# MONEY DEMAND, THE CAGAN MODEL, TESTING RATIONAL EXPECTATIONS vs ADAPTIVE EXPECTATIONS : THE CASE OF TURKEY

A THESIS PRESENTED BY ILKER MUSLU
TO
THE INSTITUTE OF
ECONOMICS AND SOCIAL SCIENCES
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS
FOR THE DEGREE OF MASTER OF
ECONOMICS

BILKENT UNIVERSITY
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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Economics.

Assist.Prof.Dr. Kıvılcım METİN

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Economics.

Assist.Prof.Dr. Fatma TAŞKIN

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Economics.

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# ABSTRACT

## MONEY DEMAND, THE CAGAN MODEL, TESTING RATIONAL EXPECTATIONS vs ADAPTIVE EXPECTATIONS: THE CASE OF TURKEY

**İLKER MUSLU**
**Master of Economics**
**Supervisor: Assist.Prof.Dr. Kıvılcım METİN**
**July, 1995**

This thesis considers the demand for money under conditions of high inflation in Turkey during the period 1986:1-1995:3. We test whether the monetary and inflationary experiences of Turkey can be adequately characterized by the Cagan (1956) model, using an econometric procedure which is reliant only on the assumption that forecasting errors are stationary. We also examine the hypothesis that monetary policy was conducted in such a way as to maximize the inflation tax revenue. Finally we test the Cagan model with the additional assumption of rational expectations for Turkey for the considered period.

Keywords: Adaptive Expectations, Cointegration, Hyperinflation, Inflation Tax, Money Demand, Rational Expectations, Unit Root.

# ÖZ

## PARA TALEBİ, CAGAN MODELİ, RASYONEL BEKLENTİLERİN UYARLANABİLİR BEKLENTİLERE KARŞI TEST EDİLMESİ: TÜRKİYE UYGULAMASI

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**Temmuz, 1995**

Bu tez Türkiye'de 1986:1-1995:3 dönemindeki yüksek enflasyon koşulları altındaki para talebini incelemektedir. Tezde, Türkiye'de bu dönemde yaşanan parasal ve enflasyonist tecrübelerin Cagan'm (1956) modeli ile tam olarak nitelenmesinin mümkün olup olmadığı ekonometrik bir yöntem kullanılarak test edilmiştir. Ayrıca söz konusu dönemde otoritelerce yürütülen para politikasının enflasyon vergisini maksimize edecek şekilde olduğu hipotezi incelenmiştir. Son olarak da Cagan'ın modeli rasyonel beklentiler varsayılarak Türkiye için yukarıdaki dönem gözönüne alınarak test edilmiştir.

Anahtar Kelimeler: Birim Kök, Hiperenflasyon, Enflasyon Vergisi, Kointegrasyon, Para Talebi, Rasyonel Beklentiler, Uyarlanabilir Beklentiler.

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# I - INTRODUCTION

Cagan (1956) formulated a specific version of the demand for money function and a specific hypothesis about the formation of inflationary expectations. Cagan's paper posed and dealt with questions about the role of money in generating inflation. His paper produced results that have had wide range of applications in the context of monetary approach to inflation. Cagan confined his study to hyperinflations where, he argued, fluctuations in the price level and the inflation rate swamped those in real income or the rate of return on capital goods. Hence, he formulated a demand for real money balances function in which the only argument was the expected inflation rate. Further, Cagan assumed adaptive expectations about inflation.

Cagan (1956) deals with the relation between changes in the quantity of money and price level during hyperinflations. Cagan defines hyperinflations as beginning in the month the rise in prices exceeds 50 percent and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year.

This thesis considers the demand for money under conditions of high inflation in Turkey during the period 1986:1-1995:3. We test whether the monetary and inflationary experiences of Turkey can be adequately characterized by the Cagan (1956) model, using an econometric procedure which is reliant only on the assumption that forecasting errors are stationary. Engle (1982) demonstrates that forecasting errors would be stationary under adaptive expectations.

Turkey has not experienced such a hyperinflation in the Cagan's sense, but high rates of inflation have been seen in Turkey during the period 1986-1995. Taylor and Phylaktis (1991) examined the demand for money under conditions of high inflation in some Latin American countries, during the 1970s and 1980s, using Cagan's hyperinflation model and these countries also have not experienced hyperinflation in the Cagan's sense. If the Cagan model is applicable to Turkey, then it can be a powerful tool of analysis in understanding the features of the monetary experiences of Turkey.

It is well known that generating inflation through printing money can be viewed as a means of raising revenue for the authorities-an inflation tax. Cagan (1956) shows that, in the context of the hyperinflation model, the revenue from the inflation tax, which results from money creation by the authorities, is maximized by a certain percentage rate of increase in prices and money. In the thesis, we test the hypothesis that the authorities expanded the money supply in such a way as to maximize the inflation tax revenue in Turkey for the considered period.

During the period 1986-1995, excluding 1994, Turkey experienced a stable annual inflation rate of sixty percent to seventy percent. This can be taken as a clue for rational expectations. This encouraged us to derive a test of the Cagan (1956) model with the additional assumption of rational expectations for Turkey for the considered period. Under the additional assumption of rational expectations, this implication of the hyperinflation model is a particular case of a general result for present value models discussed by Campbell and Shiller (1987).

In this thesis, section II gives a brief overview of the Turkish economy. Cagan's hyperinflation model is explained in section III and the methodology is described in section IV. In section V estimation results are presented and we concluded the results of the thesis in section VI.

## II - AN OVERVIEW OF TURKISH ECONOMY

After a period of economic and politic difficulties, some mixed stabilization and liberalization policies were announced by the Turkish government in January 1980. The announced policies aimed a new adjustment path with a new export lead development strategy. Main topics of the policies were the convertibility of the Turkish lira, flexible exchange rate policy and export promotions. As a component of the programme, there was a major devaluation of the Turkish lira in January 1980. The 1980 programme also included positive real interest rate policy. As a result of these policies in 1981-1983 period the inflation rate did not exceed 36%.

In the period 1984-1987, the average inflation rate was around 40%. In April 1986, the Central Bank set up an Interbank market for one and two week maturities and introduced overnight transaction in May 1986. In 1986, the Central Bank introduced for the first time the policy approach of targeting a monetary aggregate. Money in wider sense (M2) was selected to be kept on a growth path during the year. In 1986, M2 grew 38.6%, which was close to the target level. In 1986, M1 had a growth of 62.5% and reserve money had a growth of 32.8% and the consumer price inflation achieved 34.6%. For 1987 the monetary authorities targeted growth of M2 at 30 percent which was considered consistent with an expansion of 5 percent and an inflation rate of 25 percent. The Central Bank had planned 28 percent growth of the reserve money which was the main instrument to control M2. But reserve money growth was nearly 50 percent in 1987 and consumer price inflation was 38.9 percent. In 1987, M1 growth was 58.3% and M2 growth was 37.6%.

Growing public sector deficits has been a traditional problem of Turkish economy for many years. Largeness and incapability of the public economy has been the main causes of the growing public sector deficits. Deficits of the State Economic Enterprises (SEEs) and wasting the public worker's fee resources can be shown as examples to the incapability of the public economy. High debt interest payments, insufficient adjustment of prices of State Economic Enterprises to increased costs and a large increase in the public sector wage bill were the main factors behind the growing public sector deficits.

In view of accelerating inflation and instability in financial markets, monetary policy was severely tightened in 1988. Deposit interest rates were raised to encourage financial savings and reduce the share of currency and sight deposits in M2. But, in spite of this tightening policy, targets were exceeded by substantial amount in 1988. M1, M2 and reserve money growth were 39.7%, 77.5% and 67.5%, respectively. Consumer price inflation reached 75.4% in 1988.

In 1989 as a result of decree number 32 that is put into use, Turkish lira has become completely convertible across other foreign currencies and financial capital has become completely free to enter and leave the country. At this point to gain the macro financial balance in the country, exchange rate and domestic interest rate became integrated and real return from interest rate was higher than the real return from foreign currency. As a result, dolarization was prevented and domestic currency has been used widely and foreign capital entered the country.

For 1989, the Central Bank has abstained from announcing monetary targets. In 1989, reserve money growth accelerated due to increase in net foreign assets and due to the government's decision to grant large salary increases and to raise agricultural support prices. Reserve money growth reached 75% and M1 and M2 growth were 97.1% and 82%, respectively. In 1989, consumer price inflation was at the level of 69.6%. In the context of the programme of economic liberalization, the Turkish authorities have been aiming at placing greater reliance on monetary policy for economic stabilization purposes. However, as the Central bank is not completely autonomous and economic policy decisions are taken at the government level, it has been difficult to follow a clear anti-inflationary monetary policy.

Starting from 1990, interest rate-exchange rate balance and foreign capital inflow have directly depended on each other. In 1990 return from interest was 2.5% above the return from foreign currency and this caused 3000 million dollars of foreign capital inflow. In 1991 the return from interest over return from foreign currency fell to -3.3% and this caused 3020 million dollars of capital to leave the country. From this time after, return from interest have been always above the return from foreign currency and in 1992 and 1993 there have been seen net foreign capital inflow. In 1993 Total

Capital Movements item has reached 9279 million dollars and this value is 5.6% of GNP in 1993.

In this way it was aimed to cover public sector's deficits by savings from outside. But this also brought about increase in volume of imports. Public sector was continuing high interest rate policy by domestic credit and high interest rate was bringing about financial capital entrance into the economy. But this procedure has directly affected the goods market and there have been seen many cycling in the real production sectors. This was because that under the above procedure what was determining the exchange rate was not the international good and service trade, but it was the capital movements that depended on speculative demand determining the exchange rate. As a result exports have fallen and imports have risen. In 1989 the ratio of exports to imports was 0.736 but in 1993 the ratio has fallen to 0.516. Inflation has reached an average of 68.2% in the period 1988-1992. Monetary policy aimed at maintaining orderly conditions in financial markets. The Central bank, however, was again obliged to finance the PSBR, and hence fiscal imbalance induced rapid growth in the monetary aggregates. In the period 1988-1992, M1, M2 and reserve money growth reached an average of 62%, 67% and 58%, respectively.

Strong output growth in 1992 and 1993, led by domestic demand, brought about a widening current account deficit and rising foreign indebtedness. Inflationary pressures intensified, partly in response to the further increase in public sector deficits to very high levels. In 1993 real GNP growth averaged 6.75%, the trade deficit rose to 12% of GNP and public sector borrowing requirement (PSBR) rose to 16% of GNP. Annual consumer price inflation averaged 66% in 1993, compared with 70% in 1992. At the end of 1993, international creditworthiness was downrated and the Turkish lira drastically depreciated. M1, M2 and reserve money growth were 53%, 43% and 60%, respectively in 1993.

Starting in 1994, Turkish economy have undergone the most important crisis of the last 15 years. The crisis has started in the first months of 1994 in finance market and it has spread to the real part of the economy in a little time. The main causes of the crisis has been shown as the growing public sector deficits and the incorrect steps towards liberalization.

On 5 April 1994, the government announced a new programme. Prices of goods and services produced by SEEs were immediately raised by 110 percent. The new programme also envisages accelerated closure and privatization of SEEs, a decrease in public sector real wages and other unspecified public expenditure cuts. After the announcement of the package the Turkish lira depreciated further by about 35%, to some 60% below the level at the beginning of the year. Also the economic expansion stopped and there was a short term increase in inflation. Higher inflation, public sector wage restraint and labour shedding eroded real household incomes and depressed private consumption. Consumer price inflation was 126%, and wholesale price inflation was 150% in 1994. Public sector borrowing requirement fell to 8% of GNP. In 1994, M1, M2 and reserve money growth reached 85%, 132% and 85%, respectively. In April 1995, the annual consumer price inflation achieved 94%. And in April 1995, the three months M1, M2 and reserve money growth ratios achieved 15.6%, 19.2% and 20%, respectively.

