STOCHASTIC SHELTER SITE LOCATION UNDER MULTI-HAZARD SCENARIOS

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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In some cases, natural disasters happen successively (e.g. a tsunami following an earthquake) in close proximity of each other, even if they are not correlated. This study locates shelter sites and allocates the affected population to the established set of shelters by considering the aftershock(s) following the initial earthquake, via a three-stage stochastic mixed-integer programming model. In each stage, before the uncertainty, which is the number of affected people, in the corresponding stage is resolved, shelters are established, and after the uncertainty is resolved, affected population is allocated to the established set of shelters. To manage the inherent risk related to the uncertainty, conditional value-at-risk is utilized as a risk measure in allocation of victims to the established set of shelters. Computational results on the Istanbul dataset are presented to emphasize the necessity of considering secondary disaster(s), along with a heuristic method to improve the solution times and qualities. During these computational analyses, it is observed that the original single-objective model poses some obstacles in parameter selection. As in humanitarian operations, choosing parameters may cause conflict of interests and hence may be criticized, a multi-objective framework is developed with various formulations. Some generalizations regarding the performance and applicability of the developed formulations are discussed and finally, another heuristic for the multi-objective formulation is presented to tackle the curse of dimensionality and improve the solution times.

Keywords: Shelter Site Location, Secondary Disasters, Multi-Stage Stochastic Programming, Conditional Value-at-Risk, Multi Objective Programming.

ÖZET

ÇOKLU AFETLERİN YÖNETİMİ: RASSAL TALEP ALTINDA ÇADIRKENT YER SEÇİMİ PROBLEMİ

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Bazı doğal afetler, birbirleriyle direkt ilişkili olmasalar da, birbirlerinin yakınında gerçekleşebilir. Buna örnek olarak tsunaminin bir depremden sonra oluşması veya selden sonra bir yangın çıkması verilebilir. Bu çalışma, artçıların ana depremi takip ettiği durumlar için üç aşamalı karışık tam sayılı bir program geliştirerek çadırkentlerin yerleştirilmesi ve afetten etkilenen insanların bu çadırkentlere atanmasını amaçlamaktadır. Ana deprem ve artçının yarattığı talepler rassal kabul edilmekte, bu rassallık her aşamada çözümlenmeden önce çadırkentler yerleştirilmekte, çözümlendikten sonra ise afetzedeler yerleştirilmiş çadırkentlere atanmaktadırlar. Talebin rassal olmasının çadırkent kapasitelerinin aşımında yarattığı riski yönetebilmek için koşullu riske maruz değer kullanılmaktadır. İstanbul veri kümesi kullanılarak elde edilen sonuçlarla önerilen modelin gerekliliği ve önemi tartışılmış, çözüm sürelerini iyileştirmek için sezgisel bir yöntem geliştirilmiştir. Bu analizler sırasında tek amaçlı modelin parametre seçimi konusunda problemler yaratabileceği gözlemlenmiş, bu problemleri çözmek içinse çok amaçlı bir model geliştirilmiştir. Çok amaçlı modelin performans ve uygulanabilirliği üzerinde analizler yapılmış, çeşitli varyasyonları incelenerek çözüm sürelerini azaltmak ve daha büyük veri kümeleri ile çözümler elde edebilmek için bir sezgisel yöntem daha geliştirilmiştir.

Anahtar sözcükler: Çadırkent Yer Seçimi, İkincil Afetler, Çok Aşamalı Rassal Modelleme, Koşullu Riske Maruz Değer, Çok Amaçlı Programlama.

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Chapter 1

Introduction

Humanitarian logistics is concerned with delivering relief materials and providing services such as sheltering, medical care and much more in cases of disasters of various kinds. As some disasters might cause people to lose their homes and compel them to seek safe accommodation alternatives, it is of great importance to determine the best shelter site combination, which can be considered as one of the critical applications of location problems in the context of Disaster Operations Management (DOM).

From the beginning of the 20$^{\text{th}}$ century, more than the current population of the world has been affected by various natural disasters [1], and the literature on DOM has grown remarkably to manage the consequences and the risks of those disasters. We can observe in surveys such as Altay and Green [2], Caunhye et al. [3], Galindo and Batta [4], Hoyos et al. [5] and many more, that the location studies make up a great part of this literature.

As an extension to this profuse literature, in this thesis, we are concerned with providing sheltering to the disaster victims. We consider not just the initial disaster but the disaster(s) that might follow it. As there are numerous examples on secondary disasters following the initial one, e.g. tsunamis coupled with nuclear meltdown following an earthquake as in Tōhoku Earthquake in

2011, we analyze the effects of locating shelter sites following a disaster and a possible secondary disaster, which is formally called multi-hazard in the related literature ([6, 7]). As one cannot make any generalizations on the magnitude or the possible consequences of consecutive disasters, we aim to increase the disaster preparedness and improve the effectiveness of the response by incorporating multi-hazard nature of disasters in selecting the best possible combination of shelter sites to provide disaster victims with services at an acceptable level. Following this approach, we propose a three-stage stochastic shelter site location program, where we consider the number of victims that each disaster, initial and the secondary, creates is random.

Having defined the foundation of our problem, in the following chapter, we discuss a particular application of the stochastic shelter site location problem in a specific disaster context, i.e. earthquakes, while motivating it by referring to the related literature and the past examples of disasters exhibiting the features of multi-hazard phenomenon and conclude by emphasizing the necessity of considering secondary disasters in Disaster Operations Management context.

In Chapter 3, we define the problem formally and provide it in full detail, then present the formulation along with emphasizing its ability to model the real-life applications of disaster operations and expectations of both the decision maker (e.g. government authorities) and the disaster victims.

In Chapter 4, we present a novel dataset that contains demand scenarios for the earthquakes and the aftershocks following them. As no dataset can be found on multi-hazard disasters, we discuss the details regarding the generation of a particular dataset of demand scenarios for a district of Istanbul, Turkey.

In Chapter 5, the computational studies performed on the generated dataset using the proposed model are presented and the features of the solutions and the effect of parameter selection are discussed. Moreover, a heuristic methodology is proposed to solve the problem with scenario sets having higher cardinality. The chapter is concluded with a discussion on how our formulation performs when it is compared against more *traditional* approaches.

In Chapter 6, we discuss the shortcomings of the single-objective formulation and its burden on the decision maker as selecting the parameters for it can be quite challenging in a humanitarian context, and propose a multi-objective formulation for the stochastic shelter site location problem under multi-hazard scenarios. We discuss the selection of objectives by referring to the single-objective formulation.

In Chapter 7, we present the computational studies performed on the same dataset using the setting and formulation proposed in Chapter 6 with an ϵ-constraint method tailored for this formulation, and design another heuristic method to solve the multi-objective formulation with scenario sets having higher cardinality. We conclude by discussing the performance of the heuristic method using the instances with smaller cardinalities.

The thesis ends with a conclusion chapter providing an overview and guidelines for future research directions.

Chapter 2

Problem Definition and Related Literature

From the dawn of civilization, disasters, regardless of man-made or natural, had shaped the human culture of sheltering. But the industrialization, making fast urbanization possible, resulted in disorganized and densely populated cities, in which there is no regard for and sheltering from possible disasters. Balcik and Beamon [8] denote that the number of people affected by disasters between 2000-2004 was 33% more than the preceding five year period and a difference of seven million people affected in disasters occurred in 2004 and in 2005, suggesting an increasing trend which requires considerable attention. Fortunately, this need has drawn sufficient consideration to the humanitarian logistics and disaster operations management initiatives. Currently, mathematical modeling and optimization, statistical analysis, simulation and many more tools of them are heavily used to improve and shape the modern human's culture of sheltering.

Disasters are split into two main classes: man-made and natural. Both of these subclasses can be further divided according to their rapidness of onset, as slow onset and sudden onset. For example, a terrorist attack is a man-made and sudden onset disaster, while drought is a natural and slow onset disaster. As we aim to be able to respond to a disaster as soon as it happens, we are more concerned with sudden onset disasters - slow onset ones provide enough time for preparation throughout the occurrence due to our ability to observe and plan for them. This proposes a more general classification of disaster management operations.

McLoughlin [9] classifies the Disaster Operations Management (DOM) literature into four main phases: (i) mitigation, (ii) preparedness, (iii) response, and (iv) recovery. Phases (i) and (ii) refer to pre-disaster, phases (iii) and (iv) refer to post-disaster operations. The mitigation phase involves the actions taken in order to prevent and mitigate the consequences of a possible disaster. The preparedness phase includes plans for specific cases and provides effective responses to disasters. After a disaster has occurred, the aim in the response phase is to provide the affected population with relief goods and primary needs, such as water, food, medical care, shelter, and etc. Lastly, the aim of the recovery phase is to recover all the damaged (infra)structure in order to ensure the normal functioning of the affected population.

So, to be able to respond to drought in the most effective and efficient way, a Decision Maker (DM) should spend more time on the pre-disaster operations of DOM to mitigate the possible effects and prepare plans for alternative consequences of the disaster to make the response and recovery as easy and swift as possible. On the other hand, a DM in an earthquake setting, in addition to performing the best in pre-disaster operations, should spend more time on the post-disaster operations and respond to the disaster in a quick and effective fashion in order to minimize the casualties.

2.1 Role of Shelter Sites and Their Innate Stochasticity

Given a disaster which results in people losing their homes and other means of accommodation, it is of great importance to provide safe, prompt and sustainable sheltering. These shelters are not just means of accommodation for the disaster victims but a place for them to recover from the disaster by being together with people they greet on a routine basis and hence reduce their vulnerability. In these facilities, victims are provided food, water, and medical care and can continue their lives with dignity, expediting the recovery from the disaster significantly. Considering this problem of selecting shelter sites, given its significance, one should plan and prepare in a systematic manner for a disaster. This makes the shelter site location problem one of the fundamental facility location problems in DOM.

In this thesis, the emphasis is on the people who cannot stay in their homes after a disaster has occurred and seek accommodation in temporary shelters. In order to accommodate the disaster victims, one has to devote certain safe areas, that are preferably close to densely populated regions, to establish temporary shelters. Usually, this decision of choosing candidate shelter locations is made before a disaster occurs. Unfortunately, for sudden onset disasters, e.g. earthquake or tsunami, it is impossible to forecast the number of victims that a disaster will create, implying it is important to take demand uncertainty into account and not work with deterministic demand assumptions for resource planning, i.e. selecting shelters to be established, in the preparedness phase. So, in reality, a DM decides on the location of the shelters to be established after a disaster occurs but before the observation of the actual demand, making the consideration of demand variability a vital part of this process.

As facility location decisions are often costly and irreversible –in our problem, an established shelter cannot be closed as there will be disaster victims already staying there– and since the parameters, such as demand, that they abide may fluctuate, stochastic modeling is very relevant [10]. While reviews by Owen and Daskin [11] and Current et al. [12] examine both deterministic and stochastic facility location models, Snyder [10] and Caunhye et al. [3] discuss only stochastic nature of facility location problems, agreeing that the complexity of location problems are captured best by stochastic modeling. So, we essentially define our problem as stochastic shelter site location problem.

2.2 Considering Multi-Hazards

In some cases of disasters, the size of the displaced population may grow larger because of the secondary disaster(s) following the initial one. For 1999 Marmara Earthquake, secondary disasters were a disastrous fire at the Tüpraş petroleum refinery, tsunamis in the Marmara sea, and a strong aftershock in Düzce [13]. When the nature of consecutive disasters are analyzed, it can be observed that the initial and secondary disasters might be of same types (e.g. aftershocks following an earthquake as in Illapel Earthquake, 2015) or of different types (e.g. tsunamis coupled with nuclear meltdown following an earthquake as in Tōhoku Earthquake, 2011) while no generalization can be made on the magnitude or the possible consequences of the corresponding disasters.

In the literature, this phenomenon of having consecutive disasters is called multi-hazard, which is represented as the combination of various hazards in a defined area [6, 7]. Projecting this to the shelter site location problem, the decision of establishing some combination of the candidate shelters becomes more complicated as the demand uncertainty created by the initial disaster couples with the demand uncertainty created by the possible secondary disaster(s). So, we revise the definition of our problem as stochastic shelter site location under multi-hazard disasters.

2.2.1 Necessity of Differentiating the Stages

In this thesis, we aim to investigate the effect of the secondary disasters on the stochastic shelter site location problem. We discuss this extension in an earthquake specific case, implying our initial and secondary disasters are both earthquakes – secondary earthquake is called an aftershock. Since we do not consider only one disaster but a sequence of disasters, a suitable modeling methodology is required. So, to model the innate stochastic nature of the initial earthquake and the possible aftershocks, we propose a three-stage stochastic mixed-integer programming (MIP) model that decides on the locations of shelters, where we group shelters as the first stage shelters, the shelters that are established right after the initial shock, and the second stage shelters, the shelters that are established –if needed– after an aftershock.

As it is in the real setting, we assume that the DM decides on the location of the shelter sites in the first stage, that is after the initial earthquake but before the realization of actual demand. In the first stage, the disaster victims also choose the nearest shelter from open set of shelters and travel there. Note that the allocation decisions of the disaster victims to open shelters are made implicitly as the victims travel to the nearest open shelter in any case, hence the allocation decisions coincide with the location decisions. The DM cannot assign a district to a farther shelter as victims do not and would not act out of their interests after any disaster.

Once the disaster victims are located to the shelters after the realization of the demand in the second stage, first decision in the second stage is whether or not to establish new shelter(s) to meet the demand that a possible aftershock might create. Then, in the same stage, similar to the initial shock setting, allocation decision of victims to the shelter sites are finalized in accordance with the nearest assignment methodology. Finally, in the third stage, after the uncertainty on the demand of the aftershock is resolved, the utilization of established shelters are observed.

2.2.2 Necessity of Mimicking Real Life

In creating a methodology of locating shelter sites for hosting disaster victims, it is important to consider the features of the network, particularly the capacity of the shelter sites. 1999 Marmara Earthquake provides an example for the case where the population hosted in the shelters exceeds the shelter capacities by as much as 40% [14]. The problems that were observed in 1999 Marmara Earthquake motivated an international study, JICA-IMM joint work [15], and numerous papers by authors located in Turkey, such as Görmez et al. [16], Kılcı et al. [14], and Cavdur et al. [17].

In order to model the behavior of the disaster victims in a more realistic manner, we assume that the disaster victims in the same neighborhood will always travel to the same and the nearest shelter. From a psychological point of view, it is possible that a certain portion of the disaster victims who choose to reside in shelter sites after the secondary shock may not choose to travel to the nearest shelter but a farther shelter that has been established after the initial shock but before the secondary shock, i.e. some portion of the population affected by the secondary shock may choose to travel farther to be with their neighbors. Since this approach would require parametric analysis on the portion of population that embraces such a choice, we preserve the nearest assignment idea throughout this study.

When the disaster victims are *always* assigned to the nearest shelter without demand division, the shelter capacities may be exceeded. So, we define the risk in this setting as the capacity of a shelter being exceeded.

2.3 Related Literature

With an enormous literature on facility location, the application of those models to humanitarian logistics is abundant as reviews by Altay and Green [2], Simpson and Hancock [18], and Galindo and Batta [4] suggest. Özdamar et al. [19], Kovács and Spens [20], and Leiras et al. [21] reiterate.

Moreover, as also discussed throughout the definition of our problem, review papers by Ortuño et al. [22], Liberatore et al. [23], and Grass and Fisher [24] indicate the essence of the effects that stochasticity creates in humanitarian logistics. The review by Liberatore et al. [23] defines the risks and uncertainties associated with disasters in depth, and furthermore, discusses the sources of uncertainties in disasters and how to model them. Grass and Fisher [24], on the other hand, survey only two-stage stochastic models in disaster management in depth and provide details on the general framework. These surveys provide a basis for the significance of our problem and help us to find the crucial and essential research directions to pursue.

The facility location problems in the context of humanitarian logistics may be classified as: (i) emergency medical location problem, (ii) relief material (warehouse) location problem, and (iii) shelter site location problem [14]. Existing literature covers categories (i) and (ii) extensively, leaving category (iii) rather unexplored. In this work, we focus on category (iii). Next, we survey the literature further by dividing it into two main parts; deterministic and stochastic studies in humanitarian logistics, with an emphasis on location problems.

2.3.1 Deterministic Facility Location Problems

The relevant deterministic studies are summarized in Tables 2.1a and 2.1b. The first column in each table introduces the article; the second column states if the study is single-objective or multi-objective (denoted as S/M); the third and fourth columns denote the objective(s) and decision(s) of the study, respectively; and lastly the fifth column denotes if the proposed model is solved directly with a commercial solver or the author(s) devise a methodology. In humanitarian logistics studies various types of costs are considered, so we use following abbreviations in Tables 2.1a and 2.1b: TC is the relief material transportation cost; LC is the facility location cost; IC is the inventory holding cost; PC is the penalty cost of unsatisfied demand; and DC is the cost for destroyed or surplus material. Note that if a study only uses costs in its objective function, we classify it as a single-objective study.

Jia et al. [25] propose three heuristics to solve the model they suggest in their previous study, [34], which determines the locations of medical supply facilities for large-scale emergencies. Salman and Gül [26] propose a multi-period extension of this problem which also decides on the capacities of emergency service facilities for large-scale emergencies. They provide an MIP model and analyze the performance of it on a case study for Istanbul, Turkey.

Campbell and Jones [27] and Galindo and Batta [29] aim to minimize costs of

Table 2.1a: Deterministic Location Studies in Humanitarian Logistics, Types (i) and (ii)

Article	Single/Multi Objective	Objective(s)	Decision(s)	Solution Method
[25]	S	Demand coverage	Location, allocation	Heuristic
[26]	M	Travel and waiting times, LC	Location, capacity	MIP solver
[27]	S	TC	Location, prepositioning	Heuristic
[28]	S	Response time	Location, prepositioning, routing	MIP solver
[29]	S	TC, DC	Location, prepositioning	MIP solver
[30]	S	TC, PC	Location, prepositioning, routing	Two-phase heuristic
[31]	M	Travel time, # first-aiders, unmet demand	Location, routing	Heuristic
[16]	M	Distance, # facilities	Location, inventory	MIP solver
[32]	M	Unmet demand, travel time	Routing	Heuristic
[33]	M	TC, LC, IC, satisfied demand	Location, routing	MIP solver

locating warehouses and prepositioning relief supplies. While [29] assumes that the probabilities for potential facilities being destroyed is given, [27] discusses the trade-off between having relief supplies located closer to the disaster area, for faster delivery, and the supplies being at risk because of closeness to the disaster area, and extend their study to networks with already existing set of prepositioning facilities. Duran et al. [28], on the other hand, minimize the expected average response time by adding routing of relief supplies.

Lin et al. [32] propose a multi-objective integer program for delivery of prioritized relief items from a central warehouse in disaster relief operations and solve it using two different heuristics. Since supplying relief items from a central depot for longer time horizons is costly, Lin et al. [30] extend [32] by locating temporary depots and prepositioning relief supplies in the temporary depots, decreasing the transportation costs.

Table 2.1b: Deterministic Location Studies in Humanitarian Logistics, Type (iii)

Article	Single/Multi Objective	Objective(s)	Decision(s)	Solution Method
[14]	S	Shelter weight	Location, allocation	MIP solver
[35]	S	Evacuation time	Location, allocation, evacuation	2^{nd} order cone programming
[36]	S	Evacuation time	Location, allocation, evacuation	Genetic algorithm
[37]	M	Weighted distance, maximum cover	Location, allocation	MIP solver
[38]	M	Distance, risk, evacuation time	Location, allocation, evacuation	MIP solver
[39]	M	Distance, risk, evacuation time	Location, allocation, evacuation	MIP solver

Abounacer et al. [31], Rath and Gutjahr [33], and Görmez et al. [16] provide multi-objective warehouse location models. [31] and [33] consider routing of relief supplies but [16] allocates relief supplies directly to demand points. [31] and [33] develop epsilon-constraint based heuristics to find the Pareto front, while [16] proposes a bi-level program to manage the multi-objective structure.

Kılcı et al. [14] address the problem of locating shelters for an earthquake case for Istanbul, Turkey. Using predetermined set of weights for shelters (weight of a shelter is simply an indicator for its overall service level), they maximize the minimum weight of the established shelters. Bayram et al. [35] and Kongsomsaksakul et al. [36] propose models to minimize the total evacuation time by locating shelters and assigning evacuees to shelters. While [35] assigns evacuees to the nearest shelter sites, within a given degree of tolerance, [36] proposes a bi-level program with the upper level deciding on the shelter locations and the lower level deciding on the assignment of evacuees to shelters.

Lastly, Alçada-Almeida et al. [38] propose a multi-objective location-evacuation model to locate emergency shelters and identify evacuation routes with lower and upper limits on shelter utilizations and predefined number of shelters.

Coutinho-Rodrigues et al. [39] extend [38] by introducing varying objectives and not limiting the number of shelters to be opened. Chanta and Sangsawang [37] investigate a bi-objective model which determines the locations of shelters to serve a region suffering from a flood disaster.

2.3.2 Stochastic Facility Location Problems

Table 2.2a: Stochastic Single-Objective Location Studies in Humanitarian Logistics, Types (i) and (ii)

Article	# of stages	Objective(s)	Decision(s)	Uncertainty	Solution Method
[40]	2	LC, VC, AC_e	F: Location, # vehicles; S: Allocation	Demand	MIP solver
[8]	2	Satisfied demand	F: Location, preposition; S: Demand satisfaction	Demand, cost, time	MIP solver
[41]	2	LC, MC_e	F: Location, preposition; S: Allocation	Demand	MIP solver
[42]	2	IC, OP, TC_e, LC_e, PC_e	F: Location, preposition; S: Allocation, location	Demand, capacity, time, cost	Heuristic
[43]	2	LC, MC, PC_e, DC_e, TC_e	F: Location, preposition; S: Allocation	Demand, inventory, transport network	L-Shaped method
[44]	2	Accessibility	F: Location, capacity; S: Allocation	Demand, accessibility	Integer L-Shaped method

Tables 2.2a - 2.2d summarize the relevant stochastic studies. Tables 2.2a and 2.2b include single-objective studies where Tables 2.2c and 2.2d include multi-objective studies. The first column introduces the article; the second column states if the model is two-stage or three-stage; the third and fourth columns denote the objective(s) and decision(s) of the study, respectively, where F stands for the first stage, S for the second stage, and T for the third stage; the fifth

column indicates the uncertain parameters; and lastly the sixth column denotes if the proposed model is solved directly with a commercial solver or the author(s) devise a methodology.

