ROBUST AIRLINE SCHEDULING WITH CONTROLLABLE CRUISE TIMES AND CHANCE CONSTRAINTS

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

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July, 2012

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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July, 2012

This is a study on robust airline scheduling where flight block times are considered in two parts as cruise time and non-cruise time. Cruise times are controllable and non-cruise times are random variables. Cruise time controllability is used together with idle time insertion to handle uncertainty to guarantee passenger connection service levels while ensuring minimum costs. The nonlinearity of these cost functions are handled by representing them via second order conic inequalities. The uncertainty in non-cruise times are modeled through chance constraints on passenger connection service levels, which are expressed using second order conic inequalities using the closed form equations. Congestion levels of origin and destination airports are used to decide variability for each flight. Computational study shows exact solutions can be obtained by commercial solvers in seconds for a single hub schedule and in minutes for a 4-hub daily schedule of a major US carrier.

Keywords: chance constraints, congestion, airline scheduling, cruise time controllability, passenger connections, service level.

ÖZET

ŞANS KISITLI VE DENETLENEBİLİR UÇUŞ SÜRELERİNE SAHİP DAYANIKLI HAVAYOLU ÇİZELGELEME MODELİ

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Bu çalışma uçuş blok zamanlarının seyir süresi ve seyir dışı süre olarak iki kısımda incelendiği dayanıklı bir çizelgeleme üzerinedir. Seyir zamanları kontrol edilebilir karar değişkenleriyken seyir dışı süreler rassal değişkenler olarak alınmıştır. Seyir zamanlarındaki kontrol edilebilirlik ve boş süre yerleştirme beraber kullanılarak belirsizlikleri dengelemek ve yolcu bağlantı hizmet seviyelerini en düşük maliyetle garanti altına almak amaçlanmaktadır. Doğrusal olmayan maliyet fonksiyonları ve şans kısıtları ikinci dereceden konik eşitsizlikler ile ifade edilerek eniyi çözümler hızlıca elde edilebilmiştir. Büyük bir Amerikan havayolları için yapılan sayısal hesaplamalar, tek ana üslü çizelge için saniyeler, 4 üslü çizelge içinse dakikalar içinde çözüm elde edilebildiğini göstermiştir.

Anahtar sözcükler: şans kısıtları, uçuş çizelgeleme, dayanıklı çizelgeleme, gürbüz çizelgeleme, konik eşitsizlikler.

Acknowledgement

First of all, I wish to thank my thesis supervisors Prof. Selim Aktürk and Asst. Prof. Sinan Gürel for their time, help and patience. I consider it an honor to work with them and this thesis would not have been possible without their guidance.

I also would like to acknowledge the financial support of The Scientific and Technological Research Council of Turkey (TUBITAK) for the Graduate Study Scholarship Program they awarded.

I am truly indebted and thankful to my family for their constant support and encouragement throughout my life and my education. They always believed in me, even at times that I did not.

I would like to thank all my classmates for all the good times we shared while getting through our first experience of graduate school. Special thanks go to my office friends Fırat, Görkem and Feyza for they have enriched my time at Bilkent University in a way that cannot be expressed by words.

Last but not least, I want to thank Efe for his constant support, for cheering me up during the stressful times and for bearing with all my mood swings.

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Chapter 1

Introduction

Using operations research tools in airline industry became popular after the deregulation of US airline industry which resulted in a high competition among carriers. Companies started to lose money when trying to keep up with the prices of low-cost carriers and consequently they started to employ operations research methods to increase their profits. Airline scheduling is one of the major operations research tools that is currently being used in airline industry.

An airline schedule provides information for a specified set of flights such as origin and destination, the arrival and departure times or the assigned aircraft and crew. Developing schedules for airline operations is a challenging mathematical programming problem considering the competitive environment, operations consisting of many steps and expensive resources. One of the major challenges in airline scheduling models is that problem sizes are very large. Considering the schedule generation, fleet assignment, crew assignment and passenger itineraries in a single model will necessitate the use of millions of variables and constraints.

Generating flight schedules is not enough. There are many disruptions that occur which cause operational delays and decrease schedule performances. Examples to disruptions can be given as problems with aircraft, crew unavailability, gate shortages, security delays, unexpected delays during the loading of a plane, weather conditions or even natural disasters. All of these disruptions have different effects on the flight block times. Accurately reflecting the effect of a disruption on flight block times is not an easy task.

Another challenge is the managing of these disruptions. Alternative options have been developed but this is not an easy procedure. A delay resulting from a natural disaster may result in cancellation of all following flights making it impossible to continue the original schedule whereas a delay due to crew illnesses can sometimes be solved easily by using back-up crews or switching crew pairs. It is important for a disruption management model to include as many recovery options and alternatives as possible but this results in an increase in the problem complexity. Schedules that can handle these delays which are generated in reasonable time are needed. This makes it important to have flexible or robust schedules. Robust schedules are less vulnerable to disruptions and are easier to repair in case of a disruption.

Airline practitioners try to maintain this flexibility by inserting idle time into schedules. However, inserting more idle time than needed is not favorable since expensive equipment such as aircraft is kept idle. Another tool is to change the speed of the aircraft. Aircraft can fly faster to reduce the cruise time in exchange for increased fuel costs. However, these decisions are made locally and the propagation of delays are not taken into account in majority of cases. In this study, we develop a model that uses both idle time insertion and aircraft speed control to output a robust schedule of minimum costs that satisfy given passenger connection service levels. Thinking over all passenger connections also allows us to consider delay propagation. This thesis provides several major contributions to airline scheduling literature.

First of all, it is very important to take variability in block times into account when studying robust airline scheduling. Flight cruise times are not affected significantly by variability so we start by taking an initial schedule, where a flight block-time is considered in two parts: cruise time and non-cruise time. Non-cruise times are taken as random variables and the uncertainty is modeled through chance constraints. In this study, fast and exact solutions to this large size model of probabilistic constraints and non-linear cost components are provided. Chance constraints are transformed to second order conic inequalities from the closed form expressions for these probabilistic constraints. The nonlinearity arising from the cost function is also transformed using second order conic inequalities. Therefore, we are able to solve a nonlinear mixed integer model in reasonable computation time using commercial solvers like Cplex.

Another important contribution is incorporating origin and destination information of a flight when calculating non-cruise time variability. It is known that airport congestion levels are different than each other, and an aircraft taking off from a non-hub location spends lot less time for take off compared to an aircraft that originates from a hub location; with the same concept applying to landing times. Therefore, the variability of non-cruise times in this study are calculated separately for each flight, depending on the origin-destination pairs.

To continue with, idle times actually are very expensive to put into a schedule since aircrafts are not utilized during that time slot and a lot of revenue is lost. In most situations, it can be cheaper not to insert idle time to the schedule but cover the delay time by making the aircraft fly faster in exchange for increased fuel costs. In this thesis, uncertainty is covered through both idle time insertion and speed controllability to achieve minimum costs.

Moreover, in this study we superimpose the aircraft network and passenger connection network. Considering them together rather than having a sequential approach allows us to achieve more realistic results and more accurately evaluate the interaction between these two networks.

In the next chapter, an extensive literature review is provided. Detailed backgrounds on robustness, airline scheduling, disruption management techniques in airline scheduling, cruise time and fuel cost relationship and conic programming are given.

In Chapter 3, the problem environment is described. Extensive information on problem parameters and decision variables are given. The properties of the random variable in the model is described in detail. The structure of the service level decisions and the fuel cost function is analyzed. In addition, a numerical example is provided to explain how the model works on an example.

Chapter 4 is devoted to the problem formulation and the mathematical model. The closed form expression of the chance constraints and the conic reformulation of the model are also described in this chapter.

An extensive computational study is given in Chapter 5. In two separate sections, results for a single hub schedule and a 4-hub schedule of a major US carrier are discussed. Computation time analysis is done for the two schedules separately. Finally, the thesis is concluded in Chapter 6.

Chapter 2

Literature Review

In this chapter, a literature review on related research areas to this thesis will be provided. In the following sections, background information on robustness, airline scheduling, disruption management in airline operations, flight cruise time controllability and fuel cost information and second order cone programming are summarized.

2.1 Robustness

Robustness incorporated into a system tries to ensure the system performs well even if the conditions do not fit the previous assumptions and there are perturbations or uncertainty. However, it is not in stone what makes up for a robust solution. There are different metrics used to quantify the robustness of a solution and also different methodologies are used to incorporate robustness into a model.

Beyer and Sendhoff (2007) worked on a comprehensive survey on robust optimization. Their work includes information on methods to measure and evaluate robustness and the different approaches to robust optimization in literature. They also discuss benefits and shortcomings of the different methods. Bertsimas et al. (2011) also conducted an extensive study on robust optimization. In their work, they adress important issues such as tractability of robust optimization problems, the probability guarantees of problem solutions and the flexibility provided by robust optimization.

In our study, we use chance constraints to handle the uncertainty to output a robust schedule. There are many studies on chance constrained programming and researchers took many different approaches. Luedtke and Ahmed (2008) work on checking feasibility of regions defined by chance constraints by developing a Monte Carlo based sample approximation. Nemirovski and Shapiro (2006) develop computationally tractable convex approximations for the chance constrained problems. They extend their work to cases where the data distributions are not known exactly but belong to a convex compact set. Calafiore and Ghaoui (2006) discuss linear programming models for radial distributions of the data and for data that is known to belong to a given set of distributions.

2.2 Airline Scheduling Process

An airline scheduling process consists of a series of operations that follow each other. The first step in the process is the generation of an initial schedule which answers the questions of which markets to serve in what frequency. Then fleet assignment problem follows where each flight is assigned an aircraft type. The output information from this step is used in aircraft maintenance routing problem where the aircraft from airline's fleet is assigned to a flight considering the maintenance requirements. In the last step, crew assignment problem is considered to assign crew to each flight incurring minimal cost. Extensive information on flight operations of airlines can be found in Barnhart and Cohn (2004); Midkiff et al. (2009) for interested readers.

Considering all these steps in the scheduling process result in problems that are not manageable since considering the schedule generation, fleet assignment, maintenance routing, crew assignment and passenger itineraries in a single model will necessitate the use of millions of variables and constraints. Some researchers take a sequential approach to get optimization results that are closer to a full optimization model. Another approach is to combine several of these problems in a single integrated model to get better results. Various integrated models are introduced in Papadakos (2009) with compared solutions to classic approaches in literature. Still, solving these problems deterministically result in unforeseen operational costs since uncertainties such as delays and disruptions are not considered.

2.3 Disruption Management in Airline Operations

During the implementation of airline schedules, many disruptions are faced that compromise matching up with the initial schedule and result in operational delays. The continuous increase in fleet sizes, number of flights and number of passengers result in congestions which make the effects of delays very significant. With new destinations added each day, amount of passenger connections also grow and impacts of delay propagation cannot be avoided. All of these problems necessitate robust schedule generations that are less vulnerable to these delays, or recovery methods that help handling these delays in a short response time. An extensive review for irregular airline operations can be found in Barnhart (2009); Clausen et al. (2010). Two main methods for handling disruptions are robust planning and recovery models.

2.3.1 Robust Planning

Robust scheduling is a proactive scheduling model that is more flexible to schedule disruptions and offers a plan that reduces the impacts of a disruption in case one happens. Robust schedules can offer better use of resources and considerable cost savings for airlines. Ageeva and Clarke (2000) provide a wide study on how airline optimization problems can be made robust and suggests new methods for building robust schedules. Robust airline scheduling has been studied by many researchers with different metrics used to define and incorporate robustness into schedules.

Airlines put slack times into the schedules to ensure robustness most of the time. However, slack time is keeping expensive aircraft idle losing efficiency which is not preferred. Moreover, putting slack time simply without an overall analysis of the system fails to capture later effects of uncertainties and delays in the network. There are a few studies addressing the problem of slack distribution and its effects on schedule performances.

In the robust aircraft maintenance routing study of Lan et al. (2006), flight delays are categorized into two as propogated and nonpropogated delay. An aircraft routing is a sequence of flights flown by a single aircraft, so a delay in one of these flights propogates to the following if there is no slack time in between. The authors suggest that propogated delay can be reduced by assigning slack optimally to aircraft routings.

Chiraphadhanakul and Barnhart (2011) study a model that re-allocates existing schedule slack to achieve a more robust schedule. They propose alternative objective functions that result in more robust solutions with respect to different performance evaluation metrics. They use delay propogation and passenger delays as metrics to evaluate the resulting schedules. The study shows that minor schedule adjustments to the original schedule can result in significant overall schedule performance improvements.

Ahmadbeygi et al. (2010) conducted a study to reduce delay propogation by redistributing existing slack, while leaving the original fleeting and crew scheduling decisions unchanged. They show that re-allocating the existing slack to the flight connections that are most prone to delay propagation, downstream impacts can be reduced without changing planned crew or fleeting costs and exceeding planned budgets.

There are few other studies addressing the later affects of delays. Delay propagation for airline networks are analyzed and robustness measures are developed in Arıkan et al. (2012). They use a stochastic model that captures the randomness in the block-time of a flight and the propogation of this randomness through the flight network. Dunbar et al. (2012) developed a formulation to minimize propogated delay costs while integrating aircraft routing and crew pairing problems.

Schedule performances and affects of schedule delays on these are other areas of study under robust airline scheduling. Arıkan and Deshpande (2012) analyze the impact of scheduled block time to the on time performances. Burke et al. (2010) develop a multi-objective robust scheduling approach where they consider schedule reliability and schedule flexibility as two robust schedule objectives and show that increased flexibility and reliability improve on-time performances.

Various methods are used to capture uncertainty in flight times in the robust scheduling models. We use chance constraints to model the uncertainty, which is studied by few other researchers. Sohoni et al. (2011) take an alternative approach and model block-time distributions using chance constraints and perturb departure times of an initial schedule to achieve improved passenger and network service levels and also maximize operational profits. To solve the model, they develop linear approximations on chance constraints. Marla and Barnhart (2010) employ two approaches to robust airline optimization focusing on the aircraft routing problem, the extreme value-based approach and chance constrained programming approach and they provide trade-offs between the different models. Chance-constrained programming is an old technique appearing in the work of Charnes and Cooper (1959). In our study, we also model the variability using chance constraints.

2.3.2 Recovery Models

Recovery models or rescheduling models are more of a reactive scheduling measure that focuses on reoptimising a schedule after a disruption occurs. As airline transportation industry grows, frequency of disruptions increase and recovery decisions should be made in minutes of times, which makes this an area of importance. One major difficulty of recovery problems is to satisfy constraints such as maintenance requirements, balance requirements, crew union constraints and passenger itinerary constraints and to generate recovery options in seconds of time. Building robust schedules make it easier to recover a schedule in case of a disruption. A summary on recovery literature is provided for interested readers.

