HİLAL TAŞKAN OPTIMAL DISCRETIONARY MONETARY POLICY UNDER PERSISTENT-TRANSITORY CONFUSION OVER COST SHOCK BILKENT UNIVERSITY 2023

OPTIMAL DISCRETIONARY MONETARY POLICY UNDER PERSISTENT-TRANSITORY CONFUSION OVER COST SHOCK

A Master's Thesis

by
HİLAL TAŞKAN

Department of
Economics
İhsan Doğramacı Bilkent University
Ankara
July 2023

To my mother

OPTIMAL DISCRETIONARY MONETARY POLICY UNDER PERSISTENT-TRANSITORY CONFUSION OVER COST SHOCK

The Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

HİLAL TAŞKAN

In Partial Fulfillment of the Requirements for the Degree of
MASTER OF ARTS IN ECONOMICS

THE DEPARTMENT OF
ECONOMICS
İHSAN DOĞRAMACI BİLKENT UNIVERSITY
ANKARA

July 2023

OPTIMAL DISCRETIONARY MONETARY POLICY UNDER PERSISTENT-TRANSITORY CONFUSION OVER COST SHOCK

By Hilal Taşkan

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Ali Hakan Kara

Advisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Burçin Kısacıkoğlu

Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Bedri Kamil Onur Taş

Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

Refet Soykan Gürkaynak
Director

ABSTRACT

OPTIMAL DISCRETIONARY MONETARY POLICY UNDER
PERSISTENT-TRANSITORY CONFUSION OVER COST SHOCK

Taşkan, Hilal
M.A., Department of Economics
Supervisor: Prof. Dr. Ali Hakan Kara

July 2023

In this study, I model the debated statements of Federal Reserve regarding transitory inflation in New Keynesian context and imperfect information about cost shock, where it is formulated as sum of unobserved persistent and transitory components. Specifically, I consider the case, in which policy maker and private agents are uninformed about the components and subject to a gradual recognition from observation using a Kalman filter. Policy maker solves the inference problem to learn the components, and, in turn, private agents rely on central bank speeches to learn policy maker's inferences. Then, based on policy maker's estimation, private agent's problem and, so New Keynesian relations are formulated. The responses with filtered variables incorporate progressive learning. In particular, when the true realization of the shock and its perception is persistent, the increase (decline) in inflation (output), therefore in interest rate is less compared to that of perfect information responses, creating a relatively desirable outcome for the policy maker. If instead shock is perceived as transitory and communicated as such, which was the case in Fed's statements, the response of inflation, hence the required policy response is significantly more pronounced. This leads me to conclude that it may not be the best response to announce that shock is transitory while it is not the case. Then, I continue studying welfare loss under discretionary policy when the estimation and the actual realization is persistent. I find that loss under imperfect information with an inferred persistent shock is smaller relative to that of perfect information when variance of the noise increases, and shock becomes less persistent. Lastly, I extend the model by introducing a signal on transitory component.

Keywords: Persistent-Transitory Shock, Imperfect Information, Learning, Monetary Policy

ÖZET

GEÇİCİ-KALICI MALİYET ŞOKLARI AYIRT EDİLEMEDİĞİNDE TAKDİRE DAYALI OPTİMAL PARA POLİTİKASI

Taşkan, Hilal
Yüksek Lisans, İktisat Bölümü
Tez Danışmanı: Prof. Dr. Ali Hakan Kara

Temmuz 2023

Bu çalışmada ABD Merkez Bankası'nın geçici enflasyon ile ilgili çokça tartışılan açıklamalarını Yeni Keynesyen bağlamında ve maliyet şokunun geçici ve kalıcı gözlenemeyen şokların toplamı olduğu eksik bilgi ortamında modelliyorum. Özellikle, politika yapıcıların ve özel sektörün bileşenler hakkında bilgi sahibi olmadığı ve bileşenlerin Kalman filtresi kullanılarak gözlemden yola çıkılarak kademeli olarak öğrenildiği durumu ele alıyorum. Politika yapıcı, bileşenleri öğrenmek için kestirim problemini çözmek durumundadır ve buna karşılık, özel sektör, politika yapıcının tahminini öğrenmek için merkez bankası iletişiminden faydalanmaktadır. Ardından, politika yapıcının tahminine bağlı olarak, özel sektörün optimizasyon problemi ve dolayısıyla Yeni Keynesyen ilişkileri belirlenir. Etki-tepki grafikleri bilgi eksikliğinin bir sonucu olarak kademeli öğrenme kaydetmektedir. Eğer şok kalıcıysa ve tahmini de kalıcı olarak gerçekleşiyorsa, enflasyonun ve dolayısıyla faizin (çıktı açığının) tam bilgi modeline kıyasla daha az yükseldiğini (düştüğünü) gözlemliyorum, ki bu politika yapıcı için daha tercih edilebilir bir sonuçtur. Bunun yerine, şok geçici olarak tahmin edilirse, enflasyonun tepkisi, dolayısıyla gerekli politika tepkisi önemli ölçüde daha fazladır. Bu durum, şok kalıcıyken geçici olarak adlandırmanın en iyi cevap olamayabileceği sonucuna varmamı sağlamaktadır. Daha sonra, çalışmama ihtiyati para politikası kapsamındaki refah kaybını şok kalıcıyken ve tahmini de bu şekildeyken inceleyerek devam ediyorum. Geçici şokun varyansı arttığında veya şok daha az kalıcı hale geldiğinde, çıkarsanan kalıcı şok ile eksik bilgi altındaki kaybın, mükemmel bilgiye göre daha az olduğunu bulmaktayım. Son olarak, modelimi geçici bileşene ilişkin sinyal tanıtarak genişletiyorum.

Anahtar Kelimeler: Geçici-Kalıcı Şoklar, Öğrenme, Eksik Bilgi, Para Politikası

ACKNOWLEDGMENTS

I would like to express my sincere gratitude to my advisor, Hakan Kara for his guidance during this process. I am deeply thankful for his support and trust during my undergraduate and masters studies. I appreciate for his kind patience especially in formulating the research question.

I am deeply grateful to Burçin Kısacıkoğlu for his continuous guidance and support for always being available for my questions. Without his help, it was definitely impossible to finish writing this thesis. I will always be indebted to him.

I am thankful to Refet Gürkaynak and Sang Seok Lee for their insightful comments during my presentations and support in various ways.

I would like to extend my inmost appreciation and respect to my entire family for never saying no to me and for their unlimited love and support throughout my life.

Last but not the least, I am appreciative of the support of my dearest friend Gaye Aşkın. I cannot overstate my gratitude to her for always being beside me and keeping me mentally sound. I am also immensely grateful to Çağla İplikçi. This stressful task and two years in masters would have been more difficult without her friendship and endless care and support.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The discussion on mixing permanent and transitory components is considerable. The focus on this literature is to incorporate situations in which a stochastic shock is formulated as the sum of a non-stationary process and a transitory white noise component none of which are ever reliably known. That is, only the realized level of the shock is observed and this observation with its all past trajectory does not permit agents to separate the two components. Notice that the existence of transitory shock exclusively leads to this information problem. In other words, it is exactly the transitory shock that prevents agents from backing out the permanent shock from the observation about the level.

Then what actually differentiates the two is that the transitory shock is expected to disappear in the next period, which is the defining feature of a white noise component provided that there is no other transitory shock hitting the economy. Contrary to transitory shocks, the permanent-random walk component stays at its past value. This, in turn, imply that the expected value of a change in permanent component vanishes. Note that both permanent and transitory components can be replaced by a stationary process to describe situations, in which agents are subject to untangling persistent and transitory shocks at any period in time.

The standard evaluation of persistent shocks is that long run values of the fundamentals may be subject to a shift after such a shock. In contrast, a transitory shock may lead to a deviation from the long run values, but are not expected to change the target value. This leads policy responses to be subject to a similar understanding. That is, transitory shocks seem to be associated with more benign and not lasting policy responses relative to the response to a persistent shock.

The brief description above turns out to be quite relevant given transcripts of Chair Powell's Press Conference in 2021. That is, the close reading of those statements indicates that upward pressure in inflation was initially described as one-time, transitory, transient, or temporary increases in prices. Then, it was announced that they retired this word and accepted that it became more pronounced, permanent or persistent. This seemingly confused and mixed statements about the components of the shock is then explained through the existence of uncertainty that policy makers faced when it comes to identifying transitory and persistent shocks separately.

This discussion leads me to think that there is a quite obvious information problem that the policy maker faces. However, I may rethink the possibility that it was the best response for Fed to say that shock is transitory as it was not the proper time to tighten and transitory shocks are considered to be requiring little or no response. Regardless of such a possibility, it turns out to be interesting for central bankers to think about to what extent officials release their estimations or beliefs about the components of the shock.

Now, in this paper, I ask what the optimal discretionary monetary policy is when there is persistent-transitory confusion over a cost-push shock. In particular, the response of monetary policy under discretion is analyzed by incorporating imperfectly informed policy maker and private agents about transitory or persistent changes in cost shock. Here, I adopt imperfect information framework and assume no superior information on central bank or private agents. This is because it seems arguably more realistic to introduce informational friction that is intrinsically created by confusion over the components of the shock, which cannot be backed-out with certainty by any agent. Given its nature, it seems difficult to motivate what makes one side to distinguish its components. Therefore, all the information sets are symmetric incomplete in this paper.

Given this incompleteness, agents are subject to signal extraction problem to form and update their beliefs about the components of the shock. I now assume that it is only the policy maker that is involved in this inference problem. Then, the policy maker communicates its inferences and they become available for private agents, as this was the case in FOMC statements. This approach implicitly regards Fed as sufficiently credible and reliable. This turns out be an innocuous assumption when initial phases of pandemic is in consideration.

Therefore, in this paper, I study the implications of misperception about persistent component of the cost shock in the context of standard New Keynesian model in the first part. I firstly analyze how responses of inflation and output when unobserved states are gradually learned are shaped. Note that all the comparisons are with respect to responses under perfect information. I find underestimated and hump-shaped responses when there is a persistent shock and policy maker perceives that shock is persistent. If there is a transitory shock and policy maker still thinks that shock is persistent, then the complete depletion of the shock requires time even though the shock is truly iid by its nature. If policy maker thinks that shock is transitory and it is indeed transitory, then the expected immediate responses and dissipating nature of the shock is observed.

This evaluation, in turn, leads me to conclude that when there is a persistent shock, agents can learn this in two ways. Firstly, they may estimate that shock is persistent, and be subject to progressive recognition of this persistent component. Secondly, they may misconceive the shock and think that it is transitory. However, as they realize that shock does not vanish next period, they may think that there is a sequence of transitory shock in every period.

Then, I discuss a welfare question under discretionary monetary policy. I evaluate the loss when shock is estimated and announced to be persistent relative to the case in which it is actually persistent. That is to consider the loss under imperfect and perfect information about the persistent component. This turns out to be of interest since what matters for agents is to learn about the persistent component, which is impossible given the addition of white noise. Also, impulse-response analysis shows that when the perception and true realization of the shock is persistent, responses of variables of interest are underestimated and hump-shaped. My main findings in this part illustrate that the performance of welfare loss under imperfect information relative to its perfect information counterpart depends only on the parameters that describe the cost shock process, which are variance of the shocks and the degree of persistency. Then, simulating those parameters one by one indicate that loss with inferred persistent component performs better when persistence of the shock decreases, and variance of the noise increases.

Then, I extend the model via the addition of signal on transitory component. This is particularly to motivate Fed officials initial statements about the shock. That is, as the transcripts show, inflation developments are initially regarded as largely driven by transitory factors. Then, what makes policy makers to perceive most of the movements as temporary is of interest. Indeed, in the first part, I capture this via low Kalman gain. That is, low learning parameter implies that observed changes in the realized level of the cost shock is considered as transitory, and there is progressive learning by the accumulation of new information.

I check whether the addition of signal helps improving the responses of inflation and output relative to perfect information and imperfect information with a single observation. Surely, this part allows me to check the responses with respect to a noise shock.

1.1.1 Literature Review

This work is related to a few strands of the literature. I begin with the literature on the expression of shocks with two independent random components, that is permanent component with a noise. Muth (1960) illustrates that when shocks are decomposed as permanent/random walk and a transitory/white noise process, the current prediction of future values of the shock uses all past observations, hence optimal expectations turn out to be adaptive and subject to slow learning process. This is important to note especially in forward-looking New Keynesian setting, which is the case in this paper. Continuing with another seminal empirical work, Stock and Watson (2007) present a model of inflation, in which it is formulated as the sum of a permanent stochastic trend component and a serially uncorrelated transitory component.

Multiple studies have incorporated persistent-transitory confusion into productivity growth to explore the impact of imperfect information on household behaviour. Blanchard et al. (2013) express productivity driven by two unobserved permanent and transitory shock in DSGE (New Keynesian) model. In addition to the observation on the realized level, they introduce a signal on permanent component and let consumers solve the signal extraction problem. This is different from my way of modelling the shock process as both shocks are themselves are persistent here. Differently from mine, Foerster and Sarte (2020) study the a similar productivity process in one-sector neoclassical growth model. They also show that the stability of the model dynamics and signal extraction problem that household faces are two independent problems. In other words, solution still follows Blanchard and Kahn (1980), allowing me to adopt this as my solution concept as well.

