 2: Set the new D set as $D = \mathcal{T}$. Set $\mathcal{T} = \emptyset$.

6. For all $d \in D$, find the shortest path on weighted graph $\mathcal{G}$, with weights taken from the cost matrix C. Construct a path matrix $\mathcal{P}$ with weighted shortest paths for $d \in D$ and apply steps 2 and 3. At the end of each

iteration of step 3 in Phase 2, recalculate path matrix $\mathcal{P}$ for the new D set. Repeat step 6 until D is empty.

7. Taking the new cost matrix into consideration, repeat Steps 5-6 (Phase 2) until all demands are allocated.

In the next chapter, we will analyze the performance of our ILP formulation with various test schemes and the performance of the two heuristic algorithms we have defined in this chapter, LCS and ICPA. We will conduct our tests on numerous network topologies and also test the performance of combining our heuristic algorithms with our ILP formulation on a real-life network topology.

Algorithm 2 Iterative Common Path Allocation Algorithm

BEGIN FIRST PHASE
INITIALIZE Sets: $D = \{d : (u(d), v(d))\}$, Parameters: $T(d)$, G, $S(d) = 0$, Matrices: $\mathcal{P}$, C
SET C := $\{ c_{\{i,j\}} = 0 | \ \forall \{i,j\} \in E : c_{\{i,j\}} = M \ otherwise \ \}$
SET UPath(d) := Shortest path of d from source $u(d)$ to sink $v(d)$ on graph $\mathcal{G}$ where all edges have a weight of 1
SET $\mathcal{P}$:=Matrix of shortest paths UPath(d), for all $d \in D$.
SET h:= The edge which occurs most in $\mathcal{P}$.
while $D \neq \emptyset$ **do**
  Find h
  Find all d with most number of edges in UPath(d) s.t. $h \in UPath(d)$
  Set X:=$\{d|$ UPath(d) has most number of edges s.t.$h \in UPath(d)\}$
  if $|X| \geq 1$ **then**
    Find $max\{T(d')\} | \ d' \in X$
    $d \leftarrow d'$
  end if
  Calculate $c'_e = c_e + T(d) + G$ for each $e \in UPath(d)$
  Update $c_{\{i,j\}} = \max_{e \in Path(d)} \{c'_e\}$, for each $\{i,j\} \in UPath(d)$
  $S(d) = \max_{e \in Path(d)} \{c'_e\} - T(d)$
  $D \leftarrow D \setminus \{d\}$
  Find all $d' \in D$ s.t. $UPath(d') \cap UPath(d) \neq \emptyset$ and set $\mathcal{T} = \{d'\}$ for all such d'
  $D \leftarrow D \setminus \mathcal{T}$
end while
END FIRST PHASE
BEGIN SECOND PHASE
SET $D \leftarrow \mathcal{T}$ and $\mathcal{T} \leftarrow \emptyset$
SET WPath(d) := Shortest path of d from source $u(d)$ to sink $v(d)$ on graph $\mathcal{G}$ where edge weights come from C
SET $\mathcal{P}$:=Matrix of shortest paths WPath(d), for all $d \in D$.
while $D \neq \emptyset$ **do**
  Find h
  Find all d with most number of edges in WPath(d) s.t. $h \in WPath(d)$
  Set X:=$\{d|$ WPath(d) has most number of edges s.t.$h \in WPath(d)\}$
  if $|X| \geq 1$ **then**
    Find $max\{T(d')\} | \ d' \in X$
    $d \leftarrow d'$
  end if
  Calculate $c'_e = c_e + T(d) + G$ for each $e \in WPath(d)$
  Update $c_{\{i,j\}} = \max_{e \in Path(d)} \{c'_e\}$, for each $\{i,j\} \in WPath(d)$
  $S(d) = \max_{e \in Path(d)} \{c'_e\} - T(d)$
  $D \leftarrow D \setminus \{d\}$
  Find all $d' \in D$ s.t. $WPath(d') \cap WPath(d) \neq \emptyset$ and set $\mathcal{T} = \{d'\}$ for all such d'
  $D \leftarrow D \setminus \mathcal{T}$
  Recalculate $\mathcal{P}$ with new C
end while
REPEAT PHASE 2 Until $\mathcal{T} = \emptyset$ and $D = \emptyset$

Chapter 5

Performance Analysis for RSA ILP without Predefined Set of Routes

In order to analyze the performance of the ILP formulation suggested in the Section 3.3: Routing and Spectrum Allocation without Predefined Set of Paths, 7 sets of experiments are conducted. The first 5 sets of experiments are designed to test the effect of sparsity and to find out the maximum number of node-connection request number combination that the ILP formulation can handle within 3600 seconds CPU time limit. For each set of experiment, random graph topologies are created for node numbers of N=$\{8, 14, 25, 50, 100\}$ and the formulation is tested for different combinations of N and D, where D is the traffic load –or total demand number– where D=$\{4, 10, 30, 50, 75, 100\}$. Each $d \in D$ has a demand size generated from the distribution Uniform(0,50). The same test scenarios of N-D are used for analyzing the effect of sparsity of the graph topology. 4 different sparsity scenarios consisting of sparsity coefficients $\{1, 0.75, 0.5, 0.25\}$ are used. A sparsity coefficient of 1 represents a complete network and a sparsity coefficient of 0.25 represents a very sparse network where 25% of all potential edges are present in the network. In the last two sets of experiments, the maximum number of commodities that the ILP formulation can handle on a real-life

network, Deutsche Telekom Network, and the effects of Heuristics+ILP formulation combinations on run time values are tested. For DT network experiment, Deutsche Telekom network consisting of 14 nodes and 23 edges, and connection request sets of D=$\{12, 15, 20, 25, 27\}$ are used. Each commodity of $d \in D$ has a demand size generated from the distribution Uniform(0,50). 5 instances of each traffic load variation are generated for the tests. The runtime and optimality gap performance analyses of our proposed LCS and ICPA heuristic algorithms are also tested on DT network. Computational tests are created in IBM ILOG CPLEX 12.6.0.0 for ILP formulation and in MATLAB for Heuristic algorithms. Tests are conducted on a 2.10 GHz Intel core i7 computer.

To the best of our knowledge, run time analyses are conducted only for formulations of [9], [10], and [14] in the literature and there is no previous work that tests the effects of sparsity of graphs on SLICE networks. Christodoulopoulos et.al [10] test their formulation of RSA with predefined set of routes for two cases of network topologies, a small 6 node network topology and generic 14 node DT network topology on LINDO API. Two cases of traffic demand D=4 for low load and D=30 for high load case are used. It is stated that for small network topology, the formulation finds a solution in average 3.25 seconds for low load, and in average 503.19 for high load demand case. For DT network topology, the formulation could not produce any results in a reasonable amount of time.

Klinkowski et.al [14] test their ILP formulation with predefined sets on a small 6 node network with traffic loads of $D = \{15, 30, 45, 60, 90\}$, where demand sizes are taken from $\{5, 15, 30\}$. Average run times range from 13 seconds to 12747 seconds for low and high load cases for ILP formulation. Their proposed heuristic algorithm AFA-CA has optimality gap ranging from 2.49% to 7.02% with respect to the objective function value of their ILP formulation with predefined set of paths. They take tests with ILOG CPLEX v.12.2 on a 2.27 GHz Intel i3 computer. No further analysis on run time values and gap analysis for larger network topologies are present in [14].

Paul [9] tests his formulations, ILP1 that represents RSA without predefined set of routes and ILP2 that represents RSA with predefined set of routes on

topologies of 8,12,15 node networks and 14 node DT network. Traffic demands of 8,12,15,18 and 20 commodities are used. ILP1 formulation that finds a global optimum solution can handle up to 15 commodities for 15 node generic network topology and 20 commodities for 14 node DT network topology. Although ILOG CPLEX is used for tests in [9], the version of CPLEX and information on the computer used for tests are not stated.

5.1 Results for Varying Sparsity Coefficient on Random Topologies

Objective function values and run time results of the experiments for varying sparsity on different demand number-node number combinations can be seen in Tables 5.1, 5.2, 5.3, and 5.4. The first column in the tables denotes the sparsity coefficient used for the network topology. A sparsity coefficient of 1 corresponds to a fully connected network, a sparsity coefficient of 0.75 corresponds to a connected network where 75% of edges are present, a sparsity coefficient of 0.5 corresponds to a connected network where 50% of edges are present, and a sparsity coefficient of 0.25 corresponds to a connected network where 25% of edges are present. The second column denotes the traffic load (i.e. number of demands) used for the experiments and the third column denotes the number of nodes used. The fourth column gives the run time value (in CPU seconds) for *Sparsity-Demand Number-Node Number* combination used for the experiment. The last column gives the objective value for the respective experiment. If the optimal solution is not reached within the time limit of 3600 seconds, the optimality gap as reported by CPLEX is provided. CPLEX code for ILP model which creates random demand attributes and random network topologies for different sparsity levels and node numbers can be seen in Appendix B.1.

The performance analysis of our formulation for RSA without predefined set of routes resulted that for fully connected networks (sparsity coefficient of 1), the formulation can handle up to 100 commodities in 25 node networks, up to 50

Sparsity (F)	Demand Number (D)	Node Number (N)	Run Time (CPU Seconds)	Objective function value
1	4	8	0,02	43
1	10	8	0,06	89
1	30	8	173,90	97
1	50	8	3597,99	247
1	4	14	0,03	24
1	10	14	0,22	96
1	30	14	2,70	94
1	50	14	47,45	95
1	75	14	3600,97	(gap 2,32%)
1	4	25	0,06	46
1	10	25	0,61	49
1	30	25	7,92	47
1	50	25	29,77	48
1	75	25	81,84	49
1	100	25	850,48	49
1	4	50	0,41	79
1	10	50	3,03	78
1	30	50	34,91	91
1	50	50	118,67	47
1	75	50	3600,00	(gap 23,11%)
1	4	100	1,55	91
1	10	100	13,66	95
1	30	100	173,78	48
1	50	100	3600,00	(gap 48,07%)

Table 5.1: Test Results: Run Time and Objective Function Values for Network Topologies with Sparsity Coefficient 1 consisting of $\{8, 14, 25, 50, 100\}$ nodes and traffic load of $\{4, 10, 30, 50, 75, 100\}$ Demands. Gaps reported at the objective function values denote that the results could not be attained due to 3600 CPU seconds time limit.

