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Kantian equilibria of a class of Nash bargaining games

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Abstract

We study Kantian equilibria of an n-player bargaining game, which is a modified version of the well-known divide-the-dollar game. We first show that the Kantian equilibrium exists under fairly minimal assumptions. Second, if the bankruptcy rule used satisfies equal treatment of equals, and is almost nowhere proportional, then only equal division can prevail in any Kantian equilibrium. On the other hand, we show that an "anything goes" type result emerges only under the proportional rule. Finally, using hybrid bankruptcy rules that we construct in a novel fashion, we can characterize the whole equilibrium set. Our results highlight the interactions between institutions (axiomatic properties of division rules) and agents' equilibrium behavior.

1 | INTRODUCTION

Immanuel Kant's moral philosophy is structured around his notion of "Categorical Imperative," which is an unconditional and absolute ethical principle for all rational beings. The first formulation of Categorical Imperative states that one is to "act only in accordance with that maxim through which you can at the same time will that it become a universal law" (Kant, 1996). It suggests that one should decide to take some particular action after considering the counterfactual: What would happen if all rational agents would also take the same action? If one would rationally "will" themselves to take the action in a world where every rational

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agent is ready to implement the same action after her, then taking this action is morally acceptable.[1] This emphasis on *universalization* and specifying what *universalization* of an action means for the other agents are crucial to introduce Kantian reasoning into economic modeling.[2]

One way of doing so is to incorporate agents who have *preferences* in compliance with Kantian morality. An influential line of such literature has been cultivated by Alger and Weibull (2013). These authors introduce an alternative agent type, *homo moralis*, whose preferences lie in between the selfish concern of maximizing own payoff and the moral concern of choosing an action which would lead to the greatest possible payoff if every agent copies her action.

Another way of incorporating Kantian reasoning into economic modeling is to assume that the agents follow an optimization protocol based on Kantian morality. Roemer (2010, 2015, 2019) develops an equilibrium concept called the "Kantian equilibrium," which captures a practical mathematical representation of Kantian ethical agenda: Given an action profile, when an agent thinks about whether there is a profitable/beneficial deviation for her in a Kantian manner, she evaluates the profile of actions that would occur if everyone deviated like her. Here, a deviation materializes by changing the action to some other, which can be seen as a certain multiple of the original action. In the more commonly used version of Kantian equilibrium, for a given action profile where an action is actually a number (such as effort levels to be exerted or bids to be made), an agent can deviate from his action by multiplying it with some factor $\alpha > 0$ (e.g., initially exerting an effort level of 10, and deviating to 30, i.e., three times 10).[3] The Kantian counterfactual requires him to consider what happens if the other agents also change their actions in the same way. Then, an action profile is said to be a Kantian equilibrium if no agent prefers that everyone changes his/her action by the same factor $\alpha > 0$. This is in stark contrast to optimizing à la Nash, where each agent considers the counterfactual "What would happen if solely I changed my strategy while the other agents kept theirs fixed?" (Roemer, 2019).

As also noted by Sher (2020), constant-sum games are seen as poor candidates for Kantian equilibrium since competition rather than coordination is emphasized in such games. Indeed, the literature on the application of Kantian equilibrium (or Kantian reasoning) has been almost exclusively on contributions to the public goods games and tragedy of commons scenarios in environmental problems.[4] Here, we study the Kantian equilibrium of a bargaining game, which has a constant-sum nature. To the best of our knowledge, this is the first attempt to investigate the implications of Kantian optimization in a bargaining game and compare the equilibrium outcomes with those under Nashian optimization. Our results show, in contrast with the commonly held view, that the Kantian equilibrium may also have a promise in games that have a constant-sum nature.

[1]This interpretation of Categorical Imperative equates *maxim* with *action*. Within Kantian ethics, it is possible to consider cases where different *maxims* can lead to the same action, or a specific *maxim* suggesting different actions.

[2]The application of Kantian reasoning to economics dates back to Laffont (1975). He articulates the idea of a model which is composed of agents who strictly follow Kantian morality and expect others to behave like them, and informally discusses what would be different if some macroeconomic models had Kantian agents.

[3]It is important to emphasize that this definition employs the Kantian reasoning by considering deviations that change the given strategy profile in a multiplicative fashion. Although there are other ways to conceptualize what a deviation is in a Kantian manner (e.g., defining in an additive fashion), we treat this multiplicative version as the default. All general commentary concerning our findings on Kantian equilibrium pertains to only this version.

[4]An interested reader is referred to Roemer (2010, 2015), Ghosh and Van Long (2015), Grafton et al. (2017), Eichner and Pethig (2020), and Dutta et al. (2021) for public goods applications; and Grafton et al. (2017), Bezin and Ponthiere (2019), Planas (2018), and Long (2020) for the tragedy of commons problems.

At this point, some background information on the game we study is in order. In an attempt to provide a strategic justification for the axiomatic Nash bargaining solution (Nash, 1950), Nash (1953) introduced what was later called the *Nash Demand Game (NDG)*. The *Divide-the-Dollar (DD)* game is a simplified version of the *NDG*, where bargaining frontier is linear and the bargaining set is symmetric. In the *DD* game, n agents simultaneously declare their demands on a dollar. If the sum of demands is less than or equal to one, then everyone receives his demand, whereas if the sum of demands is larger than one, then everyone receives zero. This simple game is frequently used in economics, political science, and international relations, likely because it carries the two defining characteristics of a canonical bargaining situation: (i) joint interest in reaching an agreement and (ii) conflict of interest over which agreement to reach (see Binmore, 1998). However, the Nash equilibrium set of the *DD* game may cause disappointment for those who use this game to make sharp predictions: any strategy profile where the demands add up to one (i.e., the whole bargaining frontier) constitutes a Nash equilibrium. In other words, there are infinitely many Nash equilibria. Among them, the one that induces an equal division of the dollar is arguably the most reasonable one. Some scholars provided arguments in favor of equal division, referring to its normative appeal, focality, symmetry, or evolutionary stability.[5] There is also a strong experimental support for equal division in symmetric bargaining games.[6] Starting with Brams and Taylor (1994), some scholars attempt to modify the rules of the *DD* game so as to match the equilibrium prediction with the common sense prediction (i.e., equal division).

1.1 | Summary of results and contribution

In this paper we propose modifying the rules of the *DD* game by applying a bankruptcy rule when the players' demands are not jointly feasible (also see Ashlagi et al., 2012). Our framework is different than other modifications in terms of the optimization concept that the players employ: they are assumed to be Kantian in the sense formulated in Roemer (2010). Accordingly, we focus on the Kantian equilibria of the modified game by bringing the axiomatic properties of different bankruptcy rules into the picture. We, first, show the existence of Kantian equilibrium under a fairly weak assumption (i.e., equal treatment of equals [*ETE*]) on the bankruptcy rule used in the game (for a discussion on the existence of Kantian equilibrium in general, the reader is referred to Sher, 2020). Any division rule which satisfies *ETE* induces a Kantian equilibrium, where equal division is the equilibrium outcome. Second, we show that the use of the proportional rule, arguably the most prominent bankruptcy rule among all, leads to an *anything goes* type result: any efficient division can be supported in Kantian equilibrium. Importantly, we show that there also exist division rules, other than the proportional rule, which satisfy *ETE*, but still induce unequal division in equilibrium. We introduce two properties which separately eliminate these cases. Finally, we construct a family of bankruptcy rules in a novel fashion, with the help of which we span the set of all possible efficient divisions in Kantian equilibria. We show how (i) the moral reasoning embraced by the agents affects the strategic interaction and (ii) the axiomatic properties of bankruptcy rules (i.e., institutions) influence agents' behavior and equilibrium outcomes.

[5]The examples include Nash (1953), Schelling (1960), Young (1993), and Skyrms (1996).

[6]Some of such experiments are Nydegger and Owen (1975) and Roth and Malouf (1979).

This paper contributes to three different lines of work. First, to the best of our knowledge, it is the first paper to study Kantian equilibrium in a bargaining game. Second, it contributes to the *Nash Program* in a novel fashion.[7] Harsanyi (1974) details on the aims of the Nash Program and writes: "Nash (1953) has suggested that we can obtain a clear understanding of the alternative solution concepts proposed for cooperative games and can better identify and evaluate the assumptions to make about the players' bargaining behavior if we reconstruct them as equilibrium points in suitably defined bargaining games, treating the latter formally as non-cooperative games" (as cited in Serrano, 2005). The underlying idea is to enhance the relevance of a cooperative solution by reaching it from different points of view, which—in this context—boils down to establishing non-cooperative foundations for equal division as the equilibrium outcome in our bargaining game (Serrano, 2021). We achieve this reconciliation without resorting to the Nash equilibrium or its refinements. As such, what we do in the paper, that is providing a strategic support to the Nash bargaining solution via Kantian equilibrium, is novel. Finally, since our game addresses bankruptcy situations that can arise as an outcome of strategic interaction, it contributes to a relatively small literature on strategic bankruptcy games.

