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SUPERCONDUCTIVITY IN HIGH FREQUENCY FIELDS

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Fundamentals of BCS and GLAG theories of superconductivity are reviewed with a focus on high-frequency properties of bulk superconductors, superconducting films and superconducting cavities. Superconductivity as a macroscopic quantum coherent state. Supercurrents and persistent currents. Condensate and excitations. Complex penetration depth and RF losses. Mechanisms of Q degradation in superconducting cavities at increasing a.c. field amplitude. Depairing effects and vortex nucleation mechanisms. Surface superconductivity and tilted vortices. Material parameters and factors responsible for ultimate performance of superconducting resonators.

Keywords: Superconductivity; Radiofrequency; Cavities; Surface impedance

Superconductivity bears its applications owing to zero d.c. resistance and extremely small a.c. losses, magnetic flux expulsion (the Meissner effect) and the possibility of generating very high magnetic fields, flux quantization and extreme sensitivity of superconducting currents to weak electromagnetic fields (the Josephson effect).

First decisive measurement of zero resistance in a superconductor was done by Holst[1] in the Kamerlingh Onnes Laboratory.[2]

A variety of superconducting materials with critical temperatures in the range 1–23 K have been found following this seminal work of Kamerlingh Onnes and, most recently, resulted in the discovery of superconducting materials with critical temperature of superconducting transition above 100 K.[3,4]

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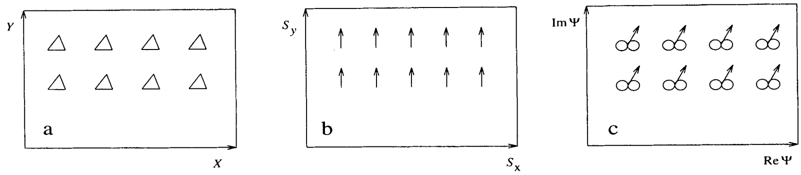

FIGURE 1 Macroscopic order in crystals (a), ferromagnets (b) and superconductors (c).

Superconductivity is one of most remarkable phenomena in the condensed matter physics (and in physics in general). It originates from the specific kind of electron ordering on a macroscopic scale similar to ordering of molecules in crystals (Figure 1(a)) or atomic spins in ferromagnets (Figure 1(b)). In contrast to those classically describable types of order, in superconductors the *phases* of electronic wave function Ψ corresponding to pair of electrons order in the Re Ψ, Im Ψ space (Figure 1(c)) making a macroscopic quantum state, a condensate.

All types of ordering are of quantum nature. Crystalline order is not considered generally as a wonder (which it is), and similarly the magnetic ordering of spins in solids is readily accepted. Superconducting order is less easy to comprehend since it relates to a non-classical entity, Ψ. It lasted almost 50 years until the superconductivity theory was developed by Bardeen, Cooper and Schrieffer[5] and accepted the general recognition as the "BCS model". Behavior of superconductors in the electromagnetic fields is adequately described within the GLAG (Ginzburg–Landau–Abrikosov–Gorkov) theory.[6–8] Basic properties of superconductors are well understood within the BCS–GLAG scheme[9] and are listed schematically in Figure 2. In Table I, we list characteristic representatives of types of superconducting materials, and their basic critical parameters (critical temperature T_c and critical magnetic field H_c).

Origin of superconductivity relates to the interaction between electrons and quantized vibrations of a crystalline lattice, the phonons. In a ground state of a metal, electrons from the outer atomic shells of atoms delocalize to form a "Fermi liquid", a state in which all electrons are confined within the "Fermi sphere" located in the momentum space at the center of the Brillouin zone (Figure 3(a)). Phonons share states within the full Brillouin zone (Figure 3(b)). These are