## III - CAGAN'S HYPERINFLATION MODEL

Cagan (1956) deals with the relation between changes in the quantity of money and price level during hyperinflations. An outstanding characteristic of such periods is the decline in the real value of the quantity of money-real cash balances (*M/P*). Cagan defines hyperinflations as beginning in the month the rise in prices exceeds 50 percent and as ending in the month before the monthly rise in prices drops below that amount and stays below for at least a year. The theory developed by Cagan (1956) involves an extension of the Cambridge cash-balances equation. That equation asserts that real cash balances remain proportional to real income (*Y*) under given conditions ($M/P = kY$; *k is a constant*).

Cagan (1956) discusses that individuals' desired real cash balances depend on numerous variables. The main variables that affect an individual's desired real cash balances are his wealth in real terms, his current real income and the expected returns from each form in which wealth can be held, including money. Desired real cash balances change in the same direction as real wealth and current real income and in the direction opposite to changes in the return on assets other than money. A specification of the amount of real cash balances that individuals want to hold for all values of the variables listed above defines a demand function for real cash balances. Other variables usually have only minor effects on desired real cash balances and can be omitted from the demand function. This demand function and the other demand and supply functions that characterize the economic system simultaneously determine the equilibrium amount of real cash balances.

In one theory of this determination-the quantity theory of money-the absolute level of prices is independently determined as the ratio of the quantity of money supplied to a given level of desired real cash balances. Individuals can not change the nominal amount of money in circulation, but, according to the quantity theory of money, they can influence the real value of their cash balances by attempting to reduce or increase their balances. In this attempt they bid the prices of goods and services up or down, respectively, and thereby alter the real value of cash balances.

Cagan (1956) discusses that during hyperinflation the amount of real cash balances changes drastically. At first sight these changes may appear to reflect changes in individuals' preferences for real cash balances, but these changes in real cash balances may reflect instead changes in the variables that affect the desired level of balances. Cagan observed that two of the main variables affecting individuals' desired level, wealth in real terms and real income, were relatively stable during hyperinflation, at least compared with the large fluctuations in real cash balances. Thus he decided to look for large changes in the only remaining variable, which is the expected returns on various forms of holding wealth, to explain large fluctuations in the desired level of real cash balances. Changes in the return on an asset affect real cash balances only if there is a change in the difference between the expected return on the asset and that on money. If this difference rises, individuals will substitute the asset for part of their cash balances. So Cagan turned to a more detailed consideration of the difference in return on money and on various alternatives to holding money-the cost of holding cash balances.

Cagan also observed that the only cost of holding cash balances that fluctuate widely enough to account for the drastic changes in real cash balances during hyperinflation is the rate of depreciation in the value of money or, equivalently, the rate of changes in prices. This observation suggested the hypothesis that changes in real cash balances in hyperinflation result from variations in the expected rate of change in prices. Cagan assumed that desired real cash balances are equal to actual real cash balances at all times. This means that any discrepancy that may exist between the two is erased almost immediately by movements in the price level. He also assumed that the expected rate of change in prices is revised per period of time in proportion to the difference between the actual rate of change in prices and the rate of change that was expected.

Cagan's model is composed of two equations, an equation giving the demand for money and an equation describing the formation of expectations. The monetary equilibrium is given by

$$M/P = c\exp(-\alpha\pi^{*}), \qquad (3.1)$$

where $c$ and $\alpha$ are constant terms and $\pi^*$ is the expected rate of inflation. The higher expected inflation, the lower will be the demand for real money balances. Two important assumptions are implicit in this formulation. The first is that output is given and thus is part of the constant term $c$. The second is that the real interest rate is constant and thus also included in the constant term $c$. The main rationale for this functional form is convenience, though it appears consistent with the data from hyperinflations. In an equilibrium the real money stock must be equal to money demand, and ( 3.1 ) can be interpreted as an equilibrium equation. An implication of the above relation is that variations in the expected rate of changes in prices have the same effect on real cash balances in percentage terms regardless of the absolute amount of the balances. This follows from the fact that equation ( 3.1 ) is a linear relation between the expected rate of change in prices and the logarithm of real cash balances.

From equation ( 3.1 ), the elasticity of demand for real cash balances with respect to the expected change in prices can be written as

$$\frac{d(M/P)}{d\pi^*} \cdot \frac{\pi^*}{M/P} = -\alpha\pi^*,$$

where $\alpha\pi^*$ is a pure number. The elasticity is proportional to the expected rate of change in prices. It is positive when expected inflation rate is negative, and negative when expected inflation rate is positive.

The second equation Cagan used describes the formation of expectations. Cagan assumed adaptive expectations about inflation. Under adaptive expectations, expectations of inflation are adjusted according to

$$d\pi^*/dt = b(\pi - \pi^*), \qquad (3.2)$$

where $\pi$ is the actual inflation rate. If current inflation exceeds expected inflation, expected inflation increases. The coefficient $b$ reflects the speed at which individuals revise their expectations. Note that the expected inflation depends only on past inflation. Equation ( 3.2 ) can be integrated to yield

$$\pi^*_t = b\int^t_{-\infty} \pi_s \exp[b(s - t)]\, ds.$$

Given the dynamics of money growth, equations ( 3.1 ) and ( 3.2 ) determine the dynamics of inflation.

Cagan (1956) studied if inflation will converge to $\sigma$ or it will take off on its own toward hyperinflation when money growth is constant at rate $\sigma$. To answer this question differentiate equation ( 3.1 ) after taking logarithms. This gives

$$\sigma - \pi = -\alpha(d\pi^*/dt). \qquad ( 3.3 )$$

Eliminating $d\pi^*/dt$ between ( 3.2 ) and ( 3.3 ) gives the relation

$$\sigma - \pi = -\alpha b(\pi - \pi^*). \qquad ( 3.4 )$$

Cagan showed that a self-generating inflation is impossible if the product of the parameters $\alpha$ and $b$ is less than unity. Thus econometric estimates of these two parameters provide vital evidence on the stability of the inflationary process. Cagan estimated his model using data on seven hyperinflations and was not able to reject the hypothesis that the stability conditions were satisfied. He found out that, in these seven hyperinflations, the sensitivity of the demand for money to the expected rate of inflation and the sensitivity of the expected inflation rate to the actual rate are both small enough to rule out a self-generating inflation.

If $\alpha b > 1$, then the equilibrium is unstable. In the unstable case, depending on the initial conditions, the economy can have either accelerating inflation or accelerating deflation. Thus whether there can be hyperinflation under constant money growth depends on the parameters $\alpha$ and $b$, which reflect respectively the elasticity of money demand and the speed of revision of expectations. Why is the equilibrium unstable if

$\alpha b > 1$? If $b$ is large, higher inflation leads money holders to quickly revise upward their expectations of inflation and thus to attempt to reduce their money holdings; given money growth, this leads to further inflation, further revisions, and accelerating inflation. If $\alpha$ is large, an increase in inflation that leads to an upward revision of expected inflation has a strong negative effect on money demand, leading again to accelerating inflation. Accordingly, if individuals have adaptive expectations, it is possible for hyperinflation to result not from accelerating money growth but rather from a self-generating unstable process.

Cagan (1956) also studied the maximum amount of revenue, that is available, from the inflation tax, if the equilibrium is stable. The inflation tax is the tax imposed on money holders as a result of inflation, i.e., it is the loss in the value of money holders' real balances. Inflation tax is equal to

$$I = \frac{dP/dt}{P}\frac{M}{P} = \pi\frac{M}{P}. \qquad (3.5)$$

Using equation ( 3.1 ) and the fact that in steady state (without growth) $\pi^{*} = \sigma$ gives

$$I = \pi c \exp(-\alpha\sigma).$$

Accordingly, steady state inflation tax is maximized when $\sigma = 1/\alpha$. So the percentage increase in prices and money, which maximizes the revenue from the inflation tax, is just equal to $(100/\alpha)\%$.

## IV - METHODOLOGY

### 4.1. Stationarity

A stochastic process is said to be *stationary*, if the joint and conditional probability distributions of the process are unchanged if displaced in time. In practice, it is more usual to deal with *weak sense* stationarity, restricting attention to the means, variances and covariances of the process ( Spanos 1986 ). Consider a simple time series model as follows:

$$y_t = \alpha y_{t-1} + \varepsilon_t,$$

where $\varepsilon_t$ is the uncorrelated disturbance term with zero mean and constant variance. In such a model, if $\alpha$ is less than 1 in absolute value, the observations fluctuate around zero. Such series in econometrics is said to be stationary. On the other hand, if the absolute value of $\alpha$ is greater than 1, the model is explosive.

Then, a stochastic process $y_t$ is said to be *stationary* if:

$$E(y_t) = \text{constant} = \mu;$$
$$Var(y_t) = \text{constant} = \sigma^2;$$

and:

$$Cov(y_t y_{t+j}) = \sigma_j.$$

Thus the means and the variances of the process are constant over time, while the value of the covariance between two periods depends only on the gap between the periods, and not the actual time at which this covariance is considered. If one or more of the conditions above are not fulfilled, the process is nonstationary.

Equivalently a time series is said to be *stationary* if:

$$y_t = E(y_t) + \varepsilon_t,$$

and

$$E(\varepsilon_t) = 0,$$
$$E(\varepsilon^2_t) = \sigma^2,$$
$$E(\varepsilon_t\varepsilon_s) = 0 \qquad t \neq s.$$

Therefore a stationary series is said to tend to fluctuate around its mean with broadly constant amplitude. Whereas a nonstationary time series will have a time varying mean and variance so that cannot be referred without reference of some particular time period.

An important type of a nonstationary stochastic process is the process which is called *random walk.* The main assumption is that, every current observation consist of its own previous value plus a random disturbance term and disturbance terms are identically distributed independent random variables:

$$y_t = y_{t-1} + \varepsilon_t,$$
$$E(\varepsilon_t) = \mu,$$
$$E(\varepsilon^2_t) = \sigma^2,$$
$$E(\varepsilon_t\varepsilon_s) = 0 \qquad t \neq s.$$

Another example of a nonstationary stochastic process is:

$$y_t = a + y_{t-1} + \varepsilon_t, \quad a \neq 0,$$

where $\varepsilon_t$ is defined as before as a series of identically distributed independent random variables and $a$ is constant. This stochastic process is called a *random walk with drift.*

If the errors $\varepsilon_t$ are identically distributed independent random variables with zero means, then the stochastic process $\varepsilon_t$ is called a *white noise process.* In economics, the form of nonstationarity in a time series may well be evident from an examination of the series. If the form of nonstationarity is a propensity of the series to move in one direction, we will call this tendency a *trend.*

A series may drift slowly upwards or downwards purely as a result of the effects of stochastic or random shocks. This is true for the random walk process. The variance of this process increases over time and also the correlation between neighbouring values increases over time. These results imply that there may be long periods in which the process takes values well away from its mean value. Such series is called a time series with a *stochastic trend.*

Another example of a developing tendency in a nonstationary stochastic process is where the mean of the process is itself a specific function of time. If such a function is linear then the process can be described as:

$$y_t = \mu_t + \varepsilon_t,$$

where:

$$\mu_t = \alpha + \beta t,$$

or:

$$y_t = \alpha + \beta t + \varepsilon_t.$$

In this case it is said the process has a *deterministic trend.* A mixed stochastic-deterministic trend process is also possible. That is, the process can be described as:

$$y_t = \alpha + \beta t + y_{t-1} + \varepsilon_t.$$

In these expressions, it has been assumed that the expected values of $\varepsilon_t$ are zero and that the stochastic process $\varepsilon_t$ is white noise, but these conditions may be relaxed to allow for autocorrelation in the series of $\varepsilon_t$.

