

UTILIZATION OF ANALYTIC HIERARCHY PROCESS  
IN THE LONG - RANGE AIRCRAFT SELECTION PROBLEM  
OF TURKISH AIRLINES

A THESIS  
SUBMITTED TO THE DEPARTMENT OF MANAGEMENT  
AND THE GRADUATE SCHOOL OF BUSINESS ADMINISTRATION  
OF BILKENT UNIVERSITY  
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE  
OF MASTER OF BUSINESS ADMINISTRATION

By  
HAKAN KAYALIK  
FEBRUARY, 1991

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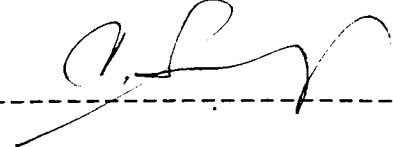
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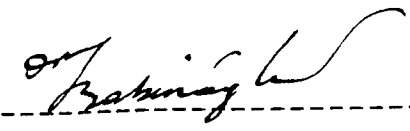
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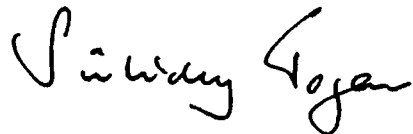
  
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Approved for the Graduate School of Business Administration.

Prof. Dr. Sübidey Togan



## **ABSTRACT**

### **UTILIZATION OF ANALYTIC HIERARCHY PROCESS IN THE LONG-RANGE AIRCRAFT SELECTION PROBLEM OF TURKISH AIRLINES**

**HAKAN KAYALIK**

**Master of Business Administration**

**Supervisor : Asst. Prof. Dr. CAN ŞİNGA**

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**Analytic Hierarchy Process (AHP)** developed by T.L. Saaty has received widespread attention in the literature. It has been applied in various decision making cases.

This study presents the AHP with its application to a recent multi-criteria decision problem of Turkish Airlines. Among three long-range commercial jet aircrafts, the best one for Turkish Airlines will be chosen using the AHP.

**Keywords:** Analytic Hierarchy Process, hierarchical structure, pairwise comparison.

## **ÖZET**

### **TÜRK HAVA YOLLARI İÇİN UZUN MENZİLLİ UÇAK SEÇİMİNDE ANALİTİK HİYERARŞİ METODUNUN KULLANIMI**

**HAKAN KAYALIK**

**Yüksek Lisans Tezi**

**Tez Yöneticisi : Y. Doç. Dr. CAN ŞİNGA**

**Şubat 1991, 43 Sayfa**

T.L. Saaty tarafından bulunan Analitik Hiyerarşi Metodu (AHM) büyük ilgi görmüş ve birçok "karar verme" durumunda kullanılmıştır.

Bu çalışmada, AHM tanıtılmakta ve Türk Hava Yolları'nın karşılaşmış olduğu bir "çok-kriterli karar verme" olayına uygulanışı sunulmaktadır. Herbiri uzun menzilli ticari uçak olan üç alternatif arasından en iyi olanı, AHM kullanılarak seçilecektir.

**Anahtar Kelimeler:** Analitik Hiyerarşi Metodu, hiyerarşik yapı, ikili mukayese.

## **ACKNOWLEDGEMENTS**

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I am also thankful to the **Planning and Coordination Department of Turkish Airlines Inc.** for their support in data gathering.

Finally, I would like to express my gratitude to **my family** as they have always stood by me.

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## **CHAPTER 1**

### **INTRODUCTION :**

The use of human judgement in decision making models is receiving an increasing attention in the last decade.

One method which has a growing presence in the literature is the **Analytic Hierarchy Process (AHP)** developed by **Thomas L. Saaty** (1977,1980). It has been applied in many problem areas ranging from predicting oil prices and planning for a national waterway (Saaty and Vargas, 1982) to various marketing decisions (Wind and Saaty, 1980) and financial decisions (Srinivasan and Kim, 1986). A comprehensive review of non-financial applications of the AHP is available in Zahedi (1985). Saaty's method has been recommended and used by many other researchers since its establishment. (Please see the references.)

### **1.1 Purpose of the Thesis:**

In this thesis, the purpose is to choose the best long-range commercial aircraft for Turkish Airlines among three alternatives; namely, Airbus A340-300, Boeing 747-400 Combi and Mc Donnell Douglas MD-11, utilizing the Analytic Hierarchy Process.

The recent long-range aircraft selection case of Turkish

Airlines will be taken as a multi-criteria decision problem to which the AHP can be applied. After a brief illustration of the method, the decision problem will be solved and the result will be compared with the actual decision of Turkish Airlines.

## **1.2 Organization of the Thesis:**

This thesis consists of **five chapters** including this introductory chapter.

In the second chapter, the method will be briefly illustrated. Theoretical details related to "matrix theory" and "Statistics" will be disregarded. Since enough references analyzing, praising or criticising the AHP are provided in the literature, main emphasis of this study will be given to the application. Therefore, only a short critique of the method will take place in this chapter.

In the following chapter, the problem will be specified. The criteria which are used in our case to evaluate a long-range commercial aircraft and the necessary data related to the alternatives will be presented in detail. After that, the hierarchical structure of the problem will be constructed.