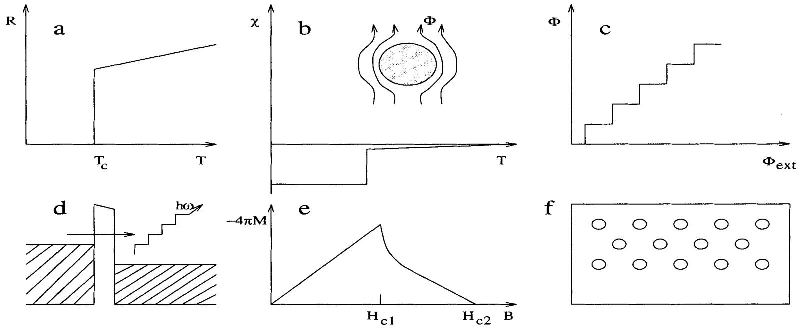

FIGURE 2 Basic properties of superconductors. (a) Zero resistance, (b) Meissner effect, (c) flux quantization, (d) Josephson effect and (e and f) mixed state and Abrikosov vortices.

TABLE I Basic critical parameters of superconducting materials

Superconductor	T_c (K)	H_c (T)	*Structure/type*
Nb	9.3	0.2	Elemental superconductors
Nb_3Ge	23	38	Intermetallics
Nb–Ti	10	10	Alloys
$Ba_{1-x}K_xBiO_3$	30		Perovskites
$Sn_xMo_6S_8$	14	50	Chevrel phases
$CeCu_2Si_2$	0.6	2.4	Heavy fermions
$ErRh_4B_4$	8.7		Magnetic superconductors
$LuNi_2B_2C$	16		Borocarbides
PdH(Cu, Ag, Au)	17		Palladium-hydrogen
BEDT–TTF	14		Organic superconductors
$La_{2-x}Sr_xCuO_4$	35		Oxides
$YBa_2Cu_3O_7$	95	200	"YBCO"
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110		"BISCO"
$Tl_2Ba_2Ca_2Cu_3O_{10}$	125		Thallium-compound
$HgBa_2Ca_2Cu_3O_8$	133		Mercury-compound
Cs_xC_{60}	43	20	Fullerenes

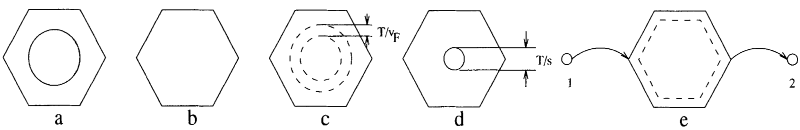

FIGURE 3 (a) "Vacuum" of electrons, (b) "vacuum" of phonons (virtual phonons), (c) electronic excitations and the Fermi surface, (d) phononic excitations (thermal phonons) and (e) interaction between electrons 1 and 2 mediated by phonon vacuum deformation.

virtual particles, since they do not contribute to thermal part of free energy and are not detected in an experiment. Thermal excitations, the electrons near the Fermi surface (Figure 3(c)) and phonon states near the origin of the momentum space (Figure 3(d)) account for physical properties of non-superconducting metals. In a superconductor, there appears an attractive interaction between electrons mediated by deformation of the phonon vacuum (Figure 3(e)) which leads to the formation of the bound states of electrons near the Fermi surface. Such pairs form a condensate whereas non-paired electrons remain as "quasiparticles".

The concentration of quasiparticles increases with the increasing temperature thus decreasing the concentration of pairs. Quasiparticle excitations in superconductor are of two types: electrons (1) and holes (2) depending on whether the momentum of excitation is larger or smaller than the Fermi momentum, p_{F}. Energy of the excitation

$$\varepsilon_{p1,2} = \sqrt{\Delta^2 + \xi_p^2}, \quad \xi_p = v_{\mathrm{F}}|p - p_{\mathrm{F}}|, \tag{1}$$

where Δ is an energy gap, the minimal energy necessary for creating a quasiparticle. This can be done either by removing an electron from the Fermi surface and placing it to the state above the Fermi surface with momentum $p > p_{\mathrm{F}}$, or by removing electron from below the Fermi surface ($p < p_{\mathrm{F}}$) thus creating a hole at $p < p_{\mathrm{F}}$ and one extra electron at the Fermi surface.