Stationarity is an important concept in time series modeling. However, many time series, in economics, are not stationary. But nevertheless, by taking first or second differences, a nonstationary series can be transformed to a stationary series. Sometimes it is necessary to difference a series more than once in order to achieve stationarity. A nonstationary series which can be transformed to a stationary series by differencing $d$ times is said to be *integrated of order d* ( Engle and Granger 1987 ). A series $y_t$ integrated of order $d$ is denoted as $y_t \sim I(d)$.

### 4.2. Unit Roots

#### 4.2.1. Introduction

As we indicated above, in the time series, a statistical time series may be difference stationary. Consider a simple difference stationary series:

$$y_t = y_{t-1} + \varepsilon_t,$$

where $\varepsilon_t$ is an independent, normal, zero mean stationary process. In such a model the effect of a shock is permanent. Any jump in $\varepsilon_t$ will cause increase in all $y_t$'s. On the other hand if the shock fades away then we assume the model to be:

$$y_t = \alpha y_{t-1} + \varepsilon_t \quad \alpha < 1.$$

Therefore, whether there is a unit root or not ( $\alpha = 1$ or $\alpha < 1$ ), becomes a very important issue for economists.

#### 4.2.2. Unit Root Tests

Suppose we wish to test the hypothesis that a variable $y_t$ is integrated of order one, that is that $y_t$ is generated by:

$$y_t = y_{t-1} + \varepsilon_t,$$

where $\varepsilon_t$ represents a series of identically distributed stationary variables with zero means.

A straightforward procedure would seem to test for $\alpha = 1$ in the model:

$$y_t = \alpha y_{t-1} + \varepsilon_t. \qquad (4.1)$$

An appropriate and simple method of testing the order of integration of $y_t$ has been proposed by Dickey and Fuller (1979) which is called the *DF test*. The DF test is a test of the hypothesis that in ( 4.1 ) $\alpha = 1$, the so-called *unit root test*. This test proposes a simple method of testing for $\alpha = 1$ or $\alpha < 1$. Instead of equation ( 4.1 ) we can write

$$\Delta y_t = \delta y_{t-1} + \varepsilon_t, \qquad (4.2)$$

where

$$\alpha = 1 + \delta.$$

Then the test is simply testing $\delta = 0$ or $\delta < 0$. If $\delta$ is significantly negative then $\alpha < 1$ and the series are time stationary. Whereas, if $\delta = 0$ then $\alpha = 1$ and the series $y_t$ has a unit root. So the Dickey-Fuller test consists of testing the negativity of $\delta$ in the ordinary least squares regression of ( 4.2 ). Rejection of the null hypothesis $\delta = 0$ in favor of the alternative $\delta < 0$ implies that $\alpha < 1$ and that $y_t$ is integrated of order zero.

Since in ( 4.2 ) we want to evaluate a hypothesis which concerns only a single parameter, the natural choice would seem to be that of a Student t-ratio. But, for equation ( 4.2 ), this ratio or statistic does not have the familiar Student t-distribution. Because of the unit root, the t-ratio does not have a limiting normal distribution. Therefore, the simulated DF critical values table are used for comparison. Critical values for the DF test statistics are tabulated in Fuller (1976), table 8.5.2.

If the null hypothesis can not be rejected, the variable $y_t$ might be integrated of order higher than zero, or might not be integrated at all. Consequently the next step would be to test the order of integration is one. Hence, we repeat the test for:

$$\Delta\Delta y_t = \delta \Delta y_{t-1} + \varepsilon_t,$$

and again our interest is in testing the negativity of $\delta$. If the null hypothesis is rejected and the alternative $\delta < 0$ can be accepted, the series $y_t \sim I(1)$. If the null hypothesis can not be rejected, we may test whether $y_t \sim I(2)$. We can continue the process until we establish an order of integration for $y_t$. But this process creates a danger of overdifferencing, which results in a very high positive value of DF test accompanied by a very high coefficient of determination for the fitted regression. Such cases indicate that either the series is integrated of some order but the test fails to discover this or, the series is not an integrated time series and differencing cannot transfer it into a stationary series.

The DF test can also be used for testing the order of integration for a variable generated as a stochastic process with drift, that is by tests on the equation:

$$\Delta y_t = a + \delta y_{t-1} + \varepsilon_t,$$

where $a$ is a constant representing drift. A modification of the DF equation which accounts for both drift and a linear deterministic trend is the following:

$$\Delta y_t = a + \beta t + \delta y_{t-1} + \varepsilon_t.$$

A weakness of the DF test is that it does not take account of possible autocorrelation in the error process. If $\varepsilon_t$ is autocorrelated, then the ordinary least squares estimates of equation ( 4.2 ) are not efficient. A simple solution, advocated by Dickey and Fuller (1981), is to use lagged left-hand side variables as additional explanatory variables to approximate the autocorrelation. This test is called *Augmented Dickey-Fuller (ADF) test*. The ADF tests involve estimating the equation:

$$\Delta y_t = \delta y_{t-1} + \sum_{i=1}^{k} \delta_i \Delta y_{t-i} + \varepsilon_t,$$

The value of $k$ must be small enough to save the degrees of freedom, but large enough to capture the autocorrelation in the error process. The testing procedure is the same as DF, with an examination of the Student t-ratio for $\delta$ and the critical values are the same as for the DF test. A modification of the ADF equation which accounts for drift is the following:

$$\Delta y_t = a + \delta y_{t-1} + \sum^{k}_{i=1}\delta_i \Delta y_{t-i} + \varepsilon_t.$$

A modification of the ADF equation which accounts for both drift and a linear deterministic trend is the following:

$$\Delta y_t = a + \beta t + \delta y_{t-1} + \sum^{k}_{i=1}\delta_i \Delta y_{t-i} + \varepsilon_t.$$

### 4.3. Cointegration Analysis

Time series $x_t$ and $y_t$ are said to be *cointegrated* of order d, b where $d \geq b \geq 0$, written as: $x_t, y_t \sim CI(d, b)$, if:

i. both series are integrated of order *d*,

ii. there exists a linear combination of these variables, say $\alpha_1 x_t + \alpha_2 y_t$, which is integrated of order *d - b*. The vector $[\alpha_1, \alpha_2]$ is called a *cointegrating vector*.

A generalization of the above definition is the following. If $x_t$ denotes an $n \times 1$ vector of series and:

i. each of them is I(d),

ii. there exists an $n \times 1$ vector $\beta$ such that $x'_t . \beta = \sim I(d - b)$, then: $x'_t . \beta \sim CI(d, b)$.

The vector $\beta$ is called the *cointegrating vector*. If $d = b = 1$ then the components of $x_t$ is I(d) and the equilibrium error will be I(0) and will not drift far from its mean.

If $x_t$ has n components, there may be more than one cointegrating vector $\beta$. It is assumed that there are *r* independent cointegrating vectors ($r \leq n - 1$) which constructs the rank of $\beta$ and is called the *cointegrating rank* ( Granger 1981 ).

Two types of tests can be used for cointegration analysis. The first cointegration test is the *Engle-Granger two step approach*. To test for cointegration between a pair of series, one can formulate the cointegration regression as,

$$y_t = \alpha_0 + \alpha_1 x_t + u_t,$$

and test if the residual $u_t$ is I(0) or not. The null hypothesis is that $x_t$, $y_t$ are not cointegrated. The DF cointegration test involve estimating the equation:

$$\Delta u_t = \delta u_{t-1} + \varepsilon_t.$$

The ADF cointegration test involve estimating the equation:

$$\Delta u_t = \delta u_{t-1} + \sum^{k}_{i=1}\delta_i \Delta u_{t-i} + \varepsilon_t.$$

The critical values of the test are the same as used for testing integration. If $\delta$ is less than the critical ADF value the null hypothesis is rejected and $x_t$, $y_t$ are cointegrated. Critical values for the ADF cointegration test statistics are tabulated in Engle and Granger (1987), table 2.

The second test employed for cointegration analysis is the *maximum likelihood procedure* suggested by Johansen (1988). This procedure analyses multicointegration directly investigating coïntegration in the vector autoregression, VAR, model. We will assume throughout that all the variables in $z_t$ are integrated of the same order, and that this order of integration is either zero or one. The VAR model can be represented, ignoring the deterministic part (intercepts, deterministic trends, seasonals, etc. ), in the form:

$$\Delta z_t = \sum^{k-1}_{i=1}\Gamma_i \Delta z_{t-i} + \Pi z_{t-k} + \varepsilon_t, \qquad (4.3)$$

where:

$\Gamma_i = -I + A_1 + \ldots + A_i$ ( I is a unit matrix ),

$\Pi = -(I - A_1 - \ldots - A_k)$

and $\varepsilon_t$ are independent $n$ dimensional Gaussian variables with zero mean and variance matrix $\Sigma$ and stationary. Since there are $n$ variables which constitute the vector $z_t$ the dimension of $\Pi$ is $n \times n$ and its rank can be at most equal to $n$. If the rank of matrix $\Pi$ is equal to $r < n$, there exists a representation of $\Pi$ such that :

$$\Pi = \alpha\beta',$$

where $\alpha$ and $\beta$ are both $n \times r$ matrices. Matrix $\beta$ is called the *cointegrating matrix* and has the property that $\beta' z_t \sim I(0)$, while $z_t \sim I(1)$. The columns of $\beta$ contain the coefficients in the $r$ cointegrating vectors. The $\alpha$ matrix is called the *adjustment or loadings matrix*, which measure the speed of adjustment of particular variables with respect to a disturbance in the equilibrium relation.

By regressing $\Delta z_t$ and $z_{t-k}$ on $\Delta z_{t-1}, \Delta z_{t-2}, \ldots, \Delta z_{t-k+1}$ we obtain residuals $R_{0t}$ and $R_{kt}$. The residual product moment matrices are,

$$S_{ij} = T^{-1}\Sigma^{T}_{t=1} R_{it} R'_{jt}, \quad i, j = 0, k. \text{ ( T = sample size ).}$$

Solving the eigenvalue problem,

$$| \mu S_{kk} - S_{k0}S^{-1}_{00}S_{0k} | = 0, \qquad (4.4)$$

yields the eigenvalues $\hat{\mu}_1 > \hat{\mu}_2 > \ldots > \hat{\mu}_n$ ( ordered from the largest to the smallest ) and associated eigenvectors $\hat{u}_i$ which may be arranged into the matrix $V = [\hat{u}_1\, \hat{u}_2 \ldots \hat{u}_n]$. The eigenvectors are normalized such that $V'S_{kk}V = I$. If the cointegrating matrix $\beta$ is of rank $r < n$, the first $r$ eigenvectors are the cointegrating vectors, that is they are the columns of matrix $\beta$. Using the above eigenvalues, the hypothesis that there are at most $r$ cointegrating vectors can be tested by calculating the loglikelihood ratio test statistics:

$$LR = -T\, \Sigma^{n}_{i=r+1} \ln(1 - \hat{\mu}_i).$$

This is called the *trace statistic* ( Johansen and Juselius 1990 ). Normally testing starts from $r = 0$, that is from the hypothesis that there are no cointegrating vectors in a VAR model. If this cannot be rejected the procedure stops. If it is rejected, it is possible to examine sequentially the hypothesis that $r \leq 1$, $r \leq 2$, and so on.