In the fourth chapter, calculations will be carried out to find out the best alternative. In addition, a sensitivity analysis will take place to explore the effects of different

judgements and ratings as well as the stability of the result given the existing circumstances.

In the final chapter, the result will be restated and this AHP application will be discussed. Furthermore, other possible uses of the "aircraft satisfaction hierarchy" presented in this study will be mentioned.

## CHAPTER 2

### THE ANALYTIC HIERARCHY PROCESS :

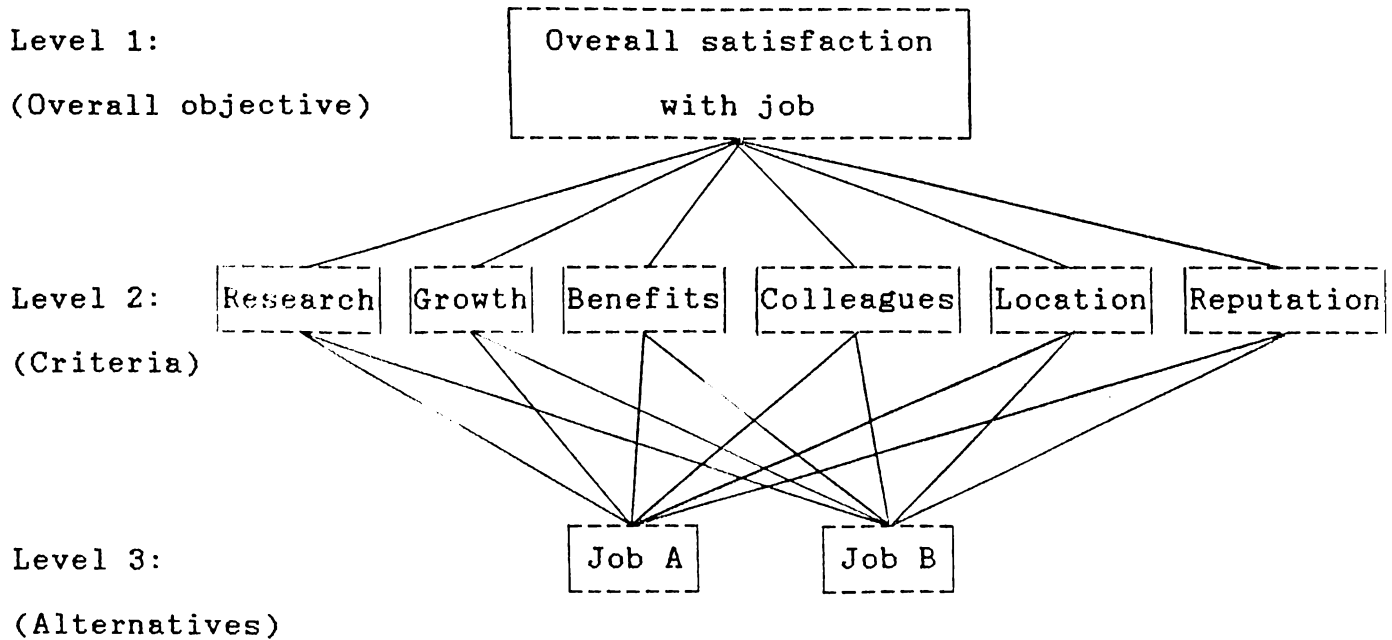
This is one of the main methods of multi-criteria decision making.

Like in every multi-criteria analysis, in this method, there is a set of alternatives (actions, activities) whose score on different criteria (attributes, measures, objectives, goals) are evaluated.

In such situations, if the precise weights of the activities are unknown, we should somehow find the relative strengths or weights (priorities) of each alternative with respect to each objective. Then, we should compose the results obtained for each objective in order to obtain a single overall priority for all the alternatives.

Generally, the objectives themselves must be prioritized or ranked in terms of another set of higher level objectives. Then, the obtained priorities are used as weighting factors for the priorities just derived for the alternatives. In many applications, this process continues by comparing the higher level objectives in terms of still higher ones and so on up to a single overall objective. Such a decision making structure is called a **hierarchical structure**. (Saaty, 1977)

**FIGURE 1**  
**Job Satisfaction Hierarchy**



A hierarchy is complete when each level connects to all elements in the next higher level. (Saaty, 1977) Therefore, FIGURE 1 is an example of a **complete hierarchy**.

### 2.1 The Pairwise Comparison Matrix:

The purpose of Saaty's method is scaling the weights of the elements (e.g. criteria or objectives) in each level of the hierarchy with respect to an element of the next higher

level. In order to do this, a matrix of pairwise comparisons of the elements is constructed. This matrix is also called "judgement matrix". The entries of this matrix are judgemental positive numerical ratios indicating the relative strengths of elements in the comparison.