Electron vacuum of normal metal transforms at superconducting transition to the condensate of Cooper pairs. Nontrivial circumstance is that this condensate, or restructured electron vacuum, can carry a current by shifting its position in the momentum space by a vector $\mathbf{p}_{\mathrm{s}}$,

$$\mathbf{j}_{\mathrm{s}} = n_{\mathrm{s}} e \mathbf{v}_{\mathrm{s}}, \tag{2}$$

where $\mathbf{v}_{\mathrm{s}} = \mathbf{p}_{\mathrm{s}}/2m$, and $n_{\mathrm{s}} = n$ at $T = 0$ (n is the total electron concentration). At increasing temperature, concentration of excitation increases and n_{s} decreases. Below T_{c}, both condensate and excitations coexist, and current is represented as a sum $\mathbf{j} = \mathbf{j}_{\mathrm{s}} + \mathbf{j}_{\mathrm{n}}$ where normal current $\mathbf{j}_{\mathrm{n}}$ is proportional to the electric field,

$$\mathbf{j}_{\mathrm{n}} = \sigma \mathbf{E} = -(\sigma/c)\partial \mathbf{A}/\partial t, \tag{3}$$

whereas the supercurrent (2) relates to vector potential according to

$$\mathbf{v}_s = \frac{1}{2m}\left(\mathbf{p}_s - \frac{2e}{c}\mathbf{A}\right), \tag{4}$$

where $\mathbf{v}_s$ is the velocity of Cooper pair.

Expression (3) explains the phenomenon of flux quantization in hollow superconducting cylinders: since the azimuthal momentum of pair in a cylinder is quantized,

$$p_\theta = 2\pi n\hbar/L, \quad n = 0, \pm 1, \pm 2, \ldots, \tag{5}$$

where L is the circumference of the cylinder cross section, so the flux $\Phi = \oint \mathbf{A}\, d\mathbf{l} = \int \mathbf{H}\, d\mathbf{S}$ does. The minimal amount of flux between successive values of n is called the "flux quantum" $\Phi_0 = hc/2e = 2 \cdot 10^{-7}\,\mathrm{G\,cm}^2$. After switching off the magnetic field, the flux is trapped inside the cylinder with the value of Φ quantized in units of Φ_0.

If the external vector potential $A = A_\theta$ is applied to the cylinder, a superconducting current will appear proportional to A. The energy of the cylinder will increase proportional to A^2 (Figure 4). There exists an infinite number of states $E_n(A)$ with various energies. These states are

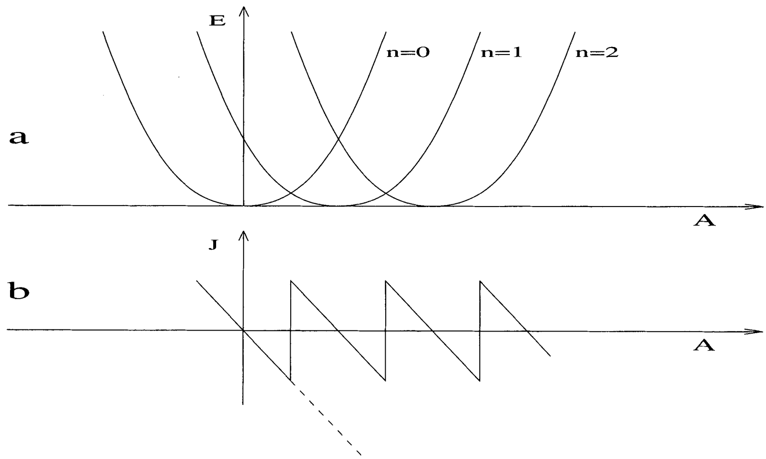

FIGURE 4 (a) Energy versus vector potential dependence and (b) current vs vector potential. Solid line corresponds to a persistent current, dashed line corresponds to the supercurrent.

