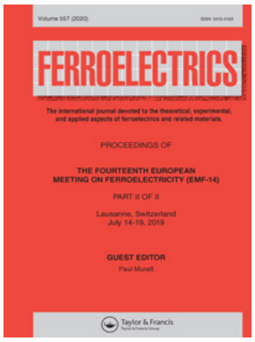

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$BaTiO_3$ based photonic time crystal and momentum stop band

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ABSTRACT

Temporally periodic photonic crystals develop an ω-k dispersion relation with momentum band gaps. While conventional photonic crystals induce forbidden bands in the frequency spectrum of photons, photonic time crystals create forbidden regions in the momentum spectrum of photons. This effect allows for enhanced control over many optical processes that require both photonic energy and momentum conservations such as nonlinear harmonic generation. The simulation results show that more intensive scatter fields can obtained in photonic space time crystal. Also, we investigate topological phase transitions of photonic time crystals systems.

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Photonic time crystal; finite element method; momentum stop band

1. Introduction

Photonic crystals (PhC) consist of spatially varying structures that are periodic on the order of the wavelength of the incident light. They exhibit an ω-k dispersion relation with the forbidden frequency zones commonly used in spontaneous emission inhibition, photon localization, high Q cavities, and wave guiding [1]. It is well known that in wave equations, space and time play similar roles, allowing for temporally periodic materials to provide optical properties complementary to the effects demonstrated by spatially periodic materials [2]. Inspired by a spice-time duality, in a series of recent papers Wilczek and Shapera considered the possibility that either classical or quantum system may display periodic motion in their lowest-energy state, that forming a time analog of crystalline spatial order - the concept of time crystal (TC) [3–6]. Since then, many researchers have made further investigations. On the other hand, TCs are the temporal analog of the PhCs [7–10]. Usually, PhCs are designed with a refractive index $n(r)$ that changes periodically in space. However, due to the unique duality of time and space in Maxwell's equations, one can think of a "photonic time-crystal" (PhTC), where the refractive index changes periodically in time, rather than in space [2–6]. Consequently, a sudden temporal change in the permittivity, ε causes time reflections, similar to a sudden change of ε in space, that causes spatial reflections [11]. When inducing time reflections in a periodic manner, one obtains interference between

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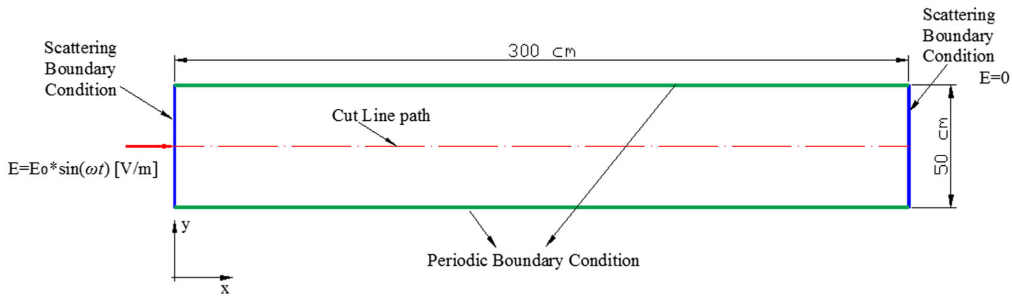


Figure 1. 1 dimensional (1D) photonic time crystal and boundary conditions.

forward propagating waves and time reversal waves, giving rise to Floquet states and dispersion bands, which are gapped in the momentum k [2, 3, 11]. These systems, are now attracting growing attention due to recent advances in fabricating dynamic optical systems and metamaterials [12]. Indeed, the progress in altering properties of materials in high frequencies will lead PhTCs to the experimental level in the near future.

In this work, we study photonic time crystals (PhTC) demonstrating ω-k dispersion relations with forbidden momentum zones, and concluded by discussing possible applications on the $BaTiO_3$ (near the ferroelectric phase transition) based PhTC.