Thengvall et al. (2001) present multi-commodity network type models for constructing a recovery schedule for all aircraft operated by a large carrier following a hub closure. Rosenberger et al. (2003) modeled the aircaft recovery problem as a set packing problem where each flight leg is either included in one route or cancelled. As an addition, they also consider airport disruptions that occur for example when weather conditions change the capacity of a given airport. In the study of Eggenberg et al. (2010) a flexible model named constraint specific recovery network is introduced for solving airline recovery problems. A dynamic programming algorithm is used for recovery network generation and a column generation algorithm is used to solve the problem. A heuristic method to solve the aircraft recovery problem which involves reassignments of aircraft to flights, delaying of flights and cancellations of flights is discussed in Love et al. (2002).

Recovery and rescheduling models provide many options to handle disruptions after the disruptions occur but uncertainties can also be considered before they occur by robust planning. The study by Eggenberg (2009) combines robustness and recovery for airline schedules. The model results in schedules that are more robust and more recoverable than the original schedule, with lower recovery costs.

2.4 Cruise Time and Fuel Costs

Fuel costs make up the most of airline operational costs and airlines developed an evaluating system named as cost index to manage fuel costs. Cost index is defined as the ratio of time related cost per minute of flight of an airline operation to the cost of fuel per kg of fuel in Airbus (1998). The cost index provides a tool to control fuel burn and trip time between the two extremes of minimum fuel burn and maximum range to minimum time and maximum fuel burn. Cost index provides an essential tool when optimizing cost by trading increased fuel burn for reduced trip time for example. It is provided as an index to the Flight Management System (FMS) of an aircraft and FMS decides all flight parameters such as cruise speed.

Still, making the trip time vs. fuel burn decision locally for each flight is not a very effective method since effects of any modification propagates through the whole flight network. A global optimization tool that considers the cost index and cruise time controllability is needed. Majority of airline scheduling and disruption management research assumes that flight cruise times are constant. In fact, flight cruise times can be altered by changing the speed of the aircraft in exchange for differing fuel costs.

In the recovery model proposed by Aktürk et al. (2012), flight cruise speeds are taken as controllable providing a recovery option after a disruption occurs. In our study, cruise time controllability is used to build a robust schedule where cruise times are taken as decision variables and they can be shortened with increased fuel costs and this is taken as a measure to reduce unnecessary slack in the schedule.

2.5 Second Order Cone Programming

In our study, instead of developing approximations, chance constraints are transformed to second order conic equations and therefore can be solved exactly and fast. As far as we know, these methods have not been applied in airline scheduling literature before. Extensive information on conic programming and conic representable functions can be found in Ben-Tal and Nemirovski (2001).

Second order cone programming has begun to be applied in optimization and operation research in recent years. Günlük and Linderoth (2010) show how to express the convex hull via conic quadratic constraints for several classes of problems. Aktürk et al. (2009) studied a conic quadratic reformulation for a machine-job assignment problem where processing times are controllable. Calafiore and Ghaoui (2006) show that for radial distributions on the data, probability constraints can be converted into second-order cone constraints.

2.6 Summary

During the implementation of airline schedules, disruptions occur that compromise the schedule and result in operational delays. The continuous increase in fleet sizes, number of flights and number of passengers result in congestions which make the effects of delays very significant. With new destinations added each day, amount of passenger connections also grow and impacts of delay propagation can not be avoided. A flexible schedule generations scheme that can handle these delays during implementation or propose a recovery alternative after the disruption is needed.

Recovery models and robust scheduling models answer these needs and there is a growing literature addressing them. Our study is on a robust scheduling approach where inserted slack time is balanced by cruise speed controllability which results in less operational costs. Few studies address slack-time redistribution but there is no research that handles cruise speed controllability as a trade-off to slack times.

We use chance constraints to model the uncertainty in flight block times. Other studies with the same approach develop approximations to solve the chance constraints whereas we transform them to second order cone equations and solve exactly in very short time.

Chapter 3

Problem Definition

Given an initial published schedule, the model that we work on uses both idle time insertion and cruise time controllability to output a robust schedule of minimum cost that satisfy given passenger connection service levels. To achieve that, aircraft routings, flight sequence and passenger itineraries of an initial schedule is used to design a more robust schedule alternative. Block time of flights in the original schedule are considered in two parts as cruise and non-cruise times. Cruise times are allowed to be shortened via speeding of the aircraft, while non-cruise times are represented with random variables and their duration change regarding the variability. The model adjusts the departure times in the original schedule by perturbing the original flight durations and inserting idle time between flights.

While building the robust schedule, the service level of passenger connections are set to achieve a desired level. A passenger connection between two flights F1 and F2 is possible if the departure time of F2 is later than and within a time interval of the arrival time of F1, the origin of F2 is the same as the destination of F1 and the destination of F2 is different than the origin of F1. Each connection has a service level expressed by chance constraints and will be described in detail later in this chapter. The overall service level of the schedule is a weighted average of individual connection service levels.

The variability of flight non-cruise times are affected by the congestion in the airports that the flights take-off from and land to. The airport congestion also affects the turnaround time the flight spends in an airport. Turnaround times also depend on aircraft types. Note that delays are not allowed in this study, so the departure time of a flight cannot be set earlier than the previous arrival time of its aircraft plus the necessary turnaround time.

In this section the parameters of the model will be given and described. The properties of the random variable will be given with the associated distribution function. The calculation of passenger service levels will be explained in detail and the cost function related to cruise time and speed change will be analyzed.

Parameters

J : set of all flight legs

T : set of aircraft

B : set of airports

t_i : the aircraft of flight $i \in J$, $t_i \in T$

O_i : origin of flight $i \in J$

D_i : destination of flight $i \in J$

FIL_i : number of passengers in flight $i \in J$

f_i^u : original cruise time duration of flight $i \in J$

TP_{ij} : turntime needed to connect passengers between flights $i, j \in J$

TA_{ij} : turntime needed to connect aircraft between flights $i, j \in J$

PAS_{ij} : normalized passenger connection level between flights $i, j \in J$

C_t : fuel burn rate of aircraft $t \in T$ in tons of fuel per minute

I_t : unit idle time cost of aircraft $t \in T$ in dollars per minute

w_i : lower bound for the departure time of flight $i \in J$

v_i : upper bound for the departure time of flight $i \in J$

f_i^l : lower bound for the cruise time of flight $i \in J$

f_i^u : upper bound for the cruise time of flight $i \in J$

P_i : set of flights that has a passenger connection with flight $i \in J$

$PAIR$: set of pairs of consecutive flights of the same aircraft

e_b : airport congestion coefficient for $b \in B$

γ : required service level

cf : fuel cost per ton of aircraft fuel

Decision Variables

x_i : departure time of flight $i \in J$

s_i : idle time after flight $i \in J$

f_i : cruise time of flight $i \in J$

γ_{ij}: service level for passenger connections between $i, j \in J$

In the model, J represents the set of flights in the initial schedule, T represents the set of aircraft and $PAIR$ represents the set of flight duals that are flown consecutively by the same aircraft. For a flight $i \in J$, t_i represents the aircraft assigned to that flight, FIL_i is the number of passengers in the flight and f_i^u

is the ideal duration of the flight which is the scheduled duration in the initial plan. This ideal duration in flight operations is decided by airlines using the cost index ratio described earlier. This duration is the result of the setting that has the minimum fuel cost. So decreasing this duration results in higher fuel costs. Since this is the minimum cost time setting, it is taken as the upper bound for the flight cruise time duration. In this model, f_i^l, is the allowed lower bound for cruise time of flight i, where f_i^l will result in the highest fuel costs in this case. There is a lower bound on the cruise time, because speeding can only be done up to some extent. Higher speeds may not comply with the aircraft specifications, or cause noise levels that are disturbing to passengers and therefore avoided.

P_i represents the set of flights for which i has an immediate passenger connection with in the destination point of i, i.e. set P_i consists of following flights that passengers from flight i continue their itineraries with. Flights having the same destination as origin of flight i are not allowed in the connection set. An important parameter is the window for departure time of flight i, which is represented by $[w_i,\ v_i]$. Ensuring the departure time of a flight is within a certain time frame might be important for marketing and demand purposes, as the model works with perturbing the original departure times of flights.

For a given aircraft $t \in T$, C_t equals the fuel burn rate for aircraft in tons per minute where cf gives the fuel cost per ton of aircraft fuel and I_t equals idle time cost of aircraft in dollars per minute. For flights $(i, j) \in PAIR$, TA_{ij} represents the turnaround time needed by the aircraft between two consecutive flights. The realized turnaround times are dependent to airport congestion coefficients that measure the congestion level of the airport that the turnaround takes place. They are also affected by the type of aircraft, since each aircraft needs a different amount of time for this operation. [1] For each $i \in J$, we have decision variables x_i, s_i and f_i, representing departure time, idle time after the flight and cruise time of the flight respectively.

[1]These congestion coefficients are provided in Table 5.3, and are explained in detail in §5.

3.1 Random Variable $\mathbf{A}_i$

The random variable in our model, A_i, for $i \in J$ represents the portion of block time except the cruise time. Arıkan and Deshpande (2012) suggest that flight block times fit a Loglaplace distribution, so A_i's are assumed to be Loglaplace variables. Each A_i is associated with two parameters of the Loglaplace distribution; α and β_i. β_i are calculated by multiplying parameter β by a function of two congestion coefficients corresponding to origin and destination airports of the flight. Therefore, the mean and variance of the random variable change depending on the airports. In other words,

$$\beta_i = \beta \cdot (e_{O_i})^4 \cdot (e_{D_i})^4$$

where O_i and D_i are the origin and destination airports of flight $i \in J$. A_i are assumed to be symmetric Loglaplace random variables, therefore the tail grows one-sided, i.e., depending on the level of variability, the mean of distribution grows.

3.1.1 Loglaplace Distribution

The properties for a symmetric Loglaplace random variable X with parameters α and $\beta_i > 0$, where e^{α} is a scale parameter and $1/\beta_i$ is the tail parameter are given as:

$$F_X(x) = \begin{cases} \frac{1}{2} e^{\frac{(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) < \alpha \\ 1 - \frac{1}{2} e^{\frac{-(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

$$f_X(x) = \begin{cases} \frac{1}{2 \cdot \beta_i \cdot x} e^{\frac{(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) < \alpha \\ \frac{1}{2 \cdot \beta_i \cdot x} e^{\frac{-(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

with quantile function

$$F_X^{-1}(p) = \begin{cases} (2p)^{\beta_i} \cdot e^{\alpha}, & \text{if } \ln(x) < \alpha \\ \frac{e^{\alpha}}{(2-2p)^{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

Proposition 3.1. *Expected value of loglaplace variable X with parameters α and β_i is finite only for $\beta_i < 1$ and has value $\frac{e^\alpha}{(1-\beta_i)\cdot(1+\beta_i)}$.*

Proof. Define δ such that $\alpha = ln(\delta)$. Then;

$$f_X(x) = \begin{cases} \frac{1}{2\beta_i\delta}\left[\frac{x}{\delta}\right]^{1/\beta_i - 1}, & \text{if } x < \delta \\ \frac{1}{2\beta_i\delta}\left[\frac{\delta}{x}\right]^{1/\beta_i + 1}, & \text{if } x \geq \delta \end{cases}$$

Using the distribution function, we can calculate expected value of X by:

$$\begin{aligned} E[X] &= \int_{-\infty}^{\infty} x \cdot f_X(x) \cdot dx \\ &= \int_0^\delta \frac{1}{2\beta_i}\left[\frac{x}{\delta}\right]^{1/\beta_i} \cdot dx + \int_\delta^\infty \frac{1}{2\beta_i}\left[\frac{\delta}{x}\right]^{1/\beta_i} \cdot dx \end{aligned}$$

Define;

$$\begin{aligned} g_1(x) &= \int_0^\delta \frac{1}{2\beta_i}\left[\frac{x}{\delta}\right]^{1/\beta_i} \cdot dx \\ g_2(x) &= \int_\delta^\infty \frac{1}{2\beta_i}\left[\frac{\delta}{x}\right]^{1/\beta_i} \cdot dx \end{aligned}$$

Then;

$$g_1(x) = \frac{\delta}{2(\beta_i + 1)}$$

whereas

$$g_2(x) = \begin{cases} \frac{-\delta}{2(\beta_i - 1)}, & \text{if } \beta_i < 1 \\ undefined, & \text{if } \beta_i = 1 \\ \infty, & \text{if } \beta_i > 1 \end{cases}$$

Then, for α and $0 < \beta_i < 1$ we get:

$$E[X] = \frac{e^\alpha}{(1-\beta_i)\cdot(1+\beta_i)}.$$

□

3.2 Service Level (γ)

In this study, aircraft routing network is considered together with the passenger connection network. For flight $i \in J$, and $j \in P_i$, TP_{ij} equals the time needed by passengers to connect between flights i and j, with associated decision variable γ_{ij} which represents the percentage of passenger connection satisfied between i and j. γ_{ij}'s are calculated using chance constraints for the above described random variable such that the probability of time between arrival of flight i and departure of flight j being greater than the required connection time TP_{ij} is at least γ_{ij}. The weighted average of these γ_{ij} values using weights PAS_{ij} needs to be greater than or equal to γ, the overall service level of the schedule. PAS_{ij} values are assigned to flight connections in a manner that they represent the percentage of a given connection among all other passenger connections based on the number of passengers connecting. These values are normalized over the whole flight network and are used as weights when calculating the overall schedule service level.

Note that the service level of the schedule is calculated using a weighted average of service levels of individual passenger connections. This provides more reasonable information on actual service levels, since the service level value of each connection is allowed to be different. In this study, we weigh the connections based on the number of passengers connecting, but a different weighing scheme such as percentage of higher class customers within all connecting passengers could be used as well. It is also possible to add lower bound constraints in the following manner to desired γ_{ij} variables to ensure a minimum level of service is satisfied in flights.

$$\gamma_{ij} \geq {\gamma_{ij}}^d \qquad i \in J, j \in P_i$$

where ${\gamma_{ij}}^d$ represents the minimum desired connection service level.

3.3 Fuel Cost Function

Airbus (1998) provides detailed information on fuel costs and the relationship between speed and fuel costs of airplanes. In this study, the fuel cost function for flight $i \in J$ is given as:

$$K_{t_i}(f_i) = \frac{C_{t_i} \cdot cf \cdot (f_i^u)^m}{f_i^{m-1}}$$

for a factor m. The fuel burn rate of the aircraft in tons per minute is multiplied with the cost per ton of fuel to get how much fuel an aircraft burns in monetary terms in one minute. This resulting cost term is used in the nonlinear cost function by multiplying it with the term $(f_i^u)^m / f_i^{m-1}$, where f_i^u stands for the initial cruise time of flight i, and f_i is the associated decision variable with the new cruise time of flight i. You can observe the trade-off between fuel cost and idle cost functions in relation to time in Figure 3.1. Note that for an amount of slack that is wanted, it is cheaper to speed the aircraft up to a point, and cover the rest of the time with inserting idle time.