In addition to the abbreviations used for different types of costs in Tables 2.1a and 2.1b, we also define the following abbreviations: VC is the cost of locating vehicles; MC is the cost of procuring relief materials; AC is the cost of transporting disaster victims to shelters; and OP is the operation cost of a warehouse/relief center. Note that if the corresponding cost is calculated as an expected cost, we denote it with a subscript, e.g. TC_e.

Table 2.2b: Stochastic Single-Objective Location Studies in Humanitarian Logistics, Type (iii)

Article	# of stages	Objective(s)	Decision(s)	Uncertainty	Solution Method
[45]	2	Evacuation time	F: Location; S: Evacuation	Demand, transport network	MIP solver
[46]	2	LC, TC_e, IC_e, PC_e, AC_e	F: Location, capacity; S: Allocation	Evacuees, costs	L-Shaped method
Our S-O Model	3	Expected-weighted shelter	F: Location; S: Allocation, Location; T: Allocation	Demand	Heuristic

For the emergency medical location problems (i); Beraldi and Bruni [40] locate emergency service vehicles in congested emergency systems using reliability constraints. Mete and Zabinsky [47] extend [40] by prepositioning the supplies and adding uncertainties in transportation time and costs, adding transportation time as an additional objective, and discuss the effectiveness of their proposed methodology using a case study for earthquake scenarios in Seattle area.

For the relief material (warehouse) location problems (ii); Balcik and Beamon [8], Chang et al. [41], and Döyen et al. [42] propose facility location models with prepositioning. All assume uncertainties in demand while [8] and [42] have additional uncertainty assumptions, e.g. in cost, time, and etc. Noyan [43] extends the facility location model by adding risk-aversion through Conditional

Table 2.2c: Stochastic Multi-Objective Location Studies in Humanitarian Logistics, Types (i) and (ii)

Article	# of stages	Objective(s)	Decision(s)	Uncertainty	Solution Method
[47]	2	F: OP; S: PC_e, Transport time	F: Location, preposition; S: Allocation	Demand, time, costs	MIP solver
[48]	2	F: TC, MC; S: TC_e, MC_e, IC_e, PC_e, shortage	F: Location, preposition; S: Allocation	Demand, costs, inventory	MIP solver
[49]	2	F: LC; S: Response time	F: Location, preposition; S: Allocation, routing	Demand, time	MIP solver
[17]	2	F: # facilities; S: Distance, unmet demand	F: # of facilities; S: Allocation	Demand	MIP solver
[50]	2	F: Transport time, risk, LC, IC; S: Transport time, unmet demand	F: Location, capacity, preposition; S: Routing	Demand, transport network	Not solved
[51]	2	F: LC, VC, OP; S: Demand coverage	F: Budget, location, # vehicles; S: Allocation	Time, costs	MIP solver
[52]	2	F: LC, IC, OP; S: PC_e, DC_e, travel time	F: Location, preposition; S: Allocation	Demand, time, cost, inventory	Heuristic
[53]	3	S: Unmet demand; T: Budget	F: Location, routing; S: Routing	Demand, capacity, transport network	MIP solver

Value-at-Risk (CVaR) where Noyan et al. [44] focus on the last mile distribution to achieve high accessibility and equity.

In multi-objective literature, Bozorgi-Amiri et al. [48], Caunhye et al. [49], Gunnec and Salman [50], and Tofighi et al. [52] also propose facility location models with prepositioning. [48] and [52] only consider transportation of relief supplies, where [49] and [50] also introduce routing decisions. All consider demand as an uncertain parameter while some address, for example, cost, time, and etc. to be uncertain. Cavdur et al. [17], Rath et al. [51] –extending Rath and Gutjahr [33]–, and Rennemo et al. [53] consider multi-objective cases. Only [53] offers a three-stage model and routing of relief supplies to demand points and focuses on the last mile distribution.

For the shelter site location problems (iii); Bayram and Yaman [45], Li et al. [46], and Li et al. [54] propose shelter location models. [46] looks at cases where the relief supplies are transported from an already existing set of depots to located shelters along with shelter capacities, where [45] and [54] consider evacuation of victims from disaster points to shelter sites. Bayram and Yaman [45], extending [35], assign evacuees to the nearest shelter sites, within a given degree of tolerance, while [54] deals with the distance traveled by evacuees in the objective function and allow evacuees to be remain unassigned.

Table 2.2d: Stochastic Multi-Objective Location Studies in Humanitarian Logistics, Type (iii)

Article	# of stages	Objective(s)	Decision(s)	Uncertainty	Solution Method
[54]	2	S: Unmet demand, travel time	F: Location; S: Evacuation	Demand, shelter, accessibility, time	Heuristic
Our M-O Model	3	Risk, minimum utilization, expected shelter, shelter weight	F: Location; S: Allocation, Location; T: Allocation	Demand	Heuristic

Our single-objective model, denoted as S-O, belongs to Table 2.2b and its extended version onto the multi-objective framework, denoted as M-O, belongs to Table 2.2d. In the single-objective formulation, we minimize the expected number of established shelters over all scenarios while aiming to establish the shelters with higher weights. In the multi-objective model, we break up the objective in the single-objective formulation into two objectives and also introduce two new objective functions, which are observed to be necessary in computational analyses of the single-objective formulation. In the next section, we discuss the details.

2.4 Extending the Literature

The above literature reveals that shelter location, especially a study that considers secondary earthquakes, is a research direction still to be explored. To the best of our knowledge, only Zhang et al. [55] consider secondary disasters directly. But the method they propose is fairly inefficient as they have to repeat their algorithm for each disaster scenario (see Su et al. [56]). While [55] allocates relief supplies to disaster nodes, we locate shelter sites and allocate disaster victims so that they receive acceptable levels of service in terms of sheltering. We manage the risks in all possible initial earthquake-aftershock scenario pairs.

Extending the above literature, we propose consideration of demand variability across different occurrences in choosing the locations of shelter sites while bearing the additional variability introduced by consecutive disasters - called multi-hazard in the relevant literature. We mimic the behavior of disaster victims in the sense that we assume that they will *always* travel to the shelter nearest to them with all of their neighbors, without any regard for the capacity limitations.

We model the multi-hazard nature of disasters via a multi-stage stochastic MIP model in the shelter site location problem. To mimic the behavior of the disaster victims, we use nearest assignment constraints as proposed in [57]. Since the capacity of the shelters may be exceeded when the disaster victims travel to the nearest shelter without any demand division, we define the risk in this setting as the capacity of a shelter being exceeded. To manage this unavoidable risk, we utilize CVaR constraints in sheltering the disaster victims and define our risk-aversion level to lie between certain limits.

We observe that considering smaller scenario sets, e.g. 100 scenarios, even with varying problem parameters may result in similar solutions between different instances and hence conclude that one should consider larger scenario sets to explain the stochastic nature of disasters and the variety of decisions made regarding the problem parameters in a more thorough sense. Hence, we propose a heuristic method to solve the problem for larger scenario sets.

In the multi-objective counterpart of our problem, we consider the same setting but save the DM from the burden of choosing parameters, as dictating performance affecting parameters in a humanitarian setting is not plausible in reality, by defining four new objectives. As it is discussed in coming sections, to improve the performance of our solutions in the single-objective model, we incorporate minimum utilization constraints on the shelters and also ask the DM to choose two parameters for the risk-aversion criterion, one of which requires some expertise. Our multi-objective model remedies this problem of parameter selection and provides a set of non-dominated solutions from which the DM can choose by prioritizing certain objectives.

Chapter 3

Single-Objective Stochastic Shelter Site Location under Multi-Hazard Scenarios

Earthquakes are disasters that are not known *a priori*. We do not know the time, effect or magnitude of an earthquake. We do not know if any aftershocks will follow the initial shock, and if it does, again we do not know the time, effect or magnitude of it. All of this uncertainty points to stochastic modeling where both the initial earthquake and the aftershock, namely the multi-hazard, are uncertain. And when this multi-hazard phenomenon does occur, the population at risk will be the disaster victims who seek shelter. Some proportion of the population at risk will seek shelter after the initial earthquake and some others will seek after the aftershock. To model this, we introduce multi-hazard methodology to shelter site location problem via a multi-stage stochastic MIP model.

3.1 Characteristics of the Problem

After an earthquake, disaster victims sharing the same neighborhood (or district) always travel to the nearest shelter. As in case of a disaster, no victim would agree to spend more time to reach a shelter than his neighbor, it is not possible to divide the demand and state that certain victims are to reside in another shelter (see Section 2.2.2 for a brief discussion). In a multi-hazard setting, this behavior reflects to both the initial earthquake and the aftershock. The fact that victims are always acting along with their interests raises a challenge on shelter capacities. When every district travels to the nearest shelter, the capacity of the established shelters may be exceeded. As it is apparent in 1999 Marmara Earthquake, having shelter utilizations as high as 140% reduces the quality of services received by the disaster victims [14].

In order to control the shelter capacities and to manage the risk of exceeding the shelter capacities, we utilize CVaR. Presented as an approach to optimize or hedge a portfolio of financial instruments to reduce risk, CVaR is also used in humanitarian logistics literature to mitigate possible risks (e.g. Noyan [43]). CVaR, in our setting, provides the DM a way of controlling the risk-aversion level, aiding in management of the over-utilization of shelters. As Rockafellar and Uryasev [58,59] discuss, value-at-risk (VaR), another approach in optimization to reduce risk, provides poor quality solutions in our setting with respect to CVaR as VaR disregards the distribution of the tail, i.e. may regard higher and smaller violations of the shelter utilizations as the same and therefore may perform worse.

Having described the problem setting, we propose a three-stage stochastic MIP model for locating shelter sites after an earthquake has occurred and an aftershock may happen. We allow the DM to tune the risk-aversion level and we incorporate nearest assignment, or nearest allocation, constraints into the model to reflect the real life choices of the disaster victims. It is assumed that the DM decides on the location of the shelter sites after an earthquake has happened and before the actual demand is observed. This is same for the first and second stages. Again in the first and second stages, as nearest assignment constraints are utilized, the assignment of districts to shelter sites are finalized. And finally after the demand realization in the third stage, the utilization of shelter sites are observable. Observe that once a shelter is established, it cannot be closed as the victims residing there will not move to other shelters. Thus, decisions made in the first stage will remain for the next stages.

3.1.1 Illustration of an Instance

We can illustrate the problem setting using Figures 3.1a–3.1e. The yellow star represents the epicenter of the initial earthquake, the red squares represent the shelters, and the blue circles represent the demand points (districts). All the demand points in Kartal, Istanbul and the epicenter of the initial earthquake can be observed in Figure 3.1a.

Once an earthquake occurs, the DM establishes the shelters, red squares, before observing the actual demand, as in Figure 3.1b, in the first stage. In the second stage, after the demand realization of the earthquake, the disaster victims travel to the nearest open shelter as in Figure 3.1c. Lines represent the allocation of the districts to the open shelters, finalized in the first stage. After the disaster victims travel to the nearest open shelters, an aftershock may hit Kartal and may require new shelters, additional red squares, to be established as in Figure 3.1d. Note that for this particular instance, three new shelters are established under some disaster scenarios. In the third stage, after the demand realization of the aftershock, the disaster victims travel to the nearest open shelter as in Figure 3.1e, decided in the second stage. Dashed lines represent the allocation of the districts to the open shelters in the third stage. As under different disaster scenarios, different shelters can be established in the second stage, third stage allocation of districts differs from scenario to scenario. This fact can be observed in Figure 3.1e as districts 4 and 13 have two dashed lines, depending on which shelter is opened in the second stage.

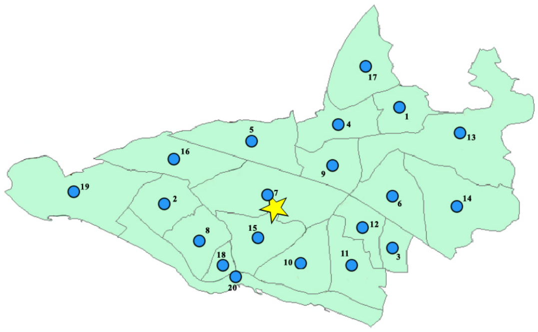

Figure 3.1a: Demand points and the epicenter of the initial earthquake

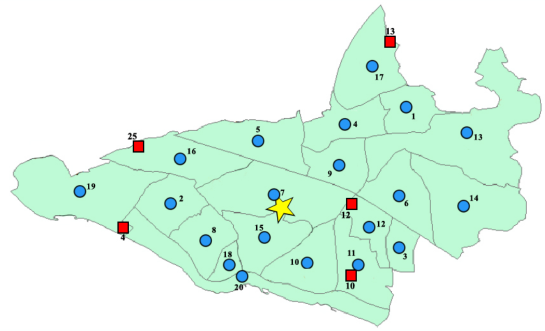

Figure 3.1b: Open shelters after an earthquake has occurred

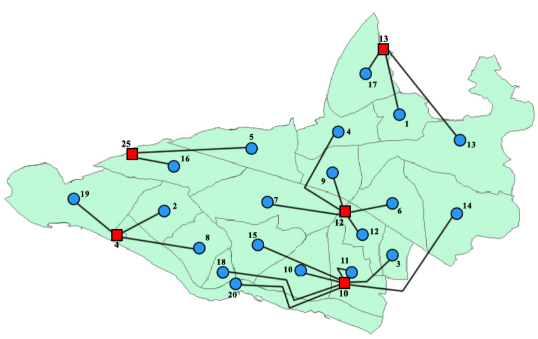

Figure 3.1c: Allocation of demand points after an earthquake has occurred

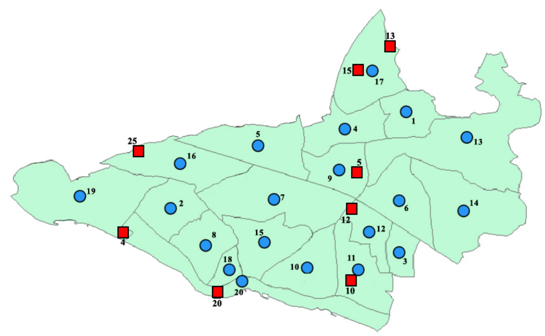

Figure 3.1d: Open shelters after the aftershock, note that some shelters were already open

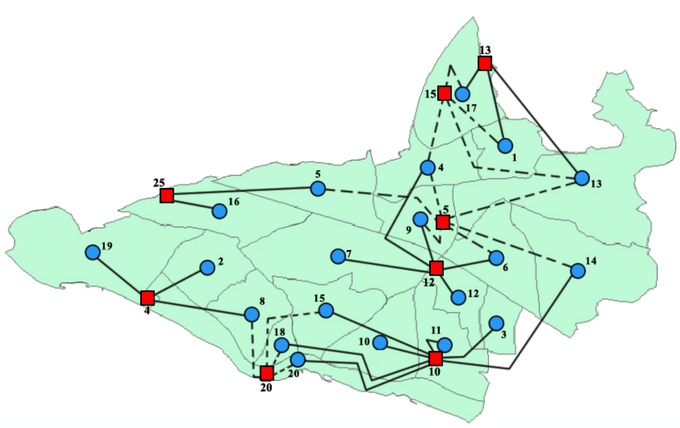

Figure 3.1e: Allocation of demand points after the aftershock and the final result of a problem instance

3.1.2 Characteristics of the Proposed Formulation

Throughout this thesis, we assume that the demand after the initial earthquake and the aftershocks is uncertain. To the best of our knowledge, in the humanitarian logistics studies, there is not any dataset that considers secondary disasters, although many do consider initial disasters (see e.g. Balcik and Beamon [8], Gunnec and Salman [50], Kılcı et al. [14], Noyan et al. [44], and Verma and Gaukler [60]). Therefore, we create a new dataset based on the network provided by Kılcı et al. [14]. As we assume that 10 different aftershocks can follow a single earthquake, we create 50 different initial earthquakes and provide a dataset of 500 earthquake and aftershock scenarios, which will be discussed in Chapter 4.

After preliminary tests with the proposed model using our dataset, we seek to improve the quality of solutions as victims are assigned to farther shelters and some shelters have utilizations as low as 3% in some instances. To remedy this, we consider including two additional set of constraints to the formulation: an upper limit on the distance between disaster victims and the assigned shelters and a minimum utilization for open shelters. These constraints provide solutions that are preferable by both the victims and the DM (e.g. government authorities), respectively.

To be in accordance with the dataset provided by Kılcı et al. [14], we assume that the set of candidate shelter locations is known in advance, all shelters have predetermined capacities and have previously assigned weights that denote their level of performance. [14] defines eligible shelter site locations, identifies the attributes of these shelter sites using ten different criteria, scales the values of respective criteria to common units and finally calculates the weights of shelter sites as a convex combination of the scaled values.

We also assume that the population of each district is concentrated in its centroid. A significant assumption is on the capacity of the shelters - we assume that under no circumstances the capacity of a shelter changes, i.e. the risk of losing convenience of any shelter is non-existent.

3.2 Mathematical Model

Consider a finite probability space (Ω, Π) where Ω is the sample space, i.e. set of elementary events (we will refer to them as scenarios hereafter), and Π is a probability measure on Ω. Let $S = \{1, \ldots, n\}$ be the index set of the scenarios, then $\Omega = \{\omega_1, \ldots, \omega_n\}$ and $\Pi(\omega_s) = p_s$ for $s \in S$. Then we use the following notation for the sets and parameters:

Sets:

I : set of districts

J : set of candidate shelter sites

S : index set of the scenarios

S_s^2 : set of scenarios sharing the same history as scenario $s \in S$ up to second stage

Parameters:

w_j: weight of candidate shelter site $j \in J$; $w_j \in (0, 1]$

p_s: probability of scenario $s \in S$

τ_j: allowed tolerance of exceeding capacity for shelter site $j \in J$

q_{is}^1: number of people affected in district $i \in I$ under scenario $s \in S$ after the initial earthquake

q_{is}^2: number of people affected in district $i \in I$ under scenario $s \in S$ after the aftershock

d_{ij}: distance between district $i \in I$ and candidate shelter site $j \in J$

α: risk-aversion parameter of CVaR

c_j: capacity of shelter site $j \in J$

For each district $i \in I$, the distances d_{ij} can be sorted non-decreasingly, thus providing an ordered sequence for the candidate shelter sites in terms of their distances to each district. We denote it by $j_i(r)$, the r-th closest candidate shelter site to district $i \in I$, $r = 1, \ldots, |J|$.

Then we define the decision variables as:

$$x_j^1 = \begin{cases} 1 & \text{if shelter } j \text{ is established in stage 1} \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in J$$

$$y_{ij}^1 = \begin{cases} 1 & \text{if district } i \text{ is assigned to shelter } j \\ & \text{in stage 1} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I, j \in J$$

$$x_{js}^2 = \begin{cases} 1 & \text{if shelter } j \text{ is established in stage 2} \\ & \text{under scenario } s \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in J, s \in S$$

$$y_{ijs}^2 = \begin{cases} 1 & \text{if district } i \text{ is assigned to shelter } j \\ & \text{under scenario } s \text{ in stage 2} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I, j \in J, s \in S$$

$$f_{js}^3 = \text{overall utilization of shelter site } j \text{ under scenario } s \quad \forall j \in J, s \in S$$

Recall the construction of this problem using the nearest assignment constraints. The definition of decision variables follows the same discussion. Since nearest assignment constraints are utilized, once the shelter sites are located, the assignment decisions are immediate. Therefore, the assignment decisions will be the same whether they are made before observing the demand or after observing the demand. But, to decide on the utilization of a shelter site, it is required to realize the uncertain demand for the whole planning horizon, which is in turn realized finally in the third stage. Hence follows the above definition of variables.

Additionally, we define random variables X_j^2 and F_j^3. Let x_{js}^2 be the realizations of the random variable X_j^2 where $x_{js}^2 = X_j^2(\omega_s)$, and let f_{js}^3 be the realizations of the random variable F_j^3 where $f_{js}^3 = F_j(\omega_s)$, $j \in J, s \in S$. Then we have the following three-stage stochastic MIP model:

$$\mathrm{P(S)} \ = \ \min \quad \sum_{s \in S} \sum_{j \in J} p_s \frac{1}{w_j} x_{js}^2 \tag{3.1}$$

$s.t.$

$$\sum_{j\in J} y^1_{ij} = 1 \qquad \forall i \in I \tag{3.2}$$

$$\sum_{k=r+1}^{|J|} y^1_{ij_i(k)} + x^1_{j_i(r)} \le 1 \qquad \forall i \in I, r = 1, \dots, |J|-1 \tag{3.3}$$

$$y^1_{ij} \le x^1_j \qquad \forall i \in I, j \in J \tag{3.4}$$

$$\sum_{j\in J} y^2_{ijs} = 1 \qquad \forall i \in I, s \in S \tag{3.5}$$

$$\sum_{k=r+1}^{|J|} y^2_{ij_i(k)s} + x^2_{j_i(r)s} \le 1 \qquad \forall i \in I, s \in S, r = 1, \dots, |J|-1 \tag{3.6}$$

$$y^2_{ijs} \le x^2_{js} \qquad \forall i \in I, j \in J, s \in S \tag{3.7}$$

$$x^1_j \le x^2_{js} \qquad \forall j \in J, s \in S \tag{3.8}$$

$$x^2_{js'} = x^2_{js} \qquad \forall j \in J, s \in S, s' \in S^2_s \tag{3.9}$$

$$CVaR_\alpha\left(F^3_j - X^2_j\right) \le \tau_j \qquad \forall j \in J \tag{3.10}$$

$$f^3_{js} = \frac{\sum_{i\in I} q^1_{is} y^1_{ij} + \sum_{i\in I} q^2_{is} y^2_{ijs}}{c_j} \qquad \forall j \in J, s \in S \tag{3.11}$$

$$x^1_j \in \{0,1\} \qquad \forall j \in J \tag{3.12}$$

$$y^1_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{3.13}$$

$$x^2_{js} \in \{0,1\} \qquad \forall j \in J, s \in S \tag{3.14}$$

$$y^2_{ijs} \in \{0,1\} \qquad \forall i \in I, j \in J, s \in S \tag{3.15}$$

$$f^3_{js} \ge 0 \qquad \forall j \in J, s \in S \tag{3.16}$$

The objective function (3.1) minimizes the weighted expected number of established shelters while aiming to establish shelters with higher weights. We achieve this goal using reciprocates of the shelter weights. Constraints (3.2) make sure that every district is allocated to only one shelter in the first stage. Constraints (3.3) are the nearest allocation constraints for the first stage as presented by Wagner and Falkson [57] where we sort the distances between districts and shelter sites in a non-decreasing manner. Constraints (3.4) assure that a district is assigned to a shelter if this shelter is established in the first stage. For ease of representation, we denote constraints (3.2)–(3.4) as the "first stage allocation constraints", so that constraints (3.5)–(3.7), which are only the projections of the same decisions, can be denoted as the "second stage allocation constraints". Constraints (3.8) are to make sure that if shelter $j \in J$ is established in the first stage, it should be kept open for any scenario at the second stage (i.e. a located shelter site cannot be closed). Constraints (3.9) are the non-anticipativity constraints. Constraints (3.10) are the CVaR constraints which check the utilizations of shelter sites and make sure that the configuration of established shelters meet the risk-aversion criterion. Constraints (3.11) define the overall utilization of a shelter in the corresponding scenario. Lastly, constraints (3.12)–(3.16) are the domain constraints.