2. Theory and simulation results

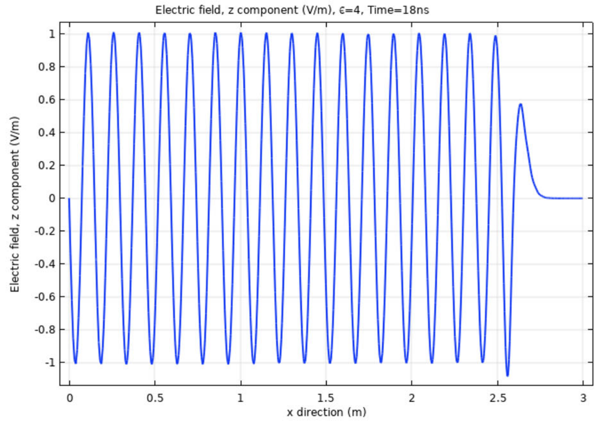
Electromagnetic wave (EM) propagation is described by the Maxwell's equation, in photonic time crystal, permittivity and permeability are the function of time, which vary with time periodically, for free source case, the Maxwell's equations can be written as [13]

$$E_x^{i+1}(j) = \frac{\varepsilon^i(j)}{\varepsilon^{i+1}(j)} E_x^i(j) - \frac{\Delta t}{\varepsilon^{i+1}(j)} \frac{H_y^{i+\frac{1}{2}}\left(j+\frac{1}{2}\right) - H_y^{i+\frac{1}{2}}\left(j-\frac{1}{2}\right)}{\Delta z} \tag{1}$$

$$H_y^{i+\frac{1}{2}}\left(j+\frac{1}{2}\right) = \frac{\mu^{i-\frac{1}{2}}\left(j+\frac{1}{2}\right)}{\mu^{i+\frac{1}{2}}\left(j+\frac{1}{2}\right)} H_y^{i-\frac{1}{2}}\left(j+\frac{1}{2}\right) - \frac{\Delta t}{\mu^{i+\frac{1}{2}}\left(j+\frac{1}{2}\right)} \frac{E_x^i(j+1) - E_x^i(j)}{\Delta z} \tag{2}$$

where $E(r,t)$ and $H(r,t)$ are the time harmonic electric and magnetic fields, respectively, ε_o and μ_o are the permittivity and permeability in free space, respectively. $\varepsilon_r(\vec{r},t)$ and $\mu_r(\vec{r},t)$ are time- and space dependent relative permittivity and permeability, respectively. $t = \left(i+\frac{1}{2}\right)\Delta t$, Δt is the time step, i is total number of time step, j is the position of grid cell. The permittivity in any grid cell is equal at same time, so $\varepsilon^i\left(j+\frac{1}{2}\right) = \varepsilon^i(j)$ $\dots\dots$, so is the permeability in any grid cell, it can be written as $\mu^i\left(j+\frac{1}{2}\right) = \mu^i(j) = \dots$

In order to understand the optical properties demonstrated by PhTCs, a material whose permittivity changes periodically in time ($BaTiO_3$ below and above phase transition - PT), we start with the study of a single temporal boundary. We define this temporal boundary as the time at which crystal material changes instantaneously from $\varepsilon_1(n_1)$ to $\varepsilon_2(n_2)$. In literature this phenomenon is called time refraction [14]. Time refraction is the result of the time continuity of electromagnetic waves which ensures the wave momentum $k = \frac{n\omega}{c}$ will be conserved regardless of the time variation of n.