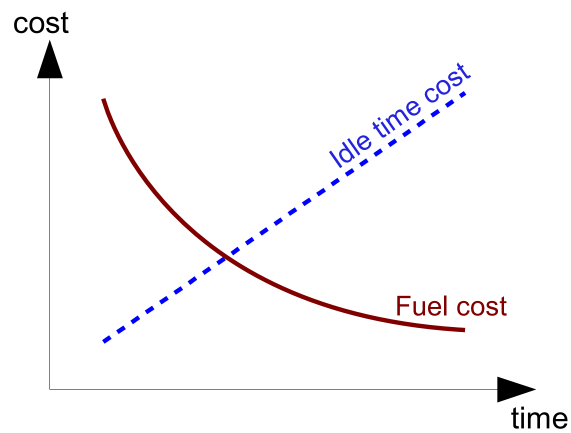

Figure 3.1: Idle time and fuel cost functions

3.4 Numerical Example

A numerical example will be provided in this section to better explain model mechanics. On a small schedule example, both the idle time insertion and the cruise time controllability mechanisms of the model will be shown. First, the flight network graph of the initial schedule will be given which shows the initial idle time distribution in the schedule and the resulting delays. Remember with our model, delays are not allowed. Following the original schedule example, a new schedule generated only using the idle time insertion mechanism of our model will be given. Speeding of the aircraft is not allowed for this case, and it is seen that even with using a better idle time distribution, delays can be avoided and costs can be improved. In the last example, both speed control and idle time insertion will be used in the schedule which will result in greater cost improvements. The

Tail #	Flight #	From	To	Dep.Time	Duration	Arr.Time	Actual Dep.	TA Time
N531AA	2303	ORD	LGA	7:35	2:05	9:40	7:35	0:39
	2336	LGA	ORD	10:30	2:15	12:45	10:30	0:41
	1053	ORD	DFW	13:15	3:00	16:15	13:33	0:40
	336	DFW	ORD	16:50	3:00	19:50	17:20	0:21
	336	ORD	LGA	20:20	2:05	22:25	20:49	
N4WPAA	2311	ORD	DFW	7:45	2:25	10:10	7:45	0:37
	2348	DFW	ORD	11:30	2:25	12:55	11:30	0:38
	1797	ORD	LGA	14:00	2:20	16:20	14:41	0:36
	1982	LGA	ORD	17:20	2:00	19:20	17:44	0:38
	1339	ORD	SAN	20:20	4:30	0:50	20:29	

Table 3.1: Published Schedule

published schedule extracted from BTS data used in this numerical example is provided in Table 3.1 and it consists only of the daily plans of two aircraft. Tail numbers of these aircraft are provided in the first column, which is followed by the assigned flights to these aircraft in the second column. The following two columns give origin and destination information for flights, where the following three columns list planned and announced departure times, flight durations and arrival times. In the next column, actual departure time information is listed, and finally turnaround times are given in the last column. Note that there are two flights with flight code 336. This is because flight 336 is a “through” flight, which is defined as a single flight from origin to destination with one or more intermediate stops.

As it can be observed, actual departure times could be different than the planned departure times, which results in delays. This is related to several issues. First of all, because of variability, actual duration of flights are realized differently than planned durations. For example, the planned duration of flight 2303 is 2 hours and 5 minutes. The non-cruise time of the flight is taken as 20 minutes of this duration. But this non-cruise time has an expected value of 27 minutes instead, because of variability. These mean times are calculated as explained in §3.1, with an α value of ln(20), a β value of 0.05 and airport congestion coefficients given in Table 5.3. Another reason for the difference is turnaround times. In some cases, the planned duration left for the aircraft between arrival time of

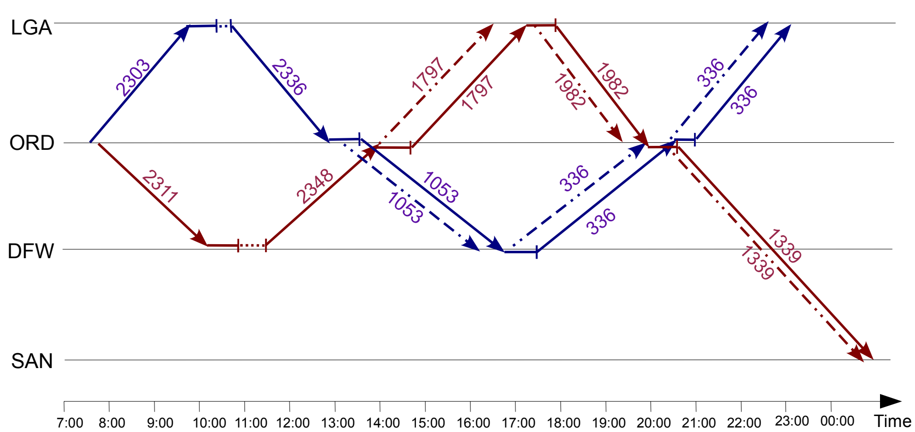

Figure 3.2: Network graph for the published schedule

a flight and the departure time of the next flight is shorter than the necessary turnaround time, which results in a delayed departure. If this time is longer than what is necessary, then there is an unnecessary idle time for the aircraft. It is also important to note that the delays propogate through the network. For example, a 10 minute delay in a given flight will affect the next flight of that aircraft as well if there is no idle time left in between two consecutive flights.

The resulting flight network is seen in Figure 3.2. Continuous lines for flights show the actual realized departure times of aircraft, where dashed lines for flights show the planned departure times. Continuous ground lines correspond to turn times of aircraft, where dashed ground lines correspond to unnecessary waiting. The idle time in the schedule are 5 minutes after flight 2303 and 35 minutes after flight 2311. It can be observed that this utilization of idle time distribution results in unnecessary waiting for some flights, whereas there is a delay in others. These delays also cause connecting passengers to miss their flights since a certain time is needed for passengers to connect to their next flight. Passenger connected flight pairs in this schedule are 2336-1053, 336-336, 336-1339, 2348-1797 and 1982-1339.

Assume schedule delay is unwanted and needs to be avoided. Intuitively, one can decide that better utilization of slack time can reduce these unnecessary cost items and avoid flight delays. In fact, Chiraphadhanakul and Barnhart (2011) support this claim with their research. The departure times for a perturbed schedule with a better utilization of idle times is drawn in Figure 3.3, where delay is completely avoided, and passenger service levels are same as the original schedule. Passenger service levels are decided based on the percentage of passengers that catch their connections. It can be seen that in the new schedule, two idle time slots are inserted after the first connecting leg of flight 336 and flight 2348, and there is no delay in the schedule.

In this schedule, idle times are put as 48 minutes after flight 2348 and 10 minutes after first connecting flight of 336. Note that total idle time seems to be more than before, but delay costs are totally avoided in this case. In cost terms, if we compare the costs of two schedules without taking delay costs into consideration, total costs increased by around 5%. But when delay costs are considered, there is a total cost saving of 32% when the second schedule is used.

Remember the total operational costs are calculated as the sum of fuel cost and idle time cost in our model. Delay costs for the original schedule are also considered for comparison, however delay costs for our model are zero. Fuel costs are calculated via the speed cost function described in §3.3. Idle time costs are calculated by multiplying the total idle time in the schedule with the unit idle time cost per minute of the assigned aircraft. Lastly, delay costs for the original schedule are calculated in the same manner, by multiplying the total delay time by the unit delay cost per minute. The improvement percentages are calculated using the formula:

$$\text{Cost Improvement} = 100 \cdot \frac{\text{Original Schedule - Proposed Model}}{\text{Original Schedule}}$$

Costs can be improved even more while preserving the service levels by the utilization of cruise time controllability. In exchange for extra fuel burn, an aircraft can fly a route faster resulting in cruise time savings. Fuel costs are nonlinear with increasing speed, therefore a balance of cruise time controllability and idle time insertion can be achieved to have a schedule with the same service level with significant idle time cost improvements. The new schedule with controlled speed times can be found in Figure 3.4.

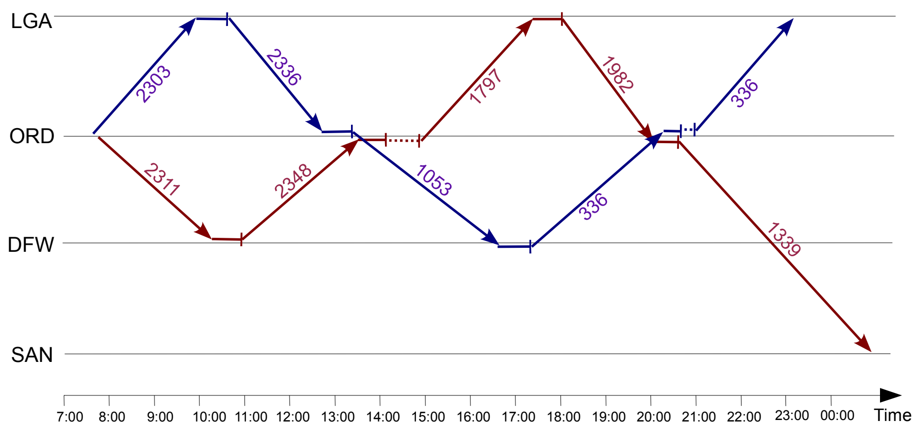

Figure 3.3: Network graph with adjusted departure times

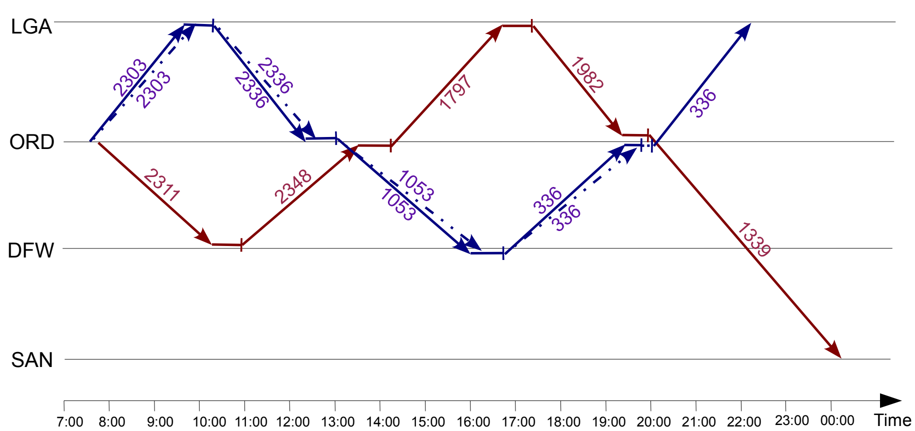

Figure 3.4: Network graph with adjusted departure times and speed control

With speeding option introduced, Flights 2303, 2336, 1053 and the first connecting part of flight 336 have decreased cruise times. The original duration of the flights are drawn in dashed lines. In this case, the idle time after flight 2348 is not needed anymore since passenger service levels could be ensured with speed control. This new schedule has 8% more fuel cost than the initial schedule, but idle time costs have improved by 74% and there is 35% total cost improvement.

The model given in the new section works with these mechanics. The objective is to achieve less costs where costs are measured in terms of idle time, fuel and delay cost components. By using cruise time control, major idle time cost and total cost savings are achieved in exchange for increase in fuel costs.

3.5 Summary

In this chapter, the problem definition along with a numerical example is given. Moreover, parameters, decision variables and the random variable in the problem are described. Several properties of the random variable, service level constraints in the model and the structure of the cost function are explained in detail. The working mechanics of the model are explained through a numerical study of an example published schedule.

Chapter 4

Problem Formulation

The model works as described in the numerical example. As mentioned earlier, the aircraft routings, flight sequence and passenger itinerary information on connections are taken from the original schedule. The departure times of flights in the initial schedule is perturbed by inserting slack into the schedule and speeding aircraft as necessary. As a result, a more robust schedule is generated that avoids delays and ensures a given level of passenger connection service levels while minimizing costs.

Airlines can make flight cruise speed decisions based on the cost index ratio defined below. But cruise time decisions affect the whole flight network through propagation. It is not possible for a pilot to make the most effective cruise time decisions locally during a flight. The balancing of cruise time reduction and idle time insertion is an even more complex problem and decisions should be made considering the whole network, so a global optimization tool such as the one described in this chapter is needed.

4.1 Mathematical Model

The mathematical formulation is provided below which includes chance constraints and nonlinear cost terms.

$$
\begin{aligned}
\min \quad & \sum_{i \in J} s_i \cdot I_{t_i} + \frac{C_{t_i} \cdot cf \cdot (f_i^u)^m}{f_i^{m-1}} && && (4.1)\\
\text{s.to} \quad & Pr[A_i + f_i \leq x_j - x_i - TP_{ij}] \geq \gamma_{ij} && i \in J, j \in P_i && (4.2)\\
& \sum_{i \in J} \sum_{j \in P_i} PAS_{ij} \cdot \gamma_{ij} \geq \gamma && && (4.3)\\
& w_i \leq x_i \leq v_i && i \in J && (4.4)\\
& x_j - x_i - TA_{ij} - f_i - E[A_i] - s_i = 0 && (i,j) \in PAIR && (4.5)\\
& f_i^l \leq f_i \leq f_i^u && i \in J && (4.6)\\
& s_i \geq 0 && i \in J && (4.7)\\
& {\gamma_{ij}}^d \leq \gamma_{ij} \leq 1 && i \in J, j \in P_i && (4.8)
\end{aligned}
$$

The objective function (4.1) aims to minimize sum of idle time and fuel costs, where the summation is over each flight in the network. The fuel costs are calculated using a nonlinear function of the cruise time for flight i and for exponent m, which was described in Chapter 3. Note that there is no term included in the model for delay costs, since delay is not allowed in our model, and delay costs are zero. Delay costs are only calculated for the original schedule for comparison purposes. In (4.2), we require the probability of the block time being not greater than the difference of departure times minus minimum passenger connection time needed to be greater than or equal to the associated service level variable. Detailed information on passenger connection service levels was given in Chapter 3. In (4.3), we require the weighted sum of γ_{ij}'s to be greater than the desired service level γ, where weights are measured by the passenger connections, PAS_{ij}. Constraints (4.2) and (4.3) work together to guarantee desired service levels, where γ_{ij}'s are decision variables in the model and this constraint applies the restriction on their values. Moreover, with (4.8), we impose the desired lower bounds to service level variables.

In (4.4), time window constraint for a flight leg is given. Remember that this restriction on departure times can be important for marketing purposes, since passenger demand highly depends on the time of a flight. For example, passengers traveling for business purposes would prefer early flights in the mornings and later flights in the evening. In (4.5), we guarantee that the minimum aircraft connection time is available between two consecutive flights of the same aircraft, using the mean value of the random variable. This is the constraint that allows our model to avoid delays, since aircraft is not allowed to take-off before the necessary time for turnaround operations has passed. In (4.6), we give the allowed boundaries for cruise time change.