Sparsity (F)	Demand Number (D)	Node Number (N)	Run Time (CPU Seconds)	Objective function value
0,75	4	8	0,02	75
0,75	10	8	0,36	91
0,75	30	8	3600,72	(gap 1,98%)
0,75	50	8	3599,55	(gap 5,03%)
0,75	75	8	3600,00	infeasible
0,75	4	14	0,03	47
0,75	10	14	0,38	84
0,75	30	14	8,39	48
0,75	50	14	1905,86	45
0,75	75	14	3600,00	(gap 1,08%)
0,75	4	25	0,14	34
0,75	10	25	1,16	47
0,75	30	25	20,97	48
0,75	50	25	29,69	48
0,75	75	25	322,59	49
0,75	100	25	3600,00	(gap 12,88%)
0,75	4	50	0,63	75
0,75	10	50	5,94	93
0,75	30	50	199,67	93
0,75	50	50	394,44	49
0,75	75	50	3600,00	(no LP file)
0,75	4	100	3,13	69
0,75	10	100	25,56	73
0,75	30	100	369,86	49
0,75	50	100	3600,00	(no LP file)

Table 5.2: Test Results: Run Time and Objective Function Values for Network Topologies with Sparsity Coefficient 0.75 consisting of $\{8, 14, 25, 50, 100\}$ nodes and Connection Requests of $\{4, 10, 30, 50, 75, 100\}$ Demands. Gaps reported at the objective function values denote that the results could not be attained due to 3600 CPU seconds time limit. "(no LP file)" at the objective function value denotes that the model file (.LP) could not be produced within 3600 CPU seconds time limit.

commodities in 8,14 and 50 node networks and up to 30 commodities in 100 node networks (see Table 5.1). For networks with sparsity coefficient of 0.75, the formulation can handle up to 10 commodities in 8 node networks, 50 commodities in 14 node networks, 75 commodities in 25 node networks, 50 commodities in 50 node networks and 30 commodities in 100 node networks (see Table 5.2). For networks with sparsity coefficient of 0.5, the formulation can handle up to 10 commodities in 8 node networks, 30 commodities in 14 node networks, 75 commodities in 25 and 50 node networks and 30 commodities in 100 node networks (see Table 5.3). For very sparse networks (sparsity coefficient 0.25), the formulation can handle up to 75 commodities in 50 node networks and up to 30 commodities in 25 and 100 node networks, but due to small numbers of edges present for small network topologies of 8 and 14 nodes, commodities beyond 30 result in infeasibility (see Table 5.4). A summary of the amount of traffic handled for different node values can be seen in Figure 5.1.

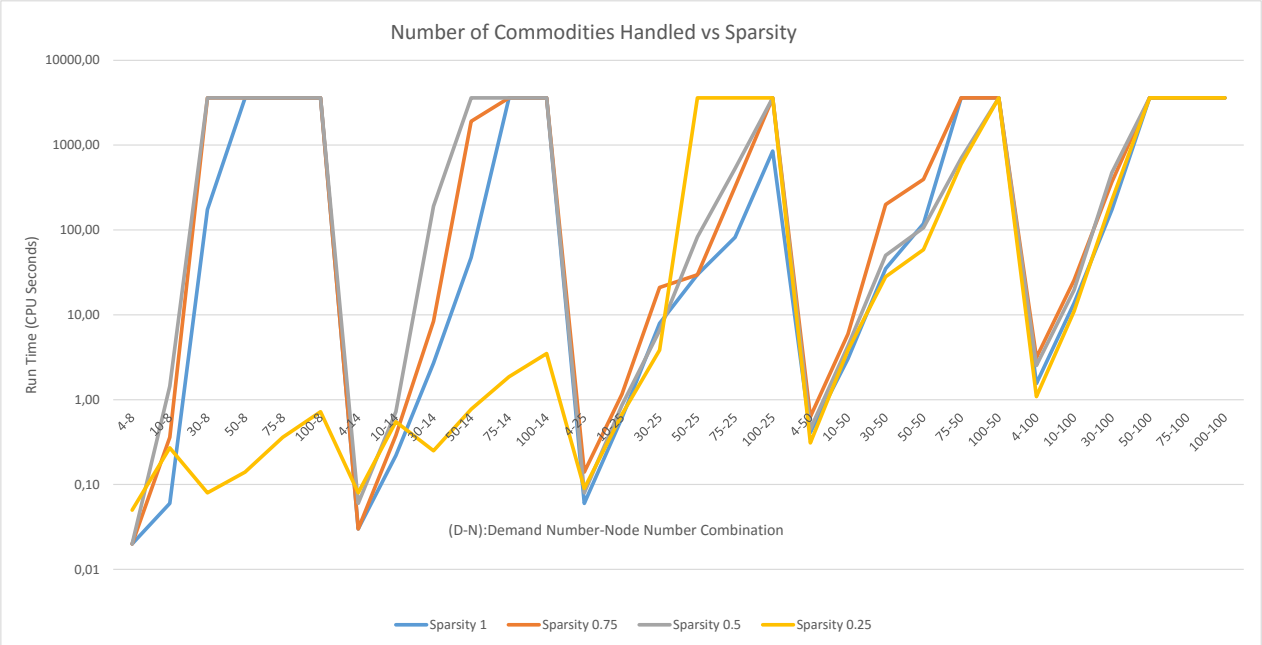


Figure 5.1: Trend of Run Time Values for Demand Number-Node Number vs Sparsity Combinations

The results show that changing sparsity levels has different impacts on run times depending on the size of the network. Searching for an optimum value in

Sparsity (F)	Demand Number (D)	Node Number (N)	Run Time (CPU Seconds)	Objective function value
0,5	4	8	0,02	28
0,5	10	8	1,44	100
0,5	30	8	3600,52	(gap 8,45%)
0,5	50	8	3604,64	(gap 74,66%)
0,5	4	14	0,06	70
0,5	10	14	0,72	90
0,5	30	14	190,31	94
0,5	50	14	3594,68	(gap 9,71%)
0,5	4	25	0,08	42
0,5	10	25	0,86	44
0,5	30	25	6,52	46
0,5	50	25	82,61	48
0,5	75	25	528,05	48
0,5	100	25	3600,00	(gap 26,03%)
0,5	4	50	0,45	55
0,5	10	50	4,31	91
0,5	30	50	50,05	99
0,5	50	50	106,45	46
0,5	75	50	696,47	49
0,5	100	50	3600,00	(no LP file)
0,5	4	100	2,53	84
0,5	10	100	19,67	98
0,5	30	100	474,73	48
0,5	50	100	3600,00	(no LP file)

Table 5.3: Test Results: Run Time and Objective Function Values for Network Topologies with Sparsity Coefficient 0.5 consisting of $\{8, 14, 25, 50, 100\}$ nodes and Connection Requests of $\{4, 10, 30, 50, 75, 100\}$ Demands. Gaps reported at the objective function values denote that the results could not be attained due to 3600 CPU seconds time limit. "(no LP file)" at the objective function value denotes that the model file (.LP) could not be produced within 3600 CPU seconds time limit.

Sparsity (F)	Demand Number (D)	Node Number (N)	Run Time (CPU Seconds)	Objective function value
0,25	4	8	0,05	89
0,25	10	8	0,27	185
0,25	30	8	0,08	infeasible
0,25	50	8	0,14	infeasible
0,25	75	8	0,36	infeasible
0,25	100	8	0,72	infeasible
0,25	4	14	0,08	72
0,25	10	14	0,55	133
0,25	30	14	0,25	infeasible
0,25	50	14	0,77	infeasible
0,25	75	14	1,86	infeasible
0,25	100	14	3,50	infeasible
0,25	4	25	0,09	44
0,25	10	25	0,67	35
0,25	30	25	3,83	45
0,25	50	25	3600,00	(gap 6,87%)
0,25	75	25	3600,00	(gap 24,57%)
0,25	100	25	3600,00	(gap 52,63%)
0,25	4	50	0,31	85
0,25	10	50	3,83	86
0,25	30	50	28,06	98
0,25	50	50	58,67	49
0,25	75	50	597,72	49
0,25	100	50	3600,00	(gap 37,11%)
0,25	4	100	1,09	99
0,25	10	100	11,00	89
0,25	30	100	225,42	49
0,25	50	100	3600,00	(gap 35,23%)

Table 5.4: Test Results: Run Time and Objective Function Values for Network Topologies with Sparsity Coefficient 0.25 consisting of $\{8, 14, 25, 50, 100\}$ nodes and Connection Requests of $\{4, 10, 30, 50, 75, 100\}$ Demands. Gaps reported at the objective function values denote that the results could not be attained due to 3600 CPU seconds time limit.

fully connected networks (sparsity coefficient 1) produce better runtime values compared to networks with average sparsity (0.75 and 0.5 sparsity coefficients) at almost every demand-node number combination variation. For small network topologies (8 and 14 node networks), increasing the connectivity of a graph both decreases the average solution time and increases the maximum number of commodities that can be handled. Searching for a solution in very sparse networks often results in infeasibility in small network topologies, when the number of commodities exceed the number of nodes in the network (see Figure 5.2).

For network topologies larger than 25 nodes, behavior of solution times compared to number of commodities to be handled show a different trend. For very sparse networks and balanced number of commodity-node number combinations, a solution is found faster compared to denser networks. However, as the number of demands exceed the number of nodes in the network, a solution cannot be found in 3600 seconds CPU time limit. Generally, performance of fully connected graphs are better than average sparsity graphs at almost every number of connection requests, similar to the outcomes of small network topologies. (An exception to this generalization occurs at 50 node network topology and 75 connection requests (Figure 5.3), where networks with 0.5 sparsity coefficient results in faster solutions than fully connected networks).