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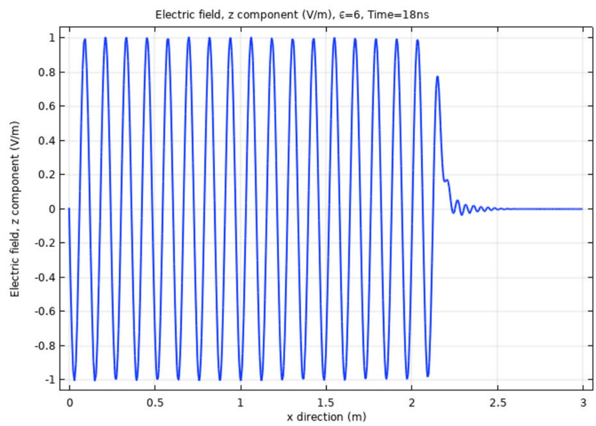

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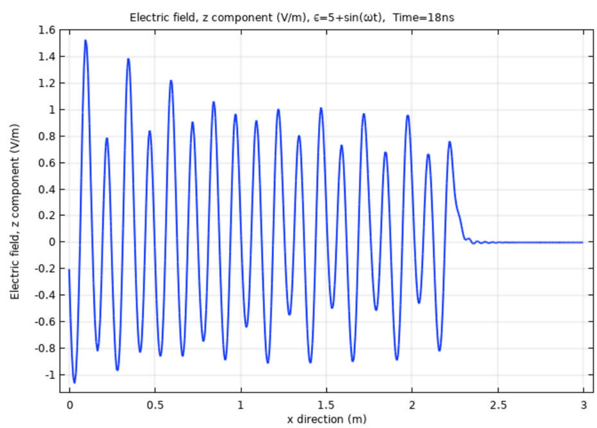


c)

Figure 2. Propagation of electromagnetic wave in 1D photonic time crystal and conventional photonic crystal, a) Electric field propagation in 1D photonic crystal with $\varepsilon(t) = 5.2$, amplitudes do not change, b) Electric field propagation 1D photonic crystal with $\varepsilon(t) = 6.5$, amplitudes do not change, c) 1D photonic space-time crystal $\varepsilon(t) = 5 + sin(\omega t)$, electric field propagation, amplitude varies with space and time.

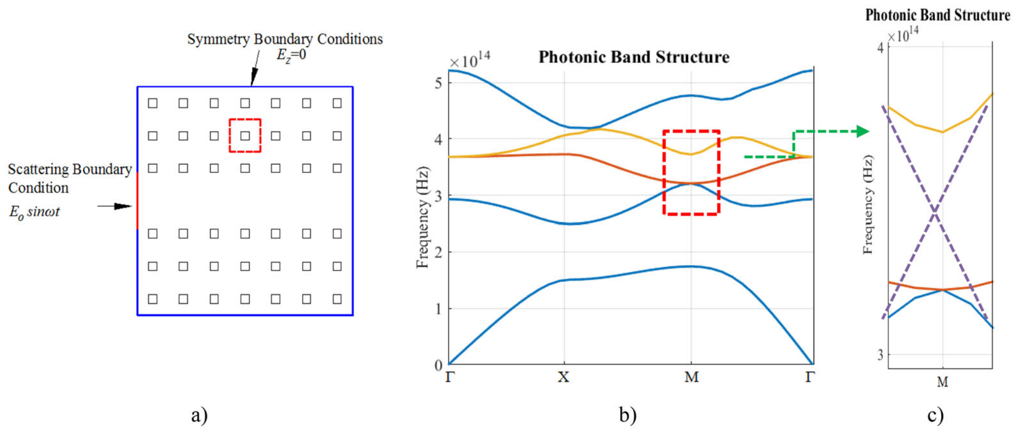


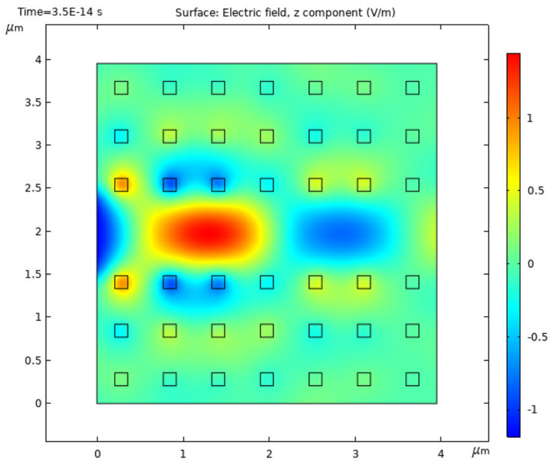
Figure 3. a) 2-dimensional (2D) photonic time crystal and boundary conditions, b) band structure of unit cell and c) the band dispersion near the Dirac point.