macroscopic since they relate to the momentum of a Bose-condensed pair. Switching between various values of n would require a "macroscopic excitation", a change of the physical vacuum in which a macroscopically large number of pairs of the order of 10^{23} per cubic centimeter, need to switch coherently between n and $n \pm 1$. Such excitations are topological in the wave-function parameter space and require an energy of the order of $(H_c^2/8\pi)\xi^3$ to be created where ξ is a coherence length of superconductor. At $T=0$, ξ equals

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta} = 0.1\text{–}1\ \mu\text{m}. \tag{6}$$

Abrikosov vortices[7] are examples of such topological excitations in the condensate. Penetration of vortex to a superconductor and its subsequent transformation to magnetic flux lines inside the cylinder when vortex leaves the cylinder, increase the flux inside the cylinder from Φ to $\Phi + \Phi_0$.

Starting from a state of the cylinder with $n=0$ and applying external magnetic field will create a current in the cylinder wall proportional to A. At the value of A corresponding to the intersection between two near parabolas in Figure 4(a), the system may switch between the states $n=0$ and $n=1$ thus decreasing the energy. If such transition does occur at any intersection between parabolas, the current as a function of A would follow the solid line depicted in Figure 4(b). If, on the other hand, a value of n is fixed, the current will increase further as depicted by a dashed line. The critical value of A at which the energies of two states n and $n+1$ equalize is extremely small since it is inversely proportional to the macroscopic parameter, the length of the ring. In a real current-carrying state of a cylinder (or in a superconducting cavity) current passes very many times by skipping through the intersections between parabolas.

In case of a constant current, full line in Figure 4(b) corresponds to the *persistent current* whereas a dashed line to the *supercurrent*. Persistent current is an equilibrium property. Such current never decays since there is no possibility of achieving a smaller energy at the given value of A. (Similar currents, but of much smaller amplitude, can also exist in normal (nonsuperconducting) metals at low temperature.[10,11])

Supercurrent is in principle finite-lived since the current-carrying state in superconductor is metastable rather than true stable state.

However, the relaxation time of such state proves to be astronomically large for any reasonable size of the superconducting device. In case of a time-dependent current, like currents in superconducting cavities, a much higher dissipation appears not related to condensate relaxation but to the Joule losses of normal current driven by the electric field.

The full current can be expressed by introducing the complex conductivity σ,[12]

$$j_\omega = \sigma E_\omega, \quad \sigma = \sigma_1(\omega) + i\sigma_2(\omega), \tag{7}$$

where $\sigma_1(\omega)$ is related to response of j_s to A_ω and $\sigma_2(\omega)$ to the electric field $E_\omega = (i\omega/c)A_\omega$. Theory of frequency and wave-vector dependent response of a superconductor to the electromagnetic field was developed by Abrikosov *et al.*,[13] and by Mattis and Bardeen.[14] In a local limit when the coherence length ξ_0 is much smaller than the characteristic size of space variation of magnetic field, the electrodynamics of superconducting state is described by the introduction of the complex "penetration depth" δ,

$$\frac{1}{\delta^2} = \frac{1}{\delta_L^2} - \frac{2i}{\delta_{sk}^2}, \tag{8}$$

where $\delta_L = (mc^2/4\pi n_s e^2)^{1/2}$ is called the London penetration depth and $\delta_s = (c^2/2\pi\sigma_2\omega)^{1/2}$ is the skin penetration depth.

