MODELLING CAPACITY EXPANSION PLANNING FOR AN OPTICAL DISC MANUFACTURING SYSTEM

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Erhan Gündüz

September, 1997

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Erdem Gündüz.

tarafından bağışlanmıştır

By

Erdem Gündüz

September, 1997

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Erdem Gündüz
M.S. in Industrial Engineering
Supervisor: Prof. Halim Doğrusöz
September, 1997

The capacity expansion problems involve determination of the optimum timing and sizing of the capacity for the facilities that have to meet a given demand function. There are various versions of the problem in the literature. In this thesis, a mathematical model for the expansion of a facility producing a single commodity is formulated. This formulation is then used for solving the capacity expansion problem for an optical disc manufacturing system, producing two types of products by the use of two different capacity types. The effects of the technological improvements and economies of scale are considered. The dynamic programming approach is used and a forward recursion algorithm is devised and coded on a personal computer.

Key words: Capacity Expansion, Dynamic Programming, Technological Improvements.

ÖZET

BİR OPTİK DİSK İMALAT SİSTEMİ İÇİN KAPASİTE GENİŞLEME PLANININ MODELLENMESİ

Erdem Gündüz
Endüstri Mühendisliği Bölümü Yüksek Lisans
Tez Yöneticisi: Prof. Halim Doğrusöz
Eylül, 1997

Kapasite genişleme problemleri, belirli bir talep fonksiyonunu karşılayacak tesislerin optimum büyüklük ve genişleme zamanlarının belirlenmesini içerir. Literatürde bu problemin bir çok çeşidi bulunmaktadır. Bu tezde, tek çeşit ürün üreten bir tesisin kapasite genişlemesinin matematiksel modeli formüle edilmiştir. Bu formülasyon, daha sonra iki çeşit kapasite tipi kullanarak, iki çeşit ürün imal eden bir optik disk üretim sisteminin kapasite genişleme probleminin çözümünde kullanılmıştır. Teknolojik gelişmelerin ve ölçek ekonomisinin etkisi de dikkate alınmıştır. Dinamik programlama yaklaşımıyla ileriye dönük ardışımlı algoritma tasarlanarak kişisel bilgisayarda kodlanmıştır.

Anahtar sözcükler: Kapasite Genişleme, Dinamik Programlama, Teknolojik Gelişmeler.

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Chapter 1

Introduction

1.1 Scope and Outline of the Study

Capacity is the maximum amount of supply that can be made available by a production facility for the demand of a product or service. The difference between the demand and the supply arises the needs for the actions to compensate this difference. The *capacity expansion* is the action which increases the existing supply capacity for demand satisfaction.

There are many situations in life where the capacity expansion decisions have to be made. Some of these decisions are daily and small decisions, like to buy a new TV set for home though there exist one already; however some of them are so serious that effects the society for many years, like building new electrical power generating stations for the expansion of the electric generation capability of a country. The former example includes the expenditure of a big investment and the decision will have impact on the country for many years. The effectiveness of this type of a decision is critical for the use of the scarce capital resources of the society.

The addition or installation of the new facilities generally have to be discrete in time, but the reaction of the demand for a product or service is a continuous process. This is because there is a big fixed cost for a reaction to change the supply capability and also there has to be a time period for this change to come into effect. The construction of a new electric plant may take years even after the expansion decision. Demand is on the other hand continuously changing with the conditions derived by the market and the surrounding environment. Therefore, the building of new capacities should take into account the change of demand during these time periods

The change of the demand pattern can best be understood with the lifetime of a new product, The demand for this new product during the introduction period has an increasing nature. This is because of the penetration process of the product into the market. After the end of the penetration process, the growth of demand for the product reaches a steady state. The product lives as long as technically and economically more attractive substitutes replaces it, from that time on the demand starts declining.

There are two main considerations to be taken into account in capacity expansion decisions: *operational* aspects and *behavioral* aspects (Freidenfelds [10]). The operational decisions are mainly the *time* and *size* of the expansion, in other words what size of expansion and when to be made questions are to be answered. In some situations the *location* of the new capacities are also important. The main behavioral aspect is the decision criteria that is value measurement to compare alternatives. Generally this measurement in the literature is the present worth. For most of the problems the time range for the solution is big such as years or months, so the time value of money have to be taken into account which justifies the usage of such a measure. There are also situations that rather than economical superiority other measurements may be more attractive for the decision maker. The management of the company can choose to install bigger capacities although it is more economical to build smaller ones, to show great management performance in the shortrun to shareholders. The literature review of the approaches to capacity expansion problems are extensively examined in Chapter 3.

The capacity expansion problem for an optical disc manufacturing plant is studied in this thesis. The facility produces CD (compact disc) including different types of formats, like Musical-CDs, CD-ROMs, CD-i, etc. Also another similar product CD-R, that is writable CD has a very fast growing demand structure. The demand of the product increases with time until it reaches a steady state amount. The production process of CD-R discs need extra processing stations added to classical CD production facilities. It is possible to manufacture CD-R and CDs with a CD-R production line, but the reverse is not true. The details of the optical disc production technology and optical disc technology are explained in Chapter 2.

The operational aspect of the problem studied is time and size of the expansions. The time and capacity of CD production line or CD-R production line that will be added to the system are the decisions to be made. For the behavioral aspect, we used a different approach rather then the classical present worth method. Time average unit cost is used which is a measure taking into account the time value of money and was first introduced by Doğrusöz and Karabakal. [14] The precise explanation of the concept is given in Chapter 4.

Another important feature of the problem studied is the technological improvements. The products and the production process emerged from high technology, resulting in continuous developments in production technics. With the help of these improvements both the capital cost of installing the facilities and the operating costs do decrease with time. This means the available production lines at a given time are more economical compared to the first invented ones and the machines in future will be better. This effect has to be taken into account in such a long-run problem in order to get reliable decisions. The construction of the model and related assumptions are explained in Chapter 4 in detail.

The dynamic programming method is used for solving the problem numerically. It is seen as the most suitable method for the studied problem. A forward recursion algorithm is developed and coded for computer. The details of the dynamic programming model are given in Chapter 5.

Solutions of problem and the sensitivity to parameters are studied in Chapters 6. Conclusion and further research topics are discussed in Chapter 7.

Chapter 2

Compact Discs and Production Technology

2.1 Compact Disc Technology

CD (Compact Disc) is an optical media that is being used for the storage of data. The compact disc technology emerged with the invention of LASER. The word LASER stands for 'Light Amplification by the Stimulated Emission of Radiation'. Lasers generate coherent light, i.e. light comprising photons with the same wavelength and in-phase. This allows the light beam to be focused to a very small spot size similar to the actual wavelength of the light itself. The advent of lasers and in particular low cost laser emitting diodes has allowed the compact disc technology to become one of the most successful consumer electronic technologies of all time.

In the late 60s, Philips developed the laser video disc. This was a 30 cm. disc capable of storing 60 mins of analogue video. A low power laser was used to read the video information stored in pits in the disc surface. The pits are simply small indentations over the surface of the plastic disc. These pits measured about 0.5 microns in width. They were arranged in a spiral pattern, like vinyl records, the radial spacing between 'tracks' being only 1.6

microns. The pit lengths and spacing are related to the video signal which they represent.

The compact disc uses exactly the same method with identical pit sizes and spacing. However, the pits are used to indicate whether a data bit is '0' or '1'. The read head is a small laser diode emitting infra red 'light' of wavelength about 780 nm. This 'light' is focused to a beam small enough to read the sub-micron pits.

The laser diode is mounted on a swivel arm which can be moved radially and has sufficiently sensitive controls to ensure that the laser follows the pits accurately, even if the CD is slightly eccentric, due perhaps to the centre hole being slightly off centre. The beam focus can be moved up and down to compensate for the disc being slightly warped. A semi-reflective mirror allows the reflected light to pass back to a photo detector. When the laser beam falls on a pit the light is scattered and very little is reflected. The changing light pattern detected is then converted into a series of zeros and ones.

The recordable CDs (CD-Rs) have a slightly different technology. CD-Recordable discs can contain the same data in the same formats as mass-produced compact discs, and they are readable on the same drives and players that read and play other compact discs. CD-R discs are created one at a time–typically in a desktop environment–by CD-R drives which use a laser to record data on special CD-R media. The laser forms microscopic "optical marks" in the light-sensitive dye layer of CD-R discs, which can be read like the microscopic pits on CD-ROMs. It is important to remember that CD-Recordable is compact disc, and as such, is identical to mass-produced compact discs in its content, capabilities, hardware required to read it, and, most importantly, its logical standard, ISO 9660.

2.1.1 Compact Disc Construction & Layout

The compact disc comprises a sandwich of a transparent polycarbonate layer, containing the pits in its upper surface, an aluminium layer forming a mirror surface to reflect the laser beam, a lacquer layer to protect the aluminium and, finally, the layer of print.

The audio data stored on a compact disc is recorded from the centre, starting at a radius of 23 mm., and extends outwards to a radius of 58 mm., the space occupied depending on the playing time. All audio CDs are CLV (constant linear velocity) discs, which means that they are played at a constant speed of between 1.2 and 1.4 m/sec. The rotation speed (rpm) will reduce from the start of the data (23 mm radius) to the outside (58 mm) of the disc by a factor of 58/23 = 2.52. This means that the pits retain the same geometry wherever they are on the disc and there will be no change in performance (including error rate) between the centre and the outside of the disc.

The annular space containing the audio data is divided into three main areas: Lead-in which contains no audio data but does contain other information relating to the audio content. It is used to allow the laser pickup head to follow the pits and synchronize to the audio data before the audio begins. Program which contains up to 74 minutes of audio data divided again into up to 99 tracks. Lead which contains data silence. All compact discs are divided up in this way. In all three areas the data is divided again into frames each of which represents 1/75 of a second of playing time. A full 74 minute disc will therefore contain 74 x 60 x 75 = 333,000 frames. The frame has little meaning for audio but is important when storing computer data on the disc.

CD-R Discs, however has an extra layer of a coated special dye. Also the reflector used in CD-R Discs are not aluminium, but gold. The dye layer is a light sensitive coating and the application of laser over it causes the pits to appear on the surface of CD-R.

2.1.2 Compact Disc Formats and Standards

The Compact Disc Products can be viewed as a group of product family, which have common technological basics but differ in some details for different applications. The discs can be used for the storage of data on computers, music, photographs, multimedia and so on. Also some of them are read-only, while the other are recordable or erasable.

There are standards for the Industry that explains the physical properties as well as software specifications for each of the product in the family. The first published standards was from Sony and Philips for Compact Disc Digital Audio (CD-A), and in 1984 they introduced the standards for CD-ROM, storage device for computer data. The related standards for the products are:

- RED BOOK : Red Book describes the physical properties of the compact disc and the encoding of digital audio data.
- YELLOW BOOK : The specifications for CD-ROM, and an extension of the standard for CD-ROM XA.
- GREEN BOOK : Written in 1987, describes the CD-interactive (CD-i) disc, player, and operating system.
- ORANGE BOOK : The Orange Book defines CD-Recordable disc in three parts: CD-MO (Magneto Optical), CD-WO (Write Once), and CD-E (Erasable) Discs.
- WHITE BOOK : The White Book defines the Video CD Specifications and published in 1993.

2.2 Production Technology

The stages of the production for CD and CD-R begin with pre-mastering and mastering stages. The Production Facilities examined here consist of the processes that is successive to the stage where the *stamper* is ready for moulding. Stamper is the mould which contains all of the negative copies of the pits for CDs over its surface. During moulding these projections over the stamper build the indentations on the plastic discs. The mastering and pre-mastering units require different technologies and a very big investment that can be justified when production in big quantities is required. The feasibility of when to build mastering facilities is yet another problem and is not examined in this study.

Premastering is where the information received from the customer begins its first stage in production. This stage is the preparation of the data in a format that makes the data ready to be processed to the master. The data is recorded generally on a CD-R Disc, but may be one of many alternative transporters like DAT (Digital Audio Tape), External Hard Disk etc. *Mastering* is the stage of the process where the data are converted into pits on a glass master. After this the master is plated over to form a negative copy that can then be used to make many plastic disc copies. The master is all of the pit and non-pit areas of the media on top of the glass disc.

The production system considered in this study begins after this point. The glass master is the mould that contain the data. This is different for all musical titles and CD-ROM. Also for CD-R there is a need to a standard master. This is placed in the injection moulding machine. Moulding is the beginning operation. Plasticized polycarbonate resin is then injected into the mold cavity under great pressure. The injected resin is then allowed to cool and solidify. The mould opens and the clear plastic disc is removed automatically and placed over the spindles. The plastic after this stage has all of the pits that define the data.

The other stage is *metallization* stage. In this stage a thin layer of metal is overcoated on the disc surface. For CDs aluminium is used as the metallizer.

The process for CD-R, however uses gold in the sputtering station. The material used for CD-R sputtering is very expensive, so the strict control of the metallization layer thickness and recycling of the scraps is important. More complex equipment is designed for CD-R lines for metallization process. This same station is naturally capable of aluminium coating.

After metallizing discs are covered with a protective layer of UV curable lacquer to prevent it from oxidizing. This is the *lacquering* station. If oxidation were to occur, the discs would become unreadable because the oxidized layer is transparent to light and the laser light from the player would not get reflected back.

These units defined so far are parts of both CD and CD-R Lines. However in CD-R production, after the moulding stage there is a process consisting of the application of dye. This is the *dye coating stage.* The cyanine dye is coated over the surface of the polycarbonate discs, cleaned and then this chemical coating is dried. After that discs go to metallization process for sputtering the gold layer.

Figure 2.1 shows the process flow schematically for a typical CD-R production line. The difference between a CD production line is just that the station with the codes B3 to B6 are removed for CD production, and at station C1 aluminium coating instead of gold is used. To produce CD on a CD-R line the products after stage B2 is directly transferred to stage C1.

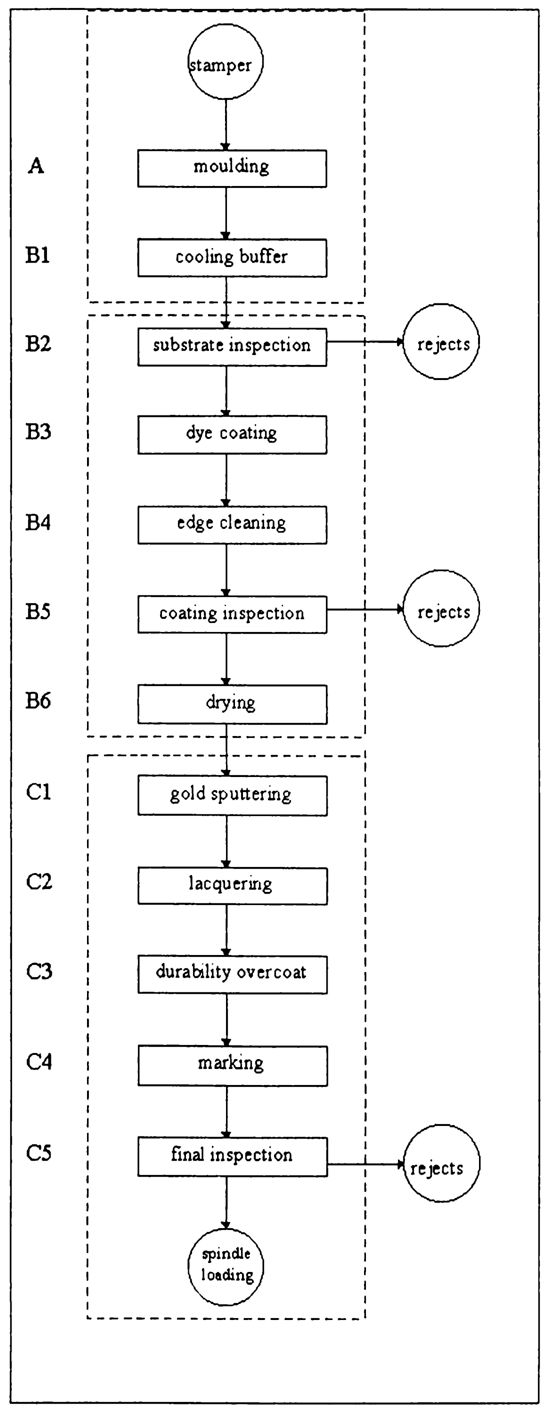

Figure 2.1: The Process Flow For CD-R Production Line.