3.2.1 Details on the Mathematical Model

For completeness, let us introduce a more precise description of CVaR for continuous variables, as presented in [58, 59]. Given that Z is a random cost:

$$\mathrm{CVaR}_\alpha(Z) = \mathbb{E}\big[Z \mid Z \geq \mathrm{VaR}_\alpha(Z)\big],$$

where

$$\mathrm{VaR}_\alpha(Z) = \min_{\eta \in \mathbb{R}} \big\{\eta : \mathbb{P}\{Z \leq \eta\} \geq \alpha\big\},$$

and $\alpha \in (0, 1)$ is a preselected confidence level to tune the risk-aversion. So, $\mathrm{CVaR}_\alpha(Z)$ is the conditional expected value exceeding the $\mathrm{VaR}_\alpha(Z)$ at the confidence level α.

In our setting, we wish to control the risk of having over-utilized shelters. To do so, we introduce the discrete random variable $F_j^3 - X_j^2$ as the cost to be minimized, where $F_j^3 - X_j^2$ may also be regarded as the loss function. Referring to the previous discussion on realizations of these random variables, this difference is positive (negative) when the realizations of the utilization of the corresponding shelter in the corresponding scenario is above (below) 100%. Note that $f^3_{js} > 0$ when $x^2_{js} = 1$, due to the minimum utilization constraints, and $f^3_{js} = 0$ when $x^2_{js} = 0$. As our aim is to keep this loss, over-utilization when positive, as small as possible, we measure the risk of this loss using CVaR. We limit $\text{CVaR}_\alpha(F^3_j - X^2_j)$ from above with τ_j, a parameter tuned by the DM as a secondary measure of risk-aversion – also a bound to control the tail of the loss distribution, and formally introduce the CVaR constraints (3.10).

We provide a more general version of CVaR, keeping Z as the random cost vector for ease of representation, and provide the linearized version of constraints (3.10) by referring to Rockafellar and Uryasev [58]:

$$\text{CVaR}_\alpha(Z) = \inf_{\eta \in \mathbb{R}} \big\{ \eta + \frac{1}{1-\alpha} \mathbb{E}([Z - \eta]_+) \big\},$$

where $[a]_+ = \max\big\{0,\, a\big\}$, $a \in \mathbb{R}$.

To linearize the CVaR constraints (3.10), referring to the above discussion, we define two new continuous decision variables, z_{js} and η_j, and replace constraints (3.10) with constraints (3.17)–(3.20) in P(S):

$$\eta_j + \frac{1}{1-\alpha} \sum_{s \in S} p_s z_{js} \leq \tau_j \qquad \forall j \in J \tag{3.17}$$

$$z_{js} \geq f^3_{js} - x^2_{js} - \eta_j \qquad \forall j \in J, s \in S \tag{3.18}$$

$$z_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{3.19}$$

$$n_j \text{ is free} \qquad \forall j \in J \tag{3.20}$$

In multi-stage stochastic models, for the scenarios having the same history up to a given stage, the decisions made at that stage must be the same. This is called non-anticipativity [61]. In the proposed model, this translates to the scenarios having the same history up to second stage should share the same decisions at that stage. In other words, the assignment of districts to shelters and establishment of new shelters in the second stage cannot differ for scenarios sharing the same initial earthquake, where the scenarios correspond to the whole horizon. To force this on the proposed model, we utilize non-anticipativity constraints. Note that these type of constraints are not necessary for the first stage decisions as they do not depend on scenarios.

To discuss the structure of non-anticipativity constraints in this context, in Figure 3.2, we first visualize the decision process. Recall that an initial earthquake triggers x^1 decisions and an aftershock triggers x^2 decisions. Also observe that assignment decisions do not depend on demand realizations and the utilization of shelters are finalized in the third stage.

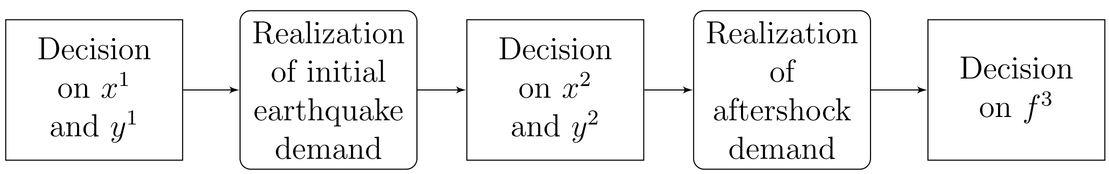

Figure 3.2: Structure of the Decision Process

By construction of the dataset, we have 10 different aftershocks following each initial shock. Since second stage shelter location decisions do not depend on the realization of aftershocks, i.e. second stage shelter location decisions only depend on the realization of initial earthquake, these decisions should be kept homogeneous throughout the aftershock scenarios sharing the same initial earthquake, hence we define set S_s^2, which is the set of scenarios sharing the same history as scenarios $s \in S$ up to second stage. Then, constraints (3.9) define this relation and make sure that the second stage shelter location decisions are homogeneous with respect to the common history, i.e. the initial earthquake.

Figure 3.3 visualizes this discussion. In accordance with Figure 3.2, location and allocation decisions in the first stage is followed by location and allocation decisions in the second stage, after the demand regarding the initial shock is realized. Finally, after the demand regarding the aftershock is realized, the third stage decisions, shelter utilizations, are finalized. In accordance with the previous discussion, the decisions on the second stage shelters should be homogeneous regardless of the realized aftershock scenario.

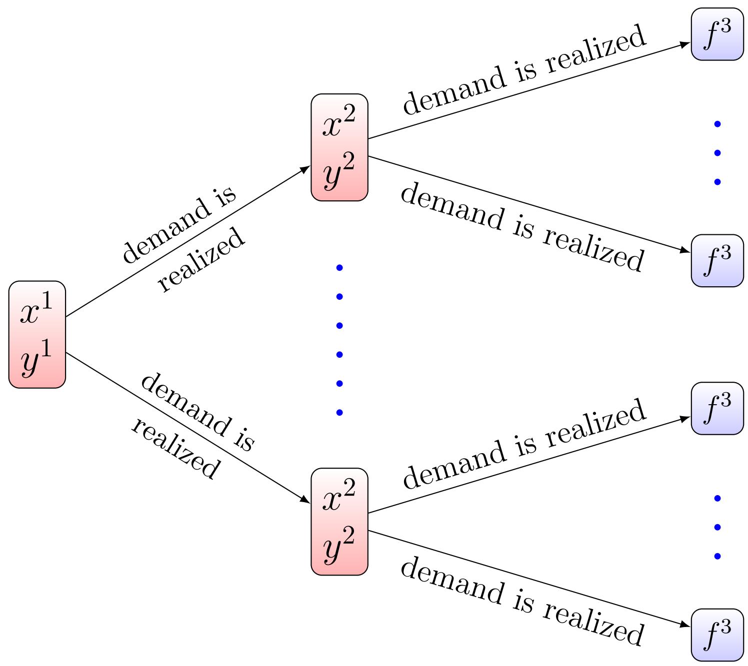

Figure 3.3: Visualization of Non-anticipativity Constraints

Referring to Figure 3.3, we can discuss that non-anticipativity constraints can also be included for the second stage allocation decisions, namely y^2, but we choose not to in our formulation as this is implied by the nearest assignment constraints.

3.2.2 Improving the Mathematical Model

As discussed earlier in this chapter, we add constraints to limit the maximum distance between the districts and the shelters and the minimum utilizations of open shelters to improve the solution qualities further. Constraints (3.21) and (3.22) limit the maximum distance between the districts and the shelters, state that no district can be forced to travel a distance more than ρ:

$$y^1_{ij} d_{ij} \leq \rho \qquad \forall i \in I, j \in J \tag{3.21}$$

$$y^2_{ijs} d_{ij} \leq \rho \qquad \forall i \in I, j \in J, s \in S \tag{3.22}$$

Constraints (3.23) and (3.24) limit the minimum utilizations of open shelters, stating that if at least one district is assigned to a shelter, then that shelter should be utilized at a level of at least υ:

$$f^3_{js} \geq \upsilon y^1_{ij} \qquad \forall i \in I, j \in J, s \in S \tag{3.23}$$
$$f^3_{js} \geq \upsilon y^2_{ijs} \qquad \forall i \in I, j \in J, s \in S \tag{3.24}$$

Then P(S) is:

$$\text{P(S)} = \min \sum_{s \in S} \sum_{j \in J} p_s \frac{1}{w_j} x^2_{js} \tag{3.1}$$

$$s.t.$$

$$\sum_{j \in J} y^1_{ij} = 1 \qquad \forall i \in I \tag{3.2}$$
$$\sum_{k=r+1}^{|J|} y^1_{ij_i(k)} + x^1_{j_i(r)} \leq 1 \qquad \forall i \in I, r = 1, \ldots, |J|-1 \tag{3.3}$$
$$y^1_{ij} \leq x^1_j \qquad \forall i \in I, j \in J \tag{3.4}$$
$$\sum_{j \in J} y^2_{ijs} = 1 \qquad \forall i \in I, s \in S \tag{3.5}$$
$$\sum_{k=r+1}^{|J|} y^2_{ij_i(k)s} + x^2_{j_i(r)s} \leq 1 \qquad \forall i \in I, s \in S, r = 1, \ldots, |J|-1 \tag{3.6}$$
$$y^2_{ijs} \leq x^2_{js} \qquad \forall i \in I, j \in J, s \in S \tag{3.7}$$
$$x^1_j \leq x^2_{js} \qquad \forall j \in J, s \in S \tag{3.8}$$
$$x^2_{js'} = x^2_{js} \qquad \forall j \in J, s \in S, s' \in S^2_s \tag{3.9}$$
$$f^3_{js} = \frac{\sum_{i \in I} q^1_{is} y^1_{ij} + \sum_{i \in I} q^2_{is} y^2_{ijs}}{c_j} \qquad \forall j \in J, s \in S \tag{3.11}$$
$$\eta_j + \frac{1}{1-\alpha} \sum_{s \in S} p_s z_{js} \leq \tau_j \qquad \forall j \in J \tag{3.17}$$
$$z_{js} \geq f^3_{js} - x^2_{js} - \eta_j \qquad \forall j \in J, s \in S \tag{3.18}$$
$$y^1_{ij} d_{ij} \leq \rho \qquad \forall i \in I, j \in J \tag{3.21}$$

$$y^2_{ijs} d_{ij} \leq \rho \qquad \forall i \in I, j \in J, s \in S \tag{3.22}$$

$$f^3_{js} \geq \upsilon y^1_{ij} \qquad \forall i \in I, j \in J, s \in S \tag{3.23}$$

$$f^3_{js} \geq \upsilon y^2_{ijs} \qquad \forall i \in I, j \in J, s \in S \tag{3.24}$$

$$x^1_j \in \{0, 1\} \qquad \forall j \in J \tag{3.12}$$

$$y^1_{ij} \in \{0, 1\} \qquad \forall i \in I, j \in J \tag{3.13}$$

$$x^2_{js} \in \{0, 1\} \qquad \forall j \in J, s \in S \tag{3.14}$$

$$y^2_{ijs} \in \{0, 1\} \qquad \forall i \in I, j \in J, s \in S \tag{3.15}$$

$$f^3_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{3.16}$$

$$z_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{3.19}$$

$$n_j \text{ is free} \qquad \forall j \in J \tag{3.20}$$

Chapter 4

Dataset

We model the demand uncertainty in our setting using a dataset consisting of earthquake and aftershock scenarios. To the best of our knowledge, in the humanitarian logistics literature, even though there are many studies providing datasets on initial disasters there is not any study that provides a dataset for both initial and secondary disasters. Therefore, we devise a new methodology to create scenarios for a district of Istanbul, Turkey.

Throughout our study, we use the network of Kartal provided by Kılcı et al. [14] (see Figures 4.1 and 4.2). Kartal has 25 candidate shelter locations with corresponding capacities provided in Table 4.1 and corresponding weights in Appendix A.1. Kartal also has 20 districts, which are given along with their populations in Appendix A.2.

Table 4.1: Shelter capacities

Shelter #	1	2	3	4	5	6	7	8	9
Capacity	24,000	45,000	25,000	60,000	60,000	25,000	30,000	75,000	25,600
Shelter #	10	11	12	13	14	15	16	17	18
Capacity	100,000	30,000	62,500	60,000	50,000	30,625	30,000	75,000	45,000
Shelter #		19	20	21	22	23	24	25	
Capacity		60,000	30,000	25,000	25,000	150,000	30,000	60,000	

We create the dataset in accordance with the JICA-IMM joint study [15]. We

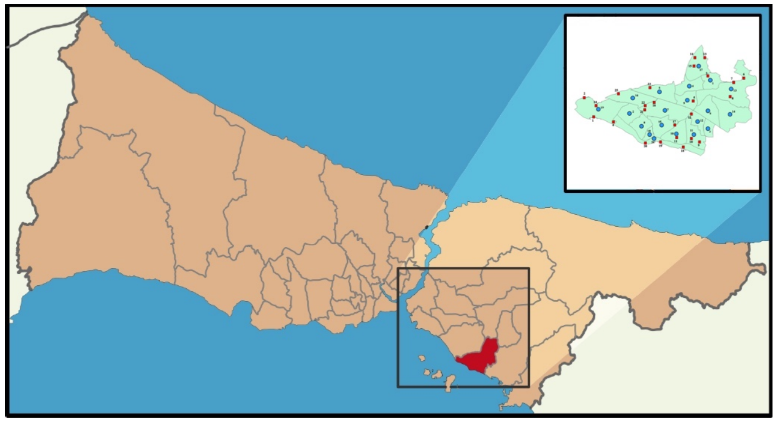

Figure 4.1: Kartal's location in Istanbul

assume that each initial earthquake will be followed by 10 different aftershocks and all of the initial earthquakes share the same epicenter, varying in magnitude. We propose 50 distinct initial earthquakes and therefore a total of 500 distinct disaster scenarios.

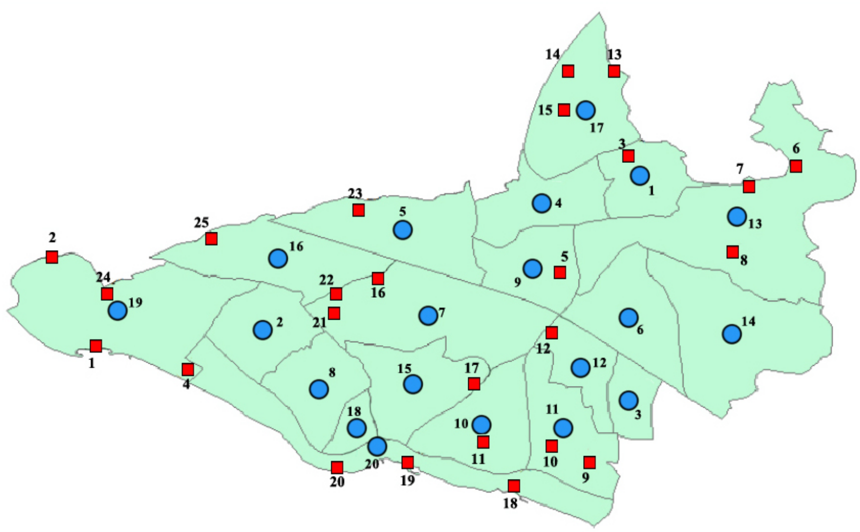

Figure 4.2: Blue circles represent the demand points (districts) and red squares represent the candidate shelter locations in Kartal

We differentiate the earthquakes in this setting according to three features: epicenter, effect radius and percent affected ratio (PAR). Our methodology regards these features and, as discussed above, uses the same epicenter for every initial earthquake. For initial earthquakes, we only decide on the effect radius and the proportion of the population in a district it affects, namely PAR. We assume that with a probability of 20%, the initial earthquake will affect the districts in 3 km radius, with a probability of 40% it will affect the districts in 4 km radius and again with a probability of 40% it will affect the districts in 5 km radius.

Table 4.2a: Effect radius, occurrence probability and PAR values of initial earthquakes

Effect radius (km)	Occurrence probability	PAR
3	16%	$\mathcal{U}$[0.4, 0.5]
	50%	$\mathcal{U}$[0.5, 0.6]
	34%	$\mathcal{U}$[0.6, 0.7]
4	16%	$\mathcal{U}$[0.5, 0.6]
	50%	$\mathcal{U}$[0.6, 0.7]
	34%	$\mathcal{U}$[0.7, 0.8]
5	16%	$\mathcal{U}$[0.6, 0.7]
	50%	$\mathcal{U}$[0.7, 0.8]
	34%	$\mathcal{U}$[0.8, 0.9]

Table 4.2b: Effect radius, occurrence probability and PAR values of aftershocks

Effect radius (km)	Occurrence probability	PAR
$\mathcal{U}$[3.9, 4.2]	16%	$\mathcal{U}$[0.32, 0.40]
	50%	$\mathcal{U}$[0.40, 0.48]
	34%	$\mathcal{U}$[0.48, 0.56]
$\mathcal{U}$[5.2, 5.6]	16%	$\mathcal{U}$[0.40, 0.48]
	50%	$\mathcal{U}$[0.48, 0.56]
	34%	$\mathcal{U}$[0.56, 0.64]
$\mathcal{U}$[6.5, 7.0]	16%	$\mathcal{U}$[0.48, 0.56]
	50%	$\mathcal{U}$[0.56, 0.64]
	34%	$\mathcal{U}$[0.64, 0.72]

The corresponding PAR values along with their probabilities for the initial earthquakes can be found in Table 4.2a, where $\mathcal{U}$[a, b] denotes a continuous uniform distribution in the interval [a, b] used to generate the PAR values, for which a $\leq$ b. It is important to note that the districts are affected inversely proportional to their distances to the epicenter in the cases of both initial earthquakes and the aftershocks.

The same idea applies to the generation of aftershocks. But since aftershocks, as in the real setting, may depend on the initial earthquake, we use the features of the initial earthquake. We assume that the epicenter of the aftershock is within a circle, which is centered at the epicenter of the initial earthquake and has a radius equal to the half of the effect radius of the initial earthquake. The aftershock's effect radius is greater than the initial earthquake's effect radius by a factor of a number generated from $\mathcal{U}$[0.3, 0.4], i.e. we multiply the effect radius of the initial earthquake by $\mathcal{U}$[1.3, 1.4] and obtain the interval for the effect radius of the aftershock, and its PAR value is 20% lower than the initial earthquake's PAR value. For example, if an initial earthquake has an effect radius of 3 km, as in the first row of Table 4.2a, the aftershock's epicenter is within 1.5 km radius of the initial earthquake's epicenter and the aftershock's features are determined as presented in the first row of Table 4.2b. The aftershock's effect radius is $3\times\mathcal{U}[1.3, 1.4] = \mathcal{U}[3.9, 4.2]$, occurrence probabilities and PAR values are as in the first row of Table 4.2b.

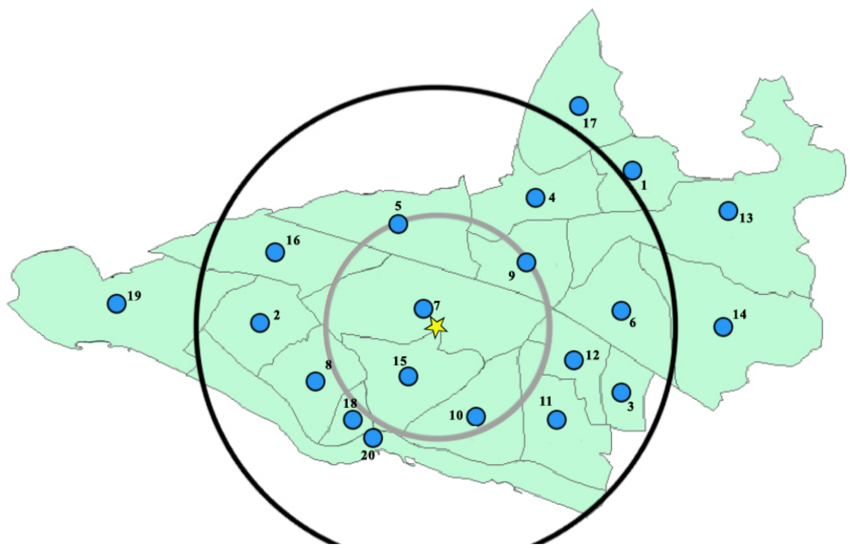

Figure 4.3: Visualization of the scenario generation methodology

The visualization of this example can be seen in Figure 4.3. The yellow star is the epicenter of the initial earthquake and its effect radius is 3 km, denoted by the black (outer) circle. Then the epicenter of the aftershock is within the gray (inner) circle, which has a radius of 1.5 km.

Chapter 5

Multi-Stage Single-Objective MIP Results

In this chapter, we present the computational experiments conducted using the proposed three-stage single-objective stochastic MIP model with the dataset described in Chapter 4. The proposed model is coded in JAVA and solved using IBM CPLEX 12.7.1. All tests were run on a Linux OS with Dual Intel Xeon E5-2690 v4 14 Core 2.6GHz processors with 128 GB of RAM.

5.1 Parameter Selection

As discussed in previous sections, some of the parameters are left to be finalized by the DM to obtain solutions of various qualities. Risk-aversion level, namely α, and allowed tolerance of exceeding capacity for each shelter site, namely $\bar{\tau}$ are to control overall risk-aversion for the shelter capacities. Note that $\forall j \in J$, τ_j is the same and we will use $\bar{\tau}$ to denote values of all τ_j in this chapter.