For 25 node network topology with connection request numbers almost equal to or smaller than the number of nodes present in the graph, and for network topologies beyond 50 nodes – except for fully connected graphs – increase in connectivity does not yield in better run time values. (See Figures 5.3 and 5.4 for the graphic of trends of run times for network topologies beyond 25 nodes). In fact, network topologies with 0.75 sparsity coefficient values result in highest solution time values compared to 0.5 and 0.25 sparsity coefficients for network topologies with nodes 25-50. One possible reason for this outcome may be the decrease in amount of paths to be considered during routing and allocation phase for networks with 0.5 sparsity coefficient than 0.75 sparsity coefficient. For 100 node networks, up to 10 demands the trend is similar to 25-50 node networks, but from 30 demands and upwards, higher run time requirement shifts towards sparser networks: 0.5 sparsity coefficient and below. The general trends of run

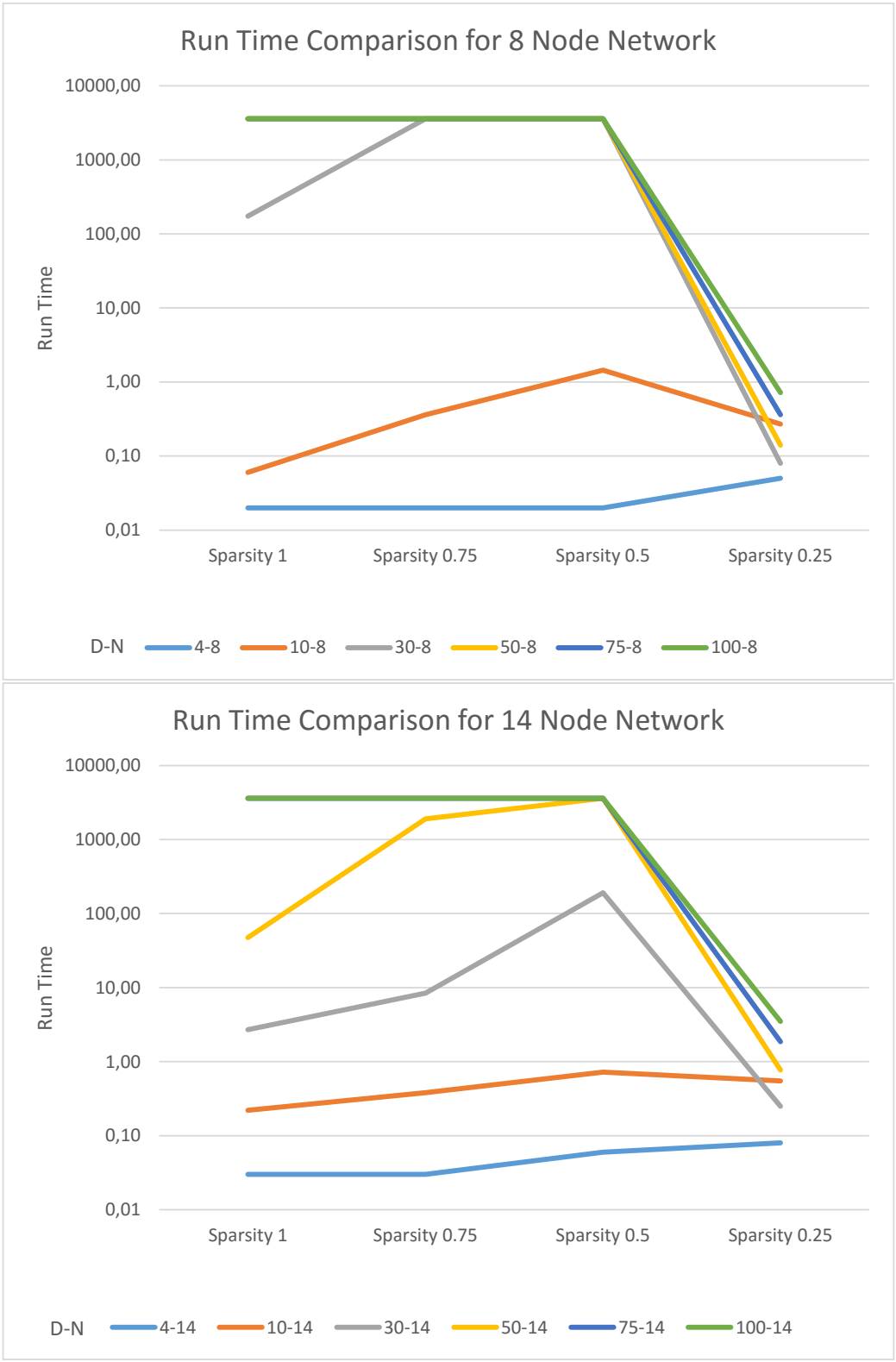
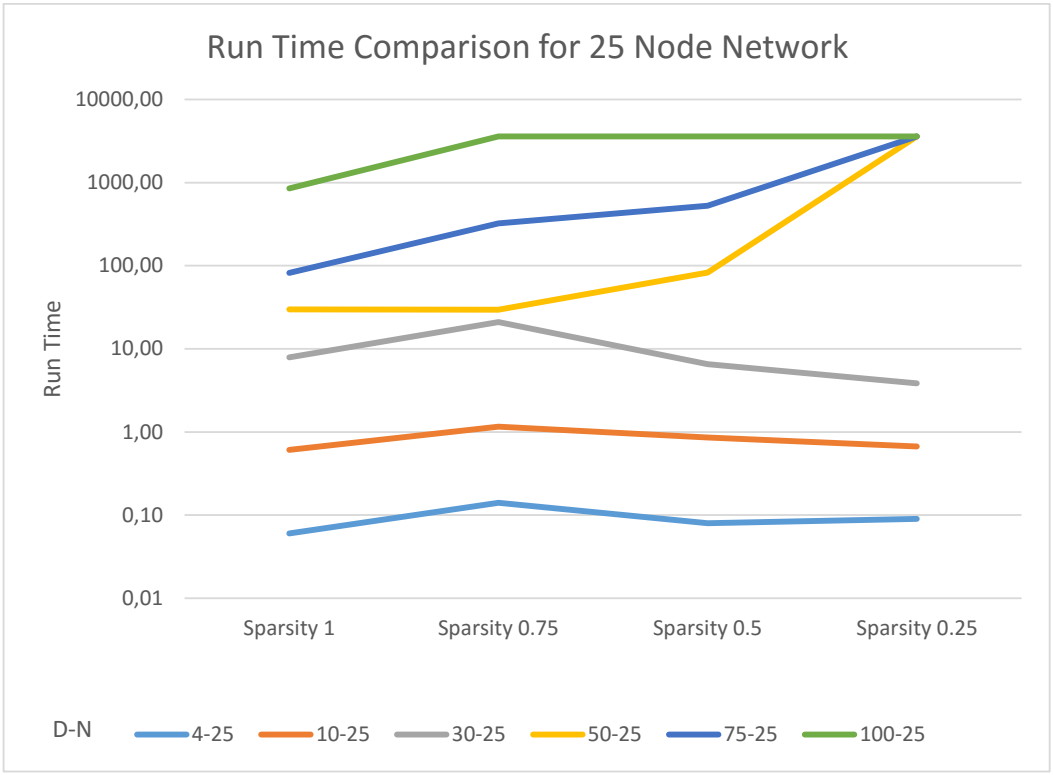


Figure 5.2: Run Time Comparison of Different Sparsity Coefficients for Small Networks Consisting of 8 and 14 Nodes49



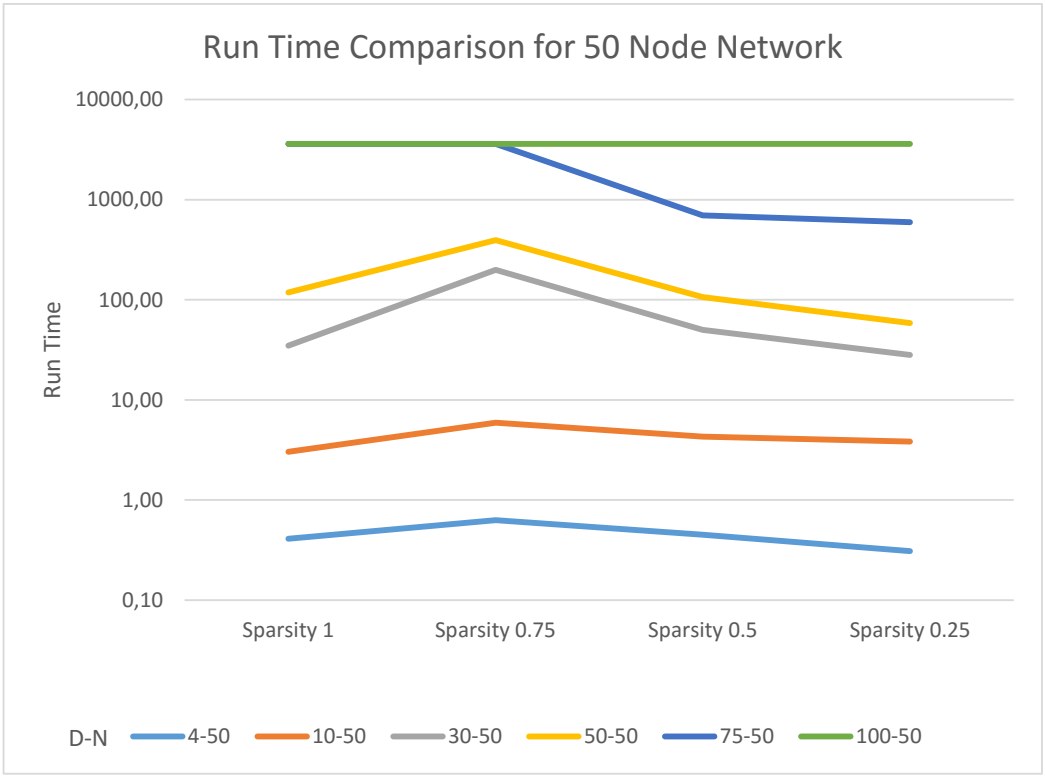


Figure 5.3: Run Time Comparison of Different Sparsity Coefficients for Networks Consisting of 25 and 50 Nodes

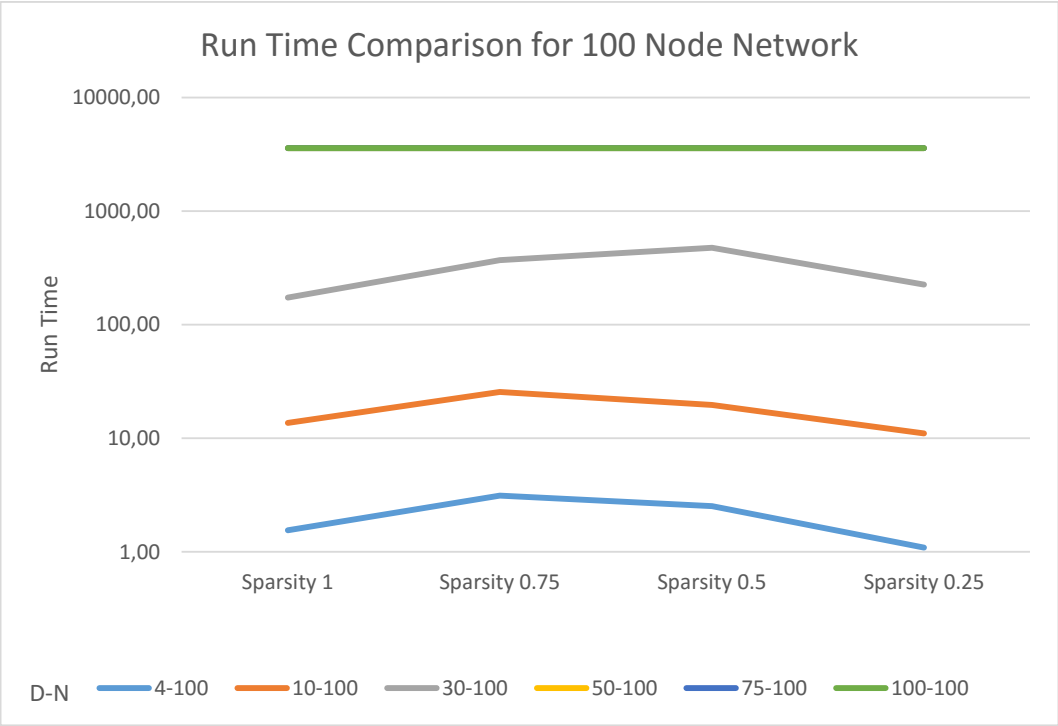


Figure 5.4: Run Time Comparison of Different Sparsity Coefficients for Large Networks Consisting of 100 Nodes

time values according to number of commodities, number of nodes present in the networks and network sparsity can be seen in Figures 5.5, 5.6, 5.7 and 5.8. Figure 5.5 gives the effects of changing demand value-node value combinations on run time values for fully connected networks. Figure 5.6 gives the effects of changing demand value-node value combinations on run time values for networks with 75% connectivity and Figures 5.7 and 5.8 represent networks with 50% and 25% connectivity, respectively.