Chapter 3

Literature Review

The problem of capacity expansion has long been the interest of the managers, and engineers; since it is a natural decision process encountered in many real life situations. However, the quantitative approaches to the problem began to be seen in the literature since the end of 1950. It is not surprising that the quantitative models and solution methods are generated from applications.

According to observations of Luss [23] the most popular parameters and decision variables in the literature for capacity expansion problem can be stated as:

The major decision variables in general for capacity expansion problems are the size, time and location of the expansion. The discount rate is critical for most of the cases and has an impact on the optimal policies, as the time considered for the problems is large. The cost of expansion have generally the form of

$$C(x) = Kx^{\alpha} \ (0 < \alpha < 1), \ x \geq 0 \tag{3.1}$$

or for fixed charge cost functions which reflect economy of scale.:

$$C(x) = \begin{cases} 0 & if \quad x = 0 \\ A + Bx & if \quad x > 0 \end{cases} \tag{3.2}$$

x : Size of the capacity expansion.

K : Base cost per capacity

α : Constant.

A : Fixed Charge (Cost per expansion).

B : Cost per unit capacity.

Three most popular demand functions in literature are the following :

$$\begin{aligned} D(t) &= \mu + \delta t \\ D(t) &= \mu e^{\delta t} \\ D(t) &= \beta\left[1 - e^{-\delta t}\right] \end{aligned} \tag{3.3}$$

It is interesting to give the application found in the Literature separately in order to view the areas research is concentrated for capacity expansion problem. This shows the motivation behind the thesis.

The references in the Literature can be examined in three groups in theoretical aspects, these are mainly single facility problems, two-facility problems, multi facility problems, and expansion via a finite set of projects as Luss classified in his Review Paper [23].

3.1 Applications

The effects of the capacity expansion decision is critical for big projects, like the heavy industries, telecommunications and electrical power plants. The first applications of operations research techniques emerged from the situations faced in those industries.

The first recognition is from the book of Manne [24]. He studied the expansions for various heavy industries in India. In the Aluminium Industry, for a growing demand pattern the trade-off between building large facilities so as to get benefit from economies of scale versus the cost of the installation before needed is optimized. The decision variables are the time and size of the expansion, and the objective function was the present worth of all costs incurred. For the Cement and Fertilizer Industries location of the facility is added to the problem as a decision variable. Related to this the costs of supplying from various production points to demand points are added to the cost pattern.

This approach is used for huge investment projects like water resource, electric power generators and communication networks that affect a big society. The decision of the size and time of the electric plants in France from 1965 to 1985 was described by Bessiere[4]. For electric power generation problem, besides the sizing and timing, the selection of the type of the plant also has an impact on the problem. There are hydro (water driven), thermal, and nuclear plants. The optimal mix of these different types are examined by Peterson [26]. O'loghaire and Himmeblau [25] studied the expansion of the water resources. The study consists of selection from a set of possible projects, and the time of the implementation of the projects in order or meet the demand for water at every time period. Many applications exist also for communication networks. The capacity expansion for networks is more complex compared to other problems. This is because there are many nodes over the network and from each node to the other ones there is an associated demand for each period. The objective is similar like the other application examples, to find the optimum expansion policy minimizing the total discounted costs and satisfying the demand for all periods. In addition, optimal *routing* according to the given link capacities are to be determined. As both the optimum link capacities and routing depend each other the problem becomes complex. Yaged [30] has solved the problem for a large communication network spanning United States by using a heuristic algorithm. Doulliez and Rao [5] have a capacity expansion model in which there are standard capacity elements with a known cost for any link in the network. They found the optimal policy using a shortest path algorithm. Luss [22] examined a situation in which there are two types of cables for the links of the network : one is the standard cable and the other is a special type cable. The special cable costs higher compared to the other type, but have the ability to satisfy two type of demand, where as the standard cable cannot. The decision process, consequently involves not just the size and time of the expansion, but the type of the cable to be installed also. There are other applications concerning the cable sizing and network multiplexing problems. The references for these subjects are : [11] and [20].

Although there are some application related papers in the literature, they generally lag behind the theory, models, and algorithms studied in the literature according to Luss [23]. The most attractive and complex studies seem the Classical works of Manne for Heavy Process Industries in India [24]. Also Luss indicates there is a need for more applications and research in this problem area.

3.2 Single Facility Problems

The problems can again be divided as Infinite-Horizon and Finite Horizon Problems. For Infinite Horizon Problems the Long-Run Optimality should be defined. Let $P1$ and $P2$ be the capacity expansion policies at times t_k and t_k. Then the definition for Long run optimality is :

Definition 1 *Policy $P1$ with scheduled expansions at t_k is long-run optimal if there exists some time t such that there is no better policy from 0 to any t_k , $t_k \geq t$.*

So the two policy $P1$ and $P2$ can both be long-run optimal.

The Single Facility problems deal with the expansion of a single type of supply. The simplest type of this problem is the deterministic linear demand case. There is a linear deterministic demand with the rate of δ. The capacity is assumed to have an infinite economic life and all of the demand should be met. The cost of adding a capacity with size x is $f(x)$. The total costs of expansions are summed up to the Present Worth and this cost figure is minimized. The expansion sizes, x are always same at each expansion because the costs and the additional projections of additional demand are identical at every shortage time. $C(x)$, the present value of all the costs can be stated as :

$$C(x) = \sum_{k=0}^{\infty} \exp(-rkx/\delta) f(x) = f(x)/[1 - \exp(-rx/\delta)] \tag{3.4}$$

The function $f(x)$ in Equation 3.4 is generally concave and reflects the economies of scale. It is in the form of Kx^{α} (Where K and α are constants, $0 < \alpha < 1$) and/or $A + Bx$ (Where A and B are constants) form. The value that equates the derivative of Equation 3.4 to 0 is then simply the optimizing x value. Manne showed using the first form of the cost function; that x increases when α decreases and decreases when discount rate, r increases. These are all expected as the decrease of α means that there are more economies of scale so it is more advantageous to build larger capacities, and as r increases the time value of money increase so to tie up money for unused capacity becomes costly [24].

Sinden [28] showed that the identical time intervals between expansions exist for the cases where Demand function $D(t)$ is arbitrary with $f(x) = Kx^{\alpha}$, and $f(x) = A + Bx$ and $D(t)$ is a linear function.

The extensions of the problem releases the assumption of the Linear Demand and the identical expansion sizes. Freidenfelds solves the problem with the Linear Demand but an initial jump D_0 and shows that the optimal expansion sizes are just $x + D_0$. Again he solves the problem using arbitrary demand curves and uses Backward and Forward Dynamic Programming Algorithms.[10] Related works can also be found in papers by Giglio [13] and Kalotay [19].

The models so far do not include the case of shortages, they assume that the demand should always be satisfied, and there cannot be imports. The paper of Erlenkotter [8] assumes unlimited imports and inventories where the demand is a linear increasing function. For the imports there is a penalty cost of $\rho > 0$, and for the inventories there is a holding cost $h > 0$ per unit of inventory per unit time. As consequence of the assumptions expansions would have been optimized when inventory is zero, and obviously the capacity after the expansion, will exceed demand. Hence, these properties of optimal solution simplify the calculations in using dynamic programming approach. The problem is solved in Manne‘s Aluminium Industry Example.

Freidenfelds [10] models the situation of the linear increasing demand with imports allowed. He solves the problem using Dynamic Programming approach. He also defines the congestion cost and solves the problem using this cost. He shows that it effects both the timing and sizing decision. This cost can be defined as:

Definition 2 *Congestion cost is the associated costs due to the existence of spare capacity in the system defined as s. It is called $v(s)$ and is a function of s.*

Hopkins [16] has examined the infinite horizon optimality in a problem which combines both capacity expansion and equipment replacement situations.

The finite horizon problems deal with generally discrete time periods $t = 1, 2, 3, .., T$ where T is the finite horizon. Generally the finite horizon problems are solved using forward dynamic programming. Smith [29] examined the impact of a finite horizon T on the expansion policy under certain assumptions. His results apply to the infinite horizon problems where time intervals of successive expansions are identical. He provided upper and lower bounds for the first expansion size as a function of T. When T increases the two bounds converge to the same value.

Rao [27] models a finite time horizon problem, with a known nondecreasing demand to be satisfied by the production in that period and/or inventory from the previous period. The total time discounted costs to be minimized include the capacity expansion, production, inventory carrying, and idle capacity costs, which are assumed to be concave. The problem is to determine an optimal capacity expansion and production schedule. Based on some interesting properties of an optimal solution, an efficient dynamic programming algorithm is constructed.

Love [21] examines n period model with known demand and separable piecewise concave production and storage costs. It is shown using the network flow concepts that for arbitrary bounds on production and inventory in each period there is an optimal schedule such that if, for any two periods, production does not equal to zero or its lower or upper bound, then the inventory level in some intermediate period equals zero or its upper or lower bound. He constructed an algorithm for searching such schedules where the bounds are $-\infty, 0,$and ∞.

Hinomoto [15] has incorporated the *Technological Improvement* concept to his model which have N- step expansions. He modelled the technological improvements as an effect in the decrease of both operating cost and capital cost. The improvement is assumed to be continuous and the decrease is related with an exponential function. The change of the capital cots is stated as:

$$W(t) = K(z)\, e^{-kt} \tag{3.5}$$

$K(z)$ is the cost of a facility at time 0 with size z. k is the rate of the technological improvement and t is time in Equation 3.5. The operating cost is similarly defined. He solved the problem first in one stage and extend it to N stages. The solution procedure is simply finding the values of decision variables when the derivatives of the present worth of the total costs equal to zero. The weak point of the model is the assumption that the number of expansion N is known. The solution procedure gives optimal values of expansion sizes, times and production by changing the number of expansions. However in real situations the number of expansions is a decision variable. This makes the model illogical for applications.

Another paper where the technological improvement are taken into account is by Gabszewicz et. al. [17] They relaxed some simplifying assumptions of

Manne's classical model with linear growing demand. The first one is instead of infinite horizon finite time period is assumed, and the *technological improvements* considered. The model instead of viewing the technological process as a continuous one, formulates it as sudden discrete changes occurring randomly in time. According to the model technology is a given production and investment cost structure. They made the definition of an obsolescence date as :

Definition 3 *Let T be a point of time such that before T there exist only one technology, say T_1, to face the capacity expansion requirements, and after T there exist a new technology, say T_2, competing with T_1. A technological progress is said to have occurred at T if, for the capacity expansion problem from T to ∞, T_2 is preferred to T_1 in terms of cost minimization objective. T is called the obsolescence date.*

In the model they assumed that the cost structure of the new technology T_2 is known with certainty at $t = 0$, but the obsolescence date may be a random variable. They solve the problem using $A + Bx$ cost structure for capital cost and a constant production cost of m. They found that when the obsolescence date T is known with certainty, depending on its position within the interval $[nx', (n+1)\,x']$, (n is the sequence of the expansion) the first decision is to build a plant of size large or smaller than the investment size x', which is optimal for the problem without technologic improvements. When T is a memoriless random variable, the optimal first decision is to build a plant size of smaller than x'.

3.3 Capacity Expansion Via A Finite Set of Projects

For this type of capacity expansion problems there are a finite number of projects available with capacities $x_1, x_2, ..., x_n$ that cost $C_1, C_2, ..., C_n$ to install and maintain forever. As a general assumption there is a demand $D(t)$ function that is nondecreasing with time. The problem is optimally sequencing the finite number of projects so as to minimize the discounted cost as well as satisfying the demand. Surely for $\sum_{i=1}^{n} x_i$ should be smaller than $D(t)$ for any t.

This type of modelling is very suitable for water reservoirs, as there are at predetermined places reservoirs with known capacities. The demand for water is to be satisfied using these finite number of reservoirs. The optimal cost policy will be the optimal sequencing of the construction of water generating facilities at these reservoirs.

For the case of $D(t)$ a linear increasing demand function $(D\,(t) = gt)$ the solution is obvious. It turns out to be optimal to order the projects according to their annual cost defined by

$$AC\,(i) = \frac{rC_i}{1 - e^{-rx_i/g}} \tag{3.6}$$

The present worth over all projects will be minimized if the projects are arranged in order of decreasing annual charge.[10]

For a more general demand pattern, Erlenkotter [7] devised a dynamic programming method for solution. He constructed a set of established projects X. The set X is described by a vector with S binary elements representing the list of implemented projects; for example for $s = 4$, $X = (1110)$ indicates that projects 1,2, and 3 have been implemented. Let $\tau(X)$ be the earliest time at which demand exceeds the available capacity $\sum_{s \in X} x_s$.

$X - i$ is the set that contains all of the projects X except project i. For all sets containing j projects, let $W_j\,(X)$ be the cost of optimally sequencing these projects. That cost can be found from

$$W_j\,(X) = \min_{i \in X} \left[W_{j-1}\,(X - i) + C_i e^{-r\tau(X-i)} \right] \tag{3.7}$$

Starting with $W_0 = 0$, W_J can successively be generated until optimal sequence for all n projects can be found.

Hopkins [16] models a situation where for each planning period t, a number of finite possible investing alternatives, associated with a unit cost for each alternative at the time the action is taken. The problem is formulated as a linear programming problem, and the infinite horizon problem is solved , first by finding the optimal solution for a finite horizon problem that includes a validation of capital stocks. Then it is used to construct a solution to the infinite-horizon problem that is identical to the finite horizon problem in early periods.

3.4 Two and Multifacility Problems

The references so far dealt with the problems consisting of only one type of product and one type of supply to satisfy the demand for that product. However the extensions of this situations include different types of demand and also different types of supply. Two different type of facility can be added, for example and they have different characteristics.

Freidenfelds [10] solves a capacity expansion model in which there are two types of demand. A standard, generally less expensive, type of capacity can serve only the standard demand, while a deluxe capacity can serve both the standard and deluxe demand. Demand for both type is assumed to grow linearly with time, there is no difference in the cost of using the two types and the cost of rearranging demand from one facility type to another is negligible. A dynamic programming algorithm is devised for the solution of the problem.

Fong and Rao [9] have modeled capacity expansion with two producing regions. There is a single commodity that can be produced in two regions. Demand in each region is nondecreasing over a finite time horizon. Demand in each region must be satisfied, either by the current production capacity there or by a shipment from the other region. Capacity expansion and shipment costs are assumed to be concave and no inventory stock is allowed. The problem is to find a schedule of capacity expansion in each region and a schedule of shipment between the two region, such that the total cost over the time horizon is to be minimized. The model is the extension of the model found in [24] differing from that, it includes two producing regions and concave transportation costs A dynamic programming algorithm is devised to solve the problem.

Kalotay [18] considers a model in which there are two types of equipments available: a general purpose equipment and a cheaper specialized equipment which could provide only one of the services. It is assumed that the ratio of the demands is independent of time and that identical economies of scale are offered in the cost of the equipments. A lower bound for the present worth of the optimal policy in the case of linearly and exponentially growing demands, and determine the conditions under which the use of specialized equipment is justified.

It is shown that in the case of linearly growing demands the specialized equipment should eventually be used, while if the demands are growing exponentially this may not be the case.

For the multifacility problems the usage of dynamic programming cannot be so useful as in the cases of the two type of facility problems. Because in the two type case the dynamic programming approach reduces the problem dimensionality to one. In various applications capacity expansions can take place in different producing locations $i = 1, 2, .., m$. Furthermore, often the products have to be shipped to different geographical regions $j = 1, 2, .., n$ at substantial transportation costs.

If the problem is formulated as a mathematical programming model so as to minimize the total transportation cost that is not discounted becomes a *transportation problem*. The capacity expansion problem is stated as finding the sequence of expansion sizes, times and locations so tat the total discounted expansion and transportation cost is minimized. There are heuristic algorithms for solving the problem and its extensions [23].

Freidenfelds and McLaughlin [11] examined a multitype problem. Specifically, they studied a cable sizing problem with several facility types in which the conversion and rearrangement costs are negligible. They developed a branch and bound algorithm assuming that there is a finite number of possible expansion sizes for each facility type. Since the decision tree is very large, they developed simple heuristic bounds that are used to eliminate branches.