Constraint on the minimum utilization of established shelters, namely υ, provides the DM a means to control of the overall utilization of established shelters. The upper limit on the distance between disaster victims and the shelters is $\rho = 4$ (km) for all instances throughout this study. The parameter settings along with respective instance IDs are presented in Table 5.1. Note that $\bar{\tau}$ values differ by 5%, e.g. Instance ID 3 has $\bar{\tau} = 15\%$ and $(\alpha, \upsilon) = (0.90, 0.10)$. Instance IDs will be used to denote the corresponding parameter sets hereafter.

Table 5.1: Parameter settings for corresponding instance IDs

(α, υ)	$\bar{\tau}$	ID
(0.90, 0.10)	0.05, 0.10, . . . , 0.25	1, . . . , 5
(0.90, 0.15)	0.05, 0.10, . . . , 0.25	6, . . . , 10
(0.95, 0.10)	0.05, 0.10, . . . , 0.25	11, . . . , 15
(0.95, 0.15)	0.05, 0.10, . . . , 0.25	16, . . . , 20

5.2 Results with the Original Dataset

For each test instance, we put a 6-hour time limit on CPLEX. The results for the dataset proposed in Chapter 4 can be found in Table 5.2. Note that the number of scenarios is 500 in Table 5.2.

In Table 5.2, the first column refers to the instance IDs. The second column is the solution time of the corresponding instance in hours. If the corresponding instance cannot be solved to optimality in 6 hours, CPU time is denoted as "> 6". The third column denotes the optimality gap of the corresponding instance if it is not solved to optimality in 6 hours. Third and fourth columns refer to the configuration of the established shelters in first and second stages, respectively. Note that in the fourth column the shelters established in the second stage are presented in their entirety, i.e. not all of them are established in every scenario group but a subset of them are. The fifth column is the best objective value and is the optimal value of the corresponding test instance if it is solved to optimality. Last column is the average walk of the disaster victims to their allocated shelters.

As seen in Table 5.2, most of the test instances are not solved to optimality in 6 hours. Two of the test instances, Instances 9 and 19, cannot be solved due to

Table 5.2: Test instances for 500 scenarios

ID	CPU (hours)	Gap (%)	First stage shelters	Second stage shelters	Objective value	Average walk (m)
1	> 6	15.37	4, 10, 12, 14, 25	5, 15, 17, 21, 22	5.962	2203
2	5.6	opt.	4, 10, 12, 14, 25	15	5.761	2213
3	> 6	23.88	10, 12, 14, 25	1, 2, 4, 5, 15, 16, 24	5.466	2290
4	> 6	0.27	10, 13, 19, 25	5, 16, 24	5.001	2368
5	> 6	25.02	8, 10, 13, 25	16	4.960	2265
6	4.8	opt.	4, 10, 12, 13, 25	5, 15, 17, 22	6.067	2279
7	1.1	opt.	4, 10, 12, 13, 25	5, 15	5.866	2288
8	> 6	18.03	4, 10, 12, 13, 25	5, 15	5.866	2288
9	> 6	*	–	–	–	–
10	> 6	0.37	10, 14, 19, 25	3, 5, 16	4.972	2230
11	> 6	10.50	4, 10, 12, 14, 25	5, 15, 17, 21	6.003	2202
12	1.4	opt.	4, 10, 12, 14, 25	15, 17	5.801	2212
13	5	opt.	4, 10, 12, 14, 25	15	5.761	2213
14	> 6	6.81	10, 13, 19, 25	5, 16, 24	5.060	2362
15	> 6	6.58	10, 13, 19, 25	16, 24	4.934	2371
16	2.3	opt.	4, 10, 12, 13, 25	5, 15, 17, 21	6.108	2277
17	0.5	opt.	4, 10, 12, 13, 25	5, 15, 20	5.912	2287
18	0.6	opt.	4, 10, 12, 13, 25	5, 15	5.866	2288
19	> 6	*	–	–	–	–
20	> 6	0.88	10, 14, 19, 25	3, 5, 16	5.095	2224

memory errors, denoted by "$*$" in the table. Those which are solved to optimality and used in comparative analyses are summarized in Tables 5.3a–5.3f. We present details on the locations of established shelters in each stage and their utilizations.

In Tables 5.3a–5.3f, the leftmost column is the open shelter's number. The second column states if the corresponding shelter is established in the first or the second stage – the same note as previous table applies here, not all of the second stage shelters are established in all scenarios or even together. Third, fourth and fifth columns denote the minimum, maximum and average utilizations of corresponding shelter, respectively. Note that the minimum (maximum) utilization of a shelter is its minimum (maximum) utilization over all scenarios. Lastly, sixth column denotes the number of scenarios where the corresponding shelter's utilization has exceeded 100%.

To see the effects of changing the minimum utilizations, namely ν, on the solutions, we compare Instances 2 and 7, provided in Tables 5.3a and 5.3b respectively. In Instance 2; $\nu = 10\%$ and in Instance 7; $\nu = 15\%$. Remaining two

Table 5.3a: Instance ID 2, 500 scenarios

Shelter	Stage	Minimum util (%)	Maximum util (%)	Average util (%)	Above 100% util
4	First	39.27	116.87	81	115
10	First	40.89	96.64	70.73	0
12	First	67.19	112	93.72	142
14	First	10.42	100.44	70.56	1
25	First	32.64	64.85	53.90	0
15	Second	11.51	48.85	29.05	0

Table 5.3b: Instance ID 7, 500 scenarios

Shelter	Stage	Minimum util (%)	Maximum util (%)	Average util (%)	Above 100% util
4	First	39.27	116.87	81	115
10	First	40.89	96.64	70.50	0
12	First	66.78	112	93.03	142
13	First	19.89	83.70	61.45	0
25	First	32.64	64.85	53.65	0
5	Second	16.80	60.35	32.74	0
15	Second	15.25	91.80	49.85	0

Table 5.3c: Instance ID 17, 500 scenarios

Shelter	Stage	Minimum util (%)	Maximum util (%)	Average util (%)	Above 100% util
4	First	39.27	113.93	80.80	114
10	First	40.89	94.27	70.35	0
12	First	66.78	112	93.03	142
13	First	19.89	83.70	61.45	0
25	First	32.64	64.85	53.65	0
5	Second	16.80	60.35	32.74	0
15	Second	15.25	91.80	49.85	0
20	Second	16.63	26.74	22.28	0

Table 5.3d: Instance ID 16, 500 scenarios

Shelter	Stage	Minimum util (%)	Maximum util (%)	Average util (%)	Above 100% util
4	First	39.27	112.66	79.32	43
10	First	40.89	92.27	69.05	0
12	First	66.78	107.35	91.71	87
13	First	19.89	83.70	61.32	0
25	First	32.64	63.53	53.40	0
5	Second	15.40	60.35	31.03	0
15	Second	15.25	91.80	49.85	0
17	Second	17.19	24.76	20.96	0
21	Second	58.65	77.26	65.83	0

Table 5.3e: Instance ID 18, 500 scenarios

Shelter	Stage	Minimum util (%)	Maximum util (%)	Average util (%)	Above 100% util
4	First	39.27	116.87	81	115
10	First	40.89	96.64	70.50	0
12	First	66.78	112	93.03	142
13	First	19.89	83.70	54.57	0
25	First	32.64	64.85	53.65	0
5	Second	16.80	60.35	32.74	0
15	Second	15.25	91.80	49.85	0

Table 5.3f: Instance ID 13, 500 scenarios

Shelter	Stage	Minimum util (%)	Maximum util (%)	Average util (%)	Above 100% util
4	First	39.27	116.87	81	115
10	First	40.89	96.64	70.73	0
12	First	67.19	112	93.72	142
14	First	10.42	100.44	70.56	1
25	First	32.64	64.85	53.90	0
15	Second	11.51	48.85	29.05	0

parameters, namely α and $\bar{\tau}$, are the same in both instances. As the minimum utilization increases, for this particular case, configuration of first stage shelters differs only by one shelter, a bigger shelter is opened instead of a smaller one, and its effect can be observed as a decrease in the average utilizations of unchanged set of open shelters. In the second stage, the model chooses to establish another shelter under some scenarios.

The change in the set of open shelters in the first stage can be explained by the higher minimum utilization constraint. Utilizations of Shelters 14 and 15 in Instance 2 are both smaller than 15%. Opening Shelter 13 instead of Shelter 14 in the first stage changes the nearest allocation configuration and provides even more districts to be allocated to Shelter 15 in the third stage so that its minimum utilization is at least 15%.

If we compare Instances 2 and 7 with respect to objective values and the average walks (see Table 5.2), we can say that Instance 2 provides a better quality solution than Instance 7 as its objective value and average walk are smaller than those of Instance 7, in addition to the fact that the average utilizations of first stage shelters are higher, with a trade-off on the solution time as solution time of Instance 2 is almost half of the solution time of Instance 7.

To see the effects of changing the risk-aversion parameter, namely α, on the solutions, we compare Instances 7 and 17, provided in Tables 5.3b and 5.3c respectively. In Instance 7; $\alpha = 90\%$ and in Instance 17; $\alpha = 95\%$, and remaining two parameters, namely ν and $\bar{\tau}$, are the same for both instances. As the DM becomes more risk-averse, the number of established shelters in the second stage increases to lower the higher utilizations of the established shelters in Instance 7, since we define the risk in our setting as the capacities of shelters being exceeded too much.

Even though more shelters are established in the second stage, the average utilizations in Instance 17 are not bigger than the average utilizations in Instance 7. It can be observed that for some shelters, specifically Shelters 12, 13 and 25 in the first stage and Shelters 5 and 15 in the second stage, the statistics on the utilizations do not differ in Instance 17 with respect to Instance 7. This is due to the nearest allocation constraints. In addition to Instance 7, only Shelter 20 is established in Instance 17 in the second stage. For districts allocated to Shelters 12, 13 and 25 in the second stage and to Shelters 5 and 15 in the third stage, opening Shelter 20 does not alter the nearest allocation configuration and therefore has no effect on the utilization statistics of aforementioned shelters. But some of the districts allocated to Shelters 4 and 10 in the third stage can be allocated to Shelter 20 and thus can change utilization statistics of Shelters 4 and 10.

To see the effects of changing the allowed tolerance parameter, namely $\bar{\tau}$, on the solutions, we compare Instances 17 and 16, provided in Tables 5.3c and 5.3d respectively. In Instance 17; $\bar{\tau} = 10\%$ and in Instance 16; $\bar{\tau} = 5\%$. Remaining two parameters, namely α and ν, are the same for both instances. As discussed previously, $\bar{\tau}$ is used as a secondary measure of risk-aversion in this setting. But in contrast to α, increasing $\bar{\tau}$ decreases risk-aversion. First indicator of this fact is the decrease in the number of established shelters and the optimal value of Instance 17. Also, as a result of higher risk-aversion, the utilizations exceed 100% in less scenarios in Instance 16 with respect to Instance 17. Note that the Figures 3.1a–3.1e in Chapter 3 are the visualizations of Instance 17.

Lastly, as an interesting observation, we present Instances 7 and 18, provided in Tables 5.3b and 5.3e respectively. In both of the instances $\nu = 15\%$. With respect to α, Instance 18 is more risk-averse than Instance 7 as its α is bigger, but with respect to $\bar{\tau}$, Instance 7 is more risk-averse than Instance 18 as its $\bar{\tau}$ is smaller. And both instances have the same solution. The same phenomenon can also be observed in Instances 2 and 13, presented in Tables 5.3a and 5.3f, respectively. Instances 2 and 13 also share the same $\nu = 10\%$ and the same solutions.

5.3 Results with Smaller Datasets

As we cannot solve all of the instances to optimality within the time bound with 500 scenarios, we decrease the cardinality of the scenario set and experiment with smaller sets. Since we propose that 10 different aftershocks may follow an initial earthquake, to create smaller scenario sets, we first choose a smaller set of initial earthquakes from the original set of initial earthquakes and include the corresponding aftershocks to generate the whole scenario set. For example, for a set of 250 scenarios, we choose 25 initial earthquakes out of 50 initial earthquakes randomly, and include the aftershocks corresponding to those initial earthquakes. The same methodology applies to generating a scenario set of cardinality 100. Table 5.4 presents the test instances for a scenario set of cardinality 250.

Table 5.4: Test instances for 250 scenarios

ID	CPU (sec)	First stage shelters	Second stage shelters	Objective value	Average walk (m)
1	4,212	4, 10, 12, 13, 25	5, 17, 21	6.090	2285
2	1,364	4, 10, 12, 13, 25	5, 17	5.786	2299
3	1,086	4, 10, 12, 13, 25	5	5.745	2300
4	4,035	10, 13, 19, 25	2, 5, 16, 24	5.194	2371
5	9,263	10, 13, 19, 25	16	5.015	2380
6	1,863	4, 10, 12, 13, 25	5, 15, 17, 20, 22	6.192	2284
7	684	4, 10, 12, 13, 25	5, 15, 20	5.888	2297
8	960	4, 10, 12, 13, 25	5, 15	5.842	2298
9	9,247	4, 10, 13, 23	3, 5, 11, 15, 16, 17	5.503	2277
10	676	10, 13, 19, 25	16	5.015	2380
11	3,186	4, 10, 12, 13, 25	5, 17, 21	6.149	2281
12	1,112	4, 10, 12, 13, 25	5, 17	5.827	2298
13	880	4, 10, 12, 13, 25	5	5.745	2300
14	3,229	10, 12, 13, 25	2, 4, 5, 16, 24	5.279	2383
15	1,613	10, 13, 19, 25	5, 16	5.057	2377
16	1,429	4, 10, 12, 13, 25	5, 15, 17, 21, 22	6.246	2280
17	602	4, 10, 12, 13, 25	5, 15, 20, 22	5.946	2294
18	739	4, 10, 12, 13, 25	5, 15	5.842	2298
19	2,495	4, 10, 12, 13, 25	5, 15	5.842	2298
20	2,286	10, 13, 19, 25	5, 16	5.057	2377

We observe that as the cardinality of the scenario set decreases, the solution times decreases drastically and all of the instances can be solved to optimality. The longest solution time in the test runs with a scenario set of cardinality 250 is around 2.5 hours and the smallest solution time is 10 minutes.

Table 5.5: Test instances for 100 scenarios

ID	CPU (sec)	First stage shelters	Second stage shelters	Objective value	Average walk (m)
1	161	4, 10, 12, 13, 25	5, 22	6.165	2236
2	139	4, 10, 12, 13, 25	5, 17	6.016	2247
3	145	4, 10, 13, 19, 25	21	5.848	2276
4	118	10, 13, 19, 25	5, 16, 24	4.961	2337
5	84	10, 13, 19, 25	16, 24	4.855	2346
6	63	4, 10, 12, 13, 25	5, 15, 21	6.195	2247
7	64	4, 10, 12, 13, 25	15, 20	6.060	2258
8	81	4, 10, 13, 19, 25	21	5.848	2276
9	122	4, 10, 13, 19, 25	17	5.805	2279
10	55	10, 13, 19, 25	5, 16	4.855	2339
11	209	4, 10, 12, 13, 25	5, 21	6.165	2236
12	148	4, 10, 12, 13, 25	5, 17	6.016	2247
13	106	4, 10, 12, 13, 25	5	5.914	2250
14	113	8, 10, 13, 25	4, 5, 16	5.130	2284
15	86	10, 13, 19, 25	16, 24	4.855	2346
16	47	4, 10, 12, 13, 25	5, 15, 21	6.195	2247
17	64	4, 10, 12, 13, 25	15, 21	6.090	2253
18	68	4, 10, 12, 13, 25	15	5.945	2261
19	79	4, 10, 13, 19, 25	17	5.805	2279
20	53	10, 13, 19, 25	5, 16	4.855	2339

In Tables 5.4 and 5.5, for most of the instances, the first stage shelters do not vary much but the second stage shelters do. With respect to optimal solutions in Table 5.2, three out of eight instances differ in the configuration of open first stage shelters.

In Table 5.4, Instances 3 and 13; 5 and 10; 8, 18 and 19; and 15 and 20 share the same solutions, respectively. In Table 5.5, Instances 2 and 12; 3 and 8; 5 and 15; 6 and 16; 9 and 19; and 10 and 20 share the same solutions, respectively. So as we decrease the size of the scenario set, varying nature of the earthquakes and the aftershocks cannot be represented thoroughly, and therefore we prefer to use larger datasets.

5.4 A Heuristic Solution Methodology

As discussed previously, the solution times exceed 6-hour limit for most of the presented test instances with 500 scenarios. For instances solved to optimality under 6 hours, the average solution time is 2.7 hours where the average gap for test instances that cannot be solved to optimality under 6 hours is 10.77%, excluding the two instances that cannot be solved due to memory errors.

Since we need as many different scenarios as possible to represent the varying nature of earthquakes and aftershocks, we wish to solve the proposed model with a larger dataset and as we observe in Tables 5.4 and 5.5, the solution times significantly improve as the cardinality of the scenario set decreases. We utilize this fact in the construction of the proposed heuristic. We define R to be a reduced set of the original dataset, which we call as S, such that $|R| < |S|$ and $R \subset S$.

The proposed heuristic blends different approaches used in stochastic optimization. In each iteration of the proposed heuristic, we solve $\mathrm{P}(R)$ and obtain a set of first stage variables and call it x^{1*}. Then we fix the first stage variables in $\mathrm{P}(S)$, namely x^1, to x^{1*} and check for feasibility. Note that $\mathrm{P}(R)$ can be defined as a *group subproblem* with adjusted probabilities as Sandıkçı et al. [62] propose. As in $\mathrm{P}(S)$, the probability of each scenario is equal to $1/|S|$, in $\mathrm{P}(R)$, the probability of each scenario is equal to $1/|R|$.

As we solve *group subproblems*, we may encounter same set of first stage variables. If we have already evaluated x^{1*}, we prefer not to evaluate it again. For this purpose, Ahmed [63] suggests using no-good cuts to eliminate a solution from a solution pool. This is another approach we utilize in this heuristic. Since we check the feasibility of x^{1*} to $\mathrm{P}(S)$, there is no need to check the same x^{1*} again, and we can confidently eliminate it from the solution pool. We perform this elimination using no-good cut constraint in each iteration by adding it to

P(R).

$$\sum_{j:x_j^{1*}=0} x_j^1 + \sum_{j:x_j^{1*}=1} (1 - x_j^1) \geq 1 \tag{5.1}$$

We store all no-good cuts, corresponding to x^{1*} obtained in each iteration, in a cut pool $\mathcal{C}$. As we solve P(R), we add all of the no-good cuts, in the form of (5.1), in the cut pool $\mathcal{C}$ as constraints. After we conclude that x^{1*} is not feasible for P(S), we add its no-good cut to the cut pool $\mathcal{C}$. So in each iteration, we obtain a new and unique set of open first stage shelters from P(R). After we solve P(R) in each iteration, we fix x^1 variables in P(S) to x^{1*} and solve it for feasibility check. The heuristic stops when we find a combination of first stage shelters that is feasible for the original problem, i.e. x^{1*} is feasible for P(S).

In each iteration, the reduced set R, where $|R| = \kappa$, is randomly selected from the set S. A more formal representation of the proposed heuristic is provided in Algorithm 1. Note that, hereafter, *opt*$[\,\cdot\,]$ implies the optimization of problem $\cdot$ and gives the optimal value of it.

Algorithm 1 Heuristic for the single-objective formulation

Require: S.

Cut pool $\mathcal{C} \leftarrow \emptyset$. Let κ be the cardinality of the reduced set. *bool* $\leftarrow TRUE$.
while *bool* **do**
  Create $R \subset S$ randomly, with $|R| = \kappa$.
  Solve P(R) regarding $\mathcal{C}$. Let x^{1*} be an optimal first stage decision of P(R).
  Solve P(S) by fixing $x^1 = x^{1*}$. $\mathcal{V} :=$ *opt*[P(S)].
  if P(S) is feasible **then**
    bool $\leftarrow FALSE$.
  end if
  if *bool* **then**
    Add no-good cut $\sum_{j:x_j^{1*}=0} x_j^1 + \sum_{j:x_j^{1*}=1} (1 - x_j^1) \geq 1$ to $\mathcal{C}$.
  end if
end while
return $\mathcal{V}$

5.4.1 Heuristic Results

In Tables 5.6a – 5.6f, we present the solution time, objective value, optimality gap and iteration numbers of the tests for instances that are solved to optimality, referring to Table 5.2. As we choose set R randomly in each iteration, we perform with six different random seeds for each test instance to obtain different R sets in each iteration. We use this procedure to discuss the effects of randomness and perform analyses on the subset selection from the original dataset. Note that using random seeds in JAVA enables users to recreate the same stream of random numbers, i.e. we use same stream of random R sets through all test instances as long as the random number generator is set to the same value in corresponding tests.

In Tables 5.6a – 5.6f, the first column provides the instance IDs and their corresponding solution times, optimality gaps, and number of iterations in the second, third and fourth columns, respectively. Iteration number in a test corresponds to the number of times the smaller problem is solved – or the number of no-good cuts in the cut pool $\mathcal{C}$. Note that $\kappa = 100$ and we solve for 500 scenario in the results presented in Tables 5.6a – 5.6f.

An occurring trend in Tables 5.6a – 5.6f is the positive correlation between the gaps and the solution times for the instances. This is the main reason we present the results of the proposed heuristic with different random seeds for scenario subset selection. The combination of scenarios in set R affects the performance of the proposed heuristic. But again, performing the proposed heuristic for six different random seeds – for Instances 2, 12 and 13, i.e. instances where none of the solutions are exact – still has lower solution times with respect to the results in Table 5.2. So the proposed heuristic can be performed numerous times and the best solution it provides can be chosen as a near-optimal solution.