By reconstructing equal division as a Kantian equilibrium, we reassure both the cooperative and non-cooperative sides of Kantian optimization. While Roemer (2010, 2015, 2019) argues that one needs to incorporate the *social cooperation* aspect in agents' optimization processes, he also emphasizes that cooperation is a way of acting which can involve self-interested agents (Roemer, 2019). He specifically tries to build Kantian optimization on self-interest and trust, mentioning:

> Playing the strategy that one would like everyone to play is, for me, motivated by the common knowledge assumption [...] and trust, not by a concern for the welfare of the group as a whole. It entails a recognition that cooperation can make me better off (incidentally, it makes all of us better off). But that parenthetical fact is not or need not be the motivation for my playing 'cooperatively.' (as cited in Sher, 2020, pp. 34–35)

So, although Kantian equilibrium is a way to include social cooperation in a solution concept by imposing universal deviations, its ties to self-interest are also foundational. This also reiterates the strategic reasoning employed by Kantian optimizers. Indeed, as we are going to show in our results, it is possible to make yourself better off, at the expense of making someone else worse off with universal deviations in bargaining games.

1.2 | Organization of the paper

The organization of the paper is as follows: Section 2 reviews the relevant literature with a special emphasis on the *DD* game and its modified versions. Section 3 introduces the model and necessary definitions. Section 4 presents equilibrium analyses and results. Section 5 presents an

[7]Recently, the Nash Program has been applied to provide strategic foundations for bankruptcy rules: the constrained equal awards (*CEA*) rule (Tsay & Yeh, 2019), the Talmud rule (Moreno-Ternero et al., 2020), and the TAL family of rules (Moreno-Ternero et al., 2021).

equilibrium analysis under the alternative, additive definition of the Kantian equilibrium. Section 6 ends the paper with concluding remarks.

2 | LITERATURE REVIEW

Our paper falls into two strands of literature in bargaining and distribution games: (i) *DD* game and its modified versions and (ii) bankruptcy/claims games. We focus on the former in this section since it is the closest one to our work among the two. Due to the way we revise the punishment clause in the standard *DD* game and the divisions rules and axioms we utilize, our game can be seen as a bankruptcy game too.[8]

As we mentioned in Section 1, despite its appealing characteristics, the *DD* game suffers from the multiplicity of Nash equilibria. To overcome this problem and induce equal division as the unique equilibrium outcome, researchers apply different methods to modify the game, by changing its rules in a *reasonable* fashion. In his seminal contribution, Nash (1953) initially suggests to introduce perturbations to the probability function, which decides whether a pair of demands is feasible or not. He informally discusses that the limit of each perturbed game's equilibrium converges to equal division as the perturbations to the probability function approaches to the original probability function. Later, Abreu and Pearce (2015) formalize this idea and specify the conditions for this convergence result to hold.

We can classify the other papers into three groups: The ones that (i) add new stages to the game, (ii) modify the punishment clause (i.e., for not reaching an agreement) by changing the rule which distributes the dollar, and (iii) change the strategy space to include other variables that are different from declaring demands. In the first group, if the sum of demands is larger than the dollar, then the game continues with a second stage. Brams and Taylor's (1994) *DD*2, Cetemen and Karagözoğlu (2014), Karagözoğlu and Rachmilevitch (2018), and Rachmilevitch (2020) fall into this category. The second group of papers modifies the punishment clause by changing the division (or the payment) rule. Brams and Taylor's (1994) *DD*1, Anbarcı (2001), Ashlagi et al. (2012), and Rachmilevitch (2017) are some papers that fall into the second category. The current paper also belongs to this category.

Among the studies mentioned above, Ashlagi et al. (2012) are the one closest to ours. More precisely, both papers study essentially the same bargaining game but with different equilibrium concepts—a difference that turned out to be a crucial one, a difference that called for very different set of axioms and led to stark differences in equilibria. Further comparisons will be presented in Section 4.

3 | THE MODEL

In this section, we present the model and the necessary definitions in three subsections. Section 3.1 describes the bargaining game that we study. Section 3.2 presents the definitions of bankruptcy problems, bankruptcy rules, and the axioms we employ in the equilibrium analysis. Finally, Section 3.3 presents the definition of Kantian equilibrium, which we use throughout the paper.

[8]Some contributions to this line of work are Chun (1989), Chang and Hu (2008), Kıbrıs and Kıbrıs (2013), Karagözoğlu (2014), and Hagiwara and Hanato (2021).

3.1 | Bargaining game

In a bargaining game, denoted by Γ, a finite set of agents $N = \{1, 2, ..., n\}$ try to divide a finite, real-valued estate $E > 0$ among themselves. The value of the estate, E, and the set of agents, N, are fixed. Agents have strictly monotonic preferences over the amounts of the estate they receive. Every agent $i \in N$ claims $c_i \in C_i = \mathbb{R}_{++}$, over the estate as a strategy, where C_i denotes his strategy set. The set of strategy profiles, or claim (or demand) vectors, is denoted by $C = C_1 \times C_2 \times \cdots \times C_n$.

The payoff structure of Γ is determined by an estate division rule, $F : C \to \mathbb{R}_+^N$. It associates every strategy profile $c \in C$ with an awards vector $F(c) \in \mathbb{R}_+^N : \sum_{i=1}^n F_i(c) \leq E$ and $\forall\, i \in N : F_i(c) \leq c_i$, where $F_i(c)$ denotes the amount of the estate the agent i receives under the strategy profile c. If $\sum_{i=1}^n c_i \leq E$, then $F_i(c) = c_i$ for every $i \in N$. Note that the whole estate does not have to be distributed in the case of strict inequality. Until this point, our game coincides with the *DD* game except that we do not restrict the value of E to 1. The main modification we make is related to the punishment clause. If $\sum_{i=1}^n c_i \geq E$, then we treat this situation as a bankruptcy problem (see Ashlagi et al., 2012, for a similar treatment). In this case, the whole estate will be distributed (i.e., $\sum_{i=1}^n F_i(c) = E$). Naturally, in this case, F will behave as a bankruptcy rule. Note that F denotes the division rule, which is defined for all possible claims vectors independent of whether the sum of claims is less than or greater than E. Later, to avoid confusion, we denote a generic bankruptcy rule that will be used when $\sum_{i=1}^n c_i \geq E$, with R. Hence, we can say that R is embedded in F. Finally, we assume that the number of players, their preferences, the value of the estate, and the estate division rule are all common knowledge among the players.

In Section 3.2, we formally define bankruptcy problems and provide details on division rules and axioms applied in such problems. Axioms that were not defined in earlier work and are introduced in this paper will appear later in Section 4, when the need for them arises.

3.2 | Bankruptcy problems, division rules, and axioms

In a *DD* game, if the sum of claims (or demands) is larger than 1, then every agent receives zero. In this paper, in line with the literature on the modifications of the *DD* game reviewed in Section 2, we change this punishment clause. In particular, even if $\sum_{i=1}^n c_i > E$, we still allocate the whole estate, using a bankruptcy rule. Now, we present the definitions of a bankruptcy problem and a bankruptcy rule. First, some notation: in a bankruptcy (or claims) problem, a finite, real-valued estate $E > 0$ has to be distributed among a set, N, of agents who have claims over E, where N is taken to be a finite subset of natural numbers $\mathbb{N}$, generally $\{1, ..., n\}$. The claim of an agent $i \in N$ is denoted by $c_i \in \mathbb{R}_+$.

Definition 3.1 (Bankruptcy problem). A *bankruptcy problem* is a pair $(c, E) \in \mathbb{R}_+^N \times \mathbb{R}_+$, where $c \equiv (c_i)_{i \in N}$ is the claims vector and $\sum c_i \geq E$. We denote the set of all such problems with ζ^N.

Bankruptcy problems were first formally studied in O'Neill (1982). For excellent reviews of this literature, the reader is referred to Thomson (2019).

Definition 3.2 (Bankruptcy rule). A *bankruptcy rule* is a function that associates with each bankruptcy problem $(c, E) \in \zeta^N$ an awards vector $R(c, E) \in \mathbb{R}_+^N$, such that $\sum R_i(c, E) = E$ and $0 \leq R_i(c, E) \leq c_i$ for all $i \in N$.

Note that two properties are embedded in this definition. First, any bankruptcy rule satisfies *efficiency* (i.e., $\sum R_i(c, E) = E$). Second, any bankruptcy rule satisfies *zero lower bound* and *claims boundedness* (i.e., $0 \leq R_i(c, E) \leq c_i$). Hence, we will not explicitly list them in the inventory of axioms. In our proofs, these defining properties of bankruptcy rules will be implicitly used if need be.

3.2.1 | Inventory of bankruptcy rules

Here, we present the definitions of some prominent bankruptcy rules that also appear in our equilibrium analysis.

The *proportional rule* is possibly the most prominent bankruptcy rule. The idea of proportionality as a criterion for justice dates back to Aristotle. In *Nicomachean Ethics*, Aristotle establishes a close connection between justice and proportionality: "… the just is—the proportional; the unjust is what violates the proportion." The proportional rule distributes the endowment proportionally with respect to claims.

Definition 3.3 (Proportional rule $[P]$). For each $(c, E) \in \zeta^N$ the *proportional rule* distributes the endowment, E, as $P_i(c, E) = \lambda_p c_i$, where $\lambda_p = \frac{E}{\sum c_v}$. Throughout the paper, $\sum c_v$ will be used to denote the sum of all claims in a claims vector. So, v is the summation index.

The *constrained equal a rule* distributes the endowment as equally as possible subject to a constraint, which is "no one should receive more than what he claimed."