In the thesis we used time average unit cost per unit product measure for objective function, which is a quite different approach compared to the generally used present worth of all cash flows concept in literature. Also, technological improvements are modeled as continous reductions in cost parameters which is similar to the study of Hinomoto. [15]. The major difference in construction is that Hinomoto takes number of expansion as a predetermined value, however it is a decision variable in our model.

Chapter 4

Formulation of the Problem

4.1 Formulation of the Capacity Expansion Problem

The investment analysis problem consists of determining the optimal times and sizes of the expansions for a production facility that has two main group of products. Two main product groups are CD family and CD-R family. The differences between these two groups and other related technical informations about products are described in Chapter 3.

There is a deterministically known future demand for both types of products. The demand is assumed to be a non-decreasing function saturating at a predetermined demand level in the long-run. The demand should be met for each time period by using only the capacities installed. The amount of production should be equal to the demand for each time period. The inventory is not allowed. No inventories is, in fact not a simplifying assumption for CD family of products, it is the natural result of the production technique. The produced CDs transport the software (If it is a musical CD, then the music title or if it is CD-ROM, then the computer software) so each produced CD corresponds to a specific product. It is not reasonable to hold inventories because of this reason for CD family. CD-R is on the other hand is blank media, it is sold as a product to be used for storing information. Therefore it is feasible for CD-R family products to be manufactured to inventory. However it is also assumed that the inventories are not allowable for CD-R Products to simplify the model.

The expansions are planned at discrete time units, in other words the expansions are executed at the beginning of year 1997, 1998 and so on, not in between these discerete units. The years are modeled as integer time units as 0, 1, 2,..,n. where $time = 0$ is the beginning year of the expansion analysis.

It is also assumed that the capacities once installed can work infinitely without any deterioration. The operating costs of the facilities remain same for all time periods.

There are two types of capacities, one is the CD production line, and the other one is the CD-R production line. The CD production line can only manufacture the products included in CD products family, whereas CD-R production line has the ability two produce both CD and CD-R product families. There is a cost function associated for the two capacity types. The capacities are discrete units, it means there is a fixed known capacity of a single production line that can change with time. The capacity installed can be the integer multiples of that single machines' capacity. The cost of the installation of a production line has a fixed amount and a variable amount that changes with the capacities installed. It is assumed that the time needed to install a production line is zero, in other words as soon as the installation is made, the machine can begin the operation.

There is an operating cost for both CD and CD-R products. The operating cost has also a fixed and variable components.

Technological developments in the sector result in the reduction of both capital and operating costs. The effect of the technological developments is assumed to be continuous with time. The major result of technological improvements is the decrease of the *cycle time* of the machines with increasing time. Here, cycle time is defined to be the time needed to produce a unit of the product. As the cost of a single machine is assumed to be constant, than the result of cycle time reduction is a decrease in the capital cost per unit capacity.

The criteria for choosing the best investment plan from the alternatives defines the objective function of the problem. As the time horizon of the problem is defined as time period of several years, the time value of money has to be taken into account. The time value is calculated with the help of discounting which makes the cash flows at different times equivalent to each other. If the cash inflows and outflows are continuous and continuously compounded with the discount rate r, \$ per \$ per time unit (time unit is year for the problem considered), the relations between the present worth P; future worth F; and, uniform series A becomes:

$$F = Pe^{rt} \text{ and } P = Fe^{-rt}$$

$$A = P\left[\frac{e^{rt}\left(e^{r}-1\right)}{\left(e^{rt}-1\right)}\right] \text{ and } P = A\left[\frac{e^{rt}-1}{e^{rt}\left(e^{r}-1\right)}\right]$$

$$A = F\left[\frac{\left(e^{r}-1\right)}{e^{rt}-1}\right] \text{ and } F = \left[\frac{e^{rt}-1}{\left(e^{r}-1\right)}\right]$$

These and similar other traditional formula related to discounting can be found in any Engineering Economy book [6]. However these formula may not work for some conditions and the need for new measures arises. These alternative measures were developed by Doğrusöz and Karabakal [14]. They modeled the investments as growth processes composed of two separate parts; one is *productive growth* and the other is *reproductive growth.* Production can be positive or negative, i.e., production is either the cash inflow or cash outflow. Reproduction can also be negative or positive representing the excess cash or borrowed cash reproducing itself. This production rate is called as a function $a(d,t)$; d is the decision variables and t is time.

In most of the practical situations, $a(d,t)$ is the difference between cash inflow rate, $r(d,t)$, and cash outflow rate, $m(d,t)$, i.e. $a(d,t) = r(d,t) -$

$m(d,t)$. The determination of $r(d,t)$ is generally not possible. If this function is assumed to be independent of the decision variables than minimization of annual worth or present worth of cost can be used.

They developed an alternative measure called *time average unit cost per unit product or service.* This is useful, for the capacity expansion problem, as for this problem the production rate of the system can be well defined but the price is unknown for the future periods. As the $r(d,t)$ function cannot be explicitly defined because of this, net present worth or annual worth concept cannot be used.

There is another problem with the usage of present or annual worth concept. The time horizon is not a predetermined constant, the costs of different time horizons should be compared. To overcome these difficulties time average unit cost per unit product, c is defined to be the price where pay all cash outlays which is defined in formula 4.1 :

$$c\int_0^T q(d,t)\, e^{-rt}dt = \int_0^T m(d,t)\, e^{-rt}dt + \sum_0^T C(d,t)\, e^{-rt} \tag{4.1}$$

$$c = \frac{\int_0^T m(d,t)\, e^{-rt}dt + \sum_0^T C(d,t)\, e^{-rt}}{\int_0^T q(d,t)\, e^{-rt}dt} \tag{4.2}$$

$q(d,t)$: Production rate as a function of decision variable d and time t.

In these formulations $C(d,t)$ is the amount of investment executed at discrete time units t. Then the aim of the model is to minimize c. By minimizing c the optimal time average cost per unit product can be found. This measure also gives more insight to the decision maker than a present worth concept for this problem, as the cost per unit product can be known. The market price expectations and calculated cost can readily be compared to determine whether the investment is worthed.

4.2 Construction of Parameters and Functions of the Model

4.2.1 Demand Functions

One of the most critical points of the problem is the correct prediction of the future demand, as it is the main data in giving the investment decision. The demand functions are assumed to be non-decreasing and saturating at a steady state demand level in the long-run. The function is mathematically formulated as a two step exponential function. Let D_0 be the demand rate at time $t = 0$, δ be the exponential rate of change in demand rate, D_s be the saturating level of demand rate and Φ be the inflection point. The function is then defined as Equation 4.3

$$\begin{aligned} D(t) &= \begin{cases} D_0 e^{\delta t} & ,\text{if } t < \Phi \\ D_\Phi + (D_s - D_\Phi)\left(1 - e^{-\delta(t-\Phi)}\right) & ,\text{if } t \geq \Phi \end{cases} \\ \text{where } D_\Phi &= D_0 e^{\delta\Phi}, \text{ is demand at time } t = \Phi \end{aligned} \tag{4.3}$$

For the determination of the parameters of demand functions, the past data structure of CD-R and CD demands in the world are examined. The demand functions of CD and CD-R do not behave in the same way because of the core differences in the usage areas of these products.

CD Product Family group consist of two main products, CD-Audio and CD-ROM's. Audio CDs are used mainly for recording of musical titles. They replaced the classical prerecorded audio cassettes in Europe and North America Market almost completely. These regions are the places where CD-Recorders have penetrated into the market. Turkey has a time phase in the introduction of this new technology which is approximately a 10 years time period. The sales data from 1993 to 1997 for Turkey shows an exponential increase which is very similar to the introduction period of Audio-CD's in North America. The Figure 4.1 shows how the music market in USA moved from compact

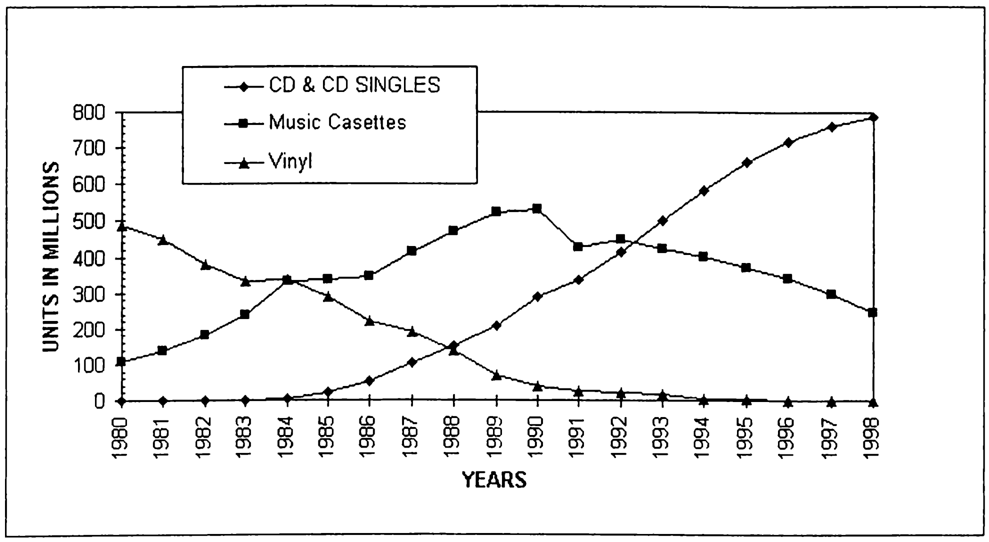

Figure 4.1: The Music Shipments in USA

cassettes and vinyl records to CD [2]. D_0 value is determined for CD as 530,000 CDs/year, from the real data in hand. There is a strong expectation and trend that all pre-recorded compact audio cassettes will be replaced with CDs. There will be still a room for audio cassettes in the future, it will be used for archieving and related amateur recording purposes. It seems that a new technology MD(Mini-Disc) can be a good substitute audio cassettes for home recording purposes. But because of the increasing penetration of CD-Players, the pre-recorded music market will eventually be dominated by this product. This is what happened in Europe and America. It seems logical that demand pattern will behave in a similar manner in our case.

There is a dramatic increase in CD-ROM demand because of the increasing usage of PC's (personal computers) and the appearance of CD-ROM Drives as a standard part of the configuration with the decreasing price. Although the increase is dramatic, the relative amount of demand for this product is low in CD Product Group. It is also logical to assume that this minority will remain. The networking and the Internet, if continues to grow (there seems no obstacle for this growth) in the same manner, the software and related computer-based information will be transferred by using the downloading facilities provided through this global network. The effect of the network will be heavier on computer based transporters like CD-ROM. According to MMIS (Magnetic Media Information Services) Report in 1996 approximately 1/6 of the whole World-wide CD-type discs sales consist of CD-ROMs. [1] Taking into account the current capacity of the music market dominated by compact audio cassettes, and adding to it the increasing CD-ROM sales; though it at first glance seems a very optimistic forecast D_s is expected to be 60,000,000 CDs per year.

The write-once CD concept was introduced first in 1989 by **Taiyo Yuden,** a medium-sized Japanese manufacturer, and made a big success story till that point by filling the gap in data storage field presenting a 650 MB storage capacity with a medium which can be readable in almost any CD-ROM drive. The forecasts of the people in the sector was always very conservative, generally being far away from the real sales data CD-R products realized. CD-R, in the first parts of its lifetime was accepted by the professional sound studios, as a recording medium for the masters. The booming of CD-R began after the big reduction in CD-R Recorders prices. (The prices are still going down) This allowed CD-R to be a cost effective media for backingup computer information. It even replaced CD-ROM in small lot applications, as for its alternative CD Production requires a fixed cost for the setup of production and larger lead time as it should be manufactured in a plant. For small lots of production CD-R is a less-expensive and fast solution. Although there is a big jump in the demand, most of the expertes of the Industry believe that this increase will not go on in that manner and in 5 or 6 years it will reach to a steady state. The worldwide CD-R Sales (till year 1997 actual data, others are forecasts) are presented in Figure 4.2 [1]. In year 2000 it is expected that the demand will be in 1 billion units and reach the steady state position.

There are many reasons behind those forecasts. One of the main reason is that there are many alternative products in removable data storage, these are different types of optical, magneto optical and solid state systems. There is a limit that an optical system can reach in capacity, however in the long run there will be new products with better performances than the optical devices for data storage. The need for bigger storage and retrieval speed seems technically not

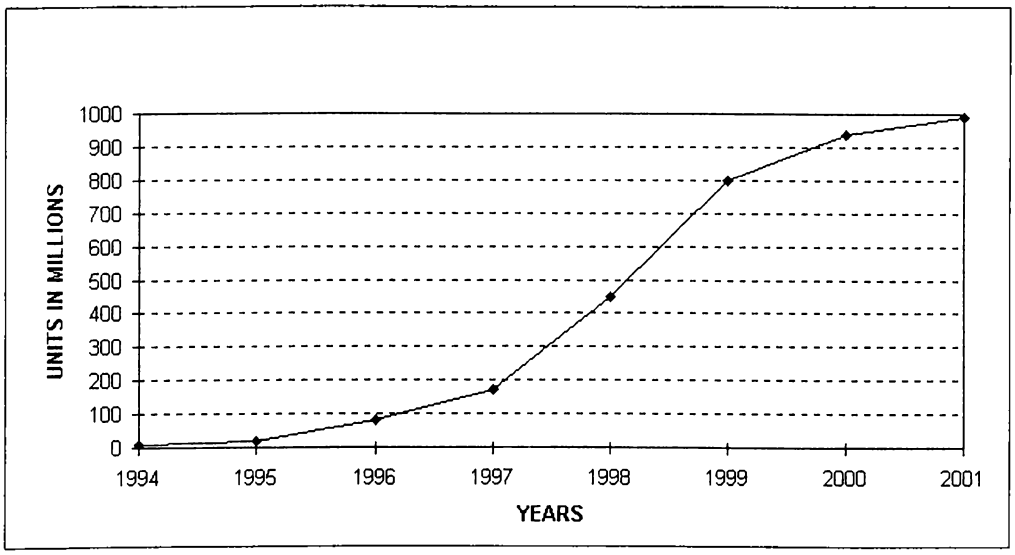

Figure 4.2: The worldwide CD-R Sales (After 1997 the data is forecasted)

possible to be met by CD-R.

There is another product called DVD, now being entered to the consumer market recently. DVD is an improved version of CD product family which can be read and written by DVD equipments, although they are in same dimensions with CD, but uses different type of laser. The product will have more than one side, which also increases the capacity. The storage capacity of 4 GB (gigabyte) per side can be achieved. This allows the recording of even a complete motion picture. Though CD-Audio will not be much effected, because CD gives enough capacity for audio, CD-ROMs and CD-Rs are expected to be replaced by DVD. The recording of motion pictures will enlarge the optical disc market as a new business replacing the classical VHS players in home usage.

According to the data and forecasts given by MMIS, a similar behaving pattern for CD-R Demand is constructed. The major difference with CD-R and CD demand is that, CD-R will have a very speed increase in its early periods and will eventually reach the saturation point.

In reality after the saturation point, demand rate should begin to decrease. From this point on demand rate will be lower than the peak level reached.