In terms of performance among the six different random seeds for scenario subset selection, third random seed performs well enough for all test instances summarized in Tables 5.6a – 5.6f since it provides the optimal solutions for the five of the instances with small gaps among the others, and also has less number

Table 5.6a: Seed 1

ID	Solution time (sec)	Gap (%)	# of iterations
2	539	0.1	3
6	653	0	3
7	581	0	3
12	658	0.1	3
13	621	0.1	3
16	621	0	3
17	582	0	3
18	441	0	3

Table 5.6b: Seed 2

ID	Solution time (sec)	Gap (%)	# of iterations
2	1,657	9	1
6	841	0	4
7	686	0	4
12	802	0.1	4
13	1,929	7	1
16	610	0	4
17	538	0	4
18	551	0	4

Table 5.6c: Seed 3

ID	Solution time (sec)	Gap (%)	# of iterations
2	413	0.1	2
6	421	0	2
7	276	0	2
12	487	0.1	2
13	445	0.1	2
16	271	0	2
17	260	0	2
18	292	0	2

Table 5.6d: Seed 4

ID	Solution time (sec)	Gap (%)	# of iterations
2	220	0.1	1
6	482	0	2
7	357	0	3
12	210	0.1	1
13	185	0.1	1
16	295	0	2
17	396	0	3
18	411	0	3

Table 5.6e: Seed 5

ID	Solution time (sec)	Gap (%)	# of iterations
2	207	0.1	1
6	337	0	1
7	545	0	4
12	204	0.1	1
13	2,151	5	7
16	196	0	1
17	182	0	1
18	587	0	4

Table 5.6f: Seed 6

ID	Solution time (sec)	Gap (%)	# of iterations
2	195	0.1	1
6	191	0.1	1
7	167	0	1
12	201	0.3	1
13	210	0.1	1
16	173	0	1
17	170	0.1	1
18	139	0	1

of iterations for those instances. So we use the third random seed to solve the remaining instances, namely the instances that cannot be solved to optimality under 6 hours. Table 5.7 provides the solution times, gaps – with respect to the solutions provided in Table 5.2, i.e. best objective value – and number of iterations for all test instances along with the information on the solutions as in Table 5.2. A negative gap value states that the proposed heuristic provides a better solution with respect to the best objective value in Table 5.2 whereas a positive gap value states that the proposed heuristic provides a worse solution. Note that the gaps are per thousand and * denotes that the corresponding gap

Table 5.7: Summary of results for $\kappa = 100$ and 500 scenarios with the proposed heuristic

ID	Solution time (sec)	Gap (‰)	# of iter	First stage shelters	Second stage shelters	Objective value	Average walk (m)
1	708	- 0.6	1	4, 10, 12, 13, 25	5, 7, 21	5.958	2281
2	413	1	2	4, 10, 12, 13, 25	5	5.766	2289
3	906	4	1	4, 10, 13, 23	5, 15, 17	5.488	2253
4	200	0	1	10, 13, 19, 25	4, 5, 16, 24	5.001	2366
5	411	- 13.2	2	10, 13, 19, 25	16, 24	4.895	2370
6	421	0	2	4, 10, 12, 13, 25	5, 15, 17, 21	6.067	2279
7	276	0	2	4, 10, 12, 13, 25	5, 15	5.866	2288
8	372	0	2	4, 10, 12, 13, 25	5, 15	5.866	2288
9	371	*	2	4, 10, 13, 23	3, 5, 11, 15, 17	5.287	2274
10	759	0	4	10, 14, 19, 25	3, 5, 16	4.972	2230
11	339	0.9	1	4, 10, 12, 13, 25	5, 17, 22	6.008	2278
12	487	1	2	4, 10, 12, 13, 25	5, 17	5.807	2288
13	445	1	2	4, 10, 12, 13, 25	5	5.766	2289
14	385	- 0.1	1	10, 13, 19, 25	5, 16, 24	5.060	2364
15	448	0	2	10, 13, 19, 25	16, 24	4.934	2371
16	271	0	2	4, 10, 12, 13, 25	5, 15, 17, 21	6.108	2277
17	260	0	2	4, 10, 12, 13, 25	5, 15, 20	5.912	2287
18	292	0	2	4, 10, 12, 13, 25	5, 15	5.866	2288
19	334	*	2	4, 10, 13, 23	3, 5, 11, 15, 17	5.382	2271
20	506	0.3	3	10, 12, 13, 25	5, 15, 16	5.097	2379

value cannot be calculated as those instances could not be solved before due to memory errors.

The solutions found in Table 5.7 are similar to those in Table 5.2 and the proposed heuristic performs better in terms of solution times. The proposed heuristic also provides us solutions for the instances where CPLEX cannot solve due to memory errors. Although we do not know the optimal solutions for all of the instances presented in Table 5.7, comparing with the results of CPLEX after 6 hours, we can say that the proposed heuristic performs well in terms of solution times as the maximum solution time is 15 minutes, averaging around 7 minutes. Note that we disregard the time it takes to choose the seed that we will run the remaining parameter sets with since it is up to the DM to choose the number of seeds to perform the initial comparison.

5.5 Value of the Three-Stage Model

To reiterate the value of using our proposed model in cases of consecutive disasters, we compare it with its *common* counterpart, where we separate the initial earthquake and the aftershock in the decision making process, i.e without relating the aftershocks to initial earthquakes. We model this problem using two two-stage stochastic MIPs. The first program, namely *F1*, includes the decisions for the initial earthquakes, and the second program, namely *F2*, includes the decisions for the aftershocks.

5.5.1 Comparison Methodology

We still assume that the locations of shelter sites are decided before any earthquake occurs. To solve this *common* counterpart problem, we first solve *F1* for all of the initial earthquake scenarios, i.e. we assume that the DM disregards the probability of having aftershocks while deciding for the locations of the shelter sites for the initial earthquake. And as we know the set of the aftershocks that can happen after any initial earthquake, we then solve *F2* for each set of possible aftershocks following each initial earthquake.

Since there are 50 different initial earthquakes and 10 aftershocks following each initial earthquake in the dataset, we solve *F1* considering all of the initial earthquakes, i.e. solve it once and for 50 different initial earthquakes, and we solve *F2* for all initial earthquake-aftershocks scenario pairs, i.e. solve it 50 times for 10 aftershocks, each time with a different set of aftershocks.

As the CVaR constraints bind all of the stages together, we cannot directly use the same τ_j values for the decomposed *common* counterpart problems. If we do so, we may obtain higher violations of the shelter utilizations with respect to the optimal solutions. Therefore, we decompose the τ_j values as τ'_j for *F1* and as τ''_{js} for *F2* in a given test using the optimal solutions in Table 5.2. Note that $\mathcal{T}_k = \left[\tau''_{k1},\ldots,\tau''_{ks}\right]$, $k \in J, s \in S$, where it is defined as the vector of decomposed τ_k

values to be used in *F2*, i.e. τ_k corresponding to the second stage utilizations.

Referring to the original model and the definitions in Chapter 3, F1(S) is:

$$
\begin{aligned}
&\min \quad && \sum_{j\in J}\frac{1}{w_j}x_j^1 && \\
&s.t. && && \\
& && (3.2) – (3.4),\ 3.13,\ 3.15,\ 3.17 && \\
& && CVaR_\alpha\left(F_j - x_j^1\right) \le \tau_j' && \forall j \in J \\
& && f_{js}^3 = \frac{\sum_{i\in I} q_{is}^1 y_{ij}^1}{c_j} && \forall j \in J, s \in S
\end{aligned}
$$

and F2(S) is:

$$
\begin{aligned}
&\min \quad && (3.1) && \\
&s.t. && && \\
& && (3.5) – (3.7),\ 3.9,\ 3.14,\ 3.16,\ 3.17 && \\
& && x_j^{1*} \le x_{js}^2 && \forall j \in J, s \in S \\
& && CVaR_\alpha\left(F_j - X_j^2\right) \le \mathcal{T}_j && \forall j \in J \\
& && f_{js}^3 = \frac{\sum_{i\in I} q_{is}^1 y_{ij}^{1*} + \sum_{i\in I} q_{is}^2 y_{ijs}^2}{c_j} && \forall j \in J, s \in S
\end{aligned}
$$

As you can see in models *F1* and *F2*, we do not need the “second stage allocation constraint” for y_{ij}^1 in *F2* since we feed the solution provided from *F1* to *F2*. Note that we denote x_j^1 and y_{ij}^1 as x_j^{1*} and y_{ij}^{1*} in *F2*, respectively, to indicate that they are given. Also note that, we first solve *F1* for each of the initial earthquakes and then solve F2($\bar{s}_i$) for the corresponding set of aftershocks, where $\bar{s}_i$ is the set of possible aftershocks that may follow the i-th initial earthquake, $|\bar{s}_i| = 10$, $i = 1, \ldots, 50$.

5.5.2 Results of the Comparison

The solutions obtained using the previous section's methodology are presented in Table 5.8.

Table 5.8: Comparison of objective values and walks of the proposed model and its *common* counterpart

ID	Optimal value Average objective	Maximum walk	Average walk	# of Infea- sibilities
2	5.761 8.679	3,571 3,903	2,213 2,251	0
6	6.067 8.699	3,903 3,903	2,279 2,228	3
7	5.866 8.741	3,903 3,903	2,288 2,245	0
12	5.801 8.778	3,571 3,903	2,212 2,250	0
13	5.761 7.919	3,571 3,903	2,213 2,272	0
16	6.108 8.348	3,903 3,903	2,277 2,228	5
17	5.912 8.396	3,903 3,903	2,287 2,251	1
18	5.866 7.746	3,903 3,903	2,288 2,273	1

The first column in Table 5.8 refers to the ID of the corresponding test instance. In the first row of the second column, we present optimal value of the corresponding test instance, and in the second row of the second column we present the average objective value of 50 *common* counterpart problems. Recall that we solve *F2* for 50 different aftershock sets for each test instance since we have 50 different initial earthquakes in the original dataset. In the third and fourth columns, we present the maximum and average walk values of the proposed model and the *common* counterpart problems, respectively. Note that third column denotes the maximum of the maximum walk values of 50 *common* counterpart problems. In the last column, we denote the cases of infeasibility, out of 50 scenarios.

As it can be observed in Table 5.8, there are some cases of infeasibilities caused by decomposing τ_j to τ'_j and τ''_{js}, i.e. decomposing stages, where in the proposed model we observe and manage all of the stages simultaneously and can meet the risk-aversion level for all of the planning horizon. In other words, we fail to achieve the level of risk-aversion obtained with our proposed model using the *common* counterpart model even though more shelters are established in the overall.

In all of the instances, the proposed model dominates the *common* counterpart model in terms of the objective value but almost always is dominated in terms of the average walks. This is mainly due to the large number of shelters established in the *common* counterpart model. Regardless, we can easily state that the proposed model performs better than the *common* counterpart model as the excess amount of established shelters does not seem to improve the average walk values considerably.

We present details of the comparison using Instance 16 as an example in Table 5.9. In the first column in Table 5.9, we average the number of shelters established over all scenarios. In the second column, we present the maximum of the maximum utilizations of shelters. In the third column, we average the average shelter utilizations. And lastly, in the fourth column we present the number of scenarios where the utilization of a shelter is above 100%.

Table 5.9: Comparison of the proposed model with the *common* counterpart model for Instance 16

	Average # of overall shelters	Maximum util (%)	Average util (%)	# of scenarios above 100% util
Proposed model	5.34	112.66	58.05	130
Common counterpart model	7.93	120.63	54.67	146

As it is apparent from Table 5.9, it is more costly for the DM to not incorporate the aftershocks in the decision making process. In terms of the objective function and the number of established shelters, proposed model outperforms the *common* counterpart model. The proposed model has a higher average utilization with smaller maximum utilization and smaller number of cases where a shelter is utilized more than 100%. All these statistics emphasize that the proposed model performs better than the *common* counterpart model.

Chapter 6

Multi-Objective Stochastic Shelter Site Location under Multi-Hazard Scenarios

Fairness and efficient utilization of resources are of primary concern in humanitarian operations, and using a single-objective might not always be sufficient to guarantee a decision satisfying both requirements. On one hand, the DM might prefer solutions that are optimal in the systematic sense, on the other hand, the victims of the disaster might prefer solutions that are optimal in the individual sense, usually two concepts in conflict. To remedy this, multi-objective programming is extensively used in the humanitarian logistics literature, as discussed in Chapter 2.

6.1 Characteristics of the Multi-Objective Problem

For the stochastic shelter site location problem, we preserve the same problem setting while introducing additional objectives and decisions to provide the DM with a set of *non-dominated* solutions from which a decision can be made regarding the priorities and expected performances.

For this setting, suppose that L-many different objective functions are considered, $Z_l(\mathbf{x})$, $l = 1, \ldots, L$, where $\mathbf{x}$ is the solution vector and $Z_l(.)$ is the l-th objective function. Then, this problem can be modeled as:

$$\begin{aligned} \min \quad & Z(\mathbf{x}) = \big[Z_1(\mathbf{x}), \ldots, Z_L(\mathbf{x})\big] \\ s.t. \quad & \mathbf{x} \in \mathcal{X} \end{aligned}$$

where $\mathcal{X}$ is the feasible set of the problem.

As in general, there is not a single solution that optimizes all objectives simultaneously, so the notion of *optimality* is replaced with the notion of *Pareto optimality* (or *Pareto efficiency*) and the goal of such a problem becomes determining the *efficient* (or *non-dominated*) solutions [64, 65].

6.1.1 Drawbacks of the Single-Objective

There is more than one type of concern in extending this formulation to a multi-objective framework. In the single-objective version, the DM needs to specify two parameters for the risk-aversion level and one parameter for the minimum utilization level beforehand, which moderately depend on the disaster and its effect. Hence, especially in a humanitarian context, it is hard to select those parameters. Nonetheless, to analyze the system, we can perform numerous parameter analyses but still, in reality, we cannot forecast the demand a disaster might create, making such analyses fairly redundant in this context.

To remove the responsibility of parameter selection from the DM, we propose a multi-objective framework and leave only one parameter selection to the DM, namely the risk-aversion parameter α. As this value is taken as either 90%, 95%, or 98% in the related risk-averse decision making literature, the DM only needs to make a choice between these options. In our computational tests in this framework, and recall that also in the single-objective framework, we use and compare with $\alpha = 0.90$ and $\alpha = 0.95$.

6.1.2 Considered Objectives

We define four objectives as:

$$
\begin{aligned}
Z_1(\mathbf{x}) &= \mathrm{CVaR}_\alpha\Big(\sum_{j \in J} \big[F_j - X_j^2\big]_+\Big) \\
Z_2(\mathbf{x}) &= u \\
Z_3(\mathbf{x}) &= W_{max} + W_{min} \\
Z_4(\mathbf{x}) &= \sum_{s \in S} \sum_{j \in J} p_s x_{js}^2
\end{aligned}
$$

where $Z_1(\mathbf{x})$ is the risk of exceeding shelter capacities, $Z_2(\mathbf{x})$ is the minimum utilization over all open shelters, $Z_3(\mathbf{x})$ is the sum of maximum and minimum weights of the opened shelters, and $Z_4(\mathbf{x})$ is the expected number of established shelters. So, naturally, $Z_1(\mathbf{x})$ and $Z_4(\mathbf{x})$ are the objectives to be minimized, whereas $Z_2(\mathbf{x})$ and $Z_3(\mathbf{x})$ are the objectives to be maximized. Note that $\mathbf{x}$ is a feasible solution for our problem.

The four objectives introduced above can be grouped according to their intended audience: objectives $Z_2(\mathbf{x})$ and $Z_4(\mathbf{x})$ are intended for the DM. Second objective is valuable as the DM might want to have smallest utilization of open shelters bigger than a specified level, hence this statistic is regarded as an objective and a measure of efficiency in the solutions. Fourth objective is again valuable to the DM as it is desirable to have smaller number of shelters, both for means of serving and cost of establishing them, assuming homogeneous establishment costs.

In the objective $Z_2(\mathbf{x})$, we consider the minimum utilization of established shelters rather than the average utilization as looking at the average may not be able to save the DM from lower utilization values, see discussion in Section 3.1

regarding the minimum utilization constraints.

Furthermore, objectives $Z_1(\mathbf{x})$ and $Z_3(\mathbf{x})$ are intended for the victims. First and third objectives are valuable to the victims as they would like to reside in shelters that are not too crowded and have more from the certain established standards, i.e. they prefer shelters to have larger weights. Recall that the weight of a shelter is an indicator of its performance level.

In the objective $Z_3(\mathbf{x})$, we consider the sum of the maximum and minimum weights of the established shelters rather than averaging the weights. This way, it is computationally more preferable and performs well enough in practice, i.e. if we were to consider the average weights of the established shelters, the number of candidate solutions that particular objective can take would be finite but very large – we will later discuss that having such a smaller set is preferable in this context. We also make an important distinction in considering the maximum and minimum weights of the established shelters. In practice, the DM naturally would like to have the best configuration of the first stage shelters since the second stage shelters may not be established in some of the scenarios. So, we implement this requirement in our model by considering the maximum weight only for the first stage shelters and the minimum weight for shelters established in either stage, i.e. we are aiming to have a better configuration of the first stage shelters by doing so.

6.2 Mathematical Model

We can reformulate our problem with the improvements proposed above as follows, starting with the decision variables – we present them in their entirety:

$$x_j^1 = \begin{cases} 1 & \text{if shelter } j \text{ is established in stage 1} \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in J$$

$$y^1_{ij} = \begin{cases} 1 & \text{if district } i \text{ is assigned to shelter } j \\ & \text{in stage 1} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I, j \in J$$

$$x^2_{js} = \begin{cases} 1 & \text{if shelter } j \text{ is established in stage 2} \\ & \text{under scenario } s \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in J, s \in S$$

$$y^2_{ijs} = \begin{cases} 1 & \text{if district } i \text{ is assigned to shelter } j \\ & \text{under scenario } s \text{ in stage 2} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I, j \in J, s \in S$$

$$f^3_{js} = \text{overall utilization of shelter site } j \text{ under scenario } s \quad \forall j \in J, s \in S$$

$$b_j = \begin{cases} 1 & \text{if shelter } j \text{ has the maximum weight} \\ 0 & \text{otherwise} \end{cases} \qquad \forall j \in J$$

$$u = \text{minimum utilization rate}$$

$$W_{min} = \text{minimum of the weights of the shelters established in the second stage}$$

$$W_{max} = \text{maximum of the weights of the shelters established in the first stage}$$

Then we have the following model, where for some parts we refer to the single-objective formulation – again we present them in their entirety for completeness:

$$\overline{\mathrm{P}}(\mathrm{S}) =$$

$$\min \ \mathrm{CVaR}_\alpha\Big(\sum_{j \in J} \big[F^3_j - X^2_j\big]_+ \Big) \tag{O1}$$

$$\max \ u \tag{O2}$$

$$\max \ W_{max} + W_{min} \tag{O3}$$

$$\min \ \sum_{s \in S} \sum_{j \in J} p_s x^2_{js} \tag{O4}$$

$s.t.$

$$\sum_{j\in J} y^1_{ij} = 1 \qquad \forall i \in I \tag{3.2}$$

$$\sum_{k=r+1}^{|J|} y^1_{ij_i(k)} + x^1_{j_i(r)} \leq 1 \qquad \forall i \in I, r = 1,\dots,|J|-1 \tag{3.3}$$

$$y^1_{ij} \leq x^1_j \qquad \forall i \in I, j \in J \tag{3.4}$$

$$\sum_{j\in J} y^2_{ijs} = 1 \qquad \forall i \in I, s \in S \tag{3.5}$$

$$\sum_{k=r+1}^{|J|} y^2_{ij_i(k)s} + x^2_{j_i(r)s} \leq 1 \qquad \forall i \in I, s \in S, r = 1,\dots,|J|-1 \tag{3.6}$$

$$y^2_{ijs} \leq x^2_{js} \qquad \forall i \in I, j \in J, s \in S \tag{3.7}$$

$$x^1_j \leq x^2_{js} \qquad \forall j \in J, s \in S \tag{3.8}$$

$$x^2_{js'} = x^2_{js} \qquad \forall j \in J, s \in S, s' \in S^2_s \tag{3.9}$$

$$f^3_{js} = \frac{\sum_{i\in I} q^1_{is} y^1_{ij} + \sum_{i\in I} q^2_{is} y^2_{ijs}}{c_j} \qquad \forall j \in J, s \in S \tag{3.11}$$

$$f^3_{js} \geq u x^2_{js} \qquad \forall j \in J, s \in S \tag{6.1}$$

$$W_{min} \leq w_j x^2_{js} + \left(1 - x^2_{js}\right) \qquad \forall j \in J, s \in S \tag{6.2}$$

$$W_{max} \geq w_j x^1_j \qquad \forall j \in J \tag{6.3}$$

$$w_j x^1_j \geq b_j W_{max} \qquad \forall j \in J \tag{6.4}$$

$$\sum_{j\in J} b_j \geq 1 \tag{6.5}$$

$$x^1_j \in \{0,1\} \qquad \forall j \in J \tag{3.12}$$

$$y^1_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{3.13}$$

$$x^2_{js} \in \{0,1\} \qquad \forall j \in J, s \in S \tag{3.14}$$

$$y^2_{ijs} \in \{0,1\} \qquad \forall i \in I, j \in J, s \in S \tag{3.15}$$

$$f^3_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{3.16}$$

$$b_j \in \{0,1\} \qquad \forall j \in J \tag{6.6}$$

$$W_{max}, W_{min}, u \geq 0 \tag{6.7}$$

Going over the objectives first, the objective function (O1) minimizes the over-utilization of shelters with a previously decided the risk-aversion level of

α. The second objective function (O2) maximizes the minimum utilization over all shelters. The third objective function (O3) maximizes the sum of maximum and minimum weights of established shelters and the fourth objective function (O4) minimizes the expected number of established shelters. Recall that we define W_{max} as the maximum of the weights of the shelters established in the first stage and W_{min} as the minimum of the weights of the shelters established in the second stage to increase the overall performance of opened shelters, i.e. let the shelter with the highest weight to be used for two stages of assignment and let the shelter with the lowest weight to be used only for one stage of assignment, hence lower the cases of inefficiency.

As in the single-objective counterpart, recall that constraints (3.2)–(3.4) are the "first stage allocation constraints", and constraints (3.5)–(3.7) are the "second stage allocation constraints". Constraints (3.8) make sure that if shelter $j \in J$ is established in the first stage, it should be kept open for any scenario at the second stage. Constraints (3.9) are the non-anticipativity constraints. Constraints (3.11) define the overall utilization rate of a shelter in the corresponding scenario.

Different from the single-objective counterpart, constraints (6.1) find the minimum utilization value over all established shelter – its linearization is explained next. Constraints (6.2) calculate the minimum weight of the shelters established in the second stage. Constraints (6.3)–(6.5) calculate the maximum weight of the shelters established in the first stage, they make sure that at least one of the open shelter have the maximum weight amongst the others – $b_j = 1$ if j-th shelter has the largest weight among other open shelter, note that more than one shelter can have the corresponding $b_j = 1$, given they all have the highest weight and are opened. Linearization of constraints (6.4) is also explained in detail in coming parts. Finally, the remaining are the domain constraints.