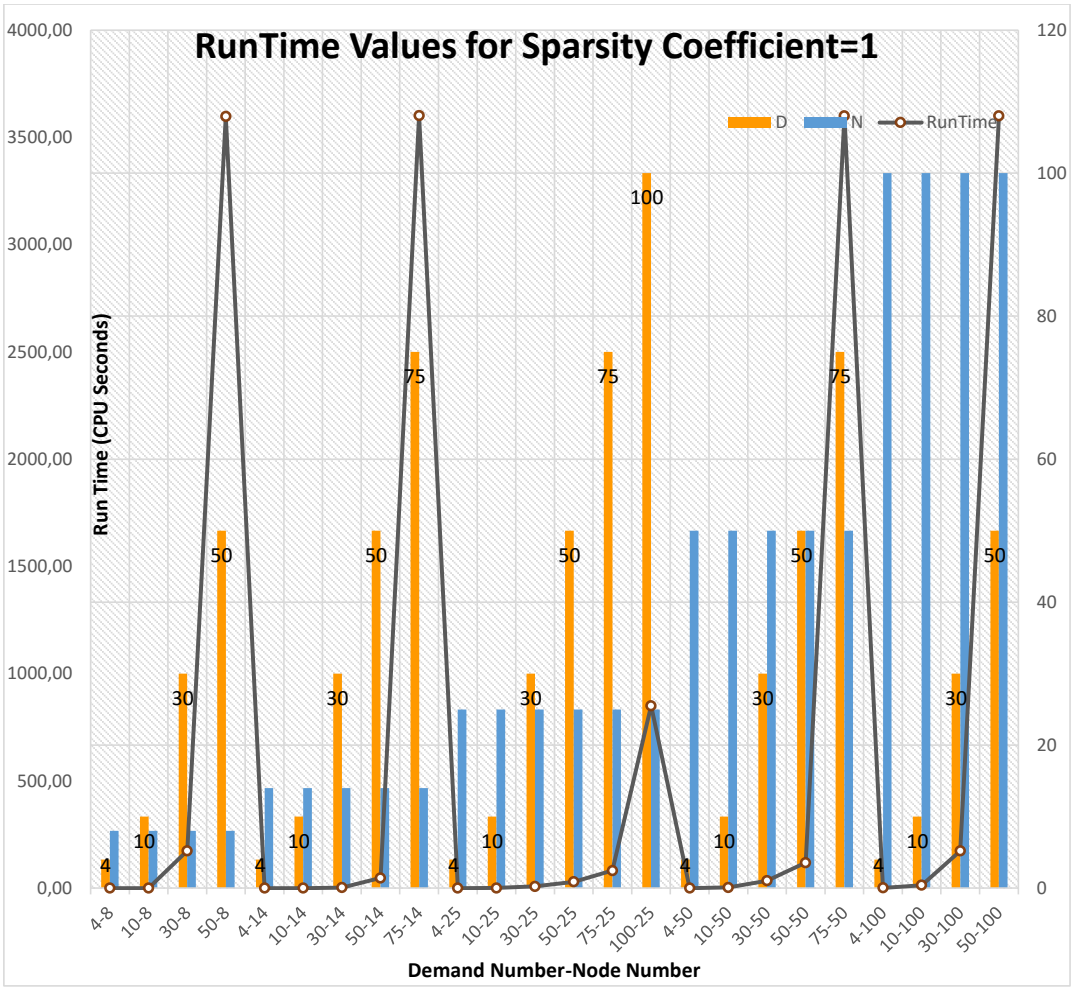


Figure 5.5: Effects of increasing connection requests on run time values at various network topologies for fully connected networks

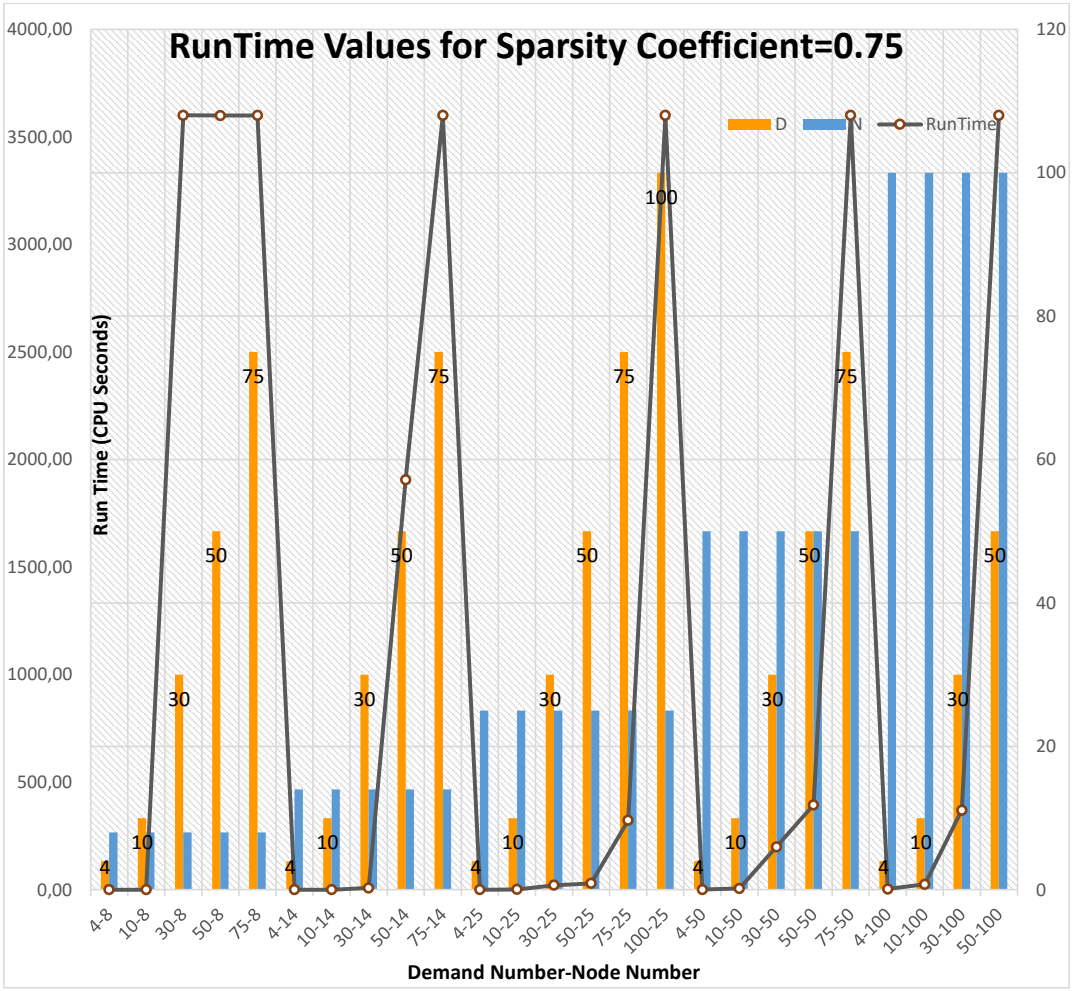


Figure 5.6: Effects of increasing connection requests on run time values at various network topologies for sparsity coefficient of 0.75

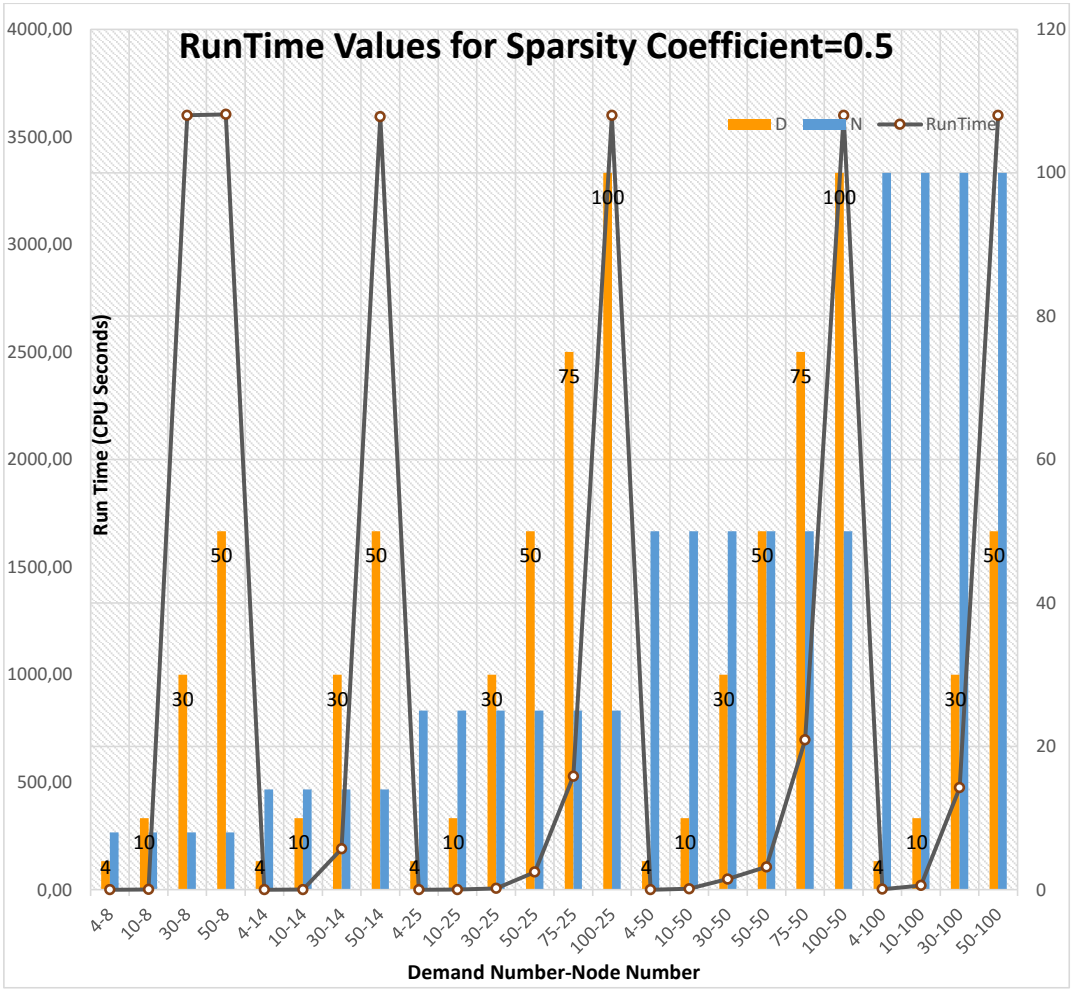


Figure 5.7: Effects of increasing connection requests on run time values at various network topologies for sparsity coefficient of 0.5