Proceeding with asset pricing models with imperfect information, Gilchrist and Saito (2008) analyze the gradual learning of technology growth with two aforementioned components for the design of an interest rate rule to indicate the benefits to responding to the asset price gap based on the information structure. In their model, both the policy maker and private sector need to learn asset price changes as a result of movements in trend growth. As opposed to mine, they let both the policy maker and private sector engage in signal extraction problem by assuming certainty equivalence to check cases, in which policy maker/private agents are un/informed about two components separately.

Then, I continue with a sequence of papers focusing on the impact of central bank communication, transparency, credibility and signaling in order for me to study Fed's statements on a shock, which I model it as a cost shock. While Baeriswyl and Cornand (2018) concentrate on optimal response of a central bank with respect to a cost-push shock in relation to its disclosure (i.e., signal on the monetary instrument), and Tamura (2018) uses the Bayesian approach to model a variety of information policies allowing central banks to create information asymmetries against the private sector; I do not allow learning from each other's actions and decisions and work with symmetric information sets. Similar to my evaluation of FOMC statements about persistent-transitory shocks, Baker and Lam (2022) use a text-based measure to show a decreasing credibility of Fed's signal and increasing inflation expectations as a result of persistency in US inflation.

Lastly, my work is related to studies on monetary policy and incomplete information in New Keynesian (NK) core. Several papers introduce zero lower bound constraint in NK environment to create information problem, while my model has it via inseparability of persistent shocks from the transitory ones. Lee (2020) studies the duration of ZLB episode and the size of deflation and output gap losses when information on demand and supply shocks is incomplete only on the part of private agents and policy rate is constrained at zero. While information on discount rate and marginal cost shocks is incomplete only on the part of policy maker with policy rate approaching zero, Gust et al. (2015) show that optimal discretionary policy reacts to signals about the increase in the equilibrium real rate relatively less than it would when far from ZLB. Importantly, Erceg and Levin (2003) combine imperfect credibility and imperfect information on long run inflation target of central bank in the following way: They introduce an optimal filtering problem on the side of private agents to separate transitory movements from the persistent ones to forecast future inflation target. Here, information problem is only on the part of private agents.

1.1.2 Narratives

To motivate my research question further, below I provide some evidence from Chair Powell's press conference transcripts. On January, 2021, Chair Powell (2021) has made the following statements about transitory inflation:

On January 27, 2021,

... —*because people will be enthusiastic that the pandemic is over, potentially, and that that could also create some upward pressure on inflation. Now, again, we would see that as something likely to be transient and not to be very large. In both cases, we don't see those as either lasting or particularly large... —we would view as very likely to be transient effects on inflation*

On March 17, 2021,

Beyond these base effects, we could also see upward pressure on prices if spending rebounds quickly as the economy continues to reopen, particularly if supply bottlenecks limit how quickly production can respond in the near term. However, these one-time increases in prices are likely to have only transient effects on inflation.

...

I would note that a transitory rise in inflation above 2 percent, as seems likely to occur this year, would not meet this standard

When it comes to explain average inflation targeting framework: *We want inflation at 2 percent, and not on a transitory basis. And we want inflation on track to be moderately—to run moderately above 2 percent for some time.*

On April 28, 2021,
However, these one-time increases in prices are likely to have only transitory effects on inflation.

On June 16, 2021,
As these transitory supply effects abate, inflation is expected to drop back toward our longer-run goal, and the median inflation projection falls from 3.4 percent this year to 2.1 percent next year and 2.2 percent in 2023.

On July 28, 2021, Chair Powell is asked to define the word transitory:
The concept of "transitory" is really this: It is that the increases will happen. We're not saying they will reverse. That's not what "transitory" means. It means that the increases in prices will happen, so there will be inflation but that the process of inflation will stop so that—so that there won't be further—when, when we think of inflation, we really think of inflation going up year upon year upon year upon year. So what, what I mean by "transitory" is just something that doesn't leave a permanent mark on the inflation process.

On November 3, 2021, Chair Powell eventually retires the word:
So "transitory" is a word that people have had different understandings of. For some, it carries a sense of "short lived," and that's, you know, there's a real time component—measured in months, let's say. Really, for us, what "transitory" has meant is that if something is transitory, it will not leave behind it permanently or very persistently higher inflation. So that's why we, you know, we took a step back from "transitory." We said "expected to be transitory," first of all, to show

uncertainty around that—we've always said that, by the way, in other contexts; we just hadn't done it in the statement—but also to acknowledge, really, that it means different things to different people.

CHAPTER 2

THE MODEL

2.1 Households

The economy has identical and infinitely-lived households. The representative household maximizes the following lifetime utility:

$$\mathcal{U} = E_o \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{2.1}$$

$$U(C_t, N_t) = ln(C_t) + \nu ln(1 - N_t), \tag{2.2}$$

where E_o denotes the conditional expectation operator, β is the discount factor satisfying $0 < \beta < 1$. The period utility depends on consumption C_t, and separable leisure term $1 - N_t$ with ν indicating the disutility from working.

Now, the household faces the following budget constraint in period t stating that its disposable income must cover the spending on consumption goods and net purchases of financial assets:

$$C_t + \frac{B_t}{P_t} = \frac{W_t N_t}{P_t} + (1 + R_{t-1})\frac{B_{t-1}}{P_t} + \frac{\Pi}{P_t}, \tag{2.3}$$

and

$$B_{-1} > 0$$

where P_t is the price level, B_t is the nominal bond holding, W_t is the wage, R_{t+1} is the nominal interest rate between period t and t+1, and Π_t is the profit from firms in period t. The first-order conditions yield the familiar Euler equation for consumption connecting the marginal cost of foregoing one unit of consumption in the current period ($\Lambda = C_t^{-1}$) to the expected discounted marginal benefit:

$$C_t^{-1} = \beta(1 + R_t)E_t\left\{C_{t+1}^{-1}\frac{P_t}{P_{t+1}}\right\}, \tag{2.4}$$

and the intratemporal labor consumption condition indicating that consumption and leisure are utility substitutes hence equating the marginal rate of substitution between consumption and leisure to the real wage:

$$\psi(1 - N_t)^{-1} = C_t^{-1}\frac{W_t N_t}{P_t}. \tag{2.5}$$

2.2 Firms and Price Setting

2.2.1 Final Goods Production

The final good Y_t is aggregated by constant elasticity of substitution technology of the Dixit-Stiglitz form in the following way:

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2.6}$$

where $Y_t(i)$ denotes the continuum of differentiated intermediate goods for all $i \in [0, 1]$ and $\epsilon > 0$ is the elasticity of substitution for intermediate goods. Moreover, final good producers are perfectly-competitive in markets for goods and factors. Therefore, they minimize the cost of production taking the price $P_t(i)$ of each intermediate good $Y_t(i)$ as given, which leas to the following demand for intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{2.7}$$

Lastly, the price of the final good that is the marginal cost of production, determines the aggregate price index of the economy given that perfect competition drives the finished goods-producing firm's profits to zero in equilibrium:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}\, di \right)^{\frac{1}{1-\epsilon}}. \tag{2.8}$$

2.2.2 Intermediate Goods Production

Each intermediate good is produced by a continuum of monopolistically competitive firms, indexed by $i \in [0, 1]$. Notice that this notation basically says that good i is a production of firm i. Now, a typical intermediate goods firm faces three constraints. First of all, every firm i works with a simple linear production technology given as:

$$Y_t(i) = Z_t N_t(i), \tag{2.9}$$

where Z_t is the stochastic total factor productivity (TFP) common to all firms with $E\{Z\} = 1$.

Second of all, firm i faces a demand function that directly associates with aggregate demand Y_t and varies inversely with the price $P_t(i)$ of $Y_t(i)$ as given by equation (7). Note that as the market for factors of production is perfectly competitive, each firm chooses labor $N_t(i)$ taking the wages as given. Thus, the first-order condition for this static problem simply suggests that each firm faces the same real marginal cost equaling to real wage that accounts for productivity:

$$MC_t = \frac{W_t P_t}{Z_t}. \tag{2.10}$$

Now, note that the intermediate good sector is subject to monopolistic competition, hence the intermediate goods firms have market power to set prices that maximize their expected discounted profits. Therefore, the third constraint is as a result of price stickiness. I consider one of the most frequently used approaches to model this rigidity, which is the Rotemberg (1982) a quadratic cost of adjusting nominal prices between periods, measured in terms of the final good Y_t and given by:

$$\frac{\phi}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - 1\right)^2 Y_{t+j} \tag{2.11}$$

Given the constraints of each firm i, I now provide the details of the price-setting problem below.

2.2.3 Firm's Price Setting Problem: Rotemberg (1982)

Following Oh (2020), firm i's profit maximization problem based on Rotemberg pricing subject to demand is as follows:

$$\max_{P_{t+j}(i)} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \left[\left(\frac{P_{t+j}(i)}{P_{t+j}} - \frac{MC_{t+j}}{P_{t+j}} M_{t+j}\right) Y_{t+j}(i) - \frac{\phi}{2}\left(\frac{P_{t+j}(i)}{P_{t+j-1}(i)} - 1\right)^2 Y_{t+j}\right], \tag{2.12}$$

$$s.t. \quad Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t,$$

The first-order condition corresponding to the optimal price with the imposition of symmetric equilibrium i.e., $Y_t(i) = Y_t$ and $P_t(i) = P_t$ for all i as all intermediate goods firms face the same profit maximization problem is as follows:

$$(\epsilon - 1) = \epsilon mc_t M_t - \phi \pi_t(\pi_t + 1) + \phi \beta E_t \pi_{t+1}(\pi_{t+1} + 1). \tag{2.13}$$

where $\Lambda_{t,t+j} \equiv \beta^j \frac{C_{t+j}}{C_t}$ is the stochastic discount factor for real payoffs of the households, and ϕ is the adjustment cost parameter which determines the degree of nominal price rigidity. Now, M_t, which is of the interest of this paper shocks to the intermediate goods-producing firms' desired markups of price over marginal cost; and in equilibrium, it can be expressed as cost-push shocks in the New Keynesian model introduced by Clarida et al. (1999). Based on this equivalence, I interchangeably use marginal cost, markup and cost shocks to refer to the same thing. In this model, this shock is formulated as the sum of a persistent and transitory component that follows:

$$ln(M_t) = ln(\tau_t) + \delta_t, \tag{2.14}$$

$$ln(\tau_t) = \rho ln(\tau_{t-1}) + \upsilon_t. \tag{2.15}$$

where $0 \leq \rho < 1$, and δ_t and υ_t are normal variates with zero means and variances σ_δ^2 and σ_υ^2, respectively.

2.3 Information Structure

I consider the cases in which both the private sector and the policy maker are endowed with perfect information first and later imperfect information regarding the components of the cost-push shock. That is, under full information, agents observe both the transitory component δ_t and the persistent component τ_t separately. Under imperfect information, they only observe M_t or the sum of the two $(\tau_t + \delta_t)$ and this observation does not allow them to differentiate two components. In other words, they need to form beliefs and learn whether changes are persistent or transitory from the information about what has been the realization of the cost shock M_t.

Note that the first case describes a situation where both agents (i.e., private sector and cb) have symmetric full information set whereas in the latter it is symmetric incomplete. It is incomplete in the sense that their observation does not reveal the current value of persistent and transitory components of the shock. It is symmetric in the sense that both agents have the same observation, that is to say that both know the same thing. Such incompleteness of information requires agents to infer the unobserved states.

In this model, this inference or information processing is characterized by the signal-extraction problem to be solved by the policy maker. That is, it is the central bank who engages in inference problem even though both are endowed with incomplete information. Then, the policy maker communicates its inferences or filtered variables as it is not an uncommon practice for central banks to publish or announce such information (see Introduction).

This assumption may seem restrictive. However, the availability of the central bank inferences to private agents turn out to be quite useful as it limits the degree of higher order beliefs (see Carboni and Ellison (2011)) and prevents the complications that may arise in partial information with forward-looking variables (see Svensson and Woodford (2003)). This assumption turns out to be reasonable as well since both agents share the same information set, which is a singleton consisting of M_t only, it suffices for one agent to solve the extraction problem. Lastly, the assumption that information sets are symmetric seems arguably more realistic as it may be more difficult to motivate a superior information set on one side given the extent of the uncertainty to making anticipations in the period that is modeled in this paper.

Below, I firstly present the perfect information solution as a benchmark to compare it with imperfect information model.