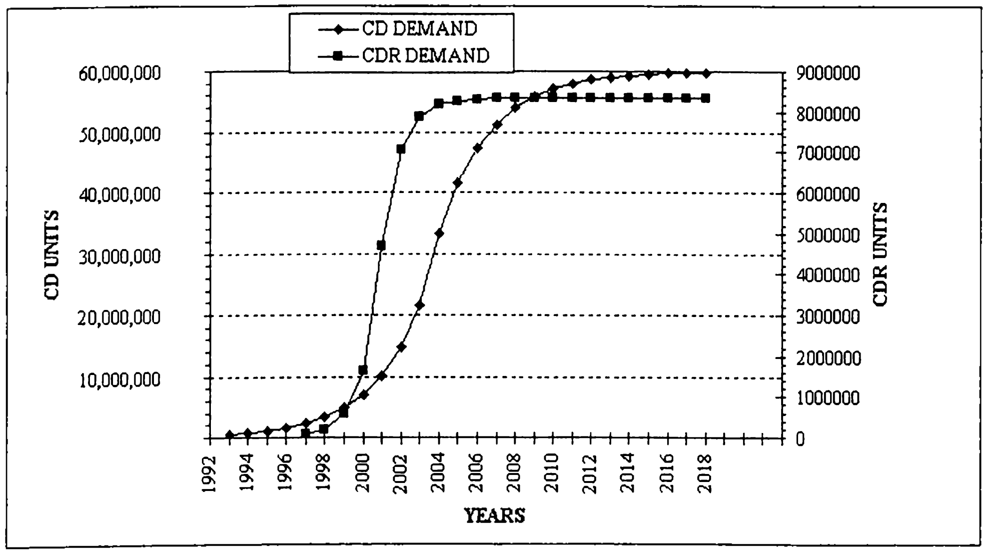

Figure 4.3: Demand Functions for CD and CD-R

After that period, expansion decisions will not be considered as there will never be need for new capacities. The demand function considered in our model simply ignores those decreasing periods of demand rate function and assumes a nondecreasing demand. If the planning horizon T is close to the time periods where decrease in demand begins, then this nondecreasing demand rate function will be very similar to the real demand rate for the time horizon considered.

The demand functions are demonstrated in Figure 4.3. The parameters of demand are summarised in Tables 4.2 and 4.3.

4.2.2 Cost Parameters

Capital Cost Parameters

The capital cost is the amount of money spent for the installation of the facilities. The capacity of a single production line is a known value and is a function of the *cycle time*. The calculation of the capacity for a single line depend on

for CD		for CD-R	
uptime	80%	uptime	80%
yield	100%	yield	80%

Table 4.1: Uptime and Yields for Production Lines

the *uptime* and *yield* of the line. *Uptime* is the percentage of time the machine is actually in operation, and *yield* is the percentage of the good products system can produce. If *cycle time* is given in seconds then the calculation of the capacity per year denoted Ω is given as follows :

$$\Omega = (((3600 * 24 * uptime) / cycle) \, yield) \, 350$$

The price of machinery and equipment do change with time , as new features or new improvements are incorporated by the equipment manufacturer firms to give advantages to the users. Generally the effect of improvements varies among industrial sectors. In the heavy industries the installation period and usage periods of the facilities are large, and so the improvements are slow. The effects of the improvements could be ignored in small time periods. However in plastics and electronics industry, for example, there is a continuous and rapid improvement in the type and characteristics of machinery, equipments and even in raw materials. In those types of the sectors even in a one year period a new technology can make the equipment in hand as an obsolescent. The price of the installation can increase because of the increase in the complexity of the machines or decrease as the methods used in the production of these facilities may improve or the production rate of the new machines can be much more higher resulting in a decreased cost per capacity.

The values for uptime and yields are defined in Table 4.1 for the problem examined. The days in a year is assumed to be 350 in capacity calculations. Yields for CD-R is lower, this is because of the complexity in the spinning and other extra production processes.

Capacities can be installed as the multiples of a single lines' capacity. The cost of the facilities have two components, one of them is the fixed charge, the other one is the variable part propotional to the capacity, y. The cost of a single line then can be expressed as $A(t)+B(t)y$, where $A(t)$ the constant part of the cost does not change with time. However, $B(t)$ decreases as the cycle time of the new facilities decrease because of the technological improvements, however $B(t)y$ does not change. This means the total cost of a single line is constant over time, but the capital cost per unit capacity decreases.

$A(t)$ consist of the costs of know-how transfer, technical trainings, off-line quality control equipments, the costs related with the building (like clean air devices, devices for pressured air, etc) and other costs that are not directly related with the amount of capacity. These are the costs incurred at every expansion independent of the capacity. $B(t)$ is the unit cost per unit capacity of the direct manufacturing equipments; these costs are effected with the size of the capacity. A is determined as \$ per each expansion and $B(t)$ as \$ per capacity per expansion. Capacity is calculated as unit products per year.

The reduction of cycle time is modeled as a decay function with a lower bound. The reduction of the cycle time is very attractive for CD machines, from the first introduction of these production lines till now the cycle time improved from 10 seconds to 4 seconds. The major manufacturers of the equipments expect it to be smaller than 3 sec. in the near future [12]. The $B(t)$ function directly depends on *cycle time*, so the function of the change with time for cycle time defines the change for $B(t)$. Figure 4.4 shows the cycletime variation with years and Figure 4.5 shows the variation of $B(t)$ versus time. The values of these parameters are given in Tables 4.2 and 4.3.

The cycle time at time t, which is $\mu(t)$ is defined as :

$$\mu(t) = \mu_s + (\mu_0 - \mu_s)\, e^{-k_B t} \tag{4.4}$$

Where μ_0 is the initial cycle time and μ_s is the limiting cycle time.

The capacity of a single machine, if installed at time t, $\Omega(t)$ is :

$$\Omega(t) = (((3600 * 24 * uptime)/\mu(t))\,yield)\,350 \tag{4.5}$$

As Equation 4.5 relates capacity directly with cycletime, and the cost of a single machine is constant over time, then $B(t)$ is defined like Equation 4.4 :

$$B(t) = B_s + (B_0 - B_s)\,e^{-k_B t} \tag{4.6}$$

As far as initial value of $B(t)$ which is B_0 is determined, B_s, the limiting point of $B(t)$ in Equation 4.6 can be calculated as :

$$B_s = B_0 \frac{\mu_0}{\mu_s} \tag{4.7}$$

The excess capacity of a CD-R line can also be used for the production of CDs. The capacity for CD and CD-R production of the line is different, however; so there is a conversion factor which converts the excess capacity define in terms of CD-R units/year to CD units/year which is taken as 2. This conversion factor is calculated as the current cycle time of CD-R line divided by cycle time of CD line which is about 2.5 , but the assumption behind reducing it to 2 lies on the compensation for the set-up times spent for converting the CD-R line for CD production.

Operating Cost Parameters

There are various factors affecting the operating cost; direct and indirect labor costs, direct and indirect materials cost, and general expenditures. The determination of the operating expenses for CD is simpler because of the existing data, the operating cost of CD-R, however are calculated from the data obtained from various major equipment manufacturers. There is a fixed charge of the operating cost, which is independent of the production quantity. These are the costs that have to be paid if the facility is in operation, generally these costs include indirect labor, material costs and indirect general expenditures

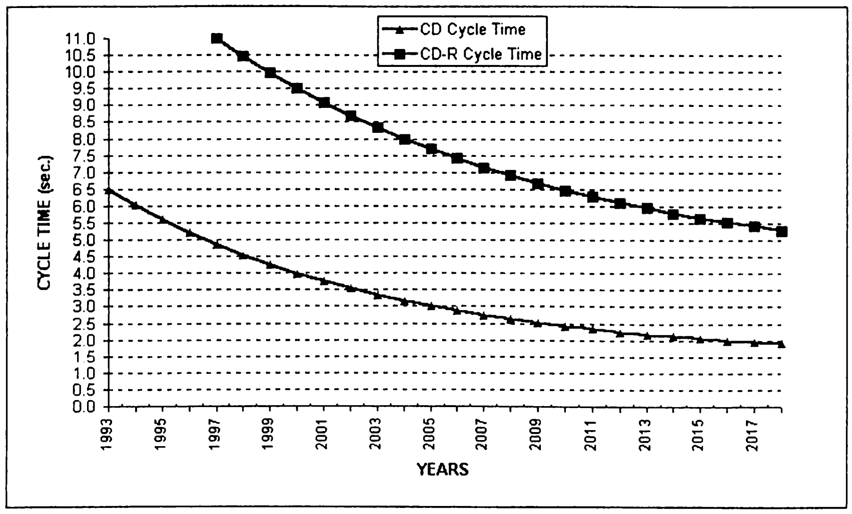

Figure 4.4: Cycle Time Versus Years for CD and CD-R

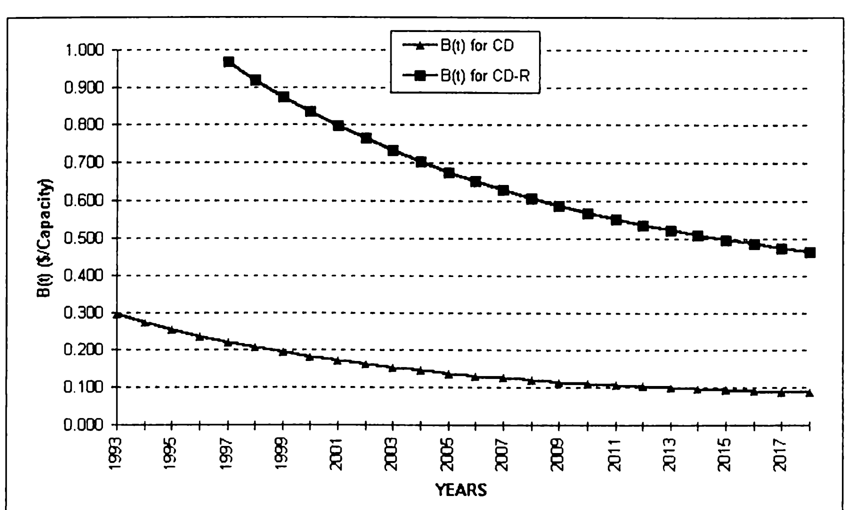

Figure 4.5: B(t) versus Years for CD and CD-R

(the majority of that component is maintenance cost and other general expenditures not directly related to CD manufacturing). The cost is defined as

$$m\,(t,p) = \alpha + \beta_0 e^{-k_\beta t} p \tag{4.8}$$

α is \$ per unit time, as unit time is years, it becomes \$ per year, and β is \$ per unit time per unit product. p is the production rate which is units per year. $m\,(t,p)$ is the operating cost per unit time.

As can be seen from the formulation α is assumed to be constant over time, but the variable component changes. The major difference in the technology that effect the cost is the introduction of new materials and the decrease in the consumption of the materials, so the technological innovations have main effect over the variable part of the operating cost. [3] The change of β with time can be observed in Figure 4.6.

The parameters for the operating cost are shown in Tables 4.2 and 4.3.

Interest rate defined as r is selected to be 0.1, and the planning period T is 25 for CD, 21 years for CD-R Problem. This is because of the 4 years time phase between the generation of demands for these two product family.

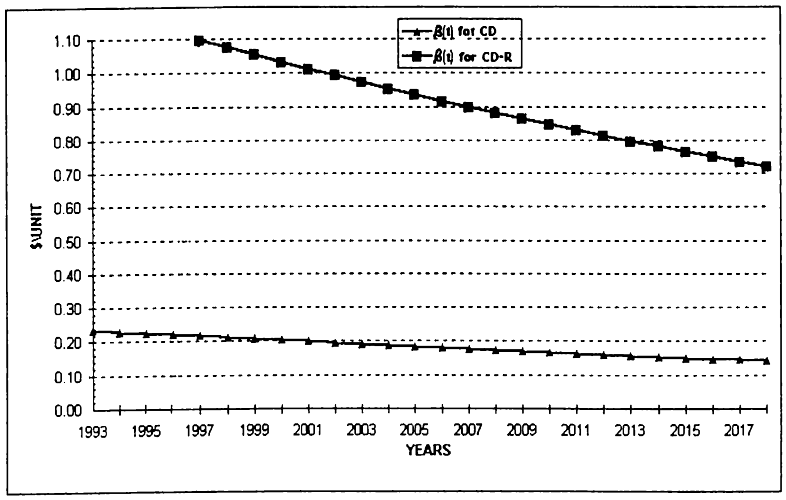

Figure 4.6: The Variable Part of Operating Cost Versus Time For CD and CD-R

D_0	D_s	Φ	δ
90000	8330000	4	1.05
Parameters for Demand			
α	β_0	k_β	
90000	1.10	0.02	
Parameters for Operating Cost			
A	B_0	B_S	k_B
800000	0.966	0.351	0.08
Parameters for Capital Cost			
μ_0	μ_s		
11	4		
Parameters for CycleTime			
$T = 21$ years, $r = 0.1$			

Table 4.2: Parameters for CD-R Capacity Expansion Problem

D_0	D_s	Φ	δ
530000	60000000	10	0.37
Parameters for Demand			
α	β_0	k_β	
120000	0.24	0.02	
Parameters for Operating Cost			
A	B_0	B_S	k_B
400000	0.296	0.068	0.1
Parameters for Capital Cost			
μ_0		μ_s	
6.5		1.5	
Parameters for CycleTime			
$T = 25$ years, $r = 0.1$			

Table 4.3: Parameters for CD Capacity Expansion Problem

Chapter 5

Mathematical Model For Capacity Expansion Problem

The model constructed in this study is aimed to solve two versions of the problem. The first version assumes that there are technological innovations that affect only the capital costs. The operating cost is assumed to be a constant rate, same for all periods. This first model is a simplification of the real problem..

The second version of the model that reflects the real life situation in a more realistic sense takes into account the technological innovations that affect the operating costs as well. Because of the time dependant operating costs, the facilities are made up of machines which may have different operating costs. This complicates the problem. To simplify the complication, lower and upper estimates for operating cost are used in the second part of the model.

5.1 Mathematical Model When There is a Single Capacity Type

5.1.1 Model for Constant Operating Costs

Figure 5.1 shows a simple non linear increasing demand function, with times and sizes of capacity expansions. The first expansion is made at time t_0 which is in a size that can meet the demand increase from time t_0 to time t_1. As seen from the figure; n^{th} expansion is executed at time t_{n-1} with a size of $y\left(t_{n-1}\right)$ which is in an amount greater than or equal to the increase of demand between time t_{n-1} and t_n. If it is assumed that capacities are available in every unit; not discrete, than the expansion sizes will be just equal to the demand increase between two consecutive expansions.

The variables of the model are:

t : time unit.

$D(t)$: Demand function, the demand for the product at time t.

$C(y\left(t\right))$: The capital cost of the installation of the facility with capacity $y\left(t\right)$.

$m\left(t\right)$: Operating cost per unit time at time t.

r : The discount rate.

t_i : The time of the $(i+1)^{th}$ expansion.

$y\left(t_i\right)$:The size of $(i+1)^{th}$ expansion.

The Operating Cost Function $m\left(t\right)$, in this section is defined with a constant coefficient β, that is equal to $\beta p\left(t\right)$; where $p\left(t\right)$ is the amount of production at time t. As production is always equal to demand, the operating cost at time t becomes $\beta D\left(t\right)$. In the original problem modeled however as explained

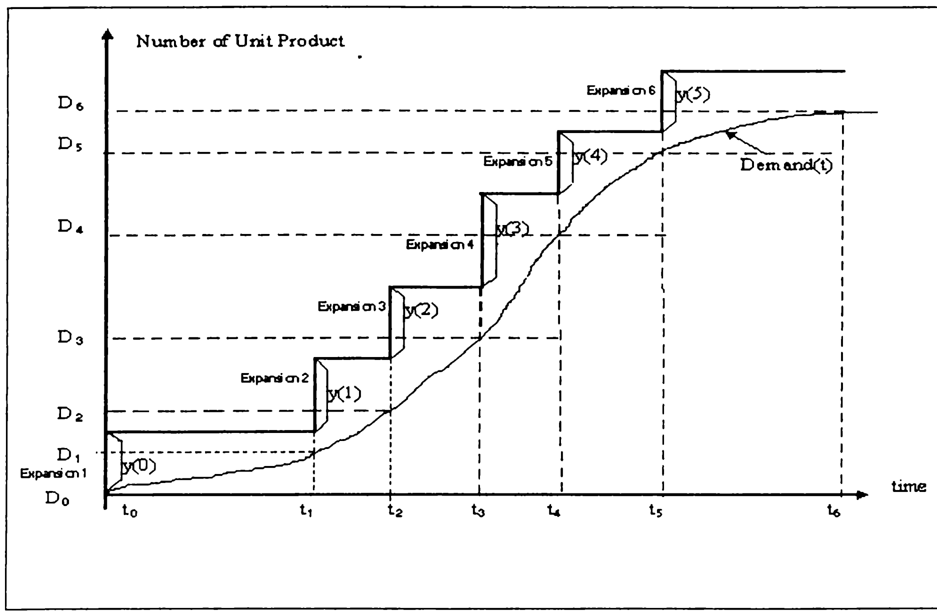

Figure 5.1: The time and sizes of Capacity Expansions.

in Chapter 4, this cost is changing with time because of the technological improvements, and also there is another component of the operating cost that does not depend on the production amount. The solution provided with the constant cost assumption will be improved in the forthcoming section to solve the exact problem.