4.1.1 Routing Feasibility

It might be important to ensure in a schedule that there is at least a time of Φ hours left between the arrival of the aircraft from the last flight of the day and the departure of the same aircraft for the first flight of the next day for crew or routing feasibility. In fact, in some cases of the original published schedule used in computations, this rule had been broken several times. To maintain this feasibility, we define two parameters for aircraft $t \in T$. Let t^f be the first flight of aircraft t in the morning where t^l be the last flight of aircraft t in the evening, $t^f \in J$, $t^l \in J$. Then by adding the below constraint, the routing feasibility can be satisfied via decreasing of cruise times and elimination of unnecessary idle times.

$$x_{t^f} - x_{t^l} \geq \Phi \qquad t \in T$$

4.1.2 Challenges for Solving the Model

The solution of the model makes for a challenge. There is nonlinearity in the objective function, and there are probabilistic constraints in the model. In previous literature, chance constraints were handled with approximations, but we intend to solve them in their exact form. The methodology is to first transform these probabilistic constraints into their closed form expressions and then transform them to second order conic equations. Nonlinear costs are also transformed to conic form. The next section explains this methodology in detail.

4.2 Conic Reformulation of the Model

Using a conic formulation of the model allows us to solve for the chance constraints exactly to optima, as opposed to using approximations. To achieve the conic reformulation, the nonlinear cost expressions and the probabilistic constraints in the model are rewritten as second order cone constraints. To transform the probabilistic constraints into conic equations we first need to write them in close form. Then the closed form expression will be transformed to second order conic equations.

4.2.1 Closed Form Expressions for the Chance Constraints

The closed form expression is written using the quantile function, i.e. the inverse cumulative distribution function of the random variable. Unfortunately, the quantile function does not have a closed form expression for all probability distributions.

If the cumulative distribution function (CDF) has a closed form expression, the quantile function can be derived inverting the cumulative distribution function using different methods such as the bisection method. For other cases, algorithms based on polynomial approximations are available. Examples to distributions with available closed-form distribution functions are exponential, logistic, log-logistic, tukey lambda, uniform, etc. Quantile functions to several distribution functions can be found in Appendix §B for reference.

As stated earlier, we will be using the loglaplace distribution to represent the random variable in our model. Remember the quantile function for a loglaplace random variable X with parameters α and β_i is derived as:

$$F^{-1}(p) = \begin{cases} (2p)^{\beta_i} \cdot e^{\alpha}, & \text{if } \ln(x) < \alpha \\ \frac{e^{\alpha}}{(2-2p)^{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

Denote

$$f_1(p) = 2^{\beta_i} \cdot e^{\alpha} \cdot p^{\beta_i}$$
$$f_2(p) = \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-p)^{\beta_i}}$$

Also remember the chance constraint of the form:

$$Pr[A_i \leq x_j - x_i - TP_{ij} - f_i] \geq \gamma_{ij}$$

Property. For $i \in J, j \in P_i$;

$$Pr[A_i \leq x_j - x_i - TP_{ij} - f_i] \geq \gamma_{ij}$$

is equivalent to

$$x_j - x_i - TP_{ij} - f_i \geq F^{-1}(\gamma_{ij})$$

Proposition 4.1. *$f_2(p) \geq f_1(p)$ for $0 \leq p \leq 1$.*
Furthermore, $f_2(p) = f_1(p)$ for $p = \frac{1}{2}$.

Proof. Consider

$$f_2(p) - f_1(p) = e^{\alpha} \left(\frac{1 - 4^{\beta_i} \cdot p^{\beta_i} \cdot (1-p)^{\beta_i}}{2^{\beta_i} \cdot (1-p)^{\beta_i}} \right)$$

Notice that, $e^{\alpha} > 0$, $2^{\beta_i} \cdot (1-p)^{\beta_i} > 0$.
Also $1 - 4^{\beta_i} \cdot p^{\beta_i} \cdot (1-p)^{\beta_i} \geq 0$ always holds since $p^{\beta_i} \cdot (1-p)^{\beta_i} \leq \frac{1}{4^{\beta_i}}$ is always true.
Furthermore, $f_2(p) = f_1(p)$ at $p = \frac{1}{2}$. $\square$

Then for our chance constraint equation for $i \in J, j \in P_i$ for a loglaplace r.v. with parameters α and β_i, we can write using Proposition 4.1:

$$F^{-1}(\gamma_{ij}) = \begin{cases} 2^{\beta_i} \cdot e^{\alpha} \cdot \gamma_{ij}^{\beta_i}, & \text{if } 0 \leq \gamma_{ij} \leq \frac{1}{2} \\ \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-\gamma_{ij})^{\beta_i}}, & \text{if } \frac{1}{2} \leq \gamma_{ij} \leq 1 \end{cases}$$

Two different conic formulations are provided below because of the properties of the loglaplace random variable. Remember for flight $i \in J$, in 3.1 it is shown that mean of loglaplace random variable A_i is finite only for $\beta_i < 1$. In order to achieve finite mean, we assume $\beta_i < 1$ when reformulating the problem in Conic Reformulation A with second order cone constraints. Conic reformulation of the model for $\beta_i > 1$, which is named as Conic Reformulation B is discussed in §4.4.

4.3 Conic Reformulation A

This is the conic reformulation of the model where $\beta_i < 1$ for for flight $i \in J$. The conic representation of the chance constraints will be given first, which will be followed by the conic representation of the cost function.

4.3.1 Conic Representation of Chance Constraints

Remember $\beta_i < 1$ in this first conic reformulation.

Proposition 4.2. *For $i \in J, j \in P_i$, $f_2(\gamma_{ij})$ is a strictly convex function when $0 < \beta_i < 1$.*

Proof. Second derivative of $f_2(\gamma_{ij})$ is positive for $0 < \beta_i < 1$. The result follows. □

Proposition 4.3. *For $i \in J, j \in P_i$, $(x_j - x_i - TP_{ij} - f_i) \geq F^{-1}(\gamma_{ij})$ is SOCP representable if $0 < \beta_i < 1$ and $\frac{1}{2} \leq \gamma_{ij} \leq 1$.*

Proof. Replace

$$x_j - x_i - TP_{ij} - f_i \geq F^{-1}(\gamma_{ij})$$

in problem with

$$x_j - x_i - TP_{ij} - f_i \geq \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-\gamma_{ij})^{\beta_i}}.$$

This case will be binding in our problem if $\gamma_{ij} \geq \frac{1}{2}$. This lower bound on γ_{ij} is applied to achieve convexity of the constraint as shown in Proposition 4.2. This is SOCP-represantable. Denote $x_j - x_i - TP_{ij} - f_i$ by σ_{ij}. Let the constant $\lambda = \frac{e^{\alpha}}{2^{\beta_i}}$. Then we can write:

$$\lambda \cdot z_{ij} \leq \sigma_{ij} \cdot \overline{\gamma}_{ij}^{\beta_i}$$

where $\overline{\gamma}_{ij} = 1 - \gamma_{ij}$. β_i can be written as $\frac{a_i}{b_i}$ for integers a_i and b_i. This is written as:

$$\lambda^{b_i} \leq {\sigma_{ij}}^{b_i} \cdot {\overline{\gamma}_{ij}}^{a_i}$$

Since λ^{b_i} is a constant, we can write this equation as:

$$(\sqrt[2^l]{\lambda^{b_i}})^{2^l} \leq {\sigma_{ij}}^{b_i} \cdot {\overline{\gamma}_{ij}}^{a_i}$$

Choose l such that

$$l = \lceil \log_2(a_i + b_i) \rceil .$$

Then, the constraint is SOCP representable due to Ben-Tal and Nemirovski (2001). □

Note that we require $\gamma_{ij} \geq \frac{1}{2}$ for each passenger connection, which is a desirable situation since it means a service level of at least 0.5 is guaranteed for each connection. This restriction does not apply in Conic Reformulation B.

4.3.2 Conic Representation of the Speeding Cost Function

The cost function for flight i and for a factor m is represented as following:

$$K_{t_i}(f_i) = \frac{C_{t_i} \cdot cf \cdot (f_i^u)^m}{f_i^{m-1}}.$$

Proposition 4.4. *For $i \in J$, the fuel cost function $K_{t_i}(f_i)$ is SOCP representable.*

Proof. The term $C_{t_i} \cdot cf \cdot (f_i^u)^m$ is constant for each flight $i \in J$, so it can be represented as Ψ_i. This cost function appears only in the objective function, and can be written as a conic inequality.

First, introduce a dummy variable q_i for each $i \in J$ to represent the cost component in the objective function. The objective function is now linear and is written as:

$$min \sum_{i \in J} s_i \cdot I_{t_i} + q_i$$

Then, we add the following constraints to the model for each i∈J:

$$\Psi_i \leq q_i \cdot f^{m-1}$$
$$q_i \geq 0$$

Define $n = \lceil \log_2 m \rceil$. Then;

$$\left(\sqrt[\frac{1}{2^n}]{\Psi_i} \right)^{2^n} \leq q_i \cdot f^{m-1}$$

which is SOCP represantable due to Ben-Tal and Nemirovski (2001). □

4.3.3 Conic Formulation of the Model

After the above described changes, the model becomes:

$$
\begin{aligned}
\min \quad & \sum_{i \in J} s_i \cdot I_{t_i} + q_i && && (4.9)\\
\text{s.to} \quad & {\sigma_{ij}}^{b_i} \cdot {\overline{\gamma}_{ij}}^{a_i} \geq (\sqrt[2^l]{\lambda^{b_i}})^{2^l} && i \in J, j \in P_i && (4.10)\\
& x_j - x_i - TP_{ij} - f_i = \sigma_{ij} && i \in J, j \in P_i && (4.11)\\
& \overline{\gamma}_{ij} = 1 - \gamma_{ij} && i \in J, j \in P_i && (4.12)\\
& \sum_{i \in J} \sum_{j \in P_i} PAS_{ij} \cdot \gamma_{ij} \geq \gamma && && (4.13)\\
& \Psi_i \leq q_i \cdot {f_i}^{m-1} && i \in J && (4.14)\\
& x_j - x_i - TA_{ij} - f_i - E[A_i] - s_i = 0 && (i,j) \in PAIR && (4.15)\\
& w_i \leq x_i \leq v_i && i \in J && (4.16)\\
& f_i^l \leq f_i \leq f_i^u && i \in J && (4.17)\\
& 0.5 \leq \gamma_{ij} \leq 1 && i \in J, j \in P_i && (4.18)\\
& s_i \geq 0 && i \in J && (4.19)\\
& q_i \geq 0 && i \in J && (4.20)
\end{aligned}
$$

The objective function (4.9) is slightly different than in the original model objective function (4.1) because of the conic transformation of the fuel cost function. The original objective is now represented with this objective equation and constraints (4.14) and (4.20). The chance constraints in (4.2) are now represented by the conic form constraints in (4.10), (4.11) and (4.12). Constraint (4.13) is the service level constraint that is available as the initial mathematical model constraint (4.3).

Constraints (4.15), (4.16), (4.17) and (4.19) are same as the original mathematical formulation. Note that the restriction of γ_{ij}'s to be greater than 0.5 is applied on constraint (4.18). In the previous constraint (4.8), this lower bound setting was different. Actually, it is possible to apply the lower bound parameters ${\gamma_{ij}}^d$ here as long as they satisfy $0.5 \leq {\gamma_{ij}}^d \leq 1$.

This resulting model is solvable via commercial solvers in reasonable computation time due to the conic quadratic formulation and can easily be used by airline practitioners.

4.4 Conic Reformulation B

This is the conic reformulation of the model where $\beta \geq 1$ for flight $i \in J$. We can formulate the problem for $\beta \geq 1$ if we use the geometric mean of Loglaplace random variable for calculations to avoid infinity. The conic representation of the chance constraints will be given for this case. Conic formulation of the cost function is same as in the Conic Reformulation A.

4.4.1 Conic Representation of Chance Constraints

Remember from §4.3 by Proposition 4.1 for $i \in J, j \in P_i$ for a loglaplace r.v. with parameters α and β_i, we can write:

$$F^{-1}(\gamma_{ij}) = \begin{cases} 2^{\beta_i} \cdot e^{\alpha} \cdot \gamma_{ij}^{\beta_i}, & \text{if } 0 \leq \gamma_{ij} \leq \frac{1}{2} \\ \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-\gamma_{ij})^{\beta_i}}, & \text{if } \frac{1}{2} \leq \gamma_{ij} \leq 1 \end{cases}$$

Denote

$$f_1(\gamma_{ij}) = 2^{\beta_i} \cdot e^{\alpha} \cdot \gamma_{ij}^{\beta_i}$$
$$f_2(\gamma_{ij}) = \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-\gamma_{ij})^{\beta_i}}$$

Proposition 4.5. *For $i \in J, j \in P_i$, $F^{-1}(\gamma_{ij})$ is a convex function when $\beta_i \geq 1$.*

Proof. Both $f_1(\gamma_{ij})$ and $f_2(\gamma_{ij})$ are convex for $0 \leq \gamma_{ij} \leq 1$. The result follows. □

Proposition 4.6. *For $i \in J, j \in P_i$, $(x_j - x_i - TP_{ij} - f_i) \geq F^{-1}(\gamma_{ij})$ is SOCP representable if $\beta_i \geq 1$.*

Proof. Introduce a 0-1 variable z_{ij} such that:

$$z_{ij} = \begin{cases} 0, & \text{if } \gamma_{ij} < \frac{1}{2} \\ 1, & \text{if } \gamma_{ij} \geq \frac{1}{2} \end{cases}$$

Replace the constraint

$$x_j - x_i - TP_{ij} - f_i \geq F^{-1}(\gamma_{ij})$$

in the problem with the equations 1, 2 and 3 below:

1.
$$x_j - x_i - TP_{ij} - f_i \geq 2^{\beta_i} \cdot e^{\alpha} \cdot \gamma_{ij}^{\beta_i}$$

   This case will be binding in our problem if $\gamma_{ij} < \frac{1}{2}$. Denote $x_j - x_i - TP_{ij} - f_i$ by σ_{ij}. Let the constant $\omega = 2^{\beta_i} \cdot e^{\alpha}$. We can denote β_i as $\frac{a_i}{b_i}$ for integers a_i and b_i such that $\mathrm{a}_i \geq \mathrm{b}_i$. Then we can write:

$$\sigma_{ij} \geq \omega \cdot \gamma_{ij}^{\frac{a_i}{b_i}}$$

   If we take powers of b for both sides, we get:

$$\sigma_{ij}^{b_i} \geq \omega^{b_i} \cdot \gamma_{ij}^{a_i}$$

   Then for $k_i = \lceil \log_2(a_i) \rceil$ we can write

$$\sigma_{ij}^{b_i} \cdot \gamma_{ij}^{(2^{k_i} - a_i)} \geq \omega^{b_i} \cdot \gamma_{ij}^{2^{k_i}}$$

   which is SOCP representable due to Ben-Tal and Nemirovski (2001).