2.3.1 The Linearized System under Perfect Information

Under perfect information, agents can distinguish persistent component from transitory component of the exogenous markup shock, M_t. That is, agents observe the two shocks separately in addition to the realized level. Let $z_t = ln(\frac{Z_t}{Z})$, measure the percentage deviation of each stationary variable from its steady-state level (i.e., $\pi_t = ln(\pi_t)$, $m_t = ln(M_t)$ etc.). Hence, the model is characterized by an intertemporal IS equation, and New Keynesian Phillips curve, which themselves are log-linear approximation of first-order conditions of household and firm optimization problems, an instrument rule given by Taylor rule and cost shock process:

$$x_t = E_t x_{t+1} - [R_t - E_t \pi_{t+1}], \tag{2.16}$$

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(\epsilon - 1)}{\phi} mc_t + \frac{(\epsilon - 1)}{\phi} m_t,$$

$$R_t = \theta \pi_t, \tag{2.17}$$

$$m_t = \tau_t + \delta_t, \tag{2.18}$$

$$\tau_t = \rho \tau_{t-1} + \upsilon_t, \tag{2.19}$$

Now, under full information, agents know what drives this cost shock. More specifically, they know the true driving process, hence face the following expectations-augmented aggregate supply relations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \kappa \tau_t, \tag{2.20}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \kappa \delta_t, \tag{2.21}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \kappa (\tau_t + \delta_t). \tag{2.22}$$

Here, π_t denotes inflation, E_t the expectations operator conditional on information in period t, x_t the output gap (i.e., the deviation of actual output from natural or flexible price output), and it is seen that a firm's optimal price depends on real marginal cost, which is assumed to be proportional to output gap, $mc_t = 2x_t$ and m_t a stochastic shock term expressed as the sum of transitory white noise component and persistent component following stationary AR(1) process with $0 \leq \rho < 1$, and δ_t and v_t normal variates with zero means and variances σ_δ^2 and σ_v^2, respectively. One also notes that $\kappa = \frac{(\epsilon-1)}{\phi}$ and ϵ measures the elasticity of substitution for intermediate goods, ϕ is the adjustment cost parameter which determines the degree of nominal price rigidity and $\theta > 1$, satisfying Taylor principle.

2.3.2 Policy maker's Inference Problem under Imperfect Information

Under imperfect information, $(\tau_t + \delta_t)$ must be inferred from the observation m_t. I assume that Fed update its inferences based on the steady-state Kalman filter and private agents follow publicly available documents and cb official's speeches to learn the filtered variables. Now, from the perspective of Fed, the measurement and state-transition equations are as follows:

$$(Observation)\; m_t = \tau_t + \delta_t, \tag{2.23}$$

$$(State - Transition)\; \tau_t = \rho\tau_{t-1} + v_t, \tag{2.24}$$

Let $E[\tau_t|m_t, m_{t-1}, ...]$ denote $\tau_{t|t}$.

$$\tau_{t|t} = \lambda m_t + (1-\lambda)\rho\tau_{t-1|t-1}, \tag{2.25}$$

where the gain, λ, is given by $\frac{\psi-(1-\rho^2)+\psi\sqrt{(1-\rho^2)^2\frac{1}{\psi^2}+1+\frac{2}{\psi}+2\rho^2\frac{1}{\psi}}}{2+\psi-(1-\rho^2)+\psi\sqrt{(1-\rho^2)^2\frac{1}{\psi^2}+1+\frac{2}{\psi}+2\rho^2\frac{1}{\psi}}}$ and ψ measures the signal-to-noise ratio: $\psi \equiv \frac{\sigma_v^2}{\sigma_\delta^2}$. Note that the Kalman updating equation can be restated as

$$\tau_{t|t} = \rho\tau_{t-1|t-1} + \lambda(m_t - \tau_{t-1|t-1}),$$

$$\tau_{t|t} = \rho\tau_{t-1|t-1} + \lambda a_t,$$

where a_t is the forecast error. Now, given $\tau_{t|t}$, the inference about the shock to the transitory component
$\tau_{t|t} = E[\delta_t|m_t, m_{t-1}, ...]$, is given by

$$\delta_{t|t} = m_t - \tau_{t|t}, \tag{2.26}$$

and the inference about the shock to the persistent component
$v_{t|t} = E[v_t|m_t, m_{t-1}, ...]$, is given by

$$v_{t|t} = \tau_{t|t} - \rho\tau_{t-1|t-1}. \tag{2.27}$$

2.3.2.1 Belief Responses to a Shock to Persistent and Transitory Components

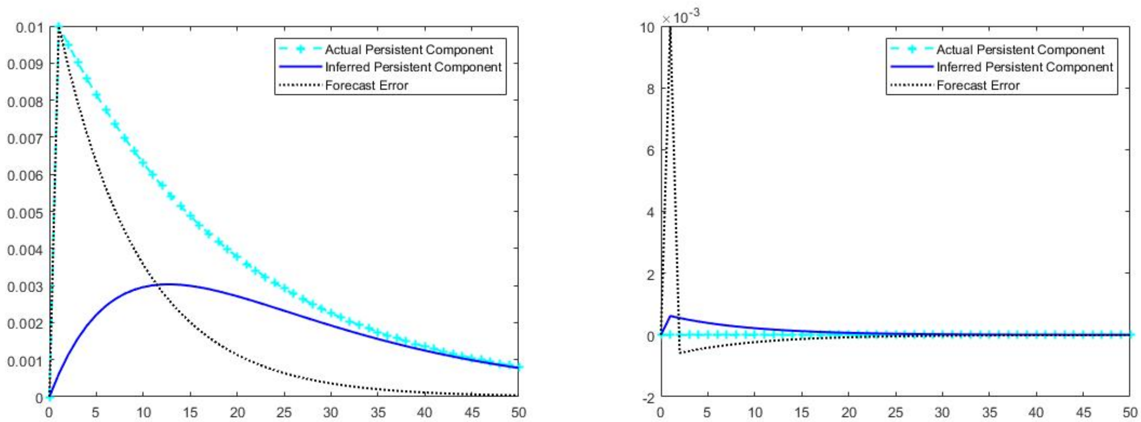

Figure 1: Belief Responses
Note: The straight line is the inference about the persistent component of the cost shock: $E[\tau_t|m_t, m_{t-1}, ...]$ and the dashed light blue line $- - +$ indicates the response of the realization of the persistent component τ_t. The dotted line is the forecast error.

Following Gilchrist and Saito (2008), I now discuss how Fed's inferences and actual components respond to a 1 percent increase in the persistent and transitory components separately. To discuss the responses above, the parameter values related to the shock process used in these experiments (described in the following section) imply that Fed initially infers that observed changes in cost shock is mostly transitory, even when these movements are in fact generated by a shock to the persistent component. Notice that this figure also illustrates the response of the forecast error as it is key in driving decisions leading Fed to adjust its behavior, based on the magnitude and direction of the error, determined by the Kalman gain. It is observed that forecast error persists since Fed treats shocks mostly transitory, that is to be overly optimistic. This is to reflect FOMC statements that initially regarded inflation developments driven by the factors that are expected to be temporary.

Now, the left panel shows the response of estimated persistent component with respect to a 1 percent persistent shock. Although the shock introduced here moves the persistent component, Fed initially reads most of the observed increase as transitory. Gradually, as Fed accumulates more observations, it revises their inferences

The right panel of Figure 1 illustrates the response of Fed's inferred persistent component with respect to a positive shock to actual transitory component. Although the shock moves the transitory component only and has no impact on actual persistent component, Fed initially attribute some of the changes to be persistent. Over time, learning takes place and belief response dissipates. Now, since Fed accurately assesses the nature of the shock in this case, forecast error dies out as well. However, the mistake in this case is associated with a negative forecast error as there is still a weight on persistent component meaning that it is -though very little- likely that shock is persistent. Now, next section presents how inferences interact with otherwise standard New Keynesian relations.

2.3.2.2 Fed's Communication on the Estimated States and the Linearized System

I present two cases in which Fed communicates its estimation about each component of the shock. In the first case, Fed says shock is perceived to be persistent i.e., $m_t = \tau_{t|t}$ whereas in the second case Fed says shock is expected to be transitory i.e., $m_t = \delta_{t|t}$. Hence, agents face the following AS relations:

$$E_t\pi_t = \beta E_t\pi_{t+1} + \kappa E_t x_t + \kappa\tau_{t|t}, \tag{2.28}$$

$$E_t\pi_t = \beta E_t\pi_{t+1} + \kappa E_t x_t + \kappa\delta_{t|t}, \tag{2.29}$$

$$E_t\pi_t = \beta E_t\pi_{t+1} + \kappa E_t x_t + \kappa(\tau_{t|t} + \delta_{t|t}). \tag{2.30}$$

The existence of $E_t x_t$ and $E_t\pi_t$ in equations above illustrates the information problem that is faced.

2.3.2.3 Calibration

I follow a fairly standard calibration of preferences and the price-setting structure. The discount factor is $\beta = 0.99$. The elasticity of substitution between goods is $\epsilon = 11$. The Rotemberg price adjustment cost parameter is $\phi = 7.7250$. This parameter is indeed implied by Calvo probability of keeping prices unchanged, which is parameterized as 0.75. I set the standard deviation of the shock to the transitory component at $\sigma_\delta = 0.01$, the standard deviation of the shock to the persistent component at $\sigma_v = 0.001$, and the AR(1) coefficient on the persistent component of cost-shock at $\rho = 0.95$. This parametrization mean that the signal-to-noise ratio in section 2.3.2 is $\psi = 0.01$. Then, the Kalman gain that is consistent with the above choices is $\lambda = 0.06138$. As aforementioned, low gain captures Fed's communication. Lastly, coefficient on inflation in monetary policy rule is set to be 1.5.

2.3.3 Impulse Responses

In this section, I report impulse response functions to cost-shocks to explore the role of imperfect information and its effects on output, inflation, and the policy rate. That is, the responses of variables of interest are shown when the relevant driving process is given by equations (2.25)-(2.27) in imperfect information model and (2.23)-(2.24) in perfect information. Note that for reference, I plot impulse responses for a version of the model with the true states. Also, all shocks are one percent standard deviation in size.

2.3.3.1 One-Time Shock to Persistent Component of Cost Shock

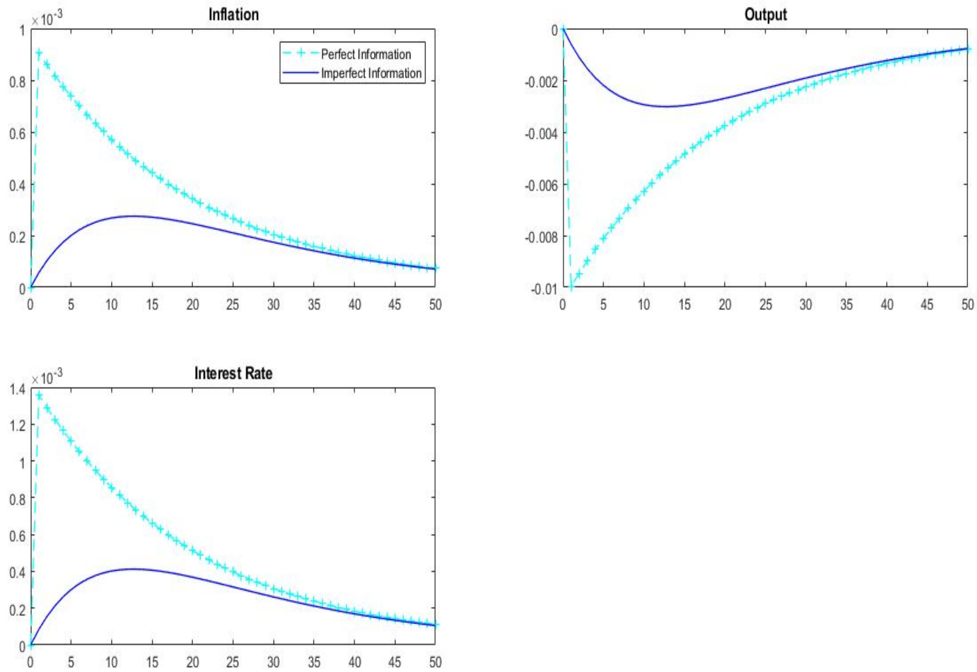

Figure 2: Response to a persistent shock

I begin by discussing the responses when agents face NKPCs in equations (2.28) and (2.20). That is to analyze the responses through filtered persistent component $\tau_{t|t}$ in NKPC first and then true persistent component τ_t with respect to a shock to actual persistent component, v_t. Note that this is the case in which Fed accurately assesses the nature of the shock. In other words, the shock is actually to persistent component and it is perceived to be persistent. Now, the persistent shock leads inflation to gradually rise after the shock and attain its peak several quarters later, and then gradually decline in imperfect information. Similarly, responses of output gap declines gradually and displays a delayed peak. The response of monetary policy follows a similar path as well. This means that the responses are hump-shaped under imperfect information, which reflects learning about (and hence asymptotes to) the true level over time. That is, the hump-shape is generated from adopting imperfect information through a progressive learning of the persistence in the cost shock. Now, one notes that the responses of perceived variables are not only hump-shaped, but also underestimated relative to the actual realizations as shown by perfect information results, because when information is imperfect, it is suspected that the shock is transitory even if the nature of the shock is evaluated correctly.