The $D(t)$ function is defined as a continuous function of time, however the expansion times t_i's are considered to be discrete. In other words, the capacity expansion decisions can be taken for example at the end of the year (or any other time unit), not in between them. So in the mathematical model t_i's are defined as integer time periods $0, 1, ..., n$.

The measure of effectiveness to be minimized is the *time average unit cost per unit product.* This measure was explained in Chapter 4. The calculation of the costs of the installation and operation of the facility is then:

c : time average unit cost per unit product.

The operating costs and the expansion costs, in other words the investment costs are summed up and discounted to the present value with a positive discount rate r. Also this cost figure is equated to the total production from time 0 up to time T, which is the time horizon. The value c is then the time averaged unit cost per unit production that can be regarded as the price for the product which will pay off all the cash outflows in the given time period.

$$\int_0^T cD(t)e^{-rt}dt = \int_0^T \beta D(t)e^{-rt}dt + \sum_{i=0}^{n-1} \{C[y(t_i)]\} e^{-rt_i} \tag{5.1}$$

$i = 1, 2, ...n.$

Where $t_{n-1:}$:is the time of the last expansion in $(0, T)$, and $t_0 = 0$.

y(t_i) is the size of expansion, and as there are no inventories and the demand increase should be met by the production at that period, the production rate at any time t is equal to the demand rate at time t.

From the Equation 5.1 c can be calculated as:

$$c = \frac{\int_0^T \beta D(t)e^{-rt}dt + \sum_{i=0}^{n-1} \{C[y(t_i)]\} e^{-rt_i}}{\int_0^T D(t)e^{-rt}dt} \tag{5.2}$$

Then the objective is to minimize c.

In the model as there are no inventories and it is assumed that demand must be satisfied, then the production rate is equal to demand rate $D(t)$. The denominator in Equation 5.2 is in fact the integration of the discounted production function.

Dynamic Programming Model

We use dynamic programming approach for solving the capacity expansion problem defined in 5.1.1. The reason for choosing the dynamic programming approach is the fact that its structure best suits the problem. The problem has a time dependent nature and at certain points of time a decision is taken place, so it seems reasonable that the decision variables at each time stage can decompose the problem. Dynamic programming is in a sense solving many problems containing small number of decision variables each for one problem with many decision variables. In certain types of problems this approach is compositionally more advantageous.

In general the structure of a dynamic programming model is made up of the following:

1. a sequence of state variables.
2. a sequence of decision variables. (or control variables)
3. recurrence relations.

In our model the number of capacity expansions is taken as stage variables. In the model t_n time of the $(n)^{th}$capacity expansion is the state variable, and x_n, the time elapsed units between the n^{th} and $(n-1)^{th}$ expansions as the decision variable. The stage transformation function which determines t_{n-1} when current state is t_n and the action taken is x_n, can be stated as:

$$t_{n-1} = P_n(x_n, t_n) \tag{5.3}$$

$P_n(x_n, t_n)$ is the *state transition function*, which is $t_{n-1} = t_n - x_n$. There is a cost associated with each transformation as a function of the current and the previous state and the decision variable of the current state can be defined as c_n:

$$c_n = c_n(x_{n,}t_{n,}t_{n-1}) \tag{5.4}$$

As in 5.3 t_{n-1} is a function of x_n and t_n, 5.4 can be rewritten as:

$$c_n = c_n(x_{n,}t_n) \tag{5.5}$$

$f_n(t_n)$ is the minimum time average cost per unit product up to time t_n in stage n is called as the *state function.*

In order to solve the problem using dynamic programming the definition of the functional equation or the *recursive relation* should be identified after the determination of the state, and decision variables as well as stage costs. The functional equation is then:

$$f_n(t_n) = \min_{1 \le x_n} \left\{ \begin{bmatrix} f_{n-1}(t_{n-1})\, z(t_{n-1}, r) + \\ c_n(x_{n,}t_n) \end{bmatrix} \frac{1}{z(t_n, r)} \right\} \tag{5.6}$$

Where function z defined in Equation 5.6 is :

$$z(T, r) = \int_0^T D(t) e^{-rt} dt \tag{5.7}$$

where $n = 1, 2, ...$ and $t_n = n, n+1, ...\infty$.

Stage, n : Number of expansions up to time t_n.

State, t_n : Time of the $(n+1)^{th}$ expansion.

Decision variable at stage n, x_n : The time elapsed between t_{n-1} and t_n..

$t_0 = 0$ is the time of the initial installation which is defined as the first expansion.

In all parts of the model as there are no inventories and demand must be satisfied, production rate at any time t is equal to demand rate at time t.

The lower bound for x_n is 1, as the time between two expansions can be at least one time unit. This is because of the assumption that any expansion decision is given at discrete time units.

State Transition Function

$$t_{n-1} = t_n - x_n \tag{5.8}$$

where $n = 1, 2, \ldots$ and $t_n = n, n+1, \ldots$

When the values of t_{n-1} is substituted in Equation 5.6, it becomes:

$$f_n(t_n) = \min_{1 \leq x_n \leq t_n - n + 1} \left\{ \begin{bmatrix} f_{n-1}(t_n - x_n)\, z(t_n - x_n, r) + \\ c_n(x_n, t_n) \end{bmatrix} \frac{1}{z(t_n, r)} \right\} \tag{5.9}$$

The earliest time possible for the $(n-1)^{th}$ expansion is the time $(n-1)$. This defines an upper bound for x_n, which is $(t_n - (n-1))$ or $(t_n - n + 1)$, as stated in Equation 5.9.

Stage Costs The function $c_n(x_n, t_n)$ can be stated as:

$$c_n(x_n, t_n) = \int_{t_n - x_n}^{t_n} \beta D(t) e^{-rt} dt + C(y(t_{n-1})) e^{-r(t_n - x_n)} \tag{5.10}$$

If it is assumed that expansions are available in any quantity; it means the expansion sizes are not discrete then, $y(t_{n-1})$: Capacity of the n^{th} expansion at time $t_n - x_n$ or t_{n-1}, and it can be stated as:

$$y(t_{n-1}) = D(t_n) - D(t_{n-1}) \tag{5.11}$$

$$y\,(t_n - x_n) = D\,(t_n) - D\,(t_n - x_n) \tag{5.12}$$

As explained in Chapter 4, however the capacity is available in only discrete units. At any time, there is a determined capacity per unit time of a single machine, and the capacity to be installed can only be multiples of that amount. This is the difference with the expansion model shown in Figure 5.1 and the Expansion Problem of the Optical Disc Plant.

For this case, the formulation should be slightly changed. The technologically available capacity of a single machine at time t was defined to be as $\Omega(t)$ as a function of its the cycletime improvements in Equations 4.4 and 4.5. Also let the number of machines installed be π_n; then $y(t_n - x_n)$ in Equation 5.12 becomes

$$\begin{aligned} y(t_n - x_n) &= \pi_n \Omega(t_n - x_n) \\ \pi_n &\geq \left(D\,(t_n) - \sum_{i=0}^{i=n-2} y(t_i) \right) \setminus \Omega(t_n - x_n) \\ & \quad \pi_n \text{ nonnegative integer} \end{aligned} \tag{5.13}$$

The $\Omega(t_n - x_n)$ function was defined in Equations 4.4 and 4.5.

Specific Forms of Demand and Cost Function $D(t)$ and $C(x)$ functions have to be identified. These were explicitly defined in Chapter 4. The demand function was defined in Equation 4.3. Demand is an exponential function reaching to a steady state value and cost of the facility is a linear function of capacity with a fixed charge A. In other words:

$$C\,(y, t) = A + B\,(t)\,y\,(t) \tag{5.14}$$

The Equation 5.10 can be rewritten as :

$$c_n(x_n, t_n) = \beta \int_{t_n - x_n}^{t_n} D(t)\, e^{-rt} dt + (A + B(t_n - x_n)\, y(t_n - x_n))\, e^{-r(t_n - x_n)} \tag{5.15}$$

The functional form of $B(t)$ was defined in Equation 4.6. The Integral can be solved for specific demand function defined in 4.3 as :

$$\int_{t_n - x_n}^{t_n} D(t)\, e^{-rt} dt = \begin{cases} \frac{D_0}{(\delta - r)} \left(e^{t_n(\delta - r)} - e^{(t_n - x_n)(\delta - r)} \right) & \text{, if } t_n < \Phi \\ \frac{D_0}{(\delta - r)} \left(e^{\Phi(\delta - r)} - e^{(t_n - x_n)(\delta - r)} \right) + & \text{, if } t_n \geq \Phi \text{ and} \\ \frac{D_s}{-r} \left(e^{-rt_n} - e^{-r\Phi} \right) + & t_n - x_n < \Phi \\ \frac{(D_s - D_\Phi) e^{\delta\Phi}}{(\delta + r)} \left(e^{-t_n(\delta + r)} - e^{-\Phi(\delta + r)} \right) & \\ \frac{D_s}{-r} \left(e^{-rt_n} - e^{-r(t_n - x_n)} \right) + & \text{, if } t_n \geq \Phi \text{ and} \\ \frac{(D_s - D_\Phi) e^{\delta\Phi}}{(\delta + r)} \left(e^{-t_n(\delta + r)} - e^{-(t_n - x_n)(\delta + r)} \right) & t_n - x_n \geq \Phi \end{cases} \tag{5.16}$$

The integration in Equation 5.16 is used directly in Equation 5.15 and 5.7. With these functions in hand $f_n(t_n)$ function can be calculated.

Forward Algorithm With recursive function, it is possible to calculate the values of $f_n(t_n)$ for each n and for all corresponding values of t_n. Let t_n^* be the value that minimizes $f_n(t_n)$ for all possible values of t_n. Then we define function $F(n)$ as :

$$F(n) = \min_{n \leq t_n \leq T} f_n(t_n) \text{ or } F(n) = f_n(t_n^*) \tag{5.17}$$

For all possible values of n, the minimizing value of $F(n)$ is the optimal number of expansion which is called as n^* in Equation 5.18 After setting these relations with the help of the forward recursions, the optimal number of expansion can be found.

$$F(n^*) = \min_{1\leq n\leq T} F(n) \tag{5.18}$$

Algorithm 5.1 (Forward Algorithm) $f(0) = 0$, *so algorithm recursively calculate* $f_n(t_n)$:

For $s = 1, .., T$

$$f_1(t_1 = s, x_1 = s) = c_1(x_1, t_1)(1/z(t_1, r))$$

For $n = 2, ..., T$

For $t_n = n, ..., T$

For $x_n = 1, ..., t_n - n + 1$

$$f_n(t_n, x_n) = \begin{bmatrix} f_{n-1}(t_n - x_n) z(t_n - x_n, r) + \\ c_n(x_n, t_n) \end{bmatrix} (1/z(t_n - x_n, r))$$

$$f_n(t_n) = \min f_n(t_n, x_n) : 1 \leq x_n \leq t_n - n + 1$$

$$F(n) = \min f_n(t_n) : n \leq t_n \leq T \;\; F(n^*) = \min F(n) : 1 \leq n \leq T$$

After computing n^*, $F(n^*)$ the optimal policy is determined by backward substitution.

5.1.2 Mathematical Model for Time Dependent Operating Cost

In the model proposed we are going to release the assumption of the constant operating cost. This is due to the fact that there are new innovations and improvements continuously on the machinery and equipments, so as time passes new alternatives come into the market place that effects operating costs of the new facilities. We are going to define the operating cost as a function of time.

In the previous section, we defined the operating cost as m , a constant figure that is the operating cost per unit produced. In the original model, however the operating cost function has two components, one is the fixed cost, i.e.. the operating cost which do not change with the production quantity. The motivation behind that type of fixed cost component in operating cost is to account for the existence of costs like technicians salary, spare parts etc. that is not effected with increasing or decreasing production. The other part is a varying component depending on the production quantity. Here these costs are the direct material cost, energy costs, etc. Even if the capacity of the facility remains same, these costs is directly proportional to the amount of the capacity used which is the production quantity.

The rate of operating cost function is then:

$$m(p) = \alpha + \beta p\left(t\right) \tag{5.19}$$

$p\left(t\right)$: The production rate at time t..

α : The fixed rate of operating cost of the facility.

β : Operating cost per unit production of the facility.

The parameters of the operating cost function should change when the size of the facility changes.

The cost function in Equation 5.19 then becomes when technological improvements are taken into account to the format below:

$$m(p,t) = \alpha + \beta\left(t\right) D\left(t\right) \tag{5.20}$$

t : The time unit.

As production level is always equal to demand,then p is equal to $D\left(t\right)$. As explained in Chapter 4, the variable part of the operating cost component

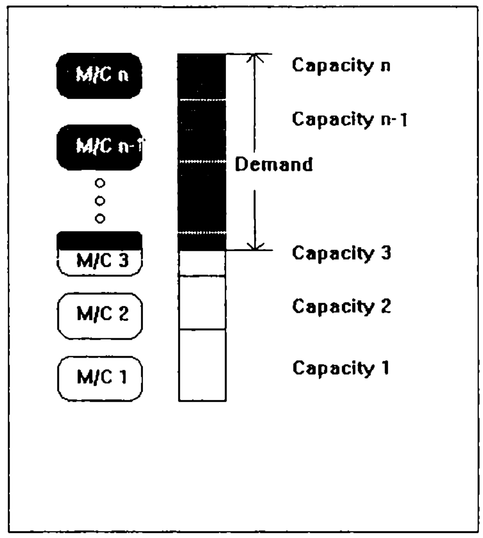

Figure 5.2: The Assignment of Demand to n Machines in the System

do change with time. This change is modeled as an exponentially decreasing function. This function was defined in Equation 4.8.

Dynamic Programming Model

There are n types of capacity in the system between the time interval (t_{n-1}, t_n), because each of the facilities have different operating cost functions. The total amount of production is divided into n parts, and so depending on the operating costs of each type of the facility and the amount of production assigned to them the total operating cost for that period is determined which is shown as Oc_n. It is the operating cost function at stage n.

The last facility added to the system will have the least operating cost, and the first installed machine will have the largest operating cost. It is logical as the aim is to minimize the cost, to use the most recently built machines for the production.

Figure 5.2 shows how the demand at time t is assigned to the machines. There are n machines in the system, the machine installed at n^{th} expansion is

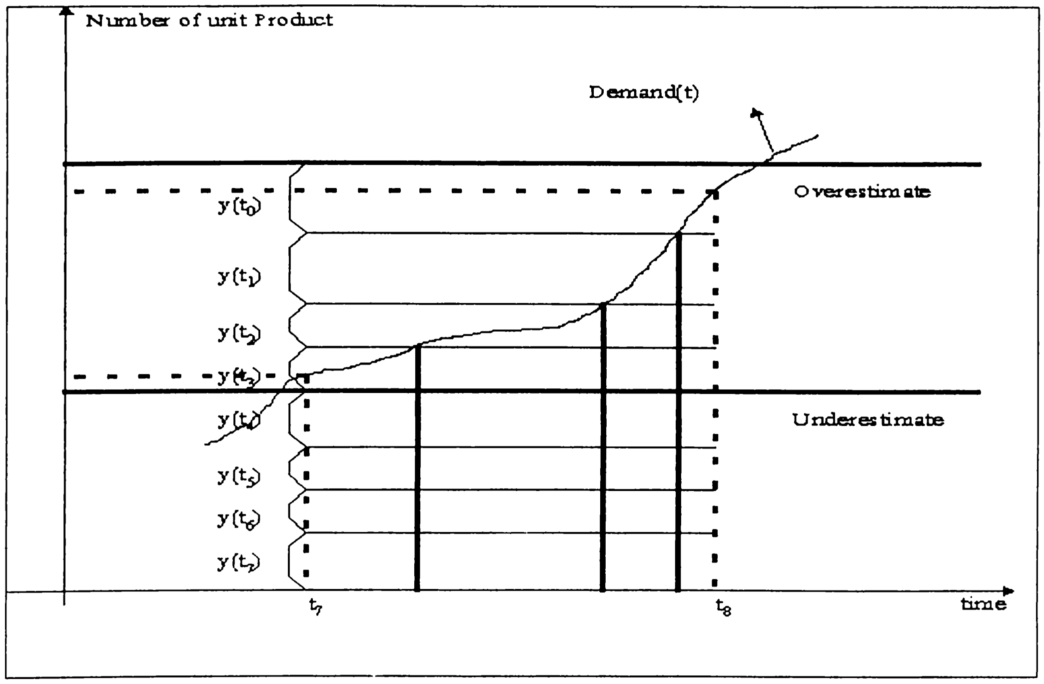

Figure 5.3: A Sample Capacity Expansion Figure showing the Underestimate and Overestimate Calculation.

called machine n, the one installed at expansion $n-1^{th}$ expansion as machine $n-1$ and so on. demand which is $D(t)$, is assigned to the machines beginning from n. The figure shows the situation where $n-4$ machines are used in full capacity. machine 3 is not used at its full capacity and machines 1 and 2 are not working because of the excess capacity in hand. During the period between the two consecutive expansions, demand increases. With the increasing demand the assignment shown in the figure may change, for example machine 3 may become fully utilized, and the remaining demand may be assigned to Machine 2.