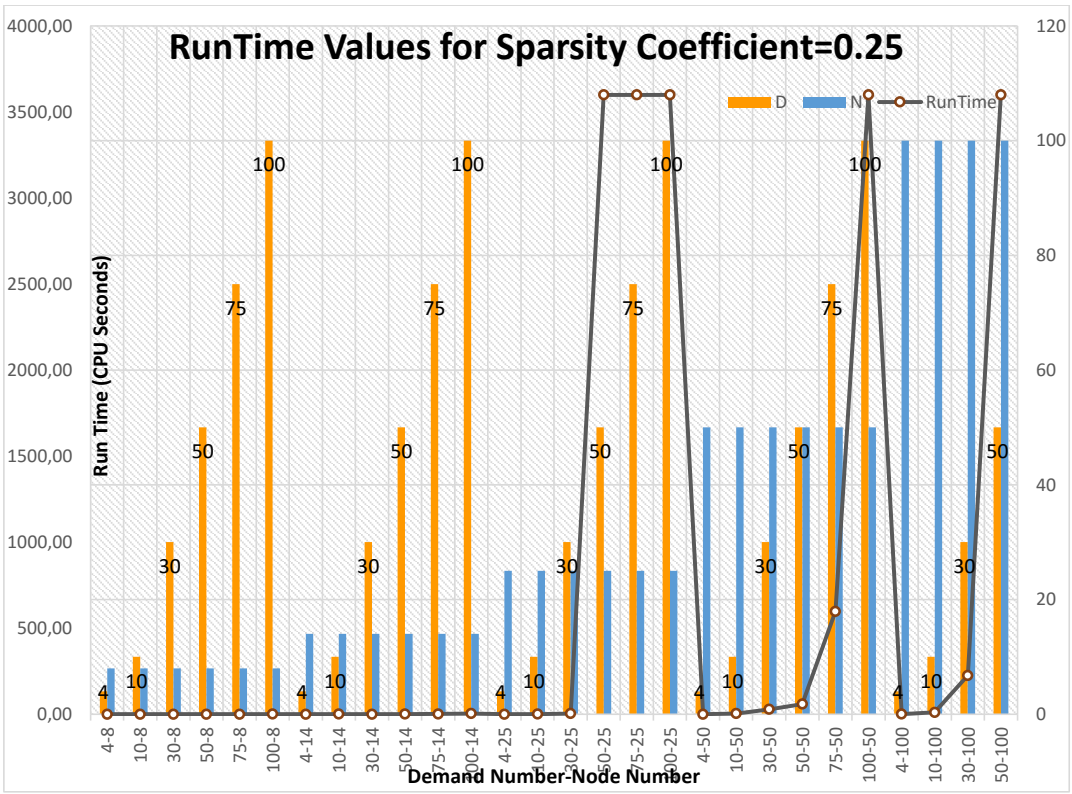


Figure 5.8: Effects of increasing connection requests on run time values at various network topologies for sparsity coefficient of 0.25

5.2 Results for a Real Network Topology: Deutsche Telekom Network

To test the performance of our proposed ILP model and heuristic algorithms, Deutsche Telekom Network is chosen as a real-life network topology. Deutsche Telekom (DT) Network consists of 14 nodes and 23 edges and the map of the DT graph can be seen in Figure 5.9. Computational tests are conducted on DT

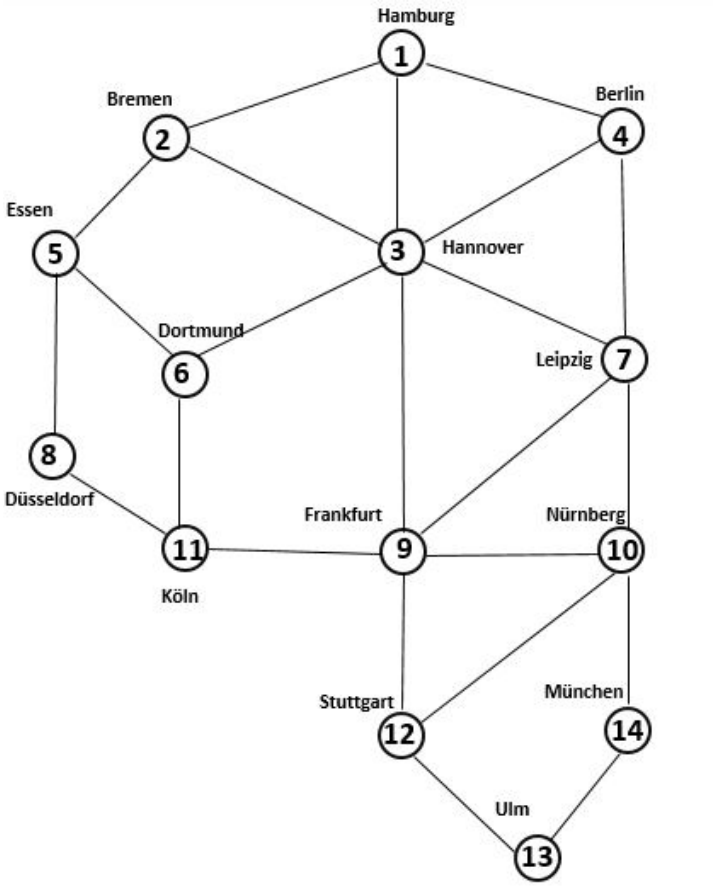


Figure 5.9: Deutsche Telekom Network

network for $D = \{12, 15, 20, 25, 27\}$ traffic loads. For each traffic load $d \in D$, 5 instances are created with demand sizes generated from the distribution Uniform(0,50). CPLEX code for ILP model which creates random demand attributes on DT network topology can be seen in Appendix B.2. Run time values of ILP, LCS algorithm and ICPA algorithm are calculated for traffic loads with identical

demand attributes (source, sink and demand sizes). The results of the experiments can be seen in Table 5.5. The first three columns in Table 5.5 gives the instance, node number in the graph and the demand number used as traffic load. "ILP Run Time" column gives the solution time values for the ILP formulation in CPU seconds. "LCS Algorithm Run Time" and "ICPA Algorithm Run Time" columns denote the run time values in CPU seconds to obtain a solution from LCS and ICPA algorithms respectively. "ILP Objective Value" column gives the exact solution values (or the gap from optimality declared by CPLEX) and "LCS Algorithm Objective Value" and "ICPA Algorithm Objective Value" columns give the solution values of the heuristic algorithms. "LCS Gap" and "ICPA Gap" columns are the optimality gaps of heuristic solutions from the exact solution of ILP. To calculate the percentage gaps in the last two columns, formulas of $\frac{\text{“}LCSAlgorithmObjectiveValue\text{”}}{\text{“}ILPObjectiveValue\text{”}} - 1$ and $\frac{\text{“}ICPAAlgorithmObjectiveValue\text{”}}{\text{“}ILPObjectiveValue\text{”}} - 1$ are used.

INSTANCE	NETWORK TOPOLOGY (DEUTSCHE TELEKOM)	DEMAND NUMBER (D)	ILP RUN TIME	LCS ALGORITHM RUN TIME	ICPA ALGORITHM RUN TIME	ILP OBJ VALUE	LCS ALGORITHM OBJ VALUE	ICPA ALGORITHM OBJ VALUE	LCS GAP	ICPA GAP
1	14	12	1,96	0,24	0,51	75	99	90	32,00%	20,00%
2	14	12	2,33	0,22	0,42	76	129	103	69,74%	35,53%
3	14	12	0,76	0,17	0,33	46	66	52	43,48%	13,04%
4	14	12	1,77	0,22	0,38	61	96	95	57,38%	55,74%
5	14	12	2,31	0,19	0,35	78	122	122	56,41%	56,41%
1	14	15	3,80	0,23	0,51	61	82	73	34,43%	19,67%
2	14	15	4,60	0,21	0,50	80	95	94	18,75%	17,50%
3	14	15	29,59	0,21	0,46	76	101	101	32,89%	32,89%
4	14	15	28,84	0,26	0,53	79	109	114	37,97%	44,30%
5	14	15	7,85	0,23	0,44	76	76	110	0,00%	44,74%
1	14	20	885,87	0,35	0,52	75	103	117	37,33%	56,00%
2	14	20	28,32	0,29	0,77	116	132	117	13,79%	0,86%
3	14	20	1974,20	0,26	0,68	106	194	130	83,02%	22,64%
4	14	20	28,61	0,33	0,66	87	138	135	58,62%	55,17%
5	14	20	315,88	0,37	0,49	64	111	118	73,44%	84,38%
1	14	25	117,52	0,35	0,86	112	156	130	39,29%	16,07%
2	14	25	2028,93	0,43	0,93	122	208	202	70,49%	65,57%
3	14	25	3253,70	0,38	0,80	138	175	146	26,81%	5,80%
4	14	25	1301,89	0,38	0,86	90	145	133	61,11%	47,78%
5	14	25	809,84	0,44	0,94	110	199	187	80,91%	70,00%
1	14	27	960,59	0,46	1,01	151	197	277	30,46%	83,44%
2	14	27	3685,07	0,47	1,06	124 (gap23,58%)	180	136	45,16%	9,68%
3	14	27	3602,06	0,41	0,83	105 (gap 8,65%)	170	156	61,90%	48,57%
4	14	27	3602,78	0,39	0,85	115 (gap 21,05%)	181	162	57,39%	40,87%
5	14	27	3601,81	0,40	0,96	95 (gap 13,83%)	152	144	60,00%	51,58%