There is also a likelihood ratio test known as the *maximum eigenvalue test* in which the null hypothesis of $r$ cointegrating vectors is tested against the alternative of $r + 1$ cointegrating vectors. The corresponding test statistic is:

$$LR = -T \ln(1 - \hat{\mu}_r).$$

These tests are asymptotically distributed as a (n - r) dimensional Brownian motion with covariance matrix $I$ ( Johansen 1992 ). The critical values of these tests are tabulated by Johansen and Juselius (1990).

## V - EMPIRICAL RESULTS

In this section, we first test the applicability of the Cagan model for Turkey, using a cointegration test which depends on the only assumption that forecasting errors are stationary. The hypothesis that the authorities, in Turkey during 1986-1995, expanded the money supply, on average, in such a way as to maximize the inflation tax revenue is tested using a likelihood ratio test. Finally we test whether the Cagan model can be coupled with rational expectations hypothesis for Turkey for the considered period.

### 5.1. The Model

Denoting the logarithm of nominal money balances and prices by *m* and *p* respectively, the Cagan model, discussed in section III, can be written, ignoring the constant term:

$$(m - p)_t = -\alpha\pi^*_t + \psi_t,$$

where $\psi_t$ denotes elements of money demand not captured by the model. Using $\Delta p^e_{t+1}$ as a representation of expected inflation rate instead of $\pi^*_t$, the above equation can be written as

$$(m - p)_t = -\alpha\Delta p^e_{t+1} + \psi_t. \qquad (5.1)$$

Cagan's insight is that under extreme inflationary conditions, real money holdings will be largely determined by inflationary expectations, with the components of $\psi_t$ playing a relatively minor role in their determination. So according to Cagan, $\psi_t$ will be stationary under extreme inflationary conditions. Replacing expected with actual inflation in ( 5.1 ):

$$(m - p)_t = -\alpha\Delta p_{t+1} + \varepsilon_{t+1}, \qquad (5.2)$$

where $\varepsilon_{t+1} = [\psi_t + \alpha(\Delta p_{t+1} - \Delta p^e_{t+1})]$. Now, suppose that, under conditions of very high and accelerating inflation, the growth rate in real money balances and the rate of change of inflation are each stationary processes. This would imply that $(m - p)_t$ and $\Delta p_t$ are each first difference stationary or, in the terminology of Engle and Granger (1987), integrated of order one, I(1). Adding $\alpha\Delta p_t$ to both sides of ( 5.2 ) we have

$$(m - p)_t + \alpha\Delta p_t = -\alpha\Delta^2 p_{t-1} + \varepsilon_{t-1}. \qquad (5.3)$$

If we assume that expectational errors ( $\Delta p_{t+1} - \Delta p^e_{t+1}$ ) are stationary, then $\varepsilon_{t+1}$ is stationary. Since $\alpha\Delta^2 p_{t+1}$ and $\varepsilon_{t+1}$ are both stationary, equation ( 5.3 ) implies that the linear combination $[(m - p)_t + \alpha\Delta p_t]$ must also be stationary, even though $(m - p)_t$ and $\Delta p_t$ are individually non-stationary. Hence, real money balances and inflation are cointegrated (Engle and Granger, 1987) with a cointegrating parameter ( after normalization on real balances ) just equal to $\alpha$. Thus, a simple test of the applicability of the hyperinflation model lies in testing whether or not real money balances and inflation are cointegrated. If we find out that real money balances and inflation are cointegrated, we will find out that $\varepsilon_{t+1}$ is stationary. With the assumption that expectational errors are stationary, this will support that $\psi_t$ is stationary.

## 5.2. The Data Set

The data set consists of monthly observations for the period 1986:1-1995:3 and data are taken from the Central Bank. The variables of the model are price index and money supply. Two indices of price level are used; the Consumer Price Index (CPI) and the Wholesale Price Index (WPI). Money supply is represented by three monetary aggregates; narrow money (M1) which is currency in circulation plus demand deposits, M2 which is M1 plus time deposits and, reserve money (RM) which is currency in circulation plus reserves held by commercial banks at the Central Bank.

## 5.3. Unit Roots and Testing for the Order of Integration

The DF and ADF tests are applied to study the unit roots in the real money balance and inflation rate series. Each ADF regression initially includes twelve lagged differences to ensure that the residuals are empirically white noise. Then a sequential reduction procedure is applied to eliminate the insignificant lagged differences. The DF and ADF test results are represented below in Table 1. The DF and ADF tests are first applied to each variable for a unit root in levels. Then the same tests are applied to the first differences of the variables that have a unit root in the level specification. The DF and ADF tests are constructed for random walk, random walk with drift, and random walk with trend and drift.

L denotes the natural logarithm of variables and Δ denotes first difference of variables. ΔLCPI denotes consumer price inflation and ΔLWPI denotes wholesale price inflation. ΔΔLCPI and ΔΔLWPI denote the first differences of these inflation rate series. Real money balance is denoted in the logarithm form, in the form $(m\text{-}p)$, where $m$ and $p$ are the logarithm of nominal money balances and prices respectively. So LM1-LCPI denotes real money balances calculated using M1 and CPI. LM1-LWPI denotes real money balances calculated using M1 and WPI. LM2-LCPI denotes real money balances using M2 and CPI, etc.

**Table 1.1. DF and ADF Tests for Inflation Rate Using Consumer Price Index (CPI).**

Unit root tests for variable ΔLCPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -6.940 | -7.444 | -3.447 |
| ADF | -6.828 | -7.403 | 0.754 |

Unit root tests for variable ΔΔLCPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -12.466 | -12.405 | -12.530 |
| ADF | -9.102 | -9.058 | -9.140 |

**Table 1.2. DF and ADF Tests for Inflation Rate Using Wholesale Price Index (WPI).**

Unit root tests for variable ΔLWPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -6.422 | -6.664 | -3.394 |
| ADF | -6.422 | -6.664 | -2.573 |

Unit root tests for variable ΔΔLWPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -13.001 | -12.932 | -13.069 |
| ADF | -6.884 | -6.846 | -6.908 |

**Table 1.3. DF and ADF Tests for Real Money Balance Using M1 and CPI.**

Unit root tests for variable LM1-LCPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -2.590 | -4.063 | -0.546 |
| ADF | -2.859 | -3.879 | -0.529 |

Unit root tests for variable Δ(LM1-LCPI)

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -13.888 | -13.858 | -13.912 |
| ADF | -14.924 | -14.857 | -14.889 |

**Table 1.4. DF and ADF Tests for Real Money Balance Using M1 and WPI.**

Unit root tests for variable LM1-LWPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -3.057 | -3.217 | -0.397 |
| ADF | -3.362 | -3.444 | -0.485 |

Unit root tests for variable Δ(LM1-LWPI)

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -13.006 | -13.016 | -13.055 |
| ADF | -13.562 | -13.591 | -13.605 |

**Table 1.5. DF and ADF Tests for Real Money Balance Using M2 and CPI.**

Unit root tests for variable LM2-LCPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -2.392 | -2.764 | -0.398 |
| ADF | -2.721 | -2.795 | -0.097 |

Unit root tests for variable Δ(LM2-LCPI)

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -9.408 | -9.357 | -9.444 |
| ADF | -6.620 | -6.583 | -6.655 |

**Table 1.6. DF and ADF Tests for Real Money Balance Using M2 and WPI.**

Unit root tests for variable LM2-LWPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -2.418 | -2.394 | -0.132 |
| ADF | -1.428 | -1.346 | -0.241 |

Unit root tests for variable Δ(LM2-LWPI)

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -8.637 | -8.601 | -8.681 |
| ADF | -7.123 | -7.090 | -7.164 |

**Table 1.7. DF and ADF Tests for Real Money Balance Using RM and CPI.**

Unit root tests for variable LRM-LCPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -1.358 | -3.558 | -0.868 |
| ADF | -0.956 | -2.825 | -0.912 |

Unit root tests for variable Δ(LRM-LCPI)

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -10.405 | -10.384 | -10.377 |
| ADF | -10.572 | -10.552 | -10.538 |

**Table 1.8. DF and ADF Tests for Real Money Balance Using RM and WPI.**

Unit root tests for variable LRM-LWPI

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -2.096 | -2.841 | -0.601 |
| ADF | -2.906 | -2.841 | -0.601 |

Unit root tests for variable Δ(LRM-LWPI)

| Statistic | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| DF | -11.043 | -11.072 | -11.057 |
| ADF | -11.043 | -11.072 | -11.057 |

Critical values for the DF test statistics are obtained from Fuller (1976), table 8.5.2. Critical values are the same for both the DF and ADF test statistics and these critical values are presented in Table 2.

**Table 2. Critical Values for the DF Test Statistics for Unit Root Test.**

| Sample size = 100 | with constant | with constant and trend | without trend and constant |
|---|---|---|---|
| 1% | -3.51 | -4.04 | -2.60 |
| 5% | -2.89 | -3.45 | -1.95 |
| 10% | -2.58 | -3.15 | -1.61 |

The graphs of the variables and the graphs of the first differences of the variables are presented in the Appendices. In all cases the first differenced series do not exhibit a unit root: the I(1) hypothesis can only be rejected when the inflation and real money series are first differenced. So according to the DF and ADF test results, real money balances and inflation rate are each integrated of order one, characterized as I(1), with test statistics significant even at 1% level.

## 5.4. Testing for Cointegration (Testing for Adaptive Expectations)

The null hypothesis of no cointegration between inflation and real money balances against one available cointegrating vector is tested using both the Engle and Granger (1987) two-step procedure and Johansen's (1988) method of maximum likelihood estimation of the multi-cointegrated VAR systems.

The Engle-Granger (1987) two-step procedure involves regressing real money balances on inflation rate first, to obtain the residuals. Then the test for the null hypothesis that cointegration exists is based on testing for unit root in the regression residuals using the ADF tests. The results from the cointegrating regressions are presented in Table 3.

**Table 3. Test of Cointegration Between Real Money Balances and Inflation Rate.**

| Dependent Variable | Independent Variable | ADF Statistics |
|---|---|---|
| LM1-LCPI | ΔLCPI | -5.386 |
| LM1-LWPI | ΔLWPI | -4.764 |
| LM2-LCPI | ΔLCPI | -5.393 |
| LM2-LWPI | ΔLWPI | -4.784 |
| LRM-LCPI | ΔLCPI | -5.362 |
| LRM-LWPI | ΔLWPI | -4.770 |

ADF test statistics are initially based on regressions with twelve lags. Then a sequential reduction procedure is applied to eliminate the insignificant lagged differences. The critical values for the ADF test statistics are obtained from Engle and Granger (1987), table 2 and these critical values are presented in Table 4.

**Table 4. Critical Values for the ADF Test Statistics for Cointegration Test.**

| Statistic | 1% | 5% | 10% |
|---|---|---|---|
| ADF | -3.77 | -3.17 | -2.84 |

Real money balances seem to be cointegrated with inflation rate as ADF test statistics for testing cointegration between real money balances and inflation rate are significant even at 1% level.

All empirical models are inherently approximations of the actual data generating process and the question is whether the benchmark model ( 4.3 ) is a satisfactorily close approximation. Therefore we investigated the stochastic specification with respect to residual correlation, heteroscedasticity and normality. The residual tests are reported in Table 5. $\sigma_\varepsilon$ is the standard deviation of the residuals, $\chi^2(2)$ is the Jarque-Bera test statistic for normality, ARCH $F_{(d.f.6,58)}$ is the ARCH test for heterocedastic residuals, AR $F_{(df:6,64)}$ is the test for residual autocorrelation, *skewness* is the third moment around the mean and *excess kurtosis* is the fourth moment around the mean.