Figure 5.3 shows a sample capacity expansion where the eigth expansion to the system is being performed. At the time of the installation only four of the lines, that are the ones most recently installed, are assigned to production at full capacity. As demand rate increases with time, however the older production lines that were not in production at t_7 begins to be used. Each time point inbetween t_7 and t_8, where the number of machines in production increases, divides the interval into subintervals. For each subinterval between the two consecutive expansions, rate of operating cost is different for each of which discounted cost is a sum of integrals. Hence, discounted operating cost at each stage is the sum of those sum of integrals. This makes the exact calculation of operating cost very complex and unnecessarily laborious. Therefore an approximate computation that consist of determining lower and upper bounds is concieved.

The upper bound, which is called overestimate, it is assumed that all of the n machines at stage n is used at their full capacity. For this case the equivalent operating cost is simply the weighted averages of the operating costs of these machines. The real operating cost is less than or equal to this calculation, as there may be older machines that are not used which will decrease the operating cost.

$$m_{over}\left(t_n, D\left(t\right)\right) = \alpha + \frac{\sum_{i=1}^{n} \beta\left(t_i\right) y\left(t_i\right)}{\sum_{i=1}^{n} y\left(t_i\right)} D\left(t\right) \tag{5.21}$$

At the beginning of stage n,which is time $t_n - x_n$, it is possible to know which machines are assigned to production at their full capacity. If only the weighted averages of the machines assigned to production at time $t_n - x_n$at full capacity is used as the operating cost function, the corresponding calculation for operating cost will be smaller than the real cost. The Equation 5.22 defines overestimate at stage $n$for $n \geq 2$. When $n = 1$, $m_{over}\left(t_n\right) = m_{under}\left(t_n\right)$ which is simply $\alpha\left(0\right) + \beta\left(0\right) D\left(t\right)$.

$$\begin{aligned} m_{under}\left(t_n, D\left(t\right)\right) &= \alpha + \frac{\sum_{j=1}^{\Psi} \beta\left(t_{n+1-j}\right) y\left(t_{n+1-j}\right)}{\sum_{i=1}^{n} y\left(t_{n+1-j}\right)} D\left(t\right), \text{ where } \Psi \text{ satisfies} \\ \sum_{j=1}^{\Psi} y\left(t_{n+1-j}\right) &\leq D\left(t_{n-1}\right) \leq \sum_{j=1}^{\Psi+1} y\left(t_{n+1-j}\right) \\ 1 &\leq \Psi \leq n-1 \text{ and } \Psi \text{ is integer} \end{aligned} \tag{5.22}$$

At stage n, the actual operating cost is smaller than the cost calculated using $m_{over}\left(t_n\right)$ and larger than $m_{under}\left(t_n\right)$.

The state, stages, and decision variables and the form of the functional equation of the new model are the same as 5.1.1. The only difference is in the $c_n(x_{n,}t_n)$ function because of the difference in costs with time. At stage n, the cost of the underestimate or overestimate is used. The functional equation is again Equation 5.6. The only difference is in the calculation of $c_n(x_n, t_n)$ function. Let $m_{over}(t_n)$ be used instead of actual operating cost calculation as:

$$\begin{aligned} c_n(x_{n,}t_n) &= \int_{t_n - x_n}^{t_n} m_{over}(t_n)\, e^{-rt} dt + \\ &\quad (A + B(t_n - x_n)\, y(t_n - x_n))\, e^{-r(t_n - x_n)} \end{aligned} \tag{5.23}$$

Equation 5.23 becomes when actual calculation of $m_{under}(t_n)$ is inserted :

$$\begin{aligned} c_n(x_{n,}t_n) &= \frac{\alpha}{-r}\left(e^{-rt_n} - e^{-r(t_n - x_n)}\right) + \\ &\quad \frac{\sum_{i=1}^{n} \beta(t_i)\, y(t_i)}{\sum_{i=1}^{n} y(t_i)} \int_{t_n - x_n}^{t_n} D(t)\, e^{-rt} dt + \\ &\quad (A + B(t_n - x_n)\, y(t_n - x_n))\, e^{-r(t_n - x_n)} \end{aligned} \tag{5.24}$$

The calculation if $m_{under}(t_n)$ is used instead of overestimate figure is similar. Also rate of operating cost can be approximated by averaging the under and overestimates as $\frac{m_{under}(t_n, D(t)) + m_{over}(t_n, D(t))}{2}$. This approximation will result in smaller deviations from actual operating cost compared to using m_{under} or m_{over} seperately. In the calculations we get results of the solution for both underestimate and overestimate, however in order to see the difference between the two figures.

5.2 Mathematical Model for Two Capacity Types

The problem that should be solved in this study consists of two different product types and two different capacity types, these are CD and CD-R. The CD production line can manufacture CD formats, however CD-R production line has the capability for manufacturing of both CD and CD-R family of products.

The capital cost of CD-R production facility is always lower than the cost of a CD production line. The cycle time of the CD-R production line is lower when it is used for the production of CD-family products. It means that for CD-R production lines there are two capacity figures, one for CD type products and the other for CD-R type products. The cost of changing from CD production to CD-R production and vice versa for CD-R production line is assumed to be zero.

The model now then has new decision variables, the time and size of the facility as well as the type of the facility to be added. The addition of a new decision variable makes the model much more complex, however using the properties of the problem the dimension of the model will be the same as the single facility model.

The demand for both CD and CD-R should be met at any time is again the main assumption. The case again does not involve any inventories for both type of products.

The first property to be used is that *the CD-R type products can only be manufactured by using the CD-R production line.* As the demand for CD-R Products to be met any time, the time and sizing decision of the CD-R production lines is independent of the time and sizing of CD production lines.

This leads to the conclusion that the time and size of the expansions for CD-R production line is simply the model explained in Sections 5.1.1 and 5.1.2. The decision variables are when to make an expansion and in which size of

CD-R production line to meet the demand of the CD-R products. Then the demand function will be the demand function for CD-R products in this model.

After the optimum time and size of CD-R production lines are determined, the time and size of the expansions for CD type products should be calculated. The demand for CD type products can be met by using CD or CD-R production lines. The second property to be used at this point is that *the capital cost of a CD-R line is always more expensive than the CD line.* The optimization criterion is to minimize the cost, then to build a CD-R line for the production of CD type products will not be optimal. The problem is then again reduced to the models in Sections 5.1.1 and 5.1.2 whit a slight modification that will include the excess capacity of the CD-R lines that are installed. The demand function will be the demand function for CD type products, and the decision variables will be the size and time of CD production line to install.

The calculation of the capacity to install for the nth expansion at time n, which is $y(t_n)$ in Equation 5.13 will be modified as:

$$\begin{aligned} y(t_n - x_n) &= \pi_n \Omega(t_n - x_n) \\ \pi_n &\geq \left(D\left(t_n\right) - \sum_{i=1}^{i=n-2} y(t_i) - \left(\lambda\left(\chi(t_n) - D'\left(t_n\right)\right)\right)\right) \setminus \Omega(t_n - x_n) \\ & \pi_n \text{ nonnegative integer} \end{aligned} \tag{5.25}$$

Where $\chi(t_n)$ is the total amount of CD-R production capacity at time t_n.The expression $\left(\chi(t_n) - D'\left(t_n\right)\right)$ is the excess capacity of the available CD-R lines which can be used for manufacturing CD type products. $D'\left(t_n\right)$ is the demand for CD-R at time t_n. The λ is a constant for the conversion of the CD-R capacity to CD capacity, as the capacity of a CD-R line if it is used for CD Production will be λ multiplied by the original CD-R capacity of the machine. λ will be greater than 1, as the production of CD is faster and less complex compared to CD-R.

The model for two different product and capacity types can then be solved using two iterative dynamic programming models, the results of the first dynamic programming model will be used as an input for the second model. The dimension of the problem remains same as the single product, single capacity type problem.

Chapter 6

Results and Sensitivity Analysis

The problem consists of two subproblems that are related to each other. The optimal solutions for the capacity expansion of CD-R production facilities is solved first, and the output of the first algorithm is used as an input for the second subproblem. The excess capacities remaining from CD-R production machines can be used for CD production, by using this data in the second algorithm the optimal capacity expansion plan for CD plant is determined.

Both of the algorithms are coded in Turbo Pascal and run on a standard PC. The computational speed of the algorithms depend on the value of the fixed finite time horizon T. 25 or 30 years are reasonable horizons for the problem in hand. The computation time of the algorithms were satisfactory for those time periods like one or two minutes. For such long time planning problems this computation time is already enough, however this time can be reduced even further by using the fact that the capacities can only be certain discrete units. Because of this property, there is a limit for the number of expansions n, call it $n^{'}$; there is no need to make any expansion after $n^{'}$ as the capacity in hand can already meet the coming demand. This property holds if the demand pattern reaches to a limiting value or a steady state value as in the case of the problem examined. The algorithm as a result after setting the $n^{'}$ value will not do any calculations for n greater than or equal to $n^{'}$. This reduces the computational time to an extent depending on the problem parameters.

The algorithms are coded using array structures, resulting increase in the memory used during the execution as T increases, however for the time horizons studied even on a standard PC memory was enough to run the program. The memory requirement can also be reduced using the property described above.

6.1 Optimal Solutions of the Problem

The estimation of the parameters and the functions for the problem was described in details in Chapter 4. The Tables 4.2 and 4.3 give the parameters of the model for costs and demand curves. Because of the phase difference between the origination points of demand for CD and CD-R products, the beginning time value for CD-R is year 1997 and CD is year 1993. This means $t = 0$ time value for CD problem represent year 1993, where as for CD-R problem year 1997. The time horizon T is taken as 21 and 25 respectively for CD-R and CD in order to get the solutions for the same ending years.

The outputs show that cost function $f_n(t_n)$ decreases as t_n increases for all n. This is logical because as time increases, the capital costs and operating costs per unit product decreases because of the technological improvements. Also as demand increases the amount of installed capacities increases as time passes. The discount rate r has also an effect, as the money spent at a future time has to be discounted to present time to a smaller amount. As the time horizon T is the planning period, for any n, the optimal $f_n(t_n)$ value will be $f_n(T)$. Consequently, the optimal number of expansions will be the minimizing value of n for $f_n(T)$ for all feasible n values. The outputs of the programs give values for n^* as 3 for CD-R expansions and 6 for CD problem, where n^* is the optimal number of expansions. The Tables 6.1 and 6.2 show the numerical values of $f_n(t_n)$ function for CD problem. Table shows that the function decreases as t_n increases. The behavior of the function for CD-R problem is also the same, for CD-R problem. The number of expansions that is n, larger than 3 for

CD-R problem and 8 for CD problem are illogical and not considered. This is because of the fact that, 3 expansions for CD-R and 8 expansions for CD already builds up a total capacity greater than the maximum level demand rate reaches. Hence, to make an extra expansion becomes unnecessary. The cost function for CD-R expansions are shown in Table 6.3. The values given are calculated for overestimated operating costs.

The Tables 6.4 and 6.5 shows the optimal cost figures and optimal expansion policies. The optimizing values show that the time average unit cost per one CD-R will be 1.2150 $ and for CD 0.2390 $ when overestimate is used for the operating cost.

The optimal expansion policies and costs for the underestimates of the operating costs are given in Tables 6.6 and 6.7. The actual cost is in between the underestimated costs and overestimated costs, as can be seen from the Tables there is 0.02 $ difference between the 2 estimates in CD-R and 0.01 $ in CD. This shows that both the underestimated and overestimated model give quite good estimates to the real optimal cost value.

6.2 Sensitivity Analysis

The results in the previous section showed that both the underestimate and the overestimates are good approximations. In the sensitivity analysis studies the results are obtained by using the overestimate for the operating cost.

6.2.1 The Effect of Discount Rate

The discount rate r, is simply the time value of money or in other words the cost of the money. In countries like Turkey that has scarce investment resources, money is more expensive compared to other wealthy countries. Generally the determination of discount rate depend at which rate a firm can borrow or use money. This rate, although may not have dramatic changes in the short

$n = 1$		$n = 2$		$n = 3$		$n = 4$	
t_n	$f_n(t_n)$	t_n	$f_n(t_n)$	t_n	$f_n(t_n)$	t_n	$f_n(t_n)$
1	2.8930	1		1		1	
2	1.4620	2		2		2	
3	0.9793	3		3		3	
4	0.7366	4		4		4	
5	0.5916	5		5		5	
6	0.6348	6	0.6324	6		6	
7	0.5305	7	0.5082	7		7	
8	0.5296	8	0.4374	8		8	
9	0.5111	9	0.4160	9	0.4300	9	
10	0.5248	10	0.3831	10	0.3669	10	0.3995
11	0.5406	11	0.3671	11	0.3401	11	0.3516
12	0.5355	12	0.3415	12	0.3188	12	0.3243
13	0.4926	13	0.3240	13	0.3043	13	0.2999
14	0.4644	14	0.3059	14	0.2932	14	0.2883
15	0.4455	15	0.2988	15	0.2813	15	0.2763
16	0.4214	16	0.2891	16	0.2742	16	0.2678
17	0.4137	17	0.2820	17	0.2681	17	0.2614
18	0.3993	18	0.2765	18	0.2634	18	0.2566
19	0.3880	19	0.2723	19	0.2606	19	0.2528
20	0.3791	20	0.2689	20	0.2574	20	0.2498
21	0.3719	21	0.2661	21	0.2548	21	0.2474
22	0.3659	22	0.2639	22	0.2525	22	0.2454
23	0.3610	23	0.2620	23	0.2507	23	0.2437
24	0.3568	24	0.2604	24	0.2491	24	0.2423
25	0.3533	25	0.2591	25	0.2478	25	0.2411
Values for $n \geq 9$ are illogical.							

Table 6.1: The Numerical Values of Cost Function for CD Problem - Part 1 (Overestimated Operating Costs)

$n=5$		$n=6$		$n=7$		$n=8$	
t_n	$f_n(t_n)$	t_n	$f_n(t_n)$	t_n	$f_n(t_n)$	t_n	$f_n(t_n)$
1		1		1		1	
2		2		2		2	
3		3		3		3	
4		4		4		4	
5		5		5		5	
6		6		6		6	
7		7		7		7	
8		8		8		8	
9		9		9		9	
10		10		10		10	
11	0.3658	11		11		11	
12	0.3263	12	0.3377	12		12	
13	0.3008	13	0.3144	13	0.3174	13	
14	0.2884	14	0.2893	14	0.2964	14	
15	0.2759	15	0.2762	15	0.2827	15	
16	0.2670	16	0.2672	16	0.2728	16	
17	0.2604	17	0.2604	17	0.2656	17	
18	0.2554	18	0.2553	18	0.2618	18	
19	0.2515	19	0.2513	19	0.2576	19	
20	0.2484	20	0.2482	20	0.2540	20	0.2540
21	0.2458	21	0.2456	21	0.2510	21	0.2510
22	0.2438	22	0.2435	22	0.2487	22	0.2486
23	0.2420	23	0.2417	23	0.2467	23	0.2466
24	0.2406	24	0.2402	24	0.2450	24	0.2449
25	0.2394	25	0.2390*	25	0.2436	25	0.2435
Values for $n \geq 9$ are illogical.							