Now, Figure 3 illustrates the case in which Fed perceives that the cost-shock is solely driven by i.i.d. transitory shock hence agents continue with a filtered transitory component in NKPC, however the shock is realized on the true persistent component. That is, NKPC relations in (2.20) and (2.29) are of interest. Notice that perfect information NKPC has the true persistent component. The response of inflation under imperfect information is relatively more pronounced whereas output gap declines relatively less.

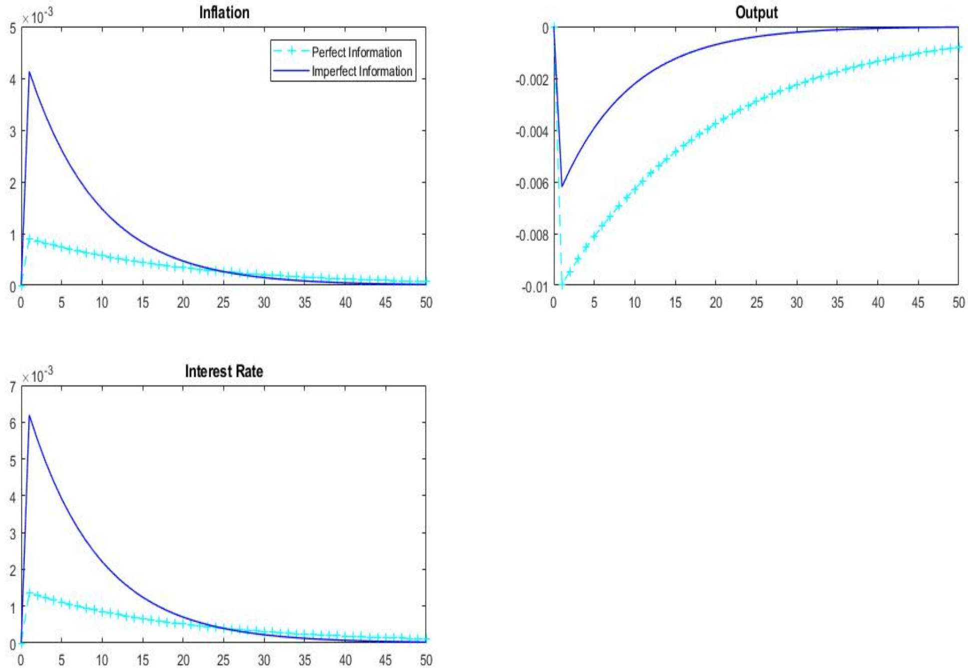

Figure 3: Response to a persistent shock

In order to indicate the intermediate case, I present when NKPC is formulated by the sum of filtered persistent and transitory components ($\tau_{t|t} + \delta_{t|t}$) in the imperfect information model and the sum of actual components ($\tau_t + \delta_t$) in the perfect information model while the shock is realized on persistent component, v_t. Figure 4 indicates that while the increase in inflation, hence the response of monetary policy is more pronounced in imperfect information relative to response under perfect information, the decline in output gap is slightly less under imperfect information.

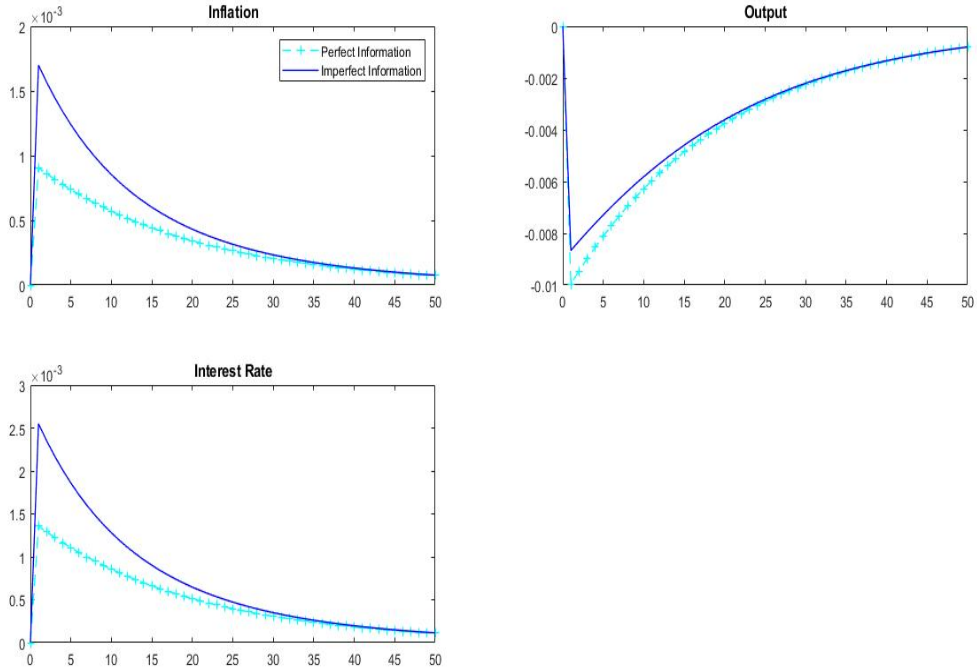

Figure 4: Response to a persistent shock

2.3.3.2 One-Time Shock to Transitory Component of Cost Shock

Now, Figure 5, which depicts the case where transitory component (estimated and actual) is in NKPC and there is a transitory shock. Given the implied calibration of Kalman gain, and Fed's accurate perception of the shock, the slight difference has been generated between the responses of the two models. That is, the expected immediate increase (decrease) in inflation and dissipating nature of the shock are observed in all responses. One also notes that the responses under both cases attain their peaks at the same quarter with a slightly more pronounced increase in inflation and policy rate and decrease in output gap under imperfect information. Secondly and more importantly, inflation and output gap change the sign under imperfect information. That is, after the peak, inflation (output) remains negative (positive) for some quarters meaning that Fed has made a negative forecast error.

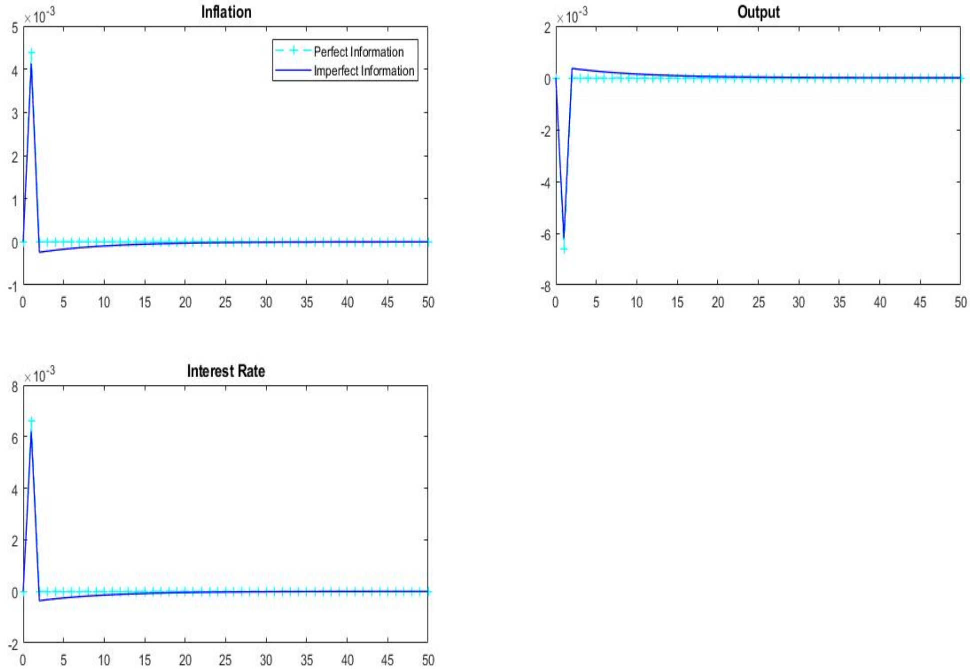

Figure 5: Response to a transitory shock

Now, Figure 6 indicate the case where shock is perceived to be persistent while it is actually transitory. In this scenario, inflation generates almost no response (i.e., it is indeed $5x10^{-}5$ and output gap declines very slightly and continues to be negative for some more period before depleting completely.

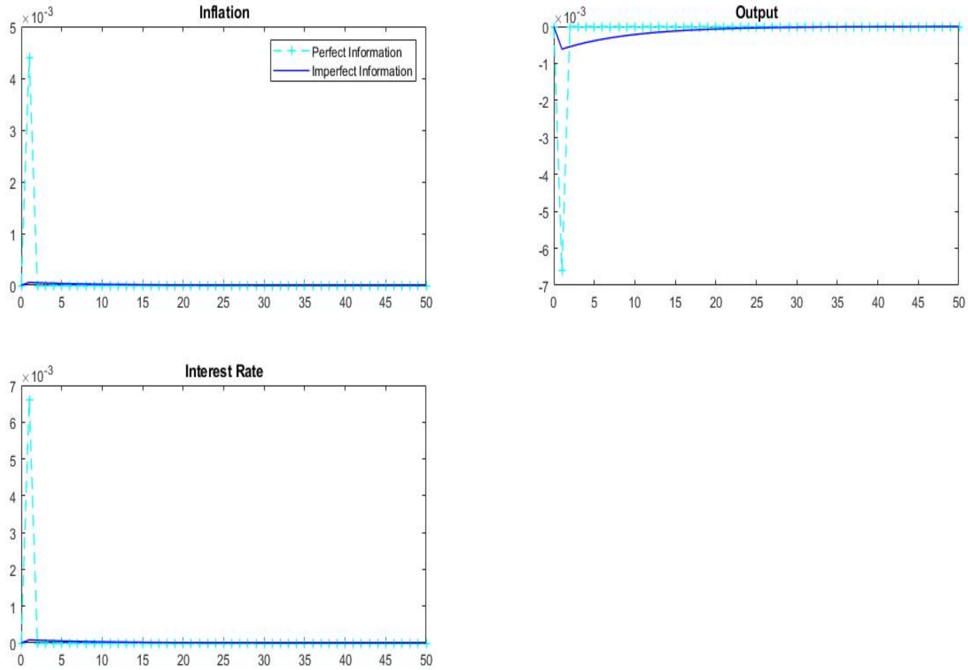

Figure 6: Response to a transitory shock

In the intermediate case, where I have $(\tau_{t|t} + \delta_{t|t})$ in the imperfect information model and $(\tau_t + \delta_t)$ in the perfect information model, one-time and i.i.d. nature of the shock is observed in Figure 7. The response of inflation and policy rate is under perfect information is more asserted in this case while output gap declines more in imperfect information.

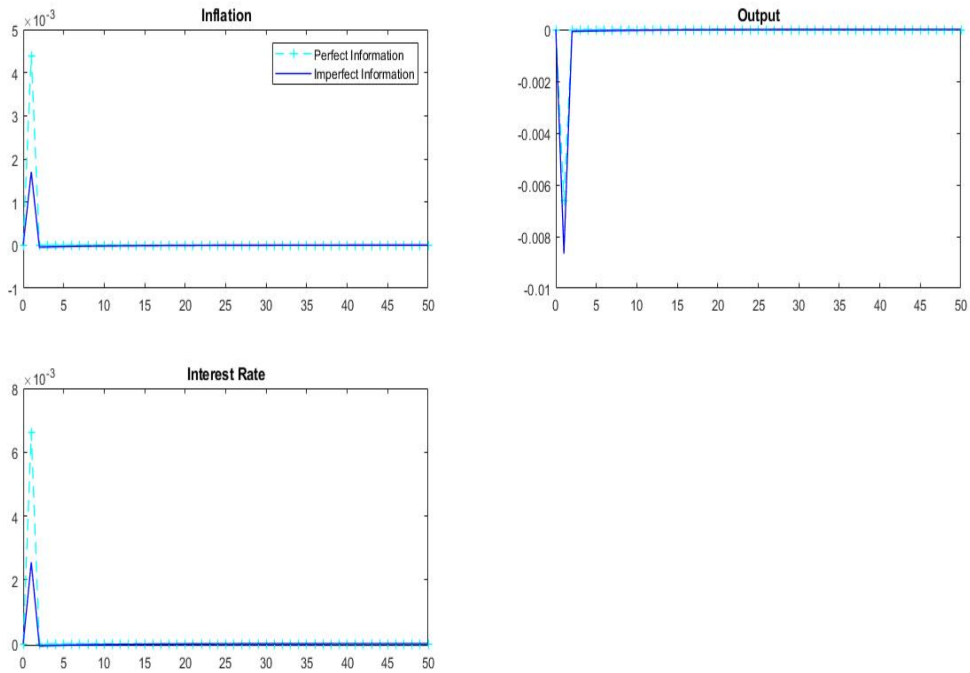

Figure 7: Response to a transitory shock

2.4 Optimal Discretionary Policy

I now discuss the optimal monetary policy when the policy maker is unable to credibly commit to future policy actions. I assume a loss function of the kind:

$$L_t = E_t \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \omega x_{t+j}^2) \tag{2.31}$$

where β is the discount factor, ω is a trade-off parameter and E_t represents the expectation conditional on time t information. Following McCallum (1999), the minimal state variable (MSV) solution under discretion given that $\tau_{t|t}$ is the only state variable is as follows:

$$E_t\pi_t = \beta E_t\pi_{t+1} + \kappa E_t x_t + \kappa\tau_{t|t},$$

$$\pi_t = \frac{-\omega}{\kappa} x_t,$$

so the conjectured solution is of the form

$$\pi_t = \gamma_1 \tau_{t|t},$$

$$x_t = \gamma_2 \tau_{t|t}.$$

Since $E_t x_{t+1} = \gamma_1 \rho \tau_{t|t}$ in this case, the MSV solution is given by

$$\pi_t = \frac{\omega\kappa}{\kappa^2 + \omega(1-\beta\rho)} \tau_{t|t}, \tag{2.32}$$

$$x_t = \frac{-\kappa^2}{\kappa^2 + \omega(1-\beta\rho)} \tau_{t|t}. \tag{2.33}$$