6.2.1 Details on the Mathematical Model

Recall that we define random variables X_j^2 and F_j^3 and let x_{js}^2 be the realizations of the random variable X_j^2 where $x_{js}^2 = X_j^2(\omega_s)$, and f_{js}^3 be the realizations of the random variable F_j^3 where $f_{js}^3 = F_j(\omega_s)$, $j \in J, s \in S$. Then, referring to the formal introduction and linearization of CVaR in Chapter 3, we directly linearize the CVaR objective (O1) by defining three new continuous decisions variables, K_{js}, z_s and η, replace objective function (O1) with O1′ and add the constraints (6.8)–(6.12) in $\overline{\mathrm{P}}$(S):

$$\eta + \frac{1}{1-\alpha}\sum_{s\in S} p_s z_s \tag{O1$'$}$$

$$z_s \geq \sum_{j\in J} K_{js} - \eta \qquad \forall s \in S \tag{6.8}$$

$$K_{js} \geq f_{js}^3 - x_{js}^2 \qquad \forall j \in J, s \in S \tag{6.9}$$

$$z_s \geq 0 \qquad \forall s \in S \tag{6.10}$$

$$K_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{6.11}$$

$$\eta \text{ is free} \tag{6.12}$$

6.2.2 Linearizing and Improving the Mathematical Model

We again add constraints to limit the maximum distance between the districts and the shelters to improve the solution qualities further. Constraints (3.21) and (3.22) limit the maximum distance between the districts and the shelters, stating that no victim in any district can be forced to travel a distance more than ρ.

To linearize constraints (6.1), we define U_{js} such that it is equal to the multiplication $u \cdot x_{js}^2$, $\forall j \in J, s \in S$. We will see that constraints (6.13) and (6.14) are enough to linearize constraints (6.1). Consider a fixed j and s. If $x_{js}^2 = 0$, then $f_{js}^3 = 0$, as no one is assigned to j-th shelter in scenario s, implying $U_{js} = 0$. $U_{js} = 0$ would imply $u \leq 1$ in constraints (6.14), which becomes redundant since u, in practice, will be closer to 0 than it is closer to 1. On the other hand, if $x_{js}^2 = 1$, than $f_{js}^3 > 0$ –otherwise we would have $u = 0$ by constraints (6.13) and (6.14)– and $U_{js} \leq f_{js}^3$, with $u \leq U_{js}$, finally giving us $u = \min_{j\in J,\ s\in S:\ x_{js}^2=1}\{U_{js}\}$ since

u is to be maximized.

$$
\begin{aligned}
& f^3_{js} \geq U_{js} && \forall j \in J, s \in S && (6.13)\\
& U_{js} \geq x^2_{js} + u - 1 && \forall j \in J, s \in S && (6.14)\\
& U_{js} \geq 0 && \forall j \in J, s \in S && (6.15)
\end{aligned}
$$

To linearize constraints (6.4), we define T_j such that it is equal to the multiplication $b_j \cdot W_{max}$, $\forall j \in J$. Following linearization works as shelter weight values are all between 0 and 1:

$$
\begin{aligned}
& w_j x^1_j \geq T_j && \forall j \in J && (6.16)\\
& T_j \leq b_j && \forall j \in J && (6.17)\\
& T_j \leq W_{max} && \forall j \in J && (6.18)\\
& T_j \geq W_{max} + b_j - 1 && \forall j \in J && (6.19)\\
& T_j \geq 0 && \forall j \in J && (6.20)
\end{aligned}
$$

Then, $\overline{\mathrm{P}}(\mathrm{S})$ is:

$\overline{\mathrm{P}}(\mathrm{S}) =$

$$
\begin{aligned}
& \min \ \eta + \frac{1}{1-\alpha} \sum_{s \in S} p_s z_s && (\mathrm{O1}')\\
& \max \ u && (\mathrm{O2})\\
& \max \ W_{max} + W_{min} && (\mathrm{O3})\\
& \min \ \sum_{s \in S} \sum_{j \in J} p_s x^2_{js} && (\mathrm{O4})
\end{aligned}
$$

$s.t.$

$$
\begin{aligned}
& \sum_{j \in J} y^1_{ij} = 1 && \forall i \in I && (3.2)\\
& \sum_{k=r+1}^{|J|} y^1_{ij_i(k)} + x^1_{j_i(r)} \leq 1 && \forall i \in I, r = 1, \ldots, |J| - 1 && (3.3)\\
& y^1_{ij} \leq x^1_j && \forall i \in I, j \in J && (3.4)
\end{aligned}
$$

$$\sum_{j\in J} y^2_{ijs} = 1 \qquad \forall i \in I, s \in S \tag{3.5}$$

$$\sum_{k=r+1}^{|J|} y^2_{ij_i(k)s} + x^2_{j_i(r)s} \leq 1 \qquad \forall i \in I, s \in S, r = 1, \ldots, |J|-1 \tag{3.6}$$

$$y^2_{ijs} \leq x^2_{js} \qquad \forall i \in I, j \in J, s \in S \tag{3.7}$$

$$x^1_j \leq x^2_{js} \qquad \forall j \in J, s \in S \tag{3.8}$$

$$x^2_{js'} = x^2_{js} \qquad \forall j \in J, s \in S, s' \in S^2_s \tag{3.9}$$

$$f^3_{js} = \frac{\sum_{i\in I} q^1_{is} y^1_{ij} + \sum_{i\in I} q^2_{is} y^2_{ijs}}{c_j} \qquad \forall j \in J, s \in S \tag{3.11}$$

$$y^1_{ij} d_{ij} \leq \rho \qquad \forall i \in I, j \in J \tag{3.21}$$

$$y^2_{ijs} d_{ij} \leq \rho \qquad \forall i \in I, j \in J, s \in S \tag{3.22}$$

$$z_s \geq \sum_{j\in J} K_{js} - \eta \qquad \forall s \in S \tag{6.8}$$

$$K_{js} \geq f^3_{js} - x^2_{js} \qquad \forall j \in J, s \in S \tag{6.9}$$

$$f^3_{js} \geq U_{js} \qquad \forall j \in J, s \in S \tag{6.13}$$

$$U_{js} \geq x^2_{js} + u - 1 \qquad \forall j \in J, s \in S \tag{6.14}$$

$$W_{min} \leq w_j x^2_{js} + \left(1 - x^2_{js}\right) \qquad \forall j \in J, s \in S \tag{6.2}$$

$$W_{max} \geq w_j x^1_j \qquad \forall j \in J \tag{6.3}$$

$$\sum_{j\in J} b_j \geq 1 \tag{6.5}$$

$$w_j x^1_j \geq T_j \qquad \forall j \in J \tag{6.16}$$

$$T_j \leq b_j \qquad \forall j \in J \tag{6.17}$$

$$T_j \leq W_{max} \qquad \forall j \in J \tag{6.18}$$

$$T_j \geq W_{max} + b_j - 1 \qquad \forall j \in J \tag{6.19}$$

$$x^1_j \in \{0,1\} \qquad \forall j \in J \tag{3.12}$$

$$y^1_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{3.13}$$

$$x^2_{js} \in \{0,1\} \qquad \forall j \in J, s \in S \tag{3.14}$$

$$y^2_{ijs} \in \{0,1\} \qquad \forall i \in I, j \in J, s \in S \tag{3.15}$$

$$f^3_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{3.16}$$

$$b_j \in \{0,1\} \qquad \forall j \in J \tag{6.6}$$

$$W_{max}, W_{min}, u \geq 0 \tag{6.7}$$

$$z_s \geq 0 \qquad \forall s \in S \tag{6.10}$$

$$K_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{6.11}$$

$$\eta \text{ is free} \tag{6.12}$$

$$U_{js} \geq 0 \qquad \forall j \in J, s \in S \tag{6.15}$$

$$T_j \geq 0 \qquad \forall j \in J \tag{6.20}$$

We refer to this set of constraints as $\mathcal{X}$ for convenience in coming parts.

Chapter 7

Multi-Stage Multi-Objective MIP Results

For multi-objective problems, as discussed in the previous chapter, generally there is not a single solution that optimizes all of the objectives simultaneously. When the concept of *optimality* is replaced with the concept of *Pareto optimality*, the *Pareto optimal* (or *efficient*, *non-dominated*) solutions make up the Pareto-frontier of the problem. In order to obtain these *non-dominated* solutions, an appropriate scalarization method (among weighting methods, constraint methods, reference point methods, or direction based methods) should be adopted. In this thesis, we adopt the ϵ-constraint method [64–66].

7.1 The ϵ-Constraint Method

Referring to the previous chapter, a generic model for our case might be as follows:

$$\begin{aligned} \min \quad & \mathrm{O1}'(\mathbf{x}) \\ s.t. \quad & \\ & \mathrm{O2}(\mathbf{x}) \geq \epsilon_2 \end{aligned}$$

$$
\begin{aligned}
&\mathrm{O3}(\mathbf{x}) \geq \epsilon_3 \\
&\mathrm{O4}(\mathbf{x}) \leq \epsilon_4 \\
&\mathbf{x} \in \mathcal{X}
\end{aligned}
$$

In this setting, systematically updating each ϵ value as required and solving the updated model iteratively, one can obtain non-dominated solutions. Note that $\mathcal{X}$ is the constraint set defined in the previous chapter. But in practice, such an approach does not guarantee that all of the Pareto efficient solutions will be generated. It is highly dependent on the choice of ϵ values, the relation of the objective functions among themselves and the step sizes for each of them [64,66].

7.1.1 Implementation for a 2-Objective Framework

Before going over a more suitable method, we revise the current practice in case of two objective functions. Consider the first two objective functions of our formulation. To find the Pareto-frontier of such a 2-objective formulation, one can solve a set of mathematical programs in a lexicographic and iterative fashion [64,67]. For a demonstration, let $P_1(\epsilon_2)$, $P_2(\epsilon_1)$ be defined in the following fashion:

$$
\begin{aligned}
\min \quad & \mathrm{O1}'(\mathbf{x}) \\
s.t. \quad & \\
& \mathrm{O2}(\mathbf{x}) \geq \epsilon_2 \\
& \mathbf{x} \in \mathcal{X}
\end{aligned}
\qquad\qquad
\begin{aligned}
\max \quad & \mathrm{O2}(\mathbf{x}) \\
s.t. \quad & \\
& \mathrm{O1}'(\mathbf{x}) \leq \epsilon_1 \\
& \mathbf{x} \in \mathcal{X}
\end{aligned}
$$

Using these predefined problems, $P_1(\epsilon_2)$ and $P_2(\epsilon_1)$, in a simple algorithmic manner, as discussed in Algorithm 2, the set of Pareto efficient solutions can be obtained. Before starting the algorithm, the *ParetoSet* is initialized as an empty set, which stores the Pareto optimal solutions. ϵ values for each subproblem and their respective step sizes are also initialized. We solve each subproblem with the updated ϵ values until no solution can be produced from the first subproblem. In that case, we note that one cannot find any more solutions which belong to $\mathcal{X}$ and satisfy constraints $\text{O2}(\mathbf{x}) \geq \epsilon_2$ with the most recent value of ϵ_2. This implies that the boundary of the Pareto-frontier is hit.

Algorithm 2 Obtain the set of Pareto-efficient solutions for a Bi-Objective Problem

$ParetoSet \leftarrow \emptyset$. $\epsilon_2 \leftarrow 0$. $k_2 \leftarrow$ *stepsize* (small enough).
while $P_1(\mathbf{x}, \epsilon_2)$ is feasible **do**
  $f_1^* := opt[P_1(\epsilon_2)]$
  $\epsilon_1 \leftarrow f_1^*$
  $f_2^* := opt[P_2(\epsilon_1)]$
  $X := \{f_1^*, f_2^*\}$
  $ParetoSet \leftarrow ParetoSet \cup X$
  $\epsilon_2 \leftarrow f_2^* + k_2$
end while
return $ParetoSet$

As our proposed formulation contains 4 distinct objective functions, we cannot utilize Algorithm 2 by adding two more subproblems, say $P_3(\cdot)$ and $P_4(\cdot)$, and have a total of three ϵ-constraints for each objective function in each subproblem, as we may possibly lose the Pareto efficiency guarantee. In order to remedy this problem, we consider a different approach.

7.1.2 Implementation for a 3-Objective Framework

Abounacer et al. [31] study the simultaneous minimization of the total transportation time of relief items ($Z_1(\mathbf{x})$), the number of required first-aiders ($Z_2(\mathbf{x})$) and the non-covered demand among all affected areas ($Z_3(\mathbf{x})$). The authors propose an exact algorithm which utilizes ϵ-constraint method to determine all the efficient solutions.

The proposed method is based on the fact that all the possible values of $Z_2(\mathbf{x})$ are known and one can exhaust all possible values of $Z_2(\mathbf{x})$ by fixing $Z_2(\mathbf{x})$ to an integer value between the predefined levels and solving for $Z_1(\mathbf{x})$ and $Z_3(\mathbf{x})$ as if the problem was a bi-objective one. For each fixed value of $Z_2(\mathbf{x})$, Algorithm 2 is followed until infeasibility. An important note here is that the authors do not find exactly the Pareto efficient solutions but a candidate set for that, from which the Pareto efficient solutions can be extracted, using Algorithm 3. Say that all of the solutions generated by Algorithm 2 is kept in a feasible set denoted as *FS*. As all of the objectives are minimization objectives, once the authors search over all of the solutions in *FS*, they can find all of the solutions that are dominated. Then, those solutions can be eliminated from the $ParetoSet$, providing the Pareto efficient set of solutions. Here, assume that FS_j^k denotes the k-th objective value of j-th solution, $k = 1,\, 2,\, 3$ and $j = 1, \ldots, |FS|$.

Algorithm 3 Obtain the Pareto Front for a 3-Objective Formulation

Require: Feasible Set (denoted by *FS*) obtained from the ϵ-constraint method

```
ParetoSet ← FS
for i = 1 : |FS| do
    for j = 1 : |FS| & j ≠ i do
        if FS_j^1 ≤ FS_i^1 & FS_j^2 ≤ FS_i^2 & FS_j^3 ≤ FS_i^3 then
            (FS_i) is dominated. ParetoSet ← ParetoSet \ {FS_i}
        else
            (FS_i) is a non-dominated solution.
        end if
    end for
end for
return ParetoSet
```

7.1.3 Implementation for a 4-Objective Framework

The methodology in the previous section can be tailored for our formulation as third and fourth objective functions are defined over a finite scale, i.e. the weight of each shelter is known and unique sums of two-combinations of those shelters can be computed, and the expected shelter number can be considered as being extracted from an ordered discrete set since the difference between that set's two consecutive entries depend on the number of scenarios. Hence, the methodology proposed in Abounacer et al. [31] can be modified for our 4-objective formulation.

The two single-objective problems that we need to consider are $B_1(\epsilon_2, \epsilon_3, \epsilon_4)$

and $B_2(\epsilon_1, \epsilon_3, \epsilon_4)$ as given below, respectively:

$$\begin{array}{ll} \min & \mathrm{O1}'(\mathbf{x}) \\ s.t. & \\ & \mathrm{O2}(\mathbf{x}) \geq \epsilon_2 \\ & \mathrm{O3}(\mathbf{x}) = \epsilon_3 \\ & \mathrm{O4}(\mathbf{x}) = \epsilon_4 \\ & \mathbf{x} \in \mathcal{X} \end{array} \qquad \begin{array}{ll} \max & \mathrm{O2}(\mathbf{x}) \\ s.t. & \\ & \mathrm{O1}'(\mathbf{x}) \leq \epsilon_1 \\ & \mathrm{O3}(\mathbf{x}) = \epsilon_3 \\ & \mathrm{O4}(\mathbf{x}) = \epsilon_4 \\ & \mathbf{x} \in \mathcal{X} \end{array}$$

As in case of the algorithm by Abounacer et al. [31], the algorithm we propose may also generate weakly dominated solutions through iterations. So, we tailor their algorithm to our case and use a version of Algorithm 3 for 4-objectives, which we present formally in Algorithm 4.

Algorithm 4 Obtain the set of candidate Pareto-efficient solutions

Require: $\mathbb{S}$ and $\mathbb{W}$

$CandidateParetoSet \leftarrow \emptyset$. $k_2 \leftarrow 10^{-3}$.
$k \leftarrow 1$
while $k \leq |\mathbb{S}|$ **do**
  $\epsilon_4 \leftarrow \mathbb{S}(k)$
  $m \leftarrow 1$
  while $m \leq |\mathbb{W}|$ **do**
    $\epsilon_3 \leftarrow \mathbb{W}(m)$
    $\epsilon_2 \leftarrow 0$
    while $B_1(\epsilon_2, \epsilon_3, \epsilon_4)$ is feasible **do**
      $f_1^* := opt[B_1(\epsilon_2, \epsilon_3, \epsilon_4)]$
      $\epsilon_1 \leftarrow f_1^*$
      $f_2^* := opt[B_2(\epsilon_1, \epsilon_3, \epsilon_4)]$
      $X := \{f_1^*, f_2^*, \epsilon_3, \epsilon_4\}$
      $CandidateParetoSet \leftarrow CandidateParetoSet \cup X$
      $\epsilon_2 \leftarrow f_2^* + k_2$
    end while
    $m \leftarrow m + 1$
  end while
  $k \leftarrow k + 1$
end while
return $CandidateParetoSet$

In Algorithm 4, we generate two sets for the possible values of third and fourth objectives, sets $\mathbb{W}$ and $\mathbb{S}$, respectively. In each iteration, we pick an element from the Cartesian set $\mathbb{W} \times \mathbb{S}$, fix the values of last two objectives to the picked element and perform bi-objective method as proposed in Section 7.1.1. Since we cannot deduce that we should stop the search for a fixed expected shelter value once an infeasibility is found at line 8, we exhaust all of the elements in the set $\mathbb{W} \times \mathbb{S}$. Since the set $\mathbb{W} \times \mathbb{S}$ has high cardinality, especially when the number of scenarios is big, computational times of this method is rather high.

7.1.4 Improved Implementation for a 4-Objective Framework

To improve our solution times for the 4-objective formulation, we introduce a dummy binary variable a_j, $j \in \mathrm{W}$, where W is the index set of the possible weight values set $\mathbb{W}$. Here note that the set $\mathbb{W}$ is recreated using combinations of shelters and sorting those values. As there are 25 candidate shelter locations in the network, we say that at least one of the open shelters will have the minimum weight and at least one of them will have the maximum weight in a non-empty set of open shelters. Hence, we generate the combinations of 25 shelters taken 2 shelter at a time and sum the corresponding shelters' weights. Then, we eliminate the repetitions and sort the values in decreasing order, finally obtaining the set $\mathbb{W}$. Exploring on this, we define following three constraints:

$$W_{max} + W_{min} = \sum_{j \in \mathrm{W}} a_j \mathbb{W}_j \tag{7.1}$$

$$\sum_{j \in \mathrm{W}} a_j = 1 \tag{7.2}$$

$$a_j \in \{0, 1\} \qquad \forall j \in \mathrm{W} \tag{7.3}$$

This version is presented in Algorithm 5, where we discuss the methodology used throughout the computational analyses of the multi-objective framework.

Algorithm 5 Obtain the set of candidate Pareto-efficient solutions - Enhanced version

Require: $\mathbb{S}$ and $\mathbb{W}$

$CandidateParetoSet \leftarrow \emptyset$. $k_2 = 10^{-3}$.
$k \leftarrow 1$
while $k \leq |\mathbb{S}|$ **do**
  $i \leftarrow 1$, $w \leftarrow 1$
  while $i \geq 1$ **do**
    if $w = 1$ **then**
      $f_3^* := opt[B_5(\mathbb{S}(k))]$. Let i such that $\mathbb{W}(i) = f_3^*$
      $w \leftarrow 0$
    end if
    $\epsilon_2 \leftarrow 0$
    while $B_3(\mathbb{S}(k),\, f_3^*,\, \epsilon_2)$ is feasible **do**
      $f_1^* := opt[B_3(\mathbb{S}(k),\, f_3^*,\, \epsilon_2)]$
      $\epsilon_1 \leftarrow f_1^*$
      $f_2^* := opt[B_4(\mathbb{S}(k),\, f_3^*,\, \epsilon_1)]$
      $\epsilon_2 \leftarrow f_2^*$
      $X := \{f_1^*,\, f_2^*,\, f_3^*,\, \mathbb{S}(k)\}$
      $CandidateParetoSet \leftarrow CandidateParetoSet \cup X$
      $\epsilon_2 \leftarrow \epsilon_2 + k_2$
    end while
    $i \leftarrow i - 1$, $f_3^* \leftarrow \mathbb{W}(i)$
  end while
  $k \leftarrow k + 1$
end while
return $CandidateParetoSet$

We first initialize the set of candidate Pareto solutions, $ParetoSet$, and the step size k_2. We set k_2 low enough such that no non-dominated solution is missed. We then start the solution generation methodology. We continue in the following fashion until we exhaust all possible values in the set $\mathbb{S}$, i.e. set of possible values for the fourth objective function:

- Find the maximum possible value for the third objective using the model $B_5(\bar{x})$ and note its objective value and index in the set $\mathbb{W}$. Set ϵ_2 to 0 so that when the problem B_3 is solved for the first time, the constraint regarding the second objective function is redundant.

- Solve the model $B_3(\bar{x}, \bar{y}, \bar{z})$, note the corresponding solution, update ϵ_1

and solve the model $B_4(\bar{x}, \bar{y}, \bar{z})$. Continue until the model $B_3(\bar{x}, \bar{y}, \bar{z})$ is infeasible.

- When infeasibility is hit in the model $B_3(\bar{x}, \bar{y}, \bar{z})$, choose the next smaller weight value from the set $\mathbb{W}$ and note its index. Continue until the set $\mathbb{W}$ is exhausted.