Table 5.5: Test Results: Run Time and Objective Function Values for Deutsche Telekom Network Topology with Connection Requests of 12,15,20,25 and 27 Demands

5.2.1 Performance Analysis for $ILP_{currentwork}$ and Heuristic Algorithms

To test the performance of our proposed ILP formulation, we used test scenarios with the same dimensions as [9]: Deutsche Telekom network topology and $D = \{12, 15, 20\}$ traffic loads. [9]'s experiments show that the average run time values are 21,89 seconds for 12 demands, 65,73 seconds for 15 demands and 896,02 seconds for 20 demands (detailed values for $ILP_{[9]}$ can be seen in Appendix A.1, results that took more than 3600 seconds are excluded while calculating average solution times). The formulation of [9] could not produce solutions for demands beyond 25. As there is no further information on the source, sink and demand size values for the demands used in instances, we could not test the performance of [9]'s and our ILP formulation on the same set of demands; hence we use the average run time values for comparison purposes. Results of performance analysis on Deutsche Telekom Network showed that our formulation can find quite fast solutions compared to [9] in the same dimension of network topology and total demand numbers. As an outcome of our computational tests, we can see that our formulation has lower average run time values compared to [9] and it can also produce solutions for 25 commodities within 3600 CPU seconds time limit.

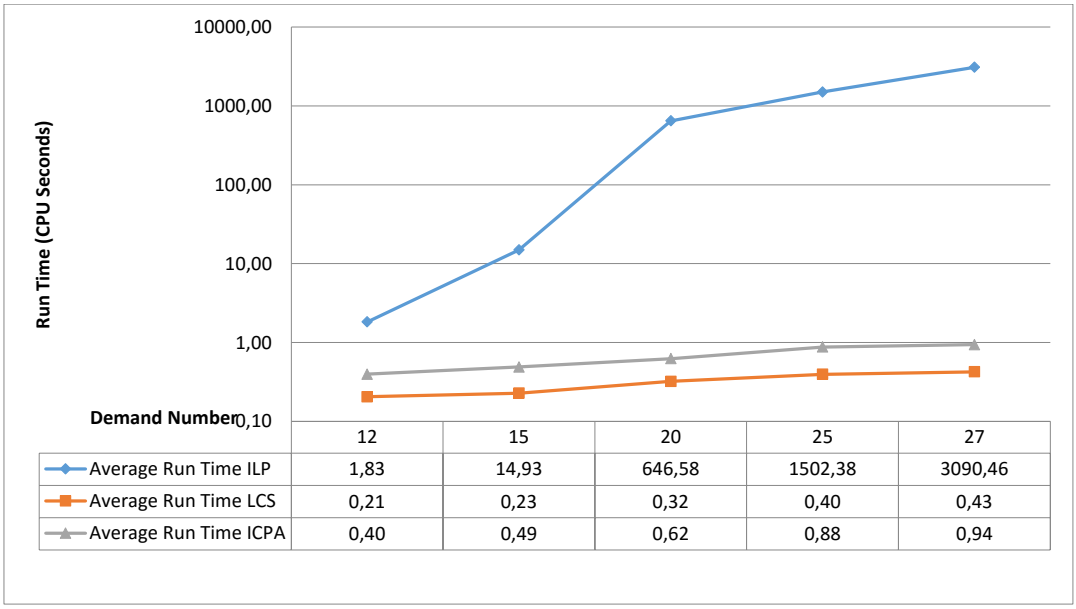
Only 1 out of 5 instances resulted in a solution within 3600 seconds for 27 commodities, hence the upper bound that our formulation can produce solutions on Deutsche Telekom network is determined as 25 commodities. Computational results show that average run times for low load cases (demand numbers up to 20 commodities) are within executable limits and solutions can be obtained on ILOG CPLEX. For high load cases (demand numbers from 20 commodities and beyond) ILP formulation can still produce results within 3600 CPU time limit boundaries up to 25 commodities. A summary table of the results in Table 5.5 consisting of average run time comparisons for $ILP_{currentwork}$ and heuristic algorithms can be seen in Table 5.6 and Figure 5.10 shows the graphical representation of $ILP_{currentwork}$, Least Cost Slot and Iterative Common Path Allocation algorithm values.

DEMAND NUMBER	AVERAGE RUN TIME ILP_currentwork (CPU SECONDS)	AVERAGE LCS ALGORITHM RUN TIME (CPU SECONDS)	AVERAGE ICPA ALGORITHM RUN TIME (CPU SECONDS)	Average Gap from Optimality LCS	Average Gap from Optimality ICPA
12	1,83	0,21	0,40	51,80%	36,14%
15	14,93	0,23	0,49	24,81%	31,82%
20	646,58	0,32	0,62	53,24%	43,81%
25	1502,38	0,40	0,88	55,72%	41,04%
27	3090,46	0,43	0,94	50,98%	46,83%

Table 5.6: Comparison of average run time values for ILP Formulations and Heuristic Algorithms for demand loads of 12, 15, 20, 25 and 27 commodities.

As a result of computational tests, it can be stated that our ILP formulation, $ILP_{currentwork}$, finds a solution faster than the formulations proposed by [9] and [10] and also can produce results for higher loads of traffic.

Heuristic algorithms LCS and ICPA find solutions very fast compared to ILP formulation, however there are large gaps from optimality for both algorithms. From the results it can be seen that LCS or ICPA cannot be used for finding close solutions to optimum results on their own, but can produce good initial solutions for the ILP model in order to limit initial search and to reduce run time values. In the next section, we analyze the impact of providing initial solutions obtained from our proposed heuristic algorithms to our ILP formulation.



	12	15	20	25	27
Average Run Time ILP	1,83	14,93	646,58	1502,38	3090,46
Average Run Time LCS	0,21	0,23	0,32	0,40	0,43
Average Run Time ICPA	0,40	0,49	0,62	0,88	0,94

Figure 5.10: Average run time of ILP Formulation and Heuristic Algorithms for traffic loads of 12, 15, 20, 25 and 27 commodities.

5.2.2 Performance Analysis for LCS+ILP and ICPA+ILP

In this section, we design an alternative experiment to test the effects of providing an initial solution to ILOG CPLEX on achieving the optimum solution. We use DT network topology and the exact same source-sink and demand size values used in the previous section as inputs for ILP model and heuristics+ILP model combinations. In order to be able to use the same data sets with the previous section as inputs for Matlab and ILOG, we used Excel as an interface between these two programs. In Table 5.7, "ILP Run Time(CPU Seconds)" column gives the solution time value for the ILP model where the attributes of demand; *source, sink and demand size* values are created internally in the model definition. The values in this column are the values in Table 5.5 and are included in this table for comparison of the effects of defining demand attributes internally in a model file versus externally through an Excel file. "ILP Run Time (input from Excel)" column gives the solution time value for the same ILP model except

with the difference that the same demand attributes are read from an Excel file. "LCS+ILP Run Time" column gives the solution time for the model where a solution is found by LCS algorithm and it is read as an initial solution along with the demand attributes from an Excel file by the ILP model, and "ICPA+ILP Run Time" column gives the solution time for the model where a solution is found by ICPA algorithm and it is read as an initial solution along with the demand attributes from an Excel file by the ILP model. "ILP Obj Value", "LCS+ILP Obj Value", "ICPA+ILP Obj Value" columns give the objective function values achieved by the models in "ILP Run Time (input from Excel)", "LCS+ILP Run Time", "ICPA+ILP Run Time" columns. The last two columns denote the decrease in run time values for Heuristic+ILP combinations. Note that all for ILP(input from Excel), LCS+ILP and ICPA+ILP columns, demand attributes are read from the Excel file; hence the percentage decrease in run time values are in reference to ILP(input from Excel), not the ILP formulation results in the first column where demand attributes are defined internally in the model. For instances of demand number 27, reading data from Excel increases the run time drastically (beyond 7000 seconds), hence in the objective function value of these instances, we provide percentage gaps from optimality reported by CPLEX at 3600 CPU seconds in the respective objective function value columns.

INSTANCE	NETWORK TOPOLOGY (DEUTSCHE TELEKOM)	DEMAND NUMBER (D)	ILP RUN TIME (CPU Seconds)	ILP RUN TIME (input from Excel)	LCS+ ILP RUN TIME (CPU Seconds)	ICPA+ILP RUN TIME (CPU Seconds)	ILP OBJ VALUE	LCS+ILP OBJ VALUE	ICPA+ILP OBJ VALUE	LCS+ILP Decrease in Run Time (%)	ICPA+ILP Decrease in Run Time(%)
1	14	12	1,96	3,58	1,98	2,78	75	75	75	44,69%	22,35%
2	14	12	2,33	3,80	4,30	2,24	76	76	76	-13,16%	41,05%
3	14	12	0,76	1,16	0,25	0,91	46	46	46	78,45%	21,55%
4	14	12	1,77	2,35	1,02	0,77	61	61	61	56,60%	67,23%
5	14	12	2,31	3,27	1,55	1,53	78	78	78	52,60%	53,21%
1	14	15	3,80	4,08	3,03	3,42	61	61	61	25,74%	16,18%
2	14	15	4,60	3,37	2,04	4,42	80	80	80	39,47%	-31,16%
3	14	15	29,59	101,19	50,25	47,75	76	76	76	50,34%	52,81%
4	14	15	28,84	25,28	26,98	31,69	79	79	79	-6,72%	-25,36%
5	14	15	7,85	9,91	4,34	4,73	76	76	76	56,21%	52,27%
1	14	20	885,87	2112,52	361,52	660,53	75	75	75	82,89%	68,73%
2	14	20	28,32	453,92	239,51	397,12	116	116	116	47,24%	12,51%
3	14	20	1974,20	2759,08	954,63	1932,38	106	106	106	65,40%	29,96%
4	14	20	28,61	78,19	79,72	45,62	87	87	87	-1,96%	41,65%
5	14	20	315,88	685,99	493,91	301,84	64	64	64	28,00%	56,00%
1	14	25	117,52	227,27	209,75	191,99	112	112	112	7,71%	15,52%
2	14	25	2028,93	3138,75	1890,11	2218,51	122	122	122	39,78%	29,32%
3	14	25	3253,70	5466,21	3943,32	3075,28	138	138	138	27,86%	43,74%
4	14	25	1301,89	3619,25	2043,55	2371,58	90	90	90	43,54%	34,47%
5	14	25	809,84	2952,38	1358,09	1814,53	110	110	110	54,00%	38,54%
1	14	27	960,59	7000+	7000+	7000+	gap 40,52%	gap 40,13%	gap 42,11%	-	-
2	14	27	3685,07	7000+	7000+	7000+	gap 35,43%	gap 34,13%	gap 34,4%	-	-
3	14	27	3602,06	7000+	7000+	7000+	gap 13,21%	gap 8,49%	gap 8,49%	-	-
4	14	27	3602,78	7000+	7000+	7000+	gap 28,12%	gap 20,69%	gap 20,69%	-	-
5	14	27	3601,81	7000+	7000+	7000+	gap 31,25%	gap 31,24%	gap 36,63%	-	-

Table 5.7: Test results of run time values for ILP and heuristic algorithms and decrease in run time values for heuristics+ILP combinations. Demand loads of 12, 15, 20, 25 and 27 commodities are used for tests. Run time value 7000+ means that the results took more than 7000 CPU seconds and were automatically killed by the system.