This results in a frequency conversion of the incident light from ω_1 to $\omega_2 = \frac{n_1\omega_1}{n_2}$ [15]. Geometry and boundary conditions of simulation are shown on Figure 1. Figure 2 demonstrate the Finite Element Method simulation that verifies the frequency conversion effect. We simulated EM wave propagation in 1D PhTC and conventional PhC in the Comsol software [16],

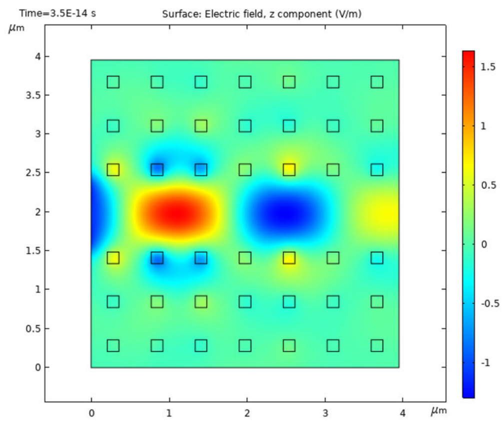
The 1-dimensional structure (1D) of Figure 1 was divided into meshes with 4328 elements by triangular elements. Scattering Boundary Condition was applied to the left and right edges of the crystal and Periodic Boundary Condition was applied to the upper and lower edges as seen in Figure 1. In the simulation, the relative permittivity of the material was changed as 5.2, 6.5 and $5 + sin(\omega t)$ respectively. On the left edge, the electric field $E_z = E_o.sin(\omega t)$V/m applied, which changes with time. Here $\omega = 2\pi.f_o$, and $f_o = 1 \times 10^9$ Hz. Total simulation time was 18ns with $\Delta t = 5 \times 10^{-11}$s in time-dependent analysis. At the end of the analysis, the results obtained for the relative permittivity 5.2, 6.5 and $5{+}sin(\omega t)$ of the propagation of the electromagnetic wave along the cut line in Figure 1 are as shown in Figure 2a–c, respectively. We concluded that in the PhTC, the EM wave is scattered everywhere, so the amplitudes vary with space and time, the amplitudes in PhTC (Figure 2c) are smaller than those in PhC (Figure 2a and b).

Figure 3 shows a 2-dimensional (2D) photonic crystal consisting of 7x7 square dielectric rods in air with a lattice constant $a = 0.565$µm and an edge of 0.15µm. In the band structure, obtained using unit cells (red square in Figure 3), a traditional photonics crystal band at $9 \text{x} 10^{14}$Hz has occurred. In the *Comsol* software [16], the 2-dimensional structure (2D) of Figure 3a was divided into meshes with 2290 elements by triangular elements.

The waveguide was created by subtracting one row from the squares. By applying $E_z = 0$V/m electric field to the blue edges of $E_z = E_o.sin(\omega t)$V/m electric field on the left red line in Figure 3, $\Delta t = 3.5 \times 10^{-17}$s steps, the simulation was performed in 1000 steps up to 3.5×10^{-14}s. The dielectric coefficient of the square bars in this structure $\varepsilon(t) = 5.2$, 6.5 and $5{+}sin(\omega t)$ for the advancement of electromagnetic wave work results as shown in Figure 4a–c.



a)



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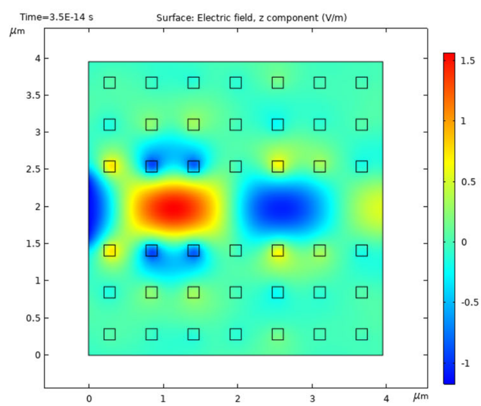

c)

