Signatures of black holes in string theory

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The effect of string theory on the four-dimensional classical Einstein equations is investigated. It is shown that the throats of nonrotating charged black holes are exact solutions of the gravitational field equations with string correction terms.

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Gravitational field equations with string correction terms have recently drawn much attention in several respects. The most important contribution of these terms is believed to change the singularity structure of the spacetime geometry. For this purpose there has been much interest in the black-hole solutions [1–10] of the higher-dimensional Einstein field equations containing higher-order curvature terms. Although not yet known, it is believed that the low-energy string action is a perturbation expansion in inverse powers of the string tension parameter. This expansion contains, in addition to the usual Einstein-Hilbert action, corrections quadratic and higher-order invariants in the massless fields, curvature tensor, and in Maxwell and dilaton fields.

Recently it has been shown that the plane-wave metrics of the Einstein theory preserve their form under string corrections at all orders [11–13]. The question arises as to whether or not there exist spherically symmetric spacetimes with the same property. In particular, nonrotating black-hole geometries are spherically symmetric and it is believed that they do not preserve their form [1].

Since gravitation is weak far from the holes, the black-hole solutions of the Einstein theory can be considered as approximate solutions of the effective field theory mentioned above. On the other hand, near the singularities, the contribution of curvature terms in the extended theory becomes important. Black-hole solutions of the Einstein theory are not any more exact solutions of the extended theory in these regions. In the general case, it is very hard, almost impossible, to consider the full extended field equations and find their exact solutions with the property that they asymptotically approach the black-hole solutions of the Einstein theory. Recently [9] it has been conjectured that metrics describing the neighborhood of the event horizon of the extreme charged black holes may solve the extended field equations exactly. Such candidates are of extreme Reissner-Nordström type and the recently found metric with a dilaton charge [8,9]. In this paper we shall show that the throats of these black-hole solutions are, in fact, exact solutions of the extended field equations.

We assume a flat internal space, an Abelian gauge field with zero components in the internal directions, and set the three-form field equal to zero. We also assume that the four-dimensional metric, the Maxwell, and dilaton fields do not depend on the internal coordinates. With these assumptions the four-dimensional low-energy action obtained from string theory is [9]

$$S=\int d^4x\sqrt{-g}\,[-R+2(\nabla\phi)^2+e^{-2\phi}F^2 + L(R_{ijkl},F_{ij},\phi)]\ , \tag{1}$$

where the Maxwell field F_{ij} is associated with a U(1) subgroup of $E_8\times E_8$ or Spin(32)/Z_2 and we set the remaining gauge fields to zero. ϕ is the dilaton field. The contribution of string theory to the classical gravitational action is through the function L. It is a perturbation expansion in inverse powers of the string tension parameter. The terms in this expansion may depend on all possible invariants constructed out of the curvature tensor R_{ijkl}, Maxwell field F_{ij}, dilaton field ϕ, and their covariant derivatives. Under these assumptions, extremizing this action with respect to the U(1) potential A_i, dilaton field ϕ, and the metric, we obtain the four-dimensional low-energy limit of the string theory (extended gravitational field equations):

$$G_{ij}-2T_{ij}=E_{ij}\ , \tag{2}$$

$$\nabla_i(e^{-2\phi}F^{ij})=E^j\ , \tag{3}$$

$$\nabla^2\phi+\tfrac{1}{2}e^{-2\phi}F^2=E\ . \tag{4}$$

The energy-momentum tensor T^{ij} corresponds to a Maxwell field coupled to the dilaton ϕ [9,10]. The second-rank symmetric tensor E_{ij}, the vector E_i, and scalar E coming from the variation of L are the string correction terms to the classical gravitational field equations. They are believed to be composed of the curvature tensor R_{ijkl}, the Maxwell field F_{ij}, the dilaton field ϕ, and of their covariant derivatives at all orders.

Although we consider the low-energy limit of string theory, our discussion in this paper applies to any theory derivable from a variational principle where the Lagrangian is an arbitrary smooth function of the Riemann tensor, Maxwell field tensor, dilaton field, and of their covariant derivatives.