If we want to compare a set of  $n$  elements in pairs according to their relative weights, the pairwise comparison matrix will be as follows:

$A_i$  : Elements (  $i=1, \dots, n$  )

$w_i$  : Weights of elements (  $i=1, \dots, n$  )

$$A = \begin{array}{c|cccc} & A_1 & A_2 & \dots & A_n \\ \hline A_1 & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ A_n & a_{n1} & a_{n2} & \dots & a_{nn} \end{array} = \begin{array}{c|cccc} & A_1 & A_2 & \dots & A_n \\ \hline A_1 & w_1/w_1 & w_1/w_2 & \dots & w_1/w_n \\ A_2 & w_2/w_1 & w_2/w_2 & \dots & w_2/w_n \\ \vdots & \vdots & \vdots & & \vdots \\ A_n & w_n/w_1 & w_n/w_2 & \dots & w_n/w_n \end{array}$$

As we see, an entry  $a_{ij}$  is an estimate of  $w_i/w_j$  (  $i, j=1, \dots, n$  ).

Saaty's technique assumes that the judgement matrix is reciprocal (i.e.  $a_{ij} = 1/a_{ji}$  ) and requires that all entries of the upper triangular half of the matrix be given (i.e.  $n(n-1)/2$  comparisons have to be made).

The normalized principal eigenvector (i.e. the normalized eigenvector of the highest eigenvalue) of this matrix is taken as the **weight (priority, preference) vector**. In matrix theory, it is proven that this vector is unique for a positive reciprocal matrix.

## 2.2 Consistency of the Matrix Data:

The eigenvalues of this matrix can be denoted by  $\lambda_i$  ( $i=1, \dots, n$ ). If our positive reciprocal matrix is **consistent** (i.e.  $a_{ij} \cdot a_{jk} = a_{ik}$ ), when we are given any row of A, we can determine the rest of the entries from this relation. In this case,  $\lambda_{\max}$ , the highest eigenvalue, will be equal to  $n$  and other eigenvalues will be zero.

In general, we have estimates of the ratios instead of real ratios in our matrix. In this case, the matrix will most probably be inconsistent (i.e.  $a_{ij} \cdot a_{jk} \neq a_{ik}$ ) and  $\lambda_{\max}$  will be greater than  $n$  while other eigenvalues will be different from zero.

In the concept of consistency, the most important point is that, **the closer the highest eigenvalue to  $n$ , the better is the matrix consistency** (Saaty, 1977).

Better matrix consistency can be achieved when the priorities of the attributes are well-defined by **experts**.



### 2.3 The Scale:

In Saaty's method, while constructing the pairwise comparison matrix, the judgements are made using a scale of 1 to 9 (see TABLE 1). The judgemental inputs can either be provided by an individual or be the collective view of several individuals.

Saaty explains his rationale of proposing this scale in the following way (1977):

"Our choice of scale hinges on the following observation. Roughly, the scale should satisfy the requirements:

1. It should be possible to represent people's differences in feelings when they make comparisons. It should represent as much as possible all distinct shades of feeling that people have.

2. If we denote the scale values by  $x_1, x_2, \dots, x_p$ , then let  $x_{i+1} - x_i = 1$ ,  $i = 1, \dots, p-1$

Since we require that the subject must be aware of all gradations at the same time and we agree with the psychological experiments (Miller, 1956) which show that an individual cannot simultaneously compare more than seven objects (plus or minus two) without being confused, we are led to choose a  $p = 7 + 2$ . Using a unit difference between successive scale values is all that we allow and using the fact that  $x = 1$  for the identity comparison, it follows that the scale values will range from 1 to 9."

The statement that the human mind is limited to  $7 \pm 2$  items for simultaneous comparison shows us that, Saaty's method is applicable to the situations where there are relatively less number of alternatives (i.e.  $n < 10$ ) and best results are obtained in 7 alternative cases.

**TABLE 1**  
**The Scale (Saaty, 1977)**

Intensity of importance	Definition	Explanation
1	Equal importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgement slightly favor one activity over another
5	Essential or strong importance	Experience and judgement strongly favor one activity over another
7	Demonstrated importance	An activity is strongly favored and its dominance is demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between the two adjacent judgements	When compromise is needed

Saaty states that, according to his studies, this scale of 1 to 9 has turned out to be the best scale among the other suggested ones (e.g. 1 - 5, 1 - 7, 1 - 15, 1 - 20), as far as the results are concerned.

#### **2.4 Clustering (Decomposition and Aggregation):**

In the previous section, we have seen that, 7 is the maximum number of elements which can be compared simultaneously with reasonable (psychological) assurance of consistency.

Assume that, we have a set of  $n$  (more than 7) elements to which we want to assign priority. In such cases, Saaty suggests that, elements must be firstly decomposed into equivalence classes of seven clusters (subsets). After that, each of these must be decomposed again into seven new clusters. This process continues until we obtain a final decomposition each of whose sets has no more than seven of the original elements.

Clustering a complex problem into hierarchical form has two advantages: (Saaty, 1977)

1. Great efficiency in making pairwise comparisons. (e.g. In a 98 element problem, if it were possible to compare them simultaneously, we would require 4753 comparisons. This number goes down to 322 with hierarchical decomposition.)