2.
$$x_j - x_i - TP_{ij} - f_i \geq \frac{e^{\alpha}}{2^{\beta_i} \cdot (1 - \gamma_{ij})^{\beta_i}} \cdot z_{ij}$$

This case will be binding in our problem if $\gamma_{ij} \geq \frac{1}{2}$. Denote $x_j - x_i - TP_{ij} - f_i$ by σ_{ij}. Let the constant $\lambda = \frac{e^\alpha}{2^{\beta_i}}$. Then we can write:

$$\lambda \cdot z_{ij} \leq \sigma_{ij} \cdot \overline{\gamma}_{ij}^{\beta_i}$$

where $\overline{\gamma}_{ij} = 1 - \gamma_{ij}$. β_i can be written as $\frac{a_i}{b_i}$ for integers a$_i$ and b$_i$. Also since z_{ij} is a 0-1 variable this inequality can be written as:

$$\lambda^{b_i} \cdot z_{ij} \leq \sigma_{ij}^{b_i} \cdot \overline{\gamma}_{ij}^{a_i}$$

Then for $l_i = \lceil \log_2(a_i + b_i) \rceil$, we can write:

$$\lambda^{b_i} \cdot z_{ij}^{2^{l_i}} \leq \sigma_{ij}^{b_i} \cdot \overline{\gamma}_{ij}^{a_i}$$

which is SOCP representable due to Ben-Tal and Nemirovski (2001) as λ^{b_i} is a constant.

3.
$$z_{ij} > \gamma_{ij} - \frac{1}{2}$$

which is the constraint to ensure that z_{ij} takes the value of 1 when $\gamma_{ij} \geq \frac{1}{2}$.

□

4.4.2 Conic Formulation of the Model

Then for this case, the conic formulation of the model becomes:

$$\min \quad \sum_{i\in J} s_i \cdot I_{t_i} + q_i \tag{4.21}$$

$$\text{s.to} \quad \lambda^{b_i} \cdot z_{ij}^{2^{l_i}} \leq \sigma_{ij}^{b_i} \cdot \overline{\gamma}_{ij}^{a_i} \qquad i \in J, j \in P_i \tag{4.22}$$

$$\sigma_{ij}^{b_i} \cdot \gamma_{ij}^{(2^{k_i}-a_i)} \geq \omega^{b_i} \cdot \gamma_{ij}^{2^{k_i}} \qquad i \in J, j \in P_i \tag{4.23}$$

$$x_j - x_i - TP_{ij} - f_i = \sigma_{ij} \qquad i \in J, j \in P_i \tag{4.24}$$

$$\overline{\gamma}_{ij} = 1 - \gamma_{ij} \qquad i \in J, j \in P_i \tag{4.25}$$

$$\sum\sum PAS_{ij} \cdot \gamma_{ij} \geq \gamma \qquad i \in J, j \in P_i \tag{4.26}$$

$$z_{ij} > \gamma_{ij} - \frac{1}{2} \qquad i \in J, j \in P_i \tag{4.27}$$

$$\Psi_i \leq q_i \cdot {f_i}^{m-1} \qquad i \in J \tag{4.28}$$

$$x_j - x_i - TA_{ij} - f_i - G[A_i] - s_i = 0 \qquad (i,j) \in PAIR \tag{4.29}$$

$$w_i \leq x_i \leq v_i \qquad i \in J \tag{4.30}$$

$$f_i^l \leq f_i \leq f_i^u \qquad i \in J \tag{4.31}$$

$${\gamma_{ij}}^d \leq \gamma_{ij} \leq 1 \qquad i \in J, j \in P_i \tag{4.32}$$

$$z_{ij} \in 0,1 \qquad i \in J, j \in P_i \tag{4.33}$$

$$s_i \geq 0 \qquad i \in J \tag{4.34}$$

$$q_i \geq 0 \qquad i \in J \tag{4.35}$$

In this version, the chance constraint (4.2) in the original mathematical model is represented by constraints (4.22), (4.23), (4.24), (4.25) and (4.27). The 0-1 variable z_{ij} is used to ensure which of the conic constraints (4.22) or (4.23) will be active depending on the value of γ_{ij}. This relation between z_{ij} and γ_{ij} is maintained by constraint (4.27).

Constraint (4.29) is also slightly different in the sense of the random variable mean used. Note that $G[A_i]$ represents the geometric mean for Loglaplace random variable here and it is used instead of the expected value $E[A_i]$. Also note that in constraint (4.32), the lower bound parameters for γ_{ij} are applied again.

4.5 Summary

In this chapter, the mathematical formulation of the model is given. This is a complex model with chance constraints and nonlinear objective function terms that makes it hard to solve. To obtain exact and fast solutions, the model is reformulated as a second order cone programming model.

Two different conic reformulations for different random variable parameters are developed and are explained in detail. Chance constraints are expressed using second order conic inequalities using their closed form equations. Nonlinear cost function in the objective is also handled by representing it via second order conic inequalities

Chapter 5

Computational Study

The aim of the study was to acquire a robust schedule in reasonable computation time. In addition to analyzing time performance, in this section, the performances of the model and the original published schedule are compared using several criteria for schedules of two different sizes. Daily schedule of a US carrier for a single hub and 4-hub will be used.

As previously defined in §4.1, a service level γ is fed into the model to achieve a minimum cost robust schedule that has a service level above γ. We chose to compare the original published schedule with our proposed robust schedule according to the sum of costs they have for the same service level. For the purposes of comparison, service levels of the original schedule are fed as input to the model, and the resulting schedule of same service values is evaluated for different cost components.

Also, as previously explained, departure time windows are given as constraints in the model. However setting time windows around the published departure times in the original schedule resulted in infeasibility because schedule delay present in the original data could not be avoided as needed. In order to get feasible solutions, departure time windows are not applied for the flights in the model. However, the departure time of the first flight for each aircraft is set to the published value in the original schedule not to diverge excessively from the published schedule.

The factors and their levels used in the study are given in Table 5.1. Computational experiments showed that the effect of the factors were monotonous. Fuel Cost represents the price for ton of jet fuel. The prices are calculated using the history of oil barrel prices. A ton of fuel is equal to 1234 liters which is approximately 326 gallons, which is around 7.8 barrels. The price of a barrel of oil fluctuated between \$60 and \$180, and currently is priced around \$100. The prices taken per barrel in the study are approximately \$75 for the lower setting and \$150 for the higher setting, which are a realistic representation of varying market prices.

		Levels	
Factor	Description	Low	High
A	Fuel Cost	\$600	\$1200
B	Compression	10%	15%
C	β	0.01	0.05
D	Connection Density	50%	100%

Table 5.1: Factor Values

Compression Level denotes the allowed percentage of cruise time compression in a flight via speeding of the aircraft. In the low setting, an aircraft is allowed to speed up to shorten the cruise time by 10%, whereas in the higher setting this value is 15%. For example, in the low setting, a flight with a cruise time duration of 120 minutes is allowed to shorten that duration by a maximum of 12 minutes.

β is the Loglaplace parameter described in Section 2, which used to adjust mean and variance of random variable. Remember, this parameter is adjusted for each flight using airport congestion coefficients.

Finally, connection density represents the percentage of the possible passenger connections realized in the network. This is basically a network density parameter and when it is set to its low value, only 50% of the possible passenger connected flights are allowed. When this factor is set high, all possible passenger connections are realized. For a given flight i, a passenger connection is possible with the consecutive flight j if destination of j is a different location than the origin of i and the scheduled departure time of j is withnin 45 minutes or 180 minutes of the scheduled arrival time of i.

There are six different types of aircraft used, each assigned different costs and capacities. The values for these parameters are provided in Table 5.2.

Aircraft Type	1	2	3	4	5	6
Idle Time Cost ($)	140	142	136	144	147	150
Fuel Burn Rate (tons/min)	0.12	0.108	0.064	0.065	0.058	0.083
Base Turntime	36	26	40	28	30	32
Number of Seats	261	262	158	159	131	190

Table 5.2: Aircraft Parameters

In Table 5.3, airport congestion coefficients used in this study are provided. These coefficients take a value between 0.8 and 1.4, latter representing the most congested airport, decided according to number of passengers visiting the airports from T-100 market data of BTS (2010b) and information in Arıkan et al. (2012). The methodology was to decide on congestion coefficients of airports based on the total passenger numbers they achieved in the year 2010. We chose 2010 since the schedules we used in the computational study are of that year. These coefficients are used for calculating the turnaround time of an aircraft and also for calculating the β_i value of a flight $i \in J$. Then, sample turnaround times were calculated for several airports, and the results were accurate to the study conducted in Arıkan et al. (2012).

To find the turntime an aircraft spends at the changeover airport, the congestion coefficients are multiplied with base aircraft turntime values. For flights $(i, j) \in PAIR$, the changeover airport will be D_i or O_j. For example, for two selected airports MIA and HDN, the turnaround times for different types of aircraft will be as given in Table 5.4. It can be observed that these values are similar to calculated averages in Arıkan et al. (2012). Moreover, if the flight is a through

Airport	Location	Coefficient	Airport	Location	Coefficient
MIA	Miami, FL	1.40	DCA	Washington, DC	1.08
ORD	Chicago, IL	1.37	SAN	San Diego, CA	1.05
LAX	Los Angeles, CA	1.35	STL	St.Louis, MO	1.05
DEN	Denver,CO	1.35	MCI	Kansas City, MO	1.02
DFW	Dallas, TX	1.32	AUS	Austin, TX	1.00
LGA	New York, NY	1.30	RDU	Raleigh/Durham, NC	1.00
BOS	Boston, MA	1.30	MSY	New Orleans, LA	0.98
ATL	Atlanta, GA	1.28	SNA	Santa Ano, CA	0.98
PHX	Phoenix, AZ	1.25	SAT	San Antonio, TX	0.95
LAS	Las Vegas, NV	1.25	RSW	Fort Myers, FL	0.95
SFO	San Fransisco, CA	1.20	SJU	San Juan, PR	0.92
MSP	Minneapolis, MN	1.15	PBI	West Palm Beach, FL	0.90
PHL	Philadelphia, PA	1.15	TUS	Tuscan, AZ	0.88
EWR	Newark, NJ	1.12	MCO	Orlando, FL	0.85
FLL	Fort Lauderdale, FL	1.12	EGE	Eagle, CO	0.85
SLC	Salt Lake City, UT	1.08	HDN	Hayden, CO	0.80

Table 5.3: Congestion Coefficients

flight, the turnaround time will be 70% of the calculated value, since number of passengers and cargo loading and unloading are significantly less in case of a connection. The reason for that is for through flights, most of the passengers and their cargo stay seated in the aircraft, where some other passengers might board the plane in the connecting airport of the through flight. Therefore, there is no unloading of the aircraft and a less number of passengers board the plane.

	Turn time (min.)	
Type	MIA	HDN
1	50.4	28.8
2	36.4	20.8
3	56	32
4	39.2	22.4
5	42	24
6	44.8	25.6

Table 5.4: Turnaround time study

Passenger connection times are taken uniformly between 25 and 40 minutes, where the number of passengers in the plane are allowed to be within 60% and 100% of full capacity, again assigned uniformly.

5.1 Schedule for Single Hub Data

The schedule in this part of the study is taken from the work of Aktürk et al. (2012). To generate this schedule, data was taken from the BTS database and was filtered to include aircrafts which originate their first flight from Chicago O'hare International Airport (ORD) and revisit ORD at least once the same day. This way, they could consider a schedule for a single hub location and work on it. This schedule has 114 flights operated by 31 different aircrafts, and can be found in Table 5.5. Note that, a similar methodology will be applied when generating the 4-hub schedule used in compuational studies.

In the given schedule, first column represents the tail number of the aircraft and the next column lists the flight numbers of the flights that are assigned to that aircraft for that day. The third and fourth columns list the departure and arrival airports of the flights respectively. Following the origin and destination information, the departure time, flight time and the arrival time of each flight can be found.

In this study, we analyze flight block times in two components as cruise time and non-cruise time. Of the flight block times given in this schedule, we took 20 minutes of this duration to be left for the non-cruise time, and the rest of the time as cruise time. As an extension, computational runs for different settings of cruise times can be done to see the results in that case.

In Table 5.6, a comparison among different cost components between model objectives and original published schedule are given. The values correspond to idle time cost improvement, increase in fuel cost, total cost improvement and improvement in total cost without including delay costs. Unit delay cost can be very hard to measure accurately, but it is evident that our model performs better costwise even when delay costs are not considered. Before analyzing the results, it is important to mention that fuel costs make up approximately 90% of the total costs for model results and they make up approximately 75% of the total cost for original schedule. As stated before, the improvement percentages are calculated

Tail No	Flight No	Departure	Arrival	Departure Time	Flight Time	Arrival Time
N530AA	398	ORD	LGA	6:15	2:14	8:29
	319	LGA	ORD	9:25	2:50	12:15
	2329	ORD	DFW	13:35	2:35	16:10
	2364	DFW	ORD	17:00	2:30	19:30
N459AA	394	ORD	LGA	6:50	2:15	9:05
	321	LGA	ORD	10:00	2:50	12:50
	366	ORD	LGA	13:55	2:20	16:15
	347	LGA	ORD	17:15	2:50	20:05
N531AA	2303	ORD	DFW	6:45	2:35	9:20
	2336	DFW	ORD	10:10	2:20	12:30
	1053	ORD	AUS	13:25	2:50	16:15
	336	AUS	ORD	17:00	2:45	19:45
	336	ORD	LGA	20:40	2:05	22:45
N4XGAA	2079	ORD	SAN	8:45	4:30	13:15
	1438	SAN	ORD	14:00	4:10	18:10
	346	ORD	LGA	19:50	2:15	22:05
N598AA	1341	ORD	SFO	7:50	4:55	12:45
	348	SFO	ORD	13:30	4:25	17:55
	1521	ORD	TUS	19:15	3:55	23:10
N439AA	2455	ORD	PHX	7:10	4:00	11:10
	358	PHX	ORD	11:55	3:30	15:25
	358	ORD	LGA	16:25	2:25	18:50
	371	LGA	ORD	20:00	2:35	22:35
N475AA	407	ORD	STL	6:20	1:10	7:30
	755	STL	ORD	8:35	1:15	9:50
	755	ORD	SAT	10:45	3:00	13:45
	408	SAT	ORD	14:30	2:40	17:10
	408	ORD	PHL	18:05	2:05	20:10
N3EEAA	876	ORD	BOS	6:35	2:10	8:45
	413	BOS	ORD	9:35	3:05	12:40
	413	ORD	SNA	13:45	4:35	18:20
	1262	SNA	ORD	19:10	3:50	23:00
N4YDAA	451	ORD	SFO	9:45	4:55	14:40
	554	SFO	ORD	15:45	4:25	20:10
N3ERAA	496	ORD	DCA	6:45	1:40	8:25
	1715	DCA	ORD	9:15	2:10	11:25
	1715	ORD	LAS	12:25	4:05	16:30
	1708	LAS	ORD	17:20	3:40	21:00
N5CLAA	1425	ORD	SNA	8:25	4:40	13:05
	556	SNA	ORD	14:00	4:00	18:00
	1940	ORD	MIA	19:25	3:00	22:25
N535AA	2460	ORD	RSW	6:45	2:45	9:30
	564	RSW	ORD	10:20	3:05	13:25
	1446	ORD	EWR	14:55	2:45	17:40
	1411	EWR	ORD	18:45	2:45	21:30
N3DMAA	568	ORD	FLL	7:25	2:55	10:20
	711	FLL	ORD	11:10	3:15	14:25
	2021	ORD	SJU	15:25	4:35	20:00
N544AA	2463	ORD	MCI	6:25	1:30	7:55
	754	MCI	ORD	8:40	1:30	10:10
	2321	ORD	DFW	11:15	2:35	13:50
	2356	DFW	ORD	14:40	2:20	17:00
	2487	ORD	DEN	17:50	2:45	20:35
N3EBAA	1565	ORD	MSP	6:40	1:30	8:10
	779	MSP	ORD	9:00	1:25	10:25
	779	ORD	SAN	11:35	4:20	15:55
	1358	SAN	ORD	16:45	3:55	20:40
	1358	ORD	BOS	21:50	2:05	23:55