Definition 3.4 (Constrained equal awards rule $[CEA]$). For each $(c, E) \in \zeta^N$ the *CEA rule* distributes the endowment, E, as $CEA_i(c, E) = min\{c_i, \lambda_{cea}\}$ where $\lambda_{cea} \in \mathbb{R}_+$ is such that $\sum min\{c_v, \lambda_{cea}\} = E$.

The *constrained equal losses (CEL) rule* distributes the loss (i.e., the discrepancy between the sum of claims and the endowment) as equally as possible subject to a constraint, which is "no one should receive a negative amount."

Definition 3.5 (Constrained equal losses rule $[CEL]$). For each $(c, E) \in \zeta^N$ the *CEL rule* distributes the endowment, E, as $CEL_i(c, E) = max\{c_i - \lambda_{cel}, 0\}$ where $\lambda_{cel} \in \mathbb{R}_+$ is such that $\sum max\{c_v - \lambda_{cel}, 0\} = E$.

The *Talmud rule* (Aumann & Maschler, 1985) applies a *hybrid* method. If the sum of claims is larger than $2E$, it distributes the endowment in a *CEA* fashion based on half-claims. If the sum of claims is smaller than $2E$, then it distributes the endowment in a *CEL* fashion based on half-claims.

Definition 3.6 (Talmud rule $[T]$). For each $(c, E) \in \zeta^N$ and $\forall\, i \in N$, the *Talmud rule* distributes the endowment, E, as

$$T_i(c, E) = \begin{cases} min\left\{\frac{c_i}{2}, \lambda_t\right\} & \text{if } E \leq \sum \frac{c_j}{2}, \\ c_i - min\left\{\frac{c_i}{2}, \lambda_t\right\} & \text{otherwise,} \end{cases}$$

where in each case, $\lambda_t \in \mathbb{R}_+$ is such that $\sum_{i \in N} T_i(c, E) = E$.

3.2.2 | Inventory of axioms

Here, we provide the definitions of the axioms we use in our equilibrium analysis. *ETE* is a very primitive fairness axiom, which stipulates that any two agents with equal claims should receive equal awards. All the bankruptcy rules that we introduced in Section 3.2.1 satisfy *ETE*.

Definition 3.7 (Equal treatment of equals). For each $(c, E) \in \zeta^N$ and all $i, j \in N$ such that $c_i = c_j$, $R_i(c, E) = R_j(c, E)$.

The next two axioms are concerned about the way awards vector reacts to certain types of changes in claims vector. Marchant (2008) labels these properties as *multiplicative invariance* and *additive invariance 2*. Here, we follow the terminology introduced by Thomson (2019). *Proportional-increase-in-claims invariance* (*PICI*; Marchant, 2008) requires that a proportional increase in all claims should not lead to any change in awards. Later, we will show that the only rule which satisfies *PICI* is the proportional rule.

Definition 3.8 (Proportional-increase-in-claims invariance). For each $(c, E) \in \zeta^N$ and for each $\alpha > 0$, $R(\alpha c, E) = R(c, E)$.

Uniform-increase-in-claims invariance (*UICI*; Marchant, 2008) requires that a uniform increase in all claims should not lead to any change in awards. As we will demonstrate in Section 5, only the *CEL* rule satisfies this axiom among the bankruptcy rules that we introduced.

Definition 3.9 (Uniform-increase-in-claims invariance). For each $(c, E) \in \zeta^N$ and for each $\alpha > 0$, $R(c + \alpha, E) = R(c, E)$.

Finally, we introduce four axioms, which were used in Ashlagi et al., 2012. We provide formal definitions of these to make the text self-contained, so that readers can easily compare our results with those in Ashlagi et al. (2012) without resorting to another source.

Claims monotonicity (*CMON*) requires that an increase in an agent's claim—ceteris paribus—should not make him worse off. All the bankruptcy rules that we introduced in Section 3.2.1 satisfy this axiom.

Definition 3.10 (Claims monotonicity). For each $(c, E) \in \zeta^N$, each $i \in N$, and each $c_i' > c_i$, $R_i((c_i', c_{-i}), E) \geq R_i(c, E)$.

Order preservation of awards (*OPA*; Aumann & Maschler, 1985) requires that the ordering of awards should conform with the ordering of claims. Again, all the bankruptcy rules that we introduced in Section 3.2.1 satisfy this axiom.

Definition 3.11 (Order preservation of awards). For each $(c, E) \in \zeta^N$ and all $i, j \in N$ such that $c_i \leq c_j$, $R_i(c, E) \leq R_j(c, E)$.

Nonbossiness (*NB*; Ashlagi et al., 2012) requires that if an agent, by changing his claim, cannot change his own award, then it must be that he cannot change anyone else's award with this change either. It was first introduced by Satterthwaite and Sonneschein (1981) in the context of implementation and social choice theory. The definition we provide here is an adapted version and belongs to Ashlagi et al. (2012). The proportional rule, the *CEA* rule, and the *CEL* rule satisfy *NB*.

Definition 3.12 (Nonbossiness). For each $(c, E) \in \zeta^N$, each $i \in N$, and c_i' such that $R_i(c, E) = R_i((c_i', c_{\text{-}i}), E)$, $R_j(c, E) = R_j((c_i', c_{\text{-}i}), E)$ for all $j \neq i$.

3.3 | Kantian equilibrium

Here, we present a generic definition of the Kantian equilibrium as well as a specific definition using the notation of our bargaining game.

Definition 3.13 (Kantian equilibrium [KE], Roemer, 2010). Consider the normal form game $G = \langle N, (A_i), (u_i) \rangle$ in which every player $i \in N = \{1, 2, ..., n\}$ chooses a strategy from a common strategy set, which is the set of positive real numbers (i.e., $\forall\ i, j \in N : S_i = S_j = \mathbb{R}_{++}$). A strategy profile $s = (s_1, s_2, ..., s_n)$ is a *Kantian equilibrium* of G if $\forall\ i \in N$, $\arg\max_{\alpha \in \mathbb{R}_+} u_i(\alpha s) = 1$.

Definition 3.14 (Kantian equilibrium of the bargaining game). In the bargaining game Γ with endowment value E, a strategy profile (or a claims vector) c is a *Kantian equilibrium* if $\forall\ i \in N \wedge \forall\ \alpha > 0$, $F_i(c, E) \geq F_i(\alpha c, E)$.

In words, a strategy profile c is a Kantian equilibrium of Γ if no agent prefers that every agent change their claims by the same factor $\alpha > 0$.

Note that in some strategic games, for a given Kantian equilibrium strategy profile c, there may be cases in which when all players' actions are scaled by some $\alpha > 0$, some player's payoff decreases while the others' remain constant. However, in the class of bargaining games studied here, as long as the division rule F satisfies efficiency, a decrease in some player's awards vector suggests an increase in some other player's awards vector due to the constant-sum nature of the game. Note that this statement would not necessarily hold if there exists an inefficient Kantian equilibrium. However, we prove in Lemma 4.1(a) that there is no such Kantian equilibrium. Therefore, such Kantian equilibrium strategy profiles do not exist in the class of bargaining games we study. Then, we can update the definition of the Kantian equilibrium in the following manner: In the bargaining game Γ with endowment value E, a strategy profile (or a claims vector) c is a *Kantian equilibrium* if $\forall\ i \in N \wedge \forall\ \alpha > 0$ such that $(\alpha c, E) \in \zeta^N$, $F_i(c, E) = F_i(\alpha c, E)$.

4 | THE RESULTS

Since we will often talk about parallel strategy profiles (or claims vectors) to a given strategy profile, it is useful to explicitly state which strategy profiles we refer to: Given that $c \in C$ is a strategy profile, a strategy profile c' is said to be parallel to c if $c' = \beta c$ for any $\beta > 0$. Now, we first present a result that simplifies our equilibrium analysis and implies that we should only be concerned with strategy profiles that create a bankruptcy problem.

Lemma 4.1. *Let Γ be a bargaining game. Let also the strategy profile $c \in C$ be a Kantian equilibrium, and the strategy profile $\bar{c} \in C$ not to be a Kantian equilibrium of Γ under the estate division rule F and the estate $E > 0$. Then, the following results hold:*

(a) *c is always efficient, that is, $\sum c_v < E$ is not possible.*
(b) *Any claims vector which is parallel to c and still generates a bankruptcy problem is also a Kantian equilibrium of Γ.*
(c) *Any claims vector which is parallel to $\bar{c}$ and still generates a bankruptcy problem is not a Kantian equilibrium of Γ either.*

It follows from Lemma 4.1(a) that we can replace F (denoting an estate division rule) with R (denoting a bankruptcy rule) in our equilibrium analysis. It is worthwhile mentioning here that the Kantian equilibrium of a standard *DD* game leads to the same multiplicity problem as the Nash equilibrium does. In particular, any strategy profile that satisfies $\sum c_v = E$ is clearly a Kantian equilibrium of the *DD* game.

The following lemma shows that there is a tight relationship between the proportional rule and the proportional-increase-in-claims-invariance property (*PICI*). This relationship will be instrumental in our equilibrium analysis.