From the results in the Table 5.7, it can be seen that the run time values between "ILP Run Time" and "ILP Run Time (input from Excel)" columns are different, but the objective function values in Tables 5.7 and 5.5 are the same. The reason for the difference in run time values is that in the "ILP Run Time(CPU Seconds)" column, demand attributes (source, sink and demand size values) are created as internal data by *execute command*, but in "ILP Run Time (input from Excel)" the same demand sets with identical source, sink and demand size values are read from Excel as external data. Reading data from an internal source vs. from an external .dat source such as Excel creates differences in terms of run times, as "internal data elements are pulled from OPL as needed and are not allocated any memory" but "external data elements are pushed to OPL and allocated memory whether it is used by the model or not" [15].

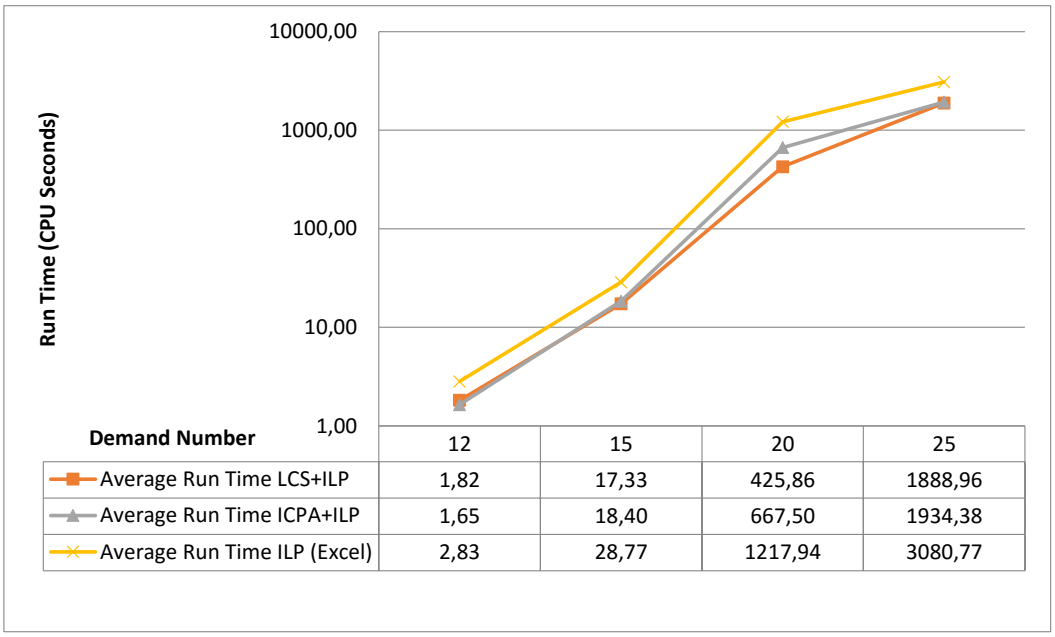
Although reading data from Excel increases run time values, providing an initial solution found via heuristic algorithms LCS and ICPA to the $ILP_{currentwork}$ on ILOG decreases run time values significantly. Note that at each iteration, the same objective function value is found for ILP, LCS+ILP and ICPA+ILP. A summary table of average run time values for ILP models with data pulled from "internal" and "external" sources and initial solutions provided by LCS and ICPA algorithms as well as average percentage decrease in run time values can be seen in Table 5.8.

DEMAND NUMBER	Average Run Time ILP_currentwork (CPU Seconds)	Average Run Time ILP_currentwork (input from Excel)	Average Run Time LCS+ILP (CPU Seconds)	Average Run Time ICPA+ILP (CPU Seconds)	Average Decrease in Run Time LCS+ILP	Average Decrease in Run Time ICPA+ILP
12	1,83	2,83	1,82	1,65	35,73%	41,88%
15	14,93	28,77	17,33	18,40	39,76%	36,03%
20	646,58	1217,94	425,86	667,50	65,03%	45,19%
25	1502,38	3080,77	1888,96	1934,38	38,69%	37,21%

Table 5.8: Test results of average run time values for ILP and heuristic algorithms and percentage decrease in run time values for heuristics+ILP combinations.

Despite the fact that ICPA gives solutions closer to the optimum than LCS, for larger network topologies than 15 nodes, the solution obtained from LCS provides a better initial solution for the ILP model than ICPA in terms of the percentage of decrease in run time. For small network topologies (12 nodes and less), solving

the problem on CPLEX already provides fast solutions; hence applying a heuristic for an initial solution is not necessary. In general, the combination of LCS+ILP provides an average of 44.8% decrease in run time and ICPA+ILP gives an average of 40% decrease when compared to solving the problem on CPLEX directly. As reading the data from Excel increased the solution times beyond 7000 CPU seconds for traffic loads of 27 and higher, we provide the percentage optimality gap reported by CPLEX at 3600 CPU seconds. From the results of traffic load of 27, it can be concluded that LCS+ILP combination decreases optimality gap by an average of 2,77% and ICPA+ILP decreases the optimality gap by an average of 1,24% around 3600 seconds. Although not tested within this thesis, a further investigation can be made on the percentage decrease in run times for traffic loads beyond 27 when initial solutions are defined manually in CPLEX formulation as internal data, rather than reading them externally from Excel. Another possible area of research could be the combination of AFA-CA heuristic algorithm of [14] with our proposed $ILP_{currentwork}$ and to test the run time values for higher traffic loads.



Demand Number	12	15	20	25
Average Run Time LCS+ILP	1,82	17,33	425,86	1888,96
Average Run Time ICPA+ILP	1,65	18,40	667,50	1934,38
Average Run Time ILP (Excel)	2,83	28,77	1217,94	3080,77

Figure 5.11: Graph of average run time with an initial solution provided: LCS+ILP and ICPA+ILP algorithms.

Chapter 6

Conclusion

The development of OFDM technology in optical networks provides huge spectrum efficiencies while transmitting connections and offers a novel area of interdisciplinary research for many branches of engineering. Motivated by the possible improvements on application of graph theory on OFDM based fiber optic networks, we investigate the Routing and Spectrum Allocation problem in optical networks in this thesis. We consider two cases: in one setting, allocation to the spectrum are made by taking a set of precomputed routes into consideration and in the other setting, we span the whole solution space without taking a predefined set of routes into consideration. We present an improved formulation for the RSA problem with predefined set of paths and propose a novel global formulation, RSA without predefined set of paths, that reaches to the global optimum.

We use 7 different computational experiment settings to test the performance of our ILP formulation and heuristic algorithms. In our settings, we use 5 different network topologies consisting of N=$\{8, 14, 25, 50, 100\}$ nodes–each generated randomly for every iteration. We test for the limitations of traffic load that can be handled on these networks and also test the effect of using different sparsity levels for network topologies and their impact on achieving a solution in terms of longevity of run time values. In the last 2 sets of experiments, we use a real-life network topology–Deutsche Telekom network consisting of 14 nodes and 23

edges– to test the amount of traffic load that can be handled and the speed of our ILP formulation. We also test the run time and results generated by our proposed heuristic algorithms, LCS and ICPA, on DT network topology. Lastly, we examine the amount of decrease on run time values when an initial solution from the results we obtain from our heuristic algorithms is provided as an input to our ILP formulation.

From the performance analysis tests we have conducted, it can be concluded that our ILP formulation provides faster solutions compared to formulations that have run time analyses in literature and also it can process much larger networks with higher traffic loads. Our heuristic algorithms could not produce results as close to optimal solution as desired, but the hybrid formulation of LCS+ILP and ICPA+ILP create an average of 44% and 40% decrease in run time values of our ILP formulation, which provides opportunity for exploration of larger networks and denser traffic loads.

There is no previous work in literature that investigates the effects of sparsity of graphs on distribution of demands in optical networks. As a result of our experimental analysis on sparsity of optical networks, it can be stated that solving RSA problem on fully connected networks generally produces better outputs. For very sparse networks where around 25% of all connections are present compared to fully connected networks, increasing traffic load often yields in infeasibility. For networks with average connectivity, having a network with 50% of all connections present generally yields in faster solutions when compared to 75% of edges being present.

For further study, we propose extensions to the Routing and Spectrum Allocation problem. AFA-CA algorithm of [14] provides solutions with small optimality gaps. For high traffic networks and large networks, combining AFA-CA's initial solutions to our $ILP_{currentwork}$ that finds the global optimum on static fiber optic networks could produce global solutions faster than our proposed algorithms. As an additional study, it would be interesting to investigate possible solution methods for the dynamic RSA problem, with demands allocated on spectrum as connection requests arrive into the system.

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Appendix A

Tables

INSTANCE	NETWORK TOPOLOGY	DEMAND SIZE	[9] ILP RUN TIME	[9] ILP OBJ VALUE
1	14	12	1,17	48
2	14	12	6,48	50
3	14	12	7,19	66
4	14	12	8,67	84
5	14	12	57,99	58
6	14	12	23,8	50
7	14	12	0,57	50
8	14	12	11,76	80
9	14	12	23,33	66
10	14	12	77,98	59
1	14	15	26,12	51
2	14	15	32,37	44
3	14	15	16,22	94
4	14	15	427,55	67
5	14	15	9,37	50
6	14	15	15,68	84
7	14	15	17,82	67
8	14	15	11,36	45
9	14	15	22,54	56
10	14	15	78,22	86
1	14	20	82,41	45
2	14	20	449,11	67
3	14	20	227,98	70
4	14	20	3600,93	100
5	14	20	2254,39	75
6	14	20	1573,12	71
7	14	20	3600,8	87
8	14	20	690,92	76
9	14	20	915,77	83
10	14	20	974,45	64

Table A.1: Run time and objective function value results for experiments conducted on DT network in [9]. Results are taken from [9] directly.