Now, implementing optimal policy:

$$R_t = (1 + \frac{\kappa}{\omega}(\frac{1-\rho}{\rho}))\frac{\omega\kappa\rho}{\omega(1-\beta\rho)+\kappa^2}\tau_{t|t}.$$

Now, when the estimated transitory shock appears in NKPC, under discretion $\delta_{t|t}$ is the only state variable in the followings:

$$E_t\pi_t = \beta E_t \pi_{t+1} + \kappa E_t x_t + \kappa \delta_{t|t},$$

$$\pi_t = \frac{-\omega}{\kappa} x_t,$$

so the conjectured solution is of the form

$$\pi_t = \zeta_1 \delta_{t|t},$$

$$x_t = \zeta_2 \delta_{t|t},$$

Since $E_t x_{t+1} = 0$ in this case, the MSV solution is given by

$$\pi_t = \frac{\omega\kappa}{\kappa^2 + \omega} \delta_{t|t}, \tag{2.34}$$

$$x_t = \frac{-\kappa^2}{\kappa^2 + \omega}\delta_{t|t}. \tag{2.35}$$

Now, implementing optimal policy:

$$R_t = \frac{\kappa^2}{\omega + \kappa^2}\delta_{t|t}.$$

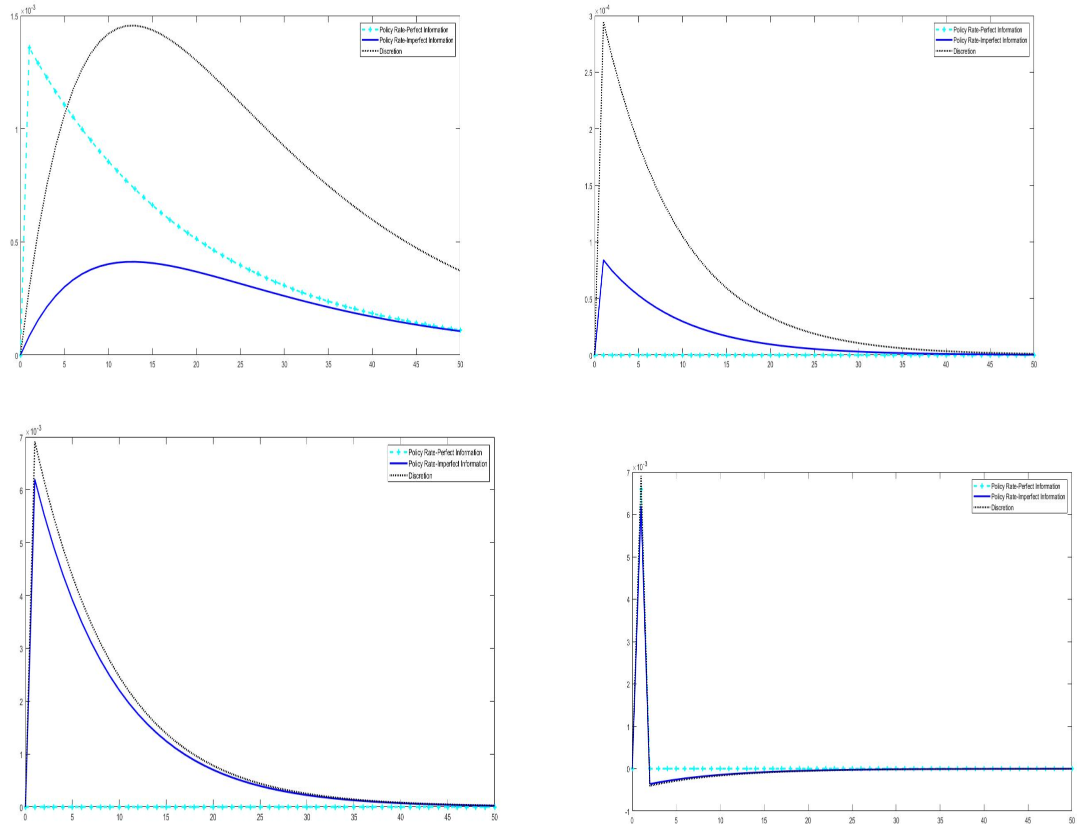

Figure 8: Policy under Discretion

Notes: Graphs in the first row plot optimal policy with filtered persistent component and other two show optimal policy with filtered transitory component with respect to persistent and transitory shocks respectively. Policy rate-perfect information is $1.5\pi_t$ whereas policy rate-Imperfect information is $1.5\pi_{t|t}$

This implementation indicate that response of discretionary policy is more pronounced relative to simple interest rate rule with perfect and imperfect information. In particular, response to a persistent shock proposes a delayed but long-lasting policy reaction under discretion. This reflects that when policy maker progressively learns that shock is persistent, it slowly tightens the policy based on this learning process.

2.4.1 Policy Evaluation

The standard approach to assess monetary policy performance is to compare average values for the period loss function, i.e. values of the unconditional expectations of the loss function in (2.31). That is,

$$L_i = E[L_{t,i}] = E[\pi_t^2] + \omega E[x_t^2]$$

for $i \in \{imperfect, perfect\}$.

2.4.1.1 Loss with Actual vs Estimated Persistent Shocks

Given the welfare criteria, the loss under imperfect information presenting the case in which Fed perceives the shock to be persistent i.e., filtered persistent component is the state is given by

$$L_{t,imperfect} = \frac{\omega\kappa^2(\omega+\kappa^2)}{(\omega(1-\beta\rho)+\kappa^2)^2}\left[\frac{\lambda^2(\sigma_\delta^2(1-\rho^2)+\sigma_v^2)}{(1-(1-\lambda)^2\rho^2)(1-\rho^2)} + \frac{2\lambda^2(1-\lambda)\rho^2\sigma_v^2}{(1-(1-\lambda)^2\rho^2)(1-(1-\lambda)\rho^2)(1-\rho^2)}\right], \quad (2.36)$$

and the loss under perfect information, i.e., the actual persistent component is the state is simply given by

$$L_{t,perfect} = \frac{\omega\kappa^2(\omega+\kappa^2)}{(\omega(1-\beta\rho)+\kappa^2)^2}\left[\frac{\sigma_v^2}{1-\rho^2}\right]. \quad (2.37)$$

These two say that trade-off parameter ω, slope of NKPC κ, discount factor β, the persistency parameter ρ and the variance of persistent component $\frac{\sigma_v^2}{1-\rho^2}$ affect the magnitude of absolute losses in both cases. Furthermore, the absolute loss under imperfect information is affected by Kalman gain λ and variance of the transitory shock σ_δ^2. Now, in order to have a meaningful comparison, I first define the welfare gap as

$$Gap = L_{t,perfect} - L_{t,imperfect},$$

Then, following Sauer (2007), I define the relative loss RL, which is the object of interest in the following way:

$$RL = \frac{L_{t,perfect}}{L_{t,imperfect}} - 1 \tag{2.38}$$

Note that $L_{imperfect} < L_{perfect}$ if and only if $RL > 0$. Note also that the relative loss is independent of ω, κ, and β and depends only on the parameters that describe the cost shock process. Now, I successively change the parameter values for ρ, σ_v^2 and σ_δ^2 as they are the ones that affect the relative performance. One also notes that another candidate is to change the Kalman gain, λ. However, λ is implied by shock variances and AR(1) coefficient, changing λ simultaneously change them preventing me from conducting comparative statics. Now, the table below reflects that the shock is perceived mostly to be transitory as this parametrization implies a low Kalman gain and also shows what values are assigned to the parameters that are kept constant when one of them is simulated.

ρ	σ_δ	σ_v
0.95	0.01	0.001

Table 1: Parameter Values

2.4.1.2 Simulation Results

This part helps evaluating the relative loss when both the policy maker and private agents face imperfect information. Notice that this evaluation can be interpreted as the case where Fed actually knows the true nature of the shock, but prefers to release the estimated or filtered versions instead of the true realizations. Hence, the following results connect the impact of communication with the welfare analysis. Therefore, I am particularly interested in cases in which loss under imperfect information is relatively smaller. That is to check when RL > 0. Below I present the simulation results for estimated and actual persistent components given that it provides the policy maker with relatively more desirable inflation-output gap trade-off as discussed previously.

Correlation of shocks: ρ

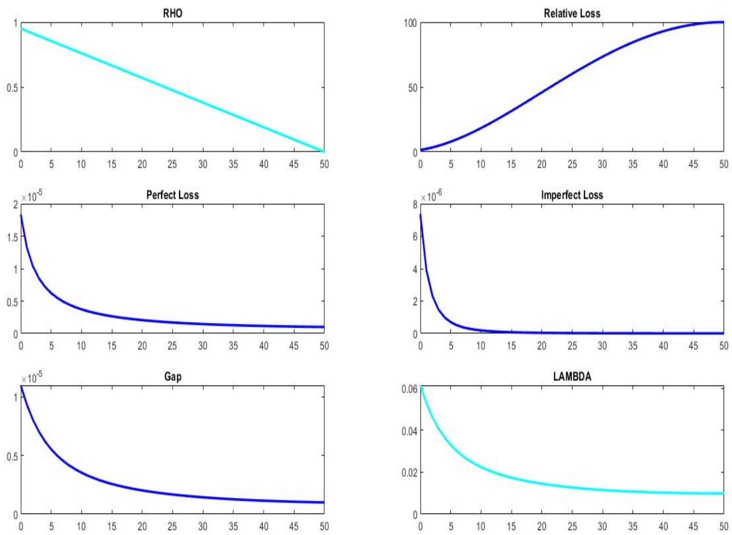

Figure 9: Variation of ρ, $L_{perfect}$ vs $L_{imperfect}$

I present the results for the variation of the persistency parameter by checking the loss from the perfect information model relative to imperfect information, RL. A positive (negative) value of RL means that the loss from the perfect information is greater (smaller) than the loss under imperfect information, while an increase (decrease) in RL implies a relative gain (loss) from imperfect information. This figure illustrates that RL increases with decreasing ρ, that is, starting the parameter from 0.95 and reducing its value consecutively implies that $L_{imperfect}$ gains relative to $L_{perfect}$. This is primarily due to the fact that the parameter for serial correlation enters equation (2.36) in multiple times through the existence of Kalman gain λ. Note that the gain is increasing in ρ, which can be easily verified in section 2.3.2. Note also that equation (2.36) is also increasing in Kalman gain. Therefore, decreasing ρ reduces the value of gain as well leading loss under imperfect information to decrease much faster relative to loss under perfect information. In fact, this result presents that as the shock becomes less persistent, hence being qualitatively more similar to transitory shock, Fed perceives the shock more correctly, as it has the tendency to regard it as transitory.

Standard Deviation of Shock to Transitory Component: σ_δ

Now, I increase the parameter value of standard deviation of shock to the transitory component. Note that the initial value is set such that Kalman gain λ is arbitrarily close to 1. Then, increasing the noise, that is to increase σ_δ successively, reduces the value of the gain and this is reflected in Figure 10. Observe firstly that change in σ_δ leads no change in $L_{perfect}$ as it does not appear in equation (2.37). Secondly, as in the previous case, σ_δ appears in multiple times in (2.36) and absent Kalman gain, $L_{imperfect}$ is increasing with σ_δ. Therefore, the net impact on $L_{imperfect}$ depends on Kalman gain as well. Now, it is seen that λ diminishes with increasing noise, i.e., σ_δ. Apparently, despite the effect of Kalman gain, relative loss increases with σ_δ, so $L_{imperfect} < L_{perfect}$.