Lemma 4.2. *A bankruptcy rule R satisfies* PICI *if and only if $R(c, E) = P(c, E)$ for all $(c, E) \in \zeta^N$.*

Proof. First, we show that *PICI* $\Rightarrow$ *P*. Pick any bankruptcy rule R, which satisfies *PICI*. From the definition of the bankruptcy problem, we have $\sum c_v \geq E$. Let α^* be the value which gives $\alpha^* \sum c_v = E$. Then, for the claims vector $\alpha^* c$, everyone gets what they claim (i.e., $\forall\ i \in N : R_i(\alpha^* c, E) = \alpha^* c_i = \frac{E}{\sum c_v} c_i$). By *PICI*, for $\alpha^* > 0$, $R(\alpha^* c, E) = R(c, E)$. So, $\forall\ i \in N : R_i(c, E) = \frac{E}{\sum c_v} c_i$. Second, we show that *P* $\Rightarrow$ *PICI*. Pick any $(c, E) \in \zeta^N$. Then, $\forall\ i \in N : R_i(c, E) = \frac{E}{\sum c_v} ci$. Pick any $\alpha > 0$ such that $(\alpha c, E) \in \zeta^N$. Then, $\forall\ i \in N : R_i(\alpha c, E) = \frac{E}{\sum \alpha c_v} \alpha c_i = \frac{E}{\alpha \sum c_v} \alpha c_i = \frac{E}{\sum c_v} c_i = R_i(c, E)$. Thus, $\forall\ \alpha > 0 : (\alpha c, E) \in \zeta^N, R(\alpha c, E) = R(c, E)$. □

Now, we present an *anything goes* result: Proposition 4.1 shows that if the proportional rule is used in Γ, then any strategy profile that creates a bankruptcy problem is a Kantian equilibrium.

Proposition 4.1. *Let $R = P$ if $(c, E) \in \zeta^N$ in Γ. Then, every strategy profile $c \in C$ such that $(c, E) \in \zeta^N$ is a Kantian equilibrium.*

Proof. By Lemma 4.2, the proportional rule is characterized by *PICI*. So, for each $(c, E) \in \zeta^N$ and for each $\alpha > 0$ such that $(\alpha c, E) \in \zeta^N$, $R(\alpha c, E) = R(c, E)$. Thus, every strategy profile, which creates a bankruptcy problem is a Kantian equilibrium. □

This result highlights a more serious multiplicity issue than the original one in the *DD* game. In that game, every strategy profile c such that $\sum c_v = E$ is a Nash equilibrium. Here in Γ, due to the nature of the Kantian equilibrium, even those strategy profiles for which $\sum c_v > E$ are Kantian equilibria. This is somewhat surprising given the success of the proportional rule in solving the multiplicity issue in the modified *DD* game in Ashlagi et al. (2012). These authors show that, under the proportional rule, there exists a unique Nash equilibrium, in which equal division prevails. We show that there are infinitely many Kantian equilibria, and any division of the estate can be supported in equilibrium. These contrasting results highlight the important differences between equilibrium concepts and the differential impact of *institutions*, which can broadly be understood as the bankruptcy rules that govern behavior. In particular, the Nash equilibrium deals with unilateral deviations, in that every agent has his own idiosyncratic set of claims vectors in $\mathbb{R}^N_+$ for evaluating his counterfactual scenarios. As such, the strong *CMON* property satisfied by the proportional rule plays an important role in bringing the unique Nash equilibrium with an equal division. Also of critical importance is the fact that the agents' strategy sets in Ashlagi et al. (2012) are bounded from above.

On the other hand, the Kantian equilibrium deals with universal deviations. In particular, when considering a deviation, an agent asks the question, "If everyone else also deviates in the same way, would I be better off?" Hence, properties like the *CMON* or *NB*, which allows a change in only one agent's claim, are useless in studying the Kantian equilibrium of Γ. Instead, a universal property like the *PICI* is needed, which characterizes the proportional rule alone. It is the proximity (or the alignment) between the nature of Kantian deviations and the *PICI* that leads to the "anything goes" result here. Put differently, the cooperative element of Kantian equilibrium leads agents to consider the counterfactual claims vectors in a common set (Roemer, 2019). Since this common set is achieved by rescaling the initial claims vector, the proportion of different claims is preserved within the set. Combining this universalization aspect of Kantian equilibrium, which is operationalized by symmetric deviations, with the proportional rule which distributes the estate proportionally with respect to claims, every efficient strategy profile turns into a Kantian equilibrium. So, under the proportional rule, any efficient strategy profile is perceived equivalent to any other strategy in the common set generated by the universalization counterfactual. This leads to the severe indeterminacy result in Proposition 4.1.

Does Γ always admit a Kantian equilibrium? In Proposition 4.2, we show that the existence is guaranteed under a primitive fairness assumption on R. A short note on terminology: In the rest of the paper, any claims vector in which every player claims the same amount will be referred as an *equal claims vector*; and any claims vector which is different from such claims vectors will be referred as an *unequal claims vector*.

Proposition 4.2 (Generic existence result). *Let R be a bankruptcy rule. If R satisfies* ETE, *then Γ has a Kantian equilibrium. Specifically,* ETE *guarantees that if every player claims the same amount, and this claims vector creates a bankruptcy problem, such a claims vector is a Kantian equilibrium.*

Proof. Suppose that a bankruptcy rule R satisfies *ETE*. For a given estate $E > 0$, pick any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$. Clearly, any such $(c, E) \in \zeta^N$ since $\sum c_\nu \geq E$. From the efficiency of bankruptcy rules, we have $\sum R_\nu(c, E) = E$. By *ETE*, $\forall\, i, j \in N : R_i(c, E) = R_j(c, E)$. Then, $\forall\, i \in N : R_i(c, E) = \frac{E}{N}$. Now, for any $\alpha > 0$ such that $(\alpha c, E) \in \zeta^N$, $\forall\, i, j \in N : R_i(\alpha c, E) = R_j(\alpha c, E) = \frac{E}{N}$ by *ETE*. So, there does not exist any α value, which makes someone better off. Thus, any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a Kantian equilibrium. □

For the equal claims vectors, the common set of the counterfactual claims vectors to which the agents can deviate also consists of equal claims vectors. So, agents with identical claims cannot deviate to any unequal claims vector. Combining this with the primitive fairness axiom of *ETE*, equal division is the only outcome within the common set. As a result, satisfying *ETE* guarantees the existence of Kantian equilibrium with equal division in Γ. *CEA,CEL*, and T are some bankruptcy rules that fall in the large family of rules given in this proposition, and these rules have Kantian equilibria *only* in the form described in the generic existence result above. In contrast, when agents consider unilateral deviations, we need *CMON* and *NB*, besides to *ETE*, for equal division to be the unique outcome in all equilibria (Ashlagi et al., 2012).

It is also worthwhile mentioning that Proposition 4.2 provides a sufficient condition for the existence. Sher (2020) shows (Proposition 2) that the Kantian equilibrium does not exist in two person zero-sum games and argues that the result can be generalized to n person constant-sum games. Note that the *ETE* property we assume rules out the assumption the author makes to show nonexistence. In Example A.2 in the appendix, we show that the violations of *ETE* can indeed result in nonexistence of Kantian equilibrium. The following example shows that *ETE* is not a necessary condition.

Example 4.1. Consider a game with four players. $E = 120$, and the bankruptcy rule R distributes E as follows: R mimics P at every claims vector except $c' = (c_1, c_2, c_3, c_4) = (50, 40, 30, 30)$ to which it assigns the awards vector, $(50, 40, 30, 0)$. Hence, it clearly violates *ETE*. Pick a strategy profile $c'' \in \zeta^N$ which is not parallel to c' (i.e., $\nexists \alpha > 0 : \alpha c' = c''$). Then, R behaves like P for c'' and any strategy profile parallel to it, while addressing $(c'', 120)$. It is easy to show that c'' is a Kantian equilibrium. Hence, the result follows.

The following proposition shows that equal-division Kantian equilibrium cannot be induced by certain types of strategy profiles. It is going to be important in our analysis on conditions that guarantee only equal-division equilibria.

Proposition 4.3. *Let R be a bankruptcy rule. A strategy profile $c \in C$ such that c is an unequal claims vector cannot induce an equal-division in a Kantian equilibrium of Γ under R.*

Proof. For a given estate $E > 0$, pick a strategy profile c such that $\exists\, i, j \in N : c_i \neq c_j$ but still $\forall\, i, j \in N : R_i(c, E) = R_j(c, E)$. Suppose for a contradiction that this strategy profile is a Kantian equilibrium. Without loss of generality, assume that $c_i < c_j$ and c_i is a minimal claim. Then, $R_i(c, E) = \frac{E}{N} \leq c_i < c_j$. Note that the other possible cases violate claims boundedness. This suggests that $\sum c_v > E$. Since $c_j > c_i$, we have $c_i = \min_{k \in N} c_k < \frac{\sum c_v}{N}$. Multiplying both sides with E and changing the sides of c_i and $\sum c_v$ guarantees the existence of some $\alpha > 0$ such that $\frac{E}{\sum c_v} < \alpha < \frac{E}{Nc_i}$. Then, $(\alpha c, E)$. Besides, $\alpha c_i < \frac{E}{N}$. So, $R_i(\alpha c, E) < \alpha c_i < E/N = R_i(c, E)$. The loss in the payoff of agent i implies that at least one player should get a higher payoff under $R(\alpha c, E)$. So, at least one player prefers everyone to change their claims by factor α. Therefore, such a strategy profile c cannot be a Kantian equilibrium. □

Is there any *other* Kantian Equilibrium strategy profile (different than the one in the generic existence result) if R satisfies *ETE*? From Proposition 4.1, we know that under P, any strategy profile that generates a bankruptcy problem is a Kantian equilibrium. Since P satisfies *ETE*, the answer to the question above is affirmative. Then, is there any other Kantian equilibria if we restrict our attention to the set of bankruptcy rules, other than the proportional rule, which satisfies *ETE*? In particular, can we have a Kantian equilibrium strategy profile that induces an unequal division in Γ, if R satisfies *ETE*? The following example shows that the answer is, again, affirmative.