2. Greater consistency under the assumption of a limited capacity of the mind for simultaneous comparison.

To decompose a hierarchy into clusters, we must first decide on which elements to group together in each cluster. This can be done according to the proximity or similarity of the elements with respect to the function they perform or property they share. (Saaty, 1977) After that, we must conduct comparisons on the clusters and on the subclusters and then recombine them to obtain the overall priorities. If this process works, the result after the decomposition should be the same as the result if there were no decomposition.

## **2.5 Obtaining the Overall Weight Vector:**

In a hierarchical structure, activities in any one level have a weight vector with respect to each criterion in the next higher level. Those weight vectors are combined as the columns of a weight matrix for that level.

The weight matrix of a level is multiplied by the weight matrix (or vector) of the next higher level. If the highest level of the hierarchy consists of a single objective, then these multiplications will result in a single weight vector which can be called the **overall weight vector**. This vector indicates the relative priority of the elements of the lowest level for accomplishing the highest objective of the hierarchy.

If one decision is required, the decision rule is to select the alternative with the highest weight. If ours is a resource allocation problem, then we can distribute the resources to the alternatives in proportion to their weights in the overall vector.

## 2.6 Critique of the Method:

We have already stated that, the Analytic Hierarchy Process of T.L. Saaty has found many applications in various fields. Nevertheless, this widespread attention has not been without some criticisms of the method.

For example, Belton and Gear (1983) have raised a question about the "consistency" concept. They have claimed that, Saaty's way of measuring consistency is a crude way with limited statistical properties. On the other hand, Saaty argues that his method provides a wise measurement of consistency (1977).

Belton and Gear (1983) have also noted that the AHP does not satisfy independence of the irrelevant alternatives. According to them, even if the decision maker assesses perfectly consistent matrices, adding another alternative to the analysis may change the ordering of the original alternatives. Saaty and Vargas (1984b) argue that independence of the irrelevant alternatives is not

necessarily a principle of good decision making. They argue that the inclusion of a new alternative should be allowed to change the original ranking because people often learn new things which may cause them to reverse previous preferences.

Another major criticism concerns the "scale" of 1 to 9 used by Saaty in the construction of pairwise comparison matrices. Watson and Freeling (1982, 1983) and French (1986) are not satisfied with the efficiency of Saaty's scale. French states as follows: "One may ask whether the questions asked to the decision maker are substantively meaningful. What does it mean to say that one alternative is demonstrably preferred to another? It may be a figure of speech that I would use for emphasis, but I am unable to give any substantive operational meaning to that phrase." As mentioned earlier, Saaty gives empirical reasons for choosing the nine-point scale. French's criticism can be countered by insisting that the scale and its implications are explained fully to the decision maker prior to any analysis.

In spite of the above mentioned criticisms, numerous applications in the literature prove the reliability of the AHP. Of course, this does not mean that Saaty's method always gives perfect results in any case. Yet, since it is easily applicable to many decision cases, make sure that, it will continue to attract the decision makers and researchers.

## CHAPTER 3

### APPLICATION OF THE ANALYTIC HIERARCHY PROCESS TO THE AIRCRAFT SELECTION PROBLEM OF TURKISH AIRLINES :

#### 3.1 Definition of the Problem and the Alternatives:

Turkish Airlines is in need of long-range passenger/freight jet aircrafts because they are planning to start new long-distance direct flights (e.g. to Tokyo, to Sydney, etc.) They currently have short-range (DC-9), medium-range (B727) and medium/long-range (A310) aircrafts in their fleet.

In 1989, Turkish Airlines and the government had some contacts with three major aircraft manufacturers; namely, Airbus, Boeing and Mc Donnell Douglas, for their future products, A340-300 (available after October 1992), 747-400 Combi (available in the second half of 1991) and MD-11 (available after May 1990), respectively. After a period of evaluation, few months ago, they declared that, Airbus A340-300 was selected to be purchased.

Beginning from this chapter, the above mentioned aircraft selection case of Turkish Airlines will be considered as a multi-criteria decision problem to which the Analytic Hierarchy Process can be applied. This chapter covers the construction of the hierarchical structure of the problem. Calculations will take place in the next chapter.

### **3.2 The Aircraft Satisfaction Hierarchy:**

The aircraft selection problem of Turkish Airlines can be represented by a three-level hierarchical structure which is quite similar to the "job satisfaction hierarchy" in FIGURE 1. The single **overall objective** at the first level is "overall satisfaction with aircraft". At the second level of the hierarchy, there are **criteria** with respect to which the **three alternatives** at the third level will be compared.

In order to **evaluate a passenger/freight aircraft** in terms of "overall satisfaction", using the six quantitative **criteria explained below** would be sufficient:

- 1) **Price:** This is a basic criterion in every purchasing decision. In this application, standard study prices declared by the manufacturers will be accepted as the aircraft delivery prices. For the sake of simplicity, it will be assumed that the alternatives are to be delivered at the same time and terms of payments are the same. When purchasing an aircraft, airframe spares, spare engines and engine spares are also to be purchased. As the amount of money to be paid for those items is assumed to be the same percentage of the aircraft delivery price for each alternative, this will not affect our comparisons.
- 2) **Passenger capacity:** This is the number of passenger seats in an aircraft. In our case, each aircraft offers a three-



class (i.e. first, business and economy (or tourist)) interior configuration with different number of seats in each class. Since this will complicate the situation, **total numbers of seats** will be used in the comparisons assuming that average ticket price is the same for each alternative.