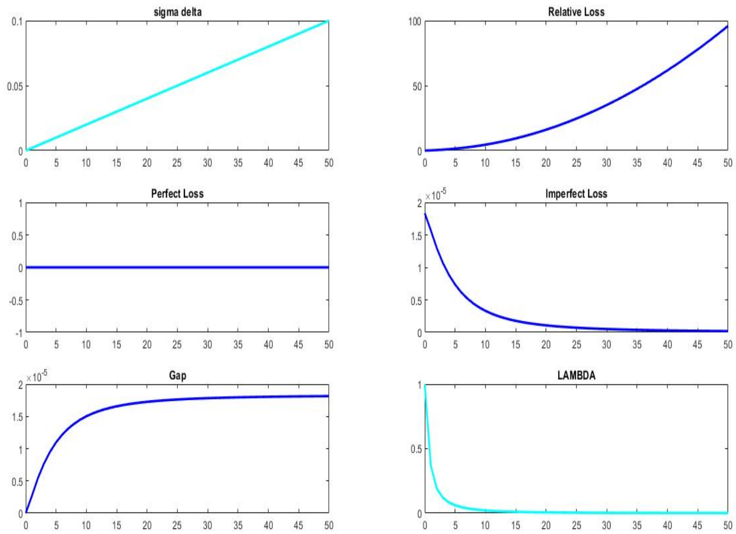

Figure 10: Variation of σ_δ, $L_{perfect}$ vs $L_{imperfect}$

Standard Deviation of Shock to Persistent Component: σ_v

The last simulation is to increase the parameter for standard deviation of shock to persistent component. Note that I set the upper bound on its value such that Kalman gain λ is below 1. This is because once λ is exactly 1, $L_{t,imperfect}$ in (2.36) reduces to

$$L_{t,imperfect} = \frac{\omega\kappa^2(\omega+\kappa^2)}{(\omega(1-\beta\rho)+\kappa^2)^2}\left[\sigma_\delta^2 + \frac{\sigma_v^2}{1-\rho^2}\right].$$

This leads to the coincidence of responses of $L_{imperfect}$ and $L_{perfect}$. Now, the figure shows that increasing σ_v means increasing the signal-to-noise ratio given by ψ, which in turn increases Kalman gain fast given the parameter values in Table 1. Given the speed of convergence of Kalman gain, loss under imperfect information quickly converges to loss under perfect information, leading RL to converge to zero. Consequently, as $L^{imperfect}$ increases in σ_v and λ and λ increases with σ_v, two model quickly yield the same loss, leading relative performance to be inconclusive.

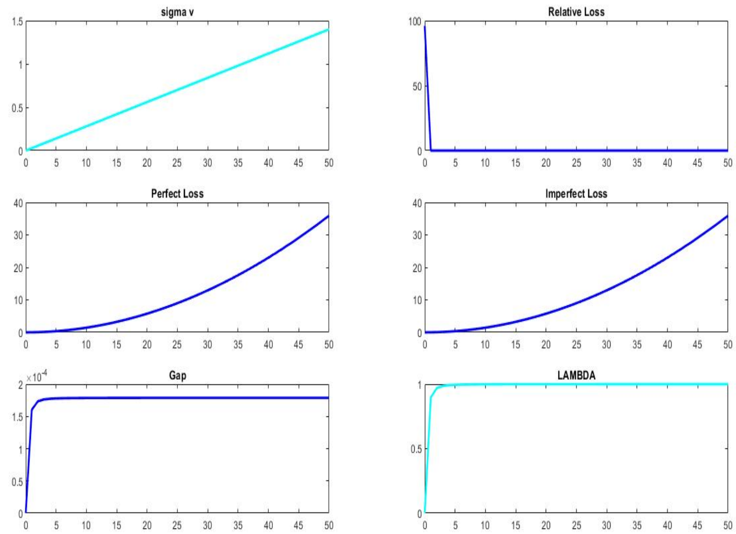

Figure 11: Variation of σ_v, $L_{perfect}$ vs $L_{imperfect}$

2.5 Extension: Signal on Transitory Component

Now, given that Fed has initially perceived the observed changes in inflation as largely driven by transitory factors, and this perception has taken place in considerable number of FOMC statements, one may suspect that Fed has received some signal on transitory component of the cost shock m_t, hence has known more about the unobserved component δ_t. Therefore, I introduce a signal of the kind in this section:

$$s_t = \delta_t + \nu_t$$

This additional observation helps motivating what led Fed to say more about the transitory component in those statements. Note importantly that it is again only the policy maker that solves the inference problem. Now, from the perspective of Fed, the measurement and state-transition equations are as follows:

$$(State - Transition)\; x_t = Ax_{t-1} + Bv_t, \tag{2.39}$$

$$(Observation)\; s_t = Cx_t + Dv_t, \tag{2.40}$$

where the state vector is x_t=(τ_t,δ_t)' and the shock vector is v_t = (v_t,δ_t,ν_t)' with 0 $\leq \rho < 1$, and δ_t, v_t, ν_t normal variates with zero means and variances σ_δ^2 and σ_v^2, and σ_ν^2,respectively and the vector of Fed observations is $s_t = (m_t,\, s_t)$. So the matrices A, B, C, D are

A=[ρ, 0; 0 0], B=[1, 0, 0; 0, 1, 0], C=[1, 1; 0, 1], and D=[0, 0, 0; 0, 0, 1]. Fed form beliefs about the unobservable components by learning from observations via the following:

$$x_{t|t} \equiv E_t[x_t] = (I - KC)Ax_{t-1|t-1} + Ks_t \tag{2.41}$$

where the matrix of Kalman gains K depends on the parameters of the productivity. The calibration in this section is as follows: the standard deviation of the transitory shock σ_δ is 0.001, the standard deviation of the persistent shock is σ_v is 0.01 and the standard deviation of the noise shock σ_ν is 0.001. The values are set such that variation in persistent component is relatively large compared to that of the noise shock so that learning is gradual, and the additional signal plays a meaningful role in the learning process, and in turn, yield dynamics. Now, I report impulse response functions to cost-shocks to explore the role of imperfect information with this additional signal and its effects on output, inflation, and the policy rate. For reference, I plot perfect information model and imperfect information model with no signal.

2.5.0.1 One-Time Shock to Persistent Component of Cost Shock

Figure 12 plots the cases where Fed releases its estimated persistent component $\tau_{t|t}$ in imperfect information models. Now, with the addition of the signal on transitory component, the responses above illustrate an initial spike at first, then learning takes place and two models under imperfect information converge. This initial spike is expected to occur in responses with respect to one time transitory shocks indeed. However, it does not die out as the shock is indeed to

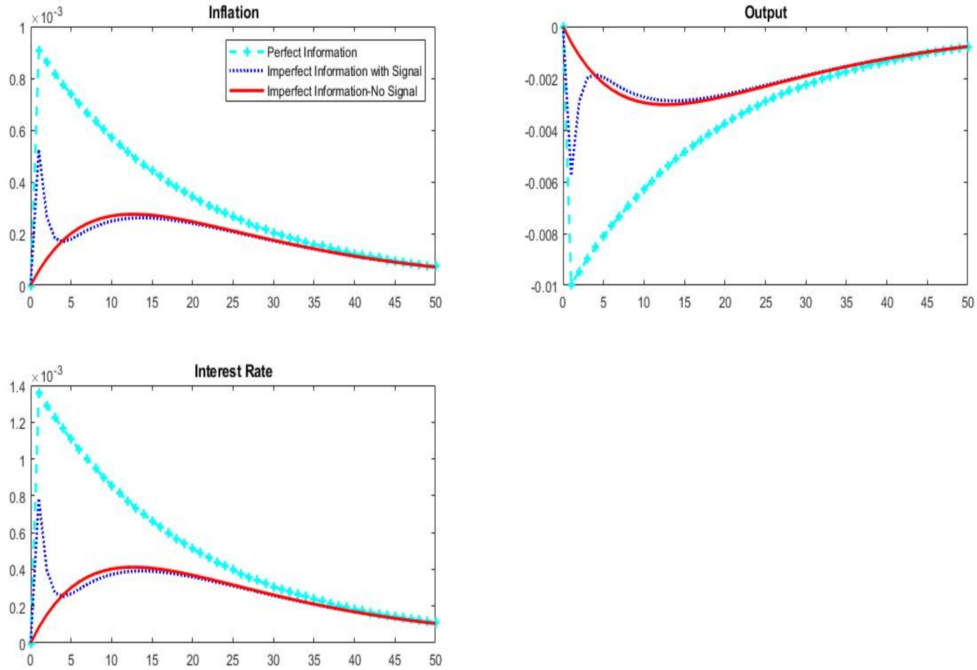

Figure 12: Response to a persistent shock

persistent component and agents in the model learn this gradually. This figure seems to be a good exercise to model initial misperception about regarding the shock to be transitory, and then learning that it is in fact persistent. Relative to responses under perfect information, the hump shaped behavior is observed in all responses but with a further delay relative no-signal model.

Now, responses when filtered transitory component $\delta_{t|t}$ is released and NKPC is given by equation (2.29). Note that perfect information model has the actual persistent component. Figure 13 shows that response of inflation is apparently larger with imperfect information no signal model while the decrease in output gap is significantly bigger under perfect information.

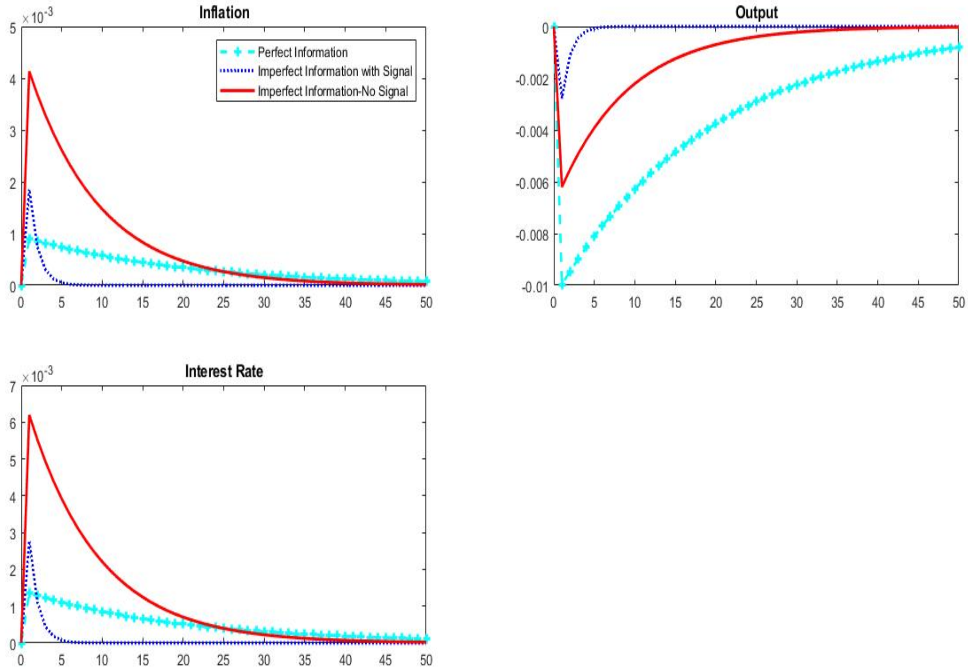

Figure 13: Response to a persistent shock

2.5.0.2 One-Time Shock to Transitory Component of Cost Shock

Now, Figure 14 presents the case in which the shock is introduced to true transitory component, δ_t, and shock is estimated to be persistent. Imperfect information models generate very little response.

Now, Figure 15 plots the case in which the relevant NKPCs are given by (2.29) under imperfect information models and (2.21) under perfect information. Again, responses in three model turn out to be similar except that inflation and output gap change the sign under imperfect information with no signal. That is, after the peak, inflation (output) remains negative (positive) for some quarters meaning that Fed has made a negative forecast error, which is not the case in the model with signal. Hence, the addition of signal captures the true nature more closely in this case.

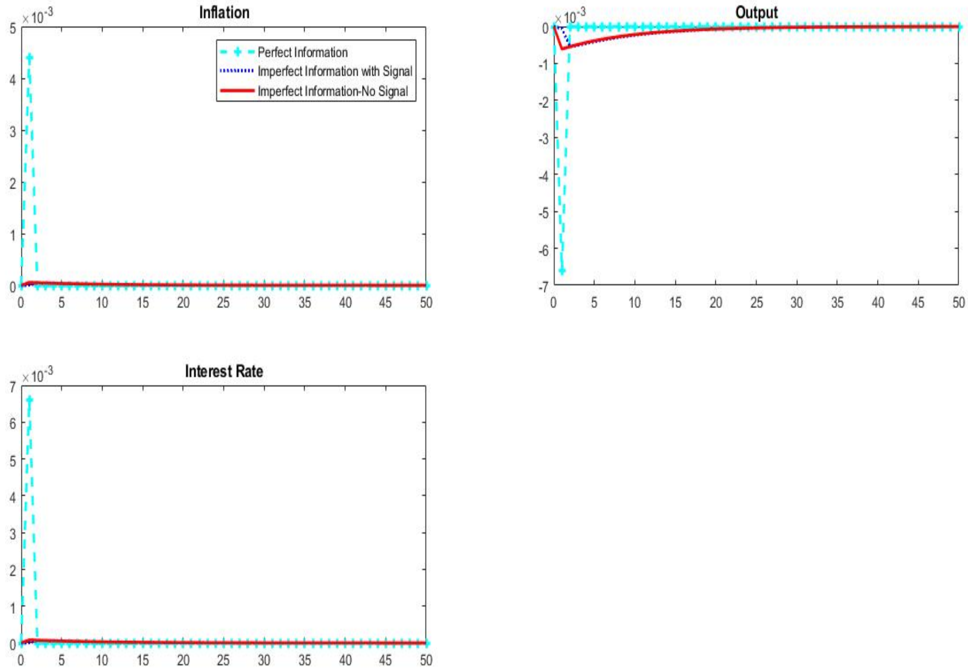

Figure 14: Response to a transitory shock

2.5.0.3 One-Time Noise Shock

In this section, notice that no response under perfect information and imperfect information with no signal is produced.