Example 4.2 (KE with an unequal division under ETE). Consider the claims problem with $N = 3$ and $E = 90$. The division rule R^* distributes E in the following fashion: If a strategy profile c, given that $(c, E) \in \zeta^N$, is equal to $c^* = (30, 40, 50)$ or it is parallel to c^*, then R^* behaves like P. For any other strategy profile, it behaves like *CEA*. This rule satisfies *ETE*. Hence, under R^*, Γ has a Kantian equilibrium with equal division (from Proposition 4.2). Moreover, it is clearly different from P. In addition to this, c^* and any c which is parallel to c^* are Kantian equilibria as well. Obviously, they are not equal-division equilibria. Note from Example A.1 that there are no other equilibria under *CEA*.

Example 4.2 shows that there can be Kantian equilibria that induce an unequal division even if we restrict our attention to the set of bankruptcy rules, other than the proportional rule, which satisfies *ETE*. Inspired by this observation, we now go further than simply excluding the proportional rule, and define a property that completely rules out proportional divisions, which are responsible for the multiplicity of equilibria, under any unequal claims vector.

Definition 4.1 (No proportionality for unequal claims vectors [NPUC]). For any $(c, E) \in \zeta^N$, a division rule R satisfies *NPUC vectors* if for any unequal claims vector c, $\exists\, i \in N : R_i(c, E) \neq P_i(c, E)$.

Note that for a bankruptcy rule, R, to be different than P, it is enough to have one bankruptcy problem in which the awards vectors of R and P do not coincide. Hence, if R satisfies *NPUC*, then $R \neq P$; but not vice versa. The next proposition shows that there is no Kantian equilibrium that induces an unequal division, under this strong property.

Proposition 4.4. *Let R be a bankruptcy rule that satisfies* ETE *and* NPUC. *Under R, Γ has no Kantian equilibrium other than the ones described in the generic existence result.*

Proof. Suppose that R satisfies *ETE* and *NPUC*. By Proposition 4.2, all equal claims vectors are Kantian equilibria. Take any unequal claims vector c. If R assigns equal division as the awards vector, by Proposition 4.3, such a c cannot be a Kantian equilibrium. The only remaining possibility for c to be a Kantian equilibrium is that R assigns an awards vector, which is different from equal division. By *NPUC*, $\exists\ i \in N : R_i(c, E) > \frac{E}{\sum c_v} c_i$. Then, $\exists\ \alpha > 0 : \frac{R_i(c,E)}{c_i} > \alpha > \frac{E}{\sum c_v}$. This suggests that $(\alpha c, E) \in \zeta^N$. Since $R_i(\alpha c, E) \leq \alpha c_i < R_i(c, E)$, the agent i is worse off under the claims vector αc. So, there must be another agent who gets more under the strategy profile αc, which implies that at least one agent prefers every player to change his claim by α. Thus, the strategy profile c is not a Kantian equilibrium. □

As we mentioned earlier, *NPUC* is a strong property. Many nonproportional bankruptcy rules fail to satisfy it since there exists at least one claims vector at which the awards vector they assign coincides with that of the proportional rule. A well-known example is the Talmud rule. When the sum of claims is equal to $2E$, the awards vector assigned by the Talmud rule coincides with that of the proportional rule. Hence, a natural question is: Can *NPUC* be weakened, yet the same result in Proposition 4.4 still holds? To answer this question, we first present the next property, which is a weakening of *NPUC*.

Definition 4.2 (Weak no proportionality for unequal claims vectors [WNPUC]). For any $(c, E) \in \zeta^N$, a division rule R satisfies *WNPUC vectors* if for any unequal claims vector c, $\exists\ i \in N \wedge \exists\ \alpha^* > 0 : R_i(\alpha^* c, E) \neq P_i(\alpha^* c, E)$ where $(\alpha^* c, E) \in \zeta^N$.

WNPUC holds if, for any bankruptcy problem, there exists a claims vector parallel to the original one and this claims vector still generates bankruptcy, and an agent whose award in the new bankruptcy problem is different from what the proportional rule gives him. As such, it is much weaker than *NPUC*. For example, the Talmud Rule, the Reverse Talmud Rule (Chun et al., 2001), all interior members of the TAL family (Moreno-Ternero & Villar, 2006), and the Reverse-TAL family of rules (Van den Brink & Moreno-Ternero, 2017) satisfy *WNPUC* but fail to satisfy *NPUC*. *CEA* and *CEL* also satisfy *WNPUC*. The next proposition shows that the result in Proposition 4.4 still follows if we replace *NPUC* with *WNPUC*.

Proposition 4.5. *Let R be a bankruptcy rule that satisfies* ETE *and* WNPUC. *Under R, Γ has no Kantian equilibrium other than the ones described in the generic existence result.*

Proof. Suppose that R satisfies *ETE* and *WNPUC*. As is the proof of Proposition 4.4, any equal claims vector is a KE; and any unequal claims vector with equal awards is not a KE. The only remaining possibility for an unequal claims vector c to be a Kantian equilibrium is that R assigns an awards vector, which is different from equal division. Suppose to the contrary that c is a Kantian equilibrium. By *WPNUC*, for c, $\exists\ i \in N \wedge \exists\ \alpha^* > 0 : R_i(\alpha^* c, E) > P_i(\alpha^* c, E)$ where $(\alpha^* c, E) \in \zeta^N$. Then, by Lemma 4.1(b), $\alpha^* c$ is also a KE. This implies that $R_i(\alpha^* c, E) = R_i(c, E) > P_i(\alpha^* c, E) = \frac{Ec_i}{\sum c_v}$. So, $\frac{R_i(c,E)}{c_i} > \frac{E}{\sum c_v}$. Then, $\exists\ \beta > 0 : \frac{R_i(c,E)}{c_i} > \beta > \frac{E}{\sum c_v}$. Thus, $(\beta c, E) \in \zeta^N$, and the payoff of agent i is smaller due to the fact that $R_i(\beta c, E) \leq \beta c_i < R_i(c, E)$. Since one agent experiences a loss in her payoff, there must be another agent who gets more under the claims vector βc. So, at least one player prefers everyone to change their claims by factor β, and c cannot be a Kantian equilibrium, which is a contradiction. Thus, any strategy profile c that does not induce equal division as the awards vector is not a Kantian equilibrium. □

Note that we have two extreme cases: On the one hand, we have the bankruptcy rules satisfying *ETE* and *NPUC* without any unequal claims vector as KE; on the other hand, we have the proportional rule in which every claims vector (inducing bankruptcy) is a Kantian equilibrium. There are also bankruptcy rules satisfying only *ETE* without having Kantian equilibria that induce equal division. However, we also know from Example 4.2 that there can be bankruptcy rules which satisfy *ETE* with an unequal claims vector as their Kantian equilibrium, given that the rule behaves like R^*. Now, the question is the following: Is it possible to construct a transition between R^* and P? For this purpose, we first would like to generalize the case in Example 4.2.

Lemma 4.3. *Let R be a bankruptcy rule, and (c^1, E) be a bankruptcy problem where $c^1 \in C$ is such that not all claims are equal to one another. For any $(c, E) \in \zeta^N, R$ distributes E in the following way: If the strategy profile $c \in C$ is equal to c^1 or it is parallel to c^1, then $R(c, E) = P(c, E)$. For any other strategy profile, R allocates the estate in a way that satisfies* NPUC *and* ETE. *Under R, Γ has some set of strategy profiles which are Kantian equilibria with unequal division. Particularly, this set only involves the strategy profile c^1 and the claims vectors parallel to it.*

Proof. Note that R satisfies *ETE* for any $(c, E) \in \zeta^N$. So, any equal claims vector is a Kantian equilibrium. Take any strategy profile $c \in C$ which is not equal to c^1 and not parallel to c^1. Then, by Proposition 4.4, all the possible Kantian equilibria of Γ have already been mentioned. So, c cannot be a Kantian equilibrium. Now, take any strategy profile c', which is equal to c^1 or parallel to c^1. Then, $R(c', E) = P(c', E)$ and $\forall\, \alpha > 0 : (\alpha c', E) \in \zeta^N, R(\alpha c', E) = P(\alpha c', E) = P(c', E)$. So, $R(c', E) = R(\alpha c', E)$ and c' is a Kantian equilibrium. Since the strategy profile c' is either equal or parallel to c^1, and c^1 is a strategy profile such that not all claims are equal to one another, Γ has a set of strategy profiles which are Kantian equilibria with an unequal division. □

We would like to extend the set of strategy profiles which induce unequal-division Kantian equilibria. To do that, we take any $c^2 \in C$ such that c^1 and c^2 are linearly independent. These strategy profiles are the ones that are not parallel to c^1. Then, we update the bankruptcy rule R as follows: R behaves like the proportional rule when the strategy profile creating the bankruptcy problem is in the span of c^1 and c^2, *span*$(\{c^1, c^2\})$. The following lemma formalizes this idea.