Finally, we formally define the problems $B_3(\bar{x}, \bar{y}, \bar{z})$, $B_4(\bar{x}, \bar{y}, \bar{z})$ and $B_5(\bar{x})$ below, respectively:

$$
\begin{array}{ll}
\min & \mathrm{O1}'(\mathbf{x}) \\
s.t. & \\
 & \mathrm{O4}(\mathbf{x}) = \bar{x} \\
 & \mathrm{O3}(\mathbf{x}) = \bar{y} \\
 & \mathrm{O2}(\mathbf{x}) \geq \bar{z} \\
 & \mathbf{x} \in \mathcal{X}
\end{array}
\qquad
\begin{array}{ll}
\max & \mathrm{O2}(\mathbf{x}) \\
s.t. & \\
 & \mathrm{O4}(\mathbf{x}) = \bar{x} \\
 & \mathrm{O3}(\mathbf{x}) = \bar{y} \\
 & \mathrm{O1}(\mathbf{x}) \leq \bar{z} \\
 & \mathbf{x} \in \mathcal{X}
\end{array}
\qquad
\begin{array}{ll}
\max & \mathrm{O3}(\mathbf{x}) \\
s.t. & \\
 & \mathrm{O4}(\mathbf{x}) = \bar{x} \\
 & (7.1) - (7.3) \\
 & \mathbf{x} \in \mathcal{X}
\end{array}
$$

After we obtain the set of candidate Pareto solutions, we use Algorithm 6 to extract the non-dominated solutions. We note here that Algorithm 6 takes negligible time to run, i.e. a fraction of a second.

Algorithm 6 Obtain the Pareto Front for a 4-Objective Formulation

Require: Feasible Set (denoted by FS) obtained from the ϵ-constraint method

$ParetoSet \leftarrow FS$
for $i = 1 : |FS|$ **do**
  for $j = 1 : |FS|$ & $j \neq i$ **do**
    if $FS_j^1 \leq FS_i^1$ & $FS_j^2 \geq FS_i^2$ & $FS_j^3 \geq FS_i^3$ & $FS_j^4 \leq FS_i^4$ **then**
      (FS_i) is dominated. $ParetoSet \leftarrow ParetoSet \setminus \{FS_i\}$
    else
      (FS_i) is a non-dominated solution.
    end if
  end for
end for
return $ParetoSet$

7.2 Computational Results

In this section, we present the computational experiments conducted using the proposed methodology for the multi-objective counterpart of the stochastic shelter site location problem. We again use the same dataset as described in Chapter 4, code the methodology in JAVA and solve using IBM CPLEX 12.7.1 with the same processing power in Chapter 5.

Although we choose to extend the original problem to its multi-objective counterpart to save the DM from the burden of parameter selection, we vary our tests not only by the value of α but provide an upper limit on the overall utilization of a shelter, which is posed through the following constraint:

$$f^3_{js} \leq M_u \qquad\qquad \forall j \in J, s \in S$$

where M_u is the upper limit on the utilization of a shelter. For the rest of the chapter, we take $M_u = 1.35$, i.e. a 135% utilization limit.

In our context, we are concerned with keeping the over-utilization value as small as possible and in the single-objective counterpart we provide upper bounds to the risk measure to do so. Since in the multi-objective framework such a way is not applicable, we directly limit the over-utilization of the shelters but do not provide a parametric analysis. We are more concerned with the distribution of the over-utilization and hence keep the CVaR objective as it is.

When $\alpha = 0.90$, for 250 scenarios, we generate 592 solutions using Algorithm 5, 172 of which are non-dominated. The average solution time for each of 592 solutions is 55 seconds and the average time it takes to find a non-dominated solution is 189 seconds.

When $\alpha = 0.95$ and the scenario set is the same, we generate again 592 solutions using Algorithm 5, but this time only 79 of those solutions are non-dominated. The average solution time of each of 592 solutions are 53 seconds and the average solution time of the non-dominated solutions is 400 seconds. As it is not possible to visualize the Pareto front of four objectives in three dimensions, we only provide the respective set of solutions in Appendices A.3 and A.4 in table form. Note that we are unable to solve the problem with 500 scenarios and hence provide a heuristic solution methodology in the following section to solve larger instances.

7.3 A Heuristic Solution Methodology for the Multi-Objective Problem

We provide a heuristic method to a generalized version of our problem. We consider a case where the DM might be unable to assign weights to shelters or may be interested in not only locating shelters but a more generally defined facility. In this case, we can disregard the shelter weights and obtain a 3-objective formulation. So, under such problem structure we can solve the problem for 250 scenarios, by disregarding the third objective function. But as in the 4-objective formulation, we cannot solve this problem for 500 scenarios. So, we propose a heuristic methodology that exploits the structure of the problem in a similar sense to the heuristic proposed for the single-objective framework (see Section 5.4).

We use the fact that the problem is much easier when the first stage shelters are fixed. Actually when we only use the second objective, namely the minimum utilization objective (O2), and disregard the other two objectives, the problem decomposes by scenario groups given that the first stage shelters are fixed – note that the objective (O3) is the objective that is not considered anymore and we have a 3-objective formulation where constraints (6.2), (6.3), (6.5), (6.6), and (6.16)–(6.20) are eliminated from the constraint set $\mathcal{X}$.

The mathematical model for objective (O2) with fixed first stage shelters is:

$$\begin{aligned} \max \quad & (O2) \\ s.t. \quad & \\ & x_j^{1*} \leq x_{js}^2 \qquad \forall j \in J, s \in S \end{aligned}$$

$$f_{js}^3 = \frac{\sum_{i \in I} q_{is}^1 y_{ij}^{1*} + \sum_{i \in I} q_{is}^2 y_{ijs}^2}{c_j} \qquad \forall j \in J, s \in S$$

plus the set of constraints (3.5)–(3.7), (3.9), (3.14)–(3.16), (3.22), (6.13)–(6.15). We call this model as $\widehat{\mathrm{P}}_1(S)$. Note that S is the original dataset with 500 scenarios.

We propose to solve $\widehat{\mathrm{P}}_1(S)$ by decomposing it by scenario groups, i.e. solve a series of $\widehat{\mathrm{P}}_1(\underline{S}_i)$ where $|\underline{S}_i| = 10$ by construction, $i = 1, \ldots, 50$ (see Section 5.4 for a similar discussion regarding the heuristic method for the single objective formulation and Sections 3.2 and 3.2.1 for the non-anticipativity constraints). In the application, we solve only one of the $\widehat{\mathrm{P}}_1(\underline{S}_i)$ to get the first stage variables – this is the first step of our construction. After we obtain an initial set of x^{1*}, we fix it and solve each $\widehat{\mathrm{P}}_1(\underline{S}_i)$ to get the minimum utilization value u^*, i.e. the value of objective (O2) corresponding to the given set of x^{1*}.

Then, we solve for the fourth objective, i.e. we aim to minimize the number of expected shelters given a constraint on the minimum utilization value – call this model as $\widehat{\mathrm{P}}_2(S)$. We state that the minimum utilization value cannot be less than u^* and obtain the corresponding objective value, say e_s^*. Finally, fixing x^{1*}, u^* and e_s^*, we solve for the first objective, namely the CVaR objective, to get the corresponding risk value, say c_r^* – call this model as $\widehat{\mathrm{P}}_3(S)$. An important observation here is that, for a fixed x^{1*}, the solution vector (c_r^*, u^*, e_s^*) is a non-dominated solution as we get the solution with the best minimum utilization value. After we obtain this initial solution vector, we perform random search in the region to find more neighboring solutions.

We formally introduce the heuristic method using Algorithm 7. To start it, we need the initial and maximum number of first stage shelters (e_{init}, e_{max}), maximum number of inner and outer iterations ($iter_{max}^{in}$, $iter_{max}^{out}$), and maximum number of inner and outer infeasibilities (inf_{max}^{in}, inf_{max}^{out}).

We then initialize the cut pool $\mathcal{C}$ and set of feasible and candidate non-dominated solutions FS as empty sets, stopping condition SC as false. Also we say that the number of first stage shelters ES is equal to e_{init}, and outer

Algorithm 7 Heuristic for the multi-objective formulation

Require: e_{init}, e_{max}, $iter^{out}_{max}$, $iter^{in}_{max}$, inf^{out}_{max}, inf^{in}_{max}.

Cut pool $\mathcal{C} \leftarrow \emptyset$. $SC \leftarrow FALSE$. $FS \leftarrow \emptyset$. $ES \leftarrow e_{init}$. $inf \leftarrow 0$. $iter^{out} \leftarrow 0$.

while $ES \leq e_{max}$ **do**

  while $iter^{out} \leq iter^{out}_{max}$ & $inf \leq inf_{max}$ **do**

    $iter^{out} \leftarrow iter^{out} + 1$.

    Randomly choose one scenario out of 50 scenarios, say k.

    Solve $\widehat{\mathrm{P}}_1(\underline{S}_k)$ regarding $\mathcal{C}$, having ES-many first stage shelters.

    Let x^{1*} be an optimal first stage decision of $\widehat{\mathrm{P}}_1(\underline{S}_k)$.

    Add no-good cuts $\displaystyle\sum_{j:x^{1*}_j=0} x^1_j + \sum_{j:x^{1*}_j=1} (1 - x^1_j) \geq 1$ to $\mathcal{C}$.

    while $!SC$ **do**

      $u^* := \mathrm{MINUT}\big(x^{1*}, (\underline{S}_1, \ldots, \underline{S}_{50})\big)$.

      $e^*_s := \mathrm{EXSHE}\big(x^{1*}, u^*, S\big)$.

      if $e^*_s < +\infty$ **then**

        $c^*_r := \mathrm{RISK}\big(x^{1*}, u^*, e^*_s, S\big)$.

        $fs \leftarrow (c^*_r, u^*, e^*_s)$

      else

        $fs \leftarrow (+\infty, +\infty, +\infty)$

      end if

      if fs is a finite vector **then**

        $FS \leftarrow FS \cup (c^*_r, u^*, e^*_s)$.

      else

        $inf \leftarrow inf + 1$.

        break

      end if

      $iter^{in} \leftarrow 1$. $inf^{in} \leftarrow 0$.

      while $iter^{in} \leq iter^{in}_{max}$ & $!SC$ & $inf^{in} \leq inf^{in}_{max}$ **do**

        Perform Algorithm 9. Obtain (c^*_r, u^*, e^*_s), SC_r and inf^{in}.

        if SC_r **then**

          $FS \leftarrow FS \cup (c^*_r, u^*, e^*_s)$.

        end if

        $iter^{in} \leftarrow iter^{in} + 1$.

        $SC \leftarrow !SC_r$.

      end while

    end while

    $SC \leftarrow FALSE$.

  end while

  $\mathcal{C} \leftarrow \emptyset$. $ES \leftarrow ES + 1$.

end while

return FS

Algorithm 8 Functions used in construction procedure for the multi-objective formulation

```
function MINUT(x^{1*}, (S_1, ..., S_50))
    u* := 1. k ← 1.
    while k ≤ 50 do
        u_int := opt[P̂_1(S_k)]
        k ← k + 1
        if u_int < u* then
            u* ← u_int
        end if
    end while
    return u*
end function
function EXSHE(x^{1*}, u*, S)
    e_s* := opt[P̂_2(S)]
    return e_s*
end function
function RISK(x^{1*}, u*, e_s*, S)
    c_r* := opt[P̂_3(S)]
    return c_r*
end function
```

infeasibility and outer iteration counters –inf and $iter^{out}$– are equal to zero.

We then, given the conditions are satisfied, randomly choose one scenario group and solve the corresponding problem to get first stage shelters x^{1*} regarding the cut pool $\mathcal{C}$ and making sure that we have ES-many first stage shelters. We then refer to functions in Algorithm 8 to obtain the initial solution.

The functions in Algorithm 8, given the set of scenarios and the first stage decisions x^{1*}, guide us to solve $\widehat{\mathrm{P}}_1(S)$ with fixed x^{1*} to get u^*, then solve $\widehat{\mathrm{P}}_2(S)$ with fixed x^{1*} and u^* to get e_s^*. Given that $\widehat{\mathrm{P}}_2(S)$ is feasible, we solve $\widehat{\mathrm{P}}_3(S)$ with fixed x^{1*}, u^*, and e_s^* to get c_r^* and report (c_r^*, u^*, e_s^*) – or report $(+\infty, +\infty, +\infty)$ to indicate infeasibility in the construction step.

If we find a solution using the functions in Algorithm 8, we then refer to Algorithm 9 for random search in the neighborhood of the found solution. The important theme in this algorithm is that we do not terminate it the first time

Algorithm 9 Random search for the multi-objective formulation

Require: u^*_{init}, e_s^{init*}, ES, S, x^{1*}, $iter^{in}$, inf^{in}.

if $iter^{in} == 1$ **then**
  $e_s \leftarrow ES + 1$. $u \leftarrow 0.01$.
else
  Define disjoint intervals (int_1, …, int_4).
  Generate random number $r \in [0, 1]$.
  if $r \in int_1$ **then**
    $e_s \leftarrow e_s - \text{constant}_1 * inf^{in}$.
  else if $r \in int_2$ **then**
    $e_s \leftarrow e_s - \text{constant}_2 * inf^{in}$.
  else if $r \in int_3$ **then**
    $u \leftarrow u + \text{constant}_3 * inf^{in}$.
  else if $r \in int_4$ **then**
    $u \leftarrow u + \text{constant}_4 * inf^{in}$.
  end if
end if
while $e_s > e_s^{init*} \mid u < u^*_{init}$ **do**
  $c_r^* := \text{RISK}\big(x^{1*}, u, e_s, S\big)$. Note the corresponding u^* and e_s^*.
  if $c_r^* < +\infty$ **then**
    $SC_r \leftarrow TRUE$.
    $fs \leftarrow (c_r^*, u^*\ e_s^*)$.
  else
    $SC_r \leftarrow FALSE$.
    $inf^{in} \leftarrow inf^{in} + 1$.
  end if
end while
return fs, SC_r and inf^{in}.

we observe an infeasibility.

If we encounter an infeasibility in Algorithm 9, we initialize the values of the second and fourth objectives according to the infeasibility count until then – we do not want to return to our starting point but want to jump to another neighbor for continuing search. Note that we solve $\widehat{\text{P}}_3(S)$ with fixed x^{1*}, u, and e_s to get $(c_r^*,\, u^*\ e_s^*)$ given that the u and e_s values are not better than the initial solution, u^*_{init} and e_s^{init*}, obtained through Algorithm 8.

Table 7.1a: Exact results for 250 scenarios and $\alpha = 0.90$

(O1)	(O2)	(O4)
0.0923	0.0836	6.00
0.0959	0.1618	6.00
0.1099	0.0836	4.96
0.3198	0.1618	5.00
0.3258	0.0857	4.00
0.3560	0.2061	5.24
0.3927	0.2473	5.76
0.4017	0.2473	5.72
0.4054	0.2061	4.36
0.4226	0.2473	5.56
0.4277	0.2473	5.52
0.4325	0.2473	5.40
0.4457	0.2473	5.32
0.4714	0.2473	4.52
0.4745	0.2473	4.44
0.4892	0.2061	4.28
0.5111	0.2061	4.24
0.5174	0.2473	4.40
0.6124	0.2473	4.36
0.6859	0.2473	4.32

Table 7.1b: Exact results for 250 scenarios and $\alpha = 0.95$

(O1)	(O2)	(O4)
0.0975	0.0836	6.00
0.0996	0.0369	5.92
0.1357	0.0836	4.84
0.2829	0.1618	6.08
0.3497	0.1618	5.00
0.3591	0.0857	4.00
0.3731	0.2061	5.24
0.4373	0.2473	5.72
0.4396	0.2473	5.44
0.4475	0.2061	4.36
0.4886	0.2473	5.32
0.4898	0.2473	5.08
0.4913	0.2473	4.44
0.5133	0.2061	4.28
0.5332	0.2061	4.24
0.5435	0.2473	4.40
0.6865	0.2473	4.36
0.7359	0.2473	4.32

7.3.1 Heuristic Results

Before going over the results for the proposed heuristic with 500 scenarios, we discuss its performance by comparing the results obtained with 250 scenarios. In Tables 7.1a and 7.1b, we present the exact Pareto solutions with 250 scenarios where $\alpha = 0.90$ and $\alpha = 0.95$, respectively. In Figures 7.1 and 7.2, we graph the Tables 7.1a and 7.1b, respectively.

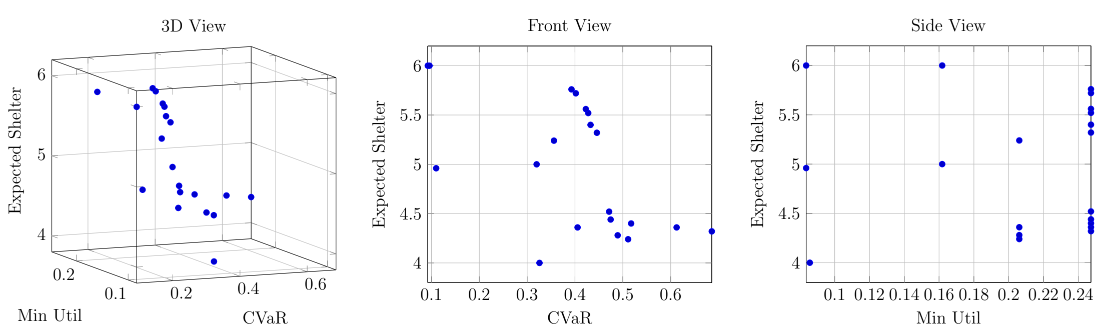

Figure 7.1: Exact results for 250 scenarios and $\alpha = 0.90$

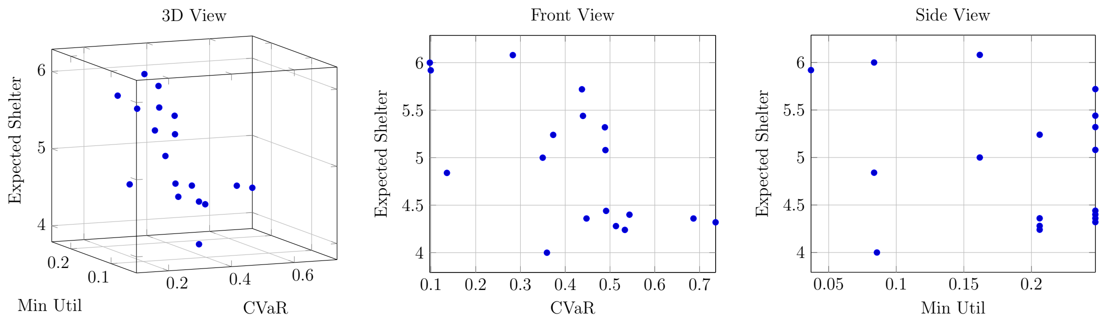

Figure 7.2: Exact results for 250 scenarios and $\alpha = 0.95$

In Tables 7.2a and 7.2b, we present the solutions found using the proposed heuristic method and in Figures 7.3 and 7.4, we graph the Tables 7.2a and 7.2b, respectively. One can observe from the figures that the non-dominated solutions for $\alpha = 0.95$ have spread more evenly with respect to the non-dominated solutions for $\alpha = 0.90$. Also observe that the in Figures 7.1 and 7.2 there seems to be more accumulation points with respect to Figures 7.3 and 7.4. So the heuristic method that we have proposed seems to be able to save the DM from choosing solutions that are relatively close to each other.

Table 7.2a: Heuristic results for 250 scenarios and $\alpha = 0.90$

(O1)	(O2)	(O4)
0.0923	0.0836	6.00
0.0959	0.1618	6.00
0.1099	0.0836	4.96
0.3198	0.1618	5.00
0.3258	0.0857	4.00
0.3560	0.2061	5.24
0.4054	0.2061	4.36
0.4745	0.2473	4.44
0.5111	0.2061	4.24
0.6859	0.2473	4.32

Table 7.2b: Heuristic results for 250 scenarios and $\alpha = 0.95$

(O1)	(O2)	(O4)
0.0975	0.0836	6.00
0.0996	0.0369	5.92
0.1357	0.0836	4.84
0.2829	0.1618	6.08
0.3497	0.1618	5.00
0.3591	0.0857	4.00
0.3731	0.2061	5.24
0.4475	0.2061	4.36
0.4913	0.2473	4.44
0.5332	0.2061	4.24
0.7359	0.2473	4.32

The average time it takes to generate one solution in Table 7.1a is 1072 seconds, in Table 7.1b is 1053 seconds, in Table 7.2a is 1056 seconds and in Table 7.2b is 576 seconds. When we compare the solutions found by the heuristic with the exact Pareto, we observe that we always find one of the Pareto-efficient solutions – this also indicates that the approach we embrace and the method of exploitation

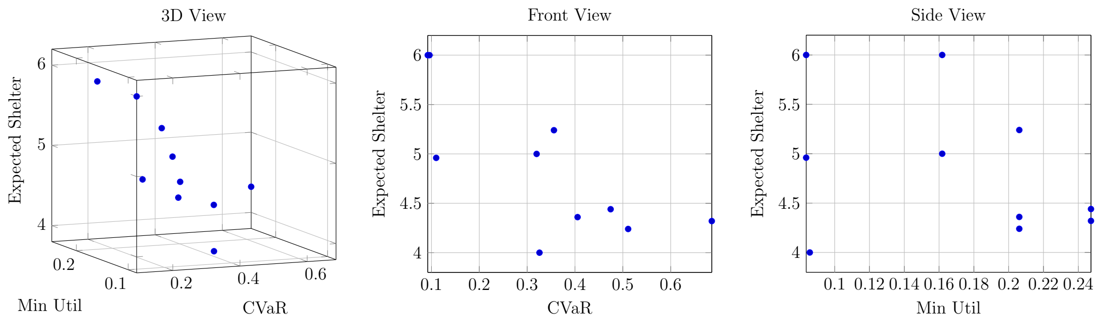

Figure 7.3: Heuristic results for 250 scenarios and $\alpha = 0.90$

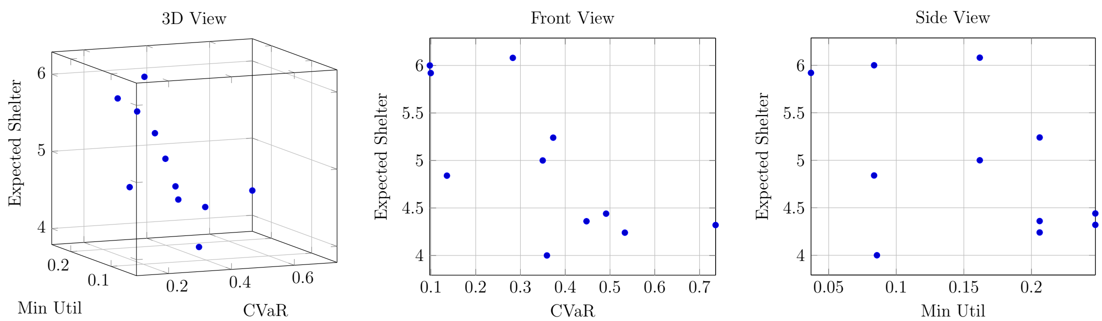

Figure 7.4: Heuristic results for 250 scenarios and $\alpha = 0.95$

is highly suitable to the problem. In the instances where $\alpha = 0.90$, we find exactly half of the solutions, but the solution times are fairly similar to the exact results. And in the instances where $\alpha = 0.95$, we generate 61% of the actual Pareto front, which is impressive considering the average time it takes to generate one such solution.