Table 6.2: The Numerical Values of Cost Function for CD Problem - Part 2 (Overestimated Operating Costs)

$n=1$		$n=2$		$n=3$	
t_n	$f_n(t_n)$	t_n	$f_n(t_n)$	t_n	$f_n(t_n)$
1	22.9748	1		1	
2	7.2932	2		2	
3	3.2822	3		3	
4	2.9107	4	2.7972	4	
5	2.3971	5	2.0409	5	2.0409
6	1.9191	6	1.6821	6	1.6994
7	1.7067	7	1.5225	7	1.5294
8	1.5904	8	1.4352	8	1.4364
9	1.5181	9	1.3809	9	1.3785
10	1.4690	10	1.3440	10	1.3392
11	1.4338	11	1.3176	11	1.3110
12	1.4074	12	1.2978	12	1.2899
13	1.3870	13	1.2824	13	1.2736
14	1.3708	14	1.2703	14	1.2606
15	1.3470	15	1.2604	15	1.2501
16	1.3380	16	1.2523	16	1.2415
17	1.3380	17	1.2456	17	1.2344
18	1.3305	18	1.2399	18	1.2283
19	1.3240	19	1.2351	19	1.2232
20	1.3186	20	1.2310	20	1.2180
21	1.3138	21	1.2275	21	1.2150*
$t_1^*=21$		$t_2^*=21$		$t_3^*=21$	
Values for $n \geq 4$ are illogical.					

Table 6.3: The Numerical Values of Cost Function for CD-R Problem (Overestimated Operating Costs)

n^*	t_n^*	$F(n^*)$
3	21	1.2150
Optimal Expansion Times and Sizes		
Expansion Time	No. of Machines Installed	Installed Capacity
0	1	1,759,418
3	2	4,473,317
4	1	2,422,943
Installed Capacity is CD-R Units Per Year.		

Table 6.4: The Optimal Solution for CD-R Capacity Expansions (Overestimated Operating Costs)

n^*	t_n^*	$F(n^*)$
6	25	0.2390
Optimal Expansion Times and Sizes		
Expansion Time	No. of Machines Installed	Installed Capacity
0	1	3,721,846
5	1	6,136,286
8	1	8,238,121
10	2	20,234,054
11	1	11,181,044
13	1	12,638,174
Installed Capacity is CD Units Per Year.		

Table 6.5: The Optimal Solution for CD Capacity Expansions (Overestimated Operating Costs).

n^*	t_n^*	$F(n^*)$
2	21	1.1898
Optimal Expansion Times and Sizes		
Expansion Time	No. of Machines Installed	Installed Capacity
0	1	1,759,418
3	2	4,473,317
4	1	2,422,943
Installed Capacity is CD-R Units Per Year.		

Table 6.6: The Optimal Solution for CD-R Capacity Expansions (Underestimated Operating Costs)

n^*	t_n^*	$F(n^*)$
5	25	0.2292
Optimal Expansion Times and Sizes		
Expansion Time	No. of Machines Installed	Installed Capacity
0	1	3,721,846
5	1	6,136,286
8	2	16,566,242
11	2	22,362,088
13	1	12,638,174
Installed Capacity is CD Units Per Year.		

Table 6.7: The Optimal Solution for CD Capacity Expansions (Underestimated Operating Costs)

runs, may change with time if duration is large just as the situation exist in the problem studied. The sensitivity of the model to the discount rate is important from this aspect. Another valuable insight gained from this analysis is the effect of the cost of money on the final cost of the product. This shows the advantages of a firm that can use smaller discount rates over the firms which have large rates; the former ones can produce the same product at lower costs.

With the increasing interest rates as mentioned above it is expected that the time average unit cost should increase as the cost of money increases. As the Tables 6.1 and 6.2 show the result is just as expected. There is 15% change in cost of a single CD-R if interest rate changes from 0.1 to 0.2. This change is very serious especially for the optical and magnetic media sector, specifically if the firm's main point of competition is price.

It is also expected that as the interest rates increases the number of expansions can decrease, this is because as the cost of money increases to tie up money by making bigger investments is disadvantageous compared to the gain taken by economies of scales. However, as can be seen from the Tables 6.1 and 6.2, the number of optimal expansions is not sensitive to discount rate. In the range examined the rate does not change the optimal expansion numbers. Only for CD expansions, for a very small rate of 0.01 the expansion times changes. This may be because of the discrete units available for expansions, and the strict increase in demand. There are already not so much alternative to meet the demand with the discrete large capacities available.

r	n^*	$F(n^*)$	**Optimal Expansion Policy**
0.01	3	1.1270	(0,3,4)
0.05	3	1.1602	(0,3,4)
0.10	3	1.2150	(0,3,4)
0.15	3	1.2857	(0,3,4)
0.20	3	1.3731	(0,3,4)
0.25	3	1.4784	(0,3,4)
0.30	3	1.6031	(0,3,4)

Table 6.8: Effect of interest rate on the Optimal Policy for CDR Problem

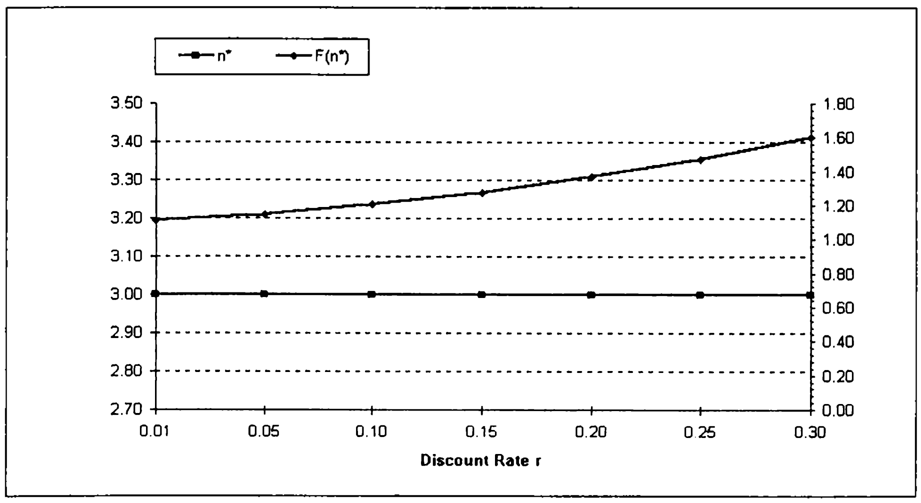

Figure 6.1: n^* and $F(n^*)$ versus Interest Rate r. (CD-R Problem)

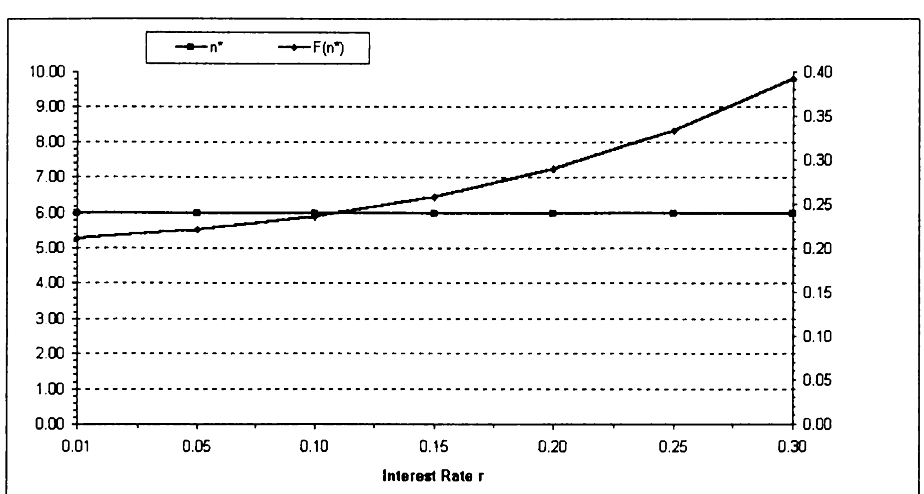

Figure 6.2: n^* and $F(n^*)$ versus Interest rate r. (CD Problem)

r	n^*	$F(n^*)$	**Optimal Expansion Policy**
0.01	6	0.2173	(0,3,6,9,11,17)
0.05	6	0.2246	(0,5,8,10,11,13)
0.10	6	0.2390	(0,5,8,10,11,13)
0.15	6	0.2603	(0,5,8,10,11,13)
0.20	6	0.2912	(0,5,8,10,11,13)
0.25	6	0.3344	(0,5,8,10,11,13)
0.30	6	0.3925	(0,5,8,10,11,13)

Table 6.9: Effect of interest rate on the Optimal Policy for CD Problem

6.2.2 Sensitivity of the Model to the Technological Developments

Sensitivity to the Technological Developments in Operating Costs

The model assumes that there is an exponential decrease in the operating cost rate, and the rate for this decreases, k_β is taken as 0.02. The sensitivity of the model to this rate shows how the expansion decision and costs are affected if the technological improvements are faster or slower than the expected rate.

The effect of the operating cost is examined in a range where k_β is taken in the range of $[0, 0.04]$. The value of 0 means that the operating cost does not change with time or there is no technological improvements affecting the operating cost. At the maximum value 0.04, the operating costs for CD-R and CD at the end of the planning horizons becomes 65 and 8 cents respectively which is a highly optimistic expectation for the reduction of costs. This range is enough for getting an insight about the possible difference among the expectations of operating cost reductions.

As expected naturally the increase of k_β reduces the operating costs, so the optimal cost decreases, however the optimal policies are not affected with this change. Only for the case of assumption that there are no technological improvements the n^* value becomes 2 and 5 respectively for CD-R and CD. This is because of the fact that the cost advantage of making an expansion in the future do not exist for this case.

k_β	n^*	$F(n^*)$	**Optimal Expansion Policy**
0	2	1.2624	(0,3)
0.010	3	1.2426	(0,3,4)
0.015	3	1.2287	(0,3,4)
0.020*	3	1.2150	(0,3,4)
0.025	3	1.2016	(0,3,4)
0.030	3	1.1883	(0,3,4)
0.035	3	1.1753	(0,3,4)
0.040	3	1.1625	(0,3,4)

Table 6.10: The effect of k_β to the Optimal Expansion Policy of CDR.

k_β	n^*	$F(n^*)$	**Optimal Expansion Policy**
0	5	0.2753	(0,5,8,10,13)
0.010	5	0.2565	(0,5,8,10,13)
0.015	6	0.2475	(0,5,8,10,11,13)
0.020*	6	0.2390	(0,5,8,10,11,13)
0.025	6	0.2309	(0,5,8,10,11,13)
0.030	6	0.2232	(0,5,8,10,11,13)
0.035	6	0.2159	(0,5,8,10,11,13)
0.040	6	0.2090	(0,5,8,10,11,13)

Table 6.11: The effect of k_β to the Optimal Expansion Policy of CD.

Sensitivity to the Technological Developments in Capital Cost

The model assumes slightly different rates for the technological development rate of the capital cost for CD-R and CD, this is mainly because of the past data in hand. The development of cycle time for CD production machines seem a bit faster than CD-Rs. This is because of the extra technical difficulties that CD-R production lines have. The developments for the bottleneck stations like gold-sputtering is not developing so rapid as the developments in the cycle time reductions of the injection molding phase. The reduction of the time in injection molding directly is a development for a CD line, but because of the bottlenecks it does not really affect the speed of a CD-R line.

In the model there is a limiting amount of the reduction of cycle times for both of the production lines. In the sensitivity analysis, the limiting cycle times are taken as constant, but the value of the rate of the development k_B is changed. This means the effect of the speed of the technological improvements are analyzed where the developments has a limiting point which does not change.

As given in Table 6.12 the k_B parameter are changed from the original value with the same percentage difference for both CD and CD-R problem. The Table gives the corresponding k_B values for each percentage change.

The results are in Tables 6.13 and 6.14. There are two effects of the technological developments in capital cost; one is the decrease of the capital cost per unit capacity, and the other is the increase of the capacity of each single discrete unit of the product lines. The fixed part of the capital cost causes economies of scale, which makes bigger expansions advantageous. On the other hand reduction of the capital cost with time can delay some of the expansions, as the cost reduction to make it later may overcome the fixed cost of that extra investment. The increase of the capacity of a single production line also effect feasible number of expansions. For CD problem another result of the capital cost decrease is that the excess capacity remained from CD-R machines which are used for CD production changes.

For the values of $k_B = 0$, that means there is no technological developments, then the optimal expansion values are 3, and 6 respectively for CD-R and CD. For CD-R, at the value of 0.048 of k_B, optimal number of expansions increase to 4. One of the production lines is installed at time 6 instead of installing 2 lines at time 4. At this value, the decrease of capital cost offsets the disadvantage of adding extra constant cost. But at the original value of k_B, optimal number of expansions again falls to 3, this is because of the fact that making four expansion becomes infeasible after reaching that value.

For CD, the optimal number of expansions decreases to 5 from 6, at 0.016 value of k_B. This decrease is the result of the increased excess capacity coming from CD-R production lines. This makes possible to meet demand with smaller capacity additions. However with further increases, the technologic improvements cause delay in expansions, and optimal number of expansion increases again. However, as capacity of unit production line increases the feasibility region of optimal number of expansion gets smaller and its optimal value falls to even 4 at k_B value of 0.160.

The Time average unit cost figures in the optimal tables do not have big variations among various values of k_B, there is a decrease with increasing k_B.

These observations show that the rate of a type of technological improvement modeled in this study, does not have a major positive effect on the cost of the product.

6.2.3 Sensitivity of the Model to Cost Parameters

Sensitivity to Capital Cost

The reduction in the capital cost structure caused by the technological improvements was modeled , however because of the market conditions the prices of the machines and equipments may go up or down during the decision making stage. This makes it important to know how the results change if the prices of

%Change	k_B for CD	k_B for CD-R
-100%	0	0
-80%	0.020	0.016
-60%	0.040	0.032
-40%	0.060	0.048
-20%	0.080	0.064
0%	0.100	0.080
20%	0.120	0.096
40%	0.140	0.112
60%	0.160	0.128
80%	0.180	0.144
100%	0.200	0.160

Table 6.12: The Values of k_B Corresponding to the Same Percentage Changes for both CD-R and CD.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-100%	3	1.2370	(0,3,4)	(1,2,2)
-80%	3	1.2369	(0,3,4)	(1,2,2)
-60%	3	1.2363	(0,3,4)	(1,2,2)
-40%	4	1.2330	(0,3,4,6)	(1,2,1,1)
-20%	4	1.2217	(0,3,4,8)	(1,2,1,1)
0%	3	1.2150	(0,3,4)	(1,2,1)
20%	3	1.2145	(0,3,4)	(1,2,1)
40%	3	1.2140	(0,3,4)	(1,2,1)
60%	3	1.2135	(0,3,4)	(1,2,1)
80%	3	1.2130	(0,3,4)	(1,2,1)
100%	3	1.2067	(0,3,5)	(1,2,1)

Table 6.13: The Effect of Change in k_B to the Optimal Policy of CD-R Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-100%	6	0.2595	(0,5,8,10,12,13)	(1,1,2,7,2,3)
-80%	5	0.2529	(0,5,8,10,12)	(1,1,2,5,4)
-60%	7	0.2470	(0,5,8,10,11,13,16)	(1,1,1,3,3,1,1)
-40%	7	0.2442	(0,5,8,9,10,12,14)	(1,1,1,1,3,1,1)
-20%	7	0.2400	(0,5,8,10,11,12,17)	(1,1,1,2,1,1,1)
0%	6	0.2390	(0,5,8,10,11,13)	(1,1,1,2,1,1)
20%	6	0.2368	(0,5,8,10,11,13)	(1,1,1,1,1,1)
40%	6	0.2345	(0,5,8,10,11,17)	(1,1,1,1,1,1)
60%	5	0.2331	(0,5,8,11,13)	(1,1,1,1,1,1)
80%	5	0.2332	(0,5,9,10,12)	(1,1,1,1,1)
100%	4	0.2317	(0,5,9,12)	(1,1,1,1)

Table 6.14: The Effect of Change in k_B to the Optimal Policy of CD Capacity Expansions.

the machines increase or decrease.