Lemma 4.4. *Let R be a bankruptcy rule, and (c^1, E) and (c^2, E) be bankruptcy problems where $c^1, c^2 \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R} : c^1 = \lambda c^2$). For any $(c, E) \in \zeta^N, R$ distributes E as follows: If the strategy profile $c \in C$ is in the span$(\{c^1, c^2\})$, then $R(c, E) = P(c, E)$, where span$(\{c^1, c^2\}) = \sum_{i=1}^{2} \lambda_i c^i$ such that $\forall\, i \in \{1, 2\} : c^i \in \{c^1, c^2\}$ and $\lambda_i \in \mathbb{R}$. For any other strategy profile, R allocates the estate in a way that satisfies* NPUC *and* ETE. *Under R, in addition to the ones described in the generic existence result, the (unequal) strategy profiles in the span$(\{c^1, c^2\})$ are also Kantian equilibria of Γ.*

Proof. Note that R satisfies *ETE* for any $(c, E) \in \zeta^N$. So, any equal claims vector is a Kantian equilibrium. Take any strategy profile $c \in C$ which is not in the $span(\{c^1, c^2\})$. Then, by Proposition 4.4, all the possible Kantian equilibria of Γ have already been mentioned. So, c cannot be a Kantian equilibrium. Now, take any strategy profile $c' \in span(\{c^1, c^2\})$ and suppose that c' is an unequal claims vector. Then, $R(c', E) = P(c', E)$. Since $c' \in span(\{c^1, c^2\})$, there exists $\lambda_1^*, \lambda_2^* \in \mathbb{R}_+$ such that $c' = \lambda_1^* c^1 + \lambda_2^* c^2$. For any strategy profile c'' which is parallel to c', we have $c'' = \lambda_1' c^1 + \lambda_2' c^2$ where $\lambda_1' = \alpha\lambda_1^*$ and $\lambda_2' = \alpha\lambda_2^*$. This suggests that $c'' \in span(\{c^1, c^2\})$ as well. Then, $\forall\, \alpha > 0 : (\alpha c', E) \in \zeta^N, R(\alpha c', E) = R(c', E)$, and c' is a Kantian equilibrium. Thus, any strategy profile which is in the $span(\{c^1, c^2\})$ is a Kantian equilibrium, and Γ has a set of strategy profiles which are Kantian equilibria with unequal division. □

We can expand the set of strategy profiles which induce unequal-division Kantian equilibria by extending the $span(\{c^1, c^2\})$. To do that, we can pick any strategy profile $c^3 \notin span(\{c^1, c^2\})$ and consider the span of c^1, c^2, and c^3. This ensures that c^1, c^2, and c^3 are linearly independent. Note that since the proportional rule is the only rule under which we have an *anything goes* result, this is the only method which can achieve this purpose. Similarly, we can iterate this process of adding new strategy profiles to the span—such that the existing claims vectors and the recently added ones are linearly independent—and modifying the bankruptcy rule R to behave as the proportional rule for the new span. Since n linearly independent vectors span $\mathbb{R}^n$, R will behave like the proportional rule for all the strategy profiles creating a bankruptcy problem when we have n linearly independent vectors $\{c^1, c^2, ..., c^n\}$.

Proposition 4.6. *Let R be a bankruptcy rule and $(c^1, E), (c^2, E), ..., (c^n, E)$ be bankruptcy problems where $c^1, c^2, ..., c^n \in C$ are linearly independent (i.e., $\nexists \lambda \in \mathbb{R}^n - \{0\} : \lambda_1 c^1 + \lambda_2 c^2 + \dots + \lambda_n c^n = 0$). For any $(c, E) \in \zeta^N, R$ distributes the estate E as follows: If the strategy profile $c \in C$ is in the $span(\{c^1, c^2, ..., c^n\})$, then $R(c, E) = P(c, E)$, where $span(\{c^1, c^2, ..., c^n\}) = \sum_{i=1}^n \lambda_i c^i$ such that $\forall\, i \in \{1, 2, ..., n\}:\ c^i \in \{c^1, c^2, ..., c^n\}$ and $\lambda_i \in \mathbb{R}$. Then, it must be that $R = P$.*

Proof. This result mainly depends on the theorem/proposition which states that if n vectors $c^1, c^2, ..., c^n \in \mathbb{R}^N$ are linearly independent, these sets of vectors span $\mathbb{R}^N$. So, given that $c^1, c^2, ..., c^n \in \mathbb{R}^N$ are linearly independent, $span(\{c^1, c^2, ..., c^n\}) = \mathbb{R}^N$. Since R distributes the estate E as the proportional rule for any $(c, E) \in \zeta^N : c \in span(\{c^1, c^2, ..., c^n\}) = \mathbb{R}^N, R(c, E) = P(c, E)$ for any $(c, E) \in \zeta^N$. □

This result is important in that it shows how we can span the whole space of Kantian equilibrium divisions from "equal-division only" in one extreme to "anything goes" in the other by varying the properties of the bankruptcy rule. Note that the set of results marking the transition between R^* and P are established so that the bankruptcy rule at hand satisfies *NPUC*. It is also possible to obtain the same set of results with any bankruptcy rule which satisfies *WNPUC*. Since the corresponding proofs are almost completely the same, we omitted them. In particular, if the rule satisfies *ETE*, and is almost nowhere proportional, then there can only be equal-division Kantian equilibria. On the other hand, if the rule is proportional everywhere (i.e., it is the proportional rule), then any division can be sustained in Kantian equilibrium (e.g., anything goes). In between these two extremes, there are bankruptcy rules that we constructed above, which induce only some unequal divisions but not others. By varying the domain on which a rule can induce unequal-division equilibria, we can go from one extreme to the other. In a sense, the analysis reported here also serves the role of a comparative static analysis on the equilibrium set, where the variable of interest is the bankruptcy rule.

5 | THE ADDITIVE DEFINITION OF KANTIAN EQUILIBRIUM

Roemer (2015) provides an alternative definition of the Kantian equilibrium, in which deviations are defined in an *additive* fashion. A natural question is: what happens to the equilibrium behavior and the axiomatic properties of the bankruptcy rules we need to use to induce various types of equilibrium divisions (in particular the equal division) under this definition? In this section, we briefly address these questions. We start by giving the alternative definition of the Kantian equilibrium with additive deviations. To avoid confusion, we label this one as Kantian* equilibrium.

Definition 5.1 (Kantian equilibrium—the additive version). In the bargaining game Γ with endowment value E, a strategy profile (or a claims vector) c is a *Kantian* equilibrium* if $\forall\, i \in N \wedge \forall\, \alpha \in \mathbb{R} : (c + \alpha) \in C, F_i(c, E) \geq F_i(c + \alpha, E)$.

Lemma 5.1. *Let Γ be a bargaining game and R be a bankruptcy rule. Let also the strategy profile $c \in C$ be a Kantian* equilibrium under the estate division rule F and $E > 0$. Then, the following results hold:*

(a) *c is always efficient; it is never the case that $\sum c_v < E$. So, we can replace F with R.*
(b) *If R satisfies* ETE, *then Γ has a Kantian* equilibrium. Specifically,* ETE *guarantees that any equal claims vector is a Kantian* equilibrium, as in the multiplicative case.*

This suggests that while agents are engaging with Kantian reasoning in an additive fashion, they should only be considering the strategy profiles which create a bankruptcy problem. In other words, given a strategy profile c, the agents should compare their awards vectors only for the strategy profiles $c + \alpha$ where $\alpha \in \mathbb{R} : (c + \alpha, E) \in \zeta^N$. Besides, the generic existence result that we highlighted in Section 4 continues to hold for the additive definition as well.

Now, let us try to find the counterpart of the proportional rule for Kantian* equilibrium. The following lemma shows that there is a tight relationship between the *CEL* rule and the uniform-increase-in-claims-invariance (*UICI*) property. This relationship will be instrumental in the corresponding equilibrium analysis.

Lemma 5.2. *A bankruptcy rule R satisfies* UICI *if $R(c, E) = CEL(c, E)$ for all $(c, E) \in \zeta^N$.*

Proof. See Marchant (2008) for the proof of this result. □

It is easy to see that the proportional rule, which we characterized above with *PICI* (see Lemma 4.2) fails to satisfy *UICI* since $\frac{E}{\sum c_v} c_i$ is clearly not invariant under uniform changes in claims. The following proposition shows that if the *CEL* rule is used in Γ, then any strategy profile that creates a bankruptcy problem is a Kantian* equilibrium.

Proposition 5.1. *Let R satisfy* UICI *for every $(c, E) \in \zeta^N$ in Γ. Then, every strategy profile $c \in C$ such that $(c, E) \in \zeta^N$ is a Kantian* equilibrium.*

Proof. The proof is similar to that of Proposition 4.1. Since R satisfies *UICI*, for all $(c, E) \in \zeta^N$ and α such that $(c + \alpha, E) \in \zeta^N, R(c, E) = R(c + \alpha, E)$. Therefore, every strategy profile, which creates a bankruptcy problem is a Kantian* equilibrium. □

Note that *UICI* employs the definition "for each $(c, E) \in \zeta^N$ and for each $\alpha > 0$, $R(c + \alpha, E) = R(c, E)$" while Kantian* equilibrium is defined by "$\forall\, \alpha \in \mathbb{R} : (c + \alpha) \in C$, $R_i(c, E) \geq R_i(c + \alpha, E)$." So, it seems as if there is a difference between the two concepts in terms of the possible strategy profiles to which any agent can deviate. For an agent to consider the counterfactual scenario with a negative α value, the sum of claims needs to be strictly greater than the estate value at that particular strategy profile c. Then, there will be some negative real number β which would make the sum of claims equal to the estate under $c - \beta$. Clearly, the set of claims vectors which any agent can potentially deviate to under $c - \beta$ includes the corresponding set under the strategy profile c if we consider only the deviations in which each agent adds some positive amount. However, due to *UICI*, the awards vector of c and $c - \beta$ are equal. So, not only $c - \beta$ being a Kantian* equilibrium implies c being a Kantian* equilibrium, but also if c is a Kantian* equilibrium, $c - \beta$ is a Kantian* equilibrium. Thus, two ways of defining the possible set of deviations are equivalent when the bankruptcy rule satisfies *UICI*.