Tail No	Flight No	Departure	Arrival	Departure Time	Flight Time	Arrival Time
N3ETAA	1704	ORD	EWR	6:35	2:05	8:40
	1883	EWR	ORD	9:30	2:40	12:10
	810	ORD	DCA	13:10	1:45	14:55
	2013	DCA	ORD	15:45	2:15	18:00
	2013	ORD	LAS	19:00	4:10	23:10
N3DYAA	1063	ORD	LAX	8:50	4:35	13:25
	874	LAX	ORD	14:30	4:15	18:45
	874	ORD	BOS	19:45	2:15	22:00
N3DRAA	1021	ORD	LAS	8:30	4:05	12:35
	1544	LAS	ORD	13:25	3:35	17:00
	1544	ORD	DCA	18:00	1:45	19:45
N5DXAA	1048	ORD	MIA	7:35	3:10	10:45
	1763	MIA	ORD	11:55	3:20	15:15
	1899	ORD	MIA	16:20	3:05	19:25
N454AA	2441	ORD	ATL	6:30	2:00	8:30
	1986	ATL	ORD	9:15	2:15	11:30
	1872	ORD	MCO	12:25	2:40	15:05
	1131	MCO	ORD	15:50	3:05	18:55
N4YMAA	1137	ORD	MSY	8:20	2:25	10:45
	1768	MSY	ORD	11:30	2:30	14:00
	1768	ORD	PHL	15:05	2:05	17:10
	1697	PHL	ORD	18:00	2:35	20:35
N467AA	1823	ORD	PBI	9:20	2:55	12:15
	2067	PBI	ORD	13:00	3:20	16:20
	2067	ORD	STL	17:15	1:10	18:25
	1186	STL	ORD	19:10	1:20	20:30
N536AA	2305	ORD	DFW	7:45	2:40	10:25
	2344	DFW	ORD	11:35	2:20	13:55
	1201	ORD	STL	14:50	1:05	15:55
	1815	STL	ORD	17:00	1:20	18:20
	1815	ORD	SLC	19:15	3:40	22:55
N420AA	1686	ORD	RDU	6:50	1:50	8:40
	2435	RDU	ORD	9:25	2:15	11:40
	2435	ORD	PHX	12:35	3:55	16:30
	1206	PHX	ORD	17:15	3:25	20:40
N546AA	1462	ORD	EWR	8:00	2:20	10:20
	1387	EWR	ORD	11:25	2:40	14:05
	1397	ORD	MCO	15:00	2:40	17:40
	1221	MCO	ORD	18:25	2:55	21:20
N4WPAA	2311	ORD	DFW	9:05	2:35	11:40
	2348	DFW	ORD	12:35	2:20	14:55
	1797	ORD	STL	15:50	1:10	17:00
	1982	STL	ORD	18:00	1:20	19:20
	1339	ORD	SAN	20:15	4:30	24:45
N5EBAA	2375	ORD	EGE	8:10	2:55	11:05
	2378	EGE	ORD	12:25	2:45	15:10
	1677	ORD	SNA	18:40	4:30	23:10
N3DUAA	2099	ORD	LAX	7:00	4:30	11:30
	1972	LAX	ORD	12:40	4:05	16:45
	1972	ORD	RDU	17:45	1:55	19:40
N3ELAA	2057	ORD	SJU	8:30	4:50	13:20
	2078	SJU	ORD	14:25	5:35	20:00
N3DTAA	2363	ORD	HDN	9:50	2:50	12:40
	2318	HDN	ORD	13:40	2:50	16:30
N412AA	2345	ORD	DFW	17:15	2:35	19:50
	2374	DFW	ORD	20:40	2:10	22:50

Table 5.5: Complete ORD Schedule

using the following formula:

$$\text{Cost Improvement} = 100 \cdot \frac{\text{Original Schedule - Proposed Model}}{\text{Original Schedule}}$$

where the percentage of cost increases are calculated using the negative of the same formula.

Factor A, i.e. the fuel price per ton of fuel, has significant effects on total cost and total fuel cost improvements. Since idle time cost contribution to total cost is lower than the fuel cost, the increase in unit fuel price ends in slightly lower idle cost improvements overall. The result is that our model achieves better total cost savings when fuel price is low as expected, but the performance of the model in improving idle time costs is only affected a little by fuel price.

An important result to see is that changing levels of Factor B does not have a statistically significant effect on model performance. This factor represents the allowed compression level on the cruise time of the flight. Similar improvement levels for different levels of compression means that cruise times are not compressed to the boundary even in the low setting. As an extension to this study, further analysis could be done to see if speed compression hits the boundaries when the exponent m used in the cost function is changed.

Factor C, i.e. β, shows another interesting result. It is observed that increase in fuel costs do not change for higher levels of β whereas all other cost improvements are decreased. The reason for less idle time cost improvements with increasing level of this factor is that higher β causes a higher variance in flight block times, which causes longer flight durations resulting in more idle time put in the schedule with higher overall idle time costs. Similarly idle times in the original schedule are utilized more, decreasing the unnecessary idle time costs of the original schedule. Overall fuel cost increase is not affected by β levels, so the decrease in idle time cost improvements reflect to total cost improvements as well.

Factor D, i.e. the connection density of the network, has a similar effect as Factor C. More passenger connected flights result in a higher need for idle times, and therefore total cost improvements decrease, since idle time cost savings is a strong advantage of our model.

Overall, it is important to observe that a 2% increase in fuel costs allows for a 60% improvement in total idle costs. This is because fuel cost is a huge part of total airline operational costs, and also cruise time controllability results in great savings from unecessary idle times.

Five replications for each factor combination is done to see if random values have any effect on objective values. The comparisons for cost improvements for different replications are given in Table 5.7, from where it can be seen values are not affected significantly from different random values.

Another measure of interest is the service level of the schedules. Results show that only significant factor affecting service level values is β. Overall average of the service level is 94%. A higher setting of β causes the average service levels to drop to 92.7%, where a lower setting of β results in service levels of 95.3% on the average. Note that these are actually good passenger connection performances for the published schedule, and our model can achieve less costs preserving these values. In fact, it will be shown later that our model achieves higher service levels if total cost is allowed to be as much as the original schedule costs.

5.1.1 Scenario Analysis

The extensive computational study results intrigues several questions on model behavior and performance. In this part, some insights into model dynamics are provided by realizing different scenarios.

5.1.1.1 What if time compression is not allowed?

Since the model works by balancing incorporating idle time into the schedule and speeding of the aircraft for flights, the performance of the model when speeding is not allowed is wondered. Fuel costs are expected to decrease to the same level with the costs of the original schedule, whereas idle costs are expected to

		Idle Time Cost Improvement			**Fuel Cost Increase**			**Total Cost Improvement**			**Total Cost Imp. Without Delay Costs**		
		Min	**Avg**	**Max**	**Min**	**Avg**	**Max**	**Min**	**Avg**	**Max**	**Min**	**Avg**	**Max**
A	0	56.8	66.4	76.2	2.2	2.8	3.7	14.1	17.6	21.1	13.8	17.4	21.1
	1	52.1	60.6	67.7	0.9	1.3	1.7	7.4	9.4	11.6	7.2	9.3	11.6
B	0	52.1	63.4	75.2	0.9	2.0	3.4	7.4	13.5	21.1	7.2	13.3	21.1
	1	52.1	63.7	76.2	0.9	2.1	3.7	7.4	13.5	21.1	7.2	13.4	21.1
C	0	57.8	65.7	76.2	0.9	2.1	3.7	9.4	14.8	21.1	9.4	14.7	21.1
	1	52.1	61.4	73.9	0.9	2.0	3.6	7.4	12.2	18.5	7.2	12.0	18.2
D	0	61.0	68.6	76.2	0.9	2.1	3.7	8.9	14.7	21.1	8.7	14.5	21.1
	1	52.1	58.5	64.0	1.3	2.0	3.1	7.4	12.3	17.7	7.2	12.2	17.6

Table 5.6: Comparison of Factor Effects (Values are in %)

	Idle Time Cost			Fuel Cost			Total Cost		
Rep.	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
1	56.2	63.8	76.2	0.9	2.1	3.7	7.8	13.6	20.9
2	54.8	64.8	75.6	0.9	2.0	3.1	7.8	13.9	21.1
3	54.4	62.8	75.8	1.0	2.0	3.5	7.6	13.4	20.9
4	52.1	62.8	74.9	1.1	2.2	3.6	7.4	13.3	20.6
5	56.4	63.4	74.4	1.1	2.1	3.3	7.8	13.4	20.7

Table 5.7: Cost comparison for different replications (%)

increase. The cost values are compared with the original schedule costs for two cases; the model with speeding allowed and not allowed. Since replications do not affect results, the comparison is done for a single set of replications, with 15% compression allowed in one case and compression is not allowed in the other case. The results in Table 5.8 show that even without cruise time controllability, a better utilization of idle times by the model result in important idle cost and total cost improvements. The reason for that is because our model acts as a global optimization tool, the network propagation effects could be considered resulting in a better placement of idle times.

			With Compression			No Compression		
A	C	D	Fuel	Idle Time	Total	Fuel	Idle Time	Total
0	0	0	3.2%	74.4%	20.7%	-	58.9%	18.1%
0	0	1	2.7%	63.2%	17.6%	-	44.0%	13.6%
0	1	0	3.3%	68.6%	16.9%	-	53.7%	14.8%
0	1	1	2.3%	58.8%	14.9%	-	42.3%	11.8%
1	0	0	1.1%	66.9%	11.3%	-	58.9%	10.7%
1	0	1	1.5%	59.2%	9.6%	-	44.0%	8.0%
1	1	0	1.4%	61.3%	8.9%	-	52.5%	8.4%
1	1	1	1.7%	56.4%	7.8%	-	39.2%	6.5%

Table 5.8: Computation results when compression is not allowed

As expected, fuel cost increases when speeding option is available, which also gives an advantage to balance the amount of idle time put into schedule, therefore resulting in better idle cost savings.

5.1.1.2 What if variability was higher?

Another issue to wonder is what will happen to model performance when variability was further increased. The expectation is that benefits of the model will be less significant as in the case of different beta values in Table 5.6 and service levels will be much lower. For this analysis, Factor C is taken to be 0.07, which is the highest possible value such that each $\beta_i > 1$ for flight $i \in J$ needed to have finite expected values as proved in Proposition 3.1. All other factors are taken as their low level values. Computation for a single parameter set resulted in a service level of 0.88, which is very low compared to average service levels that were achieved previously. Delay costs of the original schedule increased significantly since higher variance caused the block times of flights to increase. Also in the same case, when total cost of the model is taken to be equal to original schedule total cost, the model resulted in a service level of 0.98.

5.1.2 Aircraft Utilization

Computational results proved some additional benefits apart from the objective values. The results showed that for the available 31 aircraft paths in the data, the model improves makespan for almost all paths, and there is a timewise improvement in average makespan in all factor combinations. The average makespan improvement is taken for each different factor and replication combination. The mean of this improvement over all cases is 41.5 minutes, with a minimum of 28 minutes and a maximum of 59 minutes achieved in one case. Average number of paths for which the makespan improved is 30.5, with a minimum of 28 paths and a maximum of 31 paths, meaning all available paths. This gives the airlines the flexibility to add additional flights to the end of aircraft paths, which may not be possible for the original schedule. Lessening of idle times affect not only the costs, but also creates additional utilization opportunities.

5.2 Schedule for 4-Hub Data

To generate this schedule, data was taken from the BTS database and was filtered to include aircrafts which originate their first flight from four different hub locations and return to them at least once the same day. This way, we could consider a schedule for 4-hub locations. The airports that are considered as hubs are Dallas-Fort Worth International Airport (DFW), Chicago O'Hare International Airport (ORD), Miami International Airport (MIA) and New York John F. Kennedy International Airport (JFK). This schedule has 469 flights operated by 141 different aircraft.

In Table 5.9 a comparison among different cost components between model objectives and original schedule are given. The comparison is only done for two factors A and C which are the fuel cost and β, respectively. Factor B, i.e. speed compression, is taken as 15% for all runs since we saw that compression level does not affect results as the compressions on flights did not hit the boundaries. Similarly, factor D, the connection density is taken as 50% throughout, since there are many possible connections in a 4-hub problem and 100% passenger connection is not realistic. We showed that replications did not affect results in the single hub study. Still, we did 3 replications for each factor combination when calculating the results. We also did not calculate total cost improvement without delay costs seperately in this case since it was shown earlier that the model has cost improvement even when delay costs are taken as zero.

		Idle Time Cost Improvement			**Fuel Cost Increase**			**Total Cost Improvement**		
		Min	**Avg**	**Max**	**Min**	**Avg**	**Max**	**Min**	**Avg**	**Max**
A	0	59.0	61.1	62.8	2.7	2.8	3.1	14.6	15.6	16.4
	1	52.2	54.6	56.4	1.1	1.2	1.3	7.7	8.2	8.8
C	0	55.9	59.3	62.8	1.2	2.0	3.1	8.6	12.5	16.4
	1	52.2	56.4	60.7	1.1	2.0	2.9	7.7	11.3	14.9

Table 5.9: Comparison of Factor Effects (Values are in %)

It can be seen that results are very promising in the 4-hub case as well. Idle time cost savings are approximately 60%, where fuel cost increase is approximately 2%. This result is very similar to the single hub case. Intiutively, one might think that increasing the size of the schedule will deteriorate improvement percentages. This is because the bulk cost from fuel will increase highly when number of flights go up and improvements in idle time costs may not be as significant as before. However, the results show that cost savings from idle time even out nicely throughout the schedule and the improvement rates are not affected badly from increasing data size.

The results show that idle time cost improvement is decreased by increasing unit fuel costs. This is because speeding becomes more expensive in this case, and the model depends more on idle time to achieve robustness. Similarly, less speeding means lesser increase in total fuel costs, which can be observed in the results as well. Total cost improvement is also affected negatively from increasing fuel costs. Around 80% of total costs are from fuel costs, so doubling the unit fuel cost caused total cost improvements to be almost half of the previous values.

Results for Factor C are similar to the outcomes in the single hub schedule as well. Fuel cost increase is not affected by increasing β, where idle time and total cost improvements are decreased. As stated previously, this is because increased variability results in more idle time inserted in the schedule. Also, the amount of unnecessary idle time in the original schedule is decreased since with higher variability, the idle times are utilized better. The decrease in idle time cost improvements also reflect to total cost improvements.