**Table 5. Residual Misspecification Tests.**

| Equation | $\sigma_\varepsilon$ | $\chi^2$ | Skew. | Ex. kurt. | ARCH 6 F | AR 1- 6F |
|---|---|---|---|---|---|---|
| I | | | | | | |
| Δ(LM1-LCPI) | 0.0523 | 5.2473 | -0.0289 | 0.8471 | 2.1576 | 0.5801 |
| ΔΔLCPI | 0.0215 | 71.335 | 2.6623 | 12.407 | 0.0460 | 0.6436 |
| II | | | | | | |
| Δ(LM1-LWPI) | 0.0520 | 8.3987 | -0.1743 | 1.2193 | 3.1029 | 0.5226 |
| ΔΔLWPI | 0.0252 | 95.613 | 3.2407 | 18.469 | 0.0299 | 1.8788 |
| III | | | | | | |
| Δ(LM2-LCPI) | 0.0212 | 7.7048 | -0.2426 | 1.1689 | 0.4264 | 0.5078 |
| ΔΔLCPI | 0.0181 | 44.534 | 2.4360 | 13.900 | 0.0491 | 0.1359 |
| IV | | | | | | |
| Δ(LM2-LWPI) | 0.0272 | 4.8235 | -0.2228 | 0.8143 | 3.0286 | 2.5585 |
| ΔΔLWPI | 0.0215 | 51.185 | 2.6664 | 15.210 | 0.0299 | 0.8123 |
| V | | | | | | |
| Δ(LRM-LCPI) | 0.0425 | 4.0534 | 0.1385 | 0.7027 | 1.4039 | 1.0129 |
| ΔΔLCPI | 0.0229 | 81.052 | 2.9389 | 15.2843 | 0.0481 | 0.7902 |
| VI | | | | | | |
| Δ(LRM-LWPI) | 0.0374 | 8.7691 | 0.5873 | 1.4814 | 1.1245 | 0.4862 |
| ΔΔLWPI | 0.0226 | 43.956 | 2.3927 | 12.8392 | 0.0445 | 2.3344 |

The benchmark model ( 4.3 ) seems to provide a reasonably good approximation of the data generating process. There is no indication of residual autocorrelation in any of the series ( $F_{.99}$ (6,64) $\approx$ 3.12 ). ARCH 6 F did not reject homoscedasticity of residuals in any of the series ( $F_{.99}$ (6,58) $\approx$ 3.12 ). A few problems remain, such as normality of residuals are rejected for equations of inflation (*ΔΔp)* no matter which price index we used ( $\chi^2_{.99}(2) = 9.12$ ) and first differenced inflation series (*ΔΔp*) appear to be leptocurtic. Critical values of F test and chi-square test are obtained from Hines and Montgomery, 1980, table III and V.

Using the procedure suggested by Johansen (1988), cointegration between inflation and real money balances can be investigated by utilizing the VAR model. In the Johansen (1988) trace test, the null hypothesis is that there are at most *r* cointegrating vectors and it is tested against a general alternative. In the maximum eigenvalue test, the null hypothesis of *r* cointegrating vectors is tested against $r + 1$ cointegrating vectors. The hypothesis of at most zero and one cointegrating vectors are tested, respectively, and the maximum eigenvalue and the trace test statistics are presented in Table 6.

**Table 6. Johansen Cointegration Tests and Estimates**

| Variables | Eigenvalue Test Statistics | | Trace Test Statistics | | $\hat{\alpha}$ | LR ($100\alpha^{-1}=\pi$) |
|---|---|---|---|---|---|---|
| | $H_0 : r = 0$ | $H_0 : r \leq 1$ | $H_0 : r = 0$ | $H_0 : r \leq 1$ | | |
| LM1-LCPI ΔLCPI | 17.61073 | 6.41696 | 24.02769 | 6.41696 | 22.0170 | 2.44227 |
| LM1-LWPI ΔLWPI | 20.36677 | 3.451489 | 23.81827 | 3.451489 | 16.7654 | 1.69635 |
| LM2-LCPI * ΔLCPI | 15.06210 | 7.163107 | 22.22521 | 7.163107 | 22.2355 | 2.76026 |
| LM2-LWPI ΔLWPI | 20.84050 | 0.457527 | 21.29803 | 0.457527 | 21.3464 | 1.73717 |
| LRM-LCPI ΔLCPI | 15.43711 | 4.759711 | 20.19683 | 4.759711 | 23.5454 | 1.86819 |
| LRM-LWPI* ΔLWPI | 16.01599 | 8.020865 | 24.03686 | 8.020865 | 22.5000 | 2.64124 |

* 11 seasonals are included due to the criterion of having a meaningful long run equilibrium.

The critical values for the trace and maximum eigenvalue test statistics are obtained from Johansen and Juselius (1990), table A2 and these critical values are presented in Table 7.

**Table 7. Critical Values for the Trace and Maximum Eigenvalue Test Statistics**

| | Eigenvalue Test Statistics | | Trace Test Statistics | |
|---|---|---|---|---|
| Significance | $H_0 : r = 0$ | $H_0 : r \leq 1$ | $H_0 : r = 0$ | $H_0 : r \leq 1$ |
| 5% | 14.595 | 8.083 | 17.844 | 8.083 |

Applying the trace test and the maximum eigenvalue test for cointegration due to Johansen (1988), the hypothesis of at most one cointegrating vector ( $H_0 : r \leq 1$ ) can not be rejected in any case, while the hypothesis of zero cointegrating vectors ( $H_0 : r = 0$ ) is easily rejected in every case. Hence, real money balances and inflation are cointegrated with the cointegrating vector $[1, \alpha]$ ( after normalization on real balances ). This constitutes evidence in favor of the Cagan model for the Turkish case. So, assuming that agents' forecasting errors are stationary, the monetary and inflationary experiences of Turkey can be adequately characterized by the Cagan (1956) model. Table 6 also lists the estimates of $\alpha$, which is the cointegrating parameter after normalization on real balances. The estimates of $\alpha$ are calculated by normalizing the cointegrating vectors, estimated as a result of Johansen's cointegration test, on real balances.

Cagan (1956) also studied the maximum amount of revenue that is available from inflation tax. Cagan showed that, in the context of the hyperinflation model, the percentage rate of increase in prices and money, which maximizes the revenue from the inflation tax which results from money creation by the authorities, is just equal to $(100/\alpha)$%. Table 6 also lists the likelihood ratio test statistics for the null hypothesis that $100/\alpha$ is in fact equal to the average inflation rate which prevailed over the period. The likelihood ratio test statistic, constructed as in Johansen (1988), now becomes

$$LR = T \sum_{i=1}^{r} \ln\{(1 - \mu^{*}_{i})/(1 - \hat{\mu}_{i})\},$$

where $\hat{\mu}_i$ are the $r$ largest eigenvalues under no restrictions and the $\mu^{*}_{i}$ are the $r$ largest eigenvalues from solving ( 4.4 ) under the restriction that $100/\alpha$ is equal to average inflation rate which prevailed over the period. The test statistic is asymptotically distributed as chi-square with $(n-r)$ degrees of freedom. In our case, $r$ is equal to one and LR is distributed as chi-square with one degree of freedom. The critical value for chi-square with one degree of freedom at 5% level is equal to 3.84 ( Hines and Montgomery, 1980, table III, page 594 ). The hypothesis that the authorities expanded the money supply, on average, in such a way as to maximize the inflation tax revenue can not be rejected in any case at the 5% level.

## 5.5. Testing the Rational Expectations Hypothesis

If expectations are formed according to the rational expectations hypothesis, and if following Sargent (1977), we can assume $E(\psi_t \mid I_t) = 0$, where $\psi_t$ denotes elements of money demand not captured by the model as in section III, then the forecasting errors,

$$\xi_{t+1} = \Delta p_{t+1} + \alpha^{-1}(m - p)_t, \qquad (5.4)$$

should be orthogonal to information available at time t, $I_t$, that is

$$E(\xi_{t+1} \mid I_t) = 0. \qquad (5.5)$$

A way of testing ( 5.5 ) is to test for zero coefficients in a least squares projection of $\xi_{t+1}$ onto lagged values of itself ( Taylor 1991 ). Taylor (1991) demonstrates that this is equivalent to testing a set of cross-equation rational expectations restrictions on the vector autoregressive representation of $[\Delta^2 p_t, ((m - p)_t + \alpha\Delta p_t)]'$.

The test for zero coefficients in a least squares projection of $\xi_{t+1}$ onto lagged values of itself is applied and the results are presented in Table 8. Two sets of forecasting errors, $\xi_t$, were constructed-one set using the cointegration estimate of $\alpha$ as reported in Table 6 and one set constructed assuming inflation tax revenue maximization, i.e., with $\alpha = 100\pi^{-1}$. Test statistics are distributed as F(12,85) under the null hypothesis of rational expectations.

**Table 8. Tests of the Hyperinflation Model Under Rational Expectations**

| Variables | F-statistics with $\alpha$ as | |
|---|---|---|
| | Cointegration Estimate F(12,85) | $100\pi^{-1}$ F(12,85) |
| LM1-LCPI ΔLCPI | 271.513 | 270.470 |
| LM1-LWPI ΔLWPI | 148.623 | 151.222 |
| LM2-LCPI ΔLCPI | 258.756 | 258.850 |
| LM2-LWPI ΔLWPI | 152,141 | 151.550 |
| LRM-LCPI ΔLCPI | 274.795 | 274.749 |
| LRM-LWPI ΔLWPI | 149.328 | 146.045 |

In all cases the F-statistics are highly significant. In all cases, the results indicate a strong rejection of the null hypothesis of rational expectations. So it appears that the Cagan model cannot be coupled with the rational expectations for the Turkish case in the considered period. Note, however, that these results are highly dependent on the assumption that $E(\psi_t | I_t) = 0$, that is on the assumption that expectation of $\psi_t$ based on information available at time t is equal to zero. Although this assumption has a high degree of precedent in the hyperinflation literature (see, e.g. Sargent 1977), it is quite arbitrary. If $\psi_t$ is a serially correlated series, then the above assumption won't be valid (Taylor and Phylaktis 1991).

## VI - CONCLUSION

Cagan (1956) deals with the relation between changes in the quantity of money and price level during hyperinflations. The heart of Cagan's analysis is a function in which the demand for real balances depends, among other things, inversely on the expected rate of inflation. Thus, if an expanding supply of money generates inflation, that inflation lowers the demand for real balances. In the face of given nominal balances, the price level must rise in order to reduce the supply of real balances to its demand. Consequently, in hyperinflation, prices rise faster than the nominal supply of money. Cagan assumes in his model that expectations of the rate of inflation are formed adaptively.

This thesis considers the demand for money under conditions of high inflation in Turkey during the period 1986:1-1995:3. We test whether the monetary and inflationary experiences of Turkey can be adequately characterized by the Cagan (1956) model, using an econometric procedure which is reliant only on the assumption that forecasting errors are stationary. Although Turkey has not experienced hyperinflation according to Cagan's strict definition, Turkey has experienced high rates of inflation during many years. We first find out that real money balances and inflation are each first difference stationary, or I(1), using DF and ADF unit root tests. Thus, a simple test of the applicability of the hyperinflation model lies in testing whether or not real money balances and inflation are cointegrated and the cointegration test is conducted using both Engle and Granger two step approach and Johansen's cointegration.

Excluding 1994, Turkey has experienced an annual inflation ranging between 60% and 70% in the last decade. Thus we believe that, in the last decade, the economic agents can have rational expectations for inflation. Having this intuition, we derive a test of the Cagan (1956) model with the additional assumption of rational expectations for Turkey for the considered period.