**3) Cargo capacity:** According to the AEA (Association of European Airlines) standards, a baggage volume of  $0.125 \text{ m}^3$  ( $4.41 \text{ ft}^3$ ) per passenger is allocated in the containers. Cargo capacity is then calculated by multiplying the remaining volume of containers by a given cargo density. The "volume-limited payload" is the sum of "cargo", "passenger weight" (assuming standard weight of 75 kg per passenger including hand luggage) and "baggage" (20 kg per passenger).

**4) Range capability:** An aircraft which can provide a direct flight of at least 4000 nm ( $1 \text{ nm (nautical mile)} = 1.852 \text{ km}$ ) while carrying full payload can be considered as a "long-range aircraft". In our comparisons, we will utilize the data obtained from route studies showing the range capabilities of the alternatives from Istanbul under normal conditions while carrying maximum volume-limited payload.

**5) Direct operating cost (DOC):** This is the most important criterion used in the evaluation of "commercial" aircrafts. In our analysis, DOC includes the fuel cost, maintenance cost, crew (flight and cabin) allowances, navigation charges, landing fees and ground handling charges.

6) **Cabin comfort:** Like all passenger/freight aircrafts, our three alternatives meet the seating standards (e.g. seat pitch, seat width, armrest width, etc.), space requirements (e.g. interior dimensions of the passenger compartment, aisle width, etc.) and all other requirements (e.g. number, location and dimensions of doors, windows, toilets, galleys, etc.) declared by international airline associations. We will use the hypothetical data, "total numbers of seats under equivalent comfort condition", in our pairwise comparisons.

**TABLE 2**  
**Aircraft Data about the Six Criteria**

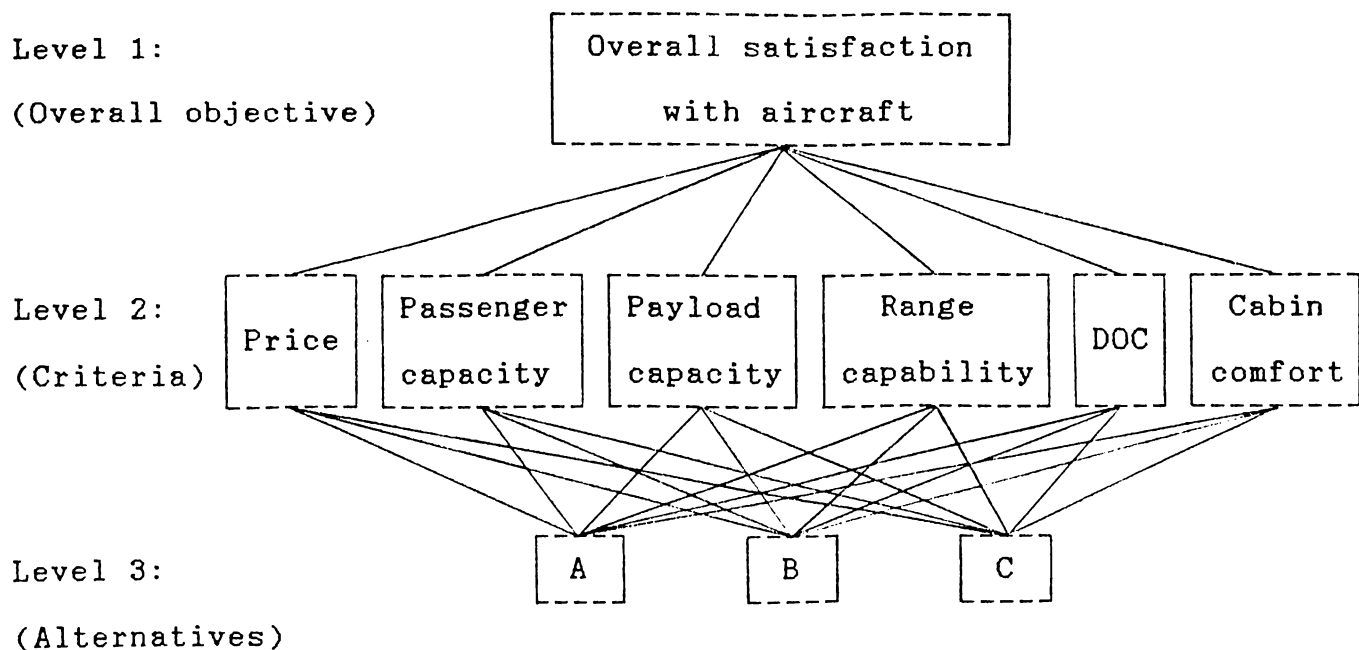
	A	B	C
	Airbus A340-300	Boeing 747-400 Combi	Mc Donnell Douglas MD-11
1) Price: (US \$ million)	92.8	131.8	96.6
2) Passenger capacity: (number of seats)	279 (18+60+201)	293 (18+56+219)	283 (18+56+209)
3) Cargo capacity: (lb) (1 lb=0.4536 kg)	30220	60780	28640
4) Range capability: (nm)	4605	4752	4818
5) DOC: (US cents per seat per nm)	2.964	3.643	3.067
6) Cabin comfort: (number of seats)	260 (20+48+192)	277 (18+56+203)	283 (18+56+209)

Alternatives, A, B and C are equipped with General Electric's CFM56-5C2 (4), CF6-80C2B1 (4) and CF6-80C2D1F (3) jet engines, respectively.