Changing levels of β also affects the service level of the schedules. For the case of low β, the average service level of the schedules is 0.93, where it is 0.96 for the case of high β setting. It is reasonable that higher variability results in smaller service levels.

It is also interesting to observe how the additional utilization arising from shortened makespans change for the 4-hub schedule case. For the 141 paths, there were time savings on 135 of the paths on the average. The average of this improvement over all cases is 33 minutes, with a minimum of 25 minutes and a maximum of 45 minutes achieved. This craeates additional aircraft utilization opportunities, like adding a new flight at the end of a given path.

5.3 Computation Time Analysis

All computations are conducted on an Intel Core i5 2410M computer with 2.30Ghz processor and 4.00 GB RAM. Problem is modeled in Java language using IBM ILOG Cplex Optimizer. Model is solved by CPLEX 12.1. In the following subsections, time analysis for the single hub and 4-hub schedules are provided.

5.3.1 Single Hub Study

Computation times are very reasonable for all factor settings. Overall average, minimum and maximum values of computation times in CPU seconds can be seen in Table 5.10. If we analyze the results we see that factors A and B do not affect

		Cpu Time (sec.)		
Factor	Level	Min	Avg	Max
A	0	2.4	6.2	12.1
	1	2.3	6.1	12.1
B	0	2.3	6.2	12.1
	1	2.4	6.1	11.7
C	0	2.3	3.9	6.4
	1	4.7	8.4	12.1
D	0	2.3	4.0	6.9
	1	4.7	8.3	12.1

Table 5.10: CPU time analysis for the single hub schedule

computation times. Factor A is the unit fuel cost and it is simply a coefficient term in the model so changing it does not change computation time. Factor B is the compression level which is the maximum amount of compression allowed for flights. We observed that compression in flights did not hit this boundary even in the low setting. Therefore, the change of this compression cap does not have any affect on computation times.

The results are different for factors C and D however. Factor C is β, and increasing it has an affect on problem complexity as variability increases. Factor D corresponds to connection level. Increasing it from 50% to 100% doubles the amount of passenger connected flights and increasing the number of connections also increases the problem complexity. To visualize this, flights can be considered as nodes and that there will be an arc for every passenger connection.

Overall, the average time for all runs is 6.6 CPU seconds. This is a very good result for a problem of that size having 31 paths and 114 flights. As it can be seen, the second order conic formulation of the chance constraints result in exact and fast solutions.

5.3.2 4-Hub Study

Computation results prove to be very good time wise for the 4-hub case as well. The size of the problem in this case is 141 paths and 469 flights. Overall average, minimum and maximum values of computation times in CPU seconds can be seen in Table 5.11. The average time for all runs is 47.5 CPU seconds in this case. It

		Cpu Time (sec.)		
Factor	Level	Min	Avg	Max
A	0	30	48.1	65.6
	1	31.9	47.4	62.6
C	0	30	32.4	34.2
	1	59.7	63	65.6

Table 5.11: CPU time analysis for the 4-hub schedule

can be observed that changing unit fuel costs does not affect computation times. However, changing β significantly affects times, almost doubling them. This is reasonable as β affects variability and increases problem complexity where unit fuel costs are merely a coefficient in the model and do not add complexity to the problem.

5.4 Summary

This chapter was devoted to the computational study of the proposed model. First, the parameters and factors that are used in the study were described and their chosen values are explained. The study that was conducted on airport congestion coefficients was presented, which provides a measure of the congestion of a given airport.

Afterwards, the computational study that was conducted on the published daily schedule of an US carrier filtered for single hub is presented. The schedule that was output from our model and the original published schedule are compared in cost terms and service levels. Following that, a scenario analysis is presented where model performance is evaluated for more extreme cases of data.

Finally, computational studies were also conducted on a 4-hub daily schedule and fast solutions were achieved to this larger problem as well. The chapter is concluded with CPU time analysis done separately for two different sizes of schedules.

Chapter 6

Conclusions and Future Work

Airline scheduling is a complex process that involves series of operations such as schedule generation, fleet assignment, aircraft maintenance routing and crew assignment. Since airline operations are expensive, global optimization tools to generate accurate schedules are very important for airline companies. Schedules are usually generated with the assumption that everything goes as planned aiming maximum utilization and minimum costs. However in the airline industry, disruptions are very expensive. For example, even a simple delay in a flight results in crews and a lot of passengers missing their connecting flights and wastes operational time of the aircraft. So uncertainties need to be foreseen and schedules should be generated to be less vulnerable to these.

Optimization in airline industry shifted from deterministic models to models with uncertainty that can work under different realizations of data conditions. Main tools for managing disruptions in airlines are building robust schedules or building rescheduling models that aim to recover the schedule after a disruption. Robust schedules are less fragile to disruptions and they are easier to recover if a disruption occurs. The trade-off for incorporating this flexibility into the schedules is increasing costs. For example, putting slack time into a schedule to have a buffer for delays results in expensive and otherwise profit generating aircraft to be sitting idle for that period of time. The aim is to find a balance of robustness and costs.

This thesis is on a robust airline scheduling model that uses tools such as idle time insertion and cruise speed controllability on a combined network of aircraft and passenger connecting flight information to guarantee given connection service levels with minimum costs. This study has many contributions to airline scheduling and optimization literature.

6.1 Summary of Thesis

We developed a global optimization tool that satisfies given passenger connection levels and avoids flight delays while minimizing the sum of fuel and idle time costs. The method used in achieving this is balancing scheduled idle time with cruise time controllability, and as far as we know this is the first time that these mechanics are combined. Fuel costs are nonlinear functions of flight speed where idle time costs are constant for unit time spent. Structure of these cost functions allows for achieving less costs when these tools are combined compared to employing them alone. Moreover, idle time in a schedule means loss of efficiency and not utilizing aircraft for that period. Combining speed controllability with idle time insertion results in less idle times put in the schedule, which allows for increased aircraft utilization.

The contributions of the study are not limited to that. Modeling the variability in the data is one of the major parts of robust schedule building. In this thesis, flight block times are considered in two separate parts as cruise and non-cruise times. Cruise times are subject to controllability where variability exists in non-cruise times. The variability in non-cruise times are assumed to be Loglaplace random variables and are modeled with chance constraints on passenger connection service levels. These chance constraints are then expressed in their closed form equations and then they are transfered to second order conic equations. Previous literature has examples of chance constrained robust scheduling models but these constraints were solved using approximations. By using a conic formulation, we can solve even large problems as a 4-hub daily schedule exactly and under minutes of time.

Another important contribution in this study is that the congestion information of airports are used to arrange the aircraft turnaround times and the variability on each flight. Each airport is assigned a congestion coefficient. For every flight, the non-cruise time variability is adjusted using the congestion coefficients of the origin and destination airport of the flight by changing the random variable parameter accordingly. For the turnaround time, the congestion coefficient of the turnaround location is used.

Remember that maintaining a desired level of passenger connection service levels is one of the objectives of this study. However, passenger connection service levels are allowed to be different for each flight as long as their weighted average is above the desired level. This gives a better chance to assess these connections according to their priority. We chose the number of connecting passengers as a measure to assign weights. By our methodology, flights with more connecting passengers are prioritized to achieve higher service levels since they have higher weights.

Since cruise speed decisions highly affect the network and needs to be decided globally, the tool we developed will prove many benefits for airlines. Moreover, any airline practitioner can run this model using commercial solvers in seconds of time. The model will help reducing the effects of propagated delay and guarantee desired passenger connection service levels.

6.2 Future Work

The benefits of this research can be extended forward with further research. A direct extension to our study would be a modification of the random variable distribution. Following the research in Arıkan and Deshpande (2012), the random variable in this study is assumed to be a Loglaplace random variable. However, it is also mentioned in the thesis that there are other distribution functions that have closed form expressions that would allow them to be represented as conic equations. Therefore, our work could be extended to see the properties and performance of the model under a different random distribution.

We presented a model that offers a robust schedule with minimized costs and guaranteed service levels using the tools of idle time insertion and cruise time controllability. In our work, it is assumed that flight routing decisions and fleet assignments are known a priori. Another important area for development is incorporating routing and fleet assignment decisions to the decision process as well. In our model, the flight sequences of aircraft paths are assumed to be known. The results show that the computation times for the model are small, which is a very big advantage for extending the model to include routing decisions and aircraft assignments. If the conic structure of our model could be preserved, exact solutions to this enlarged problem may be obtained in acceptable times. Another option could be to combine our methodology with a heuristic to solve the extended problem in short time if solving exactly turns out to be time demanding.

In this airline robust scheduling study, we tried to control the variability in the system with chance constraints. Other methods could be applied to treat the uncertainty. Scenario analysis which analyzes a large number of cases before finalizing the decision is a widely used technique to deal with uncertainty. Stochastic programming that considers a number of different scenarios at once to output a solution can also be an interesting point of view to the problem.

The airline scheduling process is a large sized complex process with many different subproblems. It is important to integrate as much of these subproblems to obtain better solutions that address the whole network with all interacting parts. For example, crew costs are a major part of the operating costs in airline industry. In our work, we did not take crew assignments and crew schedules into consideration. A problem integrating crew scheduling problem with ours that employs idle time insertion and cruise time controllability is a promising problem for future studies.

A possible other work is to integrate the aircraft maintenance routing problem. It is required by regulations that all aircraft go to maintenance after flying for a period of time. We did not consider the maintenance requirements in our model since we work with daily schedules. However, it would be a good extension for models that consider schedules with extended periods such as weekly or monthly since the model works with changing departure times of the flights.

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Appendix A

Computational Results

A.1 Single Hub Study

Table A.1: Costs for the schedule generated by the model

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
1	0	0	0	0	1	676,830	72,304	749,135
2	0	0	0	0	2	673,756	72,054	745,811
3	0	0	0	0	3	675,525	72,901	748,427
4	0	0	0	0	4	674,135	78,555	752,689
5	0	0	0	0	5	673,870	76,537	750,407
6	0	0	0	1	1	670,980	110,272	781,252
7	0	0	0	1	2	673,758	104,973	778,731
8	0	0	0	1	3	669,672	113,041	782,713
9	0	0	0	1	4	672,819	108,669	781,489
10	0	0	0	1	5	670,666	108,632	779,298
11	0	0	1	0	1	675,436	72,977	748,413
12	0	0	1	0	2	674,562	65,304	739,886

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Table A.1 – continued from previous page

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
13	0	0	1	0	3	673749	80491	754240
14	0	0	1	0	4	674,170	77,101	751,271
15	0	0	1	0	5	674,329	81,010	755,339
16	0	0	1	1	1	669,073	103,809	772,882
17	0	0	1	1	2	670,643	103,234	773,878
18	0	0	1	1	3	668,937	107,594	776,531
19	0	0	1	1	4	672,266	107,022	779,289
20	0	0	1	1	5	669,418	102,903	772,321
21	0	1	0	0	1	678,970	69,154	748,124
22	0	1	0	0	2	675,000	70,728	745,728
23	0	1	0	0	3	677,551	70,296	747,848
24	0	1	0	0	4	678,075	72,832	750,907
25	0	1	0	0	5	675,641	74,254	749,895
26	0	1	0	1	1	670,986	110,270	781,257
27	0	1	0	1	2	674,086	104,542	778,629
28	0	1	0	1	3	671,102	111,506	782,608
29	0	1	0	1	4	675,022	106,369	781,392
30	0	1	0	1	5	672,205	106,851	779,056
31	0	1	1	0	1	677,280	70,519	747,799
32	0	1	1	0	2	674,704	65,098	739,802
33	0	1	1	0	3	674,703	79,251	753,955
34	0	1	1	0	4	678,403	71,042	749,445
35	0	1	1	0	5	676,267	78,198	754,465
36	0	1	1	1	1	670,052	102,757	772,809
37	0	1	1	1	2	670,882	102,954	773,837
38	0	1	1	1	3	669,282	107,229	776,511
39	0	1	1	1	4	672,378	106,905	779,284
40	0	1	1	1	5	669,587	102,702	772,289

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Table A.1 – continued from previous page

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
41	1	0	0	0	1	1,321,527	98,288	1,419,816
42	1	0	0	0	2	1,321,144	93,661	1,414,805
43	1	0	0	0	3	1,322,966	95,993	1,418,959
44	1	0	0	0	4	1,323,616	98,994	1,422,610
45	1	0	0	0	5	1,324,223	95,982	1,420,205
46	1	0	0	1	1	1,327,334	120,754	1,448,088
47	1	0	0	1	2	1,326,078	119,705	1,445,783
48	1	0	0	1	3	1,328,866	120,953	1,449,820
49	1	0	0	1	4	1,327,542	122,591	1,450,134
50	1	0	0	1	5	1,329,085	118,435	1,447,520
51	1	0	1	0	1	1,323,509	93,801	1,417,310
52	1	0	1	0	2	1,321,717	86,422	1,408,139
53	1	0	1	0	3	1,324,605	97,195	1,421,800
54	1	0	1	0	4	1,326,665	93,902	1,420,568
55	1	0	1	0	5	1,327,493	96,536	1,424,029
56	1	0	1	1	1	1,331,000	109,229	1,440,230
57	1	0	1	1	2	1,327,779	112,807	1,440,587
58	1	0	1	1	3	1,329,337	113,726	1,443,063
59	1	0	1	1	4	1,327,220	119,493	1,446,714
60	1	0	1	1	5	1,331,327	108,576	1,439,903
61	1	1	0	0	1	1,322,213	97,357	1,419,570
62	1	1	0	0	2	1,321,169	93,659	1,414,828
63	1	1	0	0	3	1,322,965	95,993	1,418,958
64	1	1	0	0	4	1,324,454	97,933	1,422,387
65	1	1	0	0	5	1,324,239	95,981	1,420,220
66	1	1	0	1	1	1,327,352	120,753	1,448,106
67	1	1	0	1	2	1,326,076	119,705	1,445,781
68	1	1	0	1	3	1,328,855	120,955	1,449,810

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Table A.1 – continued from previous page

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
69	1	1	0	1	4	1,327,520	122,594	1,450,114
70	1	1	0	1	5	1,329,074	118,435	1,447,509
71	1	1	1	0	1	1,323,807	93,418	1,417,225
72	1	1	1	0	2	1,321,733	86,421	1,408,154
73	1	1	1	0	3	1,324,625	97,206	1,421,832
74	1	1	1	0	4	1,327,228	93,328	1,420,556
75	1	1	1	0	5	1,327,502	96,538	1,424,040
76	1	1	1	1	1	1,331,023	109,219	1,440,242
77	1	1	1	1	2	1,327,759	112,809	1,440,569
78	1	1	1	1	3	1,329,339	113,726	1,443,066
79	1	1	1	1	4	1,327,211	119,494	1,446,705
80	1	1	1	1	5	1,331,322	108,576	1,439,898