Corollary 5.1. *Let $R = CEL$ if $(c, E) \in \zeta^N$ in Γ. Then, every strategy profile $c \in C$ such that $(c, E) \in \zeta^N$ is a Kantian equilibrium*.*

This corollary and Proposition 4.1 (also Theorems 1–3 in Ashlagi et al., 2012) highlight the importance of the relationship between the equilibrium concept and the bankruptcy rule (or the axioms it satisfies) used in the game. If the proportional rule is used in Γ, then we obtain Kantian* equilibria that are identical to the ones in Proposition 4.2. We avoid a detailed proof of this result since it simply follows from the fact that P satisfies *No Equal Losses for Unequal Claims* (*NELUC*), a property we will introduce later.

Is there any *other* Kantian* equilibrium of Γ when R satisfies *ETE*? *CEL* satisfies *ETE*, and from Proposition 5.1, we know that when $R = CEL$, any strategy profile that creates a bankruptcy problem is a Kantian* equilibrium; so, the answer to the question above is affirmative. Is there any other Kantian* equilibrium if we restrict our attention to the set of bankruptcy rules, other than *CEL*, which satisfies *ETE*? In particular, can we have a Kantian* equilibrium strategy profile that induces an unequal division? The next example shows that the answer is affirmative again.

Example 5.1 (Kantian* equilibrium inducing unequal division under *ETE*). Consider the bankruptcy problem with $N = 3$ and $E = 90$. The bankruptcy rule R^+ distributes E in the following fashion: If a strategy profile c is equal to $c^+ = (32, 40, 48)$ or it is equal to $c^+ + \alpha$ for some $\alpha \in \mathbb{R} : (c^+ + \alpha, E) \in \zeta^N$, then R^+ behaves like *CEL*. For any other strategy profile, it behaves like P. This rule clearly satisfies *ETE* and, from Lemma 5.1, Γ has a Kantian* equilibrium with equal division under R^+. Note that the shortfall is 30 for c^+. For any $\alpha \in \mathbb{R} : (c^+ + \alpha, E) \in \zeta^N$, the shortfall is equal to $30 + 3\alpha$. Then, in strategy profiles c^+ and $c^+ + \alpha$, after the equal distribution of the shortfall per person, the corresponding awards vectors are $c^+ - 10$ and $c^+ + \alpha - (10 + \alpha)$. So, adding α does not have any effect on how *CEL* distributes E. Thus, for any $\alpha \in \mathbb{R} : (c^+ + \alpha, E) \in \zeta^N$, we have $CEL(c^+, E) = CEL(c^+ + \alpha, E)$, and c^+ is a Kantian* equilibrium inducing unequal division, particularly (24, 30, 36).

In what follows, we define a property on how R treats unequal claims vectors, to eliminate the sort of situations in this example. It stipulates that for every unequal claims vector, there must be some agent who receives more under R than what he would receive under *CEL*.

Definition 5.2 (No equal losses for unequal claims [NELUC]). For any $(c, E) \in \zeta^N$, a bankruptcy rule R satisfies *NELUC* if for any unequal claims vector c, $\exists\, i \in N : R_i(c, E) > c_i - \frac{\sum c_v - E}{N}$.

Note that *CEA* and P both satisfy *NELUC*. Any rule defined as a convex combination of *CEA*, *CEL*, or P also satisfies it. In the following proposition, we show that assuming *NELUC* rules out all the Kantian* equilibria other than the ones in the generic existence result.

Proposition 5.2. *Let R be a bankruptcy rule which satisfies* ETE *and* NELUC. *Under R, Γ has no Kantian* equilibrium other than the ones described under the corresponding generic existence result.*

Proof. Suppose that R satisfies *ETE* and *NELUC*. By Proposition 5.1, any equal claims vector is a Kantian* equilibrium. Take any unequal claims vector c. By *NELUC*, $\exists\, i \in N : \frac{\sum c_v - E}{n} > c_i - R_i(c, E)$. This suggests that for the agent i, $\exists\, \alpha \in \mathbb{R} : \frac{\sum c_v - E}{n} > \alpha > c_i - R_i(c, E)$. Then, $(c - \alpha, E) \in \zeta^N$. Besides, it follows from $c_i - \alpha < R_i(c, E)$ that $R_i(c - \alpha, E) \le c_i - \alpha < R_i(c, E)$ by *claims boundedness*. Then, the agent i is worse off under the strategy profile $c - \alpha$. So, there must at least exist another agent who receives more under the strategy profile $c - \alpha$, compared with c, and there is at least someone who prefers every player to change her claim by $-\alpha$ in additive fashion. Thus, the strategy profile c is not a Kantian* equilibrium. □

6 | CONCLUDING REMARKS

The literature on the applications of Kantian equilibrium almost exclusively focused on public goods and tragedy of commons type games. This is a first attempt to extend its use to bargaining games and it shows that the concept has the potential to deliver new insights in this setting as well.

We studied the Kantian equilibria of a bargaining game, which is a modified version of the well-known *DD* game. We first showed that the Kantian equilibrium of this game exists under a fairly weak assumption (i.e., *ETE*) on the bankruptcy rule used in the game. There are stark differences between the axiomatic properties of bankruptcy rules, which induce equal division of the dollar under the Nash equilibrium and the Kantian equilibrium. For instance, the proportional rule, which induced equal division in a unique Nash equilibrium, leads to infinitely many Kantian equilibria. This result highlights the importance of institutions and the incentives they provide in driving individual behavior; and shows that Kantian behavior (as operationalized in the Kantian equilibrium) does not necessarily lead to egalitarian outcomes. Furthermore, we offered a partial characterization of the family of bankruptcy rules, which induces (i) equal division in Kantian equilibrium and (ii) an anything goes result. Finally, we provided a novel method to construct hybrid bankruptcy rules that can induce any subset of the space of efficient divisions in Kantian equilibrium.

A few words on potential venues of research on the topic are in order. We assumed that all agents are Kantian. Future research may study bargaining/distribution games where the society is composed of both Kantians and Nashians and/or there is uncertainty about the player types. Such models may also set the stage for further study of the evolution of norms in bargaining/estate division games with mixed populations (see Laslier, 2020, for an example in coordination games). Bargaining games with heterogeneous *homo moralis* agents are recently studied in Bartroli and Karagözoğlu (2022), who show that the Nash equilibrium set crucially depends on agents' moral preferences. Finally, we studied multiplicative and additive versions of Kantian equilibrium. Alternatively, other variations of Kantian equilibrium by means of different functional transformations that are neither additive nor multiplicative can be studied (such as ϕ-Kantian equilibrium in Sher, 2020).

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest.

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APPENDIX A

Proof of Lemma 4.1(a). Let $E > 0$ be some given estate, and F be the estate division rule used in Γ. Suppose for a contradiction that there exists a Kantian equilibrium strategy profile $c \in C$, such that $\sum c_v < E$. Then, $E - \sum c_v > 0$. Note that since $\sum c_v < E$, it follows that (c, E) is not a bankruptcy problem, and thus $F_i(c) = c_i$. Now, consider the strategy profile αc, where $\alpha = \frac{E}{\sum c_v}$. Hence, $(\alpha c, E) \in \zeta^N$ is a bankruptcy problem since $\sum \alpha c_v = \alpha \sum c_v = E$. Notice that $\forall\, i \in N : F_i(\alpha c) = \alpha c_i > c_i = F_i(c)$, which implies that every agent $i \in N$ is strictly better off under αc (compared with c). So, there exists some $\alpha > 0$ for c such that every agent prefers every agent to change their claims by α. Thus, c cannot be a Kantian equilibrium. □

Proof of Lemma 4.1(b). Suppose $(c^*, E) \in \zeta^N$ for a given estate $E > 0$, and c^* is a Kantian equilibrium of Γ under some bankruptcy rule R. Now, take any $\beta > 0$ such that $(\beta c^*, E) \in \zeta^N$. We would like to show that the strategy profile βc^* is a Kantian equilibrium of Γ under R (i.e., $\forall\, \sigma > 0 : (\sigma\beta c^*, E) \in \zeta^N, R(\beta c^*, E) = R(\sigma\beta c^*, E)$). Since c^* is a Kantian equilibrium and $\sigma\beta > 0, \forall\, \sigma > 0 : (\sigma\beta c^*, E) \in \zeta^N, R(c^*, E) = R(\beta c^*, E) = R(\sigma\beta c^*, E)$. Thus, if c^* is a Kantian equilibrium, any $\beta > 0$ such that $(\beta c^*, E) \in \zeta^N$ is also a Kantian equilibrium. □