The data presented in TABLE 2 has been obtained from the original documents, dated October, 1989, provided by the Planning and Coordination Department of Turkish Airlines.

So far, we have determined the overall objective, the criteria and the alternatives. This means that, we are ready to construct the hierarchical structure of the problem.

**FIGURE 2**  
**Aircraft Satisfaction Hierarchy**



As illustrated in FIGURE 2, we have ended up with a "complete hierarchy".

## **CHAPTER 4**

### **CALCULATIONS OF THE APPLICATION :**

In the previous chapter, the hierarchical structure of our multi-criteria decision problem has been established.

In this chapter, calculations of this AHP application are illustrated. Computations have been carried out using the **PC-MATLAB** (a software for matrix computation, applied mathematics, time-series analysis, etc.). This software is available at Bilkent Computer Center.

#### **4.1 Comparison of the Criteria with respect to "Overall Satisfaction with Aircraft" :**

In this section, the **criterion weight vector** is obtained. In other words, calculations at the second level (i.e. criteria level) of our hierarchical structure are performed.

Firstly, a matrix of pairwise comparisons of the six criteria is constructed (see TABLE 3) with respect to the overall objective (i.e. the first level objective) of the hierarchical structure.

While making the judgements, the question to be asked is that, "How much more important is each criterion than the others as far as the overall objective is concerned ?" The judgements have been made by several experts (i.e. two

aircraft engineers, two pilots and two managers of Turkish Airlines) using the scale in TABLE 1. The judgemental entries of the matrix in TABLE 3 are the arithmetic averages of the corresponding entries provided by each expert, so they reflect the collective view of them.

**TABLE 3**  
**Pairwise Comparison Matrix of the Criteria**

	Price	Passenger capacity	Cargo capacity	Range capability	DOC	Cabin comfort
1. Price	1	3/4	3/2	3/2	1/3	3
2. Passenger capacity	4/3	1	3/2	3/2	1/3	4
3. Cargo capacity	2/3	2/3	1	5/4	1/4	2
4. Range capability	2/3	2/3	4/5	1	1/5	5/2
5. DOC	3	3	4	5	1	9
6. Cabin comfort	1/3	1/4	1/2	2/5	1/9	1

The highest eigenvalue of the matrix in TABLE 3:

$$\lambda_{\max} = 6.0261$$

As we have seen, the highest eigenvalue of the comparison matrix, 6.0261, is very close to the total number of criteria, 6. Therefore, we can claim that this comparison matrix is considerably consistent and the judgements are reliable. This is because of the fact that, the judgemental entries of this matrix have been provided by experts as explained before.

Other eigenvalues of the matrix in TABLE 3:

- 0.0073 + 0.3822 i
- 0.0073 - 0.3822 i
- 0.0060
- 0.0028 + 0.1049 i
- 0.0028 - 0.1049 i

**TABLE 4**

**Principal Eigenvector of the Comparison Matrix in TABLE 3**

Principal eigenvector	Principal eigenvector (Normalized)
0.3185	0.1419
0.3684	0.1641
0.2362	0.1052
0.2195	0.0978
1.0000	0.4453
0.1025	0.0457

**TABLE 5**  
**Weights of the Criteria**

<u>Criterion</u>	<u>Weight</u>
1	0.1419
2	0.1641
3	0.1052
4	0.0978
5	0.4453
6	0.0457

#### **4.2 Comparison of the Aircrafts with respect to the Six Criteria:**

In this section, calculations at the third level (i.e. alternative level) of the hierarchy are presented.

Firstly, pairwise comparison matrices of the aircrafts with respect to each criterion are constructed. (See TABLE 6) While doing this, there is no need to estimate the ratios (i.e. the entries of those comparison matrices) because, at this time, we have the real ratios. We can use the data provided in TABLE 2 in the construction of the matrices. As it has been done so, all matrices in TABLE 6 are consistent and consequently, weight vectors will show the "real" weights.