We know that the inflation tax is the tax imposed on money holders as a result of inflation, or it is the loss in the value of their real balances. In the thesis, we test the hypothesis that the authorities expanded the money supply in such a way as to maximize the inflation tax revenue, in Turkey for the considered period, using a likelihood ratio test statistic constructed as in Johansen (1988).

The results of this thesis suggest that Cagan's hyperinflation model does indeed provide an adequate characterization of the features of the inflationary and monetary experiences of Turkey for the period 1986:1-1995:3. Moreover, it appears that in the considered period the authorities expanded the money supply in such a way as to maximize the inflation tax revenue. Although we had the intuition that, in the last decade, the economic agents have rational expectations for inflation, it appears that the Cagan model cannot be coupled with the rational expectations hypothesis for Turkey for the considered period.

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# APPENDICES

Figure A.1. Consumer Price Inflation (ΔLCPI).

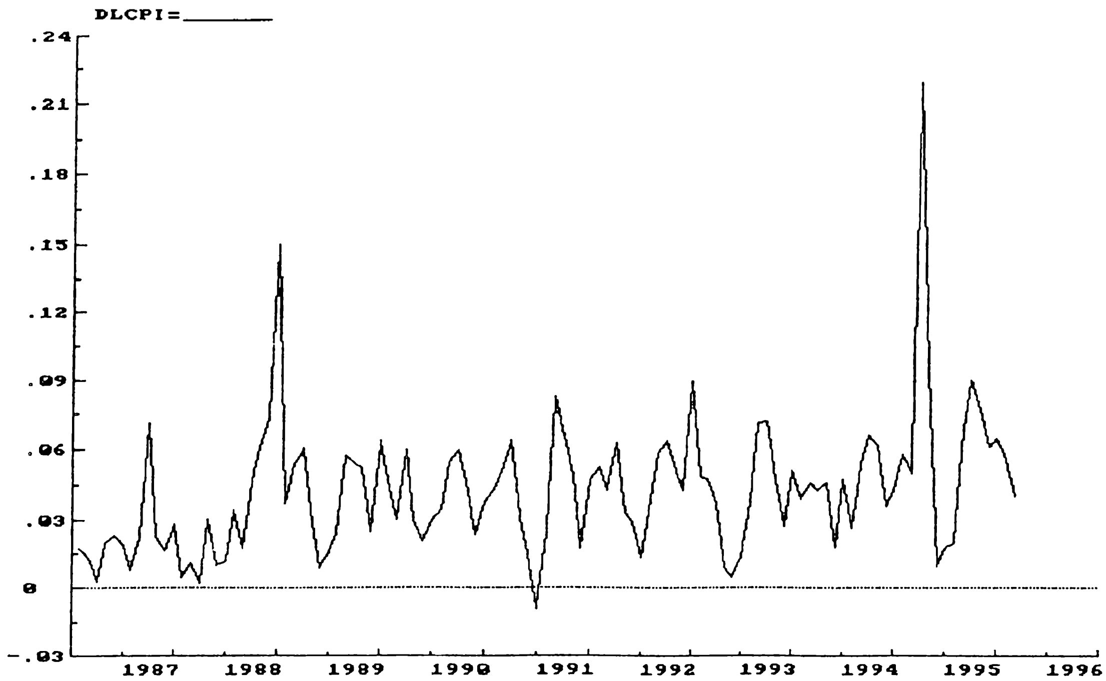


Figure A.2. First Differenced Consumer Price Inflation (ΔΔLCPI).

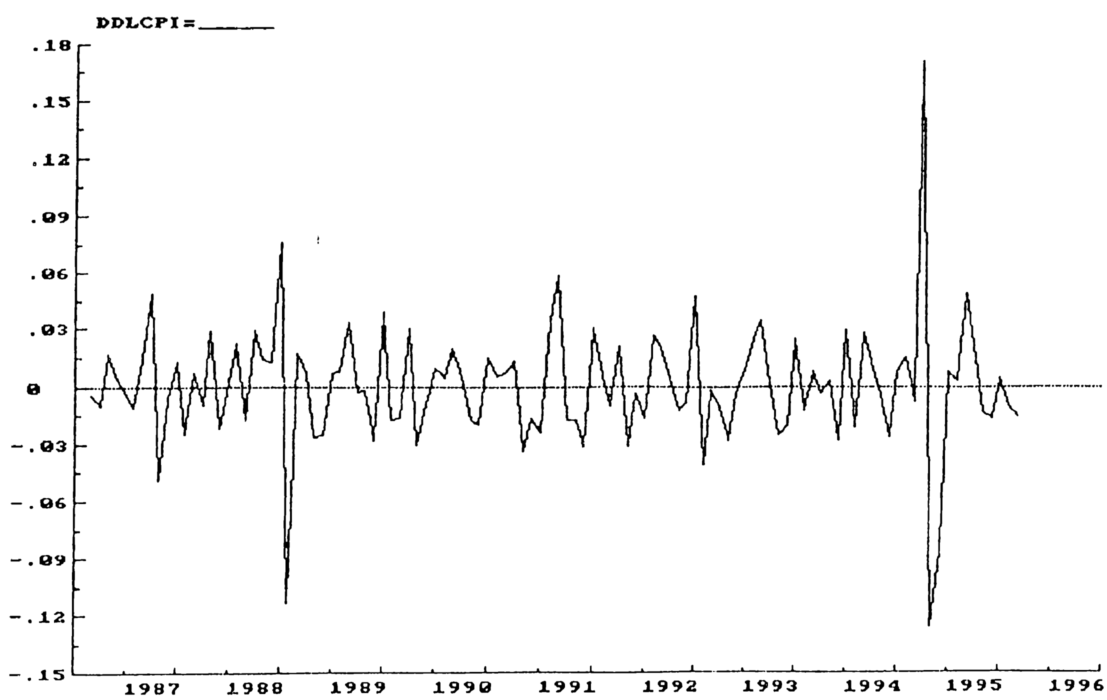

Figure A.3. Wholesale Price Inflation (ΔLWPI).

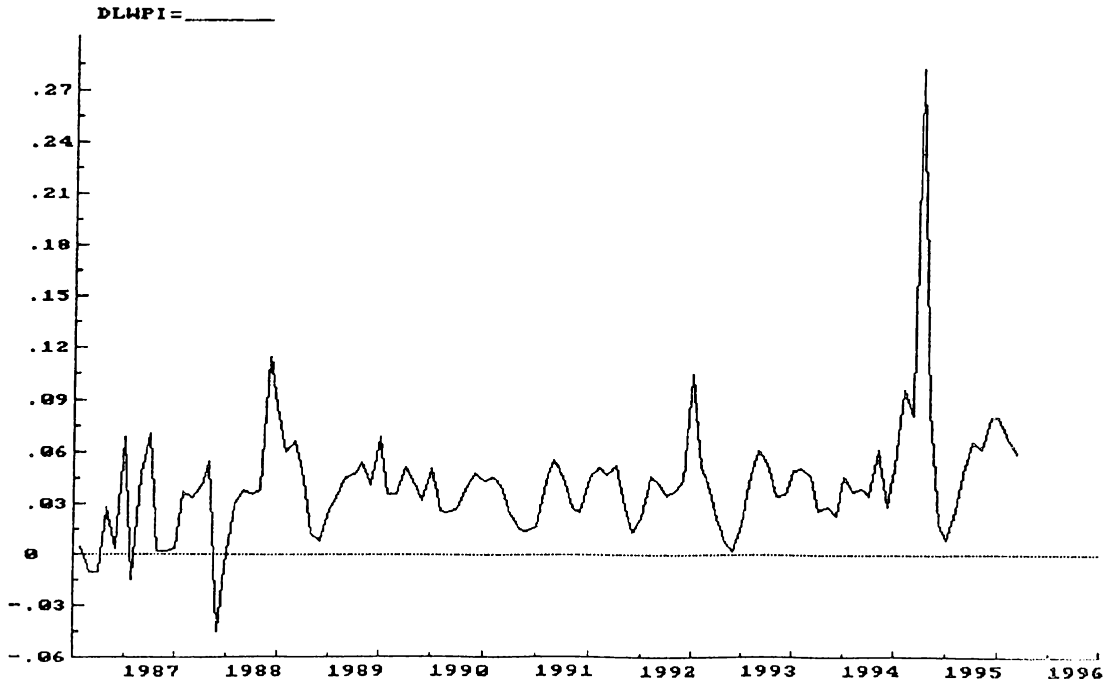


Figure A.4. First Differenced Wholesale Price Inflation (ΔΔLWPI).

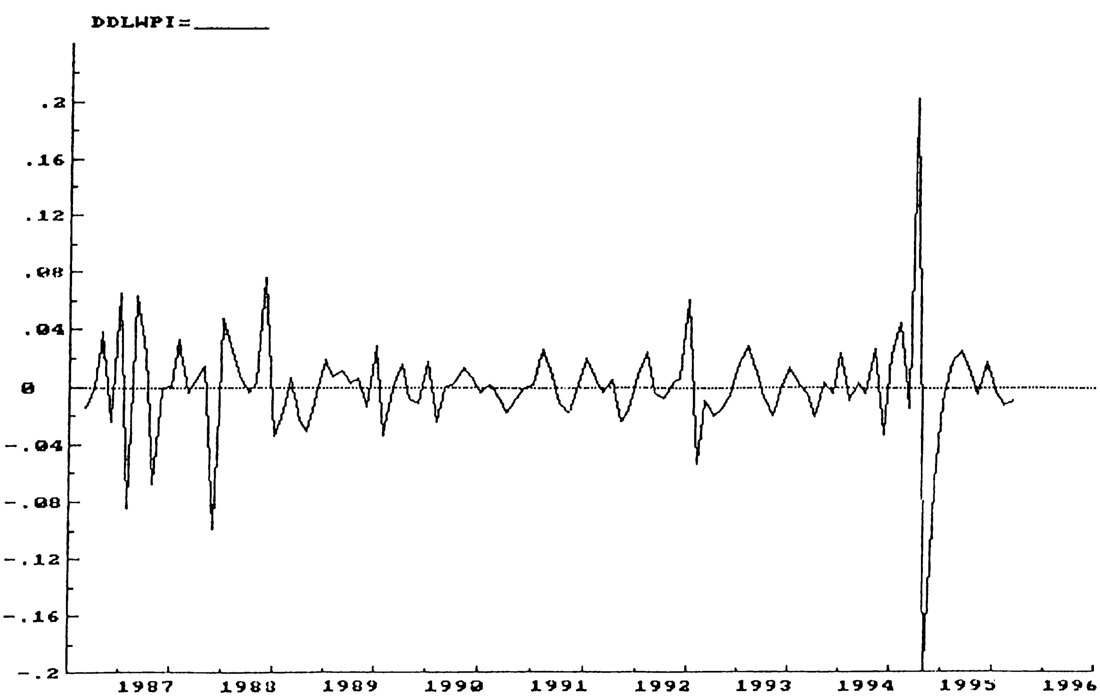

Figure A.5. Real Money Balance Using M1 and CPI (LM1-LCPI).


Figure A.6. First Differenced Real Money Balance Using M1 and CPI (Δ(LM1-LCPI)).