Now, Figure 16 presents the response of estimated persistent component with respect to a one standard deviation positive noise shock and how this transmits to inflation, output and interest rate through NKPC and IS relations. Now, it is observed that there is this initial temporary mistake in all responses. That is, inflation decreases and output gap increases at first, but then this mistake is compensated by an increase (decrease) in inflation (output) and then the shock dies out.

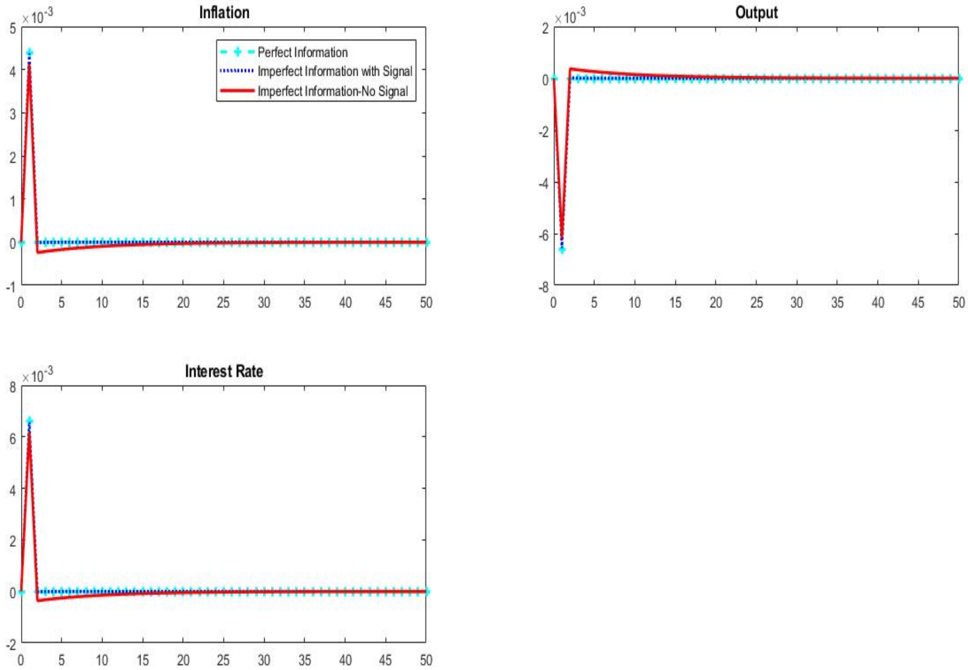

Figure 15: Response to a transitory shock

Lastly, Figure 17 illustrates the responses when filtered transitory component is relevant in NKPC. Now, positive one standard deviation shock in this case generates a temporary rise in inflation and dissipates quickly as it is learnt that it is not actually changing the true transitory component. Similarly, output gap reduces for a while, and the the shock becomes not impactful.

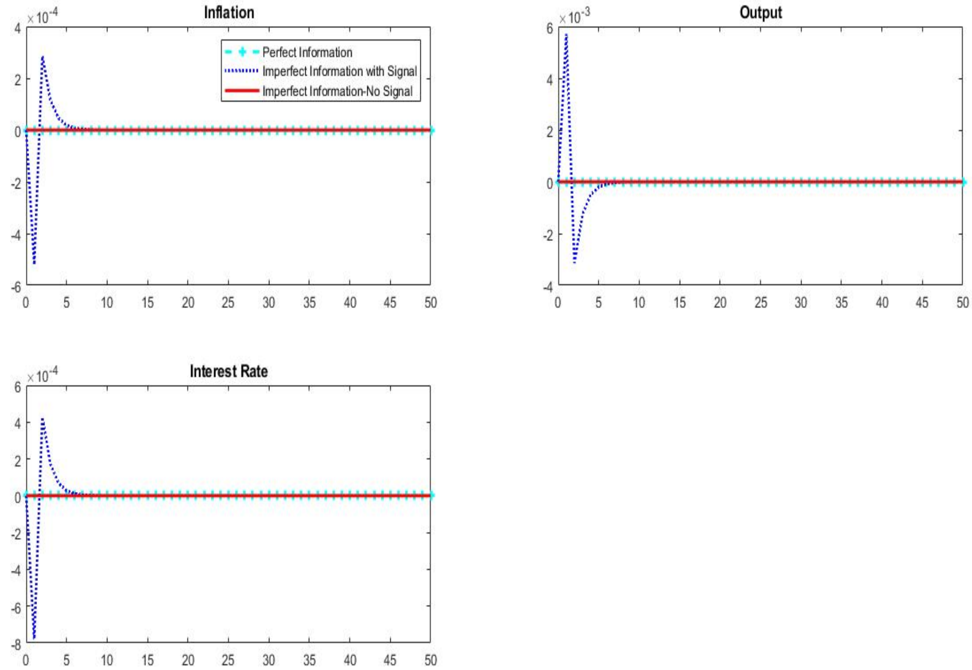

Figure 16: Response to a noise shock

2.6 Discussion

The analysis in this paper shows that responses with respect to a persistent shock when filtered persistent component is relevant reflect the hump-shaped behaviour. That is, when there is a persistent cost shock, the response of inflation and output incorporate the impact of gradual learning. Plus, the responses are underestimated relative to that of perfect information. In the intermediate case where $(\tau_t + \delta_t)$ is in the model, inflation under imperfect information is more responsive whereas output gap is less pronounced with respect to a persistent shock compared to perfect information results. On the other hand, when there is a transitory shock, the complete depletion of the shock given its iid nature, takes time with estimated persistent component. In the intermediate case where $(\tau_t + \delta_t)$ is relevant, inflation under imperfect information is less responsive whereas output gap is more pronounced compared to perfect information results. Once policy under discretion is implemented with estimated persistent component, it

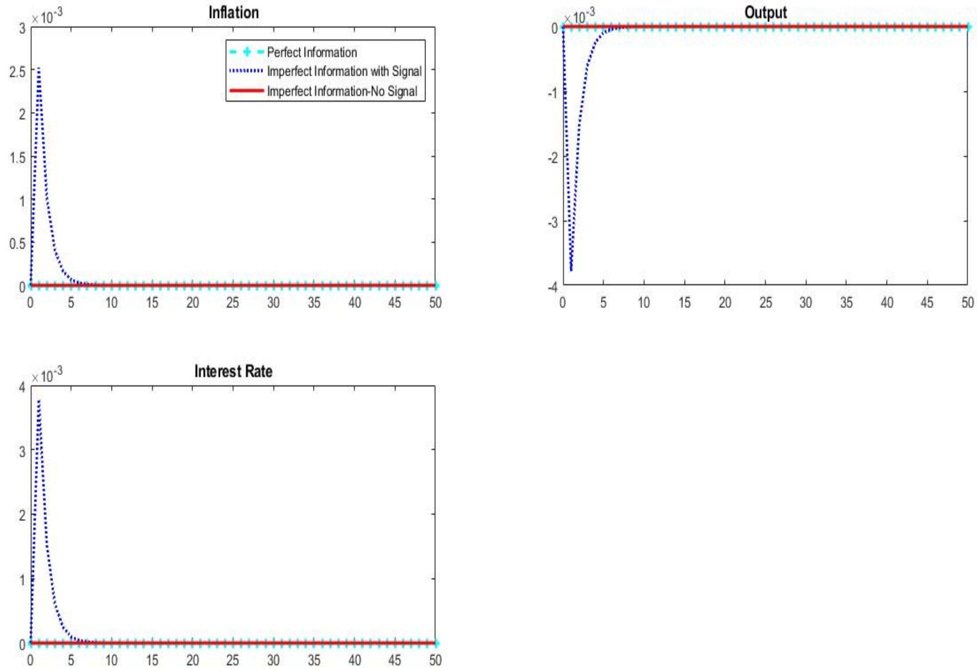

Figure 17: Response to a noise shock

is seen that responses are more pronounced response with respect to a persistent shock relative to the ones given by $1.5\pi_t$ in perfect information and $1.5\pi_{t|t}$ in imperfect information.

Turning to policy evaluation, it is observed that as $\rho \to 0$, that is to make the shock less and less persistent, loss under imperfect information is relatively less with the estimated persistent component. As the variation in noise, σ_δ increases, the relative performance is superior. That is, $L_{imperfect} < L_{perfect}$ holds when inferred persistent component is used in the computation of the value of the loss. As the variation in σ_v increases, loss with estimated persistent component does not generate a meaningful result. That is, relative loss decreases, but the results in perfect and imperfect information models converge each other quickly.

Now, in the extended model where there is a signal on transitory component, responses with respect to a persistent shock when filtered persistent component is relevant indicate an initial rise, then it converges to imperfect information model with a single observation, m_t. This figure (Figure 15) exactly captures the statements of Fed officials. Now, the responses to noise shock with estimated transitory component produces an initial increase in inflation and decrease in output, but then it is realized that it is a noise shock, all responses dissipate quickly. Notably, responses with estimated persistent component yield an initial mistake, that is, inflation (output) temporarily decreases (increases) with respect to positive noise shock. Then, it is compensated with a rise in inflation and decrease in output and shock dissipates completely.

CHAPTER 3

CONCLUSION

In this thesis, I have worked on the impact of persistent-transitory confusion in New Keynesian model. The information friction on the part of both policy maker and private agents has been introduced by the inseparability of the two components. By incorporating policy maker's communication on its estimated components, I have checked the responses of inflation output gap under imperfect information with respect to perfect information benchmark. Then, I conduct welfare analysis when shock is perceived to persistent first, and when shock is perceived to transitory second. Then, by the appropriate definition of relative loss, I have evaluated the respective performances under perfect and imperfect information by simulating parameters describing cost shock process. Lastly, I have extended the model by adding a signal on transitory component.

For future work, I can allow both the private agents and policy maker to involve in signal extraction problem. To do this, I may work with asymmetric incomplete information sets. The source of asymmetricity may be as a result of private signals on the components of the shock. That is, policy maker can receive a private signal on transitory component while the private agents can receive a private signal on persistent component. In that setting, Fed still communicates and that counts as public signal, but agents have another source of information this time. On top introducing such signals, I can explicitly model the credibility instead of assuming that agents do believe what Fed says. That is, I can endogenize credibility based on the ability of Fed to match the announcement of its beliefs about the components of the shock and its true realization. Surely, this requires a dynamic setting of the kind introduced by Gati (2022). The more Fed is mistaken, people learn that they are misguided in the next period hence, reduce the weight that they assign to signal or information coming from Fed's communication. It seems that the weight attached to agent's own private signal and public signal coming from cb speeches capture the reliability and credibility of Fed within the model.