Table A.2: Costs for the original published schedule

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
1	0	0	0	0	1	654,808	290,244	945,558
2	0	0	0	0	2	654,808	290,244	945,558
3	0	0	0	0	3	654,808	290,244	945,558
4	0	0	0	0	4	654,808	290,244	945,558
5	0	0	0	0	5	654,808	290,244	945,558
6	0	0	0	1	1	654,808	290,244	945,558
7	0	0	0	1	2	654,808	290,244	945,558
8	0	0	0	1	3	654,808	290,244	945,558
9	0	0	0	1	4	654,808	290,244	945,558
10	0	0	0	1	5	654,808	290,244	945,558
11	0	0	1	0	1	654,808	249,306	907,662

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Table A.2 – continued from previous page

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
12	0	0	1	0	2	654,808	249,306	907,662
13	0	0	1	0	3	654,808	249,306	907,662
14	0	0	1	0	4	654,808	249,306	907,662
15	0	0	1	0	5	654,808	249,306	907,662
16	0	0	1	1	1	654,808	249,306	907,662
17	0	0	1	1	2	654,808	249,306	907,662
18	0	0	1	1	3	654,808	249,306	907,662
19	0	0	1	1	4	654,808	249,306	907,662
20	0	0	1	1	5	654,808	249,306	907,662
21	0	1	0	0	1	654,808	290,244	945,558
22	0	1	0	0	2	654,808	290,244	945,558
23	0	1	0	0	3	654,808	290,244	945,558
24	0	1	0	0	4	654,808	290,244	945,558
25	0	1	0	0	5	654,808	290,244	945,558
26	0	1	0	1	1	654,808	290,244	945,558
27	0	1	0	1	2	654,808	290,244	945,558
28	0	1	0	1	3	654,808	290,244	945,558
29	0	1	0	1	4	654,808	290,244	945,558
30	0	1	0	1	5	654,808	290,244	945,558
31	0	1	1	0	1	654,808	249,306	907,662
32	0	1	1	0	2	654,808	249,306	907,662
33	0	1	1	0	3	654,808	249,306	907,662
34	0	1	1	0	4	654,808	249,306	907,662
35	0	1	1	0	5	654,808	249,306	907,662
36	0	1	1	1	1	654,808	249,306	907,662
37	0	1	1	1	2	654,808	249,306	907,662
38	0	1	1	1	3	654,808	249,306	907,662
39	0	1	1	1	4	654,808	249,306	907,662

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Table A.2 – continued from previous page

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
40	0	1	1	1	5	654,808	249,306	907,662
41	1	0	0	0	1	1,309,616	290,244	1,600,367
42	1	0	0	0	2	1,309,616	290,244	1,600,367
43	1	0	0	0	3	1,309,616	290,244	1,600,367
44	1	0	0	0	4	1,309,616	290,244	1,600,367
45	1	0	0	0	5	1,309,616	290,244	1,600,367
46	1	0	0	1	1	1,309,616	290,244	1,600,367
47	1	0	0	1	2	1,309,616	290,244	1,600,367
48	1	0	0	1	3	1,309,616	290,244	1,600,367
49	1	0	0	1	4	1,309,616	290,244	1,600,367
50	1	0	0	1	5	1,309,616	290,244	1,600,367
51	1	0	1	0	1	1,309,616	249,306	1,562,471
52	1	0	1	0	2	1,309,616	249,306	1,562,471
53	1	0	1	0	3	1,309,616	249,306	1,562,471
54	1	0	1	0	4	1,309,616	249,306	1,562,471
55	1	0	1	0	5	1,309,616	249,306	1,562,471
56	1	0	1	1	1	1,309,616	249,306	1,562,471
57	1	0	1	1	2	1,309,616	249,306	1,562,471
58	1	0	1	1	3	1,309,616	249,306	1,562,471
59	1	0	1	1	4	1,309,616	249,306	1,562,471
60	1	0	1	1	5	1,309,616	249,306	1,562,471
61	1	1	0	0	1	1,309,616	290,244	1,600,367
62	1	1	0	0	2	1,309,616	290,244	1,600,367
63	1	1	0	0	3	1,309,616	290,244	1,600,367
64	1	1	0	0	4	1,309,616	290,244	1,600,367
65	1	1	0	0	5	1,309,616	290,244	1,600,367
66	1	1	0	1	1	1,309,616	290,244	1,600,367
67	1	1	0	1	2	1,309,616	290,244	1,600,367

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Table A.2 – continued from previous page

Run	Factors				Replication	Costs		
#	A	B	C	D	#	Speeding	Idle Time	Total
68	1	1	0	1	3	1,309,616	290,244	1,600,367
69	1	1	0	1	4	1,309,616	290,244	1,600,367
70	1	1	0	1	5	1,309,616	290,244	1,600,367
71	1	1	1	0	1	1,309,616	249,306	1,562,471
72	1	1	1	0	2	1,309,616	249,306	1,562,471
73	1	1	1	0	3	1,309,616	249,306	1,562,471
74	1	1	1	0	4	1,309,616	249,306	1,562,471
75	1	1	1	0	5	1,309,616	249,306	1,562,471
76	1	1	1	1	1	1,309,616	249,306	1,562,471
77	1	1	1	1	2	1,309,616	249,306	1,562,471
78	1	1	1	1	3	1,309,616	249,306	1,562,471
79	1	1	1	1	4	1,309,616	249,306	1,562,471
80	1	1	1	1	5	1,309,616	249,306	1,562,471

Table A.3: Service levels and CPU times

Run	Factors				Replication		
#	A	B	C	D	#	Service Level	CPU Time
1	0	0	0	0	1	0.96	2.372
2	0	0	0	0	2	0.93	2.543
3	0	0	0	0	3	0.98	2.589
4	0	0	0	0	4	0.95	2.387
5	0	0	0	0	5	0.97	2.386
6	0	0	0	1	1	0.94	4.82
7	0	0	0	1	2	0.94	5.195
8	0	0	0	1	3	0.96	5.569
9	0	0	0	1	4	0.96	5.804
10	0	0	0	1	5	0.94	5.272

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Table A.3 – continued from previous page

Run	Factors				Replication		
#	A	B	C	D	#	Service Level	CPU Time
11	0	0	1	0	1	0.93	5.258
12	0	0	1	0	2	0.9	5.304
13	0	0	1	0	3	0.95	6.38
14	0	0	1	0	4	0.94	6.287
15	0	0	1	0	5	0.94	5.944
16	0	0	1	1	1	0.91	10.234
17	0	0	1	1	2	0.92	11.778
18	0	0	1	1	3	0.93	11.747
19	0	0	1	1	4	0.93	12.106
20	0	0	1	1	5	0.92	10.889
21	0	1	0	0	1	0.96	2.45
22	0	1	0	0	2	0.93	2.371
23	0	1	0	0	3	0.98	2.527
24	0	1	0	0	4	0.95	2.48
25	0	1	0	0	5	0.97	2.496
26	0	1	0	1	1	0.94	4.868
27	0	1	0	1	2	0.94	5.148
28	0	1	0	1	3	0.96	5.273
29	0	1	0	1	4	0.96	5.678
30	0	1	0	1	5	0.94	5.881
31	0	1	1	0	1	0.93	5.132
32	0	1	1	0	2	0.9	4.945
33	0	1	1	0	3	0.95	6.895
34	0	1	1	0	4	0.94	5.21
35	0	1	1	0	5	0.94	6.1
36	0	1	1	1	1	0.91	10.265
37	0	1	1	1	2	0.92	11.498
38	0	1	1	1	3	0.93	11.435

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Table A.3 – continued from previous page

Run	Factors				Replication		
#	A	B	C	D	#	Service Level	CPU Time
39	0	1	1	1	4	0.93	11.513
40	0	1	1	1	5	0.92	11.076
41	1	0	0	0	1	0.96	2.34
42	1	0	0	0	2	0.93	2.449
43	1	0	0	0	3	0.98	2.464
44	1	0	0	0	4	0.95	2.433
45	1	0	0	0	5	0.97	2.512
46	1	0	0	1	1	0.94	5.133
47	1	0	0	1	2	0.94	5.397
48	1	0	0	1	3	0.96	5.133
49	1	0	0	1	4	0.96	6.38
50	1	0	0	1	5	0.94	5.429
51	1	0	1	0	1	0.93	4.882
52	1	0	1	0	2	0.9	4.727
53	1	0	1	0	3	0.95	5.912
54	1	0	1	0	4	0.94	5.101
55	1	0	1	0	5	0.94	5.039
56	1	0	1	1	1	0.91	11.544
57	1	0	1	1	2	0.92	11.029
58	1	0	1	1	3	0.93	12.137
59	1	0	1	1	4	0.93	12.028
60	1	0	1	1	5	0.92	11.045
61	1	1	0	0	1	0.96	2.528
62	1	1	0	0	2	0.93	2.761
63	1	1	0	0	3	0.98	2.59
64	1	1	0	0	4	0.95	2.387
65	1	1	0	0	5	0.97	2.543
66	1	1	0	1	1	0.94	5.195

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Table A.3 – continued from previous page

Run	Factors				Replication		
#	A	B	C	D	#	Service Level	CPU Time
67	1	1	0	1	2	0.94	4.664
68	1	1	0	1	3	0.96	5.258
69	1	1	0	1	4	0.96	5.491
70	1	1	0	1	5	0.94	6.1
71	1	1	1	0	1	0.93	6.037
72	1	1	1	0	2	0.9	4.789
73	1	1	1	0	3	0.95	6.131
74	1	1	1	0	4	0.94	5.148
75	1	1	1	0	5	0.94	5.367
76	1	1	1	1	1	0.91	11.435
77	1	1	1	1	2	0.92	11.372
78	1	1	1	1	3	0.93	11.669
79	1	1	1	1	4	0.93	10.28
80	1	1	1	1	5	0.92	10.436

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Table A.4: Costs for the schedule generated by the model

Run	Factors		Replication	Costs		
#	A	C	#	Speeding	Idle Time	Total
1	0	0	1	2,876,478	422,594	3,299,072
2	0	0	2	2,877,855	428,477	3,306,333
3	0	0	3	2,885,933	418,306	3,304,240
4	0	1	1	2,875,377	399,674	3,275,052
5	0	1	2	2,875,285	409,261	3,284,546
6	0	1	3	2,882,122	392,245	3,274,367

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Table A.4 – continued from previous page

Run	Factors		Replication	Costs		
#	A	C	#	Speeding	Idle Time	Total
7	1	0	1	5,664,672	490,205	6,154,877
8	1	0	2	5,665,008	496,258	6,161,266
9	1	0	3	5,673,900	491,905	6,165,805
10	1	1	1	5,661,093	466,831	6,127,925
11	1	1	2	5,659,872	476,310	6,136,182
12	1	1	3	5,668,645	463,844	6,132,489

Table A.5: Costs for the original published schedule

Run	Factors		Replication	Costs		
#	A	C	#	Speeding	Idle Time	Total
1	0	0	1	2,799,926	1,124,293	3,947,806
2	0	0	2	2,799,926	1,124,293	3,947,806
3	0	0	3	2,799,926	1,124,293	3,947,806
4	0	1	1	2,799,926	997,142	3,847,139
5	0	1	2	2,799,926	997,142	3,847,139
6	0	1	3	2,799,926	997,142	3,847,139
7	1	0	1	5,599,852	1,124,292	6,747,733
8	1	0	2	5,599,852	1,124,292	6,747,733
9	1	0	3	5,599,852	1,124,292	6,747,733
10	1	1	1	5,599,852	997,142	6,647,065
11	1	1	2	5,599,852	997,142	6,647,065
12	1	1	3	5,599,852	997,142	6,647,065

Table A.6: Service levels and CPU times

Run	Factors		Replication		
#	A	C	#	Service Level	CPU Time
1	0	0	1	0.96	30
2	0	0	2	0.97	31.7
3	0	0	3	0.95	32.6
4	0	1	1	0.93	63.7
5	0	1	2	0.94	65.6
6	0	1	3	0.93	64.7
7	1	0	1	0.96	34
8	1	0	2	0.97	31.9
9	1	0	3	0.95	34.2
10	1	1	1	0.93	59.7
11	1	1	2	0.94	62.6
12	1	1	3	0.93	61.7

Appendix B

Quantile Functions

Even though we study loglaplace random variable following the research in Arıkan and Deshpande (2012), it is possible to use another probability distribution to denote non-cruise times. We derived the quantile functions of several basic distributions to be a reference for future studies. Note that not all of them are conic representable. Ben-Tal and Nemirovski (2001) is an excellent source for the properties of conic representable functions.

Exponential Distribution

If X is exponential with parameter λ:

$$F^{-1}(p, \lambda) = \frac{-\ln(1-p)}{\lambda}$$

for $0 \leq p \leq 1$.

Weibull Distribution

If X is Weibull with scale parameter λ and shape parameter k:

$$F^{-1}(p, \lambda, k) = \lambda \sqrt[k]{-\ln(1-p)}$$

for $0 \leq p \leq 1$.

Cauchy Distribution

If X is Cauchy with location parameter α and scale parameter β:

$$F^{-1}(p, \alpha, \beta) = \alpha - \frac{\beta}{\tan(\pi p)}$$

for $0 \leq p \leq 1$.

Lognormal Distribution

If X is lognormal with parameters α and β (here α is the mean and β is the standard deviation);

$$F_X(x) = \Phi\left(\frac{\ln(x) - \alpha}{\beta}\right)$$

where ϕ is the standard normal cdf. If we plug the right side in the chance constraint we get:

$$F_X(x) = \Phi\left(\frac{\ln(x_j - x_i - TP_{ij} - s_i - L_{t_i}(\mu_{t_i i})) - \alpha}{\beta}\right) = \gamma_{ij}$$

In Integral Form

For X being a lognormal random variable with parameters α and β:

$$Pr(X < k) = \int_{-\infty}^{k} \frac{1}{x\beta\sqrt{2\pi}} e^{\frac{-(\ln(x)-\alpha)^2}{2\beta^2}} \, \mathrm{d}x$$

Quantile Function

$$F^{-1}(p) = \int_0^p \left(\frac{1}{\beta\sqrt{2\pi}} \exp\left(\frac{-1}{2}\left(\frac{\ln(p) - \alpha}{\beta}\right)^2\right)\right) dp$$

Gamma Distribution

If X is gamma with parameters α and β, the cumulative distribution function is:

$$F(x, \alpha, \beta) = \int_0^x \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-x\beta}$$

which can be written as:

$$F(x, \alpha, \beta) = \frac{\gamma(\alpha, x/\beta)}{\Gamma(\alpha)} = \frac{\int_0^{x/\beta} t^{\alpha-1} e^{-t} dt}{\Gamma(\alpha)}$$

Then, we need to put the right side of the constraint in place of x. There is no closed-form expression for the Gamma quantile.

Vita

Aslıgül Serasu Duran was born on April 19, 1989 in Trabzon, Turkey. She graduated from Ankara Atatürk Anadolu Lisesi in 2005. She attended Bilkent University and graduated from the Industrial Engineering Department with high honors in 2010. In 2010, she attended Industrial Engineering Department of Bilkent University as a research assistant. Since then, she has been working with Prof. M. Selim Aktürk on her graduate study. She had been on the grant 2210 awarded by The Scientific and Technological Research Council of Turkey (TUBITAK) during her M.S. study.