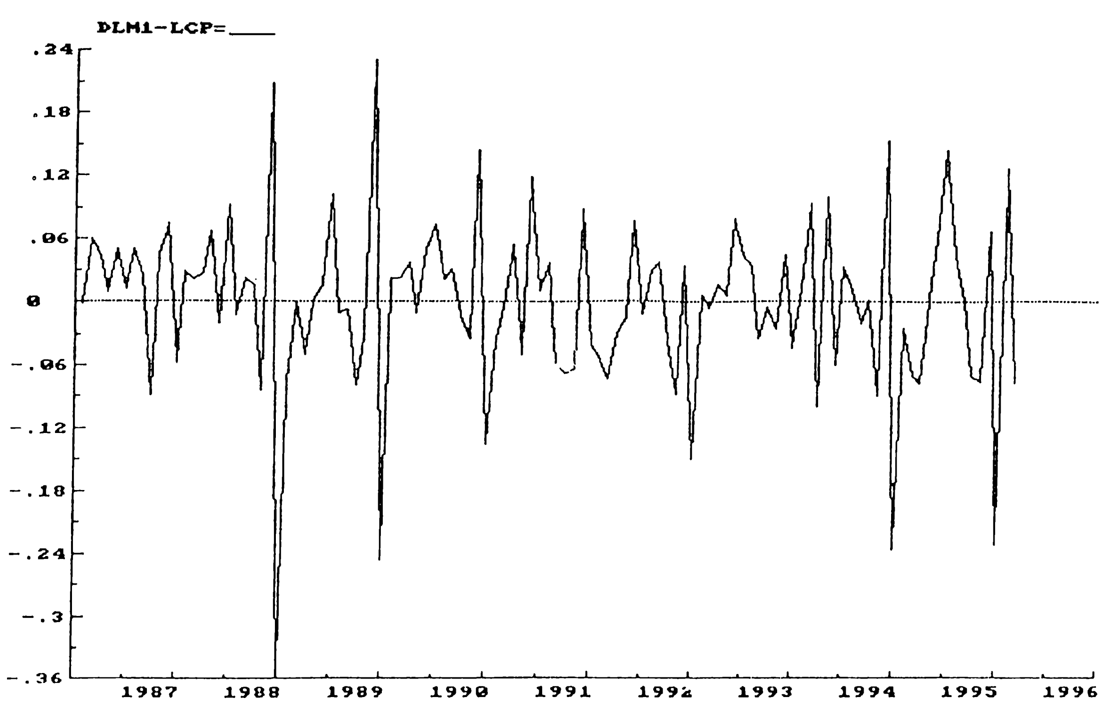

Figure A.7. Real Money Balance Using M1 and WPI (LM1-LWPI).

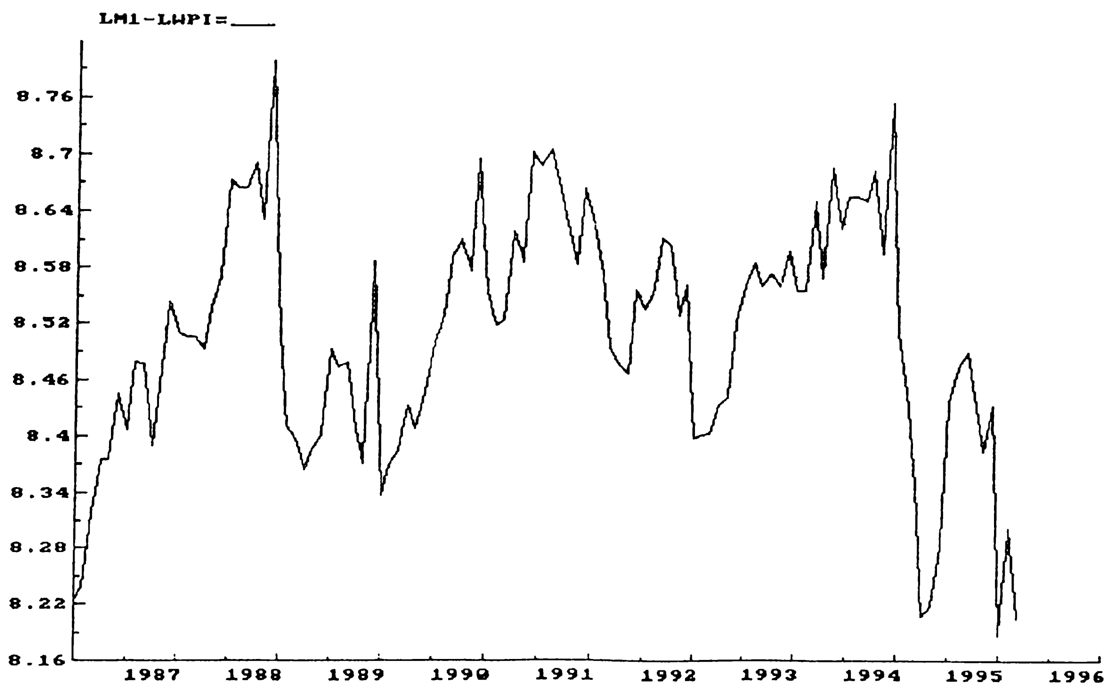


Figure A.8. First Differenced Real Money Balance Using M1 and WPI (Δ(LM1-LWPI)).

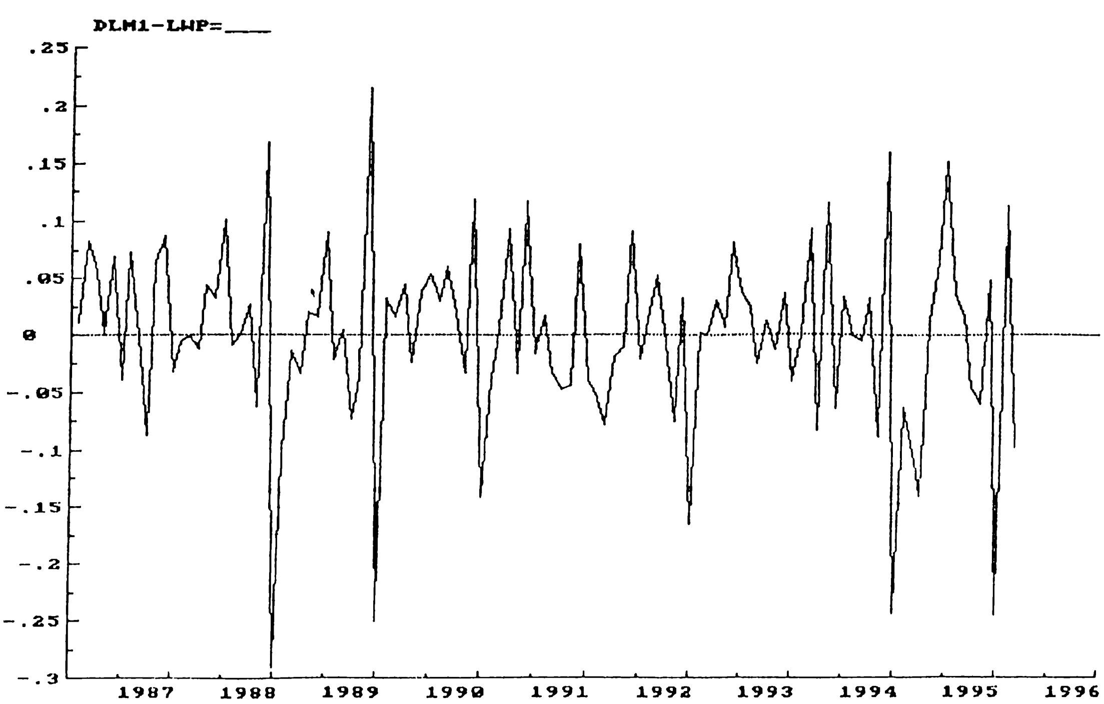

Figure A.9. Real Money Balance Using M2 and CPI (LM2-LCPI).


Figure A.10. First Differenced Real Money Balance Using M2 and CPI (Δ(LM2-LCPI)).

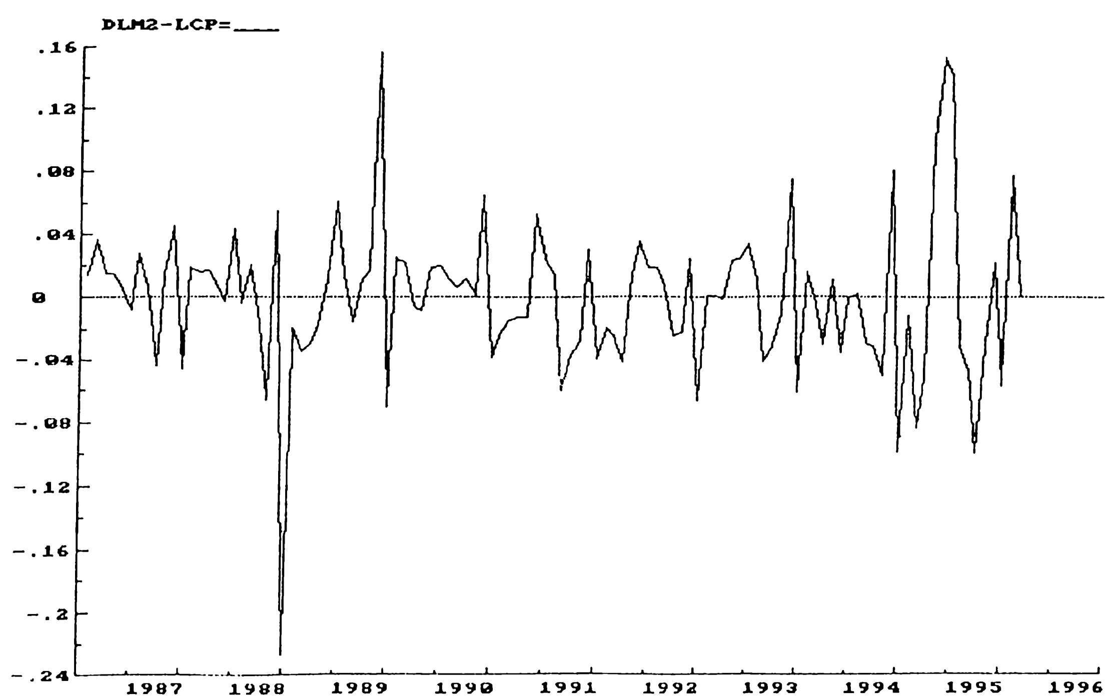

Figure A.11. Real Money Balance Using M2 and WPI (LM2-LWPI).

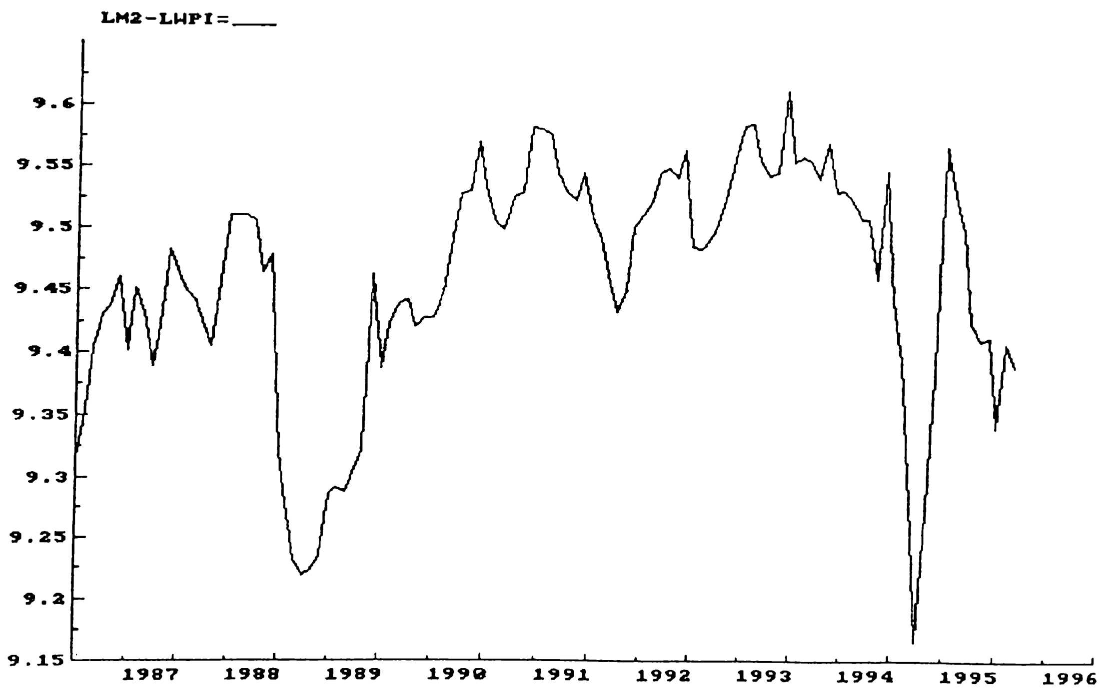


Figure A.12. First Differenced Real Money Balance Using M2 and WPI (Δ(LM2-LWPI)).