Both the parameters A and $B(t)$ are changed from its original value with the same percentage change. The effects of the two parameters are separately examined, but they are increased or decreased with the same amount for both CD and CD-R problems at the same time. The values for the same percentage change are given in Tables 6.15 and 6.16.

The effect of the change in A to the optimal policies are summarized in Tables 6.17 and 6.18. The expansion policy is unaffected even in large changes. Also the effect of the change in the optimal cost is very small.

The effect of the change in $B(t)$ to the optimal policies are in Tables 6.19 and 6.20. Again the optimal policy is unaffected in even in $\pm 50\%$ change. The optimal cost is slightly increased with increasing $B(t)$.

It can be said that the model is highly insensitive to the changes in capital costs, because even $\pm 12.5\%$ changes in the prices of machine and equipments is a serious variation of price, and in practice the changes larger than $\pm 12.5\%$ is unrealistic.

%Change	A for CD-R (\$)	$B(t)$ for CD-R (\$)
-50%	400,000	0.483
-37.5%	500,000	0.604
-25%	600,000	0.725
-12.5%	700,000	0.846
0%	800,000	0.967
12.5%	900,000	1.087
25%	1,000,000	1.208
37.5%	1,100,000	1.329
50%	1,200,000	1.450

Table 6.15: The Values of $A(t)$ and $B(t)$ Corresponding to the Same Percentage Changes for CD-R.

%Change	A for CD (\$)	$B(t)$ for CD (\$)
-50%	200,000	0.148
-37.5%	250,000	0.185
-25%	300,000	0.222
-12.5%	350,000	0.259
0%	400,000	0.296
12.5%	450,000	0.333
25%	500,000	0.370
37.5%	600,000	0.407
50%	700,000	0.444

Table 6.16: The Values of $A(t)$ and $B(t)$ Corresponding to the Same Percentage Changes for CD.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	3	1.1946	(0,3,4)	(1,2,1)
-37.5%	3	1.1997	(0,3,4)	(1,2,1)
-25%	3	1.2048	(0,3,4)	(1,2,1)
-12.5%	3	1.2099	(0,3,4)	(1,2,1)
0%	3	1.2150	(0,3,4)	(1,2,1)
12.5%	3	1.2201	(0,3,4)	(1,2,1)
25%	3	1.2252	(0,3,4)	(1,2,1)
37.5%	3	1.2304	(0,3,4)	(1,2,1)
50%	3	1.2355	(0,3,4)	(1,2,1)

Table 6.17: The Effect of Change in A to the Optimal Policy of CD-R Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	6	0.2381	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-37.5%	6	0.2372	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-25%	6	0.2363	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-12.5%	6	0.2354	(0,5,8,10,11,13)	(1,1,1,2,1,1)
0%	6	0.2390	(0,5,8,10,11,13)	(1,1,1,2,1,1)
12.5%	6	0.2399	(0,5,8,10,11,13)	(1,1,1,2,1,1)
25%	6	0.2408	(0,5,8,10,11,13)	(1,1,1,2,1,1)
37.5%	6	0.2417	(0,5,8,10,11,13)	(1,1,1,2,1,1)
50%	6	0.2426	(0,5,8,10,11,13)	(1,1,1,2,1,1)

Table 6.18: The Effect of Change in A to the Optimal Policy of CD Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	3	1.1582	(0,3,4)	(1,2,1)
-37.5%	3	1.1724	(0,3,4)	(1,2,1)
-25%	3	1.1854	(0,3,4)	(1,2,1)
-12.5%	3	1.2008	(0,3,4)	(1,2,1)
0%	3	1.2150	(0,3,4)	(1,2,1)
12.5%	3	1.2292	(0,3,4)	(1,2,1)
25%	3	1.2434	(0,3,4)	(1,2,1)
37.5%	3	1.2576	(0,3,4)	(1,2,1)
50%	3	1.2718	(0,3,4)	(1,2,1)

Table 6.19: The Effect of Change in $B(t)$ Policy of CD-R Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	6	0.2280	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-37.5%	6	0.2307	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-25%	6	0.2334	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-12.5%	6	0.2362	(0,5,8,10,11,13)	(1,1,1,2,1,1)
0%	6	0.2390	(0,5,8,10,11,13)	(1,1,1,2,1,1)
12.5%	6	0.2418	(0,5,8,10,11,13)	(1,1,1,2,1,1)
25%	6	0.2445	(0,5,8,10,11,13)	(1,1,1,2,1,1)
37.5%	6	0.2473	(0,5,8,10,11,13)	(1,1,1,2,1,1)
50%	6	0.2501	(0,5,8,10,11,13)	(1,1,1,2,1,1)

Table 6.20: The Effect of Change in $B(t)$ Policy of CD Capacity Expansions.

%Change	α for CD-R (\$)	$\beta(t)$ for CD-R (\$)
-50%	45,000	0.55
-37.5%	56,250	0.69
-25%	67,500	0.83
-12.5%	78,750	0.96
0%	90,000	1.10
12.5%	101,250	1.24
25%	112,500	1.38
37.5%	123,750	1.51
50%	135,000	1.65

Table 6.21: The Values of α and $\beta(t)$ Corresponding to the Same Percentage Changes for CD-R.

Sensitivity to Operating Cost

The same approach is used for the examination of the effect of operating cost. Tables 6.21 and 6.22 show the values of α, and $\beta(t)$ to the corresponding percentage changes from the original values.

The results for the effect of α on the optimal policy are shown in Tables 6.23 and 6.24. Changes in α doesn't effect the optimal policy, it does not even cause a major change in the optimal cost of the product.

The results for the effect of $\beta(t)$ on the optimal policy are shown in Tables 6.25 and 6.26. The optimal policy also seems insensitive to the changes in $\beta(t)$, however the optimal costs increase dramatically with increasing β.

Another very important result after those observations is about the distribution of the time average unit cost to its components. It seems that the component in the cost figure caused by the cash flows spend on capital cost (A and $B(t)$ parameters) is small. The biggest component in the cost figure is caused by the $\beta(t)$ parameter of the operating cost. The portion of the unit cost that is the result of the constant part of the operating cost, α is also small.

%Change	α for CD ($)	$\beta(t)$ for CD ($)
-50%	60,000	0.12
-37.5%	75,000	0.15
-25%	90,000	0.18
-12.5%	105,000	0.21
0%	120,000	0.24
12.5%	135,000	0.27
25%	150,000	0.30
37.5%	165,000	0.33
50%	180,000	0.36

Table 6.22: The Values of α and $\beta(t)$ Corresponding to the Same Percentage Changes for CD.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	3	1.2067	(0,3,4)	(1,2,1)
-37.5%	3	1.2088	(0,3,4)	(1,2,1)
-25%	3	1.2108	(0,3,4)	(1,2,1)
-12.5%	3	1.2130	(0,3,4)	(1,2,1)
0%	3	1.2150	(0,3,4)	(1,2,1)
12.5%	3	1.2171	(0,3,4)	(1,2,1)
25%	3	1.2190	(0,3,4)	(1,2,1)
37.5%	3	1.2213	(0,3,4)	(1,2,1)
50%	3	1.2234	(0,3,4)	(1,2,1)

Table 6.23: The Effect of Change in α to the Optimal Policy of CD-R Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	6	0.2357	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-37.5%	6	0.2365	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-25%	6	0.2374	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-12.5%	6	0.2382	(0,5,8,10,11,13)	(1,1,1,2,1,1)
0%	6	0.2390	(0,5,8,10,11,13)	(1,1,1,2,1,1)
12.5%	6	0.2398	(0,5,8,10,11,13)	(1,1,1,2,1,1)
25%	6	0.2406	(0,5,8,10,11,13)	(1,1,1,2,1,1)
37.5%	6	0.2414	(0,5,8,10,11,13)	(1,1,1,2,1,1)
50%	6	0.2423	(0,5,8,10,11,13)	(1,1,1,2,1,1)

Table 6.24: The Effect of Change in α to the Optimal Policy of CD Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	3	0.6057	(0,3,4)	(1,2,1)
-37.5%	3	0.7384	(0,3,4)	(1,2,1)
-25%	3	0.8711	(0,3,4)	(1,2,1)
-12.5%	3	0.9943	(0,3,4)	(1,2,1)
0%	3	1.2150	(0,3,4)	(1,2,1)
12.5%	3	1.2597	(0,3,4)	(1,2,1)
25%	3	1.3924	(0,3,4)	(1,2,1)
37.5%	3	1.5157	(0,3,4)	(1,2,1)
50%	3	1.6484	(0,3,4)	(1,2,1)

Table 6.25: The Effect of Change in $\beta(t)$ to the Optimal Policy of CD-R Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	5	0.1374	(0,5,8,10,13)	(1,1,1,2,1,1)
-37.5%	6	0.1628	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-25%	6	0.1882	(0,5,8,10,11,13)	(1,1,1,2,1,1)
-12.5%	6	0.2136	(0,5,8,10,11,13)	(1,1,1,2,1,1)
0%	6	0.2390	(0,5,8,10,11,13)	(1,1,1,2,1,1)
12.5%	6	0.2644	(0,5,8,10,11,13)	(1,1,1,2,1,1)
25%	6	0.2898	(0,5,8,10,11,13)	(1,1,1,2,1,1)
37.5%	6	0.3152	(0,5,8,10,11,13)	(1,1,1,2,1,1)
50%	6	0.3405	(0,5,8,10,11,13)	(1,1,1,2,1,1)

Table 6.26: The Effect of Change in $\beta(t)$ to the Optimal Policy of CD Capacity Expansions.

6.2.4 Change in Demand Function

The most important figure in the demand parameter for sensitivity studies is the long run steady state value of demand which is defined as the parameter D_s. This forecast depend heavily on subjective judgements as it is the long run value and may change significantly in future periods.

The steady state value of demand for CD was taken as 60,000,000 units/year and for CD-R it was assumed to be 8,333,000 per year. The relative percentage changes of this parameter for both CD and CD-R are given in Table 6.27. For -50% change in D_s for CD-R demand, the turning point of demand that is Φ is changed from 4 to 3. As the demand becomes decreasing at some point with Φ equals to 4, it violates the non decreasing demand assumption.

For each percentage change of D_s parameter, the optimal cost and expansion policy is shown in Table 6.28 and 6.29.

By looking at the optimal expansion policy, if there is even very large decreases in demand, the early times of the expansion are similar with the early expansion times of the original problem. This shows that there is not a very large risk to make the expansion with the outputs of the original problem. It seems that the decision to make an expansion at year 5 for CD and at year 3 for CD-R is not effected even with 50% decrease and increase in the long run steady state level of demand. A reason for this is that the rate of increase does not change, so the change in D_s do not really effect the early times of the demand function.

Another point is that after the second expansion according to the latest data in hand the demand function can be revised and the new optimal expansion policy can be determined in the future. The planned future expansion decision can be revised without a big penalty cost of a wrong forecast.

The optimal costs are lower when D_s decreases, though the difference is not is large. The reduction is because of the fact that with the rising demand it becomes possible to build large capacities that can take the advantage of economies of scale. Also with larger productions the constant part of the operating cost decreases for each unit produced.

%Change	D_s for CD-R	D_s for CD
-50%	4,165,000	30,000,000
-37.5%	5,206,250	37,500,000
-25%	6,247,500	45,000,000
-12.5%	7,288,750	52,500,000
0%	8,330,000	60,000,000
12.5%	9,371,250	67,500,000
25%	10,412,500	75,000,000
37.5%	11,453,750	82,500,000
50%	12,495,000	90,000,000

Table 6.27: The Values of D_s Corresponding to the Same Percentage Changes for CD.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	3	1.3075	(0,3,5)	(1,1,1)
-37.5%	2	1.2727	(0,3)	(1,2)
-25%	3	1.2459	(0,3,8)	(1,2,1)
-12.5%	3	1.2361	(0,3,4)	(1,2,1)
0%	3	1.2150	(0,3,4)	(1,2,1)
12.5%	3	1.2154	(0,3,4)	(1,2,2)
25%	3	1.1994	(0,3,4,6)	(1,2,2)
37.5%	4	1.1981	(0,3,4,7)	(1,2,2,1)
50%	4	1.1888	(0,3,4,5)	(1,2,2,1)

Table 6.28: The Effect of Change in D_s to the Optimal Policy of CD-R Capacity Expansions.

%Change	n^*	$F(n^*)$	Expansion Times	No of Machines Installed
-50%	4	0.2591	(0,5,7,10)	(1,1,1,1)
-37.5%	5	0.2507	(0,5,8,9,11)	(1,1,1,1,1)
-25%	6	0.2464	(0,5,8,9,10,13)	(1,1,1,1,1,1)
-12.5%	6	0.2414	(0,5,8,10,11,13)	(1,1,1,1,1,1)
0%	6	0.2390	(0,5,8,10,11,13)	(1,1,1,2,1,1)
12.5%	7	0.2353	(0,5,7,10,11,13,17)	(1,1,1,2,1,1,1)
25%	6	0.2339	(0,5,7,10,11,12)	(1,1,1,2,1,2)
37.5%	8	0.2311	(0,5,7,10,11,12,13,17)	(1,1,1,2,1,1,1,1)
50%	7	0.2303	(0,5,7,10,11,13,17)	(1,1,1,3,2,1,1)

Table 6.29: The Effect of Change in D_s to the Optimal Policy of CD Capacity Expansions.

Chapter 7

Conclusion

The capacity expansion problem for an optical disc plant is solved, where there are two different product families and two different capacity types. It is shown that, dynamic programming approach is capable of solving this rather complex problem efficiently. dynamic programming is capable of modelling the situation where capacities are available in discrete units, and demand and capital cost functions are partial equations.

Another interesting property used in modelling is the fact that by using the special structure of the model, the two capacity type and two products problem can be solved with the model consisting of one capacity type and a single product. The original problem is divided into two similar dynamic programming problem. This approach simplified the complexity of the mathematical model, and also reduced the solution time of the algorithm.

The technological improvements result in the introduction of new production lines with decreased cycle times, but with the same cost. This causes new production lines to have less cost per unit capacity. The technological improvements also reduce the operating costs. The machines installed at different times have different cost characteristics because of this. This feature complicates the calculation of operating cost, but upper and lower bounds for the real cost are constructed, and it is shown that both of the bounds are good estimates of the real cost incurred.

The sensitivity analysis of the parameters gives insight about the nature of the model and its behavior. It is shown that the model is not sensitive to cost parameters, technological improvement factors and discount rate. Although the optimal time average unit cost do change with the changes in the parameters, the optimal expansion policy is not much effected. This may be the result of allowing capacities only in discrete units.

The observations also show that the major component in the unit cost of the product is the variable part of the operating cost. The capital cost is relatively a smaller component of the unit cost. Another interesting result is that the effect of technological developments to the optimal cost for the base problem is lower than expected.

The literature on capacity expansion problems generally consist of the theoretical studies and the practical studies concerned with heavy industries. This study is useful in the sense that it will add an insight to capacity expansion problem with a practical study in the magnetic and optical media sector that show different characteristics compared to the heavy industries.

Some of the simplifying assumptions can be released and model can further be improved. The most important simplifying assumption is the fact that the demand must be met at any time with the production facility in hand. In practice, however the demand may be efficiently met by the products supplied from other sources, instead of building the capacity during the periods where demand is low. Also for CD-R problem no inventories assumption can be relaxed.

The facilities once installed operates with the same cost figure, they do not deteriorate. This is a weakness of the model. If this feature is added, the model will reflect the real life situation combining the capacity expansion with the replacement problem.

Although, there is still a room for the improvement of the existing model to reflect the real life situation more realistically, this model is still enough to help the management in decision making process. This model is also very important in the sense that it creates an insight how the unit cost of the product is effected by capital and operating costs and various other parameters.

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Vitae

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