The metric of a static and spherically symmetric space-

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time is given by

$$ds^2=-A^2dt^2+B^2dr^2+C^2(d\theta^2+\sin^2\theta\, d\phi^2)\ , \tag{5}$$

where A, B, and C depend only on r. Our convention is as follows:

$$R^i_{jkl}=\Gamma^i_{jl,k}+\Gamma^i_{mk}\Gamma^m_{jl}-(k\leftrightarrow l)\ ,$$

$R_{ij}=R^k_{ikj}$, $R=R^k_k$. The Riemann tensor corresponding to this metric in a compact form is given by

$$R_{ijkl}=g_{jl}S_{ik}-g_{jk}S_{il}+g_{ik}S_{jl}-g_{jl}S_{kj}+\eta_2 H_{ij}H_{kl}\ , \tag{6}$$

where

$$S_{ij}=\eta_0 M_{ij}+\eta_1 k_i k_j+\tfrac{1}{2}\eta_3 g_{ij}\ . \tag{7}$$

Here the scalars are given by

$$\eta_0=\frac{C^3}{AB}\left[\frac{A_{,r}C-AC_{,r}}{B}\right]_{,r}\ , \tag{8}$$

$$\eta_1=\frac{AB}{C}\left[\frac{C_{,r}}{AB}\right]_{,r}\ , \tag{9}$$

$$\eta_2=C^2-\frac{C^4}{AB}\left[\frac{A_{,r}C-AC_{,r}}{BC}\right]_{,r}\ , \tag{10}$$

$$\eta_3=-\frac{A_{,r}C_{,r}}{AB^2C}\ . \tag{11}$$

The tensor H_{ij} is antisymmetric and derivable from a vector potential A_i:

$$A_i=\cos(\theta)\delta^{\phi}_i\ , \tag{12}$$

$$H_{ij}=\nabla_i A_j-\nabla_j A_i\ . \tag{13}$$

The symmetric tensor M_{ij} is defined as

$$M_{ij}=H^k_i H_{kj}-\tfrac{1}{4}H^2 g_{ij}\ , \tag{14}$$

where

$$H^2=H^{ij}H_{ij}\ . \tag{15}$$

The spacelike vector k_i is given by

$$k_i=\nabla_i r\ . \tag{16}$$

The covariant derivatives of H_{ij} and k_i are given as

$$\nabla_l H_{ij}=\rho(-2k_l H_{ij}+k_i H_{jl}-k_j H_{il})\ , \tag{17}$$

$$\nabla_i k_j=\rho_1 g_{ij}+\rho_2 M_{ij}+\rho_3 k_i k_j\ , \tag{18}$$

where

$$\rho=\frac{C_{,r}}{C}\ , \tag{19}$$

$$\rho_1=-\frac{A_{,r}C+AC_{,r}}{2AB^2C}\ , \tag{20}$$

$$\rho_2=C^2\frac{A_{,r}C-AC_{,r}}{AB^2}\ , \tag{21}$$

$$\rho_3=-\frac{(AB)_{,r}}{AB}\ . \tag{22}$$

The covariant derivatives of H_{ij} and k_i, as seen in Eqs. (17) and (18), are expressed only by themselves and the metric tensor. Hence, any higher-order covariant derivatives of these tensors must obey this rule. Since in Eq. (6) the Riemann tensor is given in terms of H_{ij}, g_{ij}, and k_i, and the scalars depending on r, its covariant derivatives at any order obey the same rule. Hence, we have the following theorem.

Theorem 1. Covariant derivatives of the Riemann tensor R_{ijkl}, the tensor H_{ij}, and the vector k_i at any order are expressible only in terms of H_{ij}, g_{ij}, and k_i.

Since the contraction of k^i with H_{ij} vanishes, the only symmetric tensors constructable out of H_{ij}, g_{ij}, and k_i are M_{ij}, the metric tensor g_{ij}, and $k_i k_j$. Then the following theorems hold.

Theorem 2. Any second-rank symmetric tensor constructed out of the Riemann tensor, the antisymmetric tensor H_{ij}, the dilaton field $\phi=\phi(r)$, and their covariant derivatives is a linear combination of M_{ij}, g_{ij}, and $k_i k_j$. Let this symmetric tensor be E'_{ij}. Then we have

$$E'_{ij}=\sigma_1 M_{ij}+\sigma_2 g_{ij}+\sigma_3 k_i k_j\ , \tag{23}$$

where σ_1, σ_2, and σ_3 are scalars which are functions of the metric functions, invariants constructed out of the curvature tensor R_{ijkl}, H_{ij}, and on the dilaton field.