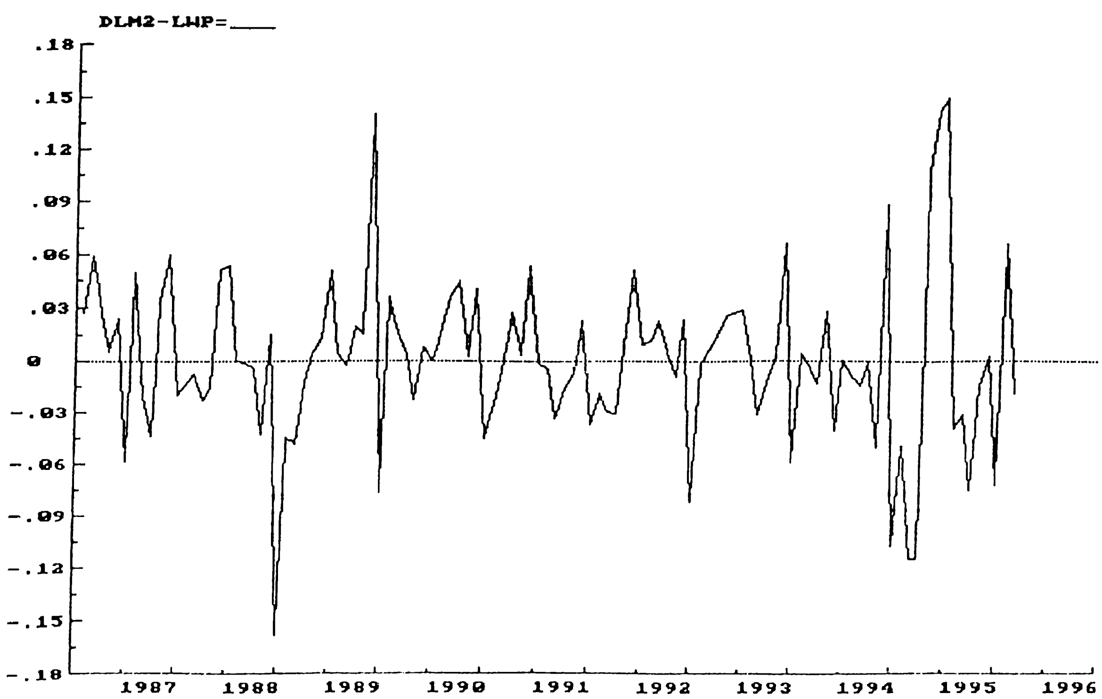

Figure A.13. Real Money Balance Using RM and CPI (LRM-LCPI).

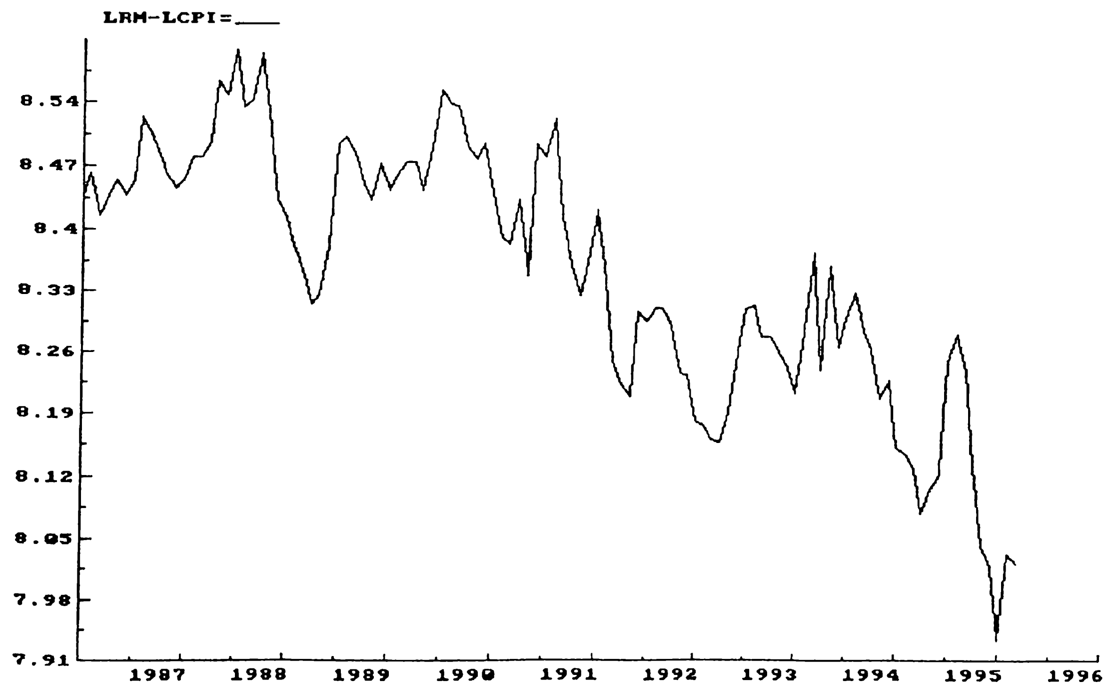


Figure A.14. First Differenced Real Money Balance Using RM and CPI (Δ(LRM-LCPI)).

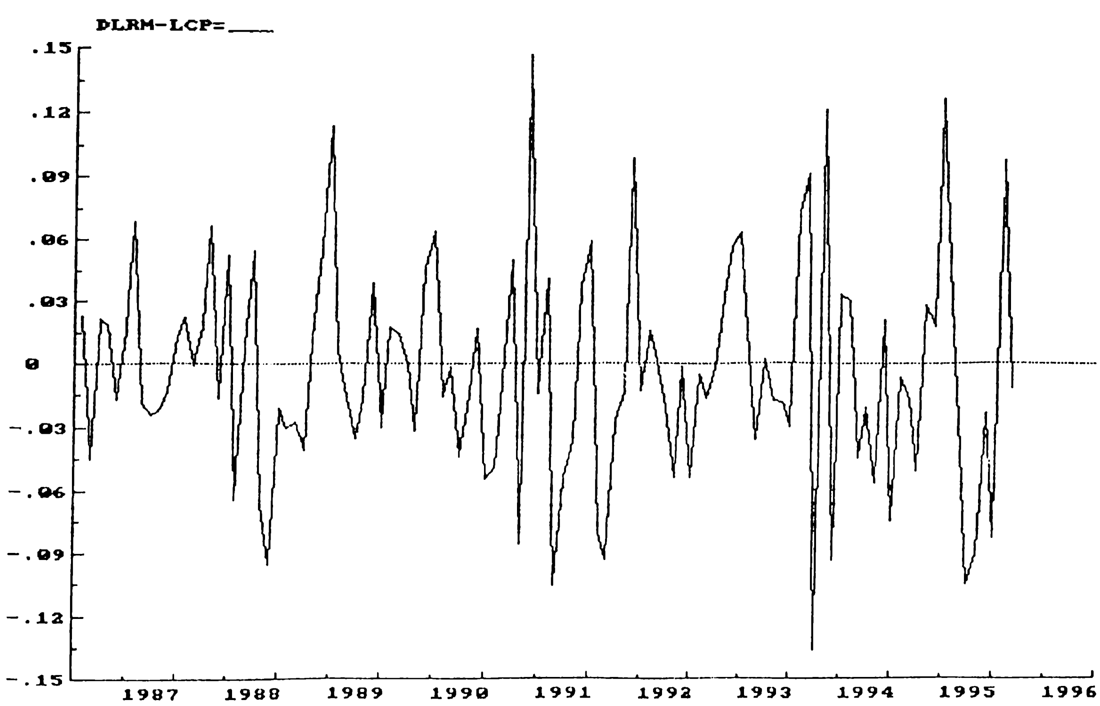

**Figure A.15. Real Money Balance Using RM and WPI (LRM-LWPI).**

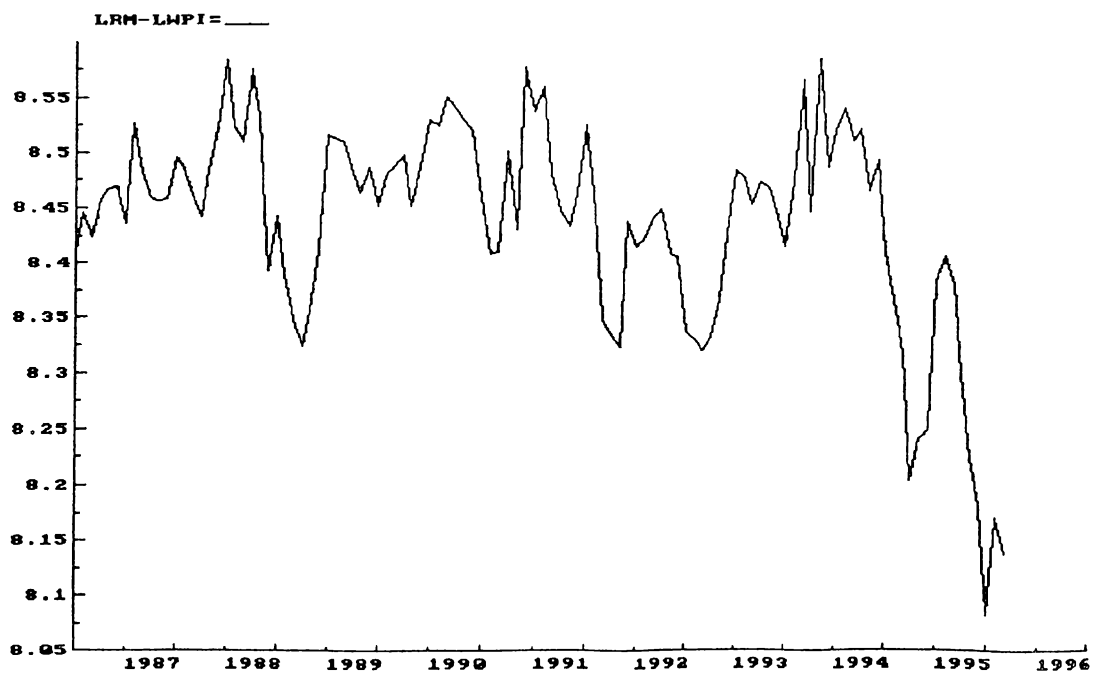


**Figure A.16. First Differenced Real Money Balance Using RM and WPI ($\Delta$(LRM- LWPI)).**