In the extreme local limit $\delta_L \ll \xi_0$ and $l \ll \xi_0$ where l is the mean free path of electron, σ_1, σ_2 are frequency dependent and, assuming that ω is below the gap frequency $2\Delta/\hbar$, equal,

$$\frac{\sigma_1}{\sigma_n} = \frac{1}{\omega}\int_\Delta^\infty \frac{\varepsilon^2 + \Delta^2 + \omega\varepsilon}{\sqrt{\varepsilon^2 - \Delta^2}\sqrt{(\varepsilon + \omega)^2 - \Delta^2}} \left(\tanh\frac{\varepsilon + \omega}{2T} - \tanh\frac{\varepsilon}{2T}\right) d\varepsilon, \tag{9}$$

$$\frac{\sigma_2}{\sigma_n} = \frac{1}{\omega}\int_{\Delta-\omega}^{\Delta} \frac{\varepsilon^2 + \Delta^2 + \omega\varepsilon}{\sqrt{\Delta^2 - \varepsilon^2}\sqrt{(\varepsilon + \omega)^2 - \Delta^2}} \tanh\frac{\varepsilon + \omega}{2T}\, d\varepsilon. \tag{10}$$

In these formulas, σ_n is the Drude conductivity in the normal state, $\sigma_n = ne^2 l/p_F$. In superconducting cavity, relation between ω, T and Δ

is such that

$$\omega \ll T \ll \Delta. \tag{11}$$

In this limit, Eqs. (9) and (10) simplify to

$$\frac{\sigma_1}{\sigma_n} \simeq \frac{2\Delta}{T} e^{-\Delta/T} \ln\frac{9T}{4\omega}, \qquad \frac{\sigma_2}{\sigma_n} \simeq \frac{\pi\Delta}{\omega}. \tag{12}$$

The value of σ_2 corresponds to the effective superelectron concentration in the dirty metal, $n_s = nl/\xi_0$. The imaginary part of conductivity σ_2 can be presented in a Drude-like form with the number of quasiparticle excitations n_{qp} substituting the number of electrons,

$$n_{qp} \sim n\frac{\Delta}{T} e^{-\Delta/T}. \tag{13}$$

Here we neglect the slow (logarithmic) frequency dependence of σ_2. The important property incipient in Eq. (13) is an exponential dependence of n_{qp} on temperature. Electrodynamics of superconducting plate of thickness d is expressed with the introduction of two surface impedances ζ_1, ζ_2 describing the relation between the tangential components of magnetic, $\mathbf{H}_t$, and electric, $\mathbf{E}_t$, fields on two surfaces of the plate,[15]

$$\mathbf{E}_t^- = \zeta_1 \mathbf{H}_t^- \times \mathbf{n} - \zeta_2 \mathbf{H}_t^+ \times \mathbf{n}, \qquad \mathbf{E}_t^+ = \zeta_2 \mathbf{H}_t^- \times \mathbf{n} - \zeta_1 \mathbf{H}_t^+ \times \mathbf{n}, \tag{14}$$

where $\mathbf{n}$ is a unit vector normal to surface and

$$\zeta_1 = \zeta \coth\frac{d}{\delta}, \quad \zeta_2 = \zeta / \sinh\frac{d}{\delta}, \qquad \zeta = -\frac{i\omega}{c}\delta. \tag{15}$$

Transparency of the metal film of thickness d equals

$$T = \left| \frac{2\zeta_2}{\zeta_2^2 - (\zeta_1 - 1)^2} \right|^2. \tag{16}$$

At thickness much smaller than the penetration depth, $d \ll |\delta|$, we receive an expression

$$T \simeq \left(\frac{4\pi|\delta|^2}{\lambda d}\right)^2, \tag{17}$$

where $\lambda = 2\pi c/\omega$ is the wavelength of the electromagnetic radiation. The quality factor Q of a cavity with the bulk superconducting walls equals in terms of δ_{L}, δ_{sk},

$$Q = \frac{c^3}{8\delta_{\mathrm{L}}^3 \sigma_2 \omega^2}. \tag{18}$$

At low temperature $T \ll \Delta$, this value is exponentially large since σ_2 is exponentially small. Values of Q of order 10^{10} are readily achieved in the Nb superconducting cavities. An important issue is the dependence of the quality factor on the amplitude of the RF power. The dependence is related to three basic mechanisms:

(1) condensate exhaustion at increasing RF amplitude;
(2) destruction of superconductivity in the walls due to nucleation of a normal phase and vortex penetration;
(3) stray losses due to foreign inclusions and imperfections in a cavity wall.