Theorem 3. Any vector constructed out of the Riemann tensor R_{ijkl}, H_{ij}, the dilaton field $\phi=\phi(r)$, and their covariant derivatives is proportional to k_i. Let this vector be E'_i. Hence, it reads

$$E'_i=\sigma k_i\ , \tag{24}$$

where σ is a scalar as σ_1, σ_2, and σ_3.

We first discuss the solutions of the Einstein field equations. The Einstein tensor is found as

$$G_{ij}=(2\eta_0+\eta_2)M_{ij}-\frac{1}{2B^2C^4}(6B^2C^4\eta_3+B^2\eta_2-4C^4\eta_1)g_{ij}+2\eta_1 k_i k_j\ . \tag{25}$$

In the Einstein theory, the form of (25) gives us an idea about the form of the energy-momentum tensor as the source for the field equations. The source may include the electromagnetic field F_{ij}, a dilaton field ϕ, and possibly a cosmological constant. In this case, the electromagnetic field may have an electric part in addition to the magnetic part, that is, F_{ij} may have the form

$$F_{ij}=e(r)\tilde{H}_{ij}+q_0 H_{ij}\ , \tag{26}$$

where $\tilde{H}_{ij}$ is the dual of the tensor H_{ij}. In the general case, we have five equations for five functions A, B, C, $\phi(r)$, and $e(r)$. In the spherically symmetric case, the function $e(r)$ is not independent. Equation (3) forces us to choose it as

$$e(r)=e_0\frac{AB}{C^2}e^{2\phi}\ , \tag{27}$$

where e_0 is a constant. The remaining four functions A, B, C, and ϕ can be solved consistently. In fact, there are various types of solutions reported so far (see, for instance, [9]).

In the presence of string correction terms in Eqs. (2)–(4), the inclusion of the electric part is problematic. The dual H_{ij} introduces a timelike vector $u_i=\delta_i^t$ into the tensor algebra discussed in theorem 1. This increases the number of symmetric tensors to five and number of vectors to two. Comparing the coefficients of these tensors and vectors in Eqs. (2) and (3), respectively, we obtain the equations for the metric functions A, B, C, $e(r)$, and the dilaton field $\phi=\phi(r)$. The total number of equations is raised to eight for these functions. In the Einstein case, since the right-hand sides of Eqs. (2)–(4) are absent, one gets a correct number of equations for these functions. In the case of the extended field equations it seems unlikely that this overdetermined system of ordinary differential equations has a solution.

In order to overcome this difficulty, we first let $e(r)=0$ and $F_{ij}=q_0 H_{ij}$, where q_0 denotes the magnetic charge. In this case, the tensor E_{ij} and the vector E_i have exactly the same forms as E'_{ij} and E'_i, respectively. Therefore, in the sequel we shall drop the primes over these tensors. By the theorems given above, we have now five equations for four functions. Again, the existence of a solution of this system of differential equations is not guaranteed. Hence, we further choose the electromagnetic field in such a way that the right-hand side of Eq. (3) automatically vanishes. This can be achieved in two ways: either F_{ij} can be set equal to zero or F_{ij} and ϕ can be taken covariantly constants. In the first case the Einstein field equations couple to a scalar field [1]. We have four equations and four unknown functions. Hence, we have a well-defined set of ordinary differential equations which may have exact solutions.

When the electromagnetic field is covariantly constant and $\phi=0$, Eq. (19) leads to $C=c_0=$ const. This simplifies the curvature functions η_0, η_1, η_2, and η_3. They read

$$\eta_1=\eta_3=0 \ , \tag{28}$$

$$\eta_2=-\eta_0+c_0^2 \ , \tag{29}$$

$$\eta_0=\frac{c_0^4}{AB}\left[\frac{A_{,r}}{B}\right]_{,r} . \tag{30}$$

Since the dilaton field ϕ is set to zero, the vector k_i does not show up in the set of tensors discussed in the above theorems. Hence, the tensor E_{ij} is a linear combination of M_{ij} and g_{ij}. This means that $\sigma_3=0$ in (2). The vector E_i or σ in Eq. (24) vanishes identically. Therefore, we have three equations for two functions A,B. This is again an overdetermined system. We shall further set $\eta_0=$ const. Under these assumptions, the extended field equations reduce to

