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The dual of the principal ideal generated by a pure p-form

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Abstract

We observe that our methods in [J. Algebra 183 (1996) 24–37] generalize to determine the dual (e.g. annihilator) of the principal ideal generated by a pure p-form.

1. Generalization of [1]

In [1] we determined the dual of the principal ideal generated by an exterior 2-form (e.g. [1, Theorem 2.3.3]). In this Note we shall observe that our methods in [1] generalize to determine the dual of the principal ideal generated by a pure p-form.

Definition 1.1. An exterior p-form $w \in \wedge^p(V)$ on a vector space V is called a pure p-form of genus g iff there exist a set of pg linearly independent vectors $x_i \in V$ such that $w = x_1 \wedge \cdots \wedge x_p + x_{p+1} \wedge \cdots \wedge x_{2p} + \cdots + x_{(g-1)p+1} \wedge \cdots \wedge x_{gp}$. Note that every 2-form is a pure form.

Let w be a pure p-form of genus g. Put $w_j = x_{(j-1)p+1} \wedge \cdots \wedge x_{jp}$ so that $w = w_1 + \cdots + w_g$. Then $[w_i + (-1)^{p-1} w_j] \wedge [w_i + w_j] = 0$. Take all possible partitions of g in the form $(i_1 j_1)(i_2 j_2) \cdots (i_r j_r)(k_1 \ldots k_{g-2r})$, $i_t \leqslant j_t$

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$(1 \leqslant t \leqslant r)$, $i_1 < \cdots < i_r$, $k_1 < \cdots < k_{g-2r}$ for all $0 \leqslant r \leqslant [g/2]$. Let $\theta(w)$ be the homogeneous ideal multiplicatively generated by generators

$$g_\alpha = \left[w_{i_1} + (-1)^{p-1} w_{j_1}\right] \wedge \cdots \wedge \left[w_{i_r} + (-1)^{p-1} w_{j_r}\right] \wedge v_{k_1} \wedge \cdots \wedge v_{k_{g-2r}}$$

where $v_{k_j} = x_i$ for some $(k_j - 1)p + 1 \leqslant i \leqslant k_j p$.

The whole machinery of [1] generalizes to prove the following analogue of [1, Theorem 2.3.3].

Theorem 1.2. $K[(w)] = \theta(w)$ (*where* $K[(w)]$ *denotes the dual or annihilator of the principal ideal* (w) *generated by* w).

References

[1] I. Dibag, Duality for ideals in the Grassmann algebra, J. Algebra 183 (1996) 24–37.