**TABLE 6**  
**Pairwise Comparison Matrices of the Aircrafts**  
**with respect to the Six Criteria**

---

**1) Price**

	A	B	C
A	1	131.8/92.8	96.6/92.8
B	92.8/131.8	1	96.6/131.8
C	92.8/96.6	131.8/96.6	1

**2) Passenger capacity**

	A	B	C
A	1	279/293	279/283
B	293/279	1	293/283
C	283/279	283/293	1

**3) Cargo capacity**

	A	B	C
A	1	30220/60780	30220/28640
B	60780/30220	1	60780/28640
C	28640/30220	28640/60780	1



**TABLE 6 (Continued)****4) Range capability**

	A	B	C
A	1	4605/4752	4605/4818
B	4752/4605	1	4752/4818
C	4818/4605	4818/4752	1

**5) Direct Operating Cost (DOC)**

	A	B	C
A	1	3.643/2.964	3.067/2.964
B	2.964/3.643	1	3.067/3.643
C	2.964/3.067	3.643/3.067	1

**6) Cabin comfort**

	A	B	C
A	1	260/277	260/283
B	277/260	1	277/283
C	283/260	283/277	1

**TABLE 7**  
**The Highest Eigenvalues and Corresponding Eigenvectors**  
**of the Six Comparison Matrices in TABLE 6**

	1	2	3	4	5	6
	Price	Passenger capacity	Cargo capacity	Range capability	DOC	Cabin comfort
	$\lambda_{\max} = 3$	$\lambda_{\max} = 3$	$\lambda_{\max} = 3$	$\lambda_{\max} = 3$	$\lambda_{\max} = 3$	$\lambda_{\max} = 3$
<b>A</b>	1.0000	0.9522	0.4972	0.9558	1.0000	0.9187
<b>B</b>	0.7041	1.0000	1.0000	0.9863	0.8136	0.9788
<b>C</b>	0.9607	0.9659	0.4712	1.0000	0.9664	1.0000

**TABLE 8**  
**Normalized Principal Eigenvectors (Weight Vectors)**  
**of the Six Comparison Matrices in TABLE 6**

	1	2	3	4	5	6
	Price	Passenger capacity	Cargo capacity	Range capability	DOC	Cabin comfort
<b>A</b>	0.3753	0.3263	0.2526	0.3249	0.3597	0.3171
<b>B</b>	0.2642	0.3427	0.5080	0.3352	0.2927	0.3378
<b>C</b>	0.3605	0.3310	0.2394	0.3399	0.3476	0.3451

#### 4.3 Obtaining the Overall Ranking of the Aircrafts:

In order to obtain the overall ranking of the aircrafts, we multiply the last matrix in TABLE 8 with the weight vector of the criteria in TABLE 5. This yields the overall weight vector in TABLE 9.

$$\begin{bmatrix} 0.3753 & 0.3263 & 0.2526 & 0.3249 & 0.3597 & 0.3171 \\ 0.2642 & 0.3427 & 0.5080 & 0.3352 & 0.2927 & 0.3378 \\ 0.3605 & 0.3310 & 0.2394 & 0.3399 & 0.3476 & 0.3451 \end{bmatrix} \cdot \begin{bmatrix} 0.1419 \\ 0.1641 \\ 0.1052 \\ 0.0978 \\ 0.4453 \\ 0.0457 \end{bmatrix} = \begin{bmatrix} 0.3398 \\ 0.3257 \\ 0.3345 \end{bmatrix}$$

TABLE 9  
Overall Weights of the Alternatives

	Alternative	Overall Weight
(1)	A (Airbus A340-300)	0.3398
(3)	B (Boeing 747-400 Combi)	0.3257
(2)	C (Mc Donnell Douglas MD-11)	0.3345

In this problem, the decision rule is to select the alternative with the highest overall weight. Therefore, Airbus A340-300 is selected for Turkish Airlines.

#### 4.4 Sensitivity Analysis:

As we have seen in TABLE 9, there are only small differences among the overall weights of the alternatives. Therefore, it would be appropriate to do a sensitivity analysis.

There is no need to deal with the weight matrix of the alternatives in TABLE 8 because it includes "real" weights instead of "judgemental" ones.

What will be done is to recalculate the overall weight vector while changing the "judgementally obtained" criterion weights. Changes in the criterion weights will be minor because it is assumed that the judgements made by these experts are stable within this context. That is, only very small changes in the criterion weights are conceivable. If we had a different group of experts, the overall weight vector might have been calculated by using the "hypothetical criterion weights" given by these "hypothetical experts". In this case (i.e. **case g** illustrated below), a detailed sensitivity analysis would be required.

However, in our case, it seems that exploring the effects of minor changes in the criterion weights on the overall weights of the alternatives is sufficient. Since "direct operating cost" has turned out to be the most important criterion, its weight will be changed and the

incremental (or discremental) amount will be distributed equally to the other weights.