$$\eta_0+c_0^2-2q_0^2=\sigma_1(\eta_0,c_0,q_0;\alpha') \ , \tag{31}$$

$$\frac{1}{2c_0^4}(\eta_0-c_0^2)=\sigma_2(\eta_0,c_0,q_0;\alpha') \ , \tag{32}$$

$$2\frac{q_0^2}{c_0^4}=E(\eta_0,c_0,q_0;\alpha') \ , \tag{33}$$

where α' is the inverse string parameter. Here we have three algebraic equations for three constants c_0, η_0, and the constant q_0. The scalars σ_1, σ_2, and E are now also constants depending upon c_0, η_0, and q_0, and on the parameters of the theory under consideration, such as the string tension. Since η_0 is a constant, Eq. (30) is exactly soluble. It reads

$$B=\frac{A_{,r}}{\sqrt{(\eta_0/c_0^4)A^2+a_1}} \ , \tag{34}$$

where a_1 is a constant. This is the only equation which determines the metric functions A and B. Although the function A appears to be arbitrary, by changing r, B can be set to any function of r. We choose it as $1/A$. With this choice of B, the above differential equation can be integrated easily and we determine the spacetime metric as

$$ds^2=-A^2dt^2+\frac{1}{A^2}dr^2+c_0^2(d\theta^2+\sin^2\theta\, d\phi^2) \ , \tag{35}$$

where

$$A^2=r\left[\frac{\eta_0}{c_0^4}r+a_1\right]+a_2^2 \ . \tag{36}$$

Here a_1 and a_2 are constants. According to whether η_0 is zero or not there are two distinct solutions.

Type (a):

$$\eta_0=a_1=0, \quad A^2=1 \ . \tag{37}$$

It is a direct product of S^2 and a two-dimensional flat Minkowski space. The vector k_i is covariantly constant; hence, it is also a spacelike Killing vector.

Type (b):

$$a_2=0, \quad A^2=r\left[\frac{\eta_0}{c_0^4}r+a_1\right] . \tag{38}$$

This metric describes a spacetime which is a direct product of a two-dimensional pseudosphere and S^2.

We now state that the metrics given above are exact solutions of the gravitational field equations with string corrections. They are all nonsingular and homogeneous spacetimes. These metrics are also the solutions of the Einstein-Maxwell equations. For instance, the second solution [type (b)] with

$$\eta_0=c_0^2=q_0^2 \tag{39}$$

is the Levi-Civita Bertotti-Robinson metric. Under the string corrections the form of these metrics is preserved but the parameters appearing in the metrics no longer satisfy the above equations (39). In this case the relations among these constants are given by Eqs. (31)–(33).

The above metrics, in particular type (b), have interesting features. They describe the geometry of black-hole solutions in the neighborhood of their outer horizons. These regions are the throats of the Einstein-Rosen bridges of the corresponding hole solutions. For example, the Levi-Civita Bertotti-Robinson universe is known to be the throat of the extreme charged black hole [14–16]. Here, extending the same idea let us find the metric describing the region near the outer horizons of the charged black holes in de Sitter space [17]. In the spherically symmetric case such a black-hole solution is given by

$$ds^2=-W^2dt^2+\frac{1}{W^2}dr^2+r^2(d\theta^2+\sin^2\theta\, d\phi^2)\,, \tag{40}$$

where

$$W^2=1-\frac{2m}{r}+\frac{Q^2}{r^2}-\lambda_0 r^2\,. \tag{41}$$

Here m, Q, and λ_0 are, respectively, the mass, charge, and the cosmological constant. In the neighborhood of the outer horizon $r=r_h+\epsilon$, where $W(r=r_h)=0$, the metric and the function W take the form

$$ds^2=-W^2dt^2+\frac{1}{W^2}d\epsilon^2+r_h^2(d\theta^2+\sin^2\theta\, d\phi^2)\,, \tag{42}$$

$$W^2=\epsilon(\alpha\epsilon+\beta)\,. \tag{43}$$

This metric remains to be the solution of the Einstein-Maxwell field equations with a cosmological constant provided

$$m=\frac{Q^2}{r_h}\,, \tag{44}$$

and α, β, and λ_0 are given by

$$\alpha=\frac{1}{r_h^4}(2Q^2-r_h^2)\,, \tag{45}$$

$$\beta=\frac{2}{r_h^3}(-r_h^2+Q^2)\,, \tag{46}$$

$$\lambda_0=\frac{2}{r_h^4}(r_h^2-Q^2)\,. \tag{47}$$

Notice that the cosmological constant is no longer an independent parameter. This limiting metric (42) is exactly of type (b) given above with the identifications