Figure 4. Propagation of electromagnetic waves in 2D photonic space-time crystals and in photonic space crystals, a) Propagation of electromagnetic waves in the photonic space crystal which is permittivity of the periodic array $\varepsilon(t) = 5.2$ b) Propagation of electromagnetic waves in the photonic space crystal which is permittivity of the periodic array $\varepsilon(t) = 6.5$ c) Propagation of electromagnetic waves in the photonic space-time crystal which is permittivity of the periodic array $\varepsilon(t) = 5 + sin(\omega t)$.

Also, we observed the topological phase in ferroelectric based PhTC. We describe PhTC displaying a topology, that form by periodically modulating the refractive index of a ferroelectric medium. PhTC have a topology very close to that of topological insulators and demonstrate in FEM simulations, that the dispersion bands in momentum is related to the relative phase between the forward- and backward-propagating waves generated by PhTC (Figure 3c). The dielectric coefficient of the square bars in this structure $\varepsilon(t) = 5.2$, 6.5 and $5 + sin(\omega t)$ for the advancement of electromagnetic wave work results as shown in Figure 4a–c.

Figure 4c shows electromagnetic wave propagation in 2D photonic space-time crystal, the relative permittivity and permeability of the period array dielectrics are $\varepsilon(t) = 5 + sin(\omega t)$ and $\mu(t) = 1$, respectively, the frequency of the source is $f = 1.9 \times 10^{14}$Hz. For comparison, Figure 4a shows the electromagnetic wave propagation in 2D photonic space crystal, the relative permittivity and permeability of the periodical array dielectrics are $\varepsilon(t) = 5.2$ and $\mu(t) = 1$, respectively. Figure 4b shows the electromagnetic wave propagation in 2D photonic space crystal, the relative permittivity and permeability of the periodical array dielectrics are $\varepsilon(t) = 6.5$ and $\mu(t) = 1$, respectively. The periodical array dielectrics in Figure 4a are darker than those in Figure 4b and c, this is because the permittivity of the periodical array dielectrics is not uniform in photonic space-time crystal, and the scatter fields in photonic space-time crystal are more intensive than those in photonic space crystal. This is because the permittivity of the periodical array dielectrics in photonic space-time crystal is not uniform, and the scatter fields in photonic space-time crystal are more intensive than those in conventional photonic crystal, namely, the larger band gaps can be obtained in photonic space-time crystal.

Theoretically, the field-dependent dielectric [17] can be designed as a photonic time crystal, yet, in high frequency, the permittivity varying with time is not obvious [18], it is very difficult to make the period of electromagnetic field equal to that of permittivity. As for some heat diffusion materials, such as nonlinear (ferroelectric materials near phase transition) compounds, the heat conductivity, mass density and specific heat vary with temperature, by adjusting temperature periodically, one might make the period of the temperature field equal to that of material parameters. Whereas, for some acoustic wave materials, one can also adjust mass density and bulk module periodically to design acoustic time crystal. By the same method, other time crystals, like mass diffusion time crystal, could be designed too. It should be pointed out that reference [19, 20] described the discrete time crystal whose period is the integer multiple of the drive period and robustness against external perturbations. Wilczek and Shapere [2, 3] stressed the periodical movement in the lowest energy state of the time crystal. Whereas, our research focused on the permittivity of ferroelectric based photonic time crystal varying with time periodically.

3. Conclusion

In this work, we proposed the concept of the photonic time crystal and photonic space-time crystal, and simulated electromagnetic wave propagation in 1D, and 2D photonic time crystal and photonic space-time crystal. The simulated results indicate that the scatter fields in photonic time crystal are more intensive than those in photonic non-crystal, and the band gaps in photonic time crystals are larger than those in conventional photonic crystals. The method we adopted provides the possibility for further investigations in other time crystals and space-time crystals.

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