Proof of Lemma 4.1(c). Suppose that $(\bar{c}, E) \in \zeta^N$ for some estate E and bankruptcy rule $R, \bar{c}$ is not a Kantian equilibrium. Then, $\exists\, \alpha > 0 : R(\alpha\bar{c}, E) \neq R(\bar{c}, E)$, where $(\alpha\bar{c}, E) \in \zeta^N$. Now, pick any $\beta > 0$ such that $(\beta\bar{c}, E) \in \zeta^N$ and $\beta \neq \alpha$. We would like to show that $\beta\bar{c}$ is not a Kantian equilibrium of Γ under R either. Suppose to the contrary that the strategy profile $\beta\bar{c}$ is a Kantian equilibrium. Then, $\forall\, \delta > 0 : (\delta\beta\bar{c}, E) \in \zeta^N, R(\beta\bar{c}, E) = R(\delta\beta\bar{c}, E)$. Let $\delta_1 = 1/\beta$ and $\delta_2 = \alpha/\beta$. This implies that $R(\beta\bar{c}, E) = R(\bar{c}, E)$ and $R(\beta\bar{c}, E) = R(\alpha\bar{c}, E)$. But then, $R(\bar{c}, E) = R(\alpha\bar{c}, E)$, which is a contradiction. So, $\beta\bar{c}$ is not a Kantian equilibrium. Hence, the result follows. □

In the following example, we explicitly solve for the Kantian equilibria of Γ under *CEA*.

Example A.1 (*CEA* rule). For each $(c, E) \in \zeta^N$, a way to calculate the *CEA* vector of the given bankruptcy problem is to begin with equal division and adjust the awards vector if an agent's award exceeds her claim. Pick any bankruptcy problem $(c, E) \in \zeta^N$. If c is such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}, (c, E) \in \zeta^N$ since $\sum c_v \geq E$. Note that *CEA* satisfies *ETE*. Hence, $\forall\, i, j \in N : CEA_i(c, E) = CEA_j(c, E)$ and $\forall\, i \in N : CEA_i(c, E) = \frac{E}{N}$. So, everyone receives equal division at such a c since every agent's awards will be less than or equal to her claim. For any $\alpha > 0 : (\alpha c, E) \in \zeta^N$, the strategy profile αc also exhibits $\alpha c_1 = \alpha c_2 = \cdots = \alpha c_N \geq \frac{E}{N}$. Again, by *ETE*, $\forall\, i, j \in N : CEA_i(\alpha c, E) = CEA_j(\alpha c, E) = \frac{E}{N}$. So, no agent prefers that every agent change their claims by the same factor $\alpha > 0$. Thus, any strategy profile c such that $c_1 = c_2 = \cdots = c_N \geq \frac{E}{N}$ is a Kantian equilibrium of Γ under *CEA*.

The other case is that c is a strategy profile in which not everyone claims the same amount (i.e., $\exists\, i \in N : \forall\, l \in N, c_i \leq c_l$ and $\exists\, j \in N : \forall\, l \in N, c_j \geq c_l$ where $c_i < c_j$). If $c_i < \frac{E}{N}$, $CEA_i(c, E) = c_i$ and $CEA_j(c, E) > \frac{E}{N}$. Then, the agent i, whose claim is a minimal claim among all claims, has an incentive to deviate to some $\alpha > 1$ such that $\alpha c_i > \frac{E}{N}$. Clearly, the strategy profile $(\alpha c, E) \in \zeta^N$. Now, under the strategy profile αc, we have $\forall\, l \in N : c_l > \frac{E}{N}$. Since *CEA* starts with giving everyone equal division and every agent's claim is greater than equal division, there does not exist any other way to distribute the loss. So, $\forall\, l \in N : CEA_l(\alpha c, E) = \frac{E}{N} > c_i = CEA_i(c, E)$. Hence, the agent i prefers everyone to change their claims by the factor $\alpha > 1$. Therefore, a strategy profile c in which the agent with a minimal claim declares a claim less than the equal division cannot be a Kantian equilibrium.

Next, consider the strategy profile in which the agent i has a claim greater than the equal division (i.e., $c_i \geq \frac{E}{N}$, and $\forall\, l \in N : c_l \geq c_i$). We have argued that every agent receives equal division under that case. Then, the agent j, whose claim is a maximal claim among all claims, has an incentive to pick some $\beta < 1$ such that $\beta c_i < \frac{E}{N}$. By choosing such a β, he aims to maintain $(\beta c, E) \in \zeta^N$ and keep his claim, c_j, above equal division; while bringing the agent i's claim to a level which is lower than the equal division. In other words, agent j seeks a $\beta < 1$ such that $\sum_{l \in N} \beta c_l \geq E, \beta c_j > \frac{E}{N}$ and $\beta c_i < \frac{E}{N}$. Note that since $c_j \geq c_l$ for any $l \in N$, and $c_j > c_i, \beta c_j \leq \frac{E}{N}$ implies that $(\beta c, E) \notin \zeta^N$. So, for such a β, we need $\sum_{l \in N} \beta c_l \geq E$ and $\beta c_i < \frac{E}{N}$. By the definition of c_i, we have $c_i < \frac{\sum c_l}{N}$. Then, by multiplying both sides with E, we have $E c_i < E \frac{\sum c_l}{N}$, which is equivalent to $\frac{E}{\sum c_l} < \frac{E}{N c_i}$. Notice that any β within this interval (i.e., $\frac{E}{\sum c_l} < \beta < \frac{E}{N c_i}$) ensures that $(\beta c, E) \in \zeta^N$ and under the claims problem $(\beta c, E), \beta c_i < \frac{E}{N} = CEA_i(c, E)$. Now, since $\beta c_i < \frac{E}{N}, CEA_i(\beta c, E) = \beta c_i < \frac{E}{N} = CEA_i(c, E)$. So, under the claims problem $(\beta c, E)$, there will be a loss to distribute among the other agents. The agent j has still a claim βc_j, which is greater than the equal division. This suggests that the award of agent j is now greater than the equal division thanks to the loss of the agent i. So, the agent j prefers everyone to change their claims by the factor $\beta < 1$. Therefore, a strategy profile c in which the agent with a minimal claim declares a claim greater than or equal to the equal division cannot be a Kantian equilibrium.

We can conclude that any strategy profile in which at least two agents have different claims is not a Kantian equilibrium under *CEA*. Hence, in any Kantian equilibrium strategy profile of Γ under *CEA*, every agent declares the same claim.

In Ashlagi et al. (2012), under *CEA*, any strategy profile where every agent claims an amount greater than or equal to the average estate is a Nash equilibrium. Notice here that in Kantian equilibria as well, all claims are greater than equal to the average estate but strategy profiles with unequal claims are not Kantian equilibria.

The following example shows that Kantian equilibrium may not exist under a bankruptcy rule which violates *ETE*.

Example A.2 (Violation of ETE and nonexistence of Kantian equilibrium). Consider the bankruptcy rule R defined as follows: For any claims vector c in which all the claims are equal, that is, $\forall\, i, j \in N : c_i = c_j$ where $i \neq j$, R distributes E in the following manner: (a) If $\sum c_i = E$, then $R_i(c, E) = R_j(c, E) = \frac{E}{n}$ for all $i, j \in N$. (b) If $\sum c_i > 2E$, then R behaves as a priority-based rule, and distributes E in a way that the priority ordering is from lower index to higher index. That is, first, it assigns agent 1 his full claim (if feasible). If there is any amount left, then agent 2 is assigned an award (again, his full claim if feasible or the residual), and so on. (c) If $E < \sum c_i \leq 2E$, then R behaves as a priority-based rule, and distributes E in a way that the priority ordering is from higher index to lower index. That is, first, it assigns agent n his full claim (if feasible). If there is any amount left, then agent $n-1$ is assigned an award (again, his full claim if feasible or the residual), and so on. (d) For any other strategy profile, that is, where all claims are not equal to one another, R behaves like CEA.

This rule clearly violates ETE. Now, we would like to show that R does not have any Kantian equilibrium. From Example A.1, we know that any unequal claims vector is not a Kantian equilibrium. Take any strategy profile c in which all claims are equal. If $\sum c_i = E$, then $c_i = c_j = \frac{E}{n}\ \forall\ i, j \in N$. In that case, at least the first agent, agent 1, and the last agent, agent n, have incentives to pick $\alpha > 1$. For instance, under the strategy profile $3c$, agent 1 is better off. So, any strategy profile which makes the sum of claims equal to E is not a Kantian equilibrium. If $\sum c_i > 2E$, like the preceding instance, then agent 1 receives more than equal division and agent n receives 0 since for any $j \in N$, $c_j > \frac{2E}{n}$. However, the agent n has an incentive to deviate to some $\alpha < 1 : E < \alpha \sum c_i \leq 2E$ so that he receives some positive awards under the strategy profile αc. So, any strategy profile such that $\sum c_i > 2E$ is not a Kantian equilibrium. Lastly, consider the case where $E < \sum c_i \leq 2E$. Then, agent 1 receives less than the equal division. By deviating to a large enough $\alpha > 1$, that is, any $\alpha > 1 : \alpha c_1 > E$, agent 1 can get the estate E completely. So, there exists some α which would make agent 1 better off under the strategy profile αc; and any c such that $E < \sum c_i \leq 2E$ is not a Kantian equilibrium. Thus, R violates ETE and does not have any Kantian equilibrium strategy profile.