$$\frac{\eta_0}{c_0^4}=\alpha,\quad a_1=\beta\,, \tag{48}$$

$$c_0=r_h,\quad q_0=Q\,. \tag{49}$$

The extended field equations (31)–(33) alter the definitions given in Eqs. (44)–(47) of α, Q, r_h, and λ_0. By the utility of this identification they now satisfy

$$r_h^4\alpha+r_h^2-2Q^2=\sigma_1(\alpha,r_h,Q;\alpha')\,, \tag{50}$$

$$\frac{1}{2r_h^2}(r_h^2\alpha-1)=\sigma_2(\alpha,r_h,Q;\alpha')\,, \tag{51}$$

$$2\frac{Q^2}{r_h^4}=E(\alpha,r_h,Q;\alpha')\,. \tag{52}$$

Hence, the above equations relate the mass and the charge to the string tension. The type (a) solution corresponds to the throat of the extreme charged hole with dilaton discussed recently in [9].

We conclude that the regions in the neighborhood of the outer horizons of the charged black holes are preserved under string corrections. String theory only changes the relations among the parameters α, Q, r_h, and the cosmological constant λ_0. They are related to the inverse string tension parameter α' through Eqs. (50)–(52).

We have found two distinct metrics which may constitute exact solutions of the gravitational field equations with string correction terms at all orders. The existence of solutions, of course, depends on the functional dependencies of σ_1, σ_2, and E on the parameters c_0, η_0, q_0 or on r_h, α, and Q. Since the extended field equations or the explicit forms of E_{ij}, E_i, and E are not known yet, it is not possible to give an answer to this existence problem. On the other hand, the gravitational field equations with the quadratic curvature terms, such as the Gauss-Bonnet term, are commonly known to be the one-loop corrections. The tensors E_{ij}, E_i, and E in this case are simpler and Eqs. (31)–(33) have consistent solutions.

We have shown that the metrics that solve the extended field equations are asymptotic forms of the charged nonrotating black holes of the Einstein theory. The mass and the charge are related to string tension. When we consider only the first-order corrections, the tensor E_{ij} is the Gauss-Bonnet term which vanishes identically for both types of metrics we found in this paper. The effect of string corrections at this order comes from Eq. (33). When the cosmological constant is set to zero, the mass, charge, and radius of the horizon turn out to be equal and they are proportional to $\sqrt{\alpha'}$. Hence, at the Planck scale we have an exact solution of the extended field equations which is the signature of a classical black hole. It is exactly the throat of the extreme charged black hole.

Inclusion of the three-form field into the field equations (2)–(4) is also possible. This will not alter our conclusion. Let $H_{ijk}=\mu k_{[i}H_{jk]}$, where μ is a function of r. One can prove the following theorem: The only antisymmetric second-rank tensor obtainable out of R_{ijkl}, H_{ij}, k_i, H_{ijk}, and their covariant derivatives is proportional to H_{ij}. Hence, in addition to the field equations (3) and (4), we have $\nabla_i(e^{-2\phi}H^{ijk})=\sigma_4 H^{jk}$, where σ_4 is a scalar as σ_1, σ_2, σ_3. Letting H_{ij} be covariantly constant, $\phi=\phi_0=$ const and $\mu=0$, we have (31)–(33) with σ's also depending on ϕ_0. In addition to these equations we also have $\sigma_4(\eta_0,c_0,q_0,\phi_0;\alpha')=0$. This leads to four algebraic equations for four constants η_0, c_0, q_0, and ϕ_0.

Extension of the results reported in this paper to higher dimensions is also possible. For instance, a D-dimensional spacetime, which is a direct product of a two-dimensional pseudosphere and S^{D-2}, is a solution of Einstein-Maxwell field equations in D dimensions. The Maxwell field is covariantly constant. It is possible to show that the Riemann tensor has exactly the same form as given in Eq. (6) with slight changes in the definitions of η_0, η_1, η_2, and η_3. The theorems given in this work remain valid. The form of the metric is preserved under string corrections. The extended field equations are again redefinitions of the parameters appearing in the theory.

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