**TABLE 10**  
**Sensitivity Analysis**

Case	Criterion Weights						Selected Aircraft
	Price	Pass.	Cargo	Range	DOC	Cabin	
<b>Ours</b>	.1419	.1641	.1052	.0978	.4453	.0457	<b>A</b>
<b>a</b>	.1319	.1541	.0952	.0878	.4953	.0357	<b>A</b>
<b>b</b>	.1219	.1441	.0852	.0778	.5453	.0257	<b>A</b>
<b>c</b>	.1519	.1741	.1152	.1078	.3953	.0557	<b>A</b>
<b>d</b>	.1619	.1841	.1252	.1178	.3453	.0657	<b>A</b>
<b>e</b>	.1719	.1941	.1352	.1278	.2953	.0757	<b>B</b>
<b>f</b>	1/6	1/6	1/6	1/6	1/6	1/6	<b>B</b>
<b>g</b>	.35	.05	.03	.25	.15	.17	<b>C</b>

Details of the cases in TABLE 10 are as follows:

a) Add 0.05 to 0.4453

Subtract 0.01 from the other weights

New weight vector of the criteria:

(0.1319, 0.1541, 0.0952, 0.0878, 0.4953, 0.0357)

New overall weight vector:  $\begin{matrix} A & B & C \\ (0.3419, & 0.3224, & 0.3357) \end{matrix}$

b) Add 0.10 to 0.4453

Subtract 0.02 from the other weights

New weight vector of the criteria:

(0.1219, 0.1441, 0.0852, 0.0778, 0.5453, 0.0257)

New overall weight vector:  $\begin{matrix} & A & B & C \\ & & & \end{matrix}$  (0.3438, 0.3193, 0.3369)

c) Subtract 0.05 from 0.4453

Add 0.01 to the other weights

New weight vector of the criteria:

(0.1519, 0.1741, 0.1152, 0.1078, 0.3953, 0.0557)

New overall weight vector:  $\begin{matrix} & A & B & C \\ & & & \end{matrix}$  (0.3378, 0.3290, 0.3332)

d) Subtract 0.10 from 0.4453

Add 0.02 to the other weights

New weight vector of the criteria:

(0.1619, 0.1841, 0.1252, 0.1178, 0.3453, 0.0657)

New overall weight vector:  $\begin{matrix} & A & B & C \\ & & & \end{matrix}$  (0.3358, 0.3322, 0.3320)

e) Subtract 0.15 from 0.4453

Add 0.03 to the other weights

New weight vector of the criteria:

(0.1719, 0.1941, 0.1352, 0.1278, 0.2953, 0.0757)

New overall weight vector:                     $\begin{matrix} A & B & C \end{matrix}$   
(0.3338, **0.3354**, 0.3308)

f) Equal weights

New weight vector of the criteria:

(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)

New overall weight vector:                     $\begin{matrix} A & B & C \end{matrix}$   
(0.3260, **0.3468**, 0.3272)

g) Hypothetical case

New weight vector of the criteria:

(0.35, 0.05, 0.03, 0.25, 0.15, 0.17)

New overall weight vector:                     $\begin{matrix} A & B & C \end{matrix}$   
(0.3443, 0.3100, **0.3457**)

From these cases, in **a**, **b**, **c** and **d**, although there are minor changes in the criterion weights, the result (i.e. selected aircraft) remains the same as in the original case. In cases **e**, **f** and **g**, there are considerable changes in the criterion weights and consequently, selected aircraft is different as expected. Especially, case **g** demonstrates that using the judgements of a different group of experts in calculations may totally change the result because the AHP is a "judgement based" method.

## CHAPTER 5

### CONCLUSION AND DISCUSSION :

In this study, the Analytic Hierarchy Process (AHP) of T.L. Saaty has been applied to the long-range aircraft selection problem of Turkish Airlines. Firstly, the criteria which can be used in properly evaluating a long-range passenger/freight aircraft have been determined. After that, the three alternatives have been presented with the essential data. Then, our decision problem has taken the form of a three-level hierarchical structure where the highest level objective is "overall satisfaction with the aircraft". After carrying out the necessary calculations using the judgements of a certain group of experts, **Airbus A340-300 has been found to be the best alternative** with an overall weight score of 33.98% . As demonstrated in the sensitivity analysis section of previous chapter, utilizing the judgements of a different group of experts may change the result since the AHP is a judgemental method. Therefore, it should be noted that the result of our case is valid given the existing circumstances.

Nevertheless, it can be claimed that, **this AHP application has given a highly sensible result** because, few



months ago, Turkish Airlines declared that they preferred Airbus A340-300 to the others. Therefore, this thesis can be considered as a good example, like many others in the literature, indicating the reliability of the AHP in real-life multi-criteria decision problems.

The aircraft satisfaction hierarchy presented in this study can also be used in the selection of some other kinds of aircrafts. For example, if we want to evaluate short-range passenger/freight aircrafts, there will be no need to make any change in the hierarchical structure. Same six criteria are valid for that kind of aircrafts; but there may be some differences in the criterion weights. The importance of "range capability" and "cabin comfort" will probably be lower when compared to the long-range case. As another example, if we need to select a cargo aircraft (also called freighter), "passenger capacity" and "cabin comfort" will definitely be disregarded. That is to say, only four criteria will take place at the second level of the hierarchy. Furthermore, "direct operating cost" will be measured in terms of "US cents per ton per nautical mile".

Unfortunately, this aircraft selection hierarchy is useless for military aircrafts (e.g. fighters, observers, etc.) and some special purpose aircrafts (e.g. fire

extinguishers, aircrafts used in agricultural works, etc.).

A new structure should be formed in each case. It would definitely be an interesting exercise to compare the F16 with some other fighters using Saaty's method.

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## **VITA**

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