Appendix B

Code

```
/*********************************************
 * OPL 12.6.0.0 Model
 * Author: Pelin Öner
 * Creation Date: 18 Ekim 2015 at 14:04
 *********************************************/
int numCities =100;                    //number of nodes present in the graph
range Cities = 1..numCities;
int demandNum =30;                     //number of connection requests
(demands)
float density= 0.5;            //sparsity coefficient
float myseed;
int upperbnddemandsize = 50;  //upper bound to determine demand size. For
visible spectrum 25600 slots are available at 12,5 GHz slot wide each

// Defining connections(edges) and routes associated with demands
 tuple arc { int i; int k;}
 tuple route {                    //demand information: specifies demand name
(as d), and source-destination pair
  int d;
  int source;
  int destination; }
{route} Routes ;
{arc} Arcs ;

{int} Demand ={d | <d,s,t> in Routes};
{int} Source [d in Demand] = {s | <d,s,t> in Routes };
{int} Destination [d in Demand] = {t | <d,s,t> in Routes };

//Defining parameters
float T[Demand] ;       //demand size: number of slots required for a
connection request d
float G = 1;                  //number of slots required as guardband
float M = sum(d in Demand)(T[d]+G);             //number of slots available in
the spectrum (given)

//Defining variables
dvar boolean X [Demand][Cities][Cities];       //decides on which arc should
the demand be sent on
dvar boolean W [Demand][Demand];               //if a shared arc exists
between two nodes, decides which demand is placed first
dvar int S [Demand];                                    //slot number of the
demand d assigned in the spectrum
dvar int sls;                                                 //for finding the
min number of slots required for feasible allocation


//Creating random arcs, routes and demand information
execute RandomGraph{
      var now = new Date();
      myseed = Opl.srand(Math.ceil(now.getTime()/1000));

      var d = 1;                                          //this part is used for
creating connection requests with random information
      while(d<=demandNum){
            var i = Opl.rand(numCities+1);
            var j = Opl.rand(numCities+1);
            if(i!=j && i!=0 && j!=0){
```

```
                Routes.add(d,i,j);
                d++;}
      }

      for(var d=1; d<=demandNum; d++){              //used for creating
random demand size
            T[d]= Opl.rand(upperbnddemandsize+1);
      }

      for(var i = 1; i <= numCities; i++){          //used for creating
random graph topologies
            for(var j = 1; j <= numCities; j++){
                var p = Math.random();
                if( p<=density && i != j){
                      Arcs.add(i,j);                    // Arc (i,j) is
generated
                }
            }
      }
}

//Model
minimize sls;
subject to{
      //routing equation 2: source node
        forall(d in Demand, i in Cities : i in Source[d])
                sum(j in Cities: <i, j> in Arcs) X[d][i][j] - sum(j in
Cities: <j, i> in Arcs) X[d][j][i] == 1;
      //routing equation 1: in between nodes
        forall (d in Demand, i in Cities: i not in Source[d] && i not in
Destination [d])
                sum(j in Cities: <i,j> in Arcs)X[d][i][j] - sum(j in Cities:
<j,i> in Arcs)    X[d][j][i] == 0;
      //routing equation 3: destination node
        forall(d in Demand, i in Cities : i in Destination[d] )
                sum(j in Cities: <i,j> in Arcs) X[d][i][j] - sum(j in
Cities: <j,i> in Arcs) X[d][j][i] == -1;

      //lowerbound for total slot number required; sls
      forall (d in Demand){
        S[d]+T[d]<=sls;
        S[d]>=0;
}

      //slot assignment equations for each demand
      forall (i,j in Cities, d1 in Demand, d2 in Demand: d1!=d2 && (<i,j> in
Arcs || <j,i> in Arcs))
       {
         W[d1][d2] + W[d2][d1] >= (X[d1][i][j] + X[d1][j][i]+ X[d2][i][j] +
X[d2][j][i])- 1;
         W[d1][d2] + W[d2][d1] <= 1;
         S[d1] + T[d1] + G <= S[d2] + M*(1-W[d1][d2]);
         S[d2] + T[d2] + G <= S[d1] + M*(1-W[d2][d1]);
       }
}
```

Figure B.1: CPLEX code for Integer Linear Programming Model of Routing and Spectrum Allocation Problem with random network topology

```
/*********************************************
 * OPL 12.6.0.0 Model
 * Author: Pelin Öner
 * Creation Date: 20 Eki 2015 at 16:35:22
 *********************************************/
int numCities = 14;                       //number of nodes present in the
graph
range Cities = 1..numCities;
int demandNum =20;                        //number of connection requests
(demands)
float myseed;
float temp;
int upperbnddemandsize = 50;  //upper bound to determine demand size. For
visible spectrum 25600 slots are available at 12,5 GHz slot wide each

// Defining connections(edges) and routes associated with demands
 tuple arc { int i; int k;}
 tuple route {                    //demand information: specifies demand name
(as d), and source-destination pair
  int d;
  int source;
  int destination; }
{route} Routes ;
{arc} Arcs ;

{int} Demand ={d | <d,s,t> in Routes};
{int} Source [d in Demand] = {s | <d,s,t> in Routes };
{int} Destination [d in Demand] = {t | <d,s,t> in Routes };

//Defining parameters
float T[Demand] ;        //demand size: number of slots required for a
connection request d
float G = 1;                    //number of slots required as guardband
float M = sum(d in Demand)(T[d]+G);             //number of slots available in
the spectrum (given)

//Defining variables
dvar boolean X [Demand][Cities][Cities];       //decides on which arc should
the demand be sent on
dvar boolean W [Demand][Demand];               //if a shared arc exists
between two nodes, decides which demand is placed first
dvar int S [Demand];                                  //slot number of the
demand d assigned in the spectrum
dvar int sls;                                            //for finding the
min number of slots required for feasible allocation

//Creating arcs, routes and random demand information
execute RandomGraph{
      var now = new Date();
      myseed = Opl.srand(Math.ceil(now.getTime()/1000));

      var t12 = Arcs.add(1,2);      //arcs are added to the graph according to
Deutsche Telekom Network Topology
      var t21 = Arcs.add(2,1);
      var t14 = Arcs.add(1,4);
      var t41 = Arcs.add(4,1);
      var t13 = Arcs.add(1,3);
```

```
	var t31 = Arcs.add(3,1);
	var t23 = Arcs.add(2,3);
	var t32 = Arcs.add(3,2);
	var t34 = Arcs.add(3,4);
	var t43 = Arcs.add(4,3);
	var t25 = Arcs.add(2,5);
	var t52 = Arcs.add(5,2);
	var t36 = Arcs.add(3,6);
	var t63 = Arcs.add(6,3);
	var t47 = Arcs.add(4,7);
	var t74 = Arcs.add(7,4);
	var t37 = Arcs.add(3,7);
	var t73 = Arcs.add(7,3);
	var t39 = Arcs.add(3,9);
	var t93 = Arcs.add(9,3);
	var t56 = Arcs.add(5,6);
	var t65 = Arcs.add(6,5);
	var t58 = Arcs.add(5,8);
	var t85 = Arcs.add(8,5);
	var t811 = Arcs.add(8,11);
	var t118 = Arcs.add(11,8);
	var t611 = Arcs.add(6,11);
	var t116 = Arcs.add(11,6);
	var t119 = Arcs.add(11,9);
	var t911 = Arcs.add(9,11);
	var t79 = Arcs.add(7,9);
	var t97 = Arcs.add(9,7);
	var t710 = Arcs.add(7,10);
	var t107 = Arcs.add(10,7);
	var t910 = Arcs.add(9,10);
	var t109 = Arcs.add(10,9);
	var t912 = Arcs.add(9,12);
	var t129 = Arcs.add(12,9);
	var t1210 = Arcs.add(12,10);
	var t1012 = Arcs.add(10,12);
	var t1213 = Arcs.add(12,13);
	var t1312 = Arcs.add(13,12);
	var t1314 = Arcs.add(13,14);
	var t1413 = Arcs.add(14,13);
	var t1410 = Arcs.add(14,10);
	var t1014 = Arcs.add(10,14);

	var d = 1;                                  //this part is used for
creating connection requests with random information
	while(d<=demandNum){
		var i = Opl.rand(numCities+1);
		var j = Opl.rand(numCities+1);
		if(i!=j && i!=0 && j!=0){
			Routes.add(d,i,j);
			d++;}
	}
	for(var d=1; d<=demandNum; d++){               //used for creating
random demand size
		T[d]= Opl.rand(upperbnddemandsize+1);
	}
}
```

```
//Model
minimize sls;
subject to{
      //routing equation 2: source node
        forall(d in Demand, i in Cities : i in Source[d])
                  sum(j in Cities: <i, j> in Arcs) X[d][i][j] - sum(j in
Cities: <j, i> in Arcs) X[d][j][i] == 1;
      //routing equation 1: in between nodes
        forall (d in Demand, i in Cities: i not in Source[d] && i not in
Destination [d])
                  sum(j in Cities: <i,j> in Arcs)X[d][i][j] - sum(j in Cities:
<j,i> in Arcs)    X[d][j][i] == 0;
      //routing equation 3: destination node
        forall(d in Demand, i in Cities : i in Destination[d] )
                  sum(j in Cities: <i,j> in Arcs) X[d][i][j] - sum(j in
Cities: <j,i> in Arcs) X[d][j][i] == -1;

      //lowerbound for total slot number required; sls
      forall (d in Demand){
        S[d]+T[d]<=sls;
        S[d]>=0;
}

      //slot assignment equations for each demand
      forall (i,j in Cities, d1 in Demand, d2 in Demand: d1!=d2 && (<i,j> in
Arcs || <j,i> in Arcs))
       {
         W[d1][d2] + W[d2][d1] >= (X[d1][i][j] + X[d1][j][i]+ X[d2][i][j] +
X[d2][j][i])- 1;
         W[d1][d2] + W[d2][d1] <= 1;
         S[d1] + T[d1] + G <= S[d2] + M*(1-W[d1][d2]);
         S[d2] + T[d2] + G <= S[d1] + M*(1-W[d2][d1]);
       }
}
```

Figure B.2: CPLEX code for Integer Linear Programming Model of Routing and Spectrum Allocation Problem with Deutsche Telekom Network Topology