Then, we can look at the performance of the heuristic with 500 scenarios, again for $\alpha = 0.90$ and $\alpha = 0.95$. In Table 7.3a, we present the heuristic results for $\alpha = 0.90$ and in Table 7.3b for $\alpha = 0.95$. In Table 7.3a, we have 11 candidate non-dominated solutions, which we hope to be on the Pareto front and in Table 7.3b, we have 12 candidate non-dominated solutions. In Figures 7.5 and 7.6, we graph the Tables 7.3a and 7.3b, respectively.

To obtain each solution in Table 7.3a, we spend 3842 seconds, and to obtain each solution in Table 7.3b, we spend 2021 seconds. As expected, the computational times increase when larger cardinality dataset is considered.

Table 7.3a: Heuristic results for 500 scenarios and $\alpha = 0.90$

(O1)	(O2)	(O4)
0.0673	0.0549	5.00
0.0688	0.0872	6.00
0.0821	0.1494	6.00
0.2090	0.0877	4.92
0.2719	0.1618	6.04
0.2872	0.0857	4.00
0.2937	0.1494	5.00
0.3409	0.1494	4.28
0.4602	0.1792	4.32
0.5001	0.1494	4.18
0.6751	0.1792	4.22

Table 7.3b: Heuristic results for 500 scenarios and $\alpha = 0.95$

(O1)	(O2)	(O4)
0.0911	0.0817	6.00
0.0916	0.0836	5.98
0.1010	0.1494	6.00
0.1075	0.0340	5.00
0.1092	0.0682	4.98
0.2884	0.1618	6.04
0.3269	0.0857	4.00
0.3284	0.1494	5.00
0.4076	0.1494	4.28
0.4817	0.1792	4.32
0.5279	0.1494	4.18
0.7314	0.1792	4.22

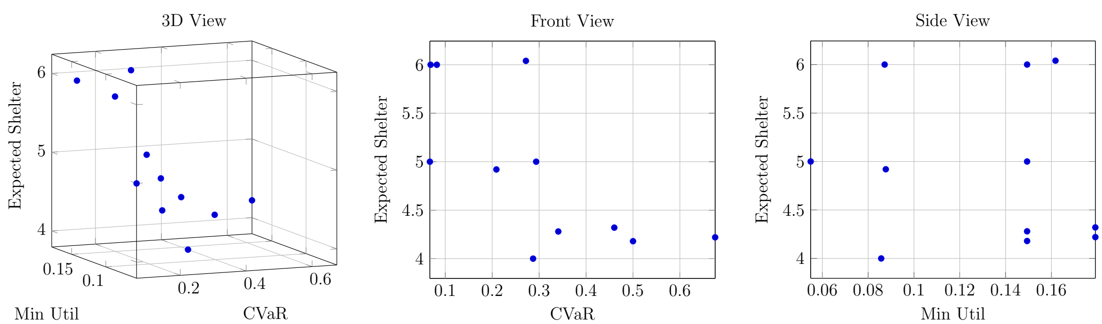

Figure 7.5: Heuristic results for 500 scenarios and $\alpha = 0.90$

7.3.2 Performance of the Heuristic Method

As we do not know the actual Pareto front for 500 scenarios, to have an idea regarding the performance of the heuristic, we present two widely used performance metrics in the literature.

Spacing (SP) metric is proposed by Schott [68] to measure the distribution of solution vectors over the Pareto front of a single set. It is computed with the

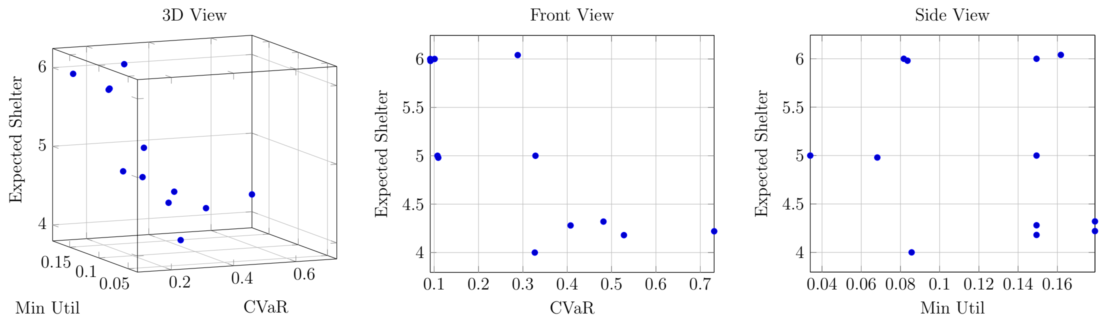

Figure 7.6: Heuristic results for 500 scenarios and $\alpha = 0.95$

standard deviation of the pairwise distances:

$$SP = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} \left(d_i - \overline{d}\right)^2}$$

where

$$d_i = \min_{j \neq i} \sum_{k=1}^{m} \mid O_k^i - O_k^j \mid \quad i = 1, 2, \ldots, n$$

and n is the number of non-dominated solutions and m is the number of objectives. In our problem, $O^i = (c_r^*,\ u^*,\ e_s^*)$ is the i-th solution obtained from the heuristic method and $\overline{d}$ is the mean of all d_i. It is desirable for SP to be small as it suggests that the solutions are distributed evenly on the solution space.

A secondary metric we use, the maximum spread (MS), proposed by Zitzler [69], is the maximum extension covered by the non-dominated solution set, defined as:

$$MS = \sqrt{\sum_{i=1}^{n} \max\left(\ || \ O^i - \bar{O}^i \ ||\right)}$$

where n is the number of non-dominated solutions and $|| \ O^i - \bar{O}^i \ ||$ is the euclidean distance between two non-dominated solutions, O^i and $\bar{O}^i$. So, the distance of each non-dominated solution to the other solutions in the non-dominated set is calculated and their maximum values are summed up. Higher values of MS

indicate that the solutions can reach further edges of the Pareto front, implying a better performance of the heuristic method.

For the spacing metric, our heuristic performs rather well. When we have 500 scenarios with $\alpha = 0.90$, we have $SP = 0.0877$, and with $\alpha = 0.95$, we have $SP = 0.1295$.

For the numbers to make sense, let's see the spacing values for the 250 scenarios. When $\alpha = 0.90$, $SP = 0.1271$ and when $\alpha = 0.95$, $SP = 0.1370$ for the exact case. When $\alpha = 0.90$, $SP = 0.1371$ and when $\alpha = 0.95$, $SP = 0.1340$ for the heuristic method.

For the maximum spread metric, the performance seems to be rather poor. When we have 500 scenarios with $\alpha = 0.90$, we have $MS = 4.7400$, and with $\alpha = 0.95$, we have $MS = 5.0483$.

Again, we provide the maximum spread values for the instances with 250 scenarios for the numbers to make sense. When $\alpha = 0.90$, $MS = 6.2998$ and when $\alpha = 0.95$, $MS = 6.0234$ for the exact case. When $\alpha = 0.90$, $MS = 4.4827$ and when $\alpha = 0.95$, $MS = 4.7636$ for the heuristic method.

So, in means of the spacing metric, the heuristic method seems to perform certainly well, implying that the obtained solutions are distributed evenly throughout the solution space. But, in means of the maximum spread metric, our method is not strong enough, implying that the proposed methodology lacks randomness, which is treated in detail in the Algorithm 9. There, we perform random search for a fixed set of first stage decision variables considering the fact that the problem is easier to solve then. The heuristic may be improved by considering local search by changing the x^{1*} vector, even though the construction part of the heuristic works well in means of generating a highly variable set of x^{1*} combinations.

This analysis implies that the heuristic is open to improvement and more random elements can be included to increase the chances of reaching to further edges of the Pareto front. Nonetheless, according to the spacing metric's results, the generated solutions are spread evenly, implying that it is easier for the DM to choose among them, i.e. there are no accumulation points and hence the solutions are easier to differentiate.

Chapter 8

Conclusion and Future Research Directions

In this study, we introduce a new modeling methodology to disaster operations management literature. We incorporate secondary disasters, e.g. aftershocks, to the shelter location decisions after an earthquake has occurred and the demand is uncertain for both of the disasters. We devise a three-stage stochastic MIP model to mimic the real setting of an earthquake. In the first stage, before observing the actual demand of the initial earthquake, we locate shelters. After the earthquake demand is realized in the second stage, the disaster victims travel to the nearest open shelter. Note that we do not assign victims to the shelters, they choose the closest open shelter and travel there without demand division. After the victims are located in the shelters, an aftershock might hit the area and create more disaster victims that require sheltering. Again, before observing the actual demand of the aftershock, we locate shelters in the second stage. Then, in the third stage, after the aftershock demand is realized, the aftershock victims travel to the nearest open shelter.

We create a set of earthquake and aftershock scenarios for Kartal, Istanbul. We use the network of Kartal introduced in Kılcı et al. [14]. We assume that 10 different aftershocks will follow each initial earthquake and create a scenario set of cardinality 500, with 50 different initial earthquakes.

As the solution times of the model with CPLEX are high, we propose a heuristic methodology. We improve the solution times drastically with the heuristic method while having small deviations from the optimal values of the instances for which we know the optimal solutions.

Comparing our proposed model with a *common* counterpart model, where we decompose the initial earthquake and the aftershock and conduct the decision making without relating the aftershocks to initial earthquakes, we show that it is important to consider secondary disasters while locating shelter sites for disaster operations management.

Considering the problems regarding the parameter selection in humanitarian operations, we extend our formulation to a multi-objective framework. We consider the risk of exceeding the shelter capacities, minimum utilization value of overall shelters, weight of established shelters and expected number of established shelters as the objectives.

To have the problem more suitable for different contexts of the facility location problem, we disregard the weight objective and propose a heuristic methodology to solve the 3-objective formulation for datasets having higher cardinality. In our tests, we observe that the proposed heuristic finds evenly distributed solutions, which is important as the decision maker can easily differentiate between the Pareto-efficient solutions, but performs relatively weak in the sense of covering the further edges of the Pareto front.

As an extension, it is worth searching for better heuristic methods as shelter location problem in the multi-objective context is an important problem in humanitarian logistics. One can also explore the risk of losing a shelter, i.e. having its capacity decreased or having it destroyed, and can search for solutions that minimizes the risk of such occurrences.

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Appendix A

Data

A.1 Shelter Weights

Shelter #	Weight
1	0.865
2	0.795
3	0.781
4	0.948
5	0.948
6	0.674
7	0.674
8	0.801
9	0.847
10	0.850
11	0.694
12	0.847
13	0.809
14	0.803
15	0.827
16	0.982

Shelter #	Weight
17	0.982
18	0.829
19	0.847
20	0.865
21	0.689
22	0.689
23	0.739
24	0.948
25	0.948

A.2 Population of Districts

Shelter #	Population
1	14,242
2	30,003
3	10,302
4	22,978
5	22,380
6	17,390
7	25,261
8	29,124
9	14,366
10	13,744
11	14,827
12	11,720
13	13,718
14	43,433
15	27,568
16	28,591
17	37,144
18	8,093

Shelter #	Population
19	30,147
20	11,649

A.3 4-Objective Results with $\alpha = 0.90$

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.5816	0.1373	1.757	4.24
0.5111	0.2061	1.637	4.24
0.5496	0.1373	1.757	4.28
0.4892	0.2061	1.637	4.28
0.5051	0.1373	1.757	4.32
0.7670	0.2473	1.637	4.32
0.7584	0.2473	1.622	4.32
0.4801	0.1373	1.757	4.36
0.6157	0.2473	1.637	4.36
0.4359	0.1373	1.757	4.40
0.5538	0.2473	1.637	4.40
0.5418	0.2473	1.622	4.40
0.3892	0.1540	1.757	4.44
0.3865	0.1663	1.729	4.44
0.5268	0.2473	1.642	4.44
0.3844	0.1540	1.757	4.48
0.3833	0.1540	1.749	4.48
0.3844	0.1663	1.729	4.48
0.5066	0.2473	1.642	4.48
0.4745	0.2473	1.637	4.48
0.3815	0.1540	1.757	4.52
0.3803	0.1540	1.751	4.52
0.3801	0.1540	1.749	4.52
0.5054	0.2473	1.642	4.52
0.4743	0.2473	1.637	4.52

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.3791	0.1540	1.757	4.56
0.3781	0.1540	1.751	4.56
0.3778	0.1540	1.749	4.56
0.5025	0.2473	1.642	4.56
0.3751	0.1540	1.757	4.60
0.3747	0.1540	1.749	4.60
0.3768	0.1663	1.729	4.60
0.4994	0.2473	1.642	4.60
0.3736	0.1540	1.757	4.64
0.3721	0.1540	1.749	4.64
0.4981	0.2473	1.642	4.64
0.3678	0.1540	1.749	4.68
0.3747	0.1663	1.729	4.68
0.4966	0.2473	1.642	4.68
0.4626	0.2473	1.637	4.68
0.3662	0.1540	1.757	4.72
0.3662	0.1663	1.729	4.72
0.4965	0.2473	1.642	4.72
0.3623	0.1540	1.757	4.76
0.3543	0.1540	1.757	4.80
0.3543	0.1663	1.729	4.80
0.4940	0.2473	1.642	4.80
0.3503	0.1540	1.757	4.84
0.3503	0.1663	1.729	4.84
0.4930	0.2473	1.642	4.84
0.3495	0.1540	1.749	4.88
0.4808	0.2473	1.642	4.88
0.3495	0.1540	1.757	4.92
0.3495	0.1663	1.729	4.92
0.4803	0.2473	1.642	4.96
0.7420	0.1818	1.791	5.00
0.4881	0.1993	1.757	5.00

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.6725	0.1818	1.791	5.04
0.4335	0.1993	1.757	5.04
0.5968	0.1818	1.791	5.08
0.4244	0.1993	1.757	5.08
0.3746	0.1993	1.729	5.08
0.4488	0.2473	1.622	5.08
0.5898	0.1818	1.791	5.12
0.4137	0.1993	1.757	5.12
0.3633	0.1993	1.729	5.12
0.4460	0.2473	1.622	5.12
0.5867	0.1818	1.791	5.16
0.5821	0.1818	1.763	5.16
0.4080	0.1993	1.757	5.16
0.3503	0.1993	1.729	5.16
0.5825	0.1818	1.791	5.20
0.5770	0.1818	1.763	5.20
0.4023	0.1993	1.757	5.20
0.5764	0.1818	1.791	5.24
0.5680	0.1818	1.763	5.24
0.3977	0.1993	1.757	5.24
0.3972	0.1993	1.749	5.24
0.4557	0.2473	1.637	5.24
0.5707	0.1818	1.783	5.28
0.5573	0.1818	1.763	5.28
0.3802	0.1993	1.757	5.28
0.5430	0.1818	1.791	5.32
0.3790	0.1993	1.757	5.32
0.4537	0.2473	1.637	5.32
0.5427	0.1818	1.785	5.36
0.5181	0.1818	1.763	5.36
0.3774	0.1993	1.757	5.36
0.3127	0.1993	1.729	5.36

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.5423	0.1818	1.791	5.40
0.5410	0.1818	1.785	5.40
0.5393	0.1818	1.783	5.40
0.5161	0.1818	1.763	5.40
0.3757	0.1993	1.757	5.40
0.3754	0.1993	1.749	5.40
0.3099	0.1993	1.729	5.40
0.4340	0.2473	1.637	5.40
0.5403	0.1818	1.791	5.44
0.5377	0.1818	1.783	5.44
0.5156	0.1818	1.763	5.44
0.3750	0.1993	1.751	5.44
0.3743	0.1993	1.749	5.44
0.3028	0.1818	1.671	5.44
0.5389	0.1818	1.785	5.48
0.5358	0.1818	1.783	5.48
0.5143	0.1818	1.763	5.48
0.3739	0.1993	1.757	5.48
0.3067	0.1993	1.729	5.48
0.4256	0.2473	1.637	5.48
0.4217	0.2473	1.622	5.48
0.5381	0.1818	1.791	5.52
0.5346	0.1818	1.783	5.52
0.5121	0.1818	1.763	5.52
0.3715	0.1993	1.757	5.52
0.3698	0.1993	1.749	5.52
0.2958	0.1818	1.671	5.52
0.4190	0.2473	1.637	5.52
0.5377	0.1818	1.791	5.56
0.3693	0.1993	1.757	5.56
0.3668	0.1993	1.749	5.56
0.3055	0.1993	1.729	5.56

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.2947	0.1818	1.671	5.56
0.4177	0.2473	1.637	5.56
0.5367	0.1818	1.791	5.60
0.3659	0.1993	1.757	5.60
0.3657	0.1993	1.749	5.60
0.2993	0.1993	1.729	5.60
0.2918	0.1818	1.671	5.60
0.4142	0.2473	1.622	5.60
0.5322	0.1818	1.783	5.64
0.3625	0.1993	1.749	5.64
0.2897	0.1818	1.671	5.64
0.4093	0.2473	1.637	5.64
0.5099	0.1818	1.763	5.68
0.3606	0.1993	1.757	5.68
0.2973	0.1993	1.729	5.68
0.4008	0.2473	1.637	5.68
0.5071	0.1791	1.763	5.72
0.3601	0.1993	1.757	5.72
0.3598	0.1993	1.749	5.72
0.2931	0.1993	1.729	5.72
0.3963	0.2473	1.637	5.72
0.3589	0.1993	1.757	5.76
0.2922	0.1993	1.729	5.76
0.2889	0.1818	1.671	5.76
0.3927	0.2473	1.637	5.76
0.5078	0.1818	1.763	5.80
0.2914	0.1993	1.729	5.80
0.2838	0.1818	1.671	5.88
0.3176	0.2061	1.642	5.92
0.5317	0.1791	1.783	5.96
0.2746	0.1818	1.671	5.96
0.1947	0.1993	1.642	5.96

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.5075	0.1818	1.763	6.00
0.4951	0.1818	1.785	6.08
0.4636	0.1818	1.783	6.08
0.4570	0.1818	1.783	6.12
0.4552	0.1818	1.791	6.16
0.4547	0.1818	1.783	6.16
0.4522	0.1818	1.785	6.20
0.4508	0.1818	1.783	6.20
0.4380	0.1818	1.791	6.28
0.4222	0.1818	1.791	6.32
0.4186	0.1818	1.763	6.32
0.4018	0.1818	1.763	6.36
0.4220	0.1818	1.791	6.40
0.3976	0.1818	1.763	6.40
0.2259	0.1818	1.671	6.40
0.3974	0.1818	1.763	6.44
0.2077	0.1818	1.671	6.44
0.1955	0.1818	1.671	6.48
0.1835	0.1818	1.671	6.52

A.4 4-Objective Results with $\alpha = 0.95$

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.6071	0.1373	1.757	4.24
0.5379	0.2061	1.637	4.24
0.5770	0.1373	1.757	4.28
0.5133	0.2061	1.637	4.28
0.5331	0.1373	1.757	4.32
0.8006	0.2473	1.637	4.32
0.7359	0.2473	1.622	4.32

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.5073	0.1373	1.757	4.36
0.6876	0.2473	1.637	4.36
0.4466	0.1373	1.757	4.40
0.5435	0.2473	1.637	4.40
0.4053	0.1540	1.757	4.44
0.4036	0.1540	1.749	4.44
0.4036	0.1663	1.729	4.44
0.5430	0.2473	1.642	4.44
0.4913	0.2473	1.637	4.44
0.4041	0.1540	1.757	4.48
0.4024	0.1540	1.751	4.48
0.5372	0.2473	1.642	4.48
0.4912	0.2473	1.637	4.48
0.4024	0.1540	1.757	4.52
0.4027	0.1663	1.729	4.52
0.4023	0.1540	1.757	4.56
0.4023	0.1663	1.729	4.60
0.4886	0.2473	1.637	4.64
0.7952	0.1818	1.791	5.00
0.5287	0.1993	1.757	5.00
0.7420	0.1818	1.791	5.04
0.4934	0.1993	1.757	5.04
0.4921	0.1993	1.729	5.04
0.6068	0.1818	1.791	5.08
0.4866	0.1993	1.757	5.08
0.3902	0.1993	1.729	5.08
0.6005	0.1818	1.791	5.12
0.4820	0.1993	1.757	5.12
0.5998	0.1818	1.791	5.16
0.4798	0.1993	1.757	5.16
0.3641	0.1993	1.729	5.16
0.5989	0.1818	1.791	5.20

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.5936	0.1818	1.763	5.20
0.4779	0.1993	1.757	5.20
0.5974	0.1818	1.785	5.24
0.5898	0.1818	1.763	5.24
0.4774	0.1993	1.749	5.24
0.3548	0.1993	1.729	5.24
0.5973	0.1818	1.785	5.28
0.5839	0.1818	1.763	5.28
0.4771	0.1993	1.757	5.28
0.3525	0.1993	1.729	5.28
0.5973	0.1818	1.791	5.32
0.3501	0.1993	1.729	5.32
0.5817	0.1818	1.763	5.36
0.3256	0.1818	1.671	5.40
0.5795	0.1818	1.763	5.44
0.3119	0.1818	1.671	5.44
0.4433	0.2473	1.637	5.44
0.4396	0.2473	1.622	5.44
0.3049	0.1818	1.671	5.48
0.4392	0.2473	1.637	5.52
0.4373	0.2473	1.622	5.52
0.4373	0.2473	1.637	5.68
0.3664	0.2061	1.642	5.92
0.2400	0.1993	1.642	5.96
0.3013	0.1818	1.671	6.00
0.4775	0.1818	1.791	6.08
0.4739	0.1818	1.783	6.08
0.4737	0.1818	1.791	6.12
0.4675	0.1818	1.791	6.16
0.4575	0.1818	1.763	6.20
0.4664	0.1818	1.791	6.24
0.4521	0.1818	1.763	6.24

CVaR	Minimum Utilization	Shelter Weight	Expected Shelter Amount
0.4656	0.1818	1.791	6.28
0.4511	0.1818	1.763	6.28
0.4449	0.1818	1.763	6.32
0.4434	0.1818	1.763	6.36
0.2513	0.1818	1.671	6.40
0.2400	0.1818	1.671	6.44
0.2182	0.1818	1.671	6.48
0.1970	0.1818	1.671	6.52