By condensate exhaustion, we mean the depairing effect of a large a.c. current. In case of a cavity formed by superconducting film sputtered over the surface of nonsuperconducting metal, depairing effect will decrease Δ and n_{s}, thus increasing the number of normal excitations and increasing the London penetration depth. Both factors may substantially reduce the value of Q before the penetration of vortices will finally destroy superconductivity.

Destruction of superconductivity is expected to grow exponentially when a.c. component of magnetic field at the surface reaches the value of critical magnetic field. Since superconducting current in a cavity is time-dependent, the supercooling and superheating critical fields will depend on frequency. The peculiarities of such dependence, as well as the relevant relaxation mechanisms responsible for superconducting state degradation, remain the important issues. In bulk superconductor, vortex penetration starts at the lower Abrikosov critical field H_{c1} and full destruction of superconductivity is achieved at the upper critical field H_{c2}.[7] Superconductivity near the surface of a metal remains up to the field $H_{c3} = 1.69 H_{c2}$.[16] The surface layer is unstable in parallel field, and "tilted" vortices[17–19] create and determine the RF losses between H_{c2} and H_{c3}.

Many factors can influence the decrease of the quality factor and its degradation in high a.c. magnetic fields. Among those, we mention the following.

1. Defects and surface contamination Inclusions of non-superconducting phases, or less-T_c phases inside the cavity wall will cause a.c supercurrents to choose a curved path to escape penetration to normal region (Figure 5(a)). The resulting increase of the current density near the inclusion will lower the overall critical field of the cavity. These factors may be particularly important for the Nb-coated cavities if the normal region penetrates through the superconducting coating. The normal substrate becomes exposed to RF radiation which greatly increases losses before the bulk critical field is reached. Assuming that low-field cavity Q is of order 10^{10} and that the normal-state $Q \sim 10^5$ we achieve the conclusion that 10^{-5} percentage of the defected surface will cause serious effect on the cavity parameters.

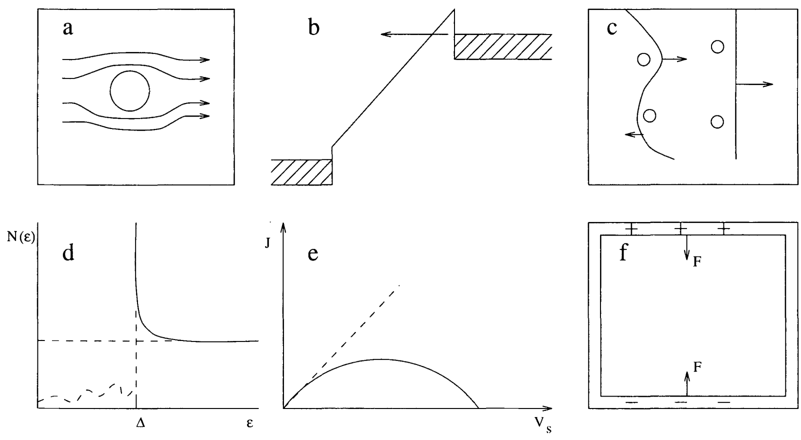

FIGURE 5 Phenomena near superconducting surface related to RF losses. (a) Supercurrents bypassing a non-superconducting inclusion, (b) cold electron emission from metal, (c) oscillation and depinning of trapped vortices, (d) quasiparticle states inside the gap, (e) depairing of critical current and (f) deformation of the cavity wall.

2. Electron ionization High a.c. electric field inside the cavity will lower potential barrier for electron tunneling from the metal (Figure 5(b)), thus causing currents inside the cavity. Such currents will contribute to a.c. losses which greatly increase at increasing a.c. amplitude.

3. Heating effects Joule losses in a cavity wall heat the wall and therefore decrease the basic parameters of a superconductor Δ and n_s. In a Nb-coated cavity the effect is expected to be less important because of much better heat conductivity of the normal metal compared to that of a superconductor.