REFERENCES

Baeriswyl, R. and C. Cornand (2018). Optimal monetary policy in response to cost-push shocks: The impact of central bank communication. *21st issue (June 2010) of the International Journal of Central Banking*. 6

Baker, J. D. and J.-P. Lam (2022). Assessing the credibility of central bank signals: The case of transitory inflation. *Economics Letters 220*, 110875. 6

Blanchard, O. J. and C. M. Kahn (1980). The solution of linear difference models under rational expectations. *Econometrica: Journal of the Econometric Society*, 1305–1311. 5

Blanchard, O. J., J.-P. L'Huillier, and G. Lorenzoni (2013). News, noise, and fluctuations: An empirical exploration. *American Economic Review 103*(7), 3045–3070. 5

Carboni, G. and M. Ellison (2011). Inflation and output volatility under asymmetric incomplete information. *Journal of Economic Dynamics and Control 35*(1), 40–51. 15

Clarida, R., J. Gali, and M. Gertler (1999). The science of monetary policy: a new keynesian perspective. *Journal of economic literature 37*(4), 1661–1707. 14

Erceg, C. J. and A. T. Levin (2003). Imperfect credibility and inflation persistence. *Journal of monetary economics 50*(4), 915–944. 7

Foerster, A. T. and P.-D. G. Sarte (2020). The effects of permanent and transitory shocks under imperfect information. *Available at SSRN 3663488*. 5

Gati, L. (2022). Talking over time-dynamic central bank communication. *Journal of Money, Credit and Banking*. 45

Gilchrist, S. and M. Saito (2008). Expectations, asset prices, and monetary policy: The role of learning. In *Asset prices and monetary policy*, pp. 45–102. University of Chicago Press. 6, 19

Gust, C., B. K. Johannsen, and D. Lopez-Salido (2015). Monetary policy, incomplete information, and the zero lower bound. 7

Lee, S. S. (2020). Information value of the interest rate and the zero lower bound. *Macroeconomic Dynamics 24*(7), 1758–1784. 6

McCallum, B. T. (1999). Role of the minimal state variable criterion in rational expectations models. *International tax and public finance 6*, 621–639. 27

Muth, J. F. (1960). Optimal properties of exponentially weighted forecasts. *Journal of the american statistical association 55*(290), 299–306. 5

Oh, J. (2020). The propagation of uncertainty shocks: Rotemberg versus calvo. *International Economic Review 61*(3), 1097–1113. 13

Powell, J. (2021). Transcript of chair powell's press conference. *Federal Open Market Committee*. 7

Sauer, S. (2007). Discretion rather than rules? when is discretionary policy-making better than the timeless perspective? 31

Stock, J. H. and M. W. Watson (2007). Why has us inflation become harder to forecast? *Journal of Money, Credit and banking 39*, 3–33. 5

Svensson, L. E. and M. Woodford (2003). Indicator variables for optimal policy. *Journal of monetary economics 50*(3), 691–720. 15

Tamura, W. (2018). Information design, signaling, and central bank transparency. *Signaling, and Central Bank Transparency (January 12, 2018)*. 6

APPENDIX

3.0.1 Linearizing the Model

$$(\epsilon - 1) = \epsilon mc_t m_t - \phi\pi_t(\pi_t + 1) + \phi\beta E_t\pi_{t+1}(\pi_{t+1} + 1)$$

$$\phi\pi_t = \phi E_t\beta\pi_{t+1} + (\epsilon - 1)m_t + (\epsilon - 1)mc_t$$

$$\pi_t = E_t\beta\pi_{t+1} + \frac{(\epsilon - 1)}{\phi}m_t + \frac{(\epsilon - 1)}{\phi}2x_t$$

.

Case 1 : Fed says shock is perceived to be persistent: $m_t = \tau_{t|t}$

$$E_t\pi_t = \beta E_t\pi_{t+1} + \kappa E_t x_t + \kappa\tau_{t|t}, \tag{3.1}$$

Case 2 : Fed says shock is perceived to be transitory: $m_t = \delta_{t|t}$

$$E_t\pi_t = \beta E_t\pi_{t+1} + \kappa E_t x_t + \kappa\delta_{t|t}, \tag{3.2}$$

3.0.2 IS

$$x_t = E_t x_{t+1} - [R_t - E_t \pi_{t+1}] \tag{3.3}$$

3.0.3 Monetary Policy

$$R_t = \theta \pi_t \tag{3.4}$$

3.0.4 Shock Process

$$\tau_{t|t} = \lambda m_t + (1-\lambda)\rho \tau_{t-1|t-1}, \tag{3.5}$$

$$\tau_{t|t} = \rho \tau_{t-1|t-1} + \lambda (m_t - \tau_{t-1|t-1}),$$

$$\tau_{t|t} = \rho \tau_{t-1|t-1} + \lambda a_t,$$

where a_t is the forecast error.

$$\delta_{t|t} = m_t - \tau_{t|t}, \tag{3.6}$$

$$v_{t|t} = \tau_{t|t} - \rho \tau_{t-1|t-1}. \tag{3.7}$$

Solution under Imperfect Information: Idea is to replace the unobserved states with the corresponding inferences. Now, the solution takes the following form:

$$X_t = A X_{t-1} + B u_t. \tag{3.8}$$

3.0.5 State Space Representation: Blanchard and Kahn (1980)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \tau_{t|t} \\ \tau_{t-1|t-1} \\ v_{t|t} \\ \delta_{t|t} \\ a_t \\ \tau_t \\ \delta_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa & 1 \end{bmatrix} \begin{bmatrix} \tau_{t-1|t-1} \\ \tau_{t-2|t-2} \\ v_{t-2|t-2} \\ \delta_{t-2|t-2} \\ a_{t-1} \\ \tau_{t-1} \\ \delta_{t-1} \\ x_t \\ \pi_t \end{bmatrix}$$

$$+ \begin{bmatrix} \lambda \\ 0 \\ \lambda \\ 1-\lambda \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} a_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} v_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \delta_t$$

3.0.6 Implementing Discretionary Policy

Minimizing CB objective with respect to NKPC with estimated persistent component:

$$\pi_t = \frac{-\omega}{\kappa} x_t,$$

Hence, the MSV solutions are given by

$$\pi_t = \frac{\omega\kappa}{\kappa^2 + \omega(1-\beta\rho)}\tau_{t|t}, \tag{3.9}$$

$$x_t = \frac{-\kappa^2}{\kappa^2 + \omega(1-\beta\rho)}\tau_{t|t}. \tag{3.10}$$

To implement optimal policy, substitute FOC into IS:

$$x_t = E_t x_{t+1} - [R_t - E_t\pi_{t+1}]$$

$$\frac{-\kappa}{\omega}\pi_t = \frac{-\kappa}{\omega}E_t\pi_{t+1} + E_t\pi_{t+1} - R_t$$

Rearranging the terms:

$$R_t = \frac{\kappa}{\omega}\pi_t + (1 - \frac{\kappa}{\omega})E_t\pi_{t+1} \tag{3.11}$$

Now, using (3.9), I have

$$E_t\pi_{t+1} = \frac{\omega\kappa}{\kappa^2 + \omega(1-\beta\rho)}E_t\tau_{t+1|t+1}$$

Notice that by law of iterated expectations $E_t\tau_{t+1|t+1} = E_t E_{t+1}\tau_{t+1} = E_t\tau_{t+1} = \rho\tau_{t|t}$ implying that

$$E_t\pi_{t+1} = \rho\pi_t$$

Therefore,

$$R_t = (1 + \frac{\kappa}{\omega}(\frac{1-\rho}{\rho}))\frac{\omega\kappa\rho}{\omega(1-\beta\rho)+\kappa^2}\tau_{t|t}.$$

Policy rule based on estimated transitory component is exercised similarly.

3.0.7 Policy Evaluation

The computation for loss with estimated persistent component in the main text

$$L_{t,imperfect} = \frac{\omega\kappa^2(\omega+\kappa^2)}{(\omega(1-\beta\rho)+\kappa^2)^2}\Bigg[\frac{\lambda^2(\sigma_\delta^2(1-\rho^2)+\sigma_v^2)}{(1-(1-\lambda)^2\rho^2)(1-\rho^2)} + \frac{2\lambda^2(1-\lambda)\rho^2\sigma_v^2}{(1-(1-\lambda)^2\rho^2)(1-(1-\lambda)\rho^2)(1-\rho^2)}\Bigg],$$

requires one to compute unconditional variance of filtered persistent component

$$\begin{aligned}
E[\tau_{t|t}]^2 &= E[\lambda m_t + (1-\lambda)\rho\tau_{t-1|t-1}]^2 \\
&= E[\lambda^2 m_t^2 + 2\lambda(1-\lambda)\rho m_t\tau_{t-1|t-1} + (1-\lambda)^2\rho^2\tau_{t-1|t-1}^2] \\
&= E[\lambda^2 m_t^2] + E[2\lambda(1-\lambda)\rho m_t\tau_{t-1|t-1}] + E[(1-\lambda)^2\rho^2\tau_{t-1|t-1}^2]
\end{aligned}$$

Notice that,

$$\lambda^2 E[m_t^2] = \lambda^2 E[\tau_t^2] + \lambda^2 E[\delta_t^2] = \lambda^2(\frac{\sigma_v^2}{1-\rho^2} + \sigma_\delta^2)$$

Now,

$$\begin{aligned}
E[m_t\tau_{t-1|t-1}] &= E[(\tau_t+\delta_t)\tau_{t-1|t-1}] \\
&= E[\tau_t\tau_{t-1|t-1}] + E[\delta_t\tau_{t-1|t-1}] \\
&= E[\tau_t\tau_{t-1|t-1}] + 0
\end{aligned}$$

Hence, it reduces to compute $E[\tau_t\tau_{t-1|t-1}]$. Now,

$$\begin{aligned}
E[\tau_t\tau_{t-1|t-1}] &= E[(\rho\tau_{t-1}+v_t)(\lambda\tau_{t-1}+\lambda\delta_{t-1}+(1-\lambda)\rho\tau_{t-2|t-2})] \\
&= E[\rho\lambda\tau_{t-1}^2 + \rho\lambda\tau_{t-1}\delta_{t-1} + (1-\lambda)\rho^2\tau_{t-1}\tau_{t-2|t-2}] \\
&= \rho\lambda E[\tau_{t-1}^2] + \rho^2(1-\lambda)E[\tau_{t-1}\tau_{t-2|t-2}]
\end{aligned}$$

Now, one needs to figure $E[\tau_{t-1}\tau_{t-2|t-2}]$.

$$\begin{aligned} E[\tau_{t-1}\tau_{t-2|t-2}] &= E[(\rho\tau_{t-2} + \upsilon_{t-1})(\lambda\tau_{t-2} + \lambda\delta_{t-2} + (1-\lambda)\rho\tau_{t-3|t-3})] \\ &= E[\rho\lambda\tau_{t-2}^2 + \rho\lambda\tau_{t-2}\delta_{t-2} + (1-\lambda)\rho^2\tau_{t-2}\tau_{t-3|t-3}] \\ &= \rho\lambda E[\tau_{t-2}^2] + \rho^2(1-\lambda)E[\tau_{t-2}\tau_{t-3|t-3}] \end{aligned}$$

Iterating similarly yields the following pattern:

$$E[\rho\lambda\tau_{t-2}^2] + \rho^2(1-\lambda)E\big[\rho\lambda E[\tau_{t-2}^2] + \rho^2(1-\lambda)[\rho\lambda E[\tau_{t-3}^2] + \rho^2(1-\lambda)E[\tau_{t-3}\tau_{t-4|t-4}] + ...]\big]$$

which is indeed equal to

$$\begin{aligned} &\rho\lambda E(\tau^2) + \rho^2(1-\lambda)\rho\lambda E(\tau^2) + \rho^2(1-\lambda)\rho^2(1-\lambda)\rho\lambda E(\tau^2) + \\ &\qquad \rho^2(1-\lambda)\rho^2(1-\lambda)\rho^2(1-\lambda)E[\tau_{t-3}\tau_{t-4|t-4}] \end{aligned}$$

Now, letting $\rho\lambda = A$ and $\rho^2(1-\lambda) = B$ and noting that $A, B < 1$ yields

$$\begin{aligned} &B^0AE(\tau^2) + B^1AE(\tau^2) + B^2AE(\tau^2) + B^3AE(\tau^2) + ... \\ &\qquad ... + B^{j-1}AE(\tau^2) + B^jE[\tau_{t-j}\tau_{t-j+1|t-j+1}] \end{aligned}$$

$$\begin{aligned} &B^0AE(\tau^2) + B^1AE(\tau^2) + B^2AE(\tau^2) + B^3AE(\tau^2) + ... \\ &\qquad ... + B^{j-1}AE(\tau^2) \end{aligned}$$

That is,

$$AE(\tau^2)[1 + B + B^2 + ... + B^{j-1}]$$

eventually leads to

$$E[\tau_t \tau_{t-1|t-1}] = \frac{\rho\lambda}{1-\rho^2(1-\lambda)}\frac{\sigma_v^2}{1-\rho^2}$$

Therefore,

$$E[\tau_{t|t}]^2 = \lambda^2(\frac{\sigma_v^2}{1-\rho^2} + \sigma_\delta^2) + 2\lambda(1-\lambda)\rho\frac{\rho\lambda}{1-\rho^2(1-\lambda)}\frac{\sigma_v^2}{1-\rho^2} + (1-\lambda)^2\rho^2 E[\tau_{t-1|t-1}^2]$$

Now, one can similarly show that $E[\tau_{t-1|t-1}^2] = E[\tau_{t|t}^2]$. Hence,

$$((1-\lambda)^2\rho^2)E[\tau_{t|t}]^2 = \lambda^2(\frac{\sigma_v^2}{1-\rho^2} + \sigma_\delta^2) + 2\lambda(1-\lambda)\rho\frac{\rho\lambda}{1-\rho^2(1-\lambda)}\frac{\sigma_v^2}{1-\rho^2}$$

$$E[\tau_{t|t}]^2 = \frac{\lambda^2}{(1-\lambda)^2\rho^2}(\frac{\sigma_v^2}{1-\rho^2} + \sigma_\delta^2) + \frac{2\rho\lambda(1-\lambda)}{(1-\lambda)^2\rho^2}\frac{\rho\lambda}{1-\rho^2(1-\lambda)}\frac{\sigma_v^2}{1-\rho^2}$$