4. Trapped vortices Abrikosov vortices may be frozen in a cavity material and pinned by defects in the wall (Figure 5(c)). RF currents create a Lorentz force on vortices. Vortex vibration at small a.c. amplitude, or vortex depinning at higher amplitude will then create additional losses which will reduce the cavity's Q and decrease its magnitude at increasing RF power.

5. Quasiparticle states inside the superconducting gap Normal-metal inclusions, point defects, linear defects such as dislocations and planar interphase boundaries may create quasiparticle excitations with energies below the gap energy. These gapless excitations (shown schematically by dashed line in Figure 5(d)) will dominate power absorption at concentration $n' \gg n_{qp}$. Since n_{qp} is exponentially small, even small amount of such gapless states which is not easily detected in the tunneling measurements of Δ, may create substantial change in the Q value of the cavity, and put a limit on maximal cavity efficiency.

6. RF harmonic generation At increasing RF amplitude, the $j(A)$ dependence becomes nonlinear (Figure 5(e)), similar to the nonlinear current-phase dependence in the Josephson effect. The nonlinear current component will create odd harmonics of a.c. electromagnetic field in the cavity. Since harmonics are out of resonance, they contribute to the reflection coefficient of the electromagnetic field from the cavity wall and therefore to the effective losses. These losses are expected to be negligible since critical currents are much smaller than the depairing current.

7. Losses in a normal substrate of Nb-coated cavities In Nb-sputtered cavities, part of RF field penetrates inside the normal metal (typically, Cu). Although the amplitude of this field in much reduced by the Meissner effect, it may compare to losses in a superconducting coating at low temperature since quasiparticle excitation concentration is small in superconductor, but remains large in a normal substrate.

8. Surface conduction Non-superconducting and even non-metallic impurities on surface of a cavity produce small currents under the RF fields. Such losses are adding to the Joule losses in a superconducting wall. Residual losses may put a limit on maximal achieved values of the cavity quality factor Q.

9. Cavity vibration and surface phonons Among exotic mechanisms limiting the cavity quality factor we mention the wall vibration which may originate from external devices near the cavity, or cavity wall deformation resulting from the (large) mechanical forces appearing between the opposite walls due to surface charge on walls proportional to normal component of the a.c. electric field (Figure 5(f)). Smaller, but intrinsic, effect arises from the (virtual) surface phonons. Any displacement of the cavity wall results in the shift of cavity resonance frequency and therefore pushes the system out of resonance. The effect may be significant just because of very large Q. A wall displacement of order λ/Q is of significance since it makes a value of δx less than the atomic size at $\lambda = 10\,\mathrm{cm}$ and $Q = 10^{10}$. An intrinsic effect of surface phonons corresponds to the displacement of order $\delta\bar{x} \sim (\hbar/M\omega_{\mathrm{D}})^{1/2}$ where M is the atomic mass and ω_{D} the Debye frequency of a crystal. Only the part of this displacement, of relative order $\omega/\omega_{\mathrm{D}}$, contributes to shift of cavity resonance which gives an estimate of the limiting Q value $Q_{\max} \sim \lambda\omega_{\mathrm{D}}(M/\hbar\omega)^{1/2}$. In Table II, we summarize the ranges of cavity frequencies and Q factors for which the above discussed effects of mechanical forces (MF) and surface phonons (SF) may become of importance.

TABLE II Effect of mechanical forces and surface phonons on superconducting cavities

Q	f (Hz)		
	10^8	10^9	10^{10}
10^{10}			SP
10^{12}	MF	SP	SP
10^{14}	MF, SP	MF, SP	SP

In conclusion, many extrinsic factors can influence the ultimate performance of high-frequency, high-power applications of superconductivity. Intrinsic mechanisms of the RF losses in superconducting cavities which are widely used in particle accelerators including the condensate exhaustion (depairing effects of the high-frequency currents) and superconductivity nucleation mechanisms including those related to surface superconductivity require